Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/194-7.3.4-u-a+b-arctanh-c-x-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [538]. This is test number [194].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (538)	0.00 (0)
Mathematica	99.63 (536)	0.37 (2)
Maple	94.61 (509)	5.39 (29)
Maxima	50.19 (270)	49.81 (268)
Fricas	48.14 (259)	51.86 (279)
Giac	32.90 (177)	67.10 (361)
Mupad	32.53 (175)	67.47 (363)
Sympy	26.95 (145)	73.05 (393)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

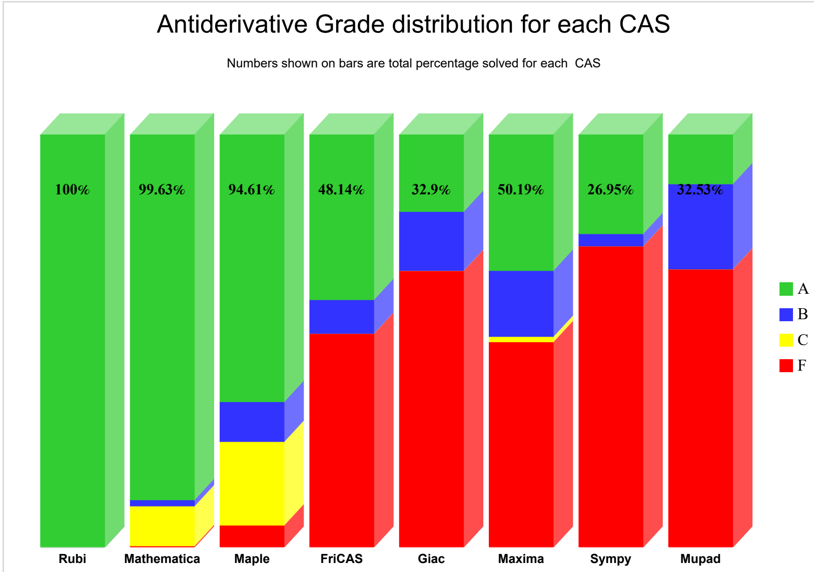
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

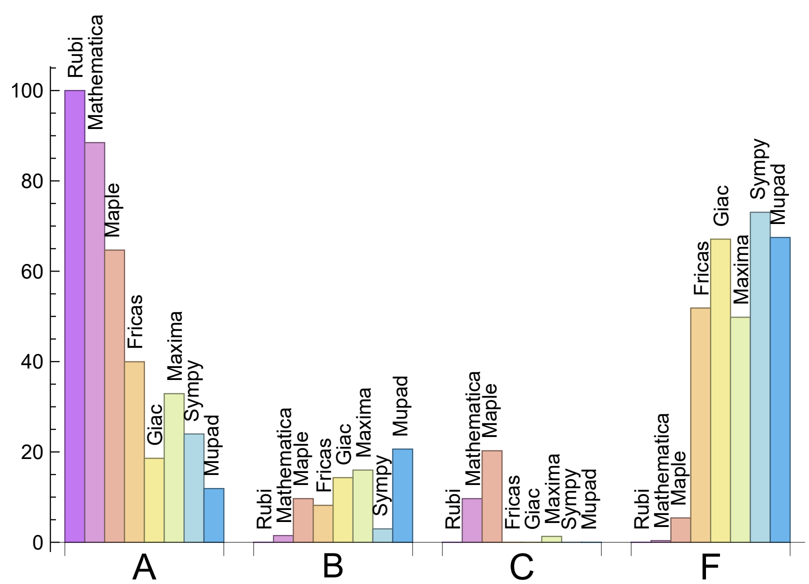
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	88.48	1.49	9.67	0.37
Maple	64.68	9.67	20.26	5.39
Fricas	39.96	8.18	0.00	51.86
Maxima	32.90	15.99	1.30	49.81
Sympy	23.98	2.97	0.00	73.05
Giac	18.59	14.31	0.00	67.10
Mupad	N/A	20.63	0.00	67.47

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	29	100.00 %	0.00 %	0.00 %
Fricas	279	98.92 %	0.00 %	1.08 %
Giac	361	86.70 %	0.00 %	13.30 %
Maxima	268	98.88 %	0.00 %	1.12 %
Sympy	393	98.47 %	1.53 %	0.00 %
Mupad	363	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

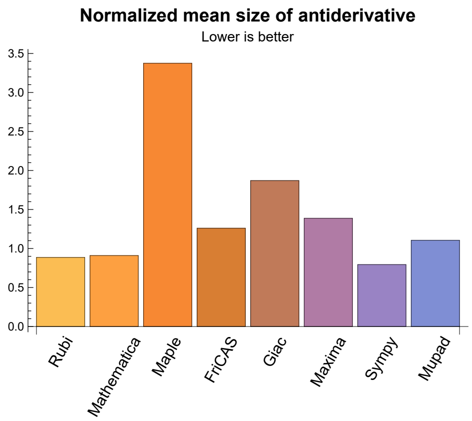
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

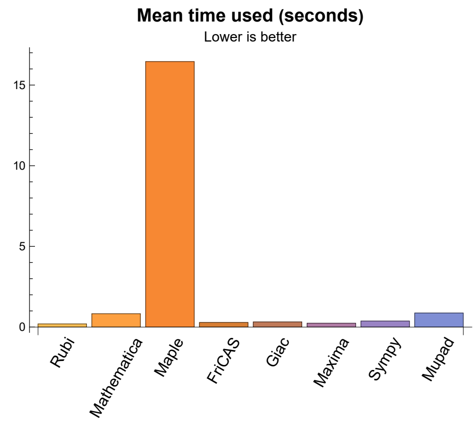
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	140.49	0.89	125.00	1.00
Mathematica	0.83	154.45	0.91	102.00	0.89
Maple	16.45	628.20	3.37	173.00	1.31
Maxima	0.24	184.60	1.39	126.00	1.23
Fricas	0.28	116.44	1.26	91.00	1.00
Sympy	0.37	82.27	0.79	37.00	0.86
Giac	0.32	203.78	1.87	122.00	1.49
Mupad	0.87	136.34	1.11	65.00	0.93

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 249, 251, 252, 254, 255, 257, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 479, 480, 485, 486, 491, 492, 511, 512, 535}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{369, 370, 372, 376, 377, 379, 383, 384, 433, 445, 504, 533\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

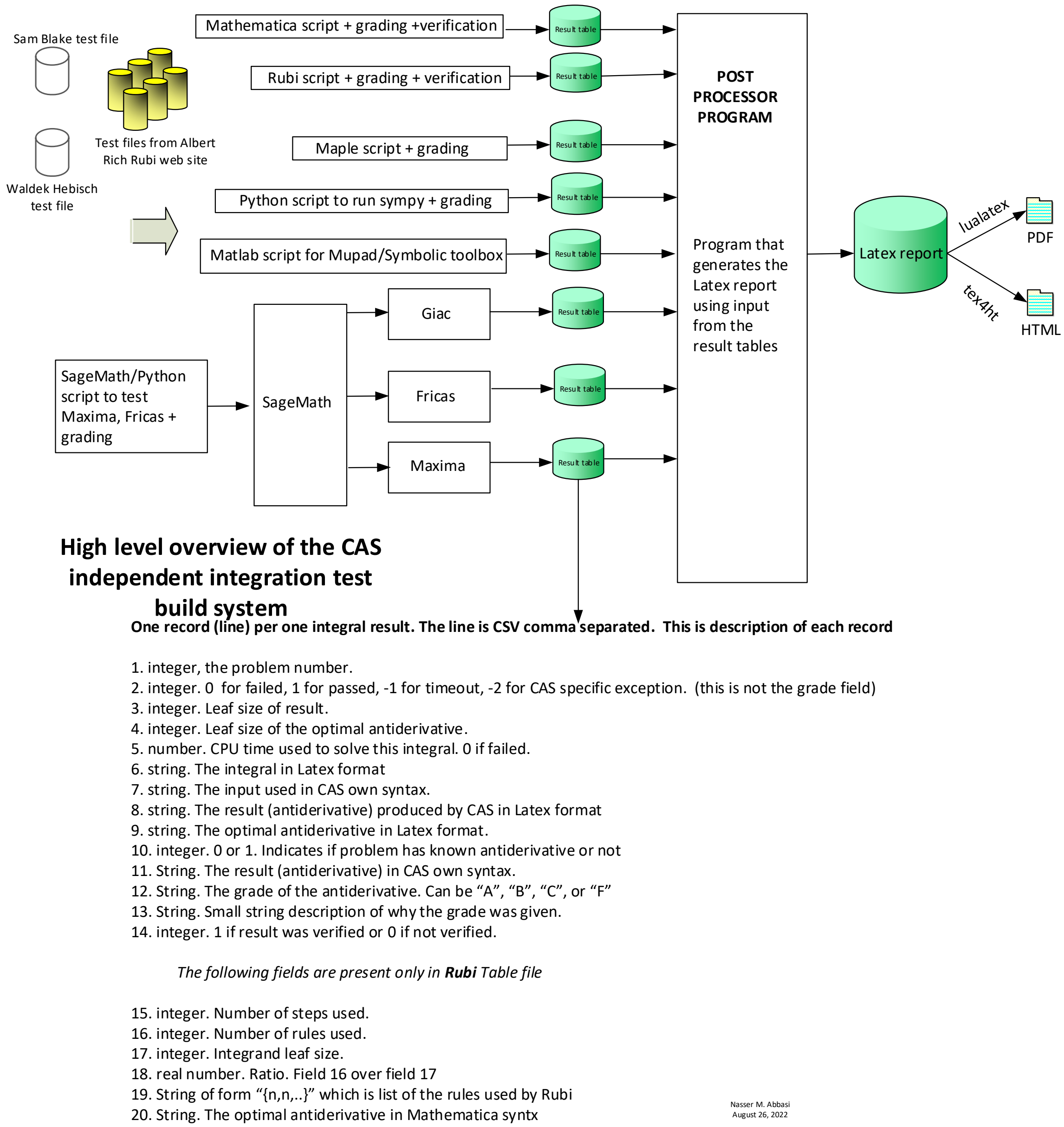
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 111, 112, 113, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 505, 507, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 529, 531, 535, 538 }

B grade: { 4, 120, 121, 383, 405, 504, 528, 530 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 99, 100, 101, 102, 108, 109, 110, 116, 117, 132, 133, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 212, 240, 246, 270, 272, 278, 312, 319, 502, 506, 508, 509, 510, 533, 534, 536, 537 }

F grade: { 527, 532 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 76, 77, 78, 84, 85, 94, 127, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 237, 241, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 265, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 307, 310, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 367, 369, 370, 371, 372, 373, 375, 377, 378, 379, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 419, 420, 421, 422, 424, 425, 426, 428, 430, 431, 432, 433, 434, 435, 436, 437, 439, 441, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 491, 492, 493, 498, 499, 500, 501, 505, 506, 507, 508, 511, 512, 517, 518, 519, 520, 521, 535 }

B grade: { 28, 40, 47, 48, 67, 71, 74, 75, 79, 83, 86, 87, 92, 93, 107, 114, 115, 118, 133, 139, 140, 181, 214, 228, 231, 232, 233, 247, 260, 262, 263, 264, 268, 305, 306, 308, 413, 418, 423, 488, 489, 490, 494, 495, 496, 502, 503, 504, 509, 510, 537, 538 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 135, 136, 137, 138, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 235, 236, 238, 239, 240, 242, 243, 245, 246, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 309, 311, 312, 313, 314, 315, 316, 317, 318, 346, 347, 364, 366, 368, 388, 427, 429, 448, 450, 497, 522, 523, 524, 525, 526, 527, 533, 534 }

F grade: { 280, 320, 348, 374, 376, 380, 381, 382, 383, 397, 404, 405, 438, 440, 442, 443, 444, 471, 475, 513, 514, 515, 516, 528, 529, 530, 531, 532, 536 }

2.1.4 Maxima

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 14, 18, 19, 20, 21, 24, 25, 30, 31, 35, 36, 42, 54, 62, 66, 69, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 227, 249, 251, 252, 254, 255, 257, 259, 261, 281, 282, 286, 287, 291, 292, 296, 302, 304, 307, 310, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 364, 366, 368, 371, 387, 388, 390, 391, 393, 395, 398, 399, 403, 406, 407, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 469, 470, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 503, 505, 506, 507, 508, 511, 512, 519, 520, 521, 525, 527, 535 }

B grade: { 4, 13, 17, 22, 23, 28, 29, 32, 33, 34, 37, 40, 41, 61, 67, 71, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 107, 114, 115, 118, 124, 125, 126, 166, 181, 228, 229, 230, 231, 232, 233, 235, 237, 239, 244, 250, 253, 256, 260, 262, 263, 264, 265, 267, 268, 269, 271, 274, 275, 276, 303, 305, 306, 308, 309, 311, 313, 314, 315, 316, 317, 345, 346, 347, 468, 472, 473, 474, 504, 513, 514, 515, 516 }

C grade: { 502, 509, 522, 523, 524, 526, 538 }

F grade: { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 68, 70, 72, 73, 74, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 236, 238, 240, 241, 242, 243, 245, 246, 247, 248, 258, 266, 270, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 312, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 510, 517, 518, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 119, 124, 125, 126, 137, 138, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 218, 219, 220, 221, 222, 223, 224, 228, 230, 232, 237, 244, 249, 251, 252, 253, 254, 255, 256, 257, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 281, 282, 286, 287, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 321, 322, 328, 329, 335, 341, 344, 345, 346, 347, 349, 350, 358, 359, 364, 366, 368, 371, 387, 388, 390, 391, 393, 395, 398, 399, 403, 406, 407, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 511, 512, 519, 520, 521, 522, 523, 524, 525, 526, 535, 538 }

B grade: { 248, 250, 258, 283, 284, 285, 288, 289, 290, 293, 294, 295, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 513, 514, 515, 516 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 306, 312, 313, 318, 319, 320, 348, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 517, 518, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.6 Sympy

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 41, 42, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 218, 219, 220, 221, 222, 223, 224, 228, 230, 232, 237, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 422, 425, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 511, 512, 522, 523, 524, 525, 526, 535 }

B grade: { 4, 13, 17, 23, 28, 34, 40, 54, 61, 62, 66, 258, 264, 302, 304, 307 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110,

111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 421, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538
}

2.1.7 Giac

A grade: { 54, 61, 62, 107, 114, 115, 124, 125, 141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 230, 237, 244, 249, 251, 252, 253, 254, 255, 256, 257, 258, 268, 269, 275, 276, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 368, 387, 390, 391, 395, 403, 411, 412, 416, 417, 421, 422, 462, 463, 464, 468, 469, 470, 479, 480, 485, 486, 491, 492, 511, 512, 513, 514, 515, 516, 519, 520, 521, 522, 524, 525, 526, 535, 538 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 66, 76, 78, 118, 126, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 248, 250, 261, 262, 302, 304, 308, 310, 314, 316, 371, 393, 498, 499, 500, 501, 505 }

C grade: { }

F grade: { 5, 6, 14, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 307, 309, 311, 312, 313, 315, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 392, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 415, 418, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 465, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 517, 518, 523, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.8 Mupad

A grade: { 141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 249, 251, 252, 254, 255, 257, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 479, 480, 485, 486, 491, 492, 511, 512, 535 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 126, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 228, 230, 232, 237, 244, 248, 250, 253, 256, 258, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 345, 346, 347, 498, 499, 500, 501, 505, 522, 523, 524, 525, 526, 538 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 306, 312, 313, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	110	121	114	124	491	103
N.S.	1	1.00	0.90	1.02	1.12	1.06	1.15	4.55	0.95
time (sec)	N/A	0.070	0.033	0.120	0.256	0.359	0.365	0.413	0.989

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	87	100	110	102	112	394	92
N.S.	1	1.00	0.91	1.04	1.15	1.06	1.17	4.10	0.96
time (sec)	N/A	0.070	0.029	0.118	0.262	0.353	0.304	0.423	0.944

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	90	99	93	100	305	83
N.S.	1	1.00	0.94	1.07	1.18	1.11	1.19	3.63	0.99
time (sec)	N/A	0.057	0.028	0.115	0.254	0.353	0.295	0.429	0.901

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	95	70	85	77	75	211	65
N.S.	1	1.00	2.16	1.59	1.93	1.75	1.70	4.80	1.48
time (sec)	N/A	0.024	0.012	0.118	0.261	0.356	0.180	0.417	0.864

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	86	0	0	0	0	-1
N.S.	1	1.00	0.90	1.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.048	0.131	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	105	0	0	0	0	-1
N.S.	1	1.00	1.01	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.032	0.155	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	76	91	89	89	95	192	75
N.S.	1	1.00	1.36	1.62	1.59	1.59	1.70	3.43	1.34
time (sec)	N/A	0.038	0.028	0.114	0.257	0.365	0.523	0.403	0.902

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	102	99	101	117	306	110
N.S.	1	1.00	0.88	1.04	1.01	1.03	1.19	3.12	1.12
time (sec)	N/A	0.063	0.031	0.117	0.259	0.362	0.731	0.436	0.900

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	112	114	110	129	401	120
N.S.	1	1.00	0.85	1.02	1.04	1.00	1.17	3.65	1.09
time (sec)	N/A	0.068	0.032	0.112	0.269	0.373	0.574	0.412	0.846

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	125	164	210	162	196	620	146
N.S.	1	1.00	0.80	1.04	1.34	1.03	1.25	3.95	0.93
time (sec)	N/A	0.119	0.045	0.208	0.255	0.368	0.470	0.414	1.043

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	115	152	184	146	177	525	134
N.S.	1	1.00	0.80	1.06	1.29	1.02	1.24	3.67	0.94
time (sec)	N/A	0.108	0.040	0.204	0.265	0.355	0.386	0.420	1.032

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	107	140	179	137	167	425	122
N.S.	1	1.00	0.83	1.09	1.39	1.06	1.29	3.29	0.95
time (sec)	N/A	0.094	0.035	0.207	0.261	0.370	0.339	0.399	0.966

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	92	102	147	114	131	330	105
N.S.	1	1.00	1.30	1.44	2.07	1.61	1.85	4.65	1.48
time (sec)	N/A	0.031	0.040	0.154	0.255	0.354	0.266	0.412	0.962

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	103	142	173	0	0	0	-1
N.S.	1	1.00	0.90	1.25	1.52	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.056	0.184	0.351	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	80	73	121	0	0	0	410	-1
N.S.	1	1.31	1.20	1.98	0.00	0.00	0.00	6.72	-0.02
time (sec)	N/A	0.092	0.059	0.214	0.000	0.000	0.000	1.151	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	143	168	0	0	0	0	-1
N.S.	1	1.00	1.04	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.047	0.220	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	103	142	157	128	158	330	116
N.S.	1	1.00	1.27	1.75	1.94	1.58	1.95	4.07	1.43
time (sec)	N/A	0.063	0.039	0.171	0.265	0.392	0.537	0.426	0.896

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	114	154	178	147	189	431	168
N.S.	1	1.00	0.78	1.05	1.21	1.00	1.29	2.93	1.14
time (sec)	N/A	0.109	0.040	0.169	0.276	0.401	0.554	0.406	1.008

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	122	166	194	156	199	532	182
N.S.	1	1.00	0.76	1.03	1.20	0.97	1.24	3.30	1.13
time (sec)	N/A	0.114	0.043	0.174	0.270	0.370	0.660	0.425	0.952

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	151	200	285	190	243	722	177
N.S.	1	1.00	0.79	1.04	1.48	0.99	1.27	3.76	0.92
time (sec)	N/A	0.134	0.054	0.216	0.258	0.368	0.560	0.437	1.054

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	142	188	265	178	235	621	165
N.S.	1	1.00	0.80	1.06	1.49	1.00	1.32	3.49	0.93
time (sec)	N/A	0.129	0.050	0.184	0.252	0.385	0.470	0.426	1.032

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	174	244	165	211	527	153
N.S.	1	1.00	0.99	1.29	1.81	1.22	1.56	3.90	1.13
time (sec)	N/A	0.074	0.040	0.158	0.267	0.366	0.398	0.422	0.978

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	115	131	219	149	182	425	136
N.S.	1	1.00	1.37	1.56	2.61	1.77	2.17	5.06	1.62
time (sec)	N/A	0.037	0.046	0.152	0.261	0.377	0.396	0.403	0.956

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	148	182	228	0	0	0	-1
N.S.	1	1.00	0.97	1.20	1.50	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.065	0.181	0.365	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	183	229	0	0	0	-1
N.S.	1	1.00	0.99	1.22	1.53	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.073	0.208	0.356	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	165	188	0	0	0	0	-1
N.S.	1	1.00	1.03	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.074	0.222	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	175	208	0	0	0	0	-1
N.S.	1	1.00	0.99	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.057	0.220	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	131	178	228	163	207	431	147
N.S.	1	1.00	1.41	1.91	2.45	1.75	2.23	4.63	1.58
time (sec)	N/A	0.072	0.043	0.162	0.258	0.362	0.584	0.414	0.949

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	140	190	250	175	233	533	233
N.S.	1	1.00	1.02	1.39	1.82	1.28	1.70	3.89	1.70
time (sec)	N/A	0.084	0.049	0.172	0.263	0.378	0.655	0.422	0.980

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	149	202	273	188	257	634	220
N.S.	1	1.00	0.76	1.03	1.39	0.96	1.31	3.23	1.12
time (sec)	N/A	0.127	0.052	0.178	0.276	0.391	0.788	0.440	1.067

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	177	234	373	222	294	817	337
N.S.	1	1.00	0.79	1.04	1.67	0.99	1.31	3.65	1.50
time (sec)	N/A	0.154	0.057	0.221	0.259	0.367	0.651	0.453	1.839

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	168	222	339	208	279	723	196
N.S.	1	1.00	0.98	1.30	1.98	1.22	1.63	4.23	1.15
time (sec)	N/A	0.126	0.084	0.175	0.264	0.352	0.554	0.437	1.077

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	159	212	326	198	269	621	185
N.S.	1	1.00	1.04	1.39	2.13	1.29	1.76	4.06	1.21
time (sec)	N/A	0.084	0.052	0.176	0.259	0.383	0.509	0.425	1.048

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	146	159	283	177	226	526	168
N.S.	1	1.00	1.36	1.49	2.64	1.65	2.11	4.92	1.57
time (sec)	N/A	0.042	0.052	0.151	0.266	0.366	0.407	0.423	1.048

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	179	222	276	0	0	0	-1
N.S.	1	1.00	0.97	1.20	1.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.076	0.218	0.355	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	194	223	281	0	0	0	-1
N.S.	1	1.00	1.09	1.25	1.58	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.082	0.215	0.353	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	143	202	293	0	0	0	-1
N.S.	1	1.00	0.92	1.29	1.88	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.076	0.221	0.346	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	197	228	0	0	0	0	-1
N.S.	1	1.00	1.04	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.084	0.238	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	248	0	0	0	0	-1
N.S.	1	1.00	0.99	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.065	0.231	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	157	214	299	191	253	532	179
N.S.	1	1.00	1.44	1.96	2.74	1.75	2.32	4.88	1.64
time (sec)	N/A	0.076	0.052	0.167	0.257	0.369	0.693	0.414	0.955

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	166	226	329	208	291	634	248
N.S.	1	1.00	1.10	1.50	2.18	1.38	1.93	4.20	1.64
time (sec)	N/A	0.092	0.060	0.167	0.271	0.360	0.821	0.422	1.235

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	175	238	353	218	301	735	260
N.S.	1	1.00	0.76	1.04	1.54	0.95	1.31	3.21	1.14
time (sec)	N/A	0.146	0.076	0.166	0.258	0.378	1.158	0.436	1.209

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	129	224	0	0	0	0	-1
N.S.	1	1.00	0.73	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.240	0.296	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	97	184	0	0	0	0	-1
N.S.	1	1.00	0.67	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.149	0.283	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	136	0	0	0	0	-1
N.S.	1	1.00	0.80	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.098	0.277	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	98	0	0	0	0	-1
N.S.	1	1.00	1.02	1.92	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.068	0.309	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	156	0	0	0	0	-1
N.S.	1	1.00	1.20	3.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.069	0.174	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	219	0	0	0	0	-1
N.S.	1	1.00	1.00	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.105	0.211	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	133	260	0	0	0	0	-1
N.S.	1	1.00	0.91	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.195	0.218	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	172	302	0	0	0	0	-1
N.S.	1	1.00	0.93	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.253	0.335	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	142	227	0	0	0	0	-1
N.S.	1	1.00	0.78	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.441	0.327	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	183	0	0	0	0	-1
N.S.	1	1.00	0.81	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.370	0.314	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	99	163	0	0	0	0	-1
N.S.	1	1.00	0.93	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.232	0.290	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	64	73	96	49	95	63	45
N.S.	1	1.00	1.12	1.28	1.68	0.86	1.67	1.11	0.79
time (sec)	N/A	0.034	0.040	0.145	0.262	0.363	0.711	0.416	1.075

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	221	0	0	0	0	-1
N.S.	1	1.00	0.81	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.257	0.201	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	260	0	0	0	0	-1
N.S.	1	1.00	0.82	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.521	0.328	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	189	303	0	0	0	0	-1
N.S.	1	1.00	0.89	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.732	0.335	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	189	272	0	0	0	0	-1
N.S.	1	1.00	0.83	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.509	0.349	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	167	228	0	0	0	0	-1
N.S.	1	1.00	0.86	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.436	0.337	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	145	208	0	0	0	0	-1
N.S.	1	1.00	0.97	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.293	0.319	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	99	114	152	84	277	114	81
N.S.	1	1.00	1.29	1.48	1.97	1.09	3.60	1.48	1.05
time (sec)	N/A	0.057	0.046	0.156	0.264	0.373	0.844	0.422	1.266

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	86	86	134	75	224	118	123
N.S.	1	1.00	1.12	1.12	1.74	0.97	2.91	1.53	1.60
time (sec)	N/A	0.040	0.042	0.139	0.259	0.352	0.772	0.418	1.089

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	147	264	0	0	0	0	-1
N.S.	1	1.00	0.91	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.312	0.229	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	186	307	0	0	0	0	-1
N.S.	1	1.00	0.85	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.808	0.232	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	220	350	0	0	0	0	-1
N.S.	1	1.00	0.82	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.926	0.348	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	78	132	91	294	161	139
N.S.	1	1.00	0.94	0.98	1.65	1.14	3.68	2.01	1.74
time (sec)	N/A	0.036	0.073	0.158	0.263	0.358	1.181	0.407	1.096

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	112	120	0	0	0	-1
N.S.	1	1.00	0.95	2.73	2.93	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.049	0.150	0.261	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	271	403	0	0	0	0	-1
N.S.	1	1.00	1.00	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	0.502	0.308	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	234	364	402	0	0	0	-1
N.S.	1	1.00	0.99	1.54	1.70	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.386	0.371	0.310	0.483	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	201	322	0	0	0	0	-1
N.S.	1	1.00	1.03	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.280	0.316	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	156	273	290	0	0	0	-1
N.S.	1	1.00	1.39	2.44	2.59	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.177	0.201	0.414	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	228	3644	0	0	0	0	-1
N.S.	1	1.00	1.19	19.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.286	2.637	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	249	3070	0	0	0	0	-1
N.S.	1	1.00	1.24	15.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.289	3.747	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	206	367	0	0	0	0	-1
N.S.	1	1.00	1.36	2.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.161	0.587	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	246	407	417	0	0	0	-1
N.S.	1	1.00	1.19	1.98	2.02	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.300	0.564	0.637	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	329	549	766	0	0	1135	-1
N.S.	1	1.00	0.92	1.54	2.15	0.00	0.00	3.19	-0.00
time (sec)	N/A	0.723	0.696	0.444	0.492	0.000	0.000	2.252	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	297	501	604	0	0	0	-1
N.S.	1	1.00	0.95	1.61	1.94	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	0.648	0.432	0.478	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	263	458	610	0	0	761	-1
N.S.	1	1.00	0.94	1.64	2.18	0.00	0.00	2.72	-0.00
time (sec)	N/A	0.469	0.431	0.468	0.482	0.000	0.000	1.586	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	227	330	464	0	0	0	-1
N.S.	1	1.00	1.30	1.89	2.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.373	0.379	0.409	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	324	1082	0	0	0	0	-1
N.S.	1	1.00	1.17	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	0.368	6.157	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	341	5974	0	0	0	0	-1
N.S.	1	1.00	1.20	21.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	0.305	6.003	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	370	1103	0	0	0	0	-1
N.S.	1	1.00	1.18	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	0.422	6.928	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	270	509	555	0	0	0	-1
N.S.	1	1.00	1.11	2.09	2.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.391	0.649	0.645	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	385	638	928	0	0	0	-1
N.S.	1	1.00	0.93	1.54	2.24	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.023	1.532	0.446	0.499	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	356	594	775	0	0	0	-1
N.S.	1	1.00	0.94	1.58	2.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.874	0.872	0.479	0.487	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	325	546	780	0	0	0	-1
N.S.	1	1.00	1.14	1.91	2.73	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.696	0.398	0.485	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	293	408	627	0	0	0	-1
N.S.	1	1.00	1.42	1.98	3.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.492	0.362	0.419	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	448	1186	0	0	0	0	-1
N.S.	1	1.00	1.26	3.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	0.437	6.668	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	479	1239	0	0	0	0	-1
N.S.	1	1.00	1.33	3.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.550	0.357	7.060	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	461	1276	0	0	0	0	-1
N.S.	1	1.00	1.20	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	0.761	6.840	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	569	1267	0	0	0	0	-1
N.S.	1	1.00	1.44	3.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.676	0.416	7.741	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	343	599	813	0	0	0	-1
N.S.	1	1.00	1.27	2.21	3.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.477	0.721	0.660	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	372	644	783	0	0	0	-1
N.S.	1	1.00	1.06	1.83	2.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.836	0.730	0.663	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	402	689	961	0	0	0	-1
N.S.	1	1.00	0.84	1.44	2.01	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	1.187	0.744	0.664	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	347	1200	0	0	0	0	-1
N.S.	1	1.00	1.05	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	0.507	8.391	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	260	1099	0	0	0	0	-1
N.S.	1	1.00	1.05	4.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	0.303	7.850	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	140	5134	0	0	0	0	-1
N.S.	1	1.00	0.81	29.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.267	6.816	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	102	769	0	0	0	0	-1
N.S.	1	1.00	1.21	9.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.127	5.433	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	132	1389	0	0	0	0	-1
N.S.	1	1.00	1.71	18.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.168	7.076	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	225	7139	0	0	0	0	-1
N.S.	1	1.00	1.39	44.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.369	10.388	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	317	1732	0	0	0	0	-1
N.S.	1	1.00	1.27	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.993	12.937	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	388	1895	0	0	0	0	-1
N.S.	1	1.00	1.16	5.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.689	0.861	14.579	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	425	1347	0	0	0	0	-1
N.S.	1	1.00	1.08	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.619	1.153	12.342	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	354	1239	0	0	0	0	-1
N.S.	1	1.00	1.07	3.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	1.074	10.819	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	295	5290	0	0	0	0	-1
N.S.	1	1.00	1.13	20.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	0.567	7.519	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	233	946	0	0	0	0	-1
N.S.	1	1.00	1.24	5.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.326	6.671	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	124	294	277	101	0	119	97
N.S.	1	1.00	1.16	2.75	2.59	0.94	0.00	1.11	0.91
time (sec)	N/A	0.096	0.074	0.302	0.272	0.344	0.000	0.399	1.251

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	254	1566	0	0	0	0	-1
N.S.	1	1.00	0.86	5.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	0.476	10.367	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	347	7294	0	0	0	0	-1
N.S.	1	1.00	0.94	19.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.584	1.436	10.641	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	452	1878	0	0	0	0	-1
N.S.	1	1.00	0.94	3.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.689	1.349	14.027	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	420	1428	0	0	0	0	-1
N.S.	1	1.00	1.03	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	1.261	11.522	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	418	5476	0	0	0	0	-1
N.S.	1	1.00	1.24	16.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	1.244	10.379	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	310	1135	0	0	0	0	-1
N.S.	1	1.00	1.17	4.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.916	9.692	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	150	391	429	164	0	226	405
N.S.	1	1.00	0.96	2.49	2.73	1.04	0.00	1.44	2.58
time (sec)	N/A	0.160	0.104	0.356	0.278	0.354	0.000	0.410	2.691

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	183	342	399	156	0	232	373
N.S.	1	1.00	1.17	2.18	2.54	0.99	0.00	1.48	2.38
time (sec)	N/A	0.135	0.075	0.351	0.292	0.386	0.000	0.404	2.164

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	376	1752	0	0	0	0	-1
N.S.	1	1.00	1.04	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	0.783	11.602	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	479	7480	0	0	0	0	-1
N.S.	1	1.00	1.07	16.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.721	1.898	12.434	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	168	321	445	203	0	333	498
N.S.	1	1.00	0.95	1.82	2.53	1.15	0.00	1.89	2.83
time (sec)	N/A	0.168	0.103	0.342	0.288	0.403	0.000	0.401	2.299

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	647	0	85	0	0	-1
N.S.	1	1.00	0.88	9.66	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.085	13.970	0.000	0.352	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	644	883	0	0	0	0	-1
N.S.	1	1.00	2.10	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	0.890	5.487	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	488	740	0	0	0	0	-1
N.S.	1	1.00	2.03	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.782	5.608	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	334	6152	0	0	0	0	-1
N.S.	1	1.00	1.75	32.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.482	10.721	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	152	1396	0	0	0	0	-1
N.S.	1	1.00	1.37	12.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.170	15.032	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	198	1811	529	160	0	172	582
N.S.	1	1.00	1.42	13.03	3.81	1.15	0.00	1.24	4.19
time (sec)	N/A	0.150	0.072	6.839	0.280	0.359	0.000	0.405	2.300

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	215	2684	796	250	0	362	930
N.S.	1	1.00	1.03	12.90	3.83	1.20	0.00	1.74	4.47
time (sec)	N/A	0.278	0.116	7.263	0.467	0.346	0.000	0.411	3.457

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	279	3557	1085	345	0	555	1304
N.S.	1	1.00	1.01	12.93	3.95	1.25	0.00	2.02	4.74
time (sec)	N/A	0.447	0.137	10.506	0.323	0.373	0.000	0.435	4.494

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	172	352	0	0	0	0	-1
N.S.	1	1.00	0.56	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.230	23.007	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	126	736	0	0	0	0	-1
N.S.	1	1.00	0.61	3.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.175	19.645	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	82	593	0	0	0	0	-1
N.S.	1	1.00	0.79	5.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.061	16.345	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	1157	0	0	0	0	-1
N.S.	1	1.00	0.92	12.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.073	23.771	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	1164	0	0	0	0	-1
N.S.	1	1.00	0.92	12.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.021	24.270	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	154	1339	0	0	0	0	-1
N.S.	1	1.00	0.81	7.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.437	0.169	26.783	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	222	602	0	0	0	0	-1
N.S.	1	1.00	0.73	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	0.390	32.815	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	233	442	0	0	0	0	-1
N.S.	1	1.00	0.61	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.604	0.273	9.951	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	172	381	0	0	0	0	-1
N.S.	1	1.00	0.66	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.166	7.063	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	112	228	0	0	0	0	-1
N.S.	1	1.00	0.85	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.080	3.137	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	754	0	155	0	0	-1
N.S.	1	1.00	0.86	6.39	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.085	10.675	0.000	0.350	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	761	0	155	0	0	-1
N.S.	1	1.00	0.86	6.45	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.020	10.926	0.000	0.414	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	172	573	0	0	0	0	-1
N.S.	1	1.00	0.72	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.296	12.428	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	250	779	0	0	0	0	-1
N.S.	1	1.00	0.66	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	1.017	13.816	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.030	1.655	18.283	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.017	0.085	3.101	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.042	0.095	2.948	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	1.446	21.549	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.016	0.850	2.809	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	1.008	2.803	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	474	423	0	0	0	0	-1
N.S.	1	1.00	1.72	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	4.533	3.745	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	394	332	0	0	0	0	-1
N.S.	1	1.00	1.84	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	2.060	3.726	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	315	243	0	0	0	0	-1
N.S.	1	1.00	2.02	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.663	3.770	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	244	158	0	0	0	0	-1
N.S.	1	1.00	2.14	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.179	0.189	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	294	210	0	0	0	0	-1
N.S.	1	1.00	1.99	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.129	3.749	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	360	317	0	0	0	0	-1
N.S.	1	1.00	1.80	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	2.228	3.697	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	435	418	0	0	0	0	-1
N.S.	1	1.00	1.67	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	4.129	3.802	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	1250	1729	0	0	0	0	-1
N.S.	1	1.00	3.25	4.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	12.286	25.626	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	1036	14121	0	0	0	0	-1
N.S.	1	1.00	3.71	50.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	10.320	14.521	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	938	1197	0	0	0	0	-1
N.S.	1	1.00	4.99	6.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	8.855	0.244	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	1034	1799	0	0	0	0	-1
N.S.	1	1.00	3.24	5.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	8.019	20.325	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	1188	26972	0	0	0	0	-1
N.S.	1	1.00	2.88	65.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	9.759	37.680	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	733	1566	0	0	0	0	-1
N.S.	1	1.00	2.67	5.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	6.548	49.974	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.033	2.754	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	70	73	76	71	335	61
N.S.	1	1.00	1.00	0.97	1.01	1.06	0.99	4.65	0.85
time (sec)	N/A	0.082	0.016	0.285	0.253	0.368	0.451	0.398	0.991

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	66	72	61	54	227	51
N.S.	1	1.00	1.25	1.05	1.14	0.97	0.86	3.60	0.81
time (sec)	N/A	0.057	0.017	0.279	0.267	0.381	0.358	0.409	0.947

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	62	65	68	63	268	53
N.S.	1	1.00	1.00	1.00	1.05	1.10	1.02	4.32	0.85
time (sec)	N/A	0.068	0.014	0.281	0.255	0.358	0.274	0.391	0.908

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	69	48	37	52	46	160	44
N.S.	1	1.00	1.72	1.20	0.92	1.30	1.15	4.00	1.10
time (sec)	N/A	0.015	0.014	0.376	0.259	0.361	0.241	0.398	0.865

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	49	47	53	49	203	40
N.S.	1	1.00	0.73	0.77	0.73	0.83	0.77	3.17	0.62
time (sec)	N/A	0.016	0.009	0.081	0.267	0.358	0.202	0.415	0.844

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	69	89	0	0	0	-1
N.S.	1	1.00	1.25	1.44	1.85	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.017	0.131	0.257	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	44	36	51	41	145	37
N.S.	1	1.00	1.00	1.16	0.95	1.34	1.08	3.82	0.97
time (sec)	N/A	0.037	0.010	0.116	0.265	0.374	0.414	0.386	0.810

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	68	78	81	0	0	330	-1
N.S.	1	1.00	1.21	1.39	1.45	0.00	0.00	5.89	-0.02
time (sec)	N/A	0.036	0.025	0.155	0.271	0.000	0.000	1.306	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	59	53	64	63	204	49
N.S.	1	1.00	1.00	1.02	0.91	1.10	1.09	3.52	0.84
time (sec)	N/A	0.058	0.014	0.184	0.259	0.367	0.388	0.397	0.850

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	71	62	61	52	46	160	61
N.S.	1	1.00	1.69	1.48	1.45	1.24	1.10	3.81	1.45
time (sec)	N/A	0.022	0.015	0.079	0.256	0.395	0.309	0.401	0.839

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	68	62	73	75	281	59
N.S.	1	1.00	1.00	0.96	0.87	1.03	1.06	3.96	0.83
time (sec)	N/A	0.067	0.017	0.206	0.258	0.362	0.552	0.404	0.876

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	113	199	190	0	0	0	-1
N.S.	1	1.00	0.70	1.23	1.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.428	0.684	0.932	0.265	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	88	182	146	109	114	522	101
N.S.	1	1.00	0.76	1.57	1.26	0.94	0.98	4.50	0.87
time (sec)	N/A	0.328	0.033	0.430	0.262	0.423	0.747	0.401	0.982

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	95	179	173	0	0	0	-1
N.S.	1	1.00	0.69	1.30	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.179	1.023	0.264	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	86	74	91	88	305	77
N.S.	1	1.00	0.69	0.91	0.78	0.96	0.93	3.21	0.81
time (sec)	N/A	0.039	0.023	0.372	0.261	0.335	0.292	0.398	0.899

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	71	154	144	0	0	0	-1
N.S.	1	1.00	0.62	1.34	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.086	0.553	0.261	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	145	663	0	0	0	0	-1
N.S.	1	1.00	0.99	4.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.036	18.243	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	156	152	0	0	0	-1
N.S.	1	1.00	1.10	1.68	1.63	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.103	0.332	0.275	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	174	736	0	0	0	0	-1
N.S.	1	1.00	1.01	4.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.054	19.018	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	93	198	188	0	0	0	-1
N.S.	1	1.00	0.80	1.71	1.62	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.210	0.154	0.256	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	82	172	164	108	102	282	246
N.S.	1	1.00	0.92	1.93	1.84	1.21	1.15	3.17	2.76
time (sec)	N/A	0.082	0.029	0.139	0.269	0.370	0.758	0.416	1.358

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	114	219	228	0	0	0	-1
N.S.	1	1.00	0.80	1.53	1.59	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.341	0.142	0.254	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	749	0	0	0	0	-1
N.S.	1	1.00	0.85	4.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.212	24.340	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	232	251	277	0	0	0	-1
N.S.	1	1.00	1.20	1.30	1.44	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.192	0.408	0.473	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.539	23.286	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.232	32.662	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.783	29.806	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.717	31.842	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.983	34.067	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.826	31.357	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.833	25.566	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	89	92	100	383	106
N.S.	1	1.00	1.00	0.94	0.93	0.96	1.04	3.99	1.10
time (sec)	N/A	0.139	0.027	0.563	0.253	0.456	0.632	0.403	1.015

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	103	76	88	77	76	240	101
N.S.	1	1.00	1.18	0.87	1.01	0.89	0.87	2.76	1.16
time (sec)	N/A	0.104	0.025	0.902	0.255	0.354	0.529	0.410	1.208

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	82	81	84	90	319	71
N.S.	1	1.00	1.00	0.95	0.94	0.98	1.05	3.71	0.83
time (sec)	N/A	0.116	0.021	0.397	0.259	0.351	0.438	0.400	0.951

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	93	68	46	68	68	176	64
N.S.	1	1.00	1.86	1.36	0.92	1.36	1.36	3.52	1.28
time (sec)	N/A	0.025	0.021	0.572	0.250	0.384	0.325	0.394	0.918

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	71	69	66	72	75	255	60
N.S.	1	1.00	0.68	0.66	0.63	0.69	0.72	2.45	0.58
time (sec)	N/A	0.032	0.017	0.266	0.257	0.340	0.279	0.400	0.907

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	82	89	106	0	0	0	-1
N.S.	1	1.00	1.17	1.27	1.51	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.024	0.304	0.275	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	64	57	66	68	249	57
N.S.	1	1.00	1.00	1.00	0.89	1.03	1.06	3.89	0.89
time (sec)	N/A	0.078	0.017	0.225	0.266	0.373	0.667	0.409	0.911

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	73	82	0	0	0	-1
N.S.	1	1.00	0.98	1.18	1.32	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.043	0.293	0.274	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	67	66	72	75	274	59
N.S.	1	1.00	1.00	0.99	0.97	1.06	1.10	4.03	0.87
time (sec)	N/A	0.075	0.019	0.277	0.269	0.365	0.471	0.391	0.878

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	89	96	112	0	0	0	-1
N.S.	1	1.00	1.16	1.25	1.45	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.033	0.300	0.267	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	80	71	81	88	265	70
N.S.	1	1.00	1.00	0.96	0.86	0.98	1.06	3.19	0.84
time (sec)	N/A	0.097	0.021	0.352	0.260	0.373	0.560	0.409	0.888

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	233	214	0	0	0	-1
N.S.	1	1.00	0.68	1.15	1.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.693	1.407	3.367	0.262	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	140	170	133	153	683	221
N.S.	1	1.00	0.69	0.90	1.09	0.85	0.98	4.38	1.42
time (sec)	N/A	0.566	0.032	1.052	0.256	0.370	0.746	0.417	1.220

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	121	213	198	0	0	0	-1
N.S.	1	1.00	0.68	1.20	1.11	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.533	0.800	1.349	0.267	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	82	120	93	116	133	473	111
N.S.	1	1.00	0.59	0.87	0.67	0.84	0.96	3.43	0.80
time (sec)	N/A	0.063	0.027	0.615	0.260	0.352	0.514	0.409	0.995

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	99	188	175	0	0	0	-1
N.S.	1	1.00	0.58	1.10	1.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.462	1.006	0.261	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	191	733	0	0	0	0	-1
N.S.	1	1.00	1.03	3.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.363	0.052	21.441	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	149	203	200	0	0	0	-1
N.S.	1	1.00	0.96	1.30	1.28	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.303	0.583	0.270	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	183	779	0	0	0	0	-1
N.S.	1	1.00	1.13	4.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.325	0.061	21.686	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	153	208	203	0	0	0	-1
N.S.	1	1.00	0.92	1.25	1.22	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.310	0.055	0.557	0.267	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	238	1124	0	0	0	0	-1
N.S.	1	1.00	1.11	5.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	0.229	22.704	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	118	233	239	0	0	0	-1
N.S.	1	1.00	0.75	1.48	1.52	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.431	0.551	0.566	0.272	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	99	206	188	132	148	440	335
N.S.	1	1.00	0.88	1.82	1.66	1.17	1.31	3.89	2.96
time (sec)	N/A	0.143	0.031	0.542	0.261	0.379	0.678	0.410	1.581

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	140	253	254	0	0	0	-1
N.S.	1	1.00	0.77	1.38	1.39	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.747	1.006	0.275	0.263	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	124	226	204	148	168	651	357
N.S.	1	1.00	0.73	1.33	1.20	0.87	0.99	3.83	2.10
time (sec)	N/A	0.609	0.033	0.539	0.265	0.349	0.928	0.413	2.720

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	183	828	0	0	0	0	-1
N.S.	1	1.00	0.74	3.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.400	30.234	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.650	54.243	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.443	41.721	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	1.285	62.162	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.538	59.206	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.800	35.987	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.696	68.779	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	79	89	82	88	97	303	80
N.S.	1	1.00	0.55	0.62	0.57	0.61	0.67	2.10	0.56
time (sec)	N/A	0.051	0.020	0.259	0.249	0.384	0.808	0.407	0.935

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	124	222	199	0	0	0	-1
N.S.	1	1.00	0.55	0.98	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.843	3.680	0.268	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	231	978	0	0	0	0	-1
N.S.	1	1.00	0.68	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.715	49.175	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	131	120	0	0	0	-1
N.S.	1	1.00	0.69	1.51	1.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.100	0.352	0.264	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	114	85	56	41	0	82
N.S.	1	1.00	1.00	2.71	2.02	1.33	0.98	0.00	1.95
time (sec)	N/A	0.050	0.029	0.296	0.265	0.356	0.806	0.000	0.956

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	99	125	0	0	0	-1
N.S.	1	1.00	0.81	1.83	2.31	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.039	0.273	0.253	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	65	22	10	22	23
N.S.	1	1.00	1.00	0.92	5.00	1.69	0.77	1.69	1.77
time (sec)	N/A	0.012	0.005	0.217	0.261	0.375	0.517	0.403	0.872

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	130	132	0	0	0	-1
N.S.	1	1.00	0.93	2.89	2.93	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.050	0.281	0.266	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	120	82	63	37	0	80
N.S.	1	1.00	1.00	2.93	2.00	1.54	0.90	0.00	1.95
time (sec)	N/A	0.056	0.031	0.303	0.276	0.381	0.506	0.000	1.049

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	170	162	0	0	0	-1
N.S.	1	1.00	0.71	2.02	1.93	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.174	0.385	0.266	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	112	728	0	0	0	0	-1
N.S.	1	1.00	0.83	5.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.085	39.980	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	5330	200	0	0	0	-1
N.S.	1	1.00	0.79	71.07	2.67	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.123	41.033	0.267	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	638	0	0	0	0	-1
N.S.	1	1.00	0.87	8.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.048	31.491	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	127	22	10	22	68
N.S.	1	1.00	1.00	0.92	9.77	1.69	0.77	1.69	5.23
time (sec)	N/A	0.017	0.005	0.227	0.262	0.348	1.003	0.386	0.953

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	1188	0	0	0	0	-1
N.S.	1	1.00	0.91	18.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.126	0.048	63.510	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	4380	237	0	0	0	-1
N.S.	1	1.00	0.92	66.36	3.59	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.138	0.158	35.338	0.272	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	1277	0	0	0	0	-1
N.S.	1	1.00	0.96	9.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.244	85.711	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	142	217	0	0	0	0	-1
N.S.	1	1.00	0.69	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.193	52.640	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	736	0	0	0	0	-1
N.S.	1	1.00	0.76	7.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.179	59.019	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	87	670	0	0	0	0	-1
N.S.	1	1.00	0.81	6.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.048	46.659	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	209	22	10	22	90
N.S.	1	1.00	1.00	0.92	16.08	1.69	0.77	1.69	6.92
time (sec)	N/A	0.017	0.005	0.328	0.259	0.351	0.677	0.391	0.903

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	83	1245	0	0	0	0	-1
N.S.	1	1.00	0.91	13.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.054	81.985	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	93	810	0	0	0	0	-1
N.S.	1	1.00	1.03	9.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.146	67.874	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	165	369	0	0	0	0	-1
N.S.	1	1.00	0.82	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.309	85.046	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	25	14	25	11
N.S.	1	1.00	1.00	0.80	0.00	1.67	0.93	1.67	0.73
time (sec)	N/A	0.019	0.007	0.717	0.000	0.329	0.701	0.391	0.874

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.943	4.135	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	21	20	7	21	9
N.S.	1	1.00	1.00	1.11	2.33	2.22	0.78	2.33	1.00
time (sec)	N/A	0.019	0.023	0.306	0.258	0.349	0.773	0.391	0.823

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.167	4.472	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.098	4.133	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	22	8	22	23
N.S.	1	1.00	1.00	1.09	2.09	2.00	0.73	2.00	2.09
time (sec)	N/A	0.017	0.005	0.282	0.264	0.356	0.466	0.386	0.781

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	0.103	4.290	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.415	4.282	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	42	22	12	22	23
N.S.	1	1.00	1.00	0.92	3.23	1.69	0.92	1.69	1.77
time (sec)	N/A	0.020	0.006	0.325	0.263	0.371	0.897	0.392	0.799

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	0.515	4.331	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	84	26	30	33
N.S.	1	1.00	1.00	1.06	0.00	4.94	1.53	1.76	1.94
time (sec)	N/A	0.022	0.009	0.279	0.000	0.364	0.868	0.402	1.053

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	64	159	177	0	0	0	-1
N.S.	1	1.00	0.59	1.46	1.62	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.103	0.754	0.266	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	134	126	65	0	0	110
N.S.	1	1.00	0.79	2.35	2.21	1.14	0.00	0.00	1.93
time (sec)	N/A	0.045	0.037	0.733	0.254	0.361	0.000	0.000	0.958

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	66	57	62	48	61	154	37
N.S.	1	1.00	1.20	1.04	1.13	0.87	1.11	2.80	0.67
time (sec)	N/A	0.027	0.027	0.306	0.252	0.435	1.077	0.402	0.913

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	134	122	64	0	255	106
N.S.	1	1.00	0.81	2.48	2.26	1.19	0.00	4.72	1.96
time (sec)	N/A	0.016	0.028	0.730	0.256	0.340	0.000	1.257	0.975

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	63	190	206	0	0	0	-1
N.S.	1	1.00	0.69	2.09	2.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.095	0.428	0.272	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	164	150	118	253	0	132
N.S.	1	1.00	0.94	2.00	1.83	1.44	3.09	0.00	1.61
time (sec)	N/A	0.105	0.066	0.366	0.259	0.360	1.972	0.000	1.153

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	213	233	0	0	0	-1
N.S.	1	1.00	0.67	1.73	1.89	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.244	0.444	0.271	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	103	735	0	0	0	0	-1
N.S.	1	1.00	0.64	4.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.099	64.104	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	93	1340	273	96	0	0	231
N.S.	1	1.00	0.99	14.26	2.90	1.02	0.00	0.00	2.46
time (sec)	N/A	0.078	0.055	51.866	0.263	0.392	0.000	0.000	1.764

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	43	153	146	66	0	140	198
N.S.	1	1.00	0.52	1.87	1.78	0.80	0.00	1.71	2.41
time (sec)	N/A	0.050	0.026	0.379	0.263	0.351	0.000	0.399	1.157

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	1340	268	95	0	88	213
N.S.	1	1.00	1.06	15.23	3.05	1.08	0.00	1.00	2.42
time (sec)	N/A	0.051	0.037	71.183	0.265	0.373	0.000	1.424	1.500

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	106	1290	0	0	0	0	-1
N.S.	1	1.00	0.78	9.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.096	116.125	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	97	4474	406	0	0	0	-1
N.S.	1	1.00	0.68	31.51	2.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.177	79.477	0.279	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	146	2444	0	0	0	0	-1
N.S.	1	1.00	0.71	11.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	0.622	402.640	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	139	806	0	0	0	0	-1
N.S.	1	1.00	0.61	3.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	0.098	107.560	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	72	1357	465	114	0	0	410
N.S.	1	1.00	0.60	11.21	3.84	0.94	0.00	0.00	3.39
time (sec)	N/A	0.098	0.033	144.892	0.297	0.359	0.000	0.000	1.708

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	91	1359	298	97	0	192	239
N.S.	1	1.00	0.76	11.42	2.50	0.82	0.00	1.61	2.01
time (sec)	N/A	0.087	0.033	166.974	0.265	0.454	0.000	0.408	1.757

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	71	1357	459	113	0	122	378
N.S.	1	1.00	0.62	11.80	3.99	0.98	0.00	1.06	3.29
time (sec)	N/A	0.072	0.024	123.619	0.282	0.358	0.000	1.523	1.718

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	135	1387	0	0	0	0	-1
N.S.	1	1.00	0.70	7.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.099	276.647	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	144	271	0	0	0	0	-1
N.S.	1	1.00	0.75	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.215	138.152	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	215	447	0	0	0	0	-1
N.S.	1	1.00	0.71	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.660	0.435	306.889	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.121	6.667	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	3.740	34.474	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	1.298	37.938	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	0	58	0	0	-1
N.S.	1	1.00	1.00	0.81	0.00	2.15	0.00	0.00	-0.04
time (sec)	N/A	0.077	0.050	5.576	0.000	0.481	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	38	0	0	-1
N.S.	1	1.00	1.00	0.93	0.00	2.71	0.00	0.00	-0.07
time (sec)	N/A	0.052	0.037	5.716	0.000	0.348	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	0	54	0	0	-1
N.S.	1	1.00	1.00	0.81	0.00	2.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.029	2.251	0.000	0.341	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.042	0.697	15.240	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.232	2.369	32.052	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	36	0	111	0	0	-1
N.S.	1	1.00	0.95	0.95	0.00	2.92	0.00	0.00	-0.03
time (sec)	N/A	0.098	0.059	6.844	0.000	0.349	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	28	0	106	0	0	-1
N.S.	1	1.00	0.89	0.78	0.00	2.94	0.00	0.00	-0.03
time (sec)	N/A	0.156	0.047	6.885	0.000	0.365	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	102	0	0	-1
N.S.	1	1.00	0.86	1.03	0.00	2.91	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.037	6.614	0.000	0.390	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.242	3.228	12.142	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	7.048	29.257	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	51	0	131	0	0	-1
N.S.	1	1.00	0.73	0.80	0.00	2.05	0.00	0.00	-0.02
time (sec)	N/A	0.194	0.064	6.861	0.000	0.440	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	43	0	135	0	0	-1
N.S.	1	1.00	0.92	0.60	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.036	6.965	0.000	0.397	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	51	0	122	0	0	-1
N.S.	1	1.00	1.00	0.88	0.00	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.174	0.044	6.822	0.000	0.384	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	2.572	20.354	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	68	0	151	0	0	-1
N.S.	1	1.00	0.75	0.70	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.101	6.911	0.000	0.414	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	84	83	0	171	0	0	-1
N.S.	1	1.00	0.70	0.69	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.068	6.872	0.000	0.361	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	101	98	0	200	0	0	-1
N.S.	1	1.00	0.66	0.64	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.054	6.876	0.000	0.338	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	112	113	0	220	0	0	-1
N.S.	1	1.00	0.63	0.64	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.054	6.898	0.000	0.362	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	128	128	0	249	0	0	-1
N.S.	1	1.00	0.61	0.61	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.056	5.888	0.000	0.369	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	110	99	71	158	239	83
N.S.	1	1.00	1.27	1.43	1.29	0.92	2.05	3.10	1.08
time (sec)	N/A	0.046	0.038	0.608	0.264	0.440	0.838	0.406	1.392

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	61	178	179	95	0	0	150
N.S.	1	1.00	0.61	1.78	1.79	0.95	0.00	0.00	1.50
time (sec)	N/A	0.053	0.039	0.783	0.269	0.365	0.000	0.000	1.195

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	75	82	71	158	239	105
N.S.	1	1.00	1.17	1.00	1.09	0.95	2.11	3.19	1.40
time (sec)	N/A	0.033	0.026	0.582	0.268	0.344	0.847	0.404	1.079

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	65	178	182	97	0	0	154
N.S.	1	1.00	0.69	1.89	1.94	1.03	0.00	0.00	1.64
time (sec)	N/A	0.033	0.030	0.771	0.264	0.341	0.000	0.000	1.368

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	81	234	268	0	0	0	-1
N.S.	1	1.00	0.63	1.81	2.08	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.124	0.916	0.272	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	94	208	204	161	549	0	183
N.S.	1	1.00	0.76	1.69	1.66	1.31	4.46	0.00	1.49
time (sec)	N/A	0.176	0.083	0.808	0.289	0.345	1.666	0.000	1.388

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	71	238	226	99	0	250	372
N.S.	1	1.00	0.56	1.87	1.78	0.78	0.00	1.97	2.93
time (sec)	N/A	0.131	0.039	1.582	0.267	0.358	0.000	0.401	1.529

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	121	2028	388	136	0	0	350
N.S.	1	1.00	0.74	12.44	2.38	0.83	0.00	0.00	2.15
time (sec)	N/A	0.185	0.051	237.513	0.286	0.422	0.000	0.000	1.971

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	71	197	206	99	0	251	319
N.S.	1	1.00	0.57	1.58	1.65	0.79	0.00	2.01	2.55
time (sec)	N/A	0.071	0.032	0.782	0.262	0.359	0.000	0.414	1.483

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	2028	392	137	0	0	358
N.S.	1	1.00	0.84	13.43	2.60	0.91	0.00	0.00	2.37
time (sec)	N/A	0.078	0.051	257.519	0.284	0.431	0.000	0.000	2.058

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	129	1392	0	0	0	0	-1
N.S.	1	1.00	0.66	7.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.235	424.507	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	127	4576	534	0	0	0	-1
N.S.	1	1.00	0.61	21.89	2.56	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.376	563.274	0.287	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	135	2088	437	140	0	341	414
N.S.	1	1.00	0.70	10.88	2.28	0.73	0.00	1.78	2.16
time (sec)	N/A	0.192	0.056	370.825	0.291	0.406	0.000	0.433	3.092

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	107	2059	657	161	0	0	831
N.S.	1	1.00	0.50	9.58	3.06	0.75	0.00	0.00	3.87
time (sec)	N/A	0.261	0.046	387.626	0.282	0.415	0.000	0.000	3.192

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	148	2047	422	140	0	342	414
N.S.	1	1.00	0.79	10.89	2.24	0.74	0.00	1.82	2.20
time (sec)	N/A	0.125	0.039	444.552	0.281	0.399	0.000	0.429	2.693

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	111	2059	663	166	0	0	736
N.S.	1	1.00	0.55	10.14	3.27	0.82	0.00	0.00	3.63
time (sec)	N/A	0.131	0.057	466.615	0.316	0.353	0.000	0.000	2.293

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	189	1533	0	0	0	0	-1
N.S.	1	1.00	0.68	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.151	3.244	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	218	351	0	0	0	0	-1
N.S.	1	1.00	0.78	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	0.427	17.687	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	152	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.296	9.583	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	6.948	4.997	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	14.593	5.284	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	118	0	0	-1
N.S.	1	1.00	0.76	0.76	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.059	1.635	0.000	0.380	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	102	0	0	-1
N.S.	1	1.00	0.83	0.83	0.00	3.52	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.075	1.962	0.000	0.414	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	0	88	0	0	-1
N.S.	1	1.00	0.81	0.81	0.00	3.26	0.00	0.00	-0.04
time (sec)	N/A	0.083	0.044	1.437	0.000	0.470	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	102	0	0	-1
N.S.	1	1.00	0.83	0.83	0.00	3.52	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.071	1.569	0.000	0.424	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	31	0	118	0	0	-1
N.S.	1	1.00	0.80	0.76	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.028	1.589	0.000	0.409	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.890	7.004	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.630	11.332	0.632	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	62	0	232	0	0	-1
N.S.	1	1.00	0.92	1.17	0.00	4.38	0.00	0.00	-0.02
time (sec)	N/A	0.129	0.096	3.660	0.000	0.343	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	76	80	54	0	231	0	0	-1
N.S.	1	1.38	1.45	0.98	0.00	4.20	0.00	0.00	-0.02
time (sec)	N/A	0.360	0.065	3.661	0.000	0.368	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	60	56	38	0	164	0	0	-1
N.S.	1	1.46	1.37	0.93	0.00	4.00	0.00	0.00	-0.02
time (sec)	N/A	0.208	0.126	3.665	0.000	0.352	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	75	54	0	225	0	0	-1
N.S.	1	1.00	1.42	1.02	0.00	4.25	0.00	0.00	-0.02
time (sec)	N/A	0.170	0.058	3.661	0.000	0.385	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	60	0	222	0	0	-1
N.S.	1	1.00	0.88	1.22	0.00	4.53	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.080	3.701	0.000	0.371	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.447	3.029	5.467	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	90	0	256	0	0	-1
N.S.	1	1.00	0.60	0.90	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.425	0.111	2.727	0.000	0.351	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	160	66	82	0	267	0	0	-1
N.S.	1	1.50	0.62	0.77	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.454	0.174	2.849	0.000	0.421	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	109	56	51	0	193	0	0	-1
N.S.	1	1.27	0.65	0.59	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.607	0.149	2.800	0.000	0.345	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	82	0	256	0	0	-1
N.S.	1	1.00	0.96	0.82	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.328	0.133	2.949	0.000	0.385	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	88	0	241	0	0	-1
N.S.	1	1.00	1.25	1.28	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.081	2.925	0.000	0.391	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.537	3.648	5.560	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	108	122	0	272	0	0	-1
N.S.	1	1.00	0.86	0.98	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.346	0.152	3.634	0.000	0.344	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	132	152	0	303	0	0	-1
N.S.	1	1.00	0.78	0.89	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.655	0.092	2.739	0.000	0.489	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	166	182	0	341	0	0	-1
N.S.	1	1.00	0.65	0.71	0.00	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.910	0.168	3.709	0.000	0.384	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	81	222	240	131	0	0	206
N.S.	1	1.00	0.60	1.66	1.79	0.98	0.00	0.00	1.54
time (sec)	N/A	0.050	0.066	0.717	0.266	0.363	0.000	0.000	1.302

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	157	2716	516	179	0	0	493
N.S.	1	1.00	0.73	12.69	2.41	0.84	0.00	0.00	2.30
time (sec)	N/A	0.117	0.083	10.109	0.287	0.390	0.000	0.000	2.490

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	143	2761	871	216	0	0	1041
N.S.	1	1.00	0.49	9.49	2.99	0.74	0.00	0.00	3.58
time (sec)	N/A	0.219	0.048	3.232	0.294	0.480	0.000	0.000	2.970

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	257	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.427	11.583	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	7.721	7.447	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	65.909	7.203	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	220	0	0	-1
N.S.	1	1.00	0.73	0.73	0.00	4.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.065	1.395	0.000	0.377	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	200	0	0	-1
N.S.	1	1.00	0.77	0.77	0.00	4.65	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.102	1.615	0.000	0.369	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	216	0	0	-1
N.S.	1	1.00	0.73	0.73	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.048	1.407	0.000	0.377	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	136	0	0	-1
N.S.	1	1.00	0.83	0.83	0.00	4.69	0.00	0.00	-0.03
time (sec)	N/A	0.087	0.079	1.388	0.000	0.420	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	40	0	216	0	0	-1
N.S.	1	1.00	1.00	0.73	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.069	1.398	0.000	0.391	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	33	0	200	0	0	-1
N.S.	1	1.00	1.00	0.77	0.00	4.65	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.108	1.560	0.000	0.379	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	216	0	0	-1
N.S.	1	1.00	0.73	0.73	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.062	1.510	0.000	0.349	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	0.987	7.383	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	2.366	8.184	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	78	0	418	0	0	-1
N.S.	1	1.00	0.84	1.16	0.00	6.24	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.128	3.872	0.000	0.376	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	86	0	413	0	0	-1
N.S.	1	1.00	0.85	1.30	0.00	6.26	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.139	3.642	0.000	0.409	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	73	121	0	447	0	0	-1
N.S.	1	1.00	0.64	1.06	0.00	3.92	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.395	3.707	0.000	0.363	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	131	0	435	0	0	-1
N.S.	1	1.00	0.93	1.47	0.00	4.89	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.099	2.720	0.000	0.358	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	79	120	163	91	0	0	-1
N.S.	1	1.00	0.57	0.86	1.17	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.067	2.714	0.473	0.364	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	160	175	0	0	0	0	-1
N.S.	1	1.00	0.81	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.332	1.783	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	99	88	72	0	0	-1
N.S.	1	1.00	0.69	1.14	1.01	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.048	1.563	0.469	0.355	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	125	154	0	0	0	0	-1
N.S.	1	1.00	0.86	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.156	1.555	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	81	30	58	0	47	-1
N.S.	1	1.00	0.91	2.53	0.94	1.81	0.00	1.47	-0.03
time (sec)	N/A	0.032	0.024	1.320	0.472	0.409	0.000	0.440	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	76	113	0	0	0	0	-1
N.S.	1	1.00	0.80	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.053	2.874	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	99	0	0	0	0	-1
N.S.	1	1.00	0.76	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.067	1.974	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	72	51	58	0	111	-1
N.S.	1	1.00	1.14	1.71	1.21	1.38	0.00	2.64	-0.02
time (sec)	N/A	0.052	0.037	1.648	0.465	0.386	0.000	0.448	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	141	0	0	0	0	-1
N.S.	1	1.00	0.92	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.485	1.773	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	160	175	0	0	0	0	-1
N.S.	1	1.00	0.78	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.301	0.688	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	188	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.528	0.970	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	151	0	0	0	0	-1
N.S.	1	1.00	0.87	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.160	0.648	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	119	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.074	1.328	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	100	158	0	0	0	0	-1
N.S.	1	1.00	1.47	2.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.064	0.648	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	131	0	0	0	0	-1
N.S.	1	1.00	0.85	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.352	0.665	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	188	231	0	0	0	0	-1
N.S.	1	1.00	1.24	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.824	0.694	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	215	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.624	0.846	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	570	0	0	0	0	0	-1
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.236	2.768	0.242	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	157	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.131	0.588	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	451	0	0	0	0	0	-1
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.272	0.631	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	146	215	0	0	0	0	-1
N.S.	1	1.00	1.43	2.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.097	0.683	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	131	190	0	0	0	0	-1
N.S.	1	1.00	1.34	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.263	0.680	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	301	386	0	0	0	0	-1
N.S.	1	1.00	1.13	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	6.806	0.730	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	2.379	4.181	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	76	144	96	94	0	0	-1
N.S.	1	1.00	1.03	1.95	1.30	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.045	1.594	0.457	0.389	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	190	0	0	0	0	-1
N.S.	1	1.00	0.88	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.200	1.111	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	27	66	39	51	0	61	-1
N.S.	1	1.00	0.63	1.53	0.91	1.19	0.00	1.42	-0.02
time (sec)	N/A	0.038	0.027	1.073	0.253	0.383	0.000	0.432	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	27	38	36	47	0	59	-1
N.S.	1	1.00	0.68	0.95	0.90	1.18	0.00	1.48	-0.02
time (sec)	N/A	0.022	0.023	1.099	0.263	0.352	0.000	0.429	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	97	157	0	0	0	0	-1
N.S.	1	1.00	0.87	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.147	1.716	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	89	132	84	107	0	155	-1
N.S.	1	1.00	1.09	1.61	1.02	1.30	0.00	1.89	-0.01
time (sec)	N/A	0.117	0.078	1.340	0.253	0.357	0.000	0.434	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	182	205	0	0	0	0	-1
N.S.	1	1.00	1.02	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	1.102	1.381	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.348	1.309	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	165	230	0	0	0	0	-1
N.S.	1	1.00	0.89	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.264	0.691	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	193	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.224	0.816	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	82	62	69	0	0	-1
N.S.	1	1.00	0.50	1.21	0.91	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.040	0.641	0.260	0.351	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	38	49	57	69	0	0	-1
N.S.	1	1.00	0.60	0.78	0.90	1.10	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.029	0.642	0.256	0.380	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	159	232	0	0	0	0	-1
N.S.	1	1.00	1.25	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.174	0.685	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	215	207	0	0	0	0	-1
N.S.	1	1.00	1.26	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.750	0.697	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	266	313	0	0	0	0	-1
N.S.	1	1.00	1.20	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	1.802	0.738	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.348	1.319	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	249	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.304	0.872	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	541	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.634	0.159	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	45	98	88	91	0	0	-1
N.S.	1	1.00	0.48	1.04	0.94	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.044	0.649	0.254	0.382	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	45	56	86	87	0	0	-1
N.S.	1	1.00	0.51	0.64	0.98	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.036	0.674	0.263	0.366	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	230	305	0	0	0	0	-1
N.S.	1	1.00	1.24	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.234	0.727	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	270	282	0	0	0	0	-1
N.S.	1	1.00	1.44	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	1.794	0.718	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	377	482	0	0	0	0	-1
N.S.	1	1.00	1.05	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	5.987	0.791	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.319	1.299	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.077	2.510	2.736	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	26	0	0	0	0	-1
N.S.	1	1.00	1.00	2.89	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.062	0.051	0.651	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.038	0.050	0.632	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	0.986	3.612	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.073	0.597	1.294	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.190	2.183	2.674	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	90	0	0	0	0	-1
N.S.	1	1.00	0.94	2.50	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	0.023	1.138	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0	-1
N.S.	1	1.00	0.91	1.77	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.062	1.253	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.400	4.260	2.674	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.598	1.286	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	4.649	2.642	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	154	0	0	0	0	-1
N.S.	1	1.00	0.63	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.087	1.145	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	86	0	0	0	0	-1
N.S.	1	1.00	0.68	1.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.115	0.100	1.233	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.327	11.320	2.721	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	178	195	0	0	0	0	-1
N.S.	1	1.00	0.73	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.478	1.621	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	79	120	128	91	0	0	-1
N.S.	1	1.00	0.58	0.88	0.94	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.047	1.373	0.460	0.410	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	160	175	0	0	0	0	-1
N.S.	1	1.00	0.82	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.321	1.378	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	99	50	72	0	0	-1
N.S.	1	1.00	0.83	1.68	0.85	1.22	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.030	1.362	0.462	0.389	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	152	0	0	0	0	-1
N.S.	1	1.00	0.82	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.179	4.122	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	113	0	0	0	0	-1
N.S.	1	1.00	0.91	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.089	1.509	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	121	188	0	0	0	0	-1
N.S.	1	1.00	0.93	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.261	1.387	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	126	141	0	0	0	0	-1
N.S.	1	1.00	0.93	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.873	1.483	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	96	90	75	0	0	-1
N.S.	1	1.00	1.13	1.37	1.29	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.049	1.793	0.468	0.361	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	222	164	0	0	0	0	-1
N.S.	1	1.00	1.16	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	1.149	1.814	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	104	116	204	93	0	0	-1
N.S.	1	1.00	0.69	0.77	1.36	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.088	1.839	0.474	0.418	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	307	183	0	0	0	0	-1
N.S.	1	1.00	1.26	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	2.314	1.924	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	268	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.967	1.520	1.095	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	175	211	0	0	0	0	-1
N.S.	1	1.00	0.62	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	0.508	0.697	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	228	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	0.768	0.728	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	135	175	0	0	0	0	-1
N.S.	1	1.00	0.77	1.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.278	0.700	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	187	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.507	3.174	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	203	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.194	0.986	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	223	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.520	0.955	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	188	231	0	0	0	0	-1
N.S.	1	1.00	1.25	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.397	0.825	0.701	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	171	0	0	0	0	-1
N.S.	1	1.00	1.05	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	1.459	0.703	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	272	215	0	0	0	0	-1
N.S.	1	1.00	0.93	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.557	0.949	1.638	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	79	140	163	106	0	0	-1
N.S.	1	1.00	0.42	0.75	0.88	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.406	0.062	1.370	0.462	0.364	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	224	195	0	0	0	0	-1
N.S.	1	1.00	0.92	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	0.669	1.376	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	61	120	67	90	0	0	-1
N.S.	1	1.00	0.75	1.48	0.83	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.046	1.348	0.460	0.360	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	173	0	0	0	0	-1
N.S.	1	1.00	0.93	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.416	4.204	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	143	132	0	0	0	0	-1
N.S.	1	1.00	0.99	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.158	1.513	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	168	205	0	0	0	0	-1
N.S.	1	1.00	0.94	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.430	1.428	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	158	145	0	0	0	0	-1
N.S.	1	1.00	0.94	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.671	1.469	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	199	220	0	0	0	0	-1
N.S.	1	1.00	1.05	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.869	1.423	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	282	164	0	0	0	0	-1
N.S.	1	1.00	1.48	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.395	2.899	1.413	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	116	126	93	0	0	-1
N.S.	1	1.00	1.11	1.23	1.34	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.058	1.436	0.473	0.400	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	474	184	0	0	0	0	-1
N.S.	1	1.00	1.95	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	4.456	1.569	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	224	193	0	0	0	0	-1
N.S.	1	1.00	0.96	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.092	0.838	5.234	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	173	0	0	0	0	-1
N.S.	1	1.00	0.93	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	152	0	0	0	0	-1
N.S.	1	1.00	0.82	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	59	74	73	0	90	-1
N.S.	1	1.00	0.55	0.66	0.83	0.82	0.00	1.01	-0.01
time (sec)	N/A	0.042	0.037	1.952	0.259	0.371	0.000	0.457	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	65	79	108	99	0	114	-1
N.S.	1	1.00	0.49	0.59	0.81	0.74	0.00	0.86	-0.01
time (sec)	N/A	0.062	0.046	1.809	0.266	0.386	0.000	0.439	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	81	99	140	121	0	138	-1
N.S.	1	1.00	0.46	0.56	0.79	0.68	0.00	0.78	-0.01
time (sec)	N/A	0.088	0.054	1.814	0.260	0.371	0.000	0.430	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	206	345	0	0	0	0	-1
N.S.	1	1.00	0.71	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.108	0.465	2.623	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	119	319	0	0	0	0	-1
N.S.	1	1.00	0.51	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.076	0.217	1.827	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	109	302	0	0	0	0	-1
N.S.	1	1.00	0.60	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.082	1.602	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	74	90	54	0	70	-1
N.S.	1	1.00	0.90	1.54	1.88	1.12	0.00	1.46	-0.02
time (sec)	N/A	0.023	0.039	1.598	0.465	0.363	0.000	0.453	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	160	90	84	0	111	-1
N.S.	1	1.00	0.61	1.52	0.86	0.80	0.00	1.06	-0.01
time (sec)	N/A	0.046	0.052	1.602	0.279	0.379	0.000	0.474	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	80	250	132	112	0	149	-1
N.S.	1	1.00	0.51	1.59	0.84	0.71	0.00	0.95	-0.01
time (sec)	N/A	0.076	0.059	1.629	0.276	0.428	0.000	0.466	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	187	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	70	84	304	105	0	0	-1
N.S.	1	1.00	0.50	0.60	2.19	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.056	0.700	0.517	0.358	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	94	118	514	139	0	0	-1
N.S.	1	1.00	0.45	0.57	2.47	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.106	0.064	0.710	0.524	0.365	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	120	152	751	169	0	0	-1
N.S.	1	1.00	0.43	0.55	2.71	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.078	0.701	0.541	0.389	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	569	0	0	0	0	0	-1
N.S.	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	2.457	1.081	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	87	105	0	134	0	0	-1
N.S.	1	1.00	0.46	0.55	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.064	0.675	0.000	0.418	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	119	153	0	176	0	0	-1
N.S.	1	1.00	0.41	0.53	0.00	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.079	0.707	0.000	0.369	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	151	201	0	214	0	0	-1
N.S.	1	1.00	0.39	0.52	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.094	0.726	0.000	0.392	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.837	8.501	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.115	0.654	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.039	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0	-1
N.S.	1	1.00	0.81	0.78	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.046	0.879	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	0	0	0	0	-1
N.S.	1	1.00	0.76	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.054	0.892	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	39	0	0	0	0	-1
N.S.	1	1.00	0.73	0.71	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.052	0.888	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.021	1.068	8.365	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.672	0.158	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0	-1
N.S.	1	1.00	0.91	1.77	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	120	0	0	0	0	-1
N.S.	1	1.00	0.87	2.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.100	2.559	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	176	0	0	0	0	-1
N.S.	1	1.00	0.85	2.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.127	0.123	2.971	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	65	232	0	0	0	0	-1
N.S.	1	1.00	0.81	2.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.156	2.938	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.021	1.203	5.941	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.765	0.158	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	86	0	0	0	0	-1
N.S.	1	1.00	0.68	1.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	180	0	0	0	0	-1
N.S.	1	1.00	0.71	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.151	3.418	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	272	0	0	0	0	-1
N.S.	1	1.00	0.85	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.168	2.857	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	99	364	0	0	0	0	-1
N.S.	1	1.00	0.93	3.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.296	4.012	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	193	1806	0	0	0	0	-1
N.S.	1	1.00	1.58	14.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.215	22.329	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	213	301	276	248	372	1471	288
N.S.	1	1.00	0.87	1.23	1.13	1.01	1.52	6.00	1.18
time (sec)	N/A	0.137	0.056	0.905	0.265	0.350	0.855	0.463	1.285

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	211	198	178	245	930	190
N.S.	1	1.00	0.89	1.25	1.17	1.05	1.45	5.50	1.12
time (sec)	N/A	0.092	0.046	0.569	0.265	0.359	0.521	0.424	1.378

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	137	131	119	155	527	118
N.S.	1	1.00	0.89	1.25	1.19	1.08	1.41	4.79	1.07
time (sec)	N/A	0.096	0.028	1.320	0.262	0.379	0.361	0.422	1.088

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	74	65	65	73	266	60
N.S.	1	1.00	1.21	1.30	1.14	1.14	1.28	4.67	1.05
time (sec)	N/A	0.047	0.012	0.145	0.252	0.360	0.563	0.425	0.942

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	662	1002	406	0	0	0	-1
N.S.	1	1.00	1.54	2.34	0.95	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.958	5.254	0.537	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	746	2227	550	0	0	0	-1
N.S.	1	1.00	1.26	3.77	0.93	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	5.025	6.229	0.525	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	1541	4047	1087	0	0	0	-1
N.S.	1	1.00	2.35	6.16	1.65	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	9.220	4.704	0.572	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	23	22	14	48	15
N.S.	1	1.00	1.00	1.12	1.35	1.29	0.82	2.82	0.88
time (sec)	N/A	0.029	0.045	1.080	0.259	0.337	0.244	0.413	0.984

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	576	221	180	0	0	0	-1
N.S.	1	1.00	3.37	1.29	1.05	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.649	6.539	0.261	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	203	278	198	0	0	0	-1
N.S.	1	1.00	1.00	1.37	0.98	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	0.029	12.847	0.261	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	260	106	119	0	0	0	-1
N.S.	1	1.00	3.02	1.23	1.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.058	1.250	0.255	0.000	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	485	679	304	0	0	0	-1
N.S.	1	1.00	1.22	1.71	0.77	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.730	3.602	0.521	0.000	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	874	937	0	0	0	0	-1
N.S.	1	1.00	3.39	3.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	14.173	4.693	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	4.274	5.486	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	2.792	3.208	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	153	356	0	71	-1
N.S.	1	1.00	1.92	0.00	2.47	5.74	0.00	1.15	-0.02
time (sec)	N/A	0.068	0.076	3.059	0.278	0.409	0.000	0.428	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	223	730	0	135	-1
N.S.	1	1.00	1.77	0.00	1.74	5.70	0.00	1.05	-0.01
time (sec)	N/A	0.233	0.214	3.197	0.479	0.382	0.000	0.430	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	401	1280	0	218	-1
N.S.	1	1.00	1.64	0.00	2.00	6.40	0.00	1.09	-0.00
time (sec)	N/A	0.720	0.500	3.155	0.482	0.523	0.000	0.435	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	639	2006	0	349	-1
N.S.	1	1.00	1.52	0.00	2.26	7.09	0.00	1.23	-0.00
time (sec)	N/A	0.896	0.987	3.302	0.485	0.562	0.000	0.436	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	97	229	0	0	0	0	-1
N.S.	1	1.00	0.52	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.217	3.998	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	90	210	0	0	0	0	-1
N.S.	1	1.00	0.62	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.075	1.837	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	63	42	0	54	-1
N.S.	1	1.00	0.81	1.41	1.70	1.14	0.00	1.46	-0.03
time (sec)	N/A	0.017	0.032	1.820	0.467	0.357	0.000	0.412	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	67	62	0	86	-1
N.S.	1	1.00	0.54	1.35	0.81	0.75	0.00	1.04	-0.01
time (sec)	N/A	0.037	0.035	1.813	0.265	0.346	0.000	0.441	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	99	82	0	118	-1
N.S.	1	1.00	0.44	1.42	0.80	0.66	0.00	0.95	-0.01
time (sec)	N/A	0.056	0.043	1.801	0.263	0.344	0.000	0.446	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4757	320	390	338	313	599
N.S.	1	1.00	0.75	15.10	1.02	1.24	1.07	0.99	1.90
time (sec)	N/A	0.533	0.089	17.533	0.272	0.364	2.392	0.556	5.593

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3739	274	306	279	0	851
N.S.	1	1.00	0.85	16.62	1.22	1.36	1.24	0.00	3.78
time (sec)	N/A	0.193	0.084	16.655	0.268	0.475	1.481	0.000	1.699

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3994	255	306	258	244	515
N.S.	1	1.00	0.74	16.17	1.03	1.24	1.04	0.99	2.09
time (sec)	N/A	0.433	0.075	11.618	0.278	0.369	0.986	0.539	2.805

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2951	174	215	202	209	557
N.S.	1	1.00	0.92	21.08	1.24	1.54	1.44	1.49	3.98
time (sec)	N/A	0.087	0.061	11.229	0.263	0.368	1.576	0.627	1.420

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2529	181	189	148	165	385
N.S.	1	1.00	1.38	24.32	1.74	1.82	1.42	1.59	3.70
time (sec)	N/A	0.137	0.018	4.360	0.270	0.439	0.665	0.464	1.794

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	0	1638	156	0	0	0	-1
N.S.	1	1.00	0.00	7.58	0.72	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.172	40.168	0.360	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	332	0	0	0	0	0	-1
N.S.	1	1.00	3.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.126	6.580	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	152	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.191	17.425	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	460	0	0	0	0	0	-1
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.338	16.757	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	299	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.086	66.382	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.227	24.642	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	1376	10161	0	0	0	0	-1
N.S.	1	1.00	2.69	19.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	4.464	7.209	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	599	1251	3574	0	0	0	0	-1
N.S.	1	1.00	2.09	5.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.563	2.672	23.526	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.142	2.031	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	613	1226	0	0	0	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	2.781	1.888	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	982	961	0	0	0	0	-1
N.S.	1	1.00	2.09	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	3.976	8.968	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	122	70	203	226	74	0	93	67
N.S.	1	1.56	0.90	2.60	2.90	0.95	0.00	1.19	0.86
time (sec)	N/A	0.220	0.042	1.605	0.308	0.367	0.000	0.407	1.155

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [497] had the largest ratio of [28]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	18	0.333
2	A	7	6	1.00	18	0.333
3	A	7	6	1.00	16	0.375
4	A	4	3	1.00	15	0.200
5	A	6	4	1.00	18	0.222
6	A	8	7	1.00	18	0.389
7	A	4	4	1.00	18	0.222
8	A	4	4	1.00	18	0.222
9	A	4	4	1.00	18	0.222
10	A	7	6	1.00	20	0.300
11	A	7	6	1.00	20	0.300
12	A	7	6	1.00	18	0.333
13	A	4	3	1.00	17	0.176
14	A	9	7	1.00	20	0.350
15	A	11	9	1.31	20	0.450
16	A	11	9	1.00	20	0.450
17	A	4	4	1.00	20	0.200
18	A	4	4	1.00	20	0.200
19	A	4	4	1.00	20	0.200
20	A	7	6	1.00	20	0.300
21	A	7	6	1.00	20	0.300
22	A	4	4	1.00	18	0.222
23	A	4	3	1.00	17	0.176
24	A	13	9	1.00	20	0.450
25	A	14	11	1.00	20	0.550

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	14	11	1.00	20	0.550
27	A	15	10	1.00	20	0.500
28	A	4	4	1.00	20	0.200
29	A	4	5	1.00	20	0.250
30	A	4	4	1.00	20	0.200
31	A	7	6	1.00	20	0.300
32	A	4	4	1.00	20	0.200
33	A	4	4	1.00	18	0.222
34	A	4	3	1.00	17	0.176
35	A	17	10	1.00	20	0.500
36	A	18	12	1.00	20	0.600
37	A	17	12	1.00	20	0.600
38	A	18	12	1.00	20	0.600
39	A	19	10	1.00	20	0.500
40	A	4	4	1.00	20	0.200
41	A	4	5	1.00	20	0.250
42	A	4	4	1.00	20	0.200
43	A	16	11	1.00	20	0.550
44	A	11	9	1.00	20	0.450
45	A	7	6	1.00	18	0.333
46	A	3	3	1.00	17	0.176
47	A	2	2	1.00	20	0.100
48	A	8	8	1.00	20	0.400
49	A	12	10	1.00	20	0.500
50	A	17	11	1.00	20	0.550
51	A	16	13	1.00	20	0.650
52	A	13	10	1.00	20	0.500
53	A	10	8	1.00	18	0.444
54	A	5	4	1.00	17	0.235
55	A	11	9	1.00	20	0.450
56	A	16	14	1.00	20	0.700
57	A	19	16	1.00	20	0.800
58	A	21	13	1.00	20	0.650
59	A	18	10	1.00	20	0.500
60	A	15	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	18	0.278
62	A	5	4	1.00	17	0.235
63	A	16	9	1.00	20	0.450
64	A	21	14	1.00	20	0.700
65	A	24	16	1.00	20	0.800
66	A	5	4	1.00	16	0.250
67	A	3	3	1.00	17	0.176
68	A	27	15	1.00	20	0.750
69	A	22	14	1.00	20	0.700
70	A	17	12	1.00	18	0.667
71	A	9	7	1.00	17	0.412
72	A	13	11	1.00	20	0.550
73	A	12	10	1.00	20	0.500
74	A	14	11	1.00	20	0.550
75	A	18	13	1.00	20	0.650
76	A	43	15	1.00	22	0.682
77	A	36	15	1.00	22	0.682
78	A	28	14	1.00	20	0.700
79	A	12	10	1.00	19	0.526
80	A	19	14	1.00	22	0.636
81	A	17	15	1.00	22	0.682
82	A	20	15	1.00	22	0.682
83	A	14	13	1.00	22	0.591
84	A	62	15	1.00	22	0.682
85	A	52	15	1.00	22	0.682
86	A	38	14	1.00	20	0.700
87	A	16	12	1.00	19	0.632
88	A	28	16	1.00	22	0.727
89	A	23	17	1.00	22	0.773
90	A	25	20	1.00	22	0.909
91	A	28	17	1.00	22	0.773
92	A	18	14	1.00	22	0.636
93	A	22	15	1.00	22	0.682
94	A	29	14	1.00	22	0.636
95	A	26	14	1.00	22	0.636

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	16	12	1.00	22	0.546
97	A	9	9	1.00	20	0.450
98	A	3	4	1.00	19	0.210
99	A	3	4	1.00	22	0.182
100	A	8	8	1.00	22	0.364
101	A	17	13	1.00	22	0.591
102	A	26	15	1.00	22	0.682
103	A	33	19	1.00	22	0.864
104	A	24	17	1.00	22	0.773
105	A	18	14	1.00	22	0.636
106	A	13	10	1.00	20	0.500
107	A	8	6	1.00	19	0.316
108	A	19	13	1.00	22	0.591
109	A	23	17	1.00	22	0.773
110	A	31	22	1.00	22	1.000
111	A	37	17	1.00	22	0.773
112	A	31	14	1.00	22	0.636
113	A	26	10	1.00	22	0.454
114	A	13	7	1.00	20	0.350
115	A	13	6	1.00	19	0.316
116	A	32	13	1.00	22	0.591
117	A	36	17	1.00	22	0.773
118	A	18	6	1.00	18	0.333
119	A	4	5	1.00	20	0.250
120	A	26	15	1.00	18	0.833
121	A	17	13	1.00	18	0.722
122	A	11	10	1.00	16	0.625
123	A	4	5	1.00	18	0.278
124	A	11	6	1.00	18	0.333
125	A	24	6	1.00	18	0.333
126	A	42	6	1.00	18	0.333
127	A	19	13	1.00	18	0.722
128	A	10	9	1.00	16	0.562
129	A	4	5	1.00	15	0.333
130	A	4	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	6	1.00	19	0.316
132	A	10	8	1.00	18	0.444
133	A	18	10	1.00	18	0.556
134	A	21	10	1.00	19	0.526
135	A	12	8	1.00	17	0.471
136	A	5	5	1.00	16	0.312
137	A	5	5	1.00	19	0.263
138	A	6	6	1.00	20	0.300
139	A	12	10	1.00	19	0.526
140	A	21	11	1.00	19	0.579
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	16	12	1.00	19	0.632
148	A	12	10	1.00	19	0.526
149	A	9	7	1.00	17	0.412
150	A	4	4	1.00	16	0.250
151	A	7	6	1.00	19	0.316
152	A	12	11	1.00	19	0.579
153	A	15	13	1.00	19	0.684
154	A	14	11	1.00	21	0.524
155	A	8	7	1.00	19	0.368
156	A	1	1	1.00	18	0.056
157	A	9	7	1.00	21	0.333
158	A	13	11	1.00	21	0.524
159	A	9	7	1.00	17	0.412
160	A	0	0	0.00	0	0.000
161	A	9	4	1.00	18	0.222
162	A	9	4	1.00	18	0.222
163	A	9	4	1.00	18	0.222
164	A	2	1	1.00	16	0.062
165	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	5	5	1.00	18	0.278
167	A	8	8	1.00	18	0.444
168	A	5	5	1.00	18	0.278
169	A	10	7	1.00	18	0.389
170	A	3	2	1.00	18	0.111
171	A	9	4	1.00	18	0.222
172	A	34	10	1.00	20	0.500
173	A	26	8	1.00	20	0.400
174	A	24	10	1.00	20	0.500
175	A	4	4	1.00	18	0.222
176	A	7	7	1.00	17	0.412
177	A	12	10	1.00	20	0.500
178	A	10	10	1.00	20	0.500
179	A	15	12	1.00	20	0.600
180	A	13	8	1.00	20	0.400
181	A	11	8	1.00	20	0.400
182	A	22	8	1.00	20	0.400
183	A	8	8	1.00	17	0.471
184	A	10	5	1.00	19	0.263
185	A	0	0	0.00	0	0.000
186	A	0	0	0.00	0	0.000
187	A	0	0	0.00	0	0.000
188	A	0	0	0.00	0	0.000
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000
191	A	0	0	0.00	0	0.000
192	A	14	4	1.00	20	0.200
193	A	14	4	1.00	20	0.200
194	A	14	4	1.00	20	0.200
195	A	3	2	1.00	18	0.111
196	A	4	3	1.00	17	0.176
197	A	10	6	1.00	20	0.300
198	A	13	9	1.00	20	0.450
199	A	9	6	1.00	20	0.300
200	A	13	9	1.00	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	10	5	1.00	20	0.250
202	A	15	7	1.00	20	0.350
203	A	59	10	1.00	22	0.454
204	A	47	8	1.00	22	0.364
205	A	44	10	1.00	22	0.454
206	A	5	4	1.00	20	0.200
207	A	9	7	1.00	19	0.368
208	A	23	12	1.00	22	0.546
209	A	20	13	1.00	22	0.591
210	A	21	15	1.00	22	0.682
211	A	19	13	1.00	22	0.591
212	A	29	13	1.00	22	0.591
213	A	27	8	1.00	22	0.364
214	A	16	8	1.00	22	0.364
215	A	42	8	1.00	22	0.364
216	A	56	9	1.00	22	0.409
217	A	12	9	1.00	19	0.474
218	A	0	0	0.00	0	0.000
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000
224	A	5	3	1.00	17	0.176
225	A	12	8	1.00	19	0.421
226	A	17	9	1.00	19	0.474
227	A	8	8	1.00	20	0.400
228	A	4	4	1.00	20	0.200
229	A	4	4	1.00	18	0.222
230	A	1	1	1.00	17	0.059
231	A	3	3	1.00	20	0.150
232	A	7	7	1.00	20	0.350
233	A	7	7	1.00	20	0.350
234	A	10	9	1.00	22	0.409
235	A	7	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	5	1.00	20	0.250
237	A	1	1	1.00	19	0.053
238	A	4	5	1.00	22	0.227
239	A	6	6	1.00	22	0.273
240	A	13	11	1.00	22	0.500
241	A	14	11	1.00	22	0.500
242	A	7	7	1.00	22	0.318
243	A	5	6	1.00	20	0.300
244	A	1	1	1.00	19	0.053
245	A	5	6	1.00	22	0.273
246	A	7	7	1.00	22	0.318
247	A	13	9	1.00	22	0.409
248	A	1	1	1.00	21	0.048
249	A	0	0	0.00	0	0.000
250	A	1	1	1.00	19	0.053
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	1	1	1.00	19	0.053
254	A	0	0	0.00	0	0.000
255	A	0	0	0.00	0	0.000
256	A	1	1	1.00	19	0.053
257	A	0	0	0.00	0	0.000
258	A	1	1	1.00	19	0.053
259	A	8	8	1.00	20	0.400
260	A	2	2	1.00	20	0.100
261	A	3	3	1.00	18	0.167
262	A	2	2	1.00	17	0.118
263	A	7	7	1.00	20	0.350
264	A	10	10	1.00	20	0.500
265	A	15	10	1.00	20	0.500
266	A	8	9	1.00	22	0.409
267	A	4	4	1.00	22	0.182
268	A	3	3	1.00	20	0.150
269	A	4	4	1.00	19	0.210
270	A	8	9	1.00	22	0.409

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	11	11	1.00	22	0.500
272	A	22	15	1.00	22	0.682
273	A	11	11	1.00	22	0.500
274	A	4	4	1.00	22	0.182
275	A	5	4	1.00	20	0.200
276	A	4	3	1.00	19	0.158
277	A	11	11	1.00	22	0.500
278	A	12	11	1.00	22	0.500
279	A	25	14	1.00	22	0.636
280	A	9	8	1.00	21	0.381
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	4	3	1.00	22	0.136
284	A	4	4	1.00	20	0.200
285	A	4	3	1.00	19	0.158
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	5	5	1.00	22	0.227
289	A	9	5	1.00	20	0.250
290	A	5	5	1.00	19	0.263
291	A	0	0	0.00	0	0.000
292	A	0	0	0.00	0	0.000
293	A	10	6	1.00	22	0.273
294	A	5	5	1.00	20	0.250
295	A	10	6	1.00	19	0.316
296	A	0	0	0.00	0	0.000
297	A	6	6	1.00	19	0.316
298	A	11	7	1.00	19	0.368
299	A	7	6	1.00	19	0.316
300	A	12	7	1.00	19	0.368
301	A	8	6	1.00	19	0.316
302	A	4	3	1.00	20	0.150
303	A	3	3	1.00	20	0.150
304	A	4	3	1.00	18	0.167
305	A	3	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	12	7	1.00	20	0.350
307	A	14	11	1.00	20	0.550
308	A	4	4	1.00	22	0.182
309	A	13	6	1.00	22	0.273
310	A	4	4	1.00	20	0.200
311	A	8	5	1.00	19	0.263
312	A	13	10	1.00	22	0.454
313	A	20	12	1.00	22	0.546
314	A	9	7	1.00	22	0.318
315	A	13	6	1.00	22	0.273
316	A	9	5	1.00	20	0.250
317	A	8	5	1.00	19	0.263
318	A	21	12	1.00	22	0.546
319	A	21	13	1.00	22	0.591
320	A	15	7	1.00	21	0.333
321	A	0	0	0.00	0	0.000
322	A	0	0	0.00	0	0.000
323	A	5	3	1.00	22	0.136
324	A	5	3	1.00	22	0.136
325	A	4	3	1.00	22	0.136
326	A	5	3	1.00	20	0.150
327	A	5	3	1.00	19	0.158
328	A	0	0	0.00	0	0.000
329	A	0	0	0.00	0	0.000
330	A	6	4	1.00	22	0.182
331	A	20	7	1.38	22	0.318
332	A	12	6	1.46	22	0.273
333	A	10	6	1.00	20	0.300
334	A	6	4	1.00	19	0.210
335	A	0	0	0.00	0	0.000
336	A	21	8	1.00	22	0.364
337	A	25	8	1.50	22	0.364
338	A	22	8	1.27	22	0.364
339	A	19	7	1.00	20	0.350
340	A	11	7	1.00	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
341	A	0	0	0.00	0	0.000
342	A	20	7	1.00	19	0.368
343	A	35	8	1.00	19	0.421
344	A	49	8	1.00	19	0.421
345	A	4	3	1.00	17	0.176
346	A	13	5	1.00	19	0.263
347	A	13	5	1.00	19	0.263
348	A	21	7	1.00	21	0.333
349	A	0	0	0.00	0	0.000
350	A	0	0	0.00	0	0.000
351	A	6	3	1.00	22	0.136
352	A	6	3	1.00	22	0.136
353	A	6	3	1.00	22	0.136
354	A	5	3	1.00	22	0.136
355	A	6	3	1.00	22	0.136
356	A	6	3	1.00	20	0.150
357	A	6	3	1.00	19	0.158
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000
360	A	13	6	1.00	20	0.300
361	A	7	4	1.00	19	0.210
362	A	22	6	1.00	20	0.300
363	A	14	7	1.00	19	0.368
364	A	9	4	1.00	22	0.182
365	A	7	5	1.00	22	0.227
366	A	5	4	1.00	22	0.182
367	A	3	3	1.00	22	0.136
368	A	2	2	1.00	20	0.100
369	A	1	1	1.00	19	0.053
370	A	1	1	1.00	22	0.045
371	A	4	4	1.00	22	0.182
372	A	3	3	1.00	22	0.136
373	A	6	4	1.00	24	0.167
374	A	11	8	1.00	24	0.333
375	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	8	5	1.00	21	0.238
377	A	8	5	1.00	24	0.208
378	A	2	2	1.00	24	0.083
379	A	13	10	1.00	24	0.417
380	A	21	8	1.00	24	0.333
381	A	13	9	1.00	24	0.375
382	A	9	6	1.00	22	0.273
383	A	10	6	1.00	21	0.286
384	A	10	6	1.00	24	0.250
385	A	9	6	1.00	24	0.250
386	A	13	9	1.00	24	0.375
387	A	0	0	0.00	0	0.000
388	A	5	4	1.00	22	0.182
389	A	2	2	1.00	22	0.091
390	A	2	2	1.00	20	0.100
391	A	1	1	1.00	19	0.053
392	A	4	4	1.00	22	0.182
393	A	6	6	1.00	22	0.273
394	A	8	6	1.00	22	0.273
395	A	0	0	0.00	0	0.000
396	A	5	4	1.00	24	0.167
397	A	11	8	1.00	24	0.333
398	A	2	2	1.00	22	0.091
399	A	2	2	1.00	21	0.095
400	A	11	8	1.00	24	0.333
401	A	5	5	1.00	24	0.208
402	A	25	13	1.00	24	0.542
403	A	0	0	0.00	0	0.000
404	A	13	9	1.00	24	0.375
405	A	13	9	1.00	24	0.375
406	A	3	3	1.00	22	0.136
407	A	2	2	1.00	21	0.095
408	A	14	10	1.00	24	0.417
409	A	12	9	1.00	24	0.375
410	A	28	13	1.00	24	0.542

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	0	0	0.00	0	0.000
412	A	0	0	0.00	0	0.000
413	A	2	2	1.00	22	0.091
414	A	2	2	1.00	21	0.095
415	A	0	0	0.00	0	0.000
416	A	0	0	0.00	0	0.000
417	A	0	0	0.00	0	0.000
418	A	3	3	1.00	22	0.136
419	A	3	3	1.00	21	0.143
420	A	0	0	0.00	0	0.000
421	A	0	0	0.00	0	0.000
422	A	0	0	0.00	0	0.000
423	A	4	4	1.00	22	0.182
424	A	4	4	1.00	21	0.190
425	A	0	0	0.00	0	0.000
426	A	11	6	1.00	22	0.273
427	A	9	5	1.00	22	0.227
428	A	7	6	1.00	22	0.273
429	A	3	3	1.00	20	0.150
430	A	2	2	1.00	19	0.105
431	A	3	3	1.00	22	0.136
432	A	6	6	1.00	22	0.273
433	A	5	4	1.00	22	0.182
434	A	5	5	1.00	22	0.227
435	A	9	5	1.00	22	0.227
436	A	21	7	1.00	22	0.318
437	A	14	5	1.00	22	0.227
438	A	45	10	1.00	24	0.417
439	A	21	7	1.00	24	0.292
440	A	29	10	1.00	24	0.417
441	A	3	3	1.00	22	0.136
442	A	10	7	1.00	21	0.333
443	A	11	8	1.00	24	0.333
444	A	11	8	1.00	24	0.333
445	A	22	11	1.00	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	6	5	1.00	24	0.208
447	A	27	7	1.00	22	0.318
448	A	24	6	1.00	22	0.273
449	A	19	7	1.00	22	0.318
450	A	4	3	1.00	20	0.150
451	A	3	2	1.00	19	0.105
452	A	7	6	1.00	22	0.273
453	A	9	7	1.00	22	0.318
454	A	9	6	1.00	22	0.273
455	A	12	7	1.00	22	0.318
456	A	15	6	1.00	22	0.273
457	A	6	5	1.00	22	0.227
458	A	24	6	1.00	22	0.273
459	A	4	2	1.00	19	0.105
460	A	3	2	1.00	19	0.105
461	A	2	2	1.00	19	0.105
462	A	2	2	1.00	19	0.105
463	A	3	2	1.00	19	0.105
464	A	4	2	1.00	19	0.105
465	A	4	3	1.00	20	0.150
466	A	3	3	1.00	20	0.150
467	A	2	2	1.00	20	0.100
468	A	1	1	1.00	20	0.050
469	A	2	2	1.00	20	0.100
470	A	3	2	1.00	20	0.100
471	A	10	7	1.00	21	0.333
472	A	5	4	1.00	21	0.190
473	A	9	4	1.00	21	0.190
474	A	14	4	1.00	21	0.190
475	A	12	8	1.00	21	0.381
476	A	5	4	1.00	21	0.190
477	A	9	4	1.00	21	0.190
478	A	14	4	1.00	21	0.190
479	A	0	0	0.00	0	0.000
480	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	2	2	1.00	21	0.095
482	A	5	3	1.00	21	0.143
483	A	6	3	1.00	21	0.143
484	A	7	3	1.00	21	0.143
485	A	0	0	0.00	0	0.000
486	A	0	0	0.00	0	0.000
487	A	3	3	1.00	21	0.143
488	A	6	4	1.00	21	0.190
489	A	7	4	1.00	21	0.190
490	A	8	4	1.00	21	0.190
491	A	0	0	0.00	0	0.000
492	A	0	0	0.00	0	0.000
493	A	4	4	1.00	21	0.190
494	A	12	7	1.00	21	0.333
495	A	14	7	1.00	21	0.333
496	A	16	7	1.00	21	0.333
497	A	7	6	1.00	28	0.214
498	A	4	4	1.00	14	0.286
499	A	4	4	1.00	14	0.286
500	A	5	5	1.00	14	0.357
501	A	5	4	1.00	12	0.333
502	A	17	5	1.00	14	0.357
503	A	25	13	1.00	14	0.929
504	A	23	11	1.00	14	0.786
505	A	1	1	1.00	20	0.050
506	A	17	5	1.00	14	0.357
507	A	17	5	1.00	16	0.312
508	A	4	4	1.00	10	0.400
509	A	17	5	1.00	12	0.417
510	A	10	7	1.00	15	0.467
511	A	0	0	0.00	0	0.000
512	A	0	0	0.00	0	0.000
513	A	5	6	1.00	16	0.375
514	A	7	9	1.00	16	0.562
515	A	8	9	1.00	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	8	9	1.00	16	0.562
517	A	3	3	1.00	15	0.200
518	A	2	2	1.00	15	0.133
519	A	1	1	1.00	15	0.067
520	A	2	2	1.00	15	0.133
521	A	3	2	1.00	15	0.133
522	A	26	15	1.00	27	0.556
523	A	14	11	1.00	27	0.407
524	A	21	15	1.00	27	0.556
525	A	7	8	1.00	25	0.320
526	A	9	8	1.00	24	0.333
527	A	14	9	1.00	27	0.333
528	A	6	6	1.00	27	0.222
529	A	5	6	1.00	27	0.222
530	A	15	14	1.00	27	0.518
531	A	10	7	1.00	27	0.259
532	A	24	16	1.00	27	0.593
533	A	22	17	1.00	22	0.773
534	A	28	12	1.00	21	0.571
535	A	0	0	0.00	0	0.000
536	A	28	14	1.00	24	0.583
537	A	20	17	1.00	24	0.708
538	A	16	7	1.56	20	0.350

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=108

$$\frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)+\frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(1+}{40c^4}$$

```
[Out] 1/4*b*d*x/c^3+1/10*b*d*x^2/c^2+1/12*b*d*x^3/c+1/20*b*d*x^4+1/4*d*x^4*(a+b*a
rctanh(c*x))+1/5*c*d*x^5*(a+b*arctanh(c*x))+9/40*b*d*ln(-c*x+1)/c^4-1/40*b*
d*ln(c*x+1)/c^4
```

Rubi [A]

time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {45, 6083, 12, 815, 647, 31}

$$\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(cx+1)}{40c^4}+\frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^3) + (b*d*x^2)/(10*c^2) + (b*d*x^3)/(12*c) + (b*d*x^4)/20 + (d
*x^4*(a + b*ArcTanh[c*x]))/4 + (c*d*x^5*(a + b*ArcTanh[c*x]))/5 + (9*b*d*Lo
g[1 - c*x])/(40*c^4) - (b*d*Log[1 + c*x])/(40*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^4(}{20\,(1}\\ &= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\\ &= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\\ &= \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\\ &= \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\\ &= \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\end{aligned}$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 0.90

$$\frac{d\left(30bcx+12bc^2x^2+10bc^3x^3+30ac^4x^4+6bc^4x^4+24ac^5x^5+6bc^4x^4(5+4cx)\tanh^{-1}(cx)+27b\log(1-cx)-3b\log(1+cx)\right)}{120c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(30*b*c*x + 12*b*c^2*x^2 + 10*b*c^3*x^3 + 30*a*c^4*x^4 + 6*b*c^4*x^4 + 2
4*a*c^5*x^5 + 6*b*c^4*x^4*(5 + 4*c*x)*ArcTanh[c*x] + 27*b*Log[1 - c*x] - 3*
b*Log[1 + c*x]))/(120*c^4)
```

Maple [A]
time = 0.12, size = 110, normalized size = 1.02

method	result
derivativedivides	$\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{db\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{db\,\mathrm{arctanh}(cx)c^4x^4}{4}+\frac{db\,c^4x^4}{20}+\frac{db\,c^3x^3}{12}+\frac{db\,c^2x^2}{10}+\frac{dbcx}{4}+\frac{9db\ln(cx-1)}{40}-\frac{db\ln(cx+1)}{40}}{c^4}$
default	$\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{db\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{db\,\mathrm{arctanh}(cx)c^4x^4}{4}+\frac{db\,c^4x^4}{20}+\frac{db\,c^3x^3}{12}+\frac{db\,c^2x^2}{10}+\frac{dbcx}{4}+\frac{9db\ln(cx-1)}{40}-\frac{db\ln(cx+1)}{40}}{c^4}$
risch	$\frac{db\,x^4(4cx+5)\ln(cx+1)}{40}-\frac{dc\,x^5b\ln(-cx+1)}{10}+\frac{dc\,x^5a}{5}-\frac{d\,x^4b\ln(-cx+1)}{8}+\frac{d\,x^4a}{4}+\frac{bd\,x^4}{20}+\frac{bd\,x^3}{12c}+\frac{bd\,x^2}{10c^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d*a*(1/5*c^5*x^5+1/4*c^4*x^4)+1/5*d*b*arctanh(c*x)*c^5*x^5+1/4*d*b*a
rctanh(c*x)*c^4*x^4+1/20*d*b*c^4*x^4+1/12*d*b*c^3*x^3+1/10*d*b*c^2*x^2+1/4*
d*b*c*x+9/40*d*b*ln(c*x-1)-1/40*d*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 121, normalized size = 1.12

$$\frac{1}{5}\,acdx^5+\frac{1}{4}\,adx^4+\frac{1}{20}\left(4\,x^5\,\mathrm{artanh}\left(cx\right)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bcd+\frac{1}{24}\left(6\,x^4\,\mathrm{artanh}\left(cx\right)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^
2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d + 1/24*(6*x^4*arctanh(c*x) + c*(2*(
c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d
```

Fricas [A]
time = 0.36, size = 114, normalized size = 1.06

$$\frac{24\,ac^5dx^5+6\,(5\,a+b)c^4dx^4+10\,bc^3dx^3+12\,bc^2dx^2+30\,bcdx-3\,bd\log\left(cx+1\right)+27\,bd\log\left(cx-1\right)+3\,(4\,bc^5dx^5+5\,bc^4dx^4)\log\left(-\frac{cx+1}{cx-1}\right)}{120\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*d*x^5 + 6*(5*a + b)*c^4*d*x^4 + 10*b*c^3*d*x^3 + 12*b*c^2*d
*x^2 + 30*b*c*d*x - 3*b*d*log(c*x + 1) + 27*b*d*log(c*x - 1) + 3*(4*b*c^5*d
*x^5 + 5*b*c^4*d*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A]
time = 0.37, size = 124, normalized size = 1.15

$$\begin{cases}\frac{acdx^5}{5}+\frac{adx^4}{4}+\frac{bcdx^5\,\mathrm{atanh}\left(cx\right)}{5}+\frac{bdx^4\,\mathrm{atanh}\left(cx\right)}{4}+\frac{bdx^4}{20}+\frac{bdx^3}{12c}+\frac{bdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{bd\log\left(x-\frac{1}{c}\right)}{5c^4}-\frac{bd\,\mathrm{atanh}\left(cx\right)}{20c^4} & \text{for } c\neq 0\\ \frac{adx^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**5/5 + a*d*x**4/4 + b*c*d*x**5*atanh(c*x)/5 + b*d*x**4*a
tanh(c*x)/4 + b*d*x**4/20 + b*d*x**3/(12*c) + b*d*x**2/(10*c**2) + b*d*x/(4
*c**3) + b*d*log(x - 1/c)/(5*c**4) - b*d*atanh(c*x)/(20*c**4), Ne(c, 0)), (
a*d*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(92) = 184$.
time = 0.41, size = 491, normalized size = 4.55

$$\frac{1}{15}\,c\left(\frac{3\left(\frac{10\,(cx+1)^4bd}{(cx-1)^4}-\frac{5\,(cx+1)^3bd}{(cx-1)^3}+\frac{15\,(cx+1)^2bd}{(cx-1)^2}-\frac{5\,(cx+1)bd}{cx-1}+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^5}{(cx-1)^5}-\frac{5\,(cx+1)^4c^5}{(cx-1)^4}+\frac{10\,(cx+1)^3c^5}{(cx-1)^3}-\frac{10\,(cx+1)^2c^5}{(cx-1)^2}+\frac{5\,(cx+1)c^5}{cx-1}-c^5}+\frac{\frac{60\,(cx+1)^4ad}{(cx-1)^4}-\frac{30\,(cx+1)^3ad}{(cx-1)^3}+\frac{90\,(cx+1)^2ad}{(cx-1)^2}-\frac{30\,(cx+1)ad}{cx-1}+6\,ad+\frac{27\,(cx+1)^4bd}{(cx-1)^4}-\frac{69\,(cx+1)^3bd}{(cx-1)^3}+\frac{79\,(cx+1)^2bd}{(cx-1)^2}-\frac{47\,(cx+1)bd}{cx-1}+10\,bd}{\frac{(cx+1)^5c^5}{(cx-1)^5}-\frac{5\,(cx+1)^4c^5}{(cx-1)^4}+\frac{10\,(cx+1)^3c^5}{(cx-1)^3}-\frac{10\,(cx+1)^2c^5}{(cx-1)^2}+\frac{5\,(cx+1)c^5}{cx-1}-c^5}-\frac{3\,bd\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^5}+\frac{3\,bd\log\left(-\frac{cx+1}{cx-1}\right)}{c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/15*c*(3*(10*(c*x + 1)^4*b*d/(c*x - 1)^4 - 5*(c*x + 1)^3*b*d/(c*x - 1)^3 +
 15*(c*x + 1)^2*b*d/(c*x - 1)^2 - 5*(c*x + 1)*b*d/(c*x - 1) + b*d)*log(-(c*
x + 1)/(c*x - 1))/((c*x + 1)^5*c^5/(c*x - 1)^5 - 5*(c*x + 1)^4*c^5/(c*x - 1
)^4 + 10*(c*x + 1)^3*c^5/(c*x - 1)^3 - 10*(c*x + 1)^2*c^5/(c*x - 1)^2 + 5*(
c*x + 1)*c^5/(c*x - 1) - c^5) + (60*(c*x + 1)^4*a*d/(c*x - 1)^4 - 30*(c*x +
 1)^3*a*d/(c*x - 1)^3 + 90*(c*x + 1)^2*a*d/(c*x - 1)^2 - 30*(c*x + 1)*a*d/(
c*x - 1) + 6*a*d + 27*(c*x + 1)^4*b*d/(c*x - 1)^4 - 69*(c*x + 1)^3*b*d/(c*x
 - 1)^3 + 79*(c*x + 1)^2*b*d/(c*x - 1)^2 - 47*(c*x + 1)*b*d/(c*x - 1) + 10*
b*d)/((c*x + 1)^5*c^5/(c*x - 1)^5 - 5*(c*x + 1)^4*c^5/(c*x - 1)^4 + 10*(c*x
 + 1)^3*c^5/(c*x - 1)^3 - 10*(c*x + 1)^2*c^5/(c*x - 1)^2 + 5*(c*x + 1)*c^5/
(c*x - 1) - c^5) - 3*b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 3*b*d*log(-(c*
x + 1)/(c*x - 1))/c^5)
```

Mupad [B]
time = 0.99, size = 103, normalized size = 0.95

$$\frac{\frac{b\,c\,d\,x}{4}-\frac{d\,\left(15\,b\,\mathrm{atanh}(c\,x)-6\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60}+\frac{b\,c^2\,d\,x^2}{10}+\frac{b\,c^3\,d\,x^3}{12}}{c^4}+\frac{d\,\left(15\,a\,x^4+3\,b\,x^4+15\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c\,d\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] ((b*c*d*x)/4 - (d*(15*b*atanh(c*x) - 6*b*log(c^2*x^2 - 1)))/60 + (b*c^2*d*x
^2)/10 + (b*c^3*d*x^3)/12)/c^4 + (d*(15*a*x^4 + 3*b*x^4 + 15*b*x^4*atanh(c*
x)))/60 + (c*d*(12*a*x^5 + 12*b*x^5*atanh(c*x)))/60
```

3.2 $\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=96

$$\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(1+cx)}{24c^3}$$

```
[Out] 1/4*b*d*x/c^2+1/6*b*d*x^2/c+1/12*b*d*x^3+1/3*d*x^3*(a+b*arctanh(c*x))+1/4*c
*d*x^4*(a+b*arctanh(c*x))+7/24*b*d*ln(-c*x+1)/c^3+1/24*b*d*ln(c*x+1)/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {45, 6083, 12, 815, 647, 31}

$$\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(cx+1)}{24c^3}+\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^2) + (b*d*x^2)/(6*c) + (b*d*x^3)/12 + (d*x^3*(a + b*ArcTanh[c*
x]))/3 + (c*d*x^4*(a + b*ArcTanh[c*x]))/4 + (7*b*d*Log[1 - c*x])/(24*c^3) +
 (b*d*Log[1 + c*x])/(24*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^3(}{12(1}\\ &= \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\\ &= \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\\ &= \frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\,\mathrm{ta}\right.\\ &= \frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\,\mathrm{ta}\right.\\ &= \frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\,\mathrm{ta}\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 87, normalized size = 0.91

$$\frac{d\left(6bcx+4bc^2x^2+8ac^3x^3+2bc^3x^3+6ac^4x^4+2bc^3x^3(4+3cx)\tanh^{-1}(cx)+7b\log(1-cx)+b\log(1+cx)\right)}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(6*b*c*x + 4*b*c^2*x^2 + 8*a*c^3*x^3 + 2*b*c^3*x^3 + 6*a*c^4*x^4 + 2*b*c
^3*x^3*(4 + 3*c*x)*ArcTanh[c*x] + 7*b*Log[1 - c*x] + b*Log[1 + c*x]))/(24*c
^3)
```

Maple [A]
time = 0.12, size = 100, normalized size = 1.04

method	result
derivativedivides	$\frac{da\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{db\,\mathrm{arctanh}(cx)c^4x^4}{4}+\frac{db\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{db\,c^3x^3}{12}+\frac{db\,c^2x^2}{6}+\frac{dbcx}{4}+\frac{7db\ln(cx-1)}{24}+\frac{db\ln(cx+1)}{24}}{c^3}$
default	$\frac{da\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{db\,\mathrm{arctanh}(cx)c^4x^4}{4}+\frac{db\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{db\,c^3x^3}{12}+\frac{db\,c^2x^2}{6}+\frac{dbcx}{4}+\frac{7db\ln(cx-1)}{24}+\frac{db\ln(cx+1)}{24}}{c^3}$
risch	$\frac{db\,x^3(3cx+4)\ln(cx+1)}{24}-\frac{dc\,x^4b\ln(-cx+1)}{8}+\frac{dc\,x^4a}{4}-\frac{d\,x^3b\ln(-cx+1)}{6}+\frac{d\,x^3a}{3}+\frac{bd\,x^3}{12}+\frac{bd\,x^2}{6c}+\frac{bdx}{4c^2}+\frac{7}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d*a*(1/4*c^4*x^4+1/3*x^3*c^3)+1/4*d*b*arctanh(c*x)*c^4*x^4+1/3*d*b*a
rctanh(c*x)*c^3*x^3+1/12*d*b*c^3*x^3+1/6*d*b*c^2*x^2+1/4*d*b*c*x+7/24*d*b*l
n(c*x-1)+1/24*d*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 110, normalized size = 1.15

$$\frac{1}{4}acdx^4+\frac{1}{3}adx^3+\frac{1}{24}\left(6x^4\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\,(cx+1)}{c^5}+\frac{3\,\log\,(cx-1)}{c^5}\right)\right)bcd+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\,(c^2x^2-1)}{c^4}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c*d*x^4 + 1/3*a*d*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*
x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d + 1/6*(2*x^3*arcta
nh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d
```

Fricas [A]
time = 0.35, size = 102, normalized size = 1.06

$$\frac{6\,ac^4dx^4+2\,(4\,a+b)c^3dx^3+4\,bc^2dx^2+6\,bcdx+bd\log\,(cx+1)+7\,bd\log\,(cx-1)+(3\,bc^4dx^4+4\,bc^3dx^3)\log\,\left(-\frac{cx+1}{cx-1}\right)}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d*x^4 + 2*(4*a + b)*c^3*d*x^3 + 4*b*c^2*d*x^2 + 6*b*c*d*x + b
*d*log(c*x + 1) + 7*b*d*log(c*x - 1) + (3*b*c^4*d*x^4 + 4*b*c^3*d*x^3)*log(
-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A]
time = 0.30, size = 112, normalized size = 1.17

$$\begin{cases}\frac{acdx^4}{4}+\frac{adx^3}{3}+\frac{bcdx^4\,\mathrm{atanh}\,(cx)}{4}+\frac{bdx^3\,\mathrm{atanh}\,(cx)}{3}+\frac{bdx^3}{12}+\frac{bdx^2}{6c}+\frac{bdx}{4c^2}+\frac{bd\log\,\left(x-\frac{1}{c}\right)}{3c^3}+\frac{bd\,\mathrm{atanh}\,(cx)}{12c^3} & \text{for } c\neq 0\\ \frac{adx^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**4/4 + a*d*x**3/3 + b*c*d*x**4*atanh(c*x)/4 + b*d*x**3*a
tanh(c*x)/3 + b*d*x**3/12 + b*d*x**2/(6*c) + b*d*x/(4*c**2) + b*d*log(x - 1
/c)/(3*c**3) + b*d*atanh(c*x)/(12*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(82) = 164$.
time = 0.42, size = 394, normalized size = 4.10

$$\frac{1}{3}c\left(\frac{\left(\frac{6(cx+1)^3bd}{(cx-1)^3}-\frac{3(cx+1)^2bd}{(cx-1)^2}+\frac{4(cx+1)bd}{cx-1}-bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^4}{(cx-1)^4}-\frac{4(cx+1)^3c^4}{(cx-1)^3}+\frac{6(cx+1)^2c^4}{(cx-1)^2}-\frac{4(cx+1)c^4}{cx-1}+c^4}+\frac{\frac{12(cx+1)^3ad}{(cx-1)^3}-\frac{6(cx+1)^2ad}{(cx-1)^2}+\frac{8(cx+1)ad}{cx-1}-2ad+\frac{5(cx+1)^3bd}{(cx-1)^3}-\frac{10(cx+1)^2bd}{(cx-1)^2}+\frac{7(cx+1)bd}{cx-1}-2bd}{\frac{(cx+1)^4c^4}{(cx-1)^4}-\frac{4(cx+1)^3c^4}{(cx-1)^3}+\frac{6(cx+1)^2c^4}{(cx-1)^2}-\frac{4(cx+1)c^4}{cx-1}+c^4}-\frac{bd\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4}+\frac{bd\log\left(-\frac{cx+1}{cx-1}\right)}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*((6*(c*x + 1)^3*b*d/(c*x - 1)^3 - 3*(c*x + 1)^2*b*d/(c*x - 1)^2 + 4*(
c*x + 1)*b*d/(c*x - 1) - b*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^4/(c
*x - 1)^4 - 4*(c*x + 1)^3*c^4/(c*x - 1)^3 + 6*(c*x + 1)^2*c^4/(c*x - 1)^2 -
 4*(c*x + 1)*c^4/(c*x - 1) + c^4) + (12*(c*x + 1)^3*a*d/(c*x - 1)^3 - 6*(c*
x + 1)^2*a*d/(c*x - 1)^2 + 8*(c*x + 1)*a*d/(c*x - 1) - 2*a*d + 5*(c*x + 1)^
3*b*d/(c*x - 1)^3 - 10*(c*x + 1)^2*b*d/(c*x - 1)^2 + 7*(c*x + 1)*b*d/(c*x -
 1) - 2*b*d)/((c*x + 1)^4*c^4/(c*x - 1)^4 - 4*(c*x + 1)^3*c^4/(c*x - 1)^3 +
 6*(c*x + 1)^2*c^4/(c*x - 1)^2 - 4*(c*x + 1)*c^4/(c*x - 1) + c^4) - b*d*log
(-(c*x + 1)/(c*x - 1) + 1)/c^4 + b*d*log(-(c*x + 1)/(c*x - 1))/c^4)
```

Mupad [B]
time = 0.94, size = 92, normalized size = 0.96

$$\frac{\frac{b\,c\,d\,x}{4}-\frac{d\,\left(3\,b\,\mathrm{atanh}(c\,x)-2\,b\,\ln(c^2\,x^2-1)\right)}{12}+\frac{b\,c^2\,d\,x^2}{6}}{c^3}+\frac{d\,\left(4\,a\,x^3+b\,x^3+4\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{12}+\frac{c\,d\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] ((b*c*d*x)/4 - (d*(3*b*atanh(c*x) - 2*b*log(c^2*x^2 - 1)))/12 + (b*c^2*d*x^
2)/6)/c^3 + (d*(4*a*x^3 + b*x^3 + 4*b*x^3*atanh(c*x)))/12 + (c*d*(3*a*x^4 +
 3*b*x^4*atanh(c*x)))/12
```

3.3 $\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=84

$$\frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{5bd\log(1-cx)}{12c^2}-\frac{bd\log(1+cx)}{12c^2}$$

```
[Out] 1/2*b*d*x/c+1/6*b*d*x^2+1/2*d*x^2*(a+b*arctanh(c*x))+1/3*c*d*x^3*(a+b*arcta
nh(c*x))+5/12*b*d*ln(-c*x+1)/c^2-1/12*b*d*ln(c*x+1)/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {45, 6083, 12, 815, 647, 31}

$$\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{5bd\log(1-cx)}{12c^2}-\frac{bd\log(cx+1)}{12c^2}+\frac{bdx}{2c}+\frac{1}{6}bdx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(2*c) + (b*d*x^2)/6 + (d*x^2*(a + b*ArcTanh[c*x]))/2 + (c*d*x^3*(a
+ b*ArcTanh[c*x]))/3 + (5*b*d*Log[1 - c*x])/(12*c^2) - (b*d*Log[1 + c*x])/(
12*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx = \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^2(3}{6-}$$

$$= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bcd)\int\frac{x^2(3}{6-}$$

$$= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bcd)\int\Big(-$$

$$= \frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)$$

$$= \frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)$$

$$= \frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.94

$$\frac{d\left(6bcx+6ac^2x^2+2bc^2x^2+4ac^3x^3+2bc^2x^2(3+2cx)\tanh^{-1}(cx)+5b\log(1-cx)-b\log(1+cx)\right)}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(6*b*c*x + 6*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 2*b*c^2*x^2*(3 + 2*
c*x)*ArcTanh[c*x] + 5*b*Log[1 - c*x] - b*Log[1 + c*x]))/(12*c^2)
```

Maple [A]
time = 0.12, size = 90, normalized size = 1.07

method	result
derivativedivides	$\frac{da\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{db\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{db\,\mathrm{arctanh}(cx)c^2x^2}{2}+\frac{db\,c^2x^2}{6}+\frac{dbcx}{2}+\frac{5db\ln(cx-1)}{12}-\frac{db\ln(cx+1)}{12}}{c^2}$
default	$\frac{da\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{db\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{db\,\mathrm{arctanh}(cx)c^2x^2}{2}+\frac{db\,c^2x^2}{6}+\frac{dbcx}{2}+\frac{5db\ln(cx-1)}{12}-\frac{db\ln(cx+1)}{12}}{c^2}$
risch	$\frac{db\,x^2(2cx+3)\ln(cx+1)}{12}-\frac{dc\,x^3b\ln(-cx+1)}{6}+\frac{dc\,x^3a}{3}-\frac{db\,x^2\ln(-cx+1)}{4}+\frac{da\,x^2}{2}+\frac{bd\,x^2}{6}+\frac{bdx}{2c}-\frac{bd\ln(cx+1}{12c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d*a*(1/3*x^3*c^3+1/2*c^2*x^2)+1/3*d*b*arctanh(c*x)*c^3*x^3+1/2*d*b*a
rctanh(c*x)*c^2*x^2+1/6*d*b*c^2*x^2+1/2*d*b*c*x+5/12*d*b*ln(c*x-1)-1/12*d*b
*ln(c*x+1))
```

Maxima [A]
time = 0.25, size = 99, normalized size = 1.18

$$\frac{1}{3}acdx^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bcd+\frac{1}{2}adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+1\right)}{c^3}+\frac{\log\left(cx-1\right)}{c^3}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c*d*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4
))*b*c*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1
)/c^3 + log(c*x - 1)/c^3))*b*d
```

Fricas [A]
time = 0.35, size = 93, normalized size = 1.11

$$\frac{4\,ac^3dx^3+2\,(3\,a+b)c^2dx^2+6\,bcdx-bd\log\left(cx+1\right)+5\,bd\log\left(cx-1\right)+\left(2\,bc^3dx^3+3\,bc^2dx^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*c^3*d*x^3 + 2*(3*a + b)*c^2*d*x^2 + 6*b*c*d*x - b*d*log(c*x + 1)
+ 5*b*d*log(c*x - 1) + (2*b*c^3*d*x^3 + 3*b*c^2*d*x^2)*log(-(c*x + 1)/(c*x
- 1)))/c^2
```

Sympy [A]
time = 0.29, size = 100, normalized size = 1.19

$$\begin{cases} \frac{acdx^3}{3}+\frac{adx^2}{2}+\frac{bcdx^3\,\mathrm{atanh}\,(cx)}{3}+\frac{bdx^2\,\mathrm{atanh}\,(cx)}{2}+\frac{bdx^2}{6}+\frac{bdx}{2c}+\frac{bd\log\left(x-\frac{1}{c}\right)}{3c^2}-\frac{bd\,\mathrm{atanh}\,(cx)}{6c^2} & \text{for}\ c\neq 0\\ \frac{adx^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**3/3 + a*d*x**2/2 + b*c*d*x**3*atanh(c*x)/3 + b*d*x**2*a
tanh(c*x)/2 + b*d*x**2/6 + b*d*x/(2*c) + b*d*log(x - 1/c)/(3*c**2) - b*d*at
anh(c*x)/(6*c**2), Ne(c, 0)), (a*d*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(72) = 144.
time = 0.43, size = 305, normalized size = 3.63

$$\frac{1}{3}c\left(\frac{\left(\frac{6\,(cx+1)^2bd}{(cx-1)^2}-\frac{3\,(cx+1)bd}{cx-1}+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^3}{(cx-1)^3}-\frac{3\,(cx+1)^2c^3}{(cx-1)^2}+\frac{3\,(cx+1)c^3}{cx-1}-c^3}+\frac{\frac{12\,(cx+1)^2ad}{(cx-1)^2}-\frac{6\,(cx+1)ad}{cx-1}+2\,ad+\frac{5\,(cx+1)^2bd}{(cx-1)^2}-\frac{8\,(cx+1)bd}{cx-1}+3\,bd}{\frac{(cx+1)^3c^3}{(cx-1)^3}-\frac{3\,(cx+1)^2c^3}{(cx-1)^2}+\frac{3\,(cx+1)c^3}{cx-1}-c^3}-\frac{bd\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}+\frac{bd\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*((6*(c*x + 1)^2*b*d/(c*x - 1)^2 - 3*(c*x + 1)*b*d/(c*x - 1) + b*d)*lo
g(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^3/(c*x - 1)^3 - 3*(c*x + 1)^2*c^3/(c
*x - 1)^2 + 3*(c*x + 1)*c^3/(c*x - 1) - c^3) + (12*(c*x + 1)^2*a*d/(c*x - 1
)^2 - 6*(c*x + 1)*a*d/(c*x - 1) + 2*a*d + 5*(c*x + 1)^2*b*d/(c*x - 1)^2 - 8
*(c*x + 1)*b*d/(c*x - 1) + 3*b*d)/((c*x + 1)^3*c^3/(c*x - 1)^3 - 3*(c*x + 1
)^2*c^3/(c*x - 1)^2 + 3*(c*x + 1)*c^3/(c*x - 1) - c^3) - b*d*log(-(c*x + 1)
/(c*x - 1) + 1)/c^3 + b*d*log(-(c*x + 1)/(c*x - 1))/c^3)
```

Mupad [B]
time = 0.90, size = 83, normalized size = 0.99

$$\frac{d\,\left(3\,a\,x^2+b\,x^2+3\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{6}-\frac{\frac{d\,\left(3\,b\,\mathrm{atanh}\left(c\,x\right)-b\,\ln\left(c^2\,x^2-1\right)\right)}{6}-\frac{b\,c\,d\,x}{2}}{c^2}+\frac{c\,d\,\left(2\,a\,x^3+2\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] (d*(3*a*x^2 + b*x^2 + 3*b*x^2*atanh(c*x)))/6 - ((d*(3*b*atanh(c*x) - b*log(
c^2*x^2 - 1)))/6 - (b*c*d*x)/2)/c^2 + (c*d*(2*a*x^3 + 2*b*x^3*atanh(c*x)))/
6
```

3.4 $\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=44

$$\frac{bdx}{2}+\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{bd\log(1-cx)}{c}$$

```
[Out] 1/2*b*d*x+1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))/c+b*d*ln(-c*x+1)/c
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6063, 641, 45}

$$\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{bd\log(1-cx)}{c}+\frac{bdx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/2 + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*c) + (b*d*Log[1 - c*x])
/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx = \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} - \frac{b\int\frac{(d+cdx)^2}{1-c^2x^2}\,dx}{2d}$$
$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} - \frac{b\int\frac{d+cdx}{\frac{1}{d}-\frac{cx}{d}}\,dx}{2d}$$
$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} - \frac{b\int\left(-d^2-\frac{2d^2}{-1+cx}\right)dx}{2d}$$
$$= \frac{bdx}{2} + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} + \frac{bd\log(1-cx)}{c}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.
time = 0.01, size = 95, normalized size = 2.16

$$adx+\frac{bdx}{2}+\frac{1}{2}acdx^2+bdx\tanh^{-1}(cx)+\frac{1}{2}bcdx^2\tanh^{-1}(cx)+\frac{bd\log(1-cx)}{4c}-\frac{bd\log(1+cx)}{4c}+\frac{bd\log\left(1-c^2x^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] a*d*x + (b*d*x)/2 + (a*c*d*x^2)/2 + b*d*x*ArcTanh[c*x] + (b*c*d*x^2*ArcTanh
[c*x])/2 + (b*d*Log[1 - c*x])/(4*c) - (b*d*Log[1 + c*x])/(4*c) + (b*d*Log[1
 - c^2*x^2])/(2*c)
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.59

method	result
derivativedivides	$\frac{da\left(\frac{1}{2}c^2x^2+cx\right)+\frac{db\,\mathrm{arctanh}(cx)c^2x^2}{2}+bcdx\,\mathrm{arctanh}(cx)+\frac{dbcx}{2}+\frac{3db\ln(cx-1)}{4}+\frac{db\ln(cx+1)}{4}}{c}$
default	$\frac{da\left(\frac{1}{2}c^2x^2+cx\right)+\frac{db\,\mathrm{arctanh}(cx)c^2x^2}{2}+bcdx\,\mathrm{arctanh}(cx)+\frac{dbcx}{2}+\frac{3db\ln(cx-1)}{4}+\frac{db\ln(cx+1)}{4}}{c}$
risch	$\frac{dbx(cx+2)\ln(cx+1)}{4}-\frac{dcb\,x^2\ln(-cx+1)}{4}+\frac{dca\,x^2}{2}-\frac{dbx\ln(-cx+1)}{2}+adx+\frac{bdx}{2}+\frac{3bd\ln(-cx+1)}{4c}+$ db ln

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d*a*(1/2*c^2*x^2+c*x)+1/2*d*b*arctanh(c*x)*c^2*x^2+b*c*d*x*arctanh(c*x
)+1/2*d*b*c*x+3/4*d*b*ln(c*x-1)+1/4*d*b*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(40) = 80.

time = 0.26, size = 85, normalized size = 1.93

$$\frac{1}{2}acdx^2+\frac{1}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bcd+adx+\frac{(2cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))bd}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*b*c*d + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 +
1))*b*d/c
```

Fricas [A]
time = 0.36, size = 77, normalized size = 1.75

$$\frac{2ac^2dx^2+2(2a+b)cdx+bd\log(cx+1)+3bd\log(cx-1)+(bc^2dx^2+2bcdx)\log\left(-\frac{cx+1}{cx-1}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*d*x^2 + 2*(2*a + b)*c*d*x + b*d*log(c*x + 1) + 3*b*d*log(c*x -
 1) + (b*c^2*d*x^2 + 2*b*c*d*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.
time = 0.18, size = 75, normalized size = 1.70

$$\begin{cases}\frac{acdx^2}{2}+adx+\frac{bcdx^2\operatorname{atanh}(cx)}{2}+bdx\operatorname{atanh}(cx)+\frac{bdx}{2}+\frac{bd\log\left(x-\frac{1}{c}\right)}{c}+\frac{bd\operatorname{atanh}(cx)}{2c} & \text{for } c\neq 0\\ adx & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**2/2 + a*d*x + b*c*d*x**2*atanh(c*x)/2 + b*d*x*atanh(c*x
) + b*d*x/2 + b*d*log(x - 1/c)/c + b*d*atanh(c*x)/(2*c), Ne(c, 0)), (a*d*x,
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(40) = 80$.
time = 0.42, size = 211, normalized size = 4.80

$$-c\left(\frac{bd\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{\left(\frac{2(cx+1)bd}{cx-1}-bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^2}{(cx-1)^2}-\frac{2(cx+1)c^2}{cx-1}+c^2}-\frac{bd\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{\frac{4(cx+1)ad}{cx-1}-2ad+\frac{(cx+1)bd}{cx-1}-bd}{\frac{(cx+1)^2c^2}{(cx-1)^2}-\frac{2(cx+1)c^2}{cx-1}+c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -c*(b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - (2*(c*x + 1)*b*d/(c*x - 1) - b*
d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^2/(c*x - 1)^2 - 2*(c*x + 1)*c^2
/(c*x - 1) + c^2) - b*d*log(-(c*x + 1)/(c*x - 1))/c^2 - (4*(c*x + 1)*a*d/(c
*x - 1) - 2*a*d + (c*x + 1)*b*d/(c*x - 1) - b*d)/((c*x + 1)^2*c^2/(c*x - 1)
^2 - 2*(c*x + 1)*c^2/(c*x - 1) + c^2))
```

Mupad [B]
time = 0.86, size = 65, normalized size = 1.48

$$\frac{d\,(2\,a\,x+b\,x+2\,b\,x\,\mathrm{atanh}(c\,x))}{2}+\frac{c\,d\,(a\,x^2+b\,x^2\,\mathrm{atanh}(c\,x))}{2}-\frac{d\,(b\,\mathrm{atanh}(c\,x)-b\,\ln\left(c^2\,x^2-1\right))}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x),x)
```

```
[Out] (d*(2*a*x + b*x + 2*b*x*atanh(c*x)))/2 + (c*d*(a*x^2 + b*x^2*atanh(c*x)))/2
 - (d*(b*atanh(c*x) - b*log(c^2*x^2 - 1)))/(2*c)
```

3.5 $\int \frac{(d+cdx)(a+b\tanh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=60

$$acdx+bcdx\tanh^{-1}(cx)+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}bd\text{PolyLog}(2,cx)$$

```
[Out] a*c*d*x+b*c*d*x*arctanh(c*x)+a*d*ln(x)+1/2*b*d*ln(-c^2*x^2+1)-1/2*b*d*polyl
og(2,-c*x)+1/2*b*d*polylog(2,c*x)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6087, 6021, 266, 6031}

$$acdx+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+bcdx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] a*c*d*x + b*c*d*x*ArcTanh[c*x] + a*d*Log[x] + (b*d*Log[1 - c^2*x^2])/2 - (b
*d*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(cd\left(a+b\tanh^{-1}(cx)\right)+\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}\right)dx$$
$$= d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+(cd)\int \left(a+b\tanh^{-1}(cx)\right)\,dx$$
$$= acdx+ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+(bcd)\int \tanh^{-1}(cx)$$
$$= acdx+bcdx\tanh^{-1}(cx)+ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)-$$
$$= acdx+bcdx\tanh^{-1}(cx)+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)-\frac{1}{2}bd\text{Li}_2(-$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.90

$$\frac{1}{2}d\left(2acx+2bcx\tanh^{-1}(cx)+2a\log(x)+b\log\left(1-c^2x^2\right)-b\text{PolyLog}(2,-cx)+b\text{PolyLog}(2,cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d*(2*a*c*x + 2*b*c*x*ArcTanh[c*x] + 2*a*Log[x] + b*Log[1 - c^2*x^2] - b*Po
lyLog[2, -(c*x)] + b*PolyLog[2, c*x]))/2
```

Maple [A]
time = 0.13, size = 86, normalized size = 1.43

method	result
derivativedivides	$da\ln(cx)+adxc+db\,\text{arctanh}(cx)\ln(cx)+bcdx\,\text{arctanh}(cx)+\frac{db\ln(cx-1)}{2}+\frac{db\ln(cx+1)}{2}$
default	$da\ln(cx)+adxc+db\,\text{arctanh}(cx)\ln(cx)+bcdx\,\text{arctanh}(cx)+\frac{db\ln(cx-1)}{2}+\frac{db\ln(cx+1)}{2}$
risch	$-\frac{\ln(-cx+1)xbcd}{2}+adxc+\ln(-cx)\,ad+\frac{\text{dilog}(-cx+1)bd}{2}+\frac{\ln(-cx+1)bd}{2}-da-db+\frac{\ln(cx+1)xbc}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] d*a*ln(c*x)+a*d*x*c+d*b*arctanh(c*x)*ln(c*x)+b*c*d*x*arctanh(c*x)+1/2*d*b*l
n(c*x-1)+1/2*d*b*ln(c*x+1)-1/2*d*b*dilog(c*x)-1/2*d*b*dilog(c*x+1)-1/2*d*b*
ln(c*x)*ln(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] a*c*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d + 1/2*b*d*integr
ate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*d*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int ac\,dx + \int \frac{a}{x}\,dx + \int bc\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x,x)
```

```
[Out] d*(Integral(a*c, x) + Integral(a/x, x) + Integral(b*c*atanh(c*x), x) + Inte
gral(b*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\operatorname{atanh}(c\,x))\,(d+c\,d\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x))/x, x)
```

3.6 $\int \frac{(d+cdx)(a+b\tanh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{d(a+b\tanh^{-1}(cx))}{x}+acd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)-\frac{1}{2}bcd\text{PolyLog}(2,-cx)+\frac{1}{2}bcd\text{PolyLog}($$

```
[Out] -d*(a+b*arctanh(c*x))/x+a*c*d*ln(x)+b*c*d*ln(x)-1/2*b*c*d*ln(-c^2*x^2+1)-1/
2*b*c*d*polylog(2,-c*x)+1/2*b*c*d*polylog(2,c*x)
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6087, 6037, 272, 36, 29, 31, 6031}

$$-\frac{d(a+b\tanh^{-1}(cx))}{x}+acd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+bcd\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTanh[c*x]))/x) + a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
- c^2*x^2])/2 - (b*c*d*PolyLog[2, -(c*x)])/2 + (b*c*d*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx &= \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{x}\right)dx\\
&= d\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+(cd)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx\\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+\\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+\\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+\\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.01

$$-\frac{ad}{x}+acd\log(x)+bcd\left(-\frac{\tanh^{-1}(cx)}{cx}+\log(cx)-\frac{1}{2}\log\left(1-c^2x^2\right)\right)+\frac{1}{2}bcd(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c*d*Log[x] + b*c*d*(-(ArcTanh[c*x]/(c*x)) + Log[c*x] - Log[1
 - c^2*x^2]/2) + (b*c*d*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

Maple [A]
time = 0.16, size = 105, normalized size = 1.50

method	result
derivativedivides	$c\left(-\frac{da}{cx}+da\ln\left(cx\right)-\frac{db\,\mathrm{arctanh}(cx)}{cx}+db\,\mathrm{arctanh}\left(cx\right)\ln\left(cx\right)+db\ln\left(cx\right)-\frac{db\ln(cx+1)}{2}-\frac{db}{}\right.$
default	$c\left(-\frac{da}{cx}+da\ln\left(cx\right)-\frac{db\,\mathrm{arctanh}(cx)}{cx}+db\,\mathrm{arctanh}\left(cx\right)\ln\left(cx\right)+db\ln\left(cx\right)-\frac{db\ln(cx+1)}{2}-\frac{db}{}\right.$
risch	$\frac{dcb\ln(-cx)}{2}-\frac{\ln(-cx+1)bcd}{2}+\frac{db\ln(-cx+1)}{2x}+\frac{dc\,\mathrm{dilog}(-cx+1)b}{2}-\frac{da}{x}+dca\ln\left(-cx\right)+\frac{bcd\ln(cx)}{2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-d*a/c/x+d*a*ln(c*x)-d*b*arctanh(c*x)/c/x+d*b*arctanh(c*x)*ln(c*x)+d*b*l
n(c*x)-1/2*d*b*ln(c*x+1)-1/2*d*b*ln(c*x-1)-1/2*d*b*dilog(c*x)-1/2*d*b*dilog
(c*x+1)-1/2*d*b*ln(c*x)*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*d*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c*d*log(x) - 1
/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a}{x^2}\,dx+\int\frac{ac}{x}\,dx+\int\frac{b\,\mathrm{atanh}\left(cx\right)}{x^2}\,dx+\int\frac{bc\,\mathrm{atanh}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a/x**2, x) + Integral(a*c/x, x) + Integral(b*atanh(c*x)/x**2, x
) + Integral(b*c*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))\,(d + c\,d\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x))/x^2, x)
```

3.7 $\int \frac{(d+cdx)(a+b\tanh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=56

$$-\frac{bcd}{2x}-\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)$$

```
[Out] -1/2*b*c*d/x-1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))/x^2+b*c^2*d*ln(x)-b*c^2*d*l
n(-c*x+1)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {37, 6083, 12, 78}

$$-\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)-\frac{bcd}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d)/x - (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*x^2) + b*c^2*d*Log
[x] - b*c^2*d*Log[1 - c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx &= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-(bc)\int \frac{d(-1-cx)}{2x^2(1-cx)}\,dx\\ &= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{1}{2}(bcd)\int \frac{-1-cx}{x^2(1-cx)}\,dx\\ &= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{1}{2}(bcd)\int \left(-\frac{1}{x^2}-\frac{2c}{x}+\frac{2c^2}{-1+cx}\right)\\ &= -\frac{bcd}{2x}-\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx\end{aligned}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 1.36

$$-\frac{d\left(2a+4acx+2bcx+2(b+2bcx)\tanh^{-1}(cx)-4bc^2x^2\log(x)+3bc^2x^2\log(1-cx)+bc^2x^2\log(1+cx)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
[Out] -1/4*(d*(2*a + 4*a*c*x + 2*b*c*x + 2*(b + 2*b*c*x)*ArcTanh[c*x] - 4*b*c^2*x
^2*Log[x] + 3*b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x]))/x^2
```

Maple [A]
time = 0.11, size = 91, normalized size = 1.62

method	result
derivativedivides	$c^2\Big(da\big(-\frac{1}{2c^2x^2}-\frac{1}{cx}\big)-\frac{db\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{db\operatorname{arctanh}(cx)}{cx}-\frac{db\ln(cx+1)}{4}-\frac{3db\ln(cx-1)}{4}-\frac{db}{2cx}+db\ln($
default	$c^2\Big(da\big(-\frac{1}{2c^2x^2}-\frac{1}{cx}\big)-\frac{db\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{db\operatorname{arctanh}(cx)}{cx}-\frac{db\ln(cx+1)}{4}-\frac{3db\ln(cx-1)}{4}-\frac{db}{2cx}+db\ln($
risch	$-\frac{db(2cx+1)\ln(cx+1)}{4x^2}-\frac{d(b\,c^2\ln(cx+1)x^2+3b\,x^2\ln(-cx+1)c^2-4b\,c^2\ln(-x)x^2-2bcx\ln(-cx+1)+4cxa+2bcx-b\ln(-}{4x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d*a*(-1/2/c^2/x^2-1/c/x)-1/2*d*b*arctanh(c*x)/c^2/x^2-d*b*arctanh(c*x)
/c/x-1/4*d*b*ln(c*x+1)-3/4*d*b*ln(c*x-1)-1/2*d*b/c/x+d*b*ln(c*x))
```

Maxima [A]
time = 0.26, size = 89, normalized size = 1.59

$$-\frac{1}{2}\left(c(\log\left(c^2x^2-1\right)-\log\left(x^2\right))+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)bcd+\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bd-\frac{acd}{x}-\frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d + 1/4*((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d - a*c*d/x -
 1/2*a*d/x^2
```

Fricas [A]
time = 0.37, size = 89, normalized size = 1.59

$$-\frac{bc^2dx^2\log\left(cx+1\right)+3\,bc^2dx^2\log\left(cx-1\right)-4\,bc^2dx^2\log\left(x\right)+2\left(2\,a+b\right)cdx+2\,ad+\left(2\,bcdx+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c^2*d*x^2*log(c*x + 1) + 3*b*c^2*d*x^2*log(c*x - 1) - 4*b*c^2*d*x^2
*log(x) + 2*(2*a + b)*c*d*x + 2*a*d + (2*b*c*d*x + b*d)*log(-(c*x + 1)/(c*x
 - 1)))/x^2
```

Sympy [A]
time = 0.52, size = 95, normalized size = 1.70

$$\begin{cases} -\frac{acd}{x}-\frac{ad}{2x^2}+bc^2d\log\left(x\right)-bc^2d\log\left(x-\frac{1}{c}\right)-\frac{bc^2d\operatorname{atanh}\left(cx\right)}{2}-\frac{bcd\operatorname{atanh}\left(cx\right)}{x}-\frac{bcd}{2x}-\frac{bd\operatorname{atanh}\left(cx\right)}{2x^2} & \text{for}\ c\neq 0\\ -\frac{ad}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] Piecewise((-a*c*d/x - a*d/(2*x**2) + b*c**2*d*log(x) - b*c**2*d*log(x - 1/c
) - b*c**2*d*atanh(c*x)/2 - b*c*d*atanh(c*x)/x - b*c*d/(2*x) - b*d*atanh(c*
x)/(2*x**2), Ne(c, 0)), (-a*d/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(52) = 104.
time = 0.40, size = 192, normalized size = 3.43

$$\left(bcd\log\left(-\frac{cx+1}{cx-1}-1\right)-bcd\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{2\,(cx+1)bcd}{cx-1}+bcd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1}+\frac{\frac{4\,(cx+1)acd}{cx-1}+2\,acd+\frac{(cx+1)bcd}{cx-1}+bcd}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] (b*c*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c*d*log(-(c*x + 1)/(c*x - 1)) + (2
*(c*x + 1)*b*c*d/(c*x - 1) + b*c*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2/
(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1) + (4*(c*x + 1)*a*c*d/(c*x - 1) + 2
*a*c*d + (c*x + 1)*b*c*d/(c*x - 1) + b*c*d)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c
*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.90, size = 75, normalized size = 1.34

$$\frac{d\left(b\,c^2\,\mathrm{atanh}(c\,x)-b\,c^2\,\ln\left(c^2\,x^2-1\right)+2\,b\,c^2\,\ln\left(x\right)\right)}{2}-\frac{\frac{d\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{2}+\frac{d\,x\,\left(2\,a\,c+b\,c+2\,b\,c\,\mathrm{atanh}(c\,x)\right)}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^3,x)
```

```
[Out] (d*(b*c^2*atanh(c*x) - b*c^2*log(c^2*x^2 - 1) + 2*b*c^2*log(x)))/2 - ((d*(a
 + b*atanh(c*x)))/2 + (d*x*(2*a*c + b*c + 2*b*c*atanh(c*x)))/2)/x^2
```

3.8 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{bcd}{6x^2}-\frac{bc^2d}{2x}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+\frac{1}{3}bc^3d\log(x)-\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(1-$$

```
[Out] -1/6*b*c*d/x^2-1/2*b*c^2*d/x-1/3*d*(a+b*arctanh(c*x))/x^3-1/2*c*d*(a+b*arct
anh(c*x))/x^2+1/3*b*c^3*d*ln(x)-5/12*b*c^3*d*ln(-c*x+1)+1/12*b*c^3*d*ln(c*x
+1)
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {45, 6083, 12, 815}

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+\frac{1}{3}bc^3d\log(x)-\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(cx+1)-\frac{bc^2d}{2x}-\frac{bcd}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d)/x^2 - (b*c^2*d)/(2*x) - (d*(a + b*ArcTanh[c*x]))/(3*x^3) - (c*
d*(a + b*ArcTanh[c*x]))/(2*x^2) + (b*c^3*d*Log[x])/3 - (5*b*c^3*d*Log[1 - c
*x])/12 + (b*c^3*d*Log[1 + c*x])/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - (bc)\int \frac{d(-2-3cx)}{6x^3\left(1-c^2x^2\right.}$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{6}(bcd)\int \frac{-2-3c}{x^3\left(1-c^2\right.}$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{6}(bcd)\int \left(-\frac{2}{x^3} - \right.$$
$$= -\frac{bcd}{6x^2} - \frac{bc^2d}{2x} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} + \frac{1}{3}bc^3d$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 0.88

$$-\frac{d\left(4a+6acx+2bcx+6bc^2x^2+2b(2+3cx)\tanh^{-1}(cx)-4bc^3x^3\log(x)+5bc^3x^3\log(1-cx)-bc^3x^3\log(1+cx)\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/12*(d*(4*a + 6*a*c*x + 2*b*c*x + 6*b*c^2*x^2 + 2*b*(2 + 3*c*x)*ArcTanh[c
*x] - 4*b*c^3*x^3*Log[x] + 5*b*c^3*x^3*Log[1 - c*x] - b*c^3*x^3*Log[1 + c*x
]))/x^3
```

Maple [A]
time = 0.12, size = 102, normalized size = 1.04

method	result
derivativedivides	$c^3\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^2x^2}\right)-\frac{db\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{db\,\mathrm{arctanh}(cx)}{2c^2x^2}-\frac{db}{6c^2x^2}-\frac{db}{2cx}+\frac{db\ln(cx)}{3}-\frac{5db\ln(cx-1)}{12}-\right.$
default	$c^3\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^2x^2}\right)-\frac{db\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{db\,\mathrm{arctanh}(cx)}{2c^2x^2}-\frac{db}{6c^2x^2}-\frac{db}{2cx}+\frac{db\ln(cx)}{3}-\frac{5db\ln(cx-1)}{12}-\right.$
risch	$-\frac{db(3cx+2)\ln(cx+1)}{12x^3}-\frac{d\left(5x^3b\ln(-cx+1)c^3-4b\,c^3\ln(-x)x^3-b\,c^3\ln(cx+1)x^3+6b\,c^2x^2-3bcx\ln(-cx+1)+6cxa+2bcx\right.}{12x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d*a*(-1/3/c^3/x^3-1/2/c^2/x^2)-1/3*d*b*arctanh(c*x)/c^3/x^3-1/2*d*b*ar
ctanh(c*x)/c^2/x^2-1/6*d*b/c^2/x^2-1/2*d*b/c/x+1/3*d*b*ln(c*x)-5/12*d*b*ln(
c*x-1)+1/12*d*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 99, normalized size = 1.01

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bcd-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\operatorname{artanh}\left(cx\right)}{x^3}\right)bd-\frac{acd}{2\,x^2}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
- 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*d - 1/2*a*c*d/x^2 - 1/3*a*d/x^3
```

Fricas [A]
time = 0.36, size = 101, normalized size = 1.03

$$\frac{bc^3dx^3\log\left(cx+1\right)-5\,bc^3dx^3\log\left(cx-1\right)+4\,bc^3dx^3\log\left(x\right)-6\,bc^2dx^2-2\,(3\,a+b)cdx-4\,ad-(3\,bcdx+2\,bd)\log\left(-\frac{cx+1}{cx-1}\right)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*d*x^3*log(c*x + 1) - 5*b*c^3*d*x^3*log(c*x - 1) + 4*b*c^3*d*x^3
*log(x) - 6*b*c^2*d*x^2 - 2*(3*a + b)*c*d*x - 4*a*d - (3*b*c*d*x + 2*b*d)*l
og(-(c*x + 1)/(c*x - 1)))/x^3
```

Sympy [A]
time = 0.73, size = 117, normalized size = 1.19

$$\begin{cases} -\frac{acd}{2x^2}-\frac{ad}{3x^3}+\frac{bc^3d\log\left(x\right)}{3}-\frac{bc^3d\log\left(x-\frac{1}{c}\right)}{3}+\frac{bc^3d\operatorname{atanh}\left(cx\right)}{6}-\frac{bc^2d}{2x}-\frac{bcd\operatorname{atanh}\left(cx\right)}{2x^2}-\frac{bcd}{6x^2}-\frac{bd\operatorname{atanh}\left(cx\right)}{3x^3} & \text{for}\: c\neq 0 \\ -\frac{ad}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*c*d/(2*x**2) - a*d/(3*x**3) + b*c**3*d*log(x)/3 - b*c**3*d*lo
g(x - 1/c)/3 + b*c**3*d*atanh(c*x)/6 - b*c**2*d/(2*x) - b*c*d*atanh(c*x)/(2
*x**2) - b*c*d/(6*x**2) - b*d*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d/(3*x**3
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(84) = 168.
time = 0.44, size = 306, normalized size = 3.12

$$\frac{1}{3}\left(bc^2d\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^2d\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{6\,(cx+1)^2bc^2d}{(cx-1)^2}+\frac{3\,(cx+1)bc^2d}{cx-1}+bc^2d\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}+\frac{\frac{12\,(cx+1)^2ac^2d}{(cx-1)^2}+\frac{6\,(cx+1)ac^2d}{cx-1}+2\,ac^2d+\frac{5\,(cx+1)^2bc^2d}{(cx-1)^2}+\frac{8\,(cx+1)bc^2d}{cx-1}+3\,bc^2d}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(b*c^2*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^2*d*log(-(c*x + 1)/(c*x -
1)) + (6*(c*x + 1)^2*b*c^2*d/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*d/(c*x - 1) +
b*c^2*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2
/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^2*a*c^2*d/(c*x -
1)^2 + 6*(c*x + 1)*a*c^2*d/(c*x - 1) + 2*a*c^2*d + 5*(c*x + 1)^2*b*c^2*d/(c
*x - 1)^2 + 8*(c*x + 1)*b*c^2*d/(c*x - 1) + 3*b*c^2*d)/((c*x + 1)^3/(c*x -
1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.90, size = 110, normalized size = 1.12

$$\frac{b\,c^3\,d\,\ln\left(x\right)}{3}-\frac{a\,c\,d}{2\,x^2}-\frac{b\,c\,d}{6\,x^2}-\frac{b\,d\,\mathrm{atanh}(c\,x)}{3\,x^3}-\frac{b\,c^3\,d\,\ln\left(c^2\,x^2-1\right)}{6}-\frac{b\,c^2\,d}{2\,x}-\frac{a\,d}{3\,x^3}-\frac{b\,c^4\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{2\,\sqrt{-c^2}}-\frac{b\,c\,d\,\mathrm{atanh}(c\,x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^4,x)
```

```
[Out] (b*c^3*d*log(x))/3 - (a*c*d)/(2*x^2) - (b*c*d)/(6*x^2) - (b*d*atanh(c*x))/(
3*x^3) - (b*c^3*d*log(c^2*x^2 - 1))/6 - (b*c^2*d)/(2*x) - (a*d)/(3*x^3) - (
b*c^4*d*atan((c^2*x)/(-c^2)^(1/2)))/(2*(-c^2)^(1/2)) - (b*c*d*atanh(c*x))/(
2*x^2)
```

3.9 $\int \frac{(d+cdx)(a+b\tanh^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=110

$$-\frac{bcd}{12x^3}-\frac{bc^2d}{6x^2}-\frac{bc^3d}{4x}-\frac{d(a+b\tanh^{-1}(cx))}{4x^4}-\frac{cd(a+b\tanh^{-1}(cx))}{3x^3}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4$$

```
[Out] -1/12*b*c*d/x^3-1/6*b*c^2*d/x^2-1/4*b*c^3*d/x-1/4*d*(a+b*arctanh(c*x))/x^4-
1/3*c*d*(a+b*arctanh(c*x))/x^3+1/3*b*c^4*d*ln(x)-7/24*b*c^4*d*ln(-c*x+1)-1/
24*b*c^4*d*ln(c*x+1)
```

Rubi [A]

time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {45, 6083, 12, 815}

$$-\frac{d(a+b\tanh^{-1}(cx))}{4x^4}-\frac{cd(a+b\tanh^{-1}(cx))}{3x^3}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4d\log(cx+1)-\frac{bc^3d}{4x}-\frac{bc^2d}{6x^2}-\frac{bcd}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d)/x^3 - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4*x) - (d*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c*d*(a + b*ArcTanh[c*x]))/(3*x^3) + (b*c^4*d*Log[x])/3 -
 (7*b*c^4*d*Log[1 - c*x])/24 - (b*c^4*d*Log[1 + c*x])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - (bc)\int \frac{d(-3-4cx}{12x^4\left(1-c^2x\right.}$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{12}(bcd)\int \frac{-3-4}{x^4\left(1-c\right.}$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{12}(bcd)\int \left(-\frac{3}{x^4} - \right.$$
$$= -\frac{bcd}{12x^3} - \frac{bc^2d}{6x^2} - \frac{bc^3d}{4x} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3}$$

Mathematica [A]
time = 0.03, size = 94, normalized size = 0.85

$$-\frac{d\left(6a+8acx+2bcx+4bc^2x^2+6bc^3x^3+2b(3+4cx)\tanh^{-1}(cx)-8bc^4x^4\log(x)+7bc^4x^4\log(1-cx)+bc^4x^4\log(1+cx)\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/24*(d*(6*a + 8*a*c*x + 2*b*c*x + 4*b*c^2*x^2 + 6*b*c^3*x^3 + 2*b*(3 + 4*
c*x)*ArcTanh[c*x] - 8*b*c^4*x^4*Log[x] + 7*b*c^4*x^4*Log[1 - c*x] + b*c^4*x
^4*Log[1 + c*x]))/x^4
```

Maple [A]
time = 0.11, size = 112, normalized size = 1.02

method	result
derivativedivides	$c^4\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{4c^4x^4}\right)-\frac{db\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{db\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{db}{12c^3x^3}-\frac{db}{6c^2x^2}-\frac{db}{4cx}+\frac{db\ln(cx)}{3}-\frac{7db}{}\right.$
default	$c^4\left(da\left(-\frac{1}{3c^3x^3}-\frac{1}{4c^4x^4}\right)-\frac{db\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{db\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{db}{12c^3x^3}-\frac{db}{6c^2x^2}-\frac{db}{4cx}+\frac{db\ln(cx)}{3}-\frac{7db}{}\right.$
risch	$-\frac{db(4cx+3)\ln(cx+1)}{24x^4}-\frac{d\left(b\,c^4\ln(cx+1)x^4+7x^4b\ln(-cx+1)c^4-8c^4b\ln(-x)x^4+6b\,c^3x^3+4b\,c^2x^2-4bcx\ln(-cx+1)+8c\right.}{24x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(d*a*(-1/3/c^3/x^3-1/4/c^4/x^4)-1/3*d*b*arctanh(c*x)/c^3/x^3-1/4*d*b*ar
ctanh(c*x)/c^4/x^4-1/12*d*b/c^3/x^3-1/6*d*b/c^2/x^2-1/4*d*b/c/x+1/3*d*b*ln(
c*x)-7/24*d*b*ln(c*x-1)-1/24*d*b*ln(c*x+1))
```

Maxima [A]
time = 0.27, size = 114, normalized size = 1.04

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\left(cx\right)}{x^3}\right)bcd+\frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\mathrm{artanh}\left(cx\right)}{x^4}\right)bd-\frac{acd}{3\,x^3}-\frac{ad}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c*d + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)
/x^3)*c - 6*arctanh(c*x)/x^4)*b*d - 1/3*a*c*d/x^3 - 1/4*a*d/x^4
```

Fricas [A]
time = 0.37, size = 110, normalized size = 1.00

$$-\frac{bc^4dx^4\log\left(cx+1\right)+7\,bc^4dx^4\log\left(cx-1\right)-8\,bc^4dx^4\log\left(x\right)+6\,bc^3dx^3+4\,bc^2dx^2+2\left(4\,a+b\right)cdx+6\,ad+\left(4\,bcdx+3\,bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(b*c^4*d*x^4*log(c*x + 1) + 7*b*c^4*d*x^4*log(c*x - 1) - 8*b*c^4*d*x^
4*log(x) + 6*b*c^3*d*x^3 + 4*b*c^2*d*x^2 + 2*(4*a + b)*c*d*x + 6*a*d + (4*b
*c*d*x + 3*b*d)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [A]
time = 0.57, size = 129, normalized size = 1.17

$$\begin{cases} -\frac{acd}{3x^3}-\frac{ad}{4x^4}+\frac{bc^4d\log\left(x\right)}{3}-\frac{bc^4d\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^4d\,\mathrm{atanh}\left(cx\right)}{12}-\frac{bc^3d}{4x}-\frac{bc^2d}{6x^2}-\frac{bcd\,\mathrm{atanh}\left(cx\right)}{3x^3}-\frac{bcd}{12x^3}-\frac{bd\,\mathrm{atanh}\left(cx\right)}{4x^4} & \text{for}\ c\neq 0 \\ -\frac{ad}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c*d/(3*x**3) - a*d/(4*x**4) + b*c**4*d*log(x)/3 - b*c**4*d*lo
g(x - 1/c)/3 - b*c**4*d*atanh(c*x)/12 - b*c**3*d/(4*x) - b*c**2*d/(6*x**2)
- b*c*d*atanh(c*x)/(3*x**3) - b*c*d/(12*x**3) - b*d*atanh(c*x)/(4*x**4), Ne
(c, 0)), (-a*d/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(94) = 188.
time = 0.41, size = 401, normalized size = 3.65

$$\frac{1}{3}\left(bc^3d\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^3d\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{6\,(cx+1)^3bc^3d}{(cx-1)^3}+\frac{3\,(cx+1)^2bc^3d}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d}{cx-1}+bc^3d\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{\frac{12\,(cx+1)^3ac^3d}{(cx-1)^3}+\frac{6\,(cx+1)^2ac^3d}{(cx-1)^2}+\frac{8\,(cx+1)ac^3d}{cx-1}+2\,ac^3d+\frac{5\,(cx+1)^3bc^3d}{(cx-1)^3}+\frac{10\,(cx+1)^2bc^3d}{(cx-1)^2}+\frac{7\,(cx+1)bc^3d}{cx-1}+2\,bc^3d}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(b*c^3*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^3*d*log(-(c*x + 1)/(c*x -
1)) + (6*(c*x + 1)^3*b*c^3*d/(c*x - 1)^3 + 3*(c*x + 1)^2*b*c^3*d/(c*x - 1)^
2 + 4*(c*x + 1)*b*c^3*d/(c*x - 1) + b*c^3*d)*log(-(c*x + 1)/(c*x - 1))/((c*
x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^
2 + 4*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^3*a*c^3*d/(c*x - 1)^3 + 6*(c
*x + 1)^2*a*c^3*d/(c*x - 1)^2 + 8*(c*x + 1)*a*c^3*d/(c*x - 1) + 2*a*c^3*d +
 5*(c*x + 1)^3*b*c^3*d/(c*x - 1)^3 + 10*(c*x + 1)^2*b*c^3*d/(c*x - 1)^2 + 7
*(c*x + 1)*b*c^3*d/(c*x - 1) + 2*b*c^3*d)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x
 + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1
))*c
```

Mupad [B]
time = 0.85, size = 120, normalized size = 1.09

$$\frac{b\,c^4\,d\,\ln\left(x\right)}{3}-\frac{a\,c\,d}{3\,x^3}-\frac{b\,c\,d}{12\,x^3}-\frac{b\,d\,\mathrm{atanh}\left(c\,x\right)}{4\,x^4}-\frac{b\,c^4\,d\,\ln\left(c^2\,x^2-1\right)}{6}-\frac{b\,c^2\,d}{6\,x^2}-\frac{b\,c^3\,d}{4\,x}-\frac{a\,d}{4\,x^4}-\frac{b\,c^5\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{4\,\sqrt{-c^2}}-\frac{b\,c\,d\,\mathrm{atanh}\left(c\,x\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x))/x^5,x)
```

```
[Out] (b*c^4*d*log(x))/3 - (a*c*d)/(3*x^3) - (b*c*d)/(12*x^3) - (b*d*atanh(c*x))/
(4*x^4) - (b*c^4*d*log(c^2*x^2 - 1))/6 - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4*x
) - (a*d)/(4*x^4) - (b*c^5*d*atan((c^2*x)/(-c^2)^(1/2)))/(4*(-c^2)^(1/2)) -
 (b*c*d*atanh(c*x))/(3*x^3)
```

3.10 $\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=157

$$\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x$$

```
[Out] 5/12*b*d^2*x/c^3+1/5*b*d^2*x^2/c^2+5/36*b*d^2*x^3/c+1/10*b*d^2*x^4+1/30*b*c
*d^2*x^5+1/4*d^2*x^4*(a+b*arctanh(c*x))+2/5*c*d^2*x^5*(a+b*arctanh(c*x))+1/
6*c^2*d^2*x^6*(a+b*arctanh(c*x))+49/120*b*d^2*ln(-c*x+1)/c^4-1/120*b*d^2*ln
(c*x+1)/c^4
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {45, 6083, 12, 1816, 647, 31}

$$\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{49bd^2\log(1-cx)}{120c^4}-\frac{bd^2\log(cx+1)}{120c^4}+\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{1}{30}bcd^2x^5+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (5*b*d^2*x)/(12*c^3) + (b*d^2*x^2)/(5*c^2) + (5*b*d^2*x^3)/(36*c) + (b*d^2*
x^4)/10 + (b*c*d^2*x^5)/30 + (d^2*x^4*(a + b*ArcTanh[c*x]))/4 + (2*c*d^2*x^
5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x]))/6 + (49*b*d^
2*Log[1 - c*x])/(120*c^4) - (b*d^2*Log[1 + c*x])/(120*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
```

```
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6(a \\
&= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6(a \\
&= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6(a \\
&= \frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4(a+b\tan \\
&= \frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4(a+b\tan \\
&= \frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4(a+b\tan
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 125, normalized size = 0.80

$$\frac{d^2\left(150bcx+72bc^2x^2+50bc^3x^3+90ac^4x^4+36bc^4x^4+144ac^5x^5+12bc^5x^5+60ac^6x^6+6bc^4x^4\left(15+24cx+10c^2x^2\right)\tanh^{-1}(cx)+147b\log(1-cx)-3b\log(1+cx)\right)}{360c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(150*b*c*x + 72*b*c^2*x^2 + 50*b*c^3*x^3 + 90*a*c^4*x^4 + 36*b*c^4*x^4
 + 144*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^4*x^4*(15 + 24*c*x +
```

```
10*c^2*x^2)*ArcTanh[c*x] + 147*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(360*c^
4)
```

Maple [A]
time = 0.21, size = 164, normalized size = 1.04

method	result
derivativedivides	$\frac{d^2 a\left(\frac{1}{6}c^6x^6+\frac{2}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^2 b \operatorname{arctanh}(cx) c^6 x^6}{6}+\frac{2d^2 b \operatorname{arctanh}(cx) c^5 x^5}{5}+\frac{d^2 b \operatorname{arctanh}(cx) c^4 x^4}{4}+\frac{d^2 b\, c^5 x^5}{30}+\frac{d^2 b\, c^4 x^4}{10}+\frac{5d^2 b}{3}}{c^4}$
default	$\frac{d^2 a\left(\frac{1}{6}c^6x^6+\frac{2}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^2 b \operatorname{arctanh}(cx) c^6 x^6}{6}+\frac{2d^2 b \operatorname{arctanh}(cx) c^5 x^5}{5}+\frac{d^2 b \operatorname{arctanh}(cx) c^4 x^4}{4}+\frac{d^2 b\, c^5 x^5}{30}+\frac{d^2 b\, c^4 x^4}{10}+\frac{5d^2 b}{3}}{c^4}$
risch	$\frac{d^2 b\, x^4 (10c^2x^2+24cx+15) \ln(cx+1)}{120}-\frac{d^2 c^2 x^6 b \ln(-cx+1)}{12}+\frac{d^2 c^2 x^6 a}{6}-\frac{d^2 c\, x^5 b \ln(-cx+1)}{5}+\frac{2d^2 c\, x^5 a}{5}+\frac{bc\, d^2}{30}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^2*a*(1/6*c^6*x^6+2/5*c^5*x^5+1/4*c^4*x^4)+1/6*d^2*b*arctanh(c*x)*c
^6*x^6+2/5*d^2*b*arctanh(c*x)*c^5*x^5+1/4*d^2*b*arctanh(c*x)*c^4*x^4+1/30*d
^2*b*c^5*x^5+1/10*d^2*b*c^4*x^4+5/36*d^2*b*c^3*x^3+1/5*d^2*b*c^2*x^2+5/12*b
*c*d^2*x+49/120*d^2*b*ln(c*x-1)-1/120*d^2*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 210, normalized size = 1.34

$$\frac{1}{6}\,ac^2d^2x^6+\frac{2}{5}\,acd^2x^5+\frac{1}{4}\,ad^2x^4+\frac{1}{180}\left(30\,x^6\operatorname{artanh}(cx)+c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log(cx+1)}{c^7}+\frac{15\,\log(cx-1)}{c^7}\right)\right)bc^2d^2+\frac{1}{10}\left(4\,x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log(c^2x^2-1)}{c^6}\right)\right)bcd^2+\frac{1}{24}\left(6\,x^4\operatorname{artanh}(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log(cx+1)}{c^5}+\frac{3\,\log(cx-1)}{c^5}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d^2*x^6 + 2/5*a*c*d^2*x^5 + 1/4*a*d^2*x^4 + 1/180*(30*x^6*arctanh
(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*
log(c*x - 1)/c^7))*b*c^2*d^2 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x
^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d^2 + 1/24*(6*x^4*arctanh(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^2
```

Fricas [A]
time = 0.37, size = 162, normalized size = 1.03

$$\frac{60\,ac^6d^2x^6+12\,(12\,a+b)c^5d^2x^5+18\,(5\,a+2\,b)c^4d^2x^4+50\,bc^3d^2x^3+72\,bc^2d^2x^2+150\,bcd^2x-3\,bd^2\log(cx+1)+147\,bd^2\log(cx-1)+3\,(10\,bc^6d^2x^6+24\,bc^5d^2x^5+15\,bc^4d^2x^4)\log\left(-\frac{cx+1}{cx-1}\right)}{360\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(60*a*c^6*d^2*x^6 + 12*(12*a + b)*c^5*d^2*x^5 + 18*(5*a + 2*b)*c^4*d^
2*x^4 + 50*b*c^3*d^2*x^3 + 72*b*c^2*d^2*x^2 + 150*b*c*d^2*x - 3*b*d^2*log(c
```

```
*x + 1) + 147*b*d^2*log(c*x - 1) + 3*(10*b*c^6*d^2*x^6 + 24*b*c^5*d^2*x^5 +
 15*b*c^4*d^2*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A]
time = 0.47, size = 196, normalized size = 1.25

$$\begin{cases} \frac{ac^2d^2x^6}{6} + \frac{2acd^2x^5}{5} + \frac{ad^2x^4}{4} + \frac{bc^2d^2x^6 \operatorname{atanh}(cx)}{6} + \frac{2bcd^2x^5 \operatorname{atanh}(cx)}{5} + \frac{bcd^2x^5}{30} + \frac{bd^2x^4 \operatorname{atanh}(cx)}{4} + \frac{bd^2x^4}{10} + \frac{5bd^2x^3}{36c} + \frac{bd^2x^2}{5c^2} + \frac{5bd^2x}{12c^3} + \frac{2bd^2 \log\left(x - \frac{1}{c}\right)}{5c^4} - \frac{bd^2 \operatorname{atanh}(cx)}{60c^4} & \text{for } c \neq 0 \\ \frac{ad^2x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**6/6 + 2*a*c*d**2*x**5/5 + a*d**2*x**4/4 + b*c**2*
d**2*x**6*atanh(c*x)/6 + 2*b*c*d**2*x**5*atanh(c*x)/5 + b*c*d**2*x**5/30 +
b*d**2*x**4*atanh(c*x)/4 + b*d**2*x**4/10 + 5*b*d**2*x**3/(36*c) + b*d**2*x
**2/(5*c**2) + 5*b*d**2*x/(12*c**3) + 2*b*d**2*log(x - 1/c)/(5*c**4) - b*d*
*2*atanh(c*x)/(60*c**4), Ne(c, 0)), (a*d**2*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(137) = 274.
time = 0.41, size = 620, normalized size = 3.95

$$\frac{1}{45}c\left(\frac{6\left(\frac{30(cx+1)^5bd^2}{(cx-1)^5} - \frac{30(cx+1)^4bd^2}{(cx-1)^4} + \frac{70(cx+1)^3bd^2}{(cx-1)^3} - \frac{45(cx+1)^2bd^2}{(cx-1)^2} + \frac{18(cx+1)bd^2}{cx-1} - 3bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^5}{(cx-1)^6} - \frac{6(cx+1)^5c^5}{(cx-1)^5} + \frac{15(cx+1)^4c^5}{(cx-1)^4} - \frac{20(cx+1)^3c^5}{(cx-1)^3} + \frac{15(cx+1)^2c^5}{(cx-1)^2} - \frac{6(cx+1)c^5}{cx-1} + c^5} + \frac{\frac{360(cx+1)^5ad^2}{(cx-1)^5} - \frac{360(cx+1)^4ad^2}{(cx-1)^4} + \frac{840(cx+1)^3ad^2}{(cx-1)^3} - \frac{540(cx+1)^2ad^2}{(cx-1)^2} + \frac{216(cx+1)ad^2}{cx-1} - 36ad^2 + \frac{162(cx+1)^5bd^2}{(cx-1)^5} - \frac{531(cx+1)^4bd^2}{(cx-1)^4} + \frac{818(cx+1)^3bd^2}{(cx-1)^3} - \frac{696(cx+1)^2bd^2}{(cx-1)^2} + \frac{300(cx+1)bd^2}{cx-1} - 53bd^2}{\frac{(cx+1)^6c^5}{(cx-1)^6} - \frac{6(cx+1)^5c^5}{(cx-1)^5} + \frac{15(cx+1)^4c^5}{(cx-1)^4} - \frac{20(cx+1)^3c^5}{(cx-1)^3} + \frac{15(cx+1)^2c^5}{(cx-1)^2} - \frac{6(cx+1)c^5}{cx-1} + c^5} - \frac{18bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^5} + \frac{18bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/45*c*(6*(30*(c*x + 1)^5*b*d^2/(c*x - 1)^5 - 30*(c*x + 1)^4*b*d^2/(c*x - 1
)^4 + 70*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 45*(c*x + 1)^2*b*d^2/(c*x - 1)^2 +
 18*(c*x + 1)*b*d^2/(c*x - 1) - 3*b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x +
1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(
c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^
2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) + (360*(c*x + 1)^5*a*d^2/(c*x - 1)^5 -
 360*(c*x + 1)^4*a*d^2/(c*x - 1)^4 + 840*(c*x + 1)^3*a*d^2/(c*x - 1)^3 - 54
0*(c*x + 1)^2*a*d^2/(c*x - 1)^2 + 216*(c*x + 1)*a*d^2/(c*x - 1) - 36*a*d^2
+ 162*(c*x + 1)^5*b*d^2/(c*x - 1)^5 - 531*(c*x + 1)^4*b*d^2/(c*x - 1)^4 + 8
18*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 696*(c*x + 1)^2*b*d^2/(c*x - 1)^2 + 300*
(c*x + 1)*b*d^2/(c*x - 1) - 53*b*d^2)/((c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x
 + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c
^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1)
 + c^5) - 18*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 18*b*d^2*log(-(c*x +
 1)/(c*x - 1))/c^5)
```

Mupad [B]
time = 1.04, size = 146, normalized size = 0.93

$$\frac{\frac{b\,c^2\,d^2\,x^2}{5} - \frac{d^2\,\left(75\,b\,\mathrm{atanh}(c\,x) - 36\,b\,\ln(c^2\,x^2-1)\right)}{180} + \frac{5\,b\,c^3\,d^2\,x^3}{36} + \frac{5\,b\,c\,d^2\,x}{12}}{c^4} + \frac{d^2\,\left(45\,a\,x^4 + 18\,b\,x^4 + 45\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{180} + \frac{c^2\,d^2\,\left(30\,a\,x^6 + 30\,b\,x^6\,\mathrm{atanh}(c\,x)\right)}{180} + \frac{c\,d^2\,\left(72\,a\,x^5 + 6\,b\,x^5 + 72\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] ((b*c^2*d^2*x^2)/5 - (d^2*(75*b*atanh(c*x) - 36*b*log(c^2*x^2 - 1)))/180 +
(5*b*c^3*d^2*x^3)/36 + (5*b*c*d^2*x)/12)/c^4 + (d^2*(45*a*x^4 + 18*b*x^4 +
45*b*x^4*atanh(c*x)))/180 + (c^2*d^2*(30*a*x^6 + 30*b*x^6*atanh(c*x)))/180
+ (c*d^2*(72*a*x^5 + 6*b*x^5 + 72*b*x^5*atanh(c*x)))/180
```

3.11 $\int x^2(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=143

$$\frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\tan\right.$$

```
[Out] 1/2*b*d^2*x/c^2+4/15*b*d^2*x^2/c+1/6*b*d^2*x^3+1/20*b*c*d^2*x^4+1/3*d^2*x^3
*(a+b*arctanh(c*x))+1/2*c*d^2*x^4*(a+b*arctanh(c*x))+1/5*c^2*d^2*x^5*(a+b*a
rctanh(c*x))+31/60*b*d^2*ln(-c*x+1)/c^3+1/60*b*d^2*ln(c*x+1)/c^3
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {45, 6083, 12, 1816, 647, 31}

$$\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{31bd^2\log(1-cx)}{60c^3}+\frac{bd^2\log(cx+1)}{60c^3}+\frac{bd^2x}{2c^2}+\frac{1}{20}bcd^2x^4+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d^2*x)/(2*c^2) + (4*b*d^2*x^2)/(15*c) + (b*d^2*x^3)/6 + (b*c*d^2*x^4)/20
 + (d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c*d^2*x^4*(a + b*ArcTanh[c*x]))/2 +
(c^2*d^2*x^5*(a + b*ArcTanh[c*x]))/5 + (31*b*d^2*Log[1 - c*x])/(60*c^3) + (
b*d^2*Log[1 + c*x])/(60*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
```

```
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)\,dx = \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5($$
$$= \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5($$
$$= \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5($$
$$= \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)$$
$$= \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)$$
$$= \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)$$

Mathematica [A]
time = 0.04, size = 115, normalized size = 0.80

$$\frac{d^2\left(30bcx+16bc^2x^2+20ac^3x^3+10bc^3x^3+30ac^4x^4+3bc^4x^4+12ac^5x^5+2bc^3x^3(10+15cx+6c^2x^2)\tanh^{-1}(cx)+31b\log(1-cx)+b\log(1+cx)\right)}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(30*b*c*x + 16*b*c^2*x^2 + 20*a*c^3*x^3 + 10*b*c^3*x^3 + 30*a*c^4*x^4
 + 3*b*c^4*x^4 + 12*a*c^5*x^5 + 2*b*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2)*ArcTan
h[c*x] + 31*b*Log[1 - c*x] + b*Log[1 + c*x]))/(60*c^3)
```

Maple [A]
time = 0.20, size = 152, normalized size = 1.06

method	result
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{1}{2}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{d^2b\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{d^2b\,\mathrm{arctanh}(cx)c^4x^4}{2}+\frac{d^2b\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{d^2b\,c^4x^4}{20}+\frac{d^2b\,c^3x^3}{6}+\frac{4d^2b\,c^2 \cdots}{15}}{c^3}$
default	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{1}{2}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{d^2b\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{d^2b\,\mathrm{arctanh}(cx)c^4x^4}{2}+\frac{d^2b\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{d^2b\,c^4x^4}{20}+\frac{d^2b\,c^3x^3}{6}+\frac{4d^2b\,c^2 \cdots}{15}}{c^3}$
risch	$\frac{d^2b\,x^3\left(6c^2x^2+15cx+10\right)\ln(cx+1)}{60}-\frac{d^2c^2x^5b\ln(-cx+1)}{10}+\frac{d^2c^2x^5a}{5}-\frac{d^2c\,x^4b\ln(-cx+1)}{4}+\frac{d^2c\,x^4a}{2}+\frac{bc\,d^2x^4}{20}- \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^2*a*(1/5*c^5*x^5+1/2*c^4*x^4+1/3*x^3*c^3)+1/5*d^2*b*arctanh(c*x)*c
^5*x^5+1/2*d^2*b*arctanh(c*x)*c^4*x^4+1/3*d^2*b*arctanh(c*x)*c^3*x^3+1/20*d
^2*b*c^4*x^4+1/6*d^2*b*c^3*x^3+4/15*d^2*b*c^2*x^2+1/2*b*c*d^2*x+31/60*d^2*b
*ln(c*x-1)+1/60*d^2*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 184, normalized size = 1.29

$$\frac{1}{5}\,ac^2d^2x^5+\frac{1}{2}\,acd^2x^4+\frac{1}{20}\left(4\,x^5\,\mathrm{artanh}\left(cx\right)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^2d^2+\frac{1}{3}\,ad^2x^3+\frac{1}{12}\left(6\,x^4\,\mathrm{artanh}\left(cx\right)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bcd^2+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\left(cx\right)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^2*d^2*x^5 + 1/2*a*c*d^2*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^
4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/12*
(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log
(c*x - 1)/c^5))*b*c*d^2 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^
2 - 1)/c^4))*b*d^2
```

Fricas [A]
time = 0.35, size = 146, normalized size = 1.02

$$\frac{12\,ac^5d^2x^5+3\,(10\,a+b)c^4d^2x^4+10\,(2\,a+b)c^3d^2x^3+16\,bc^2d^2x^2+30\,bcd^2x+bd^2\log\left(cx+1\right)+31\,bd^2\log\left(cx-1\right)+\left(6\,bc^5d^2x^5+15\,bc^4d^2x^4+10\,bc^3d^2x^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{60\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^2*x^5 + 3*(10*a + b)*c^4*d^2*x^4 + 10*(2*a + b)*c^3*d^2*x^
3 + 16*b*c^2*d^2*x^2 + 30*b*c*d^2*x + b*d^2*log(c*x + 1) + 31*b*d^2*log(c*x
 - 1) + (6*b*c^5*d^2*x^5 + 15*b*c^4*d^2*x^4 + 10*b*c^3*d^2*x^3)*log(-(c*x +
 1)/(c*x - 1)))/c^3
```

Sympy [A]
time = 0.39, size = 177, normalized size = 1.24

$$\begin{cases} \frac{ac^2d^2x^5}{5} + \frac{acd^2x^4}{2} + \frac{ad^2x^3}{3} + \frac{bc^2d^2x^5 \operatorname{atanh}(cx)}{5} + \frac{bcd^2x^4 \operatorname{atanh}(cx)}{2} + \frac{bcd^2x^4}{20} + \frac{bd^2x^3 \operatorname{atanh}(cx)}{3} + \frac{bd^2x^3}{6} + \frac{4bd^2x^2}{15c} + \frac{bd^2x}{2c^2} + \frac{8bd^2 \log\left(x - \frac{1}{c}\right)}{15c^3} + \frac{bd^2 \operatorname{atanh}(cx)}{30c^3} & \text{for } c \neq 0 \\ \frac{ad^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**5/5 + a*c*d**2*x**4/2 + a*d**2*x**3/3 + b*c**2*d*
*2*x**5*atanh(c*x)/5 + b*c*d**2*x**4*atanh(c*x)/2 + b*c*d**2*x**4/20 + b*d*
*2*x**3*atanh(c*x)/3 + b*d**2*x**3/6 + 4*b*d**2*x**2/(15*c) + b*d**2*x/(2*c
**2) + 8*b*d**2*log(x - 1/c)/(15*c**3) + b*d**2*atanh(c*x)/(30*c**3), Ne(c,
 0)), (a*d**2*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(125) = 250.
time = 0.42, size = 525, normalized size = 3.67

$$\frac{4}{15}c\left(\frac{\left(\frac{15(cx+1)^4bd^2}{(cx-1)^4} - \frac{15(cx+1)^3bd^2}{(cx-1)^3} + \frac{20(cx+1)^2bd^2}{(cx-1)^2} - \frac{10(cx+1)bd^2}{cx-1} + 2bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^4}{(cx-1)^5} - \frac{5(cx+1)^4c^4}{(cx-1)^4} + \frac{10(cx+1)^3c^4}{(cx-1)^3} - \frac{10(cx+1)^2c^4}{(cx-1)^2} + \frac{5(cx+1)c^4}{cx-1} - c^4} + \frac{\frac{30(cx+1)^4ad^2}{(cx-1)^4} - \frac{30(cx+1)^3ad^2}{(cx-1)^3} + \frac{40(cx+1)^2ad^2}{(cx-1)^2} - \frac{20(cx+1)ad^2}{cx-1} + 4ad^2 + \frac{13(cx+1)^4bd^2}{(cx-1)^4} - \frac{36(cx+1)^3bd^2}{(cx-1)^3} + \frac{41(cx+1)^2bd^2}{(cx-1)^2} - \frac{23(cx+1)bd^2}{cx-1} + 5bd^2}{\frac{(cx+1)^5c^4}{(cx-1)^5} - \frac{5(cx+1)^4c^4}{(cx-1)^4} + \frac{10(cx+1)^3c^4}{(cx-1)^3} - \frac{10(cx+1)^2c^4}{(cx-1)^2} + \frac{5(cx+1)c^4}{cx-1} - c^4} - \frac{2bd^2\log\left(-\frac{cx+1}{cx-1} + 1\right)}{c^4} + \frac{2bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 4/15*c*((15*(c*x + 1)^4*b*d^2/(c*x - 1)^4 - 15*(c*x + 1)^3*b*d^2/(c*x - 1)^
3 + 20*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 10*(c*x + 1)*b*d^2/(c*x - 1) + 2*b*d
^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^4/(c*x - 1)^5 - 5*(c*x + 1)^4*
c^4/(c*x - 1)^4 + 10*(c*x + 1)^3*c^4/(c*x - 1)^3 - 10*(c*x + 1)^2*c^4/(c*x
- 1)^2 + 5*(c*x + 1)*c^4/(c*x - 1) - c^4) + (30*(c*x + 1)^4*a*d^2/(c*x - 1)
^4 - 30*(c*x + 1)^3*a*d^2/(c*x - 1)^3 + 40*(c*x + 1)^2*a*d^2/(c*x - 1)^2 -
20*(c*x + 1)*a*d^2/(c*x - 1) + 4*a*d^2 + 13*(c*x + 1)^4*b*d^2/(c*x - 1)^4 -
 36*(c*x + 1)^3*b*d^2/(c*x - 1)^3 + 41*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 23*(
c*x + 1)*b*d^2/(c*x - 1) + 5*b*d^2)/((c*x + 1)^5*c^4/(c*x - 1)^5 - 5*(c*x +
 1)^4*c^4/(c*x - 1)^4 + 10*(c*x + 1)^3*c^4/(c*x - 1)^3 - 10*(c*x + 1)^2*c^4
/(c*x - 1)^2 + 5*(c*x + 1)*c^4/(c*x - 1) - c^4) - 2*b*d^2*log(-(c*x + 1)/(c
*x - 1) + 1)/c^4 + 2*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^4)
```

Mupad [B]
time = 1.03, size = 134, normalized size = 0.94

$$\frac{\frac{4\,b\,c^2\,d^2\,x^2}{15} - \frac{d^2\,\left(30\,b\,\mathrm{atanh}(c\,x) - 16\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60} + \frac{b\,c\,d^2\,x}{2}}{c^3} + \frac{d^2\,\left(20\,a\,x^3 + 10\,b\,x^3 + 20\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{60} + \frac{c^2\,d^2\,\left(12\,a\,x^5 + 12\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{60} + \frac{c\,d^2\,\left(30\,a\,x^4 + 3\,b\,x^4 + 30\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] ((4*b*c^2*d^2*x^2)/15 - (d^2*(30*b*atanh(c*x) - 16*b*log(c^2*x^2 - 1)))/60
+ (b*c*d^2*x)/2)/c^3 + (d^2*(20*a*x^3 + 10*b*x^3 + 20*b*x^3*atanh(c*x)))/60
 + (c^2*d^2*(12*a*x^5 + 12*b*x^5*atanh(c*x)))/60 + (c*d^2*(30*a*x^4 + 3*b*x
^4 + 30*b*x^4*atanh(c*x)))/60
```

3.12 $\int x(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=129

$$\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 3/4*b*d^2*x/c+1/3*b*d^2*x^2+1/12*b*c*d^2*x^3+1/2*d^2*x^2*(a+b*arctanh(c*x))
+2/3*c*d^2*x^3*(a+b*arctanh(c*x))+1/4*c^2*d^2*x^4*(a+b*arctanh(c*x))+17/24*
b*d^2*ln(-c*x+1)/c^2-1/24*b*d^2*ln(c*x+1)/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {45, 6083, 12, 1816, 647, 31}

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{17bd^2\log(1-cx)}{24c^2}-\frac{bd^2\log(cx+1)}{24c^2}+\frac{1}{12}bcd^2x^3+\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^2*x)/(4*c) + (b*d^2*x^2)/3 + (b*c*d^2*x^3)/12 + (d^2*x^2*(a + b*ArcT
anh[c*x]))/2 + (2*c*d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c^2*d^2*x^4*(a + b*A
rcTanh[c*x]))/4 + (17*b*d^2*Log[1 - c*x])/(24*c^2) - (b*d^2*Log[1 + c*x])/(
24*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
```

```
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)\,dx=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4(a$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4(a$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4(a$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3$$

Mathematica [A]
time = 0.04, size = 107, normalized size = 0.83

$$\frac{d^2\left(18bcx+12ac^2x^2+8bc^2x^2+16ac^3x^3+2bc^3x^3+6ac^4x^4+2bc^2x^2(6+8cx+3c^2x^2)\tanh^{-1}(cx)+17b\log(1-cx)-b\log(1+cx)\right)}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(18*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 16*a*c^3*x^3 + 2*b*c^3*x^3 +
6*a*c^4*x^4 + 2*b*c^2*x^2*(6 + 8*c*x + 3*c^2*x^2)*ArcTanh[c*x] + 17*b*Log[1
 - c*x] - b*Log[1 + c*x]))/(24*c^2)
```

Maple [A]
time = 0.21, size = 140, normalized size = 1.09

method	result
derivativedivides	$\frac{d^2a\left(\frac{1}{4}c^4x^4+\frac{2}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d^2b\,\mathrm{arctanh}(cx)c^4x^4}{4}+\frac{2d^2b\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{d^2b\,\mathrm{arctanh}(cx)c^2x^2}{2}+\frac{d^2b\,c^3x^3}{12}+\frac{d^2b\,c^2x^2}{3}+\frac{3bc\,d^2}{4}}{c^2}$
default	$\frac{d^2a\left(\frac{1}{4}c^4x^4+\frac{2}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d^2b\,\mathrm{arctanh}(cx)c^4x^4}{4}+\frac{2d^2b\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{d^2b\,\mathrm{arctanh}(cx)c^2x^2}{2}+\frac{d^2b\,c^3x^3}{12}+\frac{d^2b\,c^2x^2}{3}+\frac{3bc\,d^2}{4}}{c^2}$
risch	$\frac{d^2b\,x^2(3c^2x^2+8cx+6)\ln(cx+1)}{24}-\frac{d^2c^2x^4b\ln(-cx+1)}{8}+\frac{d^2c^2x^4a}{4}-\frac{d^2c\,x^3b\ln(-cx+1)}{3}+\frac{2d^2c\,x^3a}{3}+\frac{bc\,d^2x^3}{12}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^2*a*(1/4*c^4*x^4+2/3*x^3*c^3+1/2*c^2*x^2)+1/4*d^2*b*arctanh(c*x)*c
^4*x^4+2/3*d^2*b*arctanh(c*x)*c^3*x^3+1/2*d^2*b*arctanh(c*x)*c^2*x^2+1/12*d
^2*b*c^3*x^3+1/3*d^2*b*c^2*x^2+3/4*b*c*d^2*x+17/24*d^2*b*ln(c*x-1)-1/24*d^2
*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 179, normalized size = 1.39

$$\frac{1}{4}ac^2d^2x^4+\frac{2}{3}acd^2x^3+\frac{1}{24}\left(6x^4\,\mathrm{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bc^2d^2+\frac{1}{3}\left(2x^3\,\mathrm{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bcd^2+\frac{1}{2}ad^2x^2+\frac{1}{4}\left(2x^2\,\mathrm{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d^2*x^4 + 2/3*a*c*d^2*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*
x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^2*d^2 + 1/3*
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c*d^2 + 1/2*a*d
^2*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x
- 1)/c^3))*b*d^2
```

Fricas [A]
time = 0.37, size = 137, normalized size = 1.06

$$\frac{6\,ac^4d^2x^4+2\,(8\,a+b)c^3d^2x^3+4\,(3\,a+2\,b)c^2d^2x^2+18\,bcd^2x-bd^2\log(cx+1)+17\,bd^2\log(cx-1)+(3\,bc^4d^2x^4+8\,bc^3d^2x^3+6\,bc^2d^2x^2)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d^2*x^4 + 2*(8*a + b)*c^3*d^2*x^3 + 4*(3*a + 2*b)*c^2*d^2*x^2
 + 18*b*c*d^2*x - b*d^2*log(c*x + 1) + 17*b*d^2*log(c*x - 1) + (3*b*c^4*d^2
*x^4 + 8*b*c^3*d^2*x^3 + 6*b*c^2*d^2*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A]
time = 0.34, size = 167, normalized size = 1.29

$$\begin{cases} \frac{ac^2d^2x^4}{4} + \frac{2acd^2x^3}{3} + \frac{ad^2x^2}{2} + \frac{bc^2d^2x^4\operatorname{atanh}(cx)}{4} + \frac{2bcd^2x^3\operatorname{atanh}(cx)}{3} + \frac{bcd^2x^3}{12} + \frac{bd^2x^2\operatorname{atanh}(cx)}{2} + \frac{bd^2x^2}{3} + \frac{3bd^2x}{4c} + \frac{2bd^2\log\left(x-\frac{1}{c}\right)}{3c^2} - \frac{bd^2\operatorname{atanh}(cx)}{12c^2} & \text{for } c\neq 0 \\ \frac{ad^2x^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**4/4 + 2*a*c*d**2*x**3/3 + a*d**2*x**2/2 + b*c**2*
d**2*x**4*atanh(c*x)/4 + 2*b*c*d**2*x**3*atanh(c*x)/3 + b*c*d**2*x**3/12 +
b*d**2*x**2*atanh(c*x)/2 + b*d**2*x**2/3 + 3*b*d**2*x/(4*c) + 2*b*d**2*log(
x - 1/c)/(3*c**2) - b*d**2*atanh(c*x)/(12*c**2), Ne(c, 0)), (a*d**2*x**2/2,
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(113) = 226.
time = 0.40, size = 425, normalized size = 3.29

$$-\frac{1}{3}c\left(\frac{2bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3} - \frac{2\left(\frac{6(cx+1)^3bd^2}{(cx-1)^3} - \frac{6(cx+1)^2bd^2}{(cx-1)^2} + \frac{4(cx+1)bd^2}{cx-1} - bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^3}{(cx-1)^4} - \frac{4(cx+1)^3c^3}{(cx-1)^3} + \frac{6(cx+1)^2c^3}{(cx-1)^2} - \frac{4(cx+1)c^3}{cx-1} + c^3} - \frac{2bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^3} - \frac{\frac{24(cx+1)^3ad^2}{(cx-1)^3} - \frac{24(cx+1)^2ad^2}{(cx-1)^2} + \frac{16(cx+1)ad^2}{cx-1} - 4ad^2 + \frac{10(cx+1)^3bd^2}{(cx-1)^3} - \frac{23(cx+1)^2bd^2}{(cx-1)^2} + \frac{18(cx+1)bd^2}{cx-1} - 5bd^2}{\frac{(cx+1)^4c^3}{(cx-1)^4} - \frac{4(cx+1)^3c^3}{(cx-1)^3} + \frac{6(cx+1)^2c^3}{(cx-1)^2} - \frac{4(cx+1)c^3}{cx-1} + c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*c*(2*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 2*(6*(c*x + 1)^3*b*d^2/
(c*x - 1)^3 - 6*(c*x + 1)^2*b*d^2/(c*x - 1)^2 + 4*(c*x + 1)*b*d^2/(c*x - 1)
 - b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^3/(c*x - 1)^4 - 4*(c*x +
 1)^3*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*c^3/(c*x - 1)^2 - 4*(c*x + 1)*c^3/(c*
x - 1) + c^3) - 2*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^3 - (24*(c*x + 1)^3*a*d
^2/(c*x - 1)^3 - 24*(c*x + 1)^2*a*d^2/(c*x - 1)^2 + 16*(c*x + 1)*a*d^2/(c*x
 - 1) - 4*a*d^2 + 10*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 23*(c*x + 1)^2*b*d^2/(
c*x - 1)^2 + 18*(c*x + 1)*b*d^2/(c*x - 1) - 5*b*d^2)/((c*x + 1)^4*c^3/(c*x
- 1)^4 - 4*(c*x + 1)^3*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*c^3/(c*x - 1)^2 - 4*
(c*x + 1)*c^3/(c*x - 1) + c^3))
```

Mupad [B]
time = 0.97, size = 122, normalized size = 0.95

$$\frac{d^2\left(6ax^2+4bx^2+6bx^2\operatorname{atanh}(cx)\right)}{12} - \frac{\frac{d^2\left(9b\operatorname{atanh}(cx)-4b\ln(c^2x^2-1)\right)}{12} - \frac{3bcd^2x}{4}}{c^2} + \frac{c^2d^2\left(3ax^4+3bx^4\operatorname{atanh}(cx)\right)}{12} + \frac{cd^2\left(8ax^3+bx^3+8bx^3\operatorname{atanh}(cx)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] (d^2*(6*a*x^2 + 4*b*x^2 + 6*b*x^2*atanh(c*x)))/12 - ((d^2*(9*b*atanh(c*x) -
 4*b*log(c^2*x^2 - 1)))/12 - (3*b*c*d^2*x)/4)/c^2 + (c^2*d^2*(3*a*x^4 + 3*b
*x^4*atanh(c*x)))/12 + (c*d^2*(8*a*x^3 + b*x^3 + 8*b*x^3*atanh(c*x)))/12
```

3.13 $\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=71

$$\frac{2}{3}bd^2x+\frac{bd^2(1+cx)^2}{6c}+\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{4bd^2\log(1-cx)}{3c}$$

```
[Out] 2/3*b*d^2*x+1/6*b*d^2*(c*x+1)^2/c+1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))/c+4/
3*b*d^2*ln(-c*x+1)/c
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6063, 641, 45}

$$\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{bd^2(cx+1)^2}{6c}+\frac{4bd^2\log(1-cx)}{3c}+\frac{2}{3}bd^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (2*b*d^2*x)/3 + (b*d^2*(1 + c*x)^2)/(6*c) + (d^2*(1 + c*x)^3*(a + b*ArcTanh
[c*x]))/(3*c) + (4*b*d^2*Log[1 - c*x])/(3*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right) dx = \frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3c} - \frac{b\int \frac{(d+cdx)^3}{1-c^2x^2}dx}{3d}$$

$$= \frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3c} - \frac{b\int \frac{(d+cdx)^2}{\frac{1}{d}-\frac{cx}{d}}dx}{3d}$$

$$= \frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3c} - \frac{b\int \left(-2d^3+\frac{4d^2}{\frac{1}{d}-\frac{cx}{d}}-d^2(d+cdx)\right)}{3d}$$

$$= \frac{2}{3}bd^2x + \frac{bd^2(1+cx)^2}{6c} + \frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{4bd^2\log(1}{3c}$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 1.30

$$\frac{d^2\left(6acx+6bcx+6ac^2x^2+bc^2x^2+2ac^3x^3+2bcx(3+3cx+c^2x^2)\tanh^{-1}(cx)+6b\log(1-cx)+b\log\left(1-c^2x^2\right)\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(6*a*c*x + 6*b*c*x + 6*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + 2*b*c*x*(
3 + 3*c*x + c^2*x^2)*ArcTanh[c*x] + 6*b*Log[1 - c*x] + b*Log[1 - c^2*x^2]))
/(6*c)
```

Maple [A]
time = 0.15, size = 102, normalized size = 1.44

method	result
derivativedivides	$\frac{\frac{d^2(cx+1)^3 a}{3}+\frac{d^2 b \operatorname{arctanh}(cx) c^3 x^3}{3}+d^2 b \operatorname{arctanh}(cx) c^2 x^2+bc\, d^2 x \operatorname{arctanh}(cx)+\frac{b\, d^2 \operatorname{arctanh}(cx)}{3}+\frac{d^2 b\, c^2 x^2}{6}+bc\, d^2 x+\frac{4d^2 b \ln(}{3}}{c}$
default	$\frac{\frac{d^2(cx+1)^3 a}{3}+\frac{d^2 b \operatorname{arctanh}(cx) c^3 x^3}{3}+d^2 b \operatorname{arctanh}(cx) c^2 x^2+bc\, d^2 x \operatorname{arctanh}(cx)+\frac{b\, d^2 \operatorname{arctanh}(cx)}{3}+\frac{d^2 b\, c^2 x^2}{6}+bc\, d^2 x+\frac{4d^2 b \ln(}{3}}{c}$
risch	$\frac{d^2(cx+1)^3 b \ln(cx+1)}{6c}-\frac{d^2c^2x^3 b \ln(-cx+1)}{6}+\frac{d^2c^2x^3 a}{3}-\frac{d^2cb\,x^2\ln(-cx+1)}{2}+d^2ca\,x^2+\frac{d^2cb\,x^2}{6}-\frac{d^2bx\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*d^2*(c*x+1)^3*a+1/3*d^2*b*arctanh(c*x)*c^3*x^3+d^2*b*arctanh(c*x)*
c^2*x^2+b*c*d^2*x*arctanh(c*x)+1/3*b*d^2*arctanh(c*x)+1/6*d^2*b*c^2*x^2+b*c
*d^2*x+4/3*d^2*b*ln(c*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(63) = 126$.

time = 0.26, size = 147, normalized size = 2.07

$$\frac{1}{3}ac^2d^2x^3+\frac{1}{6}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bc^2d^2+acd^2x^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bcd^2+ad^2x+\frac{(2cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))bd^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d^2*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)
/c^4))*b*c^2*d^2 + a*c*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log
(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctanh(c
*x) + log(-c^2*x^2 + 1))*b*d^2/c
```

Fricas [A]
time = 0.35, size = 114, normalized size = 1.61

$$\frac{2ac^3d^2x^3+(6a+b)c^2d^2x^2+6(a+b)cd^2x+bd^2\log(cx+1)+7bd^2\log(cx-1)+(bc^3d^2x^3+3bc^2d^2x^2+3bcd^2x)\log\left(-\frac{cx+1}{cx-1}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*d^2*x^3 + (6*a + b)*c^2*d^2*x^2 + 6*(a + b)*c*d^2*x + b*d^2*lo
g(c*x + 1) + 7*b*d^2*log(c*x - 1) + (b*c^3*d^2*x^3 + 3*b*c^2*d^2*x^2 + 3*b*
c*d^2*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(63) = 126.
time = 0.27, size = 131, normalized size = 1.85

$$\begin{cases}\frac{ac^2d^2x^3}{3}+acd^2x^2+ad^2x+\frac{bc^2d^2x^3\operatorname{atanh}(cx)}{3}+bcd^2x^2\operatorname{atanh}(cx)+\frac{bcd^2x^2}{6}+bd^2x\operatorname{atanh}(cx)+bd^2x+\frac{4bd^2\log\left(x-\frac{1}{c}\right)}{3c}+\frac{bd^2\operatorname{atanh}(cx)}{3c} & \text{for } c\neq 0\\ ad^2x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**3/3 + a*c*d**2*x**2 + a*d**2*x + b*c**2*d**2*x**3
*atanh(c*x)/3 + b*c*d**2*x**2*atanh(c*x) + b*c*d**2*x**2/6 + b*d**2*x*atanh
(c*x) + b*d**2*x + 4*b*d**2*log(x - 1/c)/(3*c) + b*d**2*atanh(c*x)/(3*c), N
e(c, 0)), (a*d**2*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(63) = 126.
time = 0.41, size = 330, normalized size = 4.65

$$-\frac{2}{3}\left(\frac{2bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{2bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{2\left(\frac{3(cx+1)^2bd^2}{(cx-1)^2}-\frac{3(cx+1)bd^2}{cx-1}+bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^2}{(cx-1)^3}-\frac{3(cx+1)^2c^2}{(cx-1)^2}+\frac{3(cx+1)c^2}{cx-1}-c^2}-\frac{\frac{12(cx+1)^2ad^2}{(cx-1)^2}-\frac{12(cx+1)ad^2}{cx-1}+4ad^2+\frac{4(cx+1)^2bd^2}{(cx-1)^2}-\frac{7(cx+1)bd^2}{cx-1}+3bd^2}{\frac{(cx+1)^3c^2}{(cx-1)^3}-\frac{3(cx+1)^2c^2}{(cx-1)^2}+\frac{3(cx+1)c^2}{cx-1}-c^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -2/3*(2*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 2*b*d^2*log(-(c*x + 1)/(c
*x - 1))/c^2 - 2*(3*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 3*(c*x + 1)*b*d^2/(c*x
- 1) + b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c
*x + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2) - (12*(c*x + 1
)^2*a*d^2/(c*x - 1)^2 - 12*(c*x + 1)*a*d^2/(c*x - 1) + 4*a*d^2 + 4*(c*x + 1
)^2*b*d^2/(c*x - 1)^2 - 7*(c*x + 1)*b*d^2/(c*x - 1) + 3*b*d^2)/((c*x + 1)^3
*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1
) - c^2))*c
```

Mupad [B]
time = 0.96, size = 105, normalized size = 1.48

$$\frac{d^2\,\left(6\,a\,x+6\,b\,x+6\,b\,x\,\mathrm{atanh}\left(c\,x\right)\right)}{6}+\frac{c^2\,d^2\,\left(2\,a\,x^3+2\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{6}-\frac{d^2\,\left(6\,b\,\mathrm{atanh}\left(c\,x\right)-4\,b\,\ln\left(c^2\,x^2-1\right)\right)}{6\,c}+\frac{c\,d^2\,\left(6\,a\,x^2+b\,x^2+6\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

```
[Out] (d^2*(6*a*x + 6*b*x + 6*b*x*atanh(c*x)))/6 + (c^2*d^2*(2*a*x^3 + 2*b*x^3*at
anh(c*x)))/6 - (d^2*(6*b*atanh(c*x) - 4*b*log(c^2*x^2 - 1)))/(6*c) + (c*d^2
*(6*a*x^2 + b*x^2 + 6*b*x^2*atanh(c*x)))/6
```

3.14 $\int \frac{(d+cdx)^2(a+b\tanh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=114

$$2acd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tanh^{-1}(cx)+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+ad^2\log(x)+bd^2\log\left(1-\right.$$

```
[Out] 2*a*c*d^2*x+1/2*b*c*d^2*x-1/2*b*d^2*arctanh(c*x)+2*b*c*d^2*x*arctanh(c*x)+1
/2*c^2*d^2*x^2*(a+b*arctanh(c*x))+a*d^2*ln(x)+b*d^2*ln(-c^2*x^2+1)-1/2*b*d^
2*polylog(2,-c*x)+1/2*b*d^2*polylog(2,c*x)
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6087, 6021, 266, 6031, 6037, 327, 212}

$$\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+2acd^2x+ad^2\log(x)+bd^2\log\left(1-c^2x^2\right)-\frac{1}{2}bd^2\text{Li}_2(-cx)+\frac{1}{2}bd^2\text{Li}_2(cx)+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tanh^{-1}(cx)+2bcd^2x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 2*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTanh[c*x])/2 + 2*b*c*d^2*x*ArcTanh[
c*x] + (c^2*d^2*x^2*(a + b*ArcTanh[c*x]))/2 + a*d^2*Log[x] + b*d^2*Log[1 -
c^2*x^2] - (b*d^2*PolyLog[2, -(c*x)])/2 + (b*d^2*PolyLog[2, c*x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(2cd^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+c^2d^2x\left(a+b\right.\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(2cd^2\right)\int \left(a+b\tanh^{-1}(cx)\right)\,dx+\left(c^2d\right.$$

$$= 2acd^2x+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+ad^2\log(x)-\frac{1}{2}bd^2\text{Li}_2(-cx)$$

$$= 2acd^2x+\frac{1}{2}bcd^2x+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)$$

$$= 2acd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tanh^{-1}(cx)+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x$$

Mathematica [A]

time = 0.06, size = 103, normalized size = 0.90

$$\frac{1}{4}d^2\left(8acx+2bcx+2ac^2x^2+8bcx\tanh^{-1}(cx)+2bc^2x^2\tanh^{-1}(cx)+4a\log(x)+b\log(1-cx)-b\log(1+cx)+4b\log\left(1-c^2x^2\right)-2b\text{PolyLog}(2,-cx)+2b\text{PolyLog}(2,cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^2*(8*a*c*x + 2*b*c*x + 2*a*c^2*x^2 + 8*b*c*x*ArcTanh[c*x] + 2*b*c^2*x^2*
ArcTanh[c*x] + 4*a*Log[x] + b*Log[1 - c*x] - b*Log[1 + c*x] + 4*b*Log[1 - c
^2*x^2] - 2*b*PolyLog[2, -(c*x)] + 2*b*PolyLog[2, c*x]))/4
```

Maple [A]
time = 0.18, size = 142, normalized size = 1.25

method	result
derivativedivides	$\frac{d^2 a\, c^2 x^2}{2} + 2d^2 acx + d^2 a \ln(cx) + \frac{d^2 b \operatorname{arctanh}(cx) c^2 x^2}{2} + 2bc\, d^2 x \operatorname{arctanh}(cx) + d^2 b \operatorname{arctanh}(c$
default	$\frac{d^2 a\, c^2 x^2}{2} + 2d^2 acx + d^2 a \ln(cx) + \frac{d^2 b \operatorname{arctanh}(cx) c^2 x^2}{2} + 2bc\, d^2 x \operatorname{arctanh}(cx) + d^2 b \operatorname{arctanh}(c$
risch	$\frac{d^2 a\, c^2 x^2}{2} + 2d^2 acx - \frac{5d^2 a}{2} + d^2 a \ln(-cx) - \frac{d^2 \ln(-cx+1) x^2 b\, c^2}{4} - d^2 b \ln(-cx+1)\, cx + \frac{5d^2 b \ln(-}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^2*a*c^2*x^2+2*d^2*a*c*x+d^2*a*ln(c*x)+1/2*d^2*b*arctanh(c*x)*c^2*x^2+
2*b*c*d^2*x*arctanh(c*x)+d^2*b*arctanh(c*x)*ln(c*x)-1/2*d^2*b*dilog(c*x)-1/
2*d^2*b*dilog(c*x+1)-1/2*d^2*b*ln(c*x)*ln(c*x+1)+1/2*b*c*d^2*x+5/4*d^2*b*ln
(c*x-1)+3/4*d^2*b*ln(c*x+1)
```

Maxima [A]
time = 0.35, size = 173, normalized size = 1.52

$$\frac{1}{4} bc^2 d^2 x^2 \log(cx+1) - \frac{1}{4} bc^2 d^2 x^2 \log(-cx+1) + \frac{1}{2} ac^2 d^2 x^2 + 2\, acd^2 x + \frac{1}{2} bcd^2 x + \left(2\, cx \operatorname{artanh}(cx) + \log\left(-c^2 x^2 + 1\right)\right) bd^2 - \frac{1}{2} \left(\log(cx) \log(-cx+1) + \operatorname{Li}_2(-cx+1)\right) bd^2 + \frac{1}{2} \left(\log(cx+1) \log(-cx) + \operatorname{Li}_2(cx+1)\right) bd^2 - \frac{1}{4} bd^2 \log(cx+1) + \frac{1}{4} bd^2 \log(cx-1) + ad^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^2*d^2*x^2*log(c*x + 1) - 1/4*b*c^2*d^2*x^2*log(-c*x + 1) + 1/2*a*c^
2*d^2*x^2 + 2*a*c*d^2*x + 1/2*b*c*d^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*b*d^2 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^2 + 1/2*
(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^2 - 1/4*b*d^2*log(c*x + 1) +
1/4*b*d^2*log(c*x - 1) + a*d^2*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int 2ac\,dx+\int \frac{a}{x}\,dx+\int ac^2x\,dx+\int 2bc\,\mathrm{atanh}\,(cx)\,dx+\int \frac{b\,\mathrm{atanh}\,(cx)}{x}\,dx+\int bc^2x\,\mathrm{atanh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**2*(Integral(2*a*c, x) + Integral(a/x, x) + Integral(a*c**2*x, x) + Integ
ral(2*b*c*atanh(c*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(b*c**2*x*
atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,{(d+c\,d\,x)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x, x)
```

3.15 $\int \frac{(d+cdx)^2(a+b\tanh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=61

$$\frac{d^2(-1+c^2x^2)\left(a+b\tanh^{-1}(cx)\right)}{x}+(2a+b)cd^2\log(x)-bcd^2\text{PolyLog}(2,-cx)+bcd^2\text{PolyLog}(2,cx)$$

```
[Out] d^2*(c^2*x^2-1)*(a+b*arctanh(c*x))/x+(2*a+b)*c*d^2*ln(x)-b*c*d^2*polylog(2,
-c*x)+b*c*d^2*polylog(2,c*x)
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6087, 6021, 266, 6037, 272, 36, 29, 31, 6031}

$$-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2x+2acd^2\log(x)+bc^2d^2x\tanh^{-1}(cx)-bcd^2\text{Li}_2(-cx)+bcd^2\text{Li}_2(cx)+bcd^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] a*c^2*d^2*x + b*c^2*d^2*x*ArcTanh[c*x] - (d^2*(a + b*ArcTanh[c*x]))/x + 2*a
*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, -(c*x)] + b*c*d^2*PolyL
og[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(c^2d^2\left(a+b\tanh^{-1}(cx)\right) + \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x^2} + \frac{2cd^2\left(a+b\tan\right.}{x}\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(2cd^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + \left(c^2d^2\right)\int$$

$$= ac^2d^2x - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x} + 2acd^2\log(x) - bcd^2\text{Li}_2(-cx) + bcd^2$$

$$= ac^2d^2x + bc^2d^2x\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x} + 2acd^2\log(x) -$$

$$= ac^2d^2x + bc^2d^2x\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x} + 2acd^2\log(x) +$$

$$= ac^2d^2x + bc^2d^2x\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x} + 2acd^2\log(x) +$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 1.20

$$\frac{d^2\left(-a+ac^2x^2-b\tanh^{-1}(cx)+bc^2x^2\tanh^{-1}(cx)+2acx\log(x)+bcx\log(cx)-bcx\text{PolyLog}(2,-cx)+bcx\text{PolyLog}(2,cx)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-a + a*c^2*x^2 - b*ArcTanh[c*x] + b*c^2*x^2*ArcTanh[c*x] + 2*a*c*x*Lo
g[x] + b*c*x*Log[c*x] - b*c*x*PolyLog[2, -(c*x)] + b*c*x*PolyLog[2, c*x]))/
x
```

Maple [A]
time = 0.21, size = 121, normalized size = 1.98

method	result
derivativedivides	$c\Big(d^2acx - \frac{d^2a}{cx} + 2d^2a\ln(cx) + bc\,d^2x\,\text{arctanh}(cx) - \frac{d^2b\,\text{arctanh}(cx)}{cx} + 2d^2b\,\text{arctanh}(cx)\ln$
default	$c\Big(d^2acx - \frac{d^2a}{cx} + 2d^2a\ln(cx) + bc\,d^2x\,\text{arctanh}(cx) - \frac{d^2b\,\text{arctanh}(cx)}{cx} + 2d^2b\,\text{arctanh}(cx)\ln$
risch	$a\,c^2d^2x - c\,d^2a - \frac{d^2a}{x} + 2c\,d^2a\ln(-cx) - \frac{c^2d^2b\ln(-cx+1)x}{2} - bc\,d^2 + \frac{c\,d^2b\ln(-cx)}{2} + \frac{d^2b\ln(-cx+}{2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

[Out] c*(d^2*a*c*x-d^2*a/c/x+2*d^2*a*ln(c*x)+b*c*d^2*x*arctanh(c*x)-d^2*b*arctanh(c*x)/c/x+2*d^2*b*arctanh(c*x)*ln(c*x)+d^2*b*ln(c*x)-d^2*b*dilog(c*x)-d^2*b*dilog(c*x+1)-d^2*b*ln(c*x)*ln(c*x+1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")

[Out] a*c^2*d^2*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^2 + b*c*d^2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 2*a*c*d^2*log(x) - 1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^2 - a*d^2/x

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")

[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*x + b*d^2)*arctanh(c*x))/x^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int ac^{2}\,dx+\int\frac{a}{x^{2}}\,dx+\int\frac{2ac}{x}\,dx+\int bc^{2}\operatorname{atanh}{\left(cx\right)}\,dx+\int\frac{b\operatorname{atanh}{\left(cx\right)}}{x^{2}}\,dx+\int\frac{2bc\operatorname{atanh}{\left(cx\right)}}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**2,x)

[Out] d**2*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(2*a*c/x, x) + Integral(b*c**2*atanh(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(2*b*c*atanh(c*x)/x, x))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(59) = 118.
time = 1.15, size = 410, normalized size = 6.72

$$\frac{1}{6}\left(\frac{6\,ad^{2}}{\frac{(cx+1)c^{2}}{cx-1}+c^{2}}+\frac{5\,bd^{2}\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^{2}}+\frac{3\,bd^{2}\log\left(-\frac{cx+1}{cx-1}-1\right)}{c^{2}}+\left(\frac{3\,bd^{2}}{\frac{(cx+1)c^{2}}{cx-1}+c^{2}}-\frac{\frac{3\,(cx+1)^{2}bd^{2}}{(cx-1)^{2}}-\frac{12\,(cx+1)bd^{2}}{cx-1}+5\,bd^{2}}{\frac{(cx+1)^{3}c^{2}}{(cx-1)^{3}}-\frac{3\,(cx+1)^{2}c^{2}}{(cx-1)^{2}}+\frac{3\,(cx+1)c^{2}}{cx-1}-c^{2}}\right)\log\left(-\frac{cx+1}{cx-1}\right)-\frac{8\,bd^{2}\log\left(-\frac{cx+1}{cx-1}\right)}{c^{2}}-\frac{2\left(\frac{3\,(cx+1)^{2}ad^{2}}{(cx-1)^{2}}-\frac{12\,(cx+1)ad^{2}}{cx-1}+5\,ad^{2}-\frac{(cx+1)^{2}bd^{2}}{(cx-1)^{2}}+\frac{(cx+1)bd^{2}}{cx-1}\right)}{\frac{(cx+1)^{3}c^{2}}{(cx-1)^{3}}-\frac{3\,(cx+1)^{2}c^{2}}{(cx-1)^{2}}+\frac{3\,(cx+1)c^{2}}{cx-1}-c^{2}}\right)c^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*a*d^2/((c*x + 1)*c^2/(c*x - 1) + c^2) + 5*b*d^2*log(-(c*x + 1)/(c*x
- 1) + 1)/c^2 + 3*b*d^2*log(-(c*x + 1)/(c*x - 1) - 1)/c^2 + (3*b*d^2/((c*x
+ 1)*c^2/(c*x - 1) + c^2) - (3*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 12*(c*x + 1)
*b*d^2/(c*x - 1) + 5*b*d^2)/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*c^
2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2))*log(-(c*x + 1)/(c*x - 1))
 - 8*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^2 - 2*(3*(c*x + 1)^2*a*d^2/(c*x - 1)
^2 - 12*(c*x + 1)*a*d^2/(c*x - 1) + 5*a*d^2 - (c*x + 1)^2*b*d^2/(c*x - 1)^2
 + (c*x + 1)*b*d^2/(c*x - 1))/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*
c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2))*c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))\,\left(d + c\,d\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^2, x)
```

3.16 $\int \frac{(d+cdx)^2(a+b\tanh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=137

$$-\frac{bcd^2}{2x}+\frac{1}{2}bc^2d^2\tanh^{-1}(cx)-\frac{d^2(a+b\tanh^{-1}(cx))}{2x^2}-\frac{2cd^2(a+b\tanh^{-1}(cx))}{x}+ac^2d^2\log(x)+2bc^2d^2\log(x)-b$$

```
[Out] -1/2*b*c*d^2/x+1/2*b*c^2*d^2*arctanh(c*x)-1/2*d^2*(a+b*arctanh(c*x))/x^2-2*
c*d^2*(a+b*arctanh(c*x))/x+a*c^2*d^2*ln(x)+2*b*c^2*d^2*ln(x)-b*c^2*d^2*ln(-
c^2*x^2+1)-1/2*b*c^2*d^2*polylog(2,-c*x)+1/2*b*c^2*d^2*polylog(2,c*x)
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6087, 6037, 331, 212, 272, 36, 29, 31, 6031}

$$-\frac{d^2(a+b\tanh^{-1}(cx))}{2x^2}-\frac{2cd^2(a+b\tanh^{-1}(cx))}{x}+ac^2d^2\log(x)-\frac{1}{2}bc^2d^2\text{Li}_2(-cx)+\frac{1}{2}bc^2d^2\text{Li}_2(cx)-bc^2d^2\log\left(1-c^2x^2\right)+2bc^2d^2\log(x)+\frac{1}{2}bc^2d^2\tanh^{-1}(cx)-\frac{bcd^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^2)/x + (b*c^2*d^2*ArcTanh[c*x])/2 - (d^2*(a + b*ArcTanh[c*x]))/
(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x]))/x + a*c^2*d^2*Log[x] + 2*b*c^2*d^2
*Log[x] - b*c^2*d^2*Log[1 - c^2*x^2] - (b*c^2*d^2*PolyLog[2, -(c*x)])/2 + (
b*c^2*d^2*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int \left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x^3} + \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x^2} + \frac{c^2d^2\left(a+b\,\mathrm{t}\right.}{x}\right.$$

$$= d^2 \int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(2cd^2\right) \int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(c^2d^2\right)$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + ac^2d^2\log(x) -$$

$$= -\frac{bcd^2}{2x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + ac^2d^2\,\mathrm{lo}$$

$$= -\frac{bcd^2}{2x} + \frac{1}{2}bc^2d^2\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\,\mathrm{ta}\right.}{x}$$

$$= -\frac{bcd^2}{2x} + \frac{1}{2}bc^2d^2\tanh^{-1}(cx) - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{2cd^2\left(a+b\,\mathrm{ta}\right.}{x}$$

Mathematica [A]
time = 0.05, size = 143, normalized size = 1.04

$$\frac{d^2\left(-2a-8acx-2bcx-2b\tanh^{-1}(cx)-8bcx\tanh^{-1}(cx)+4ac^2x^2\log(x)+8bc^2x^2\log(cx)-bc^2x^2\log(1-cx)+bc^2x^2\log(1+cx)-4bc^2x^2\log\left(1-c^2x^2\right)-2bc^2x^2\mathrm{PolyLog}(2,-cx)+2bc^2x^2\mathrm{PolyLog}(2,cx)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a - 8*a*c*x - 2*b*c*x - 2*b*ArcTanh[c*x] - 8*b*c*x*ArcTanh[c*x] +
4*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*
x^2*Log[1 + c*x] - 4*b*c^2*x^2*Log[1 - c^2*x^2] - 2*b*c^2*x^2*PolyLog[2, -(
c*x)] + 2*b*c^2*x^2*PolyLog[2, c*x]))/(4*x^2)
```

Maple [A]
time = 0.22, size = 168, normalized size = 1.23

method	result
derivativedivides	$c^2\left(d^2a\ln(cx) - \frac{d^2a}{2c^2x^2} - \frac{2d^2a}{cx} + d^2b\,\mathrm{arctanh}(cx)\ln(cx) - \frac{d^2b\,\mathrm{arctanh}(cx)}{2c^2x^2} - \frac{2d^2b\,\mathrm{arctanh}(cx)}{cx} -\right.$
default	$c^2\left(d^2a\ln(cx) - \frac{d^2a}{2c^2x^2} - \frac{2d^2a}{cx} + d^2b\,\mathrm{arctanh}(cx)\ln(cx) - \frac{d^2b\,\mathrm{arctanh}(cx)}{2c^2x^2} - \frac{2d^2b\,\mathrm{arctanh}(cx)}{cx} -\right.$
risch	$-\frac{d^2a}{2x^2} - \frac{2c\,d^2a}{x} + c^2d^2a\ln(-cx) - \frac{bc\,d^2}{2x} + \frac{5c^2d^2b\ln(-cx)}{4} - \frac{5b\,c^2d^2\ln(-cx+1)}{4} + \frac{c\,d^2b\ln(-cx+1)}{x} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d^2*a*ln(c*x)-1/2*d^2*a/c^2/x^2-2*d^2*a/c/x+d^2*b*arctanh(c*x)*ln(c*x)
-1/2*d^2*b*arctanh(c*x)/c^2/x^2-2*d^2*b*arctanh(c*x)/c/x-5/4*d^2*b*ln(c*x-1
)-1/2*d^2*b/c/x+2*d^2*b*ln(c*x)-3/4*d^2*b*ln(c*x+1)-1/2*d^2*b*dilog(c*x)-1/
2*d^2*b*dilog(c*x+1)-1/2*d^2*b*ln(c*x)*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*d^2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^2*d^2*lo
g(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^2 + 1/4*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^2 - 2*a
*c*d^2/x - 1/2*a*d^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a}{x^3}\,dx + \int \frac{2ac}{x^2}\,dx + \int \frac{ac^2}{x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x^3}\,dx + \int \frac{2bc\,\mathrm{atanh}\,(cx)}{x^2}\,dx + \int \frac{bc^2\,\mathrm{atanh}\,(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c/x**2, x) + Integral(a*c**2/x, x)
 + Integral(b*atanh(c*x)/x**3, x) + Integral(2*b*c*atanh(c*x)/x**2, x) + In
tegral(b*c**2*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^3, x)
```

3.17 $\int \frac{(d+cdx)^2(a+b\tanh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{bcd^2}{6x^2}-\frac{bc^2d^2}{x}-\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+\frac{4}{3}bc^3d^2\log(x)-\frac{4}{3}bc^3d^2\log(1-cx)$$

```
[Out] -1/6*b*c*d^2/x^2-b*c^2*d^2/x-1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))/x^3+4/3*b
*c^3*d^2*ln(x)-4/3*b*c^3*d^2*ln(-c*x+1)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {37, 6083, 12, 90}

$$-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+\frac{4}{3}bc^3d^2\log(x)-\frac{4}{3}bc^3d^2\log(1-cx)-\frac{bc^2d^2}{x}-\frac{bcd^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^2)/x^2 - (b*c^2*d^2)/x - (d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x]))
/(3*x^3) + (4*b*c^3*d^2*Log[x])/3 - (4*b*c^3*d^2*Log[1 - c*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - (bc)\int \frac{(d+cdx)^2}{3x^3(-1+cx)}\,dx$$

$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{3}(bc)\int \frac{(d+cdx)^2}{x^3(-1+cx)}\,dx$$

$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{3}(bc)\int \left(-\frac{d^2}{x^3} - \frac{3cd^2}{x^2} - \frac{4c^2d^2}{x}\right.$$

$$= -\frac{bcd^2}{6x^2} - \frac{bc^2d^2}{x} - \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} + \frac{4}{3}bc^3d^2\log(x) -$$

Mathematica [**A**]
time = 0.04, size = 103, normalized size = 1.27

$$-\frac{d^2\left(2a+6acx+bcx+6ac^2x^2+6bc^2x^2+2b(1+3cx+3c^2x^2)\tanh^{-1}(cx)-8bc^3x^3\log(x)+7bc^3x^3\log(1-cx)+bc^3x^3\log(1+cx)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/6*(d^2*(2*a + 6*a*c*x + b*c*x + 6*a*c^2*x^2 + 6*b*c^2*x^2 + 2*b*(1 + 3*c
*x + 3*c^2*x^2)*ArcTanh[c*x] - 8*b*c^3*x^3*Log[x] + 7*b*c^3*x^3*Log[1 - c*x
] + b*c^3*x^3*Log[1 + c*x]))/x^3
```

Maple [**A**]
time = 0.17, size = 142, normalized size = 1.75

method	result
derivativedivides	$c^3\Big(d^2a\big(-\frac{1}{c^2x^2}-\frac{1}{3c^3x^3}-\frac{1}{cx}\big)-\frac{d^2b\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{d^2b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{d^2b\,\mathrm{arctanh}(cx)}{cx}-\frac{d^2b}{6c^2x^2}-\frac{d^2b}{cx}$
default	$c^3\Big(d^2a\big(-\frac{1}{c^2x^2}-\frac{1}{3c^3x^3}-\frac{1}{cx}\big)-\frac{d^2b\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{d^2b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{d^2b\,\mathrm{arctanh}(cx)}{cx}-\frac{d^2b}{6c^2x^2}-\frac{d^2b}{cx}$
risch	$-\frac{d^2b(3c^2x^2+3cx+1)\ln(cx+1)}{6x^3}-\frac{d^2\left(7x^3b\ln(-cx+1)c^3-8b\,c^3\ln(-x)x^3+b\,c^3\ln(cx+1)x^3-3b\,x^2\ln(-cx+1)c^2+6a\,c^2\right.}{6x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d^2*a*(-1/c^2/x^2-1/3/c^3/x^3-1/c/x)-d^2*b*arctanh(c*x)/c^2/x^2-1/3*d^
2*b*arctanh(c*x)/c^3/x^3-d^2*b*arctanh(c*x)/c/x-1/6*d^2*b/c^2/x^2-d^2*b/c/x
+4/3*d^2*b*ln(c*x)-1/6*d^2*b*ln(c*x+1)-7/6*d^2*b*ln(c*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(73) = 146$.
time = 0.26, size = 157, normalized size = 1.94

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)bc^2d^2+\frac{1}{2}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bcd^2-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\operatorname{artanh}\left(cx\right)}{x^3}\right)bd^2-\frac{ac^2d^2}{x}-\frac{acd^2}{x^2}-\frac{ad^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^2 + 1/2*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^2 - 1
/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b
*d^2 - a*c^2*d^2/x - a*c*d^2/x^2 - 1/3*a*d^2/x^3
```

Fricas [A]
time = 0.39, size = 128, normalized size = 1.58

$$-\frac{bc^3d^2x^3\log\left(cx+1\right)+7\,bc^3d^2x^3\log\left(cx-1\right)-8\,bc^3d^2x^3\log\left(x\right)+6\,(a+b)c^2d^2x^2+(6\,a+b)cd^2x+2\,ad^2+\left(3\,bc^2d^2x^2+3\,bcd^2x+bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c^3*d^2*x^3*log(c*x + 1) + 7*b*c^3*d^2*x^3*log(c*x - 1) - 8*b*c^3*d
^2*x^3*log(x) + 6*(a + b)*c^2*d^2*x^2 + (6*a + b)*c*d^2*x + 2*a*d^2 + (3*b*
c^2*d^2*x^2 + 3*b*c*d^2*x + b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.
time = 0.54, size = 158, normalized size = 1.95

$$\begin{cases} -\frac{ac^2d^2}{x}-\frac{acd^2}{x^2}-\frac{ad^2}{3x^3}+\frac{4bc^3d^2\log\left(x\right)}{3}-\frac{4bc^3d^2\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^3d^2\operatorname{atanh}\left(cx\right)}{3}-\frac{bc^2d^2\operatorname{atanh}\left(cx\right)}{x}-\frac{bc^2d^2}{x}-\frac{bcd^2\operatorname{atanh}\left(cx\right)}{x^2}-\frac{bcd^2}{6x^2}-\frac{bd^2\operatorname{atanh}\left(cx\right)}{3x^3} & \text{for}\: c \neq 0 \\ -\frac{ad^2}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*c**2*d**2/x - a*c*d**2/x**2 - a*d**2/(3*x**3) + 4*b*c**3*d**2
*log(x)/3 - 4*b*c**3*d**2*log(x - 1/c)/3 - b*c**3*d**2*atanh(c*x)/3 - b*c**
2*d**2*atanh(c*x)/x - b*c**2*d**2/x - b*c*d**2*atanh(c*x)/x**2 - b*c*d**2/(
6*x**2) - b*d**2*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d**2/(3*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(73) = 146.
time = 0.43, size = 330, normalized size = 4.07

$$\frac{2}{3}\left(2\,bc^2d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^2d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{2\left(\frac{3\,(cx+1)^2bc^2d^2}{(cx-1)^2}+\frac{3\,(cx+1)bc^2d^2}{cx-1}+bc^2d^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}+\frac{\frac{12\,(cx+1)^2ac^2d^2}{(cx-1)^2}+\frac{12\,(cx+1)ac^2d^2}{cx-1}+4\,ac^2d^2+\frac{4\,(cx+1)^2bc^2d^2}{(cx-1)^2}+\frac{7\,(cx+1)bc^2d^2}{cx-1}+3\,bc^2d^2}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(2*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^2*d^2*log(-(c*x + 1)
/(c*x - 1)) + 2*(3*(c*x + 1)^2*b*c^2*d^2/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*d^
2/(c*x - 1) + b*c^2*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3
 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^2
*a*c^2*d^2/(c*x - 1)^2 + 12*(c*x + 1)*a*c^2*d^2/(c*x - 1) + 4*a*c^2*d^2 + 4
*(c*x + 1)^2*b*c^2*d^2/(c*x - 1)^2 + 7*(c*x + 1)*b*c^2*d^2/(c*x - 1) + 3*b*
c^2*d^2)/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)
/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.90, size = 116, normalized size = 1.43

$$\frac{d^2\,(6\,b\,c^3\,\mathrm{atanh}(c\,x)-4\,b\,c^3\,\ln\left(c^2\,x^2-1\right)+8\,b\,c^3\,\ln\left(x\right))}{6}-\frac{\frac{d^2\,(2\,a+2\,b\,\mathrm{atanh}(c\,x))}{6}+\frac{d^2\,x\,(6\,a\,c+b\,c+6\,b\,c\,\mathrm{atanh}(c\,x))}{6}+\frac{d^2\,x^2\,(6\,a\,c^2+6\,b\,c^2+6\,b\,c^2\,\mathrm{atanh}(c\,x))}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^4,x)
```

```
[Out] (d^2*(6*b*c^3*atanh(c*x) - 4*b*c^3*log(c^2*x^2 - 1) + 8*b*c^3*log(x)))/6 -
((d^2*(2*a + 2*b*atanh(c*x)))/6 + (d^2*x*(6*a*c + b*c + 6*b*c*atanh(c*x)))/
6 + (d^2*x^2*(6*a*c^2 + 6*b*c^2 + 6*b*c^2*atanh(c*x)))/6)/x^3
```

3.18 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=147

$$-\frac{bcd^2}{12x^3}-\frac{bc^2d^2}{3x^2}-\frac{3bc^3d^2}{4x}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+\frac{2}{3}bc^4d^2\,\mathrm{lo}$$

```
[Out] -1/12*b*c*d^2/x^3-1/3*b*c^2*d^2/x^2-3/4*b*c^3*d^2/x-1/4*d^2*(a+b*arctanh(c*
x))/x^4-2/3*c*d^2*(a+b*arctanh(c*x))/x^3-1/2*c^2*d^2*(a+b*arctanh(c*x))/x^2
+2/3*b*c^4*d^2*ln(x)-17/24*b*c^4*d^2*ln(-c*x+1)+1/24*b*c^4*d^2*ln(c*x+1)
```

Rubi [A]
time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {45, 6083, 12, 1816}

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+\frac{2}{3}bc^4d^2\log(x)-\frac{17}{24}bc^4d^2\log(1-cx)+\frac{1}{24}bc^4d^2\log(cx+1)-\frac{3bc^3d^2}{4x}-\frac{bc^2d^2}{3x^2}-\frac{bcd^2}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^2)/x^3 - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (d^2*(a +
 b*ArcTanh[c*x]))/(4*x^4) - (2*c*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) - (c^2*d
^2*(a + b*ArcTanh[c*x]))/(2*x^2) + (2*b*c^4*d^2*Log[x])/3 - (17*b*c^4*d^2*L
og[1 - c*x])/24 + (b*c^4*d^2*Log[1 + c*x])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh\right.}{2x^2}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh\right.}{2x^2}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh\right.}{2x^2}$$

$$= -\frac{bcd^2}{12x^3} - \frac{bc^2d^2}{3x^2} - \frac{3bc^3d^2}{4x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh\right.}{3x^3}$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 0.78

$$-\frac{d^2\left(6a+16acx+2bcx+12ac^2x^2+8bc^2x^2+18bc^3x^3+2b(3+8cx+6c^2x^2)\tanh^{-1}(cx)-16bc^4x^4\log(x)+17bc^4x^4\log(1-cx)-bc^4x^4\log(1+cx)\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
[Out] -1/24*(d^2*(6*a + 16*a*c*x + 2*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 18*b*c^
3*x^3 + 2*b*(3 + 8*c*x + 6*c^2*x^2)*ArcTanh[c*x] - 16*b*c^4*x^4*Log[x] + 17
*b*c^4*x^4*Log[1 - c*x] - b*c^4*x^4*Log[1 + c*x]))/x^4
```

Maple [A]
time = 0.17, size = 154, normalized size = 1.05

method	result
derivativedivides	$c^4\left(d^2a\left(-\frac{2}{3c^3x^3}-\frac{1}{4c^4x^4}-\frac{1}{2c^2x^2}\right)-\frac{2d^2b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{d^2b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{d^2b\,\mathrm{arctanh}(cx)}{2c^2x^2}-\frac{17d^2b\ln(c}{24}\right.$
default	$c^4\left(d^2a\left(-\frac{2}{3c^3x^3}-\frac{1}{4c^4x^4}-\frac{1}{2c^2x^2}\right)-\frac{2d^2b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{d^2b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{d^2b\,\mathrm{arctanh}(cx)}{2c^2x^2}-\frac{17d^2b\ln(c}{24}\right.$
risch	$-\frac{d^2b\left(6c^2x^2+8cx+3\right)\ln(cx+1)}{24x^4}+\frac{d^2\left(16c^4b\ln(-x)x^4+b\,c^4\ln(cx+1)x^4-17x^4b\ln(-cx+1)c^4-18b\,c^3x^3+6b\,x^2\ln(-cx\right.}{24x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(d^2*a*(-2/3/c^3/x^3-1/4/c^4/x^4-1/2/c^2/x^2)-2/3*d^2*b*arctanh(c*x)/c^
3/x^3-1/4*d^2*b*arctanh(c*x)/c^4/x^4-1/2*d^2*b*arctanh(c*x)/c^2/x^2-17/24*d
^2*b*ln(c*x-1)+1/24*d^2*b*ln(c*x+1)-1/12*d^2*b/c^3/x^3-1/3*d^2*b/c^2/x^2-3/
4*d^2*b/c/x+2/3*d^2*b*ln(c*x))
```

Maxima [A]
time = 0.28, size = 178, normalized size = 1.21

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\ \mathrm{artanh}\left(cx\right)}{x^2}\right)bc^2d^2-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\ \mathrm{artanh}\left(cx\right)}{x^3}\right)bcd^2+\frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\,(3\,c^2x^2+1)}{x^3}\right)c-\frac{6\ \mathrm{artanh}\left(cx\right)}{x^4}\right)bd^2-\frac{ac^2d^2}{2\,x^2}-\frac{2\,acd^2}{3\,x^3}-\frac{ad^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*
d^2 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c*d^2 + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x
^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^2 - 1/2*a*c^2*d^2/x^2 - 2/3*a*c*d^
2/x^3 - 1/4*a*d^2/x^4
```

Fricas [A]
time = 0.40, size = 147, normalized size = 1.00

$$\frac{bc^4d^2x^4\log\left(cx+1\right)-17\,bc^4d^2x^4\log\left(cx-1\right)+16\,bc^4d^2x^4\log\left(x\right)-18\,bc^3d^2x^3-4\,(3\,a+2\,b)c^2d^2x^2-2\,(8\,a+b)cd^2x-6\,ad^2-(6\,bc^2d^2x^2+8\,bcd^2x+3\,bd^2)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(b*c^4*d^2*x^4*log(c*x + 1) - 17*b*c^4*d^2*x^4*log(c*x - 1) + 16*b*c^4
*d^2*x^4*log(x) - 18*b*c^3*d^2*x^3 - 4*(3*a + 2*b)*c^2*d^2*x^2 - 2*(8*a + b
)*c*d^2*x - 6*a*d^2 - (6*b*c^2*d^2*x^2 + 8*b*c*d^2*x + 3*b*d^2)*log(-(c*x +
 1)/(c*x - 1)))/x^4
```

Sympy [A]
time = 0.55, size = 189, normalized size = 1.29

$$\begin{cases} -\frac{ac^2d^2}{2x^2}-\frac{2acd^2}{3x^3}-\frac{ad^2}{4x^4}+\frac{2bc^4d^2\log\left(x\right)}{3}-\frac{2bc^4d^2\log\left(x-\frac{1}{c}\right)}{3}+\frac{bc^4d^2\ \mathrm{atanh}\left(cx\right)}{12}-\frac{3bc^3d^2}{4x}-\frac{bc^2d^2\ \mathrm{atanh}\left(cx\right)}{2x^2}-\frac{bc^2d^2}{3x^2}-\frac{2bcd^2\ \mathrm{atanh}\left(cx\right)}{3x^3}-\frac{bcd^2}{12x^3}-\frac{bd^2\ \mathrm{atanh}\left(cx\right)}{4x^4} & \text{for}\ c\neq 0\\ -\frac{ad^2}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c**2*d**2/(2*x**2) - 2*a*c*d**2/(3*x**3) - a*d**2/(4*x**4) +
2*b*c**4*d**2*log(x)/3 - 2*b*c**4*d**2*log(x - 1/c)/3 + b*c**4*d**2*atanh(c
*x)/12 - 3*b*c**3*d**2/(4*x) - b*c**2*d**2*atanh(c*x)/(2*x**2) - b*c**2*d**
```

```
2/(3*x**2) - 2*b*c*d**2*atanh(c*x)/(3*x**3) - b*c*d**2/(12*x**3) - b*d**2*a
tanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**2/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(129) = 258$.
time = 0.41, size = 431, normalized size = 2.93

$$\frac{1}{3}\left(2\,bc^3d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^3d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{2\left(\frac{6\,(cx+1)^3bc^3d^2}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3d^2}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d^2}{cx-1}+bc^3d^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{\frac{24\,(cx+1)^3ac^3d^2}{(cx-1)^3}+\frac{24\,(cx+1)^2ac^3d^2}{(cx-1)^2}+\frac{16\,(cx+1)ac^3d^2}{cx-1}+4\,ac^3d^2+\frac{10\,(cx+1)^3bc^3d^2}{(cx-1)^3}+\frac{23\,(cx+1)^2bc^3d^2}{(cx-1)^2}+\frac{18\,(cx+1)bc^3d^2}{cx-1}+5\,bc^3d^2}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(2*b*c^3*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^3*d^2*log(-(c*x + 1)
/(c*x - 1)) + 2*(6*(c*x + 1)^3*b*c^3*d^2/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^3*
d^2/(c*x - 1)^2 + 4*(c*x + 1)*b*c^3*d^2/(c*x - 1) + b*c^3*d^2)*log(-(c*x +
1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x
 + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (24*(c*x + 1)^3*a*c^3*d^
2/(c*x - 1)^3 + 24*(c*x + 1)^2*a*c^3*d^2/(c*x - 1)^2 + 16*(c*x + 1)*a*c^3*d
^2/(c*x - 1) + 4*a*c^3*d^2 + 10*(c*x + 1)^3*b*c^3*d^2/(c*x - 1)^3 + 23*(c*x
 + 1)^2*b*c^3*d^2/(c*x - 1)^2 + 18*(c*x + 1)*b*c^3*d^2/(c*x - 1) + 5*b*c^3*
d^2)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(
c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 1.01, size = 168, normalized size = 1.14

$$\frac{2\,b\,c^4\,d^2\,\ln\left(x\right)}{3}-\frac{b\,c^4\,d^2\,\ln\left(c^2\,x^2-1\right)}{3}-\frac{a\,c^2\,d^2}{2\,x^2}-\frac{b\,c^2\,d^2}{3\,x^2}-\frac{3\,b\,c^3\,d^2}{4\,x}-\frac{a\,d^2}{4\,x^4}-\frac{2\,a\,c\,d^2}{3\,x^3}-\frac{b\,c\,d^2}{12\,x^3}-\frac{b\,d^2\,\mathrm{atanh}\left(c\,x\right)}{4\,x^4}-\frac{3\,b\,c^5\,d^2\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{4\,\sqrt{-c^2}}-\frac{2\,b\,c\,d^2\,\mathrm{atanh}\left(c\,x\right)}{3\,x^3}-\frac{b\,c^2\,d^2\,\mathrm{atanh}\left(c\,x\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^5,x)
```

```
[Out] (2*b*c^4*d^2*log(x))/3 - (b*c^4*d^2*log(c^2*x^2 - 1))/3 - (a*c^2*d^2)/(2*x^
2) - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (a*d^2)/(4*x^4) - (2*a*c*d
^2)/(3*x^3) - (b*c*d^2)/(12*x^3) - (b*d^2*atanh(c*x))/(4*x^4) - (3*b*c^5*d^
2*atan((c^2*x)/(-c^2)^(1/2)))/(4*(-c^2)^(1/2)) - (2*b*c*d^2*atanh(c*x))/(3*
x^3) - (b*c^2*d^2*atanh(c*x))/(2*x^2)
```

3.19 $\int \frac{(d+cdx)^2(a+b\tanh^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=161

$$-\frac{bcd^2}{20x^4}-\frac{bc^2d^2}{6x^3}-\frac{4bc^3d^2}{15x^2}-\frac{bc^4d^2}{2x}-\frac{d^2(a+b\tanh^{-1}(cx))}{5x^5}-\frac{cd^2(a+b\tanh^{-1}(cx))}{2x^4}-\frac{c^2d^2(a+b\tanh^{-1}(cx))}{3x^3}+\frac{8}{15}$$

```
[Out] -1/20*b*c*d^2/x^4-1/6*b*c^2*d^2/x^3-4/15*b*c^3*d^2/x^2-1/2*b*c^4*d^2/x-1/5*
d^2*(a+b*arctanh(c*x))/x^5-1/2*c*d^2*(a+b*arctanh(c*x))/x^4-1/3*c^2*d^2*(a+
b*arctanh(c*x))/x^3+8/15*b*c^5*d^2*ln(x)-31/60*b*c^5*d^2*ln(-c*x+1)-1/60*b*
c^5*d^2*ln(c*x+1)
```

Rubi [A]
time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {45, 6083, 12, 1816}

$$-\frac{c^2d^2(a+b\tanh^{-1}(cx))}{3x^3}-\frac{d^2(a+b\tanh^{-1}(cx))}{5x^5}-\frac{cd^2(a+b\tanh^{-1}(cx))}{2x^4}+\frac{8}{15}bc^5d^2\log(x)-\frac{31}{60}bc^5d^2\log(1-cx)-\frac{1}{60}bc^5d^2\log(cx+1)-\frac{bc^4d^2}{2x}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2}{6x^3}-\frac{bcd^2}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^2)/x^4 - (b*c^2*d^2)/(6*x^3) - (4*b*c^3*d^2)/(15*x^2) - (b*c^4
*d^2)/(2*x) - (d^2*(a + b*ArcTanh[c*x]))/(5*x^5) - (c*d^2*(a + b*ArcTanh[c*
x]))/(2*x^4) - (c^2*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log[x]
)/15 - (31*b*c^5*d^2*Log[1 - c*x])/60 - (b*c^5*d^2*Log[1 + c*x])/60
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}-\frac{c^2d^2\left(a+b\tanh^{-1}\right.}{3x^3}$$
$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}-\frac{c^2d^2\left(a+b\tanh^{-1}\right.}{3x^3}$$
$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}-\frac{c^2d^2\left(a+b\tanh^{-1}\right.}{3x^3}$$
$$= -\frac{bcd^2}{20x^4}-\frac{bc^2d^2}{6x^3}-\frac{4bc^3d^2}{15x^2}-\frac{bc^4d^2}{2x}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{cd^2\left(a+\right.}{}$$

Mathematica [A]
time = 0.04, size = 122, normalized size = 0.76

$$-\frac{d^2\left(12a+30acx+3bcx+20ac^2x^2+10bc^2x^2+16bc^3x^3+30bc^4x^4+2b(6+15cx+10c^2x^2)\tanh^{-1}(cx)-32bc^5x^5\log(x)+31bc^5x^5\log(1-cx)+bc^5x^5\log(1+cx)\right)}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/60*(d^2*(12*a + 30*a*c*x + 3*b*c*x + 20*a*c^2*x^2 + 10*b*c^2*x^2 + 16*b*
c^3*x^3 + 30*b*c^4*x^4 + 2*b*(6 + 15*c*x + 10*c^2*x^2)*ArcTanh[c*x] - 32*b*
c^5*x^5*Log[x] + 31*b*c^5*x^5*Log[1 - c*x] + b*c^5*x^5*Log[1 + c*x]))/x^5
```

Maple [A]
time = 0.17, size = 166, normalized size = 1.03

method	result
derivativedivides	$c^5\left(d^2a\left(-\frac{1}{2c^4x^4}-\frac{1}{3c^3x^3}-\frac{1}{5c^5x^5}\right)-\frac{d^2b\,\mathrm{arctanh}(cx)}{2c^4x^4}-\frac{d^2b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{d^2b\,\mathrm{arctanh}(cx)}{5c^5x^5}-\frac{31d^2b\ln(c}{60}\right.$
default	$c^5\left(d^2a\left(-\frac{1}{2c^4x^4}-\frac{1}{3c^3x^3}-\frac{1}{5c^5x^5}\right)-\frac{d^2b\,\mathrm{arctanh}(cx)}{2c^4x^4}-\frac{d^2b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{d^2b\,\mathrm{arctanh}(cx)}{5c^5x^5}-\frac{31d^2b\ln(c}{60}\right.$
risch	$-\frac{d^2b(10c^2x^2+15cx+6)\ln(cx+1)}{60x^5}-\frac{d^2\left(b\,c^5\ln(cx+1)x^5-32c^5b\ln(-x)x^5+31x^5b\ln(-cx+1)c^5+30c^4x^4b+16b\,c^3x^3-1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(d^2*a*(-1/2/c^4/x^4-1/3/c^3/x^3-1/5/c^5/x^5)-1/2*d^2*b*arctanh(c*x)/c^
4/x^4-1/3*d^2*b*arctanh(c*x)/c^3/x^3-1/5*d^2*b*arctanh(c*x)/c^5/x^5-31/60*d
^2*b*ln(c*x-1)-1/60*d^2*b*ln(c*x+1)-1/20*d^2*b/c^4/x^4-1/6*d^2*b/c^3/x^3-4/
15*d^2*b/c^2/x^2-1/2*d^2*b/c/x+8/15*d^2*b*ln(c*x))
```

Maxima [A]
time = 0.27, size = 194, normalized size = 1.20

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\left(cx\right)}{x^3}\right)bc^2d^2+\frac{1}{12}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\mathrm{artanh}\left(cx\right)}{x^4}\right)bcd^2-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\mathrm{artanh}\left(cx\right)}{x^5}\right)bd^2-\frac{ac^2d^2}{3\,x^3}-\frac{acd^2}{2\,x^4}-\frac{ad^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^2*d^2 + 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2
+ 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 - 1) -
 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^2 - 1/3*
a*c^2*d^2/x^3 - 1/2*a*c*d^2/x^4 - 1/5*a*d^2/x^5
```

Fricas [A]
time = 0.37, size = 156, normalized size = 0.97

$$-\frac{bc^5d^2x^5\log\left(cx+1\right)+31\,bc^5d^2x^5\log\left(cx-1\right)-32\,bc^5d^2x^5\log\left(x\right)+30\,bc^4d^2x^4+16\,bc^3d^2x^3+10\left(2\,a+b\right)c^2d^2x^2+3\left(10\,a+b\right)cd^2x+12\,ad^2+\left(10\,bc^2d^2x^2+15\,bcd^2x+6\,bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(b*c^5*d^2*x^5*log(c*x + 1) + 31*b*c^5*d^2*x^5*log(c*x - 1) - 32*b*c^
5*d^2*x^5*log(x) + 30*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a + b)*c^2*d
^2*x^2 + 3*(10*a + b)*c*d^2*x + 12*a*d^2 + (10*b*c^2*d^2*x^2 + 15*b*c*d^2*x
 + 6*b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A]
time = 0.66, size = 199, normalized size = 1.24

$$\begin{cases} -\frac{ac^2d^2}{3x^3}-\frac{acd^2}{2x^4}-\frac{ad^2}{5x^5}+\frac{8bc^5d^2\log\left(x\right)}{15}-\frac{8bc^5d^2\log\left(x-\frac{1}{c}\right)}{15}-\frac{bc^5d^2\,\mathrm{atanh}\left(cx\right)}{30}-\frac{bc^4d^2}{2x}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2\,\mathrm{atanh}\left(cx\right)}{3x^3}-\frac{bc^2d^2}{6x^3}-\frac{bcd^2\,\mathrm{atanh}\left(cx\right)}{2x^4}-\frac{bcd^2}{20x^4}-\frac{bd^2\,\mathrm{atanh}\left(cx\right)}{5x^5} & \text{for}\: c \neq 0 \\ -\frac{ad^2}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**2*d**2/(3*x**3) - a*c*d**2/(2*x**4) - a*d**2/(5*x**5) + 8*
b*c**5*d**2*log(x)/15 - 8*b*c**5*d**2*log(x - 1/c)/15 - b*c**5*d**2*atanh(c
*x)/30 - b*c**4*d**2/(2*x) - 4*b*c**3*d**2/(15*x**2) - b*c**2*d**2*atanh(c*
x)/(3*x**3) - b*c**2*d**2/(6*x**3) - b*c*d**2*atanh(c*x)/(2*x**4) - b*c*d**
```

```
2/(20*x**4) - b*d**2*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**2/(5*x**5), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(141) = 282.
time = 0.42, size = 532, normalized size = 3.30

$$\frac{4}{15}\left(2\,bc^4d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^4d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{15\,(cx+1)^4bc^4d^2}{(cx-1)^4}+\frac{15\,(cx+1)^3bc^4d^2}{(cx-1)^3}+\frac{20\,(cx+1)^2bc^4d^2}{(cx-1)^2}+\frac{10\,(cx+1)bc^4d^2}{cx-1}+2\,bc^4d^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}+\frac{\frac{30\,(cx+1)^4ac^4d^2}{(cx-1)^4}+\frac{30\,(cx+1)^3ac^4d^2}{(cx-1)^3}+\frac{40\,(cx+1)^2ac^4d^2}{(cx-1)^2}+\frac{20\,(cx+1)ac^4d^2}{cx-1}+4\,ac^4d^2+\frac{13\,(cx+1)^4bc^4d^2}{(cx-1)^4}+\frac{36\,(cx+1)^3bc^4d^2}{(cx-1)^3}+\frac{41\,(cx+1)^2bc^4d^2}{(cx-1)^2}+\frac{23\,(cx+1)bc^4d^2}{cx-1}+5\,bc^4d^2}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(2*b*c^4*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^4*d^2*log(-(c*x + 1
)/(c*x - 1)) + (15*(c*x + 1)^4*b*c^4*d^2/(c*x - 1)^4 + 15*(c*x + 1)^3*b*c^4
*d^2/(c*x - 1)^3 + 20*(c*x + 1)^2*b*c^4*d^2/(c*x - 1)^2 + 10*(c*x + 1)*b*c^
4*d^2/(c*x - 1) + 2*b*c^4*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x
- 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x +
 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (30*(c*x + 1)^4*a*c^4*d^2/
(c*x - 1)^4 + 30*(c*x + 1)^3*a*c^4*d^2/(c*x - 1)^3 + 40*(c*x + 1)^2*a*c^4*d
^2/(c*x - 1)^2 + 20*(c*x + 1)*a*c^4*d^2/(c*x - 1) + 4*a*c^4*d^2 + 13*(c*x +
 1)^4*b*c^4*d^2/(c*x - 1)^4 + 36*(c*x + 1)^3*b*c^4*d^2/(c*x - 1)^3 + 41*(c*
x + 1)^2*b*c^4*d^2/(c*x - 1)^2 + 23*(c*x + 1)*b*c^4*d^2/(c*x - 1) + 5*b*c^4
*d^2)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3
/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 0.95, size = 182, normalized size = 1.13

$$-\frac{12\,a\,d^2+12\,b\,d^2\,\mathrm{atanh}(c\,x)+20\,a\,c^2\,d^2\,x^2+10\,b\,c^2\,d^2\,x^2+16\,b\,c^3\,d^2\,x^3+30\,b\,c^4\,d^2\,x^4+30\,a\,c\,d^2\,x+3\,b\,c\,d^2\,x-32\,b\,c^5\,d^2\,x^5\,\ln(x)+20\,b\,c^2\,d^2\,x^2\,\mathrm{atanh}(c\,x)+16\,b\,c^5\,d^2\,x^5\,\ln\left(c^2\,x^2-1\right)+30\,b\,c\,d^2\,x\,\mathrm{atanh}(c\,x)-30\,b\,c^4\,d^2\,x^5\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)\,\sqrt{-c^2}}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^6,x)
```

```
[Out] -(12*a*d^2 + 12*b*d^2*atanh(c*x) + 20*a*c^2*d^2*x^2 + 10*b*c^2*d^2*x^2 + 16
*b*c^3*d^2*x^3 + 30*b*c^4*d^2*x^4 + 30*a*c*d^2*x + 3*b*c*d^2*x - 32*b*c^5*d
^2*x^5*log(x) + 20*b*c^2*d^2*x^2*atanh(c*x) + 16*b*c^5*d^2*x^5*log(c^2*x^2
- 1) + 30*b*c*d^2*x*atanh(c*x) - 30*b*c^4*d^2*x^5*atan((c^2*x)/(-c^2)^(1/2)
)*(-c^2)^(1/2))/(60*x^5)
```

3.20 $\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=192

$$\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(c\right.$$

```
[Out] 3/4*b*d^3*x/c^3+13/35*b*d^3*x^2/c^2+1/4*b*d^3*x^3/c+13/70*b*d^3*x^4+1/10*b*
c*d^3*x^5+1/42*b*c^2*d^3*x^6+1/4*d^3*x^4*(a+b*arctanh(c*x))+3/5*c*d^3*x^5*(
a+b*arctanh(c*x))+1/2*c^2*d^3*x^6*(a+b*arctanh(c*x))+1/7*c^3*d^3*x^7*(a+b*a
rctanh(c*x))+209/280*b*d^3*ln(-c*x+1)/c^4-1/280*b*d^3*ln(c*x+1)/c^4
```

Rubi [A]
time = 0.13, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {45, 6083, 12, 1816, 647, 31}

$$\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{209bd^3\log(1-cx)}{280c^4}-\frac{bd^3\log(cx+1)}{280c^4}+\frac{3bd^3x}{4c^3}+\frac{1}{42}bc^2d^3x^6+\frac{13bd^3x^2}{35c^2}+\frac{1}{10}bcd^3x^5+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(4*c^3) + (13*b*d^3*x^2)/(35*c^2) + (b*d^3*x^3)/(4*c) + (13*b*d
^3*x^4)/70 + (b*c*d^3*x^5)/10 + (b*c^2*d^3*x^6)/42 + (d^3*x^4*(a + b*ArcTan
h[c*x]))/4 + (3*c*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^3*x^6*(a + b*Arc
Tanh[c*x]))/2 + (c^3*d^3*x^7*(a + b*ArcTanh[c*x]))/7 + (209*b*d^3*Log[1 - c
*x])/(280*c^4) - (b*d^3*Log[1 + c*x])/(280*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(\right.\\ &= \frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(\right.\\ &= \frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(\right.\\ &= \frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\\ &= \frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\\ &= \frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\end{aligned}$$

Mathematica [A]
time = 0.05, size = 151, normalized size = 0.79

$$\frac{d^3\left(630bcx+312bc^2x^2+210bc^3x^3+210ac^4x^4+156bc^4x^4+504ac^5x^5+84bc^5x^5+420ac^6x^6+20bc^6x^6+120ac^7x^7+6bc^4x^4(35+84cx+70c^2x^2+20c^3x^3)\tanh^{-1}(cx)+627b\log(1-cx)-3b\log(1+cx)\right)}{840c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(630*b*c*x + 312*b*c^2*x^2 + 210*b*c^3*x^3 + 210*a*c^4*x^4 + 156*b*c^4
*x^4 + 504*a*c^5*x^5 + 84*b*c^5*x^5 + 420*a*c^6*x^6 + 20*b*c^6*x^6 + 120*a*
c^7*x^7 + 6*b*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x]
+ 627*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(840*c^4)
```

Maple [A]
time = 0.22, size = 200, normalized size = 1.04

method	result
derivativedivides	$\frac{d^3 a\left(\frac{1}{7}c^7x^7+\frac{1}{2}c^6x^6+\frac{3}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^3 b \operatorname{arctanh}(cx)c^7x^7}{7}+\frac{d^3 b \operatorname{arctanh}(cx)c^6x^6}{2}+\frac{3d^3 b \operatorname{arctanh}(cx)c^5x^5}{5}+\frac{d^3 b \operatorname{arctanh}(cx)c^4x^4}{4}}{c^4}$
default	$\frac{d^3 a\left(\frac{1}{7}c^7x^7+\frac{1}{2}c^6x^6+\frac{3}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^3 b \operatorname{arctanh}(cx)c^7x^7}{7}+\frac{d^3 b \operatorname{arctanh}(cx)c^6x^6}{2}+\frac{3d^3 b \operatorname{arctanh}(cx)c^5x^5}{5}+\frac{d^3 b \operatorname{arctanh}(cx)c^4x^4}{4}}{c^4}$
risch	$\frac{d^3 b x^4(20x^3c^3+70c^2x^2+84cx+35)\ln(cx+1)}{280}-\frac{d^3c^3 b x^7\ln(-cx+1)}{14}+\frac{d^3c^3 a x^7}{7}-\frac{d^3c^2x^6 b\ln(-cx+1)}{4}+\frac{d^3c^2x^6 a}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^3*a*(1/7*c^7*x^7+1/2*c^6*x^6+3/5*c^5*x^5+1/4*c^4*x^4)+1/7*d^3*b*ar
ctanh(c*x)*c^7*x^7+1/2*d^3*b*arctanh(c*x)*c^6*x^6+3/5*d^3*b*arctanh(c*x)*c^
5*x^5+1/4*d^3*b*arctanh(c*x)*c^4*x^4+1/42*d^3*b*c^6*x^6+1/10*d^3*b*c^5*x^5+
13/70*d^3*b*c^4*x^4+1/4*d^3*b*c^3*x^3+13/35*b*c^2*d^3*x^2+3/4*b*c*d^3*x+209
/280*d^3*b*ln(c*x-1)-1/280*d^3*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 285, normalized size = 1.48

$$\frac{1}{7}ac^3d^3x^7+\frac{1}{2}ac^2d^3x^6+\frac{3}{5}acd^3x^5+\frac{1}{84}\left(12x^7\operatorname{artanh}(cx)+c\left(\frac{2c^4x^6+3c^2x^4+6x^2}{c^6}+\frac{6\log(c^2x^2-1)}{c^8}\right)\right)bc^3d^3+\frac{1}{4}ad^3x^4+\frac{1}{60}\left(30x^6\operatorname{artanh}(cx)+c\left(\frac{2(3c^4x^5+5c^2x^3+15x)}{c^6}-\frac{15\log(cx+1)}{c^7}+\frac{15\log(cx-1)}{c^7}\right)\right)bc^2d^3+\frac{3}{20}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log(c^2x^2-1)}{c^6}\right)\right)bcd^3+\frac{1}{24}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^3*d^3*x^7 + 1/2*a*c^2*d^3*x^6 + 3/5*a*c*d^3*x^5 + 1/84*(12*x^7*arct
anh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8)
)*b*c^3*d^3 + 1/4*a*d^3*x^4 + 1/60*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 +
 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*c^2*
d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
 1)/c^6))*b*c*d^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^3
```

Fricas [A]
time = 0.37, size = 190, normalized size = 0.99

$$\frac{120ac^7d^3x^7+20(21a+b)c^6d^3x^6+84(6a+b)c^5d^3x^5+6(35a+26b)c^4d^3x^4+210bc^3d^3x^3+312bc^2d^3x^2+630bcd^3x-3bd^3\log(cx+1)+627bd^3\log(cx-1)+3(20bc^7d^3x^7+70bc^6d^3x^6+84bc^5d^3x^5+35bc^4d^3x^4)\log\left(-\frac{cx+1}{cx-1}\right)}{840c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(120*a*c^7*d^3*x^7 + 20*(21*a + b)*c^6*d^3*x^6 + 84*(6*a + b)*c^5*d^3
*x^5 + 6*(35*a + 26*b)*c^4*d^3*x^4 + 210*b*c^3*d^3*x^3 + 312*b*c^2*d^3*x^2
+ 630*b*c*d^3*x - 3*b*d^3*log(c*x + 1) + 627*b*d^3*log(c*x - 1) + 3*(20*b*c
^7*d^3*x^7 + 70*b*c^6*d^3*x^6 + 84*b*c^5*d^3*x^5 + 35*b*c^4*d^3*x^4)*log(-(
c*x + 1)/(c*x - 1)))/c^4
```

Sympy [**A**]
time = 0.56, size = 243, normalized size = 1.27

$$\begin{cases} \frac{ac^3d^3x^7}{7} + \frac{ac^2d^3x^6}{2} + \frac{3acd^3x^5}{5} + \frac{ad^3x^4}{4} + \frac{bc^3d^3x^7\operatorname{atanh}(cx)}{7} + \frac{bc^2d^3x^6\operatorname{atanh}(cx)}{2} + \frac{bc^2d^3x^6}{42} + \frac{3bcd^3x^5\operatorname{atanh}(cx)}{5} + \frac{bcd^3x^5}{10} + \frac{bd^3x^4\operatorname{atanh}(cx)}{4} + \frac{13bd^3x^4}{70} + \frac{bd^3x^3}{4c} + \frac{13bd^3x^2}{35c^2} + \frac{3bd^3x}{4c^3} + \frac{26bd^3\log\left(x-\frac{1}{c}\right)}{35c^4} - \frac{bd^3\operatorname{atanh}(cx)}{140c^4} & \text{for}\: c \neq 0 \\ \frac{ad^3x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**7/7 + a*c**2*d**3*x**6/2 + 3*a*c*d**3*x**5/5 + a*
d**3*x**4/4 + b*c**3*d**3*x**7*atanh(c*x)/7 + b*c**2*d**3*x**6*atanh(c*x)/2
 + b*c**2*d**3*x**6/42 + 3*b*c*d**3*x**5*atanh(c*x)/5 + b*c*d**3*x**5/10 +
b*d**3*x**4*atanh(c*x)/4 + 13*b*d**3*x**4/70 + b*d**3*x**3/(4*c) + 13*b*d**
3*x**2/(35*c**2) + 3*b*d**3*x/(4*c**3) + 26*b*d**3*log(x - 1/c)/(35*c**4) -
 b*d**3*atanh(c*x)/(140*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(168) = 336.
time = 0.44, size = 722, normalized size = 3.76

$$\frac{1}{105}c\left(\frac{6\left(\frac{140(cx+1)^6bd^3}{(cx-1)^6} - \frac{210(cx+1)^5bd^3}{(cx-1)^5} + \frac{490(cx+1)^4bd^3}{(cx-1)^4} - \frac{455(cx+1)^3bd^3}{(cx-1)^3} + \frac{273(cx+1)^2bd^3}{(cx-1)^2} - \frac{91(cx+1)bd^3}{cx-1} + 13\,bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^7c^5}{(cx-1)^7} - \frac{7(cx+1)^6c^5}{(cx-1)^6} + \frac{21(cx+1)^5c^5}{(cx-1)^5} - \frac{35(cx+1)^4c^5}{(cx-1)^4} + \frac{35(cx+1)^3c^5}{(cx-1)^3} - \frac{21(cx+1)^2c^5}{(cx-1)^2} + \frac{7(cx+1)c^5}{cx-1} - c^5} + \frac{\frac{1680(cx+1)^6ad^3}{(cx-1)^6} - \frac{2520(cx+1)^5ad^3}{(cx-1)^5} + \frac{5880(cx+1)^4ad^3}{(cx-1)^4} - \frac{5460(cx+1)^3ad^3}{(cx-1)^3} + \frac{3276(cx+1)^2ad^3}{(cx-1)^2} - \frac{1092(cx+1)ad^3}{cx-1} + 156\,ad^3 + \frac{762(cx+1)^6bd^3}{(cx-1)^6} - \frac{3063(cx+1)^5bd^3}{(cx-1)^5} + \frac{5959(cx+1)^4bd^3}{(cx-1)^4} - \frac{6694(cx+1)^3bd^3}{(cx-1)^3} + \frac{4344(cx+1)^2bd^3}{(cx-1)^2} - \frac{1539(cx+1)bd^3}{cx-1} + 231\,bd^3}{\frac{(cx+1)^7c^5}{(cx-1)^7} - \frac{7(cx+1)^6c^5}{(cx-1)^6} + \frac{21(cx+1)^5c^5}{(cx-1)^5} - \frac{35(cx+1)^4c^5}{(cx-1)^4} + \frac{35(cx+1)^3c^5}{(cx-1)^3} - \frac{21(cx+1)^2c^5}{(cx-1)^2} + \frac{7(cx+1)c^5}{cx-1} - c^5} - \frac{78\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^5} + \frac{78\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/105*c*(6*(140*(c*x + 1)^6*b*d^3/(c*x - 1)^6 - 210*(c*x + 1)^5*b*d^3/(c*x
- 1)^5 + 490*(c*x + 1)^4*b*d^3/(c*x - 1)^4 - 455*(c*x + 1)^3*b*d^3/(c*x - 1
)^3 + 273*(c*x + 1)^2*b*d^3/(c*x - 1)^2 - 91*(c*x + 1)*b*d^3/(c*x - 1) + 13
*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^7*c^5/(c*x - 1)^7 - 7*(c*x + 1
)^6*c^5/(c*x - 1)^6 + 21*(c*x + 1)^5*c^5/(c*x - 1)^5 - 35*(c*x + 1)^4*c^5/(
c*x - 1)^4 + 35*(c*x + 1)^3*c^5/(c*x - 1)^3 - 21*(c*x + 1)^2*c^5/(c*x - 1)^
2 + 7*(c*x + 1)*c^5/(c*x - 1) - c^5) + (1680*(c*x + 1)^6*a*d^3/(c*x - 1)^6
- 2520*(c*x + 1)^5*a*d^3/(c*x - 1)^5 + 5880*(c*x + 1)^4*a*d^3/(c*x - 1)^4 -
 5460*(c*x + 1)^3*a*d^3/(c*x - 1)^3 + 3276*(c*x + 1)^2*a*d^3/(c*x - 1)^2 -
1092*(c*x + 1)*a*d^3/(c*x - 1) + 156*a*d^3 + 762*(c*x + 1)^6*b*d^3/(c*x - 1
)^6 - 3063*(c*x + 1)^5*b*d^3/(c*x - 1)^5 + 5959*(c*x + 1)^4*b*d^3/(c*x - 1)
^4 - 6694*(c*x + 1)^3*b*d^3/(c*x - 1)^3 + 4344*(c*x + 1)^2*b*d^3/(c*x - 1)^
2 - 1539*(c*x + 1)*b*d^3/(c*x - 1) + 231*b*d^3)/((c*x + 1)^7*c^5/(c*x - 1)^
7 - 7*(c*x + 1)^6*c^5/(c*x - 1)^6 + 21*(c*x + 1)^5*c^5/(c*x - 1)^5 - 35*(c*
```

```
x + 1)^4*c^5/(c*x - 1)^4 + 35*(c*x + 1)^3*c^5/(c*x - 1)^3 - 21*(c*x + 1)^2*
c^5/(c*x - 1)^2 + 7*(c*x + 1)*c^5/(c*x - 1) - c^5) - 78*b*d^3*log(-(c*x + 1
)/(c*x - 1) + 1)/c^5 + 78*b*d^3*log(-(c*x + 1)/(c*x - 1))/c^5)
```

Mupad [B]

time = 1.05, size = 177, normalized size = 0.92

$$\frac{\frac{13\,b\,c^2\,d^3\,x^2}{35}-\frac{d^3\,\left(315\,b\,\mathrm{atanh}\left(c\,x\right)-156\,b\,\ln\left(c^2\,x^2-1\right)\right)}{420}+\frac{b\,c^3\,d^3\,x^3}{4}+\frac{3\,b\,c\,d^3\,x}{4}}{c^4}+\frac{d^3\,\left(105\,a\,x^4+78\,b\,x^4+105\,b\,x^4\,\mathrm{atanh}\left(c\,x\right)\right)}{420}+\frac{c^3\,d^3\,\left(60\,a\,x^7+60\,b\,x^7\,\mathrm{atanh}\left(c\,x\right)\right)}{420}+\frac{c\,d^3\,\left(252\,a\,x^5+42\,b\,x^5+252\,b\,x^5\,\mathrm{atanh}\left(c\,x\right)\right)}{420}+\frac{c^2\,d^3\,\left(210\,a\,x^6+10\,b\,x^6+210\,b\,x^6\,\mathrm{atanh}\left(c\,x\right)\right)}{420}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] ((13*b*c^2*d^3*x^2)/35 - (d^3*(315*b*atanh(c*x) - 156*b*log(c^2*x^2 - 1)))/
420 + (b*c^3*d^3*x^3)/4 + (3*b*c*d^3*x)/4)/c^4 + (d^3*(105*a*x^4 + 78*b*x^4
 + 105*b*x^4*atanh(c*x)))/420 + (c^3*d^3*(60*a*x^7 + 60*b*x^7*atanh(c*x)))/
420 + (c*d^3*(252*a*x^5 + 42*b*x^5 + 252*b*x^5*atanh(c*x)))/420 + (c^2*d^3*
(210*a*x^6 + 10*b*x^6 + 210*b*x^6*atanh(c*x)))/420
```

3.21 $\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=178

$$\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}$$

```
[Out] 11/12*b*d^3*x/c^2+7/15*b*d^3*x^2/c+11/36*b*d^3*x^3+3/20*b*c*d^3*x^4+1/30*b*
c^2*d^3*x^5+1/3*d^3*x^3*(a+b*arctanh(c*x))+3/4*c*d^3*x^4*(a+b*arctanh(c*x))
+3/5*c^2*d^3*x^5*(a+b*arctanh(c*x))+1/6*c^3*d^3*x^6*(a+b*arctanh(c*x))+37/4
0*b*d^3*ln(-c*x+1)/c^3+1/120*b*d^3*ln(c*x+1)/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {45, 6083, 12, 1816, 647, 31}

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{37bd^3\log(1-cx)}{40c^3}+\frac{bd^3\log(cx+1)}{120c^3}+\frac{1}{30}bc^2d^3x^5+\frac{11bd^3x}{12c^2}+\frac{3}{20}bcd^3x^4+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (11*b*d^3*x)/(12*c^2) + (7*b*d^3*x^2)/(15*c) + (11*b*d^3*x^3)/36 + (3*b*c*d
^3*x^4)/20 + (b*c^2*d^3*x^5)/30 + (d^3*x^3*(a + b*ArcTanh[c*x]))/3 + (3*c*d
^3*x^4*(a + b*ArcTanh[c*x]))/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x]))/6 + (37*b*d^3*Log[1 - c*x])/(40*c^3) + (b
*d^3*Log[1 + c*x])/(120*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx = \frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+\right.$$

Mathematica [A]
time = 0.05, size = 142, normalized size = 0.80

$$\frac{d^3\left(330bcx+168bc^2x^2+120ac^3x^3+110bc^3x^3+270ac^4x^4+54bc^4x^4+216ac^5x^5+12bc^5x^5+60ac^6x^6+6bc^3x^3(20+45cx+36c^2x^2+10c^3x^3)\tanh^{-1}(cx)+333b\log(1-cx)+3b\log(1+cx)\right)}{360c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(330*b*c*x + 168*b*c^2*x^2 + 120*a*c^3*x^3 + 110*b*c^3*x^3 + 270*a*c^4
*x^4 + 54*b*c^4*x^4 + 216*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^3
*x^3*(20 + 45*c*x + 36*c^2*x^2 + 10*c^3*x^3)*ArcTanh[c*x] + 333*b*Log[1 - c
*x] + 3*b*Log[1 + c*x]))/(360*c^3)
```

Maple [A]
time = 0.18, size = 188, normalized size = 1.06

method	result
derivativedivides	$\frac{d^3 a\left(\frac{1}{6}c^6x^6+\frac{3}{5}c^5x^5+\frac{3}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{d^3 b\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{3d^3 b\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3d^3 b\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{d^3 b\operatorname{arctanh}(cx)c^3}{3}}{c^3}$
default	$\frac{d^3 a\left(\frac{1}{6}c^6x^6+\frac{3}{5}c^5x^5+\frac{3}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{d^3 b\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{3d^3 b\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3d^3 b\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{d^3 b\operatorname{arctanh}(cx)c^3}{3}}{c^3}$
risch	$\frac{d^3 b\,x^3\left(10x^3c^3+36c^2x^2+45cx+20\right)\ln(cx+1)}{120}-\frac{d^3c^3x^6 b\ln(-cx+1)}{12}+\frac{d^3c^3x^6 a}{6}-\frac{3d^3c^2x^5 b\ln(-cx+1)}{10}+\frac{3d^3c^2x^5}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^3*a*(1/6*c^6*x^6+3/5*c^5*x^5+3/4*c^4*x^4+1/3*x^3*c^3)+1/6*d^3*b*ar
ctanh(c*x)*c^6*x^6+3/5*d^3*b*arctanh(c*x)*c^5*x^5+3/4*d^3*b*arctanh(c*x)*c^
4*x^4+1/3*d^3*b*arctanh(c*x)*c^3*x^3+1/30*d^3*b*c^5*x^5+3/20*d^3*b*c^4*x^4+
11/36*d^3*b*c^3*x^3+7/15*b*c^2*d^3*x^2+11/12*b*c*d^3*x+37/40*d^3*b*ln(c*x-1
)+1/120*d^3*b*ln(c*x+1))
```

Maxima [A]
time = 0.25, size = 265, normalized size = 1.49

$$\frac{1}{6}\,ac^3d^3x^6+\frac{3}{5}\,ac^2d^3x^5+\frac{3}{4}\,acd^3x^4+\frac{1}{180}\left(30\,x^6\operatorname{artanh}(cx)+c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log(cx+1)}{c^7}+\frac{15\,\log(cx-1)}{c^7}\right)\right)bc^3d^3+\frac{3}{20}\left(4\,x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log(c^2x^2-1)}{c^6}\right)\right)bc^2d^3+\frac{1}{3}\,ad^3x^3+\frac{1}{8}\left(6\,x^4\operatorname{artanh}(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log(cx+1)}{c^5}+\frac{3\,\log(cx-1)}{c^5}\right)\right)bcd^3+\frac{1}{6}\left(2\,x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^3*d^3*x^6 + 3/5*a*c^2*d^3*x^5 + 3/4*a*c*d^3*x^4 + 1/180*(30*x^6*arc
tanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 +
 15*log(c*x - 1)/c^7))*b*c^3*d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 +
 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/8*(6*x
^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
 - 1)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
1)/c^4))*b*d^3
```

Fricas [A]
time = 0.39, size = 178, normalized size = 1.00

$$\frac{60\,ac^6d^3x^6+12\,(18\,a+b)c^5d^3x^5+54\,(5\,a+b)c^4d^3x^4+10\,(12\,a+11\,b)c^3d^3x^3+168\,bc^2d^3x^2+330\,bcd^3x+3\,bd^3\log(cx+1)+333\,bd^3\log(cx-1)+3\,(10\,bc^6d^3x^6+36\,bc^5d^3x^5+45\,bc^4d^3x^4+20\,bc^3d^3x^3)\log\left(-\frac{cx+1}{cx-1}\right)}{360\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(60*a*c^6*d^3*x^6 + 12*(18*a + b)*c^5*d^3*x^5 + 54*(5*a + b)*c^4*d^3*
x^4 + 10*(12*a + 11*b)*c^3*d^3*x^3 + 168*b*c^2*d^3*x^2 + 330*b*c*d^3*x + 3*
b*d^3*log(c*x + 1) + 333*b*d^3*log(c*x - 1) + 3*(10*b*c^6*d^3*x^6 + 36*b*c^
5*d^3*x^5 + 45*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3)*log(-(c*x + 1)/(c*x - 1)))
/c^3
```

Sympy [A]
time = 0.47, size = 235, normalized size = 1.32

$$\begin{cases} \frac{ac^3d^3x^6}{6} + \frac{3ac^2d^3x^5}{5} + \frac{3acd^3x^4}{4} + \frac{ad^3x^3}{3} + \frac{bc^3d^3x^6\operatorname{atanh}(cx)}{6} + \frac{3bc^2d^3x^5\operatorname{atanh}(cx)}{5} + \frac{bc^2d^3x^5}{30} + \frac{3bcd^3x^4\operatorname{atanh}(cx)}{4} + \frac{3bcd^3x^4}{20} + \frac{bd^3x^3\operatorname{atanh}(cx)}{3} + \frac{11bd^3x^3}{36} + \frac{7bd^3x^2}{15c} + \frac{11bd^3x}{12c^2} + \frac{14bd^3\log\left(x-\frac{1}{c}\right)}{15c^3} + \frac{bd^3\operatorname{atanh}(cx)}{60c^3} & \text{for}\ c \neq 0 \\ \frac{ad^3x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**6/6 + 3*a*c**2*d**3*x**5/5 + 3*a*c*d**3*x**4/4 +
a*d**3*x**3/3 + b*c**3*d**3*x**6*atanh(c*x)/6 + 3*b*c**2*d**3*x**5*atanh(c*
x)/5 + b*c**2*d**3*x**5/30 + 3*b*c*d**3*x**4*atanh(c*x)/4 + 3*b*c*d**3*x**4
/20 + b*d**3*x**3*atanh(c*x)/3 + 11*b*d**3*x**3/36 + 7*b*d**3*x**2/(15*c) +
 11*b*d**3*x/(12*c**2) + 14*b*d**3*log(x - 1/c)/(15*c**3) + b*d**3*atanh(c*
x)/(60*c**3), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(156) = 312.
time = 0.43, size = 621, normalized size = 3.49

$$-\frac{1}{45}\,c\left(\frac{42\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4} - \frac{6\left(\frac{60\,(cx+1)^5bd^3}{(cx-1)^5} - \frac{90\,(cx+1)^4bd^3}{(cx-1)^4} + \frac{140\,(cx+1)^3bd^3}{(cx-1)^3} - \frac{105\,(cx+1)^2bd^3}{(cx-1)^2} + \frac{42\,(cx+1)bd^3}{cx-1} - 7\,bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^4}{(cx-1)^6} - \frac{6\,(cx+1)^5c^4}{(cx-1)^5} + \frac{15\,(cx+1)^4c^4}{(cx-1)^4} - \frac{20\,(cx+1)^3c^4}{(cx-1)^3} + \frac{15\,(cx+1)^2c^4}{(cx-1)^2} - \frac{6\,(cx+1)c^4}{cx-1} + c^4} - \frac{42\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^4} - \frac{\frac{720\,(cx+1)^5ad^3}{(cx-1)^5} - \frac{1080\,(cx+1)^4ad^3}{(cx-1)^4} + \frac{1680\,(cx+1)^3ad^3}{(cx-1)^3} - \frac{1260\,(cx+1)^2ad^3}{(cx-1)^2} + \frac{504\,(cx+1)ad^3}{cx-1} - 84\,ad^3 + \frac{318\,(cx+1)^5bd^3}{(cx-1)^5} - \frac{1119\,(cx+1)^4bd^3}{(cx-1)^4} + \frac{1742\,(cx+1)^3bd^3}{(cx-1)^3} - \frac{1464\,(cx+1)^2bd^3}{(cx-1)^2} + \frac{636\,(cx+1)bd^3}{cx-1} - 113\,bd^3}{\frac{(cx+1)^6c^4}{(cx-1)^6} - \frac{6\,(cx+1)^5c^4}{(cx-1)^5} + \frac{15\,(cx+1)^4c^4}{(cx-1)^4} - \frac{20\,(cx+1)^3c^4}{(cx-1)^3} + \frac{15\,(cx+1)^2c^4}{(cx-1)^2} - \frac{6\,(cx+1)c^4}{cx-1} + c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/45*c*(42*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 - 6*(60*(c*x + 1)^5*b*d
^3/(c*x - 1)^5 - 90*(c*x + 1)^4*b*d^3/(c*x - 1)^4 + 140*(c*x + 1)^3*b*d^3/(
c*x - 1)^3 - 105*(c*x + 1)^2*b*d^3/(c*x - 1)^2 + 42*(c*x + 1)*b*d^3/(c*x -
1) - 7*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^4/(c*x - 1)^6 - 6*(c
*x + 1)^5*c^4/(c*x - 1)^5 + 15*(c*x + 1)^4*c^4/(c*x - 1)^4 - 20*(c*x + 1)^3
*c^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^4/(c*x - 1)^2 - 6*(c*x + 1)*c^4/(c*x -
1) + c^4) - 42*b*d^3*log(-(c*x + 1)/(c*x - 1))/c^4 - (720*(c*x + 1)^5*a*d^3
/(c*x - 1)^5 - 1080*(c*x + 1)^4*a*d^3/(c*x - 1)^4 + 1680*(c*x + 1)^3*a*d^3/
(c*x - 1)^3 - 1260*(c*x + 1)^2*a*d^3/(c*x - 1)^2 + 504*(c*x + 1)*a*d^3/(c*x
 - 1) - 84*a*d^3 + 318*(c*x + 1)^5*b*d^3/(c*x - 1)^5 - 1119*(c*x + 1)^4*b*d
^3/(c*x - 1)^4 + 1742*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 1464*(c*x + 1)^2*b*d^
3/(c*x - 1)^2 + 636*(c*x + 1)*b*d^3/(c*x - 1) - 113*b*d^3)/((c*x + 1)^6*c^4
/(c*x - 1)^6 - 6*(c*x + 1)^5*c^4/(c*x - 1)^5 + 15*(c*x + 1)^4*c^4/(c*x - 1)
```

```
^4 - 20*(c*x + 1)^3*c^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^4/(c*x - 1)^2 - 6*(c
*x + 1)*c^4/(c*x - 1) + c^4))
```

Mupad [B]
time = 1.03, size = 165, normalized size = 0.93

$$\frac{\frac{7\,b\,c^2\,d^3\,x^2}{15}-\frac{d^3\,\left(165\,b\,\mathrm{atanh}\left(c\,x\right)-84\,b\,\ln\left(c^2\,x^2-1\right)\right)}{180}+\frac{11\,b\,c\,d^3\,x}{12}}{c^3}+\frac{d^3\,\left(60\,a\,x^3+55\,b\,x^3+60\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{180}+\frac{c^3\,d^3\,\left(30\,a\,x^6+30\,b\,x^6\,\mathrm{atanh}\left(c\,x\right)\right)}{180}+\frac{c\,d^3\,\left(135\,a\,x^4+27\,b\,x^4+135\,b\,x^4\,\mathrm{atanh}\left(c\,x\right)\right)}{180}+\frac{c^2\,d^3\,\left(108\,a\,x^5+6\,b\,x^5+108\,b\,x^5\,\mathrm{atanh}\left(c\,x\right)\right)}{180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] ((7*b*c^2*d^3*x^2)/15 - (d^3*(165*b*atanh(c*x) - 84*b*log(c^2*x^2 - 1)))/18
0 + (11*b*c*d^3*x)/12)/c^3 + (d^3*(60*a*x^3 + 55*b*x^3 + 60*b*x^3*atanh(c*x
)))/180 + (c^3*d^3*(30*a*x^6 + 30*b*x^6*atanh(c*x)))/180 + (c*d^3*(135*a*x^
4 + 27*b*x^4 + 135*b*x^4*atanh(c*x)))/180 + (c^2*d^3*(108*a*x^5 + 6*b*x^5 +
 108*b*x^5*atanh(c*x)))/180
```

3.22 $\int x(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)\, dx$

Optimal. Leaf size=135

$$\frac{3bd^3x}{5c}+\frac{3bd^3(1+cx)^2}{20c^2}+\frac{bd^3(1+cx)^3}{20c^2}+\frac{bd^3(1+cx)^4}{20c^2}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\,\mathrm{tan}\right.}{5c^2}$$

```
[Out] 3/5*b*d^3*x/c+3/20*b*d^3*(c*x+1)^2/c^2+1/20*b*d^3*(c*x+1)^3/c^2+1/20*b*d^3*
(c*x+1)^4/c^2-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))/c^2+1/5*d^3*(c*x+1)^5*(a
+b*arctanh(c*x))/c^2+6/5*b*d^3*ln(-c*x+1)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {45, 6083, 12, 78}

$$\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{bd^3(cx+1)^4}{20c^2}+\frac{bd^3(cx+1)^3}{20c^2}+\frac{3bd^3(cx+1)^2}{20c^2}+\frac{6bd^3\log(1-cx)}{5c^2}+\frac{3bd^3x}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(5*c) + (3*b*d^3*(1 + c*x)^2)/(20*c^2) + (b*d^3*(1 + c*x)^3)/(2
0*c^2) + (b*d^3*(1 + c*x)^4)/(20*c^2) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x
]))/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (6*b*d^3*Log
[1 - c*x])/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}$$

$$= \frac{3bd^3x}{5c}+\frac{3bd^3(1+cx)^2}{20c^2}+\frac{bd^3(1+cx)^3}{20c^2}+\frac{bd^3(1+cx)^4}{20c^2}-\frac{d^3(1+cx}{}$$

Mathematica [A]
time = 0.04, size = 133, normalized size = 0.99

$$\frac{d^3\left(50bcx+20ac^2x^2+24bc^2x^2+40ac^3x^3+10bc^3x^3+30ac^4x^4+2bc^4x^4+8ac^5x^5+2bc^2x^2(10+20cx+15c^2x^2+4c^3x^3)\tanh^{-1}(cx)+49b\log(1-cx)-b\log(1+cx)\right)}{40c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(50*b*c*x + 20*a*c^2*x^2 + 24*b*c^2*x^2 + 40*a*c^3*x^3 + 10*b*c^3*x^3
+ 30*a*c^4*x^4 + 2*b*c^4*x^4 + 8*a*c^5*x^5 + 2*b*c^2*x^2*(10 + 20*c*x + 15*
c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x] + 49*b*Log[1 - c*x] - b*Log[1 + c*x]))/(4
0*c^2)
```

Maple [A]
time = 0.16, size = 174, normalized size = 1.29

method	result
derivativedivides	$\frac{d^3a\left(\frac{1}{5}c^5x^5+\frac{3}{4}c^4x^4+x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d^3b\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{3d^3b\,\mathrm{arctanh}(cx)c^4x^4}{4}+d^3b\,\mathrm{arctanh}(cx)c^3x^3+\frac{d^3b\,\mathrm{arctanh}(cx)c^2x}{2}}{c^2}$
default	$\frac{d^3a\left(\frac{1}{5}c^5x^5+\frac{3}{4}c^4x^4+x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d^3b\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{3d^3b\,\mathrm{arctanh}(cx)c^4x^4}{4}+d^3b\,\mathrm{arctanh}(cx)c^3x^3+\frac{d^3b\,\mathrm{arctanh}(cx)c^2x}{2}}{c^2}$
risch	$\frac{d^3b\,x^2\left(4x^3c^3+15c^2x^2+20cx+10\right)\ln(cx+1)}{40}-\frac{d^3c^3x^5b\ln(-cx+1)}{10}+\frac{d^3c^3x^5a}{5}-\frac{3d^3c^2x^4b\ln(-cx+1)}{8}+\frac{3d^3c^2x^4}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^3*a*(1/5*c^5*x^5+3/4*c^4*x^4+x^3*c^3+1/2*c^2*x^2)+1/5*d^3*b*arctan
h(c*x)*c^5*x^5+3/4*d^3*b*arctanh(c*x)*c^4*x^4+d^3*b*arctanh(c*x)*c^3*x^3+1/
2*d^3*b*arctanh(c*x)*c^2*x^2+1/20*d^3*b*c^4*x^4+1/4*d^3*b*c^3*x^3+3/5*b*c^2
*d^3*x^2+5/4*b*c*d^3*x+49/40*d^3*b*ln(c*x-1)-1/40*d^3*b*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(121) = 242$.
time = 0.27, size = 244, normalized size = 1.81

$$\frac{1}{5}\,ac^3d^3x^5 + \frac{3}{4}\,ac^2d^3x^4 + \frac{1}{20}\left(4\,x^5\operatorname{artanh}\left(cx\right) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2 - 1\right)}{c^6}\right)\right)bc^3d^3 + acd^3x^3 + \frac{1}{8}\left(6\,x^4\operatorname{artanh}\left(cx\right) + c\left(\frac{2\,(c^2x^3 + 3\,x)}{c^4} - \frac{3\,\log\left(cx+1\right)}{c^5} + \frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bc^2d^3 + \frac{1}{2}\left(2\,x^3\operatorname{artanh}\left(cx\right) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2 - 1\right)}{c^4}\right)\right)bcd^3 + \frac{1}{2}\,ad^3x^2 + \frac{1}{4}\left(2\,x^2\operatorname{artanh}\left(cx\right) + c\left(\frac{2\,x}{c^2} - \frac{\log\left(cx+1\right)}{c^3} + \frac{\log\left(cx-1\right)}{c^3}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^3*d^3*x^5 + 3/4*a*c^2*d^3*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*
x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^3 + a*c*d^3*x^3 + 1/8*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^2*d^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^3
```

Fricas [A]
time = 0.37, size = 165, normalized size = 1.22

$$\frac{8\,ac^5d^3x^5 + 2\,(15\,a + b)c^4d^3x^4 + 10\,(4\,a + b)c^3d^3x^3 + 4\,(5\,a + 6\,b)c^2d^3x^2 + 50\,bcd^3x - bd^3\log\left(cx + 1\right) + 49\,bd^3\log\left(cx - 1\right) + (4\,bc^5d^3x^5 + 15\,bc^4d^3x^4 + 20\,bc^3d^3x^3 + 10\,bc^2d^3x^2)\log\left(-\frac{cx+1}{cx-1}\right)}{40\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*d^3*x^5 + 2*(15*a + b)*c^4*d^3*x^4 + 10*(4*a + b)*c^3*d^3*x^3
 + 4*(5*a + 6*b)*c^2*d^3*x^2 + 50*b*c*d^3*x - b*d^3*log(c*x + 1) + 49*b*d^3
*log(c*x - 1) + (4*b*c^5*d^3*x^5 + 15*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3 + 10
*b*c^2*d^3*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A]
time = 0.40, size = 211, normalized size = 1.56

$$\begin{cases} \frac{ac^3d^3x^5}{5} + \frac{3ac^2d^3x^4}{4} + acd^3x^3 + \frac{ad^3x^2}{2} + \frac{bc^3d^3x^5\operatorname{atanh}\left(cx\right)}{5} + \frac{3bc^2d^3x^4\operatorname{atanh}\left(cx\right)}{4} + \frac{bc^2d^3x^4}{20} + bcd^3x^3\operatorname{atanh}\left(cx\right) + \frac{bcd^3x^3}{4} + \frac{bd^3x^2\operatorname{atanh}\left(cx\right)}{2} + \frac{3bd^3x^2}{5} + \frac{5bd^3x}{4c} + \frac{6bd^3\log\left(x - \frac{1}{c}\right)}{5c^2} - \frac{bd^3\operatorname{atanh}\left(cx\right)}{20c^2} & \text{for}\: c \neq 0 \\ \frac{ad^3x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**5/5 + 3*a*c**2*d**3*x**4/4 + a*c*d**3*x**3 + a*d*
*3*x**2/2 + b*c**3*d**3*x**5*atanh(c*x)/5 + 3*b*c**2*d**3*x**4*atanh(c*x)/4
 + b*c**2*d**3*x**4/20 + b*c*d**3*x**3*atanh(c*x) + b*c*d**3*x**3/4 + b*d**
3*x**2*atanh(c*x)/2 + 3*b*d**3*x**2/5 + 5*b*d**3*x/(4*c) + 6*b*d**3*log(x -
 1/c)/(5*c**2) - b*d**3*atanh(c*x)/(20*c**2), Ne(c, 0)), (a*d**3*x**2/2, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(121) = 242.

time = 0.42, size = 527, normalized size = 3.90

$$-\frac{1}{5}\left(\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}-\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}-\frac{2\left(\frac{20\,(cx+1)^4bd^3}{(cx-1)^4}-\frac{30\,(cx+1)^3bd^3}{(cx-1)^3}+\frac{30\,(cx+1)^2bd^3}{(cx-1)^2}-\frac{15\,(cx+1)bd^3}{cx-1}+3\,bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^3}{(cx-1)^5}-\frac{5\,(cx+1)^4c^3}{(cx-1)^4}+\frac{10\,(cx+1)^3c^3}{(cx-1)^3}-\frac{10\,(cx+1)^2c^3}{(cx-1)^2}+\frac{5\,(cx+1)c^3}{cx-1}-c^3}-\frac{\frac{80\,(cx+1)^4ad^3}{(cx-1)^4}-\frac{120\,(cx+1)^3ad^3}{(cx-1)^3}+\frac{120\,(cx+1)^2ad^3}{(cx-1)^2}-\frac{60\,(cx+1)ad^3}{cx-1}+12\,ad^3+\frac{34\,(cx+1)^4bd^3}{(cx-1)^4}-\frac{103\,(cx+1)^3bd^3}{(cx-1)^3}+\frac{123\,(cx+1)^2bd^3}{(cx-1)^2}-\frac{69\,(cx+1)bd^3}{cx-1}+15\,bd^3}{\frac{(cx+1)^5c^3}{(cx-1)^5}-\frac{5\,(cx+1)^4c^3}{(cx-1)^4}+\frac{10\,(cx+1)^3c^3}{(cx-1)^3}-\frac{10\,(cx+1)^2c^3}{(cx-1)^2}+\frac{5\,(cx+1)c^3}{cx-1}-c^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/5*(6*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 6*b*d^3*log(-(c*x + 1)/(c
*x - 1))/c^3 - 2*(20*(c*x + 1)^4*b*d^3/(c*x - 1)^4 - 30*(c*x + 1)^3*b*d^3/(
c*x - 1)^3 + 30*(c*x + 1)^2*b*d^3/(c*x - 1)^2 - 15*(c*x + 1)*b*d^3/(c*x - 1
) + 3*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^3/(c*x - 1)^5 - 5*(c*
x + 1)^4*c^3/(c*x - 1)^4 + 10*(c*x + 1)^3*c^3/(c*x - 1)^3 - 10*(c*x + 1)^2*
c^3/(c*x - 1)^2 + 5*(c*x + 1)*c^3/(c*x - 1) - c^3) - (80*(c*x + 1)^4*a*d^3/
(c*x - 1)^4 - 120*(c*x + 1)^3*a*d^3/(c*x - 1)^3 + 120*(c*x + 1)^2*a*d^3/(c*
x - 1)^2 - 60*(c*x + 1)*a*d^3/(c*x - 1) + 12*a*d^3 + 34*(c*x + 1)^4*b*d^3/(
c*x - 1)^4 - 103*(c*x + 1)^3*b*d^3/(c*x - 1)^3 + 123*(c*x + 1)^2*b*d^3/(c*x
 - 1)^2 - 69*(c*x + 1)*b*d^3/(c*x - 1) + 15*b*d^3)/((c*x + 1)^5*c^3/(c*x -
1)^5 - 5*(c*x + 1)^4*c^3/(c*x - 1)^4 + 10*(c*x + 1)^3*c^3/(c*x - 1)^3 - 10*
(c*x + 1)^2*c^3/(c*x - 1)^2 + 5*(c*x + 1)*c^3/(c*x - 1) - c^3))*c
```

Mupad [B]

time = 0.98, size = 153, normalized size = 1.13

$$\frac{d^3\,\left(10\,a\,x^2+12\,b\,x^2+10\,b\,x^2\,\mathrm{atanh}\left(c\,x\right)\right)}{20}-\frac{\frac{d^3\,\left(25\,b\,\mathrm{atanh}\left(c\,x\right)-12\,b\,\ln\left(c^2\,x^2-1\right)\right)}{20}-\frac{5\,b\,c\,d^3\,x}{4}}{c^2}+\frac{c^3\,d^3\,\left(4\,a\,x^5+4\,b\,x^5\,\mathrm{atanh}\left(c\,x\right)\right)}{20}+\frac{c\,d^3\,\left(20\,a\,x^3+5\,b\,x^3+20\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{20}+\frac{c^2\,d^3\,\left(15\,a\,x^4+b\,x^4+15\,b\,x^4\,\mathrm{atanh}\left(c\,x\right)\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] (d^3*(10*a*x^2 + 12*b*x^2 + 10*b*x^2*atanh(c*x)))/20 - ((d^3*(25*b*atanh(c*
x) - 12*b*log(c^2*x^2 - 1)))/20 - (5*b*c*d^3*x)/4)/c^2 + (c^3*d^3*(4*a*x^5
+ 4*b*x^5*atanh(c*x)))/20 + (c*d^3*(20*a*x^3 + 5*b*x^3 + 20*b*x^3*atanh(c*x
)))/20 + (c^2*d^3*(15*a*x^4 + b*x^4 + 15*b*x^4*atanh(c*x)))/20
```

3.23 $\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=84

$$bd^3x+\frac{bd^3(1+cx)^2}{4c}+\frac{bd^3(1+cx)^3}{12c}+\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+\frac{2bd^3\log(1-cx)}{c}$$

```
[Out] b*d^3*x+1/4*b*d^3*(c*x+1)^2/c+1/12*b*d^3*(c*x+1)^3/c+1/4*d^3*(c*x+1)^4*(a+b
*arctanh(c*x))/c+2*b*d^3*ln(-c*x+1)/c
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6063, 641, 45}

$$\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+\frac{bd^3(cx+1)^3}{12c}+\frac{bd^3(cx+1)^2}{4c}+\frac{2bd^3\log(1-cx)}{c}+bd^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] b*d^3*x + (b*d^3*(1 + c*x)^2)/(4*c) + (b*d^3*(1 + c*x)^3)/(12*c) + (d^3*(1
+ c*x)^4*(a + b*ArcTanh[c*x]))/(4*c) + (2*b*d^3*Log[1 - c*x])/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) dx = \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} - \frac{b\int \frac{(d+cdx)^4}{1-c^2x^2} dx}{4d}$$
$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} - \frac{b\int \frac{(d+cdx)^3}{\frac{1}{d}-\frac{cx}{d}} dx}{4d}$$
$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} - \frac{b\int \left(-4d^4+\frac{8d^3}{\frac{1}{d}-\frac{cx}{d}} - 2d^3(d+cdx)\right.}{4d}$$
$$= bd^3x + \frac{bd^3(1+cx)^2}{4c} + \frac{bd^3(1+cx)^3}{12c} + \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx\right.}{4c}$$

Mathematica [A]
time = 0.05, size = 115, normalized size = 1.37

$$\frac{d^3 \left(24acx+42bcx+36ac^2x^2+12bc^2x^2+24ac^3x^3+2bc^3x^3+6ac^4x^4+6bcx(4+6cx+4c^2x^2+c^3x^3)\tanh^{-1}(cx)+45b\log(1-cx)+3b\log(1+cx)\right)}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(24*a*c*x + 42*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c^3*x^3 + 2*
b*c^3*x^3 + 6*a*c^4*x^4 + 6*b*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3)*ArcTanh
[c*x] + 45*b*Log[1 - c*x] + 3*b*Log[1 + c*x]))/(24*c)
```

Maple [A]
time = 0.15, size = 131, normalized size = 1.56

method	result
derivativedivides	$\frac{\frac{d^3(cx+1)^4a}{4}+\frac{d^3b\operatorname{arctanh}(cx)c^4x^4}{4}+d^3b\operatorname{arctanh}(cx)c^3x^3+\frac{3d^3b\operatorname{arctanh}(cx)c^2x^2}{2}+bc\,d^3x\operatorname{arctanh}(cx)+\frac{b\,d^3\operatorname{arctanh}(cx)}{4}+\frac{d^3b}{}}{c}$
default	$\frac{\frac{d^3(cx+1)^4a}{4}+\frac{d^3b\operatorname{arctanh}(cx)c^4x^4}{4}+d^3b\operatorname{arctanh}(cx)c^3x^3+\frac{3d^3b\operatorname{arctanh}(cx)c^2x^2}{2}+bc\,d^3x\operatorname{arctanh}(cx)+\frac{b\,d^3\operatorname{arctanh}(cx)}{4}+\frac{d^3b}{}}{c}$
risch	$\frac{d^3(cx+1)^4b\ln(cx+1)}{8c}-\frac{d^3c^3x^4b\ln(-cx+1)}{8}+\frac{d^3c^3x^4a}{4}-\frac{d^3c^2x^3b\ln(-cx+1)}{2}+d^3c^2x^3a+\frac{d^3c^2bx^3}{12}-\frac{3d^3c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*d^3*(c*x+1)^4*a+1/4*d^3*b*arctanh(c*x)*c^4*x^4+d^3*b*arctanh(c*x)*
c^3*x^3+3/2*d^3*b*arctanh(c*x)*c^2*x^2+b*c*d^3*x*arctanh(c*x)+1/4*b*d^3*arc
tanh(c*x)+1/12*d^3*b*c^3*x^3+1/2*b*c^2*d^3*x^2+7/4*b*c*d^3*x+2*d^3*b*ln(c*x
-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(78) = 156$.
time = 0.26, size = 219, normalized size = 2.61

$$\frac{1}{4}\,ac^3d^3x^4 + ac^2d^3x^3 + \frac{1}{24}\left(6\,x^4\,\operatorname{artanh}(cx) + c\left(\frac{2\,(c^2x^3 + 3\,x)}{c^4} - \frac{3\,\log(cx+1)}{c^5} + \frac{3\,\log(cx-1)}{c^5}\right)\right)bc^3d^3 + \frac{1}{2}\left(2\,x^3\,\operatorname{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log(c^2x^2-1)}{c^4}\right)\right)bc^2d^3 + \frac{3}{2}\,acd^3x^2 + \frac{3}{4}\left(2\,x^2\,\operatorname{artanh}(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)bcd^3 + ad^3x + \frac{(2\,cx\,\operatorname{artanh}(cx) + \log(-c^2x^2+1))bd^3}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^3*d^3*x^4 + a*c^2*d^3*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^
3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^3*d^3 + 1/2*(2
*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + 3/2*a*c
*d^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*
x - 1)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1
))*b*d^3/c
```

Fricas [A]
time = 0.38, size = 149, normalized size = 1.77

$$\frac{6\,ac^4d^3x^4 + 2\,(12\,a + b)c^3d^3x^3 + 12\,(3\,a + b)c^2d^3x^2 + 6\,(4\,a + 7\,b)cd^3x + 3\,bd^3\log(cx+1) + 45\,bd^3\log(cx-1) + 3\,(bc^4d^3x^4 + 4\,bc^3d^3x^3 + 6\,bc^2d^3x^2 + 4\,bcd^3x)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d^3*x^4 + 2*(12*a + b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2
 + 6*(4*a + 7*b)*c*d^3*x + 3*b*d^3*log(c*x + 1) + 45*b*d^3*log(c*x - 1) + 3
*(b*c^4*d^3*x^4 + 4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c*d^3*x)*log(-(c*
x + 1)/(c*x - 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(73) = 146$.
time = 0.40, size = 182, normalized size = 2.17

$$\begin{cases} \frac{ac^3d^3x^4}{4} + ac^2d^3x^3 + \frac{3acd^3x^2}{2} + ad^3x + \frac{bc^3d^3x^4\operatorname{atanh}(cx)}{4} + bc^2d^3x^3\operatorname{atanh}(cx) + \frac{bc^2d^3x^3}{12} + \frac{3bcd^3x^2\operatorname{atanh}(cx)}{2} + \frac{bcd^3x^2}{2} + bd^3x\operatorname{atanh}(cx) + \frac{7bd^3x}{4} + \frac{2bd^3\log\left(x-\frac{1}{c}\right)}{c} + \frac{bd^3\operatorname{atanh}(cx)}{4c} & \text{for}\: c \neq 0 \\ ad^3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**4/4 + a*c**2*d**3*x**3 + 3*a*c*d**3*x**2/2 + a*d*
*3*x + b*c**3*d**3*x**4*atanh(c*x)/4 + b*c**2*d**3*x**3*atanh(c*x) + b*c**2
*d**3*x**3/12 + 3*b*c*d**3*x**2*atanh(c*x)/2 + b*c*d**3*x**2/2 + b*d**3*x*a
tanh(c*x) + 7*b*d**3*x/4 + 2*b*d**3*log(x - 1/c)/c + b*d**3*atanh(c*x)/(4*c
), Ne(c, 0)), (a*d**3*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(78) = 156.
time = 0.40, size = 425, normalized size = 5.06

$$-\frac{1}{3}\left(\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{6\left(\frac{4\,(cx+1)^3bd^3}{(cx-1)^3}-\frac{6\,(cx+1)^2bd^3}{(cx-1)^2}+\frac{4\,(cx+1)bd^3}{cx-1}-bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^2}{(cx-1)^4}-\frac{4\,(cx+1)^3c^2}{(cx-1)^3}+\frac{6\,(cx+1)^2c^2}{(cx-1)^2}-\frac{4\,(cx+1)c^2}{cx-1}+c^2}-\frac{\frac{48\,(cx+1)^3ad^3}{(cx-1)^3}-\frac{72\,(cx+1)^2ad^3}{(cx-1)^2}+\frac{48\,(cx+1)ad^3}{cx-1}-12\,ad^3+\frac{18\,(cx+1)^3bd^3}{(cx-1)^3}-\frac{45\,(cx+1)^2bd^3}{(cx-1)^2}+\frac{38\,(cx+1)bd^3}{cx-1}-11\,bd^3}{\frac{(cx+1)^4c^2}{(cx-1)^4}-\frac{4\,(cx+1)^3c^2}{(cx-1)^3}+\frac{6\,(cx+1)^2c^2}{(cx-1)^2}-\frac{4\,(cx+1)c^2}{cx-1}+c^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*(6*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 6*b*d^3*log(-(c*x + 1)/(c
*x - 1))/c^2 - 6*(4*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 6*(c*x + 1)^2*b*d^3/(c*
x - 1)^2 + 4*(c*x + 1)*b*d^3/(c*x - 1) - b*d^3)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^4*c^2/(c*x - 1)^4 - 4*(c*x + 1)^3*c^2/(c*x - 1)^3 + 6*(c*x + 1)^2
*c^2/(c*x - 1)^2 - 4*(c*x + 1)*c^2/(c*x - 1) + c^2) - (48*(c*x + 1)^3*a*d^3
/(c*x - 1)^3 - 72*(c*x + 1)^2*a*d^3/(c*x - 1)^2 + 48*(c*x + 1)*a*d^3/(c*x -
 1) - 12*a*d^3 + 18*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 45*(c*x + 1)^2*b*d^3/(c
*x - 1)^2 + 38*(c*x + 1)*b*d^3/(c*x - 1) - 11*b*d^3)/((c*x + 1)^4*c^2/(c*x
- 1)^4 - 4*(c*x + 1)^3*c^2/(c*x - 1)^3 + 6*(c*x + 1)^2*c^2/(c*x - 1)^2 - 4*
(c*x + 1)*c^2/(c*x - 1) + c^2))*c
```

Mupad [B]
time = 0.96, size = 136, normalized size = 1.62

$$\frac{d^3\,\left(12\,a\,x+21\,b\,x+12\,b\,x\,\mathrm{atanh}(c\,x)\right)}{12}+\frac{c^3\,d^3\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{12}-\frac{d^3\,\left(21\,b\,\mathrm{atanh}(c\,x)-12\,b\,\ln\left(c^2\,x^2-1\right)\right)}{12\,c}+\frac{c\,d^3\,\left(18\,a\,x^2+6\,b\,x^2+18\,b\,x^2\,\mathrm{atanh}(c\,x)\right)}{12}+\frac{c^2\,d^3\,\left(12\,a\,x^3+b\,x^3+12\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

```
[Out] (d^3*(12*a*x + 21*b*x + 12*b*x*atanh(c*x)))/12 + (c^3*d^3*(3*a*x^4 + 3*b*x^
4*atanh(c*x)))/12 - (d^3*(21*b*atanh(c*x) - 12*b*log(c^2*x^2 - 1)))/(12*c)
+ (c*d^3*(18*a*x^2 + 6*b*x^2 + 18*b*x^2*atanh(c*x)))/12 + (c^2*d^3*(12*a*x^
3 + b*x^3 + 12*b*x^3*atanh(c*x)))/12
```

3.24 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=152

$$3acd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}bc^2d^3x^2-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^3d^3x^3\left(a+\right.$$

```
[Out] 3*a*c*d^3*x+3/2*b*c*d^3*x+1/6*b*c^2*d^3*x^2-3/2*b*d^3*arctanh(c*x)+3*b*c*d^
3*x*arctanh(c*x)+3/2*c^2*d^3*x^2*(a+b*arctanh(c*x))+1/3*c^3*d^3*x^3*(a+b*ar
ctanh(c*x))+a*d^3*ln(x)+5/3*b*d^3*ln(-c^2*x^2+1)-1/2*b*d^3*polylog(2,-c*x)+
1/2*b*d^3*polylog(2,c*x)
```

Rubi [A]
time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6087, 6021, 266, 6031, 6037, 327, 212, 272, 45}

$$\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3x+ad^3\log(x)+\frac{1}{6}bc^2d^3x^2+\frac{5}{3}bd^3\log\left(1-c^2x^2\right)-\frac{1}{2}bd^3\text{Li}_2(-cx)+\frac{1}{2}bd^3\text{Li}_2(cx)+\frac{3}{2}bcd^3x-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 3*a*c*d^3*x + (3*b*c*d^3*x)/2 + (b*c^2*d^3*x^2)/6 - (3*b*d^3*ArcTanh[c*x])/
2 + 3*b*c*d^3*x*ArcTanh[c*x] + (3*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/2 + (c^
3*d^3*x^3*(a + b*ArcTanh[c*x]))/3 + a*d^3*Log[x] + (5*b*d^3*Log[1 - c^2*x^2
])/3 - (b*d^3*PolyLog[2, -(c*x)])/2 + (b*d^3*PolyLog[2, c*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \Bigg(3cd^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3c^2d^3x\big(a+b\,\mathrm{t}$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(3cd^3\right)\int \left(a+b\tanh^{-1}(cx)\right)\,dx+\left(3c^2d^3\right.$$

$$= 3acd^3x+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+$$

$$= 3acd^3x+\frac{3}{2}bcd^3x+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)-$$

$$= 3acd^3x+\frac{3}{2}bcd^3x-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\big($$

$$= 3acd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}bc^2d^3x^2-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)$$

Mathematica [A]
time = 0.07, size = 148, normalized size = 0.97

$$\frac{1}{12}d^3\left(36acx+18bcx+18ac^2x^2+2bc^2x^2+4ac^3x^3+36bcx\tanh^{-1}(cx)+18bc^2x^2\tanh^{-1}(cx)+4bc^3x^3\tanh^{-1}(cx)+12a\log(x)+9b\log(1-cx)-9b\log(1+cx)+18b\log\left(1-c^2x^2\right)+2b\log\left(-1+c^2x^2\right)-6b\text{PolyLog}(2,-cx)+6b\text{PolyLog}(2,cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^3*(36*a*c*x + 18*b*c*x + 18*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 36*b
*c*x*ArcTanh[c*x] + 18*b*c^2*x^2*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] +
12*a*Log[x] + 9*b*Log[1 - c*x] - 9*b*Log[1 + c*x] + 18*b*Log[1 - c^2*x^2] +
 2*b*Log[-1 + c^2*x^2] - 6*b*PolyLog[2, -(c*x)] + 6*b*PolyLog[2, c*x]))/12
```

Maple [A]
time = 0.18, size = 182, normalized size = 1.20

method	result
derivativedivides	$\frac{d^3 a\, c^3 x^3}{3}+\frac{3 d^3 a\, c^2 x^2}{2}+3 d^3 a c x+d^3 a\ln(cx)+\frac{d^3 b\,\mathrm{arctanh}(cx) c^3 x^3}{3}+\frac{3 d^3 b\,\mathrm{arctanh}(cx) c^2 x^2}{2}+3 b c\, d^3 x$
default	$\frac{d^3 a\, c^3 x^3}{3}+\frac{3 d^3 a\, c^2 x^2}{2}+3 d^3 a c x+d^3 a\ln(cx)+\frac{d^3 b\,\mathrm{arctanh}(cx) c^3 x^3}{3}+\frac{3 d^3 b\,\mathrm{arctanh}(cx) c^2 x^2}{2}+3 b c\, d^3 x$
risch	$-\frac{d^3\ln(-cx+1) x^3 b\, c^3}{6}-\frac{3 d^3\ln(-cx+1) x^2 b\, c^2}{4}-\frac{3 d^3 b\ln(-cx+1) c x}{2}+\frac{29 d^3 b\ln(-cx+1)}{12}+\frac{b\, c^2 d^3 x^2}{6}+\frac{3 b c\, d^3 x}{2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d^3*a*c^3*x^3+3/2*d^3*a*c^2*x^2+3*d^3*a*c*x+d^3*a*ln(c*x)+1/3*d^3*b*arc
tanh(c*x)*c^3*x^3+3/2*d^3*b*arctanh(c*x)*c^2*x^2+3*b*c*d^3*x*arctanh(c*x)+d
```

```
^3*b*arctanh(c*x)*ln(c*x)-1/2*d^3*b*dilog(c*x)-1/2*d^3*b*dilog(c*x+1)-1/2*d
^3*b*ln(c*x)*ln(c*x+1)+1/6*b*c^2*d^3*x^2+3/2*b*c*d^3*x+29/12*d^3*b*ln(c*x-1
)+11/12*d^3*b*ln(c*x+1)
```

Maxima [A]
time = 0.37, size = 228, normalized size = 1.50

$$\frac{1}{3}ac^3d^3x^3+\frac{3}{2}ac^2d^3x^2+\frac{1}{6}bc^2d^3x^2+3\,acd^3x+\frac{3}{2}bcd^3x+\frac{3}{2}\left(2\,cx\,\mathrm{artanh}\left(cx\right)+\log\left(-c^2x^2+1\right)\right)bd^3-\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2(-cx+1)\right)bd^3+\frac{1}{2}\left(\log\left(cx+1\right)\log\left(-cx\right)+\mathrm{Li}_2(cx+1)\right)bd^3-\frac{7}{12}bd^3\log\left(cx+1\right)+\frac{11}{12}bd^3\log\left(cx-1\right)+ad^3\log\left(x\right)+\frac{1}{12}\left(2\,bc^3d^3x^3+9\,bc^2d^3x^2\right)\log\left(cx+1\right)-\frac{1}{12}\left(2\,bc^3d^3x^3+9\,bc^2d^3x^2\right)\log\left(-cx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^3*d^3*x^3 + 3/2*a*c^2*d^3*x^2 + 1/6*b*c^2*d^3*x^2 + 3*a*c*d^3*x + 3
/2*b*c*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d^3 - 1/2*(lo
g(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^3 + 1/2*(log(c*x + 1)*log(-c*x)
 + dilog(c*x + 1))*b*d^3 - 7/12*b*d^3*log(c*x + 1) + 11/12*b*d^3*log(c*x -
1) + a*d^3*log(x) + 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(c*x + 1) -
 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(-c*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac\,dx+\int\frac{a}{x}\,dx+\int 3ac^2x\,dx+\int ac^3x^2\,dx+\int 3bc\,\mathrm{atanh}\left(cx\right)dx+\int\frac{b\,\mathrm{atanh}\left(cx\right)}{x}\,dx+\int 3bc^2x\,\mathrm{atanh}\left(cx\right)dx+\int bc^3x^2\,\mathrm{atanh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**3*(Integral(3*a*c, x) + Integral(a/x, x) + Integral(3*a*c**2*x, x) + Int
egral(a*c**3*x**2, x) + Integral(3*b*c*atanh(c*x), x) + Integral(b*atanh(c*
x)/x, x) + Integral(3*b*c**2*x*atanh(c*x), x) + Integral(b*c**3*x**2*atanh(
c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x, x)
```

3.25 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=150

$$3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(c\right.$$

```
[Out] 3*a*c^2*d^3*x+1/2*b*c^2*d^3*x-1/2*b*c*d^3*arctanh(c*x)+3*b*c^2*d^3*x*arctan
h(c*x)-d^3*(a+b*arctanh(c*x))/x+1/2*c^3*d^3*x^2*(a+b*arctanh(c*x))+3*a*c*d^
3*ln(x)+b*c*d^3*ln(x)+b*c*d^3*ln(-c^2*x^2+1)-3/2*b*c*d^3*polylog(2,-c*x)+3/
2*b*c*d^3*polylog(2,c*x)
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6087, 6021, 266, 6037, 272, 36, 29, 31, 6031, 327, 212}

$$\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3x+3acd^3\log(x)+bcd^3\log\left(1-c^2x^2\right)+\frac{1}{2}bc^2d^3x+3bc^2d^3x\tanh^{-1}(cx)-\frac{3}{2}bcd^3\text{Li}_2(-cx)+\frac{3}{2}bcd^3\text{Li}_2(cx)+bcd^3\log(x)-\frac{1}{2}bcd^3\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] 3*a*c^2*d^3*x + (b*c^2*d^3*x)/2 - (b*c*d^3*ArcTanh[c*x])/2 + 3*b*c^2*d^3*x*
ArcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/x + (c^3*d^3*x^2*(a + b*ArcTanh[c
*x]))/2 + 3*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 - c^2*x^2] - (3
*b*c*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c*d^3*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
```

```
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(3c^2d^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{3cd^3\left(a+b\,\mathrm{t}\right.}{}\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(3cd^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(3c^2d^3\right)$$

$$= 3ac^2d^3x-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3ac$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3(}{}$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3(}{}$$

Mathematica [A]

time = 0.07, size = 149, normalized size = 0.99

$$\frac{d^3\left(-4a+12ac^2x^2+2bc^2x^2+2ac^3x^3-4b\tanh^{-1}(cx)+12bc^2x^2\tanh^{-1}(cx)+2bc^3x^3\tanh^{-1}(cx)+12acx\log(x)+4bcx\log(cx)+bcx\log(1-cx)-bcx\log(1+cx)+4bcx\log\left(1-c^2x^2\right)-6bcx\mathrm{PolyLog}(2,-cx)+6bcx\mathrm{PolyLog}(2,cx)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-4*a + 12*a*c^2*x^2 + 2*b*c^2*x^2 + 2*a*c^3*x^3 - 4*b*ArcTanh[c*x] +
12*b*c^2*x^2*ArcTanh[c*x] + 2*b*c^3*x^3*ArcTanh[c*x] + 12*a*c*x*Log[x] + 4*
b*c*x*Log[c*x] + b*c*x*Log[1 - c*x] - b*c*x*Log[1 + c*x] + 4*b*c*x*Log[1 -
c^2*x^2] - 6*b*c*x*PolyLog[2, -(c*x)] + 6*b*c*x*PolyLog[2, c*x]))/(4*x)
```

Maple [A]

time = 0.21, size = 183, normalized size = 1.22

method	result
derivativedivides	$c\left(\frac{d^3 a\, c^2 x^2}{2}+3d^3acx-\frac{d^3a}{cx}+3d^3a\ln(cx)+\frac{d^3 b\,\mathrm{arctanh}(cx)c^2x^2}{2}+3bc\,d^3x\,\mathrm{arctanh}(cx)-\frac{d^3b}{}\right.$
default	$c\left(\frac{d^3 a\, c^2 x^2}{2}+3d^3acx-\frac{d^3a}{cx}+3d^3a\ln(cx)+\frac{d^3 b\,\mathrm{arctanh}(cx)c^2x^2}{2}+3bc\,d^3x\,\mathrm{arctanh}(cx)-\frac{d^3b}{}\right.$

risch	$-\frac{c^3 d^3 \ln(-cx+1) x^2 b}{4} - \frac{3c^2 d^3 b \ln(-cx+1) x}{2} + \frac{5c\, d^3 b \ln(-cx+1)}{4} + \frac{b\, c^2 d^3 x}{2} - 3bc\, d^3 + \frac{c\, d^3 b \ln(-cx)}{2} + \frac{d^3 b \ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*d^3*a*c^2*x^2+3*d^3*a*c*x-d^3*a/c/x+3*d^3*a*ln(c*x)+1/2*d^3*b*arctan
h(c*x)*c^2*x^2+3*b*c*d^3*x*arctanh(c*x)-d^3*b*arctanh(c*x)/c/x+3*d^3*b*arct
anh(c*x)*ln(c*x)-3/2*d^3*b*dilog(c*x)-3/2*d^3*b*dilog(c*x+1)-3/2*d^3*b*ln(c
*x)*ln(c*x+1)+1/2*b*c*d^3*x+5/4*d^3*b*ln(c*x-1)+3/4*d^3*b*ln(c*x+1)+d^3*b*l
n(c*x))
```

Maxima [A]
time = 0.36, size = 229, normalized size = 1.53

$$\frac{1}{4}\,bc^3d^3x^2\log\left(cx+1\right) - \frac{1}{4}\,bc^3d^3x^2\log\left(-cx+1\right) + \frac{1}{2}\,ac^3d^3x^2 + 3\,ac^2d^3x + \frac{1}{2}\,bc^2d^3x + \frac{3}{2}\left(2\,cx\,\mathrm{artanh}\left(cx\right) + \log\left(-c^2x^2+1\right)\right)bcd^3 - \frac{3}{2}\left(\log\left(cx\right)\log\left(-cx+1\right) + \mathrm{Li}_2(-cx+1)\right)bcd^3 + \frac{3}{2}\left(\log\left(cx+1\right)\log\left(-cx\right) + \mathrm{Li}_2(cx+1)\right)bcd^3 - \frac{1}{4}\,bcd^3\log\left(cx+1\right) + \frac{1}{4}\,bcd^3\log\left(cx-1\right) + 3\,acd^3\log\left(x\right) - \frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\mathrm{artanh}\left(cx\right)}{x}\right)bd^3 - \frac{ad^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^3*d^3*x^2*log(c*x + 1) - 1/4*b*c^3*d^3*x^2*log(-c*x + 1) + 1/2*a*c^
3*d^3*x^2 + 3*a*c^2*d^3*x + 1/2*b*c^2*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log
(-c^2*x^2 + 1))*b*c*d^3 - 3/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*
c*d^3 + 3/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c*d^3 - 1/4*b*c*d^3
*log(c*x + 1) + 1/4*b*c*d^3*log(c*x - 1) + 3*a*c*d^3*log(x) - 1/2*(c*(log(c
^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^3 - a*d^3/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int \frac{3ac}{x}\,dx + \int ac^3x\,dx + \int 3bc^2\,\mathrm{atanh}\,(cx)\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x^2}\,dx + \int \frac{3bc\,\mathrm{atanh}\,(cx)}{x}\,dx + \int bc^3x\,\mathrm{atanh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c/x, x) +
Integral(a*c**3*x, x) + Integral(3*b*c**2*atanh(c*x), x) + Integral(b*atanh
(c*x)/x**2, x) + Integral(3*b*c*atanh(c*x)/x, x) + Integral(b*c**3*x*atanh(
c*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^2, x)
```

3.26 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=160

$$-\frac{bcd^3}{2x}+ac^3d^3x+\frac{1}{2}bc^2d^3\tanh^{-1}(cx)+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3a$$

```
[Out] -1/2*b*c*d^3/x+a*c^3*d^3*x+1/2*b*c^2*d^3*arctanh(c*x)+b*c^3*d^3*x*arctanh(c
*x)-1/2*d^3*(a+b*arctanh(c*x))/x^2-3*c*d^3*(a+b*arctanh(c*x))/x+3*a*c^2*d^3
*ln(x)+3*b*c^2*d^3*ln(x)-b*c^2*d^3*ln(-c^2*x^2+1)-3/2*b*c^2*d^3*polylog(2,-
c*x)+3/2*b*c^2*d^3*polylog(2,c*x)
```

Rubi [A]
time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6087, 6021, 266, 6037, 331, 212, 272, 36, 29, 31, 6031}

$$-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^3d^3x+3ac^2d^3\log(x)+bc^3d^3x\tanh^{-1}(cx)-\frac{3}{2}bc^2d^3\text{Li}_2(-cx)+\frac{3}{2}bc^2d^3\text{Li}_2(cx)-bc^2d^3\log\left(1-c^2x^2\right)+3bc^2d^3\log(x)+\frac{1}{2}bc^2d^3\tanh^{-1}(cx)-\frac{bcd^3}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^3)/x + a*c^3*d^3*x + (b*c^2*d^3*ArcTanh[c*x])/2 + b*c^3*d^3*x*A
rcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[
c*x]))/x + 3*a*c^2*d^3*Log[x] + 3*b*c^2*d^3*Log[x] - b*c^2*d^3*Log[1 - c^2*
x^2] - (3*b*c^2*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c^2*d^3*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
```

```
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int \left(c^3d^3\left(a+b\tanh^{-1}(cx)\right) + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^3} + \frac{3cd^3\left(a+b\tan\right.}{x^2}\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(3cd^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(3c^2d^3\right)$$

$$= ac^3d^3x - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + 3ac^2d^3\,\mathrm{lo}$$

$$= -\frac{bcd^3}{2x} + ac^3d^3x + bc^3d^3x\tanh^{-1}(cx) - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{3cd^3}{}$$

$$= -\frac{bcd^3}{2x} + ac^3d^3x + \frac{1}{2}bc^2d^3\tanh^{-1}(cx) + bc^3d^3x\tanh^{-1}(cx) - \frac{d^3(a+b}{}$$

$$= -\frac{bcd^3}{2x} + ac^3d^3x + \frac{1}{2}bc^2d^3\tanh^{-1}(cx) + bc^3d^3x\tanh^{-1}(cx) - \frac{d^3(a+b}{}$$

Mathematica [A]
time = 0.07, size = 165, normalized size = 1.03

$$\frac{d^3\left(-2a-12acx-2bcx+4ac^3x^3-2b\tanh^{-1}(cx)-12bcx\tanh^{-1}(cx)+4bc^3x^3\tanh^{-1}(cx)+12ac^2x^2\log(x)+12bc^2x^2\log(cx)-bc^2x^2\log(1-cx)+bc^2x^2\log(1+cx)-4bc^2x^2\log\left(1-c^2x^2\right)-6bc^2x^2\mathrm{PolyLog}(2,-cx)+6bc^2x^2\mathrm{PolyLog}(2,cx)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^3*(-2*a - 12*a*c*x - 2*b*c*x + 4*a*c^3*x^3 - 2*b*ArcTanh[c*x] - 12*b*c*x
*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] + 12*a*c^2*x^2*Log[x] + 12*b*c^2*x
^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x] - 4*b*c^2*x^2
*Log[1 - c^2*x^2] - 6*b*c^2*x^2*PolyLog[2, -(c*x)] + 6*b*c^2*x^2*PolyLog[2,
 c*x]))/(4*x^2)
```

Maple [A]
time = 0.22, size = 188, normalized size = 1.18

method	result
derivativedivides	$c^2\left(d^3acx - \frac{3d^3a}{cx} - \frac{d^3a}{2c^2x^2} + 3d^3a\ln(cx) + bc\,d^3x\,\mathrm{arctanh}(cx) - \frac{3d^3b\,\mathrm{arctanh}(cx)}{cx} - \frac{d^3b\,\mathrm{arctanh}(}{2c^2x^2}\right.$
default	$c^2\left(d^3acx - \frac{3d^3a}{cx} - \frac{d^3a}{2c^2x^2} + 3d^3a\ln(cx) + bc\,d^3x\,\mathrm{arctanh}(cx) - \frac{3d^3b\,\mathrm{arctanh}(cx)}{cx} - \frac{d^3b\,\mathrm{arctanh}(}{2c^2x^2}\right.$

risch	$-\frac{c^3 d^3 b \ln(-cx+1)x}{2} - \frac{5c^2 d^3 b \ln(-cx+1)}{4} - b\,c^2 d^3 - \frac{bc\,d^3}{2x} + \frac{7c^2 d^3 b \ln(-cx)}{4} + \frac{3c\,d^3 b \ln(-cx+1)}{2x} + \frac{d^3 b \ln(-}{4x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d^3*a*c*x-3*d^3*a/c/x-1/2*d^3*a/c^2/x^2+3*d^3*a*ln(c*x)+b*c*d^3*x*arct
anh(c*x)-3*d^3*b*arctanh(c*x)/c/x-1/2*d^3*b*arctanh(c*x)/c^2/x^2+3*d^3*b*ar
ctanh(c*x)*ln(c*x)-3/2*d^3*b*dilog(c*x)-3/2*d^3*b*dilog(c*x+1)-3/2*d^3*b*ln
(c*x)*ln(c*x+1)-5/4*d^3*b*ln(c*x-1)-3/4*d^3*b*ln(c*x+1)-1/2*d^3*b/c/x+3*d^3
*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] a*c^3*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^2*d^3 + 3/2*
b*c^2*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 3*a*c^2*d^3*log(
x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^3 + 1/4
*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^3 - 3
*a*c*d^3/x - 1/2*a*d^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int ac^3\,dx + \int \frac{a}{x^3}\,dx + \int \frac{3ac}{x^2}\,dx + \int \frac{3ac^2}{x}\,dx + \int bc^3 \operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x^3}\,dx + \int \frac{3bc\operatorname{atanh}(cx)}{x^2}\,dx + \int \frac{3bc^2\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a*c**3, x) + Integral(a/x**3, x) + Integral(3*a*c/x**2, x) +
 Integral(3*a*c**2/x, x) + Integral(b*c**3*atanh(c*x), x) + Integral(b*atan
h(c*x)/x**3, x) + Integral(3*b*c*atanh(c*x)/x**2, x) + Integral(3*b*c**2*at
anh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^3, x)
```

3.27 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{bcd^3}{6x^2}-\frac{3bc^2d^3}{2x}+\frac{3}{2}bc^3d^3\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3c^2d^3\left(a+b\tanh^{-1}(c\right.}{x}$$

```
[Out] -1/6*b*c*d^3/x^2-3/2*b*c^2*d^3/x+3/2*b*c^3*d^3*arctanh(c*x)-1/3*d^3*(a+b*ar
ctanh(c*x))/x^3-3/2*c*d^3*(a+b*arctanh(c*x))/x^2-3*c^2*d^3*(a+b*arctanh(c*x
))/x+a*c^3*d^3*ln(x)+10/3*b*c^3*d^3*ln(x)-5/3*b*c^3*d^3*ln(-c^2*x^2+1)-1/2*
b*c^3*d^3*polylog(2,-c*x)+1/2*b*c^3*d^3*polylog(2,c*x)
```

Rubi [A]

time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6037, 272, 46, 331, 212, 36, 29, 31, 6031}

$$-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+ac^3d^3\log(x)-\frac{1}{2}bc^3d^3\text{Li}_2(-cx)+\frac{1}{2}bc^3d^3\text{Li}_2(cx)+\frac{10}{3}bc^3d^3\log(x)+\frac{3}{2}bc^3d^3\tanh^{-1}(cx)-\frac{3bc^2d^3}{2x}-\frac{5}{3}bc^3d^3\log\left(1-c^2x^2\right)-\frac{bcd^3}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^3)/x^2 - (3*b*c^2*d^3)/(2*x) + (3*b*c^3*d^3*ArcTanh[c*x])/2 - (
d^3*(a + b*ArcTanh[c*x]))/(3*x^3) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(2*x^2)
- (3*c^2*d^3*(a + b*ArcTanh[c*x]))/x + a*c^3*d^3*Log[x] + (10*b*c^3*d^3*Log
[x])/3 - (5*b*c^3*d^3*Log[1 - c^2*x^2])/3 - (b*c^3*d^3*PolyLog[2, -(c*x)])/
2 + (b*c^3*d^3*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = \int \left(\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^4} + \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x^3} + \frac{3c^2d^3\left(a+b\right.}{x}\right.$$

$$= d^3 \int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx + \left(3cd^3\right) \int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(3c^2d^3\right)$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{3c^2d^3\left(a+b\tan\right.}{x}$$

$$= -\frac{3bc^2d^3}{2x} - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{3c^2d^3}{}$$

$$= -\frac{3bc^2d^3}{2x} + \frac{3}{2}bc^3d^3\tanh^{-1}(cx) - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{3cd^3\left(a+b\right.}{2}$$

$$= -\frac{bcd^3}{6x^2} - \frac{3bc^2d^3}{2x} + \frac{3}{2}bc^3d^3\tanh^{-1}(cx) - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{3c}{}$$

Mathematica [A]
time = 0.06, size = 175, normalized size = 0.99

$$\frac{d^3\left(-4a-18acx-2bcx-36ac^2x^2-18bc^2x^2-4b\tanh^{-1}(cx)-18bcx\tanh^{-1}(cx)-36bc^2x^2\tanh^{-1}(cx)+12ac^3x^3\log(x)+40bc^3x^3\log(cx)-9bc^3x^3\log(1-cx)+9bc^3x^3\log(1+cx)-20bc^3x^3\log\left(1-c^2x^2\right)-6bc^3x^3\text{PolyLog}(2,-cx)+6bc^3x^3\text{PolyLog}(2,cx)\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-4*a - 18*a*c*x - 2*b*c*x - 36*a*c^2*x^2 - 18*b*c^2*x^2 - 4*b*ArcTanh
[c*x] - 18*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 12*a*c^3*x^3*Lo
g[x] + 40*b*c^3*x^3*Log[c*x] - 9*b*c^3*x^3*Log[1 - c*x] + 9*b*c^3*x^3*Log[1
 + c*x] - 20*b*c^3*x^3*Log[1 - c^2*x^2] - 6*b*c^3*x^3*PolyLog[2, -(c*x)] +
6*b*c^3*x^3*PolyLog[2, c*x]))/(12*x^3)
```

Maple [A]
time = 0.22, size = 208, normalized size = 1.18

method	result
derivativedivides	$c^3\left(-\frac{3d^3a}{cx} - \frac{d^3a}{3c^3x^3} - \frac{3d^3a}{2c^2x^2} + d^3a\ln(cx) - \frac{3d^3b\,\mathrm{arctanh}(cx)}{cx} - \frac{d^3b\,\mathrm{arctanh}(cx)}{3c^3x^3} - \frac{3d^3b\,\mathrm{arctanh}(cx)}{2c^2x^2} +\right.$
default	$c^3\left(-\frac{3d^3a}{cx} - \frac{d^3a}{3c^3x^3} - \frac{3d^3a}{2c^2x^2} + d^3a\ln(cx) - \frac{3d^3b\,\mathrm{arctanh}(cx)}{cx} - \frac{d^3b\,\mathrm{arctanh}(cx)}{3c^3x^3} - \frac{3d^3b\,\mathrm{arctanh}(cx)}{2c^2x^2} +\right.$
risch	$-\frac{3b\,c^2d^3}{2x} + \frac{29c^3d^3b\ln(-cx)}{12} - \frac{29\ln(-cx+1)b\,c^3d^3}{12} + \frac{3c^2d^3b\ln(-cx+1)}{2x} + \frac{3c\,d^3b\ln(-cx+1)}{4x^2} - \frac{bc\,d^3}{6x^2} + \frac{d^3b\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-3*d^3*a/c/x-1/3*d^3*a/c^3/x^3-3/2*d^3*a/c^2/x^2+d^3*a*ln(c*x)-3*d^3*b
*arctanh(c*x)/c/x-1/3*d^3*b*arctanh(c*x)/c^3/x^3-3/2*d^3*b*arctanh(c*x)/c^2
/x^2+d^3*b*arctanh(c*x)*ln(c*x)-1/6*d^3*b/c^2/x^2-3/2*d^3*b/c/x+10/3*d^3*b*
ln(c*x)-29/12*d^3*b*ln(c*x-1)-11/12*d^3*b*ln(c*x+1)-1/2*d^3*b*dilog(c*x)-1/
2*d^3*b*dilog(c*x+1)-1/2*d^3*b*ln(c*x)*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^3*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^3*d^3*lo
g(x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^3 +
 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
^3 - 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/
x^3)*b*d^3 - 3*a*c^2*d^3/x - 3/2*a*c*d^3/x^2 - 1/3*a*d^3/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a}{x^4}\,dx + \int \frac{3ac}{x^3}\,dx + \int \frac{3ac^2}{x^2}\,dx + \int \frac{ac^3}{x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x^4}\,dx + \int \frac{3bc\,\mathrm{atanh}\,(cx)}{x^3}\,dx + \int \frac{3bc^2\,\mathrm{atanh}\,(cx)}{x^2}\,dx + \int \frac{bc^3\,\mathrm{atanh}\,(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(a/x**4, x) + Integral(3*a*c/x**3, x) + Integral(3*a*c**2/x**
2, x) + Integral(a*c**3/x, x) + Integral(b*atanh(c*x)/x**4, x) + Integral(3
*b*c*atanh(c*x)/x**3, x) + Integral(3*b*c**2*atanh(c*x)/x**2, x) + Integral
(b*c**3*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))\,\left(d + c\,d\,x\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^4, x)
```

3.28 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{bcd^3}{12x^3}-\frac{bc^2d^3}{2x^2}-\frac{7bc^3d^3}{4x}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)$$

```
[Out] -1/12*b*c*d^3/x^3-1/2*b*c^2*d^3/x^2-7/4*b*c^3*d^3/x-1/4*d^3*(c*x+1)^4*(a+b*
arctanh(c*x))/x^4+2*b*c^4*d^3*ln(x)-2*b*c^4*d^3*ln(-c*x+1)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {37, 6083, 12, 90}

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)-\frac{7bc^3d^3}{4x}-\frac{bc^2d^3}{2x^2}-\frac{bcd^3}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^3)/x^3 - (b*c^2*d^3)/(2*x^2) - (7*b*c^3*d^3)/(4*x) - (d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(4*x^4) + 2*b*c^4*d^3*Log[x] - 2*b*c^4*d^3*Lo
g[1 - c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-(bc)\int \frac{(d+cdx)^3}{4x^4(-1+cx)}\,dx\\ &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{1}{4}(bc)\int \frac{(d+cdx)^3}{x^4(-1+cx)}\,dx\\ &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{1}{4}(bc)\int \left(-\frac{d^3}{x^4}-\frac{4cd^3}{x^3}-\frac{7c^2d^3}{x^2}\right.\\ &= -\frac{bcd^3}{12x^3}-\frac{bc^2d^3}{2x^2}-\frac{7bc^3d^3}{4x}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}+2bc^4d^3\end{aligned}$$

Mathematica [A]
time = 0.04, size = 131, normalized size = 1.41

$$-\frac{d^3\left(6a+24acx+2bcx+36ac^2x^2+12bc^2x^2+24ac^3x^3+42bc^3x^3+6b\left(1+4cx+6c^2x^2+4c^3x^3\right)\tanh^{-1}(cx)-48bc^4x^4\log(x)+45bc^4x^4\log(1-cx)+3bc^4x^4\log(1+cx)\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/24*(d^3*(6*a + 24*a*c*x + 2*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c
^3*x^3 + 42*b*c^3*x^3 + 6*b*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x
] - 48*b*c^4*x^4*Log[x] + 45*b*c^4*x^4*Log[1 - c*x] + 3*b*c^4*x^4*Log[1 + c
*x]))/x^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(85) = 170$.
time = 0.16, size = 178, normalized size = 1.91

method	result
derivativedivides	$c^4\left(d^3a\left(-\frac{1}{cx}-\frac{1}{4c^4x^4}-\frac{3}{2c^2x^2}-\frac{1}{c^3x^3}\right)-\frac{d^3b\operatorname{arctanh}(cx)}{cx}-\frac{d^3b\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{3d^3b\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{d^3b}{}\right.$
default	$c^4\left(d^3a\left(-\frac{1}{cx}-\frac{1}{4c^4x^4}-\frac{3}{2c^2x^2}-\frac{1}{c^3x^3}\right)-\frac{d^3b\operatorname{arctanh}(cx)}{cx}-\frac{d^3b\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{3d^3b\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{d^3b}{}\right.$
risch	$-\frac{d^3b\left(4x^3c^3+6c^2x^2+4cx+1\right)\ln(cx+1)}{8x^4}-\frac{d^3\left(3b\,c^4\ln(cx+1)x^4-48c^4b\ln(-x)x^4+45x^4b\ln(-cx+1)c^4-12x^3b\ln(-cx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(d^3*a*(-1/c/x-1/4/c^4/x^4-3/2/c^2/x^2-1/c^3/x^3)-d^3*b*arctanh(c*x)/c/
x-1/4*d^3*b*arctanh(c*x)/c^4/x^4-3/2*d^3*b*arctanh(c*x)/c^2/x^2-d^3*b*arcta
nh(c*x)/c^3/x^3-15/8*d^3*b*ln(c*x-1)-1/8*d^3*b*ln(c*x+1)-1/12*d^3*b/c^3/x^3
-1/2*d^3*b/c^2/x^2-7/4*d^3*b/c/x+2*d^3*b*ln(c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(85) = 170$.
time = 0.26, size = 228, normalized size = 2.45

$$-\frac{1}{2}\left(c\left(\log\left(c^{2}x^{2}-1\right)-\log\left(x^{2}\right)\right)+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)bc^{3}d^{3}+\frac{3}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^{2}}\right)bc^{2}d^{3}-\frac{1}{2}\left(\left(c^{2}\log\left(c^{2}x^{2}-1\right)-c^{2}\log\left(x^{2}\right)+\frac{1}{x^{2}}\right)c+\frac{2\,\operatorname{artanh}\left(cx\right)}{x^{3}}\right)bcd^{3}-\frac{ac^{3}d^{3}}{x}+\frac{1}{24}\left(\left(3\,c^{3}\log\left(cx+1\right)-3\,c^{3}\log\left(cx-1\right)-\frac{2\,\left(3\,c^{2}x^{2}+1\right)}{x^{3}}\right)c-\frac{6\,\operatorname{artanh}\left(cx\right)}{x^{4}}\right)bd^{3}-\frac{3\,ac^{2}d^{3}}{2\,x^{2}}-\frac{acd^{3}}{x^{3}}-\frac{ad^{3}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^3 + 3/4*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d^3 -
 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c*d^3 - a*c^3*d^3/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2
*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^3 - 3/2*a*c^2*d^3/x^2 - a
*c*d^3/x^3 - 1/4*a*d^3/x^4
```

Fricas [A]
time = 0.36, size = 163, normalized size = 1.75

$$-\frac{3\,bc^{4}d^{3}x^{4}\log\left(cx+1\right)+45\,bc^{4}d^{3}x^{4}\log\left(cx-1\right)-48\,bc^{4}d^{3}x^{4}\log\left(x\right)+6\,\left(4\,a+7\,b\right)c^{3}d^{3}x^{3}+12\,\left(3\,a+b\right)c^{2}d^{3}x^{2}+2\,\left(12\,a+b\right)cd^{3}x+6\,ad^{3}+3\,\left(4\,bc^{3}d^{3}x^{3}+6\,bc^{2}d^{3}x^{2}+4\,bcd^{3}x+bd^{3}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*b*c^4*d^3*x^4*log(c*x + 1) + 45*b*c^4*d^3*x^4*log(c*x - 1) - 48*b*
c^4*d^3*x^4*log(x) + 6*(4*a + 7*b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2 +
 2*(12*a + b)*c*d^3*x + 6*a*d^3 + 3*(4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*
b*c*d^3*x + b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(92) = 184$.
time = 0.58, size = 207, normalized size = 2.23

$$\begin{cases} -\frac{ac^{3}d^{3}}{x}-\frac{3ac^{2}d^{3}}{2x^{2}}-\frac{acd^{3}}{x^{3}}-\frac{ad^{3}}{4x^{4}}+2bc^{4}d^{3}\log\left(x\right)-2bc^{4}d^{3}\log\left(x-\frac{1}{c}\right)-\frac{bc^{4}d^{3}\operatorname{atanh}\left(cx\right)}{4}-\frac{bc^{3}d^{3}\operatorname{atanh}\left(cx\right)}{x}-\frac{7bc^{3}d^{3}}{4x}-\frac{3bc^{2}d^{3}\operatorname{atanh}\left(cx\right)}{2x^{2}}-\frac{bc^{2}d^{3}}{2x^{2}}-\frac{bcd^{3}\operatorname{atanh}\left(cx\right)}{x^{3}}-\frac{bcd^{3}}{12x^{3}}-\frac{bd^{3}\operatorname{atanh}\left(cx\right)}{4x^{4}} & \text{for}\: c \neq 0 \\ -\frac{ad^{3}}{4x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c**3*d**3/x - 3*a*c**2*d**3/(2*x**2) - a*c*d**3/x**3 - a*d**3
/(4*x**4) + 2*b*c**4*d**3*log(x) - 2*b*c**4*d**3*log(x - 1/c) - b*c**4*d**3
*atanh(c*x)/4 - b*c**3*d**3*atanh(c*x)/x - 7*b*c**3*d**3/(4*x) - 3*b*c**2*d
**3*atanh(c*x)/(2*x**2) - b*c**2*d**3/(2*x**2) - b*c*d**3*atanh(c*x)/x**3 -
 b*c*d**3/(12*x**3) - b*d**3*atanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**3/(4*x*
*4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(85) = 170$.
time = 0.41, size = 431, normalized size = 4.63

$$\frac{1}{3}\left(6\,bc^3d^3\log\left(-\frac{cx+1}{cx-1}-1\right)-6\,bc^3d^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{6\left(\frac{4\,(cx+1)^3bc^3d^3}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3d^3}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d^3}{cx-1}+bc^3d^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{\frac{48\,(cx+1)^3ac^3d^3}{(cx-1)^3}+\frac{72\,(cx+1)^2ac^3d^3}{(cx-1)^2}+\frac{48\,(cx+1)ac^3d^3}{cx-1}+12\,ac^3d^3+\frac{18\,(cx+1)^3bc^3d^3}{(cx-1)^3}+\frac{45\,(cx+1)^2bc^3d^3}{(cx-1)^2}+\frac{38\,(cx+1)bc^3d^3}{cx-1}+11\,bc^3d^3}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(6*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 6*b*c^3*d^3*log(-(c*x + 1)
/(c*x - 1)) + 6*(4*(c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^3*
d^3/(c*x - 1)^2 + 4*(c*x + 1)*b*c^3*d^3/(c*x - 1) + b*c^3*d^3)*log(-(c*x +
1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x
 + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (48*(c*x + 1)^3*a*c^3*d^
3/(c*x - 1)^3 + 72*(c*x + 1)^2*a*c^3*d^3/(c*x - 1)^2 + 48*(c*x + 1)*a*c^3*d
^3/(c*x - 1) + 12*a*c^3*d^3 + 18*(c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 + 45*(c*
x + 1)^2*b*c^3*d^3/(c*x - 1)^2 + 38*(c*x + 1)*b*c^3*d^3/(c*x - 1) + 11*b*c^
3*d^3)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2
/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [**B**]
time = 0.95, size = 147, normalized size = 1.58

$$\frac{d^3\,\left(21\,b\,c^4\,\mathrm{atanh}(c\,x)-12\,b\,c^4\,\ln\left(c^2\,x^2-1\right)+24\,b\,c^4\,\ln\left(x\right)\right)}{12}-\frac{\frac{d^3\,\left(3\,a+3\,b\,\mathrm{atanh}(c\,x)\right)}{12}+\frac{d^3\,x\,\left(12\,a\,c+b\,c+12\,b\,c\,\mathrm{atanh}(c\,x)\right)}{12}+\frac{d^3\,x^2\,\left(18\,a\,c^2+6\,b\,c^2+18\,b\,c^2\,\mathrm{atanh}(c\,x)\right)}{12}+\frac{d^3\,x^3\,\left(12\,a\,c^3+21\,b\,c^3+12\,b\,c^3\,\mathrm{atanh}(c\,x)\right)}{12}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^5,x)
```

```
[Out] (d^3*(21*b*c^4*atanh(c*x) - 12*b*c^4*log(c^2*x^2 - 1) + 24*b*c^4*log(x)))/1
2 - ((d^3*(3*a + 3*b*atanh(c*x)))/12 + (d^3*x*(12*a*c + b*c + 12*b*c*atanh(
c*x)))/12 + (d^3*x^2*(18*a*c^2 + 6*b*c^2 + 18*b*c^2*atanh(c*x)))/12 + (d^3*
x^3*(12*a*c^3 + 21*b*c^3 + 12*b*c^3*atanh(c*x)))/12)/x^4
```

3.29 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=137

$$-\frac{bcd^3}{20x^4}-\frac{bc^2d^3}{4x^3}-\frac{3bc^3d^3}{5x^2}-\frac{5bc^4d^3}{4x}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}+\frac{6}{5}bc^5d^3\,\mathrm{l}$$

```
[Out] -1/20*b*c*d^3/x^4-1/4*b*c^2*d^3/x^3-3/5*b*c^3*d^3/x^2-5/4*b*c^4*d^3/x-1/5*d
^3*(c*x+1)^4*(a+b*arctanh(c*x))/x^5+1/20*c*d^3*(c*x+1)^4*(a+b*arctanh(c*x))
/x^4+6/5*b*c^5*d^3*ln(x)-6/5*b*c^5*d^3*ln(-c*x+1)
```

Rubi [A]
time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {47, 37, 6083, 12, 153}

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\log(1-cx)-\frac{5bc^4d^3}{4x}-\frac{3bc^3d^3}{5x^2}-\frac{bc^2d^3}{4x^3}-\frac{bcd^3}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^3)/x^4 - (b*c^2*d^3)/(4*x^3) - (3*b*c^3*d^3)/(5*x^2) - (5*b*c^
4*d^3)/(4*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/(5*x^5) + (c*d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(20*x^4) + (6*b*c^5*d^3*Log[x])/5 - (6*b*c^5*
d^3*Log[1 - c*x])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4} \\
&= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4} \\
&= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4} \\
&= -\frac{bcd^3}{20x^4}-\frac{bc^2d^3}{4x^3}-\frac{3bc^3d^3}{5x^2}-\frac{5bc^4d^3}{4x}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 140, normalized size = 1.02

$$
-\frac{d^3\left(8a+30acx+2bcx+40ac^2x^2+10bc^2x^2+20ac^3x^3+24bc^3x^3+50bc^4x^4+2b(4+15cx+20c^2x^2+10c^3x^3)\tanh^{-1}(cx)-48bc^5x^5\log(x)+49bc^5x^5\log(1-cx)-bc^5x^5\log(1+cx)\right)}{40x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/40*(d^3*(8*a + 30*a*c*x + 2*b*c*x + 40*a*c^2*x^2 + 10*b*c^2*x^2 + 20*a*c
^3*x^3 + 24*b*c^3*x^3 + 50*b*c^4*x^4 + 2*b*(4 + 15*c*x + 20*c^2*x^2 + 10*c^
3*x^3)*ArcTanh[c*x] - 48*b*c^5*x^5*Log[x] + 49*b*c^5*x^5*Log[1 - c*x] - b*c
^5*x^5*Log[1 + c*x]))/x^5
```

Maple [A]
time = 0.17, size = 190, normalized size = 1.39

method	result
derivativedivides	$c^5\left(d^3a\left(-\frac{1}{5c^5x^5}-\frac{3}{4c^4x^4}-\frac{1}{2c^2x^2}-\frac{1}{c^3x^3}\right)-\frac{d^3b\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{3d^3b\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{d^3b\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{d^3}{}\right.$
default	$c^5\left(d^3a\left(-\frac{1}{5c^5x^5}-\frac{3}{4c^4x^4}-\frac{1}{2c^2x^2}-\frac{1}{c^3x^3}\right)-\frac{d^3b\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{3d^3b\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{d^3b\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{d^3}{}\right.$
risch	$-\frac{d^3b(10x^3c^3+20c^2x^2+15cx+4)\ln(cx+1)}{40x^5}+\frac{d^3(48c^5b\ln(-x)x^5+b\,c^5\ln(cx+1)x^5-49x^5b\ln(-cx+1)c^5-50c^4x^4b+10x^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(d^3*a*(-1/5/c^5/x^5-3/4/c^4/x^4-1/2/c^2/x^2-1/c^3/x^3)-1/5*d^3*b*arcta
nh(c*x)/c^5/x^5-3/4*d^3*b*arctanh(c*x)/c^4/x^4-1/2*d^3*b*arctanh(c*x)/c^2/x
^2-d^3*b*arctanh(c*x)/c^3/x^3-49/40*d^3*b*ln(c*x-1)-1/20*d^3*b/c^4/x^4-1/4*
d^3*b/c^3/x^3-3/5*d^3*b/c^2/x^2-5/4*d^3*b/c/x+6/5*d^3*b*ln(c*x)+1/40*d^3*b*
ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(121) = 242$.
time = 0.26, size = 250, normalized size = 1.82

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bc^3d^3-\frac{1}{2}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\operatorname{artanh}\left(cx\right)}{x^3}\right)bc^2d^3+\frac{1}{8}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\operatorname{artanh}\left(cx\right)}{x^4}\right)bcd^3-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\operatorname{artanh}\left(cx\right)}{x^5}\right)bd^3-\frac{ac^3d^3}{2\,x^2}-\frac{ac^2d^3}{x^3}-\frac{3\,acd^3}{4\,x^4}-\frac{ad^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*
d^3 - 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c^2*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*
x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^3 - 1/20*((2*c^4*log(c^2*x^2 -
1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^3 -
1/2*a*c^3*d^3/x^2 - a*c^2*d^3/x^3 - 3/4*a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

Fricas [A]
time = 0.38, size = 175, normalized size = 1.28

$$\frac{bc^5d^3x^5\log\left(cx+1\right)-49\,bc^5d^3x^5\log\left(cx-1\right)+48\,bc^5d^3x^5\log\left(x\right)-50\,bc^4d^3x^4-4\left(5\,a+6\,b\right)c^3d^3x^3-10\left(4\,a+b\right)c^2d^3x^2-2\left(15\,a+b\right)cd^3x-8\,ad^3-\left(10\,bc^3d^3x^3+20\,bc^2d^3x^2+15\,bcd^3x+4\,bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(b*c^5*d^3*x^5*log(c*x + 1) - 49*b*c^5*d^3*x^5*log(c*x - 1) + 48*b*c^5
*d^3*x^5*log(x) - 50*b*c^4*d^3*x^4 - 4*(5*a + 6*b)*c^3*d^3*x^3 - 10*(4*a +
b)*c^2*d^3*x^2 - 2*(15*a + b)*c*d^3*x - 8*a*d^3 - (10*b*c^3*d^3*x^3 + 20*b*
c^2*d^3*x^2 + 15*b*c*d^3*x + 4*b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A]
time = 0.66, size = 233, normalized size = 1.70

$$\begin{cases} -\frac{ac^3d^3}{2x^2} - \frac{ac^2d^3}{x^3} - \frac{3acd^3}{4x^4} - \frac{ad^3}{5x^5} + \frac{6bc^5d^3\log(x)}{5} - \frac{6bc^5d^3\log\left(x-\frac{1}{c}\right)}{5} + \frac{bc^5d^3\operatorname{atanh}(cx)}{20} - \frac{5bc^4d^3}{4x} - \frac{bc^3d^3\operatorname{atanh}(cx)}{2x^2} - \frac{3bc^3d^3}{5x^2} - \frac{bc^2d^3\operatorname{atanh}(cx)}{x^3} - \frac{bc^2d^3}{4x^3} - \frac{3bcd^3\operatorname{atanh}(cx)}{4x^4} - \frac{bcd^3}{20x^4} - \frac{bd^3\operatorname{atanh}(cx)}{5x^5} & \text{for } c \neq 0 \\ -\frac{ad^3}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**3*d**3/(2*x**2) - a*c**2*d**3/x**3 - 3*a*c*d**3/(4*x**4) -
 a*d**3/(5*x**5) + 6*b*c**5*d**3*log(x)/5 - 6*b*c**5*d**3*log(x - 1/c)/5 +
b*c**5*d**3*atanh(c*x)/20 - 5*b*c**4*d**3/(4*x) - b*c**3*d**3*atanh(c*x)/(2
*x**2) - 3*b*c**3*d**3/(5*x**2) - b*c**2*d**3*atanh(c*x)/x**3 - b*c**2*d**3
/(4*x**3) - 3*b*c*d**3*atanh(c*x)/(4*x**4) - b*c*d**3/(20*x**4) - b*d**3*at
anh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**3/(5*x**5), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(121) = 242.
time = 0.42, size = 533, normalized size = 3.89

$$\frac{1}{5}\left(6bc^4d^3\log\left(-\frac{cx+1}{cx-1}-1\right) - 6bc^4d^3\log\left(-\frac{cx+1}{cx-1}\right) + \frac{2\left(\frac{20(cx+1)^4bc^4d^3}{(cx-1)^4} + \frac{30(cx+1)^3bc^4d^3}{(cx-1)^3} + \frac{30(cx+1)^2bc^4d^3}{(cx-1)^2} + \frac{15(cx+1)bc^4d^3}{cx-1} + 3bc^4d^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5} + \frac{5(cx+1)^4}{(cx-1)^4} + \frac{10(cx+1)^3}{(cx-1)^3} + \frac{10(cx+1)^2}{(cx-1)^2} + \frac{5(cx+1)}{cx-1} + 1} + \frac{\frac{80(cx+1)^4ac^4d^3}{(cx-1)^4} + \frac{120(cx+1)^3ac^4d^3}{(cx-1)^3} + \frac{120(cx+1)^2ac^4d^3}{(cx-1)^2} + \frac{60(cx+1)ac^4d^3}{cx-1} + 12ac^4d^3 + \frac{34(cx+1)^4bc^4d^3}{(cx-1)^4} + \frac{103(cx+1)^3bc^4d^3}{(cx-1)^3} + \frac{123(cx+1)^2bc^4d^3}{(cx-1)^2} + \frac{69(cx+1)bc^4d^3}{cx-1} + 15bc^4d^3}{\frac{(cx+1)^5}{(cx-1)^5} + \frac{5(cx+1)^4}{(cx-1)^4} + \frac{10(cx+1)^3}{(cx-1)^3} + \frac{10(cx+1)^2}{(cx-1)^2} + \frac{5(cx+1)}{cx-1} + 1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/5*(6*b*c^4*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 6*b*c^4*d^3*log(-(c*x + 1)
/(c*x - 1)) + 2*(20*(c*x + 1)^4*b*c^4*d^3/(c*x - 1)^4 + 30*(c*x + 1)^3*b*c^
4*d^3/(c*x - 1)^3 + 30*(c*x + 1)^2*b*c^4*d^3/(c*x - 1)^2 + 15*(c*x + 1)*b*c
^4*d^3/(c*x - 1) + 3*b*c^4*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x
 - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x
+ 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (80*(c*x + 1)^4*a*c^4*d^3
/(c*x - 1)^4 + 120*(c*x + 1)^3*a*c^4*d^3/(c*x - 1)^3 + 120*(c*x + 1)^2*a*c^
4*d^3/(c*x - 1)^2 + 60*(c*x + 1)*a*c^4*d^3/(c*x - 1) + 12*a*c^4*d^3 + 34*(c
*x + 1)^4*b*c^4*d^3/(c*x - 1)^4 + 103*(c*x + 1)^3*b*c^4*d^3/(c*x - 1)^3 + 1
23*(c*x + 1)^2*b*c^4*d^3/(c*x - 1)^2 + 69*(c*x + 1)*b*c^4*d^3/(c*x - 1) + 1
5*b*c^4*d^3)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x
 + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) +
1))*c
```

Mupad [B]
time = 0.98, size = 233, normalized size = 1.70

$$-\frac{4ad^3 + 4bd^3\operatorname{atanh}(cx) + 20ac^2d^3x^2 + 10ac^3d^3x^3 + 10ac^5d^3x^5 + 5bc^2d^3x^2 + 12bc^3d^3x^3 + 25bc^4d^3x^4 + 12bc^5d^3x^5 + 15acd^3x + bcd^3x - 24bc^5d^3x^5\ln(x) + 20bc^2d^3x^2\operatorname{atanh}(cx) + 10bc^3d^3x^3\operatorname{atanh}(cx) + 12bc^5d^3x^5\ln(c^2x^2-1) + 15bcd^3x\operatorname{atanh}(cx) - 25bc^4d^3x^5\operatorname{atan}\left(\frac{c^2x}{\sqrt{-c^2}}\right)\sqrt{-c^2}}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^6,x)
```

```
[Out] -(4*a*d^3 + 4*b*d^3*atanh(c*x) + 20*a*c^2*d^3*x^2 + 10*a*c^3*d^3*x^3 + 10*a
*c^5*d^3*x^5 + 5*b*c^2*d^3*x^2 + 12*b*c^3*d^3*x^3 + 25*b*c^4*d^3*x^4 + 12*b
*c^5*d^3*x^5 + 15*a*c*d^3*x + b*c*d^3*x - 24*b*c^5*d^3*x^5*log(x) + 20*b*c^
2*d^3*x^2*atanh(c*x) + 10*b*c^3*d^3*x^3*atanh(c*x) + 12*b*c^5*d^3*x^5*log(c
^2*x^2 - 1) + 15*b*c*d^3*x*atanh(c*x) - 25*b*c^4*d^3*x^5*atan((c^2*x)/(-c^2
)^(1/2))*(-c^2)^(1/2))/(20*x^5)
```

3.30 $\int \frac{(d+cdx)^3(a+b\tanh^{-1}(cx))}{x^7}\,dx$

Optimal. Leaf size=196

$$-\frac{bcd^3}{30x^5}-\frac{3bc^2d^3}{20x^4}-\frac{11bc^3d^3}{36x^3}-\frac{7bc^4d^3}{15x^2}-\frac{11bc^5d^3}{12x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3c^2d^3(a+b}{4}$$

```
[Out] -1/30*b*c*d^3/x^5-3/20*b*c^2*d^3/x^4-11/36*b*c^3*d^3/x^3-7/15*b*c^4*d^3/x^2
-11/12*b*c^5*d^3/x-1/6*d^3*(a+b*arctanh(c*x))/x^6-3/5*c*d^3*(a+b*arctanh(c*
x))/x^5-3/4*c^2*d^3*(a+b*arctanh(c*x))/x^4-1/3*c^3*d^3*(a+b*arctanh(c*x))/x
^3+14/15*b*c^6*d^3*ln(x)-37/40*b*c^6*d^3*ln(-c*x+1)-1/120*b*c^6*d^3*ln(c*x+
1)
```

Rubi [A]
time = 0.13, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {45, 6083, 12, 1816}

$$-\frac{c^3d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{14}{15}bc^6d^3\log(x)-\frac{37}{40}bc^6d^3\log(1-cx)-\frac{1}{120}bc^6d^3\log(cx+1)-\frac{11bc^5d^3}{12x}-\frac{7bc^4d^3}{15x^2}-\frac{11bc^3d^3}{36x^3}-\frac{3bc^2d^3}{20x^4}-\frac{bcd^3}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d^3)/x^5 - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)/(36*x^3) - (7
*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTanh[c*x]))/(
6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(5*x^5) - (3*c^2*d^3*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x]))/(3*x^3) + (14*b*c^6*d^3*Lo
g[x])/15 - (37*b*c^6*d^3*Log[1 - c*x])/40 - (b*c^6*d^3*Log[1 + c*x])/120
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{3c^2d^3\left(a+b\tanh\right.}{4x^4}$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{3c^2d^3\left(a+b\tanh\right.}{4x^4}$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{3c^2d^3\left(a+b\tanh\right.}{4x^4}$$

$$= -\frac{bcd^3}{30x^5} - \frac{3bc^2d^3}{20x^4} - \frac{11bc^3d^3}{36x^3} - \frac{7bc^4d^3}{15x^2} - \frac{11bc^5d^3}{12x} - \frac{d^3\left(a+b\tanh^{-1}(c\right.}{6x^6}$$

Mathematica [A]

time = 0.05, size = 149, normalized size = 0.76

$$-\frac{d^3\left(60a+216acx+12bcx+270ac^2x^2+54bc^2x^2+120ac^3x^3+110bc^3x^3+168bc^4x^4+330bc^5x^5+6b\left(10+36cx+45c^2x^2+20c^3x^3\right)\tanh^{-1}(cx)-336bc^6x^6\log(x)+333bc^6x^6\log(1-cx)+3bc^6x^6\log(1+cx)\right)}{360x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -1/360*(d^3*(60*a + 216*a*c*x + 12*b*c*x + 270*a*c^2*x^2 + 54*b*c^2*x^2 + 1
20*a*c^3*x^3 + 110*b*c^3*x^3 + 168*b*c^4*x^4 + 330*b*c^5*x^5 + 6*b*(10 + 36
*c*x + 45*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x] - 336*b*c^6*x^6*Log[x] + 333*b
*c^6*x^6*Log[1 - c*x] + 3*b*c^6*x^6*Log[1 + c*x]))/x^6
```

Maple [A]

time = 0.18, size = 202, normalized size = 1.03

method	result
derivativedivides	$c^6\left(d^3a\left(-\frac{3}{4c^4x^4}-\frac{1}{3c^3x^3}-\frac{3}{5c^5x^5}-\frac{1}{6c^6x^6}\right)-\frac{3d^3b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{d^3b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{3d^3b\,\mathrm{arctanh}(cx)}{5c^5x^5}-\right.$
default	$c^6\left(d^3a\left(-\frac{3}{4c^4x^4}-\frac{1}{3c^3x^3}-\frac{3}{5c^5x^5}-\frac{1}{6c^6x^6}\right)-\frac{3d^3b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{d^3b\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{3d^3b\,\mathrm{arctanh}(cx)}{5c^5x^5}-\right.$
risch	$-\frac{d^3b\left(20x^3c^3+45c^2x^2+36cx+10\right)\ln(cx+1)}{120x^6}-\frac{d^3\left(3b\,c^6\ln(cx+1)x^6-336c^6b\ln(-x)x^6+333x^6b\ln(-cx+1)c^6+330c^5x^5b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(d^3*a*(-3/4/c^4/x^4-1/3/c^3/x^3-3/5/c^5/x^5-1/6/c^6/x^6)-3/4*d^3*b*arc
tanh(c*x)/c^4/x^4-1/3*d^3*b*arctanh(c*x)/c^3/x^3-3/5*d^3*b*arctanh(c*x)/c^5
/x^5-1/6*d^3*b*arctanh(c*x)/c^6/x^6-37/40*d^3*b*ln(c*x-1)-1/120*d^3*b*ln(c*
x+1)-1/30*d^3*b/c^5/x^5-3/20*d^3*b/c^4/x^4-11/36*d^3*b/c^3/x^3-7/15*d^3*b/c
^2/x^2-11/12*d^3*b/c/x+14/15*d^3*b*ln(c*x))
```

Maxima [A]
time = 0.28, size = 273, normalized size = 1.39

$$-\frac{1}{6}\left(\left(c^{2}\log\left(c^{2}x^{2}-1\right)-c^{2}\log\left(x^{2}\right)+\frac{1}{x^{2}}\right)c+\frac{2\,\operatorname{artanh}\left(cx\right)}{x^{3}}\right)bc^{3}d^{3}+\frac{1}{8}\left(\left(3\,c^{3}\log\left(cx+1\right)-3\,c^{3}\log\left(cx-1\right)-\frac{2\,\left(3\,c^{2}x^{2}+1\right)}{x^{3}}\right)c-\frac{6\,\operatorname{artanh}\left(cx\right)}{x^{4}}\right)bc^{2}d^{3}-\frac{3}{20}\left(\left(2\,c^{4}\log\left(c^{2}x^{2}-1\right)-2\,c^{4}\log\left(x^{2}\right)+\frac{2\,c^{2}x^{2}+1}{x^{4}}\right)c+\frac{4\,\operatorname{artanh}\left(cx\right)}{x^{5}}\right)bcd^{3}+\frac{1}{180}\left(\left(15\,c^{5}\log\left(cx+1\right)-15\,c^{5}\log\left(cx-1\right)-\frac{2\,\left(15\,c^{4}x^{4}+5\,c^{2}x^{2}+3\right)}{x^{5}}\right)c-\frac{30\,\operatorname{artanh}\left(cx\right)}{x^{6}}\right)bd^{3}-\frac{ac^{3}d^{3}}{3\,x^{3}}-\frac{3\,ac^{2}d^{3}}{4\,x^{4}}-\frac{3\,acd^{3}}{5\,x^{5}}-\frac{ad^{3}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^3*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^3 - 3/20*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^3 + 1
/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x^
2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^3 - 1/3*a*c^3*d^3/x^3 - 3/4*a*c^2*
d^3/x^4 - 3/5*a*c*d^3/x^5 - 1/6*a*d^3/x^6
```

Fricas [A]
time = 0.39, size = 188, normalized size = 0.96

$$-\frac{3\,bc^{6}d^{3}x^{6}\log\left(cx+1\right)+333\,bc^{6}d^{3}x^{6}\log\left(cx-1\right)-336\,bc^{6}d^{3}x^{6}\log\left(x\right)+330\,bc^{5}d^{3}x^{5}+168\,bc^{4}d^{3}x^{4}+10\,\left(12\,a+11\,b\right)c^{3}d^{3}x^{3}+54\,\left(5\,a+b\right)c^{2}d^{3}x^{2}+12\,\left(18\,a+b\right)cd^{3}x+60\,ad^{3}+3\,\left(20\,bc^{3}d^{3}x^{3}+45\,bc^{2}d^{3}x^{2}+36\,bcd^{3}x+10\,bd^{3}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{360\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/360*(3*b*c^6*d^3*x^6*log(c*x + 1) + 333*b*c^6*d^3*x^6*log(c*x - 1) - 336
*b*c^6*d^3*x^6*log(x) + 330*b*c^5*d^3*x^5 + 168*b*c^4*d^3*x^4 + 10*(12*a +
11*b)*c^3*d^3*x^3 + 54*(5*a + b)*c^2*d^3*x^2 + 12*(18*a + b)*c*d^3*x + 60*a
*d^3 + 3*(20*b*c^3*d^3*x^3 + 45*b*c^2*d^3*x^2 + 36*b*c*d^3*x + 10*b*d^3)*lo
g(-(c*x + 1)/(c*x - 1)))/x^6
```

Sympy [A]
time = 0.79, size = 257, normalized size = 1.31

$$\begin{cases} - \frac{a c^{3} d^{3}}{3 x^{3}} - \frac{3 a c^{2} d^{3}}{4 x^{4}} - \frac{3 a c d^{3}}{5 x^{5}} - \frac{a d^{3}}{6 x^{6}} + \frac{14 b c^{6} d^{3} \log{\left(x \right)}}{15} - \frac{14 b c^{6} d^{3} \log{\left(x - \frac{1}{c} \right)}}{15} - \frac{b c^{6} d^{3} \operatorname{atanh}{\left(c x \right)}}{60} - \frac{11 b c^{5} d^{3}}{12 x} - \frac{7 b c^{4} d^{3}}{15 x^{2}} - \frac{b c^{3} d^{3} \operatorname{atanh}{\left(c x \right)}}{3 x^{3}} - \frac{11 b c^{3} d^{3}}{36 x^{3}} - \frac{3 b c^{2} d^{3} \operatorname{atanh}{\left(c x \right)}}{4 x^{4}} - \frac{3 b c^{2} d^{3}}{20 x^{4}} - \frac{3 b c d^{3} \operatorname{atanh}{\left(c x \right)}}{5 x^{5}} - \frac{b c d^{3}}{30 x^{5}} - \frac{b d^{3} \operatorname{atanh}{\left(c x \right)}}{6 x^{6}} & \text{for}\: c \neq 0 \\- \frac{a d^{3}}{6 x^{6}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**7,x)
```

```
[Out] Piecewise((-a*c**3*d**3/(3*x**3) - 3*a*c**2*d**3/(4*x**4) - 3*a*c*d**3/(5*x
**5) - a*d**3/(6*x**6) + 14*b*c**6*d**3*log(x)/15 - 14*b*c**6*d**3*log(x -
1/c)/15 - b*c**6*d**3*atanh(c*x)/60 - 11*b*c**5*d**3/(12*x) - 7*b*c**4*d**3
/(15*x**2) - b*c**3*d**3*atanh(c*x)/(3*x**3) - 11*b*c**3*d**3/(36*x**3) - 3
*b*c**2*d**3*atanh(c*x)/(4*x**4) - 3*b*c**2*d**3/(20*x**4) - 3*b*c*d**3*ata
nh(c*x)/(5*x**5) - b*c*d**3/(30*x**5) - b*d**3*atanh(c*x)/(6*x**6), Ne(c, 0
)), (-a*d**3/(6*x**6), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(172) = 344.
time = 0.44, size = 634, normalized size = 3.23

$$\frac{1}{45}\left(42\,bc^5d^3\log\left(-\frac{cx+1}{cx-1}-1\right)-42\,bc^5d^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{6\left(\frac{60\,(cx+1)^5bc^5d^3}{(cx-1)^5}+\frac{90\,(cx+1)^4bc^5d^3}{(cx-1)^4}+\frac{140\,(cx+1)^3bc^5d^3}{(cx-1)^3}+\frac{105\,(cx+1)^2bc^5d^3}{(cx-1)^2}+\frac{42\,(cx+1)bc^5d^3}{cx-1}+7\,bc^5d^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6}{(cx-1)^6}+\frac{6\,(cx+1)^5}{(cx-1)^5}+\frac{15\,(cx+1)^4}{(cx-1)^4}+\frac{20\,(cx+1)^3}{(cx-1)^3}+\frac{15\,(cx+1)^2}{(cx-1)^2}+\frac{6\,(cx+1)}{cx-1}+1}+\frac{\frac{720\,(cx+1)^5ac^5d^3}{(cx-1)^5}+\frac{1080\,(cx+1)^4ac^5d^3}{(cx-1)^4}+\frac{1680\,(cx+1)^3ac^5d^3}{(cx-1)^3}+\frac{1260\,(cx+1)^2ac^5d^3}{(cx-1)^2}+\frac{504\,(cx+1)ac^5d^3}{cx-1}+84\,ac^5d^3+\frac{318\,(cx+1)^5bc^5d^3}{(cx-1)^5}+\frac{1119\,(cx+1)^4bc^5d^3}{(cx-1)^4}+\frac{1742\,(cx+1)^3bc^5d^3}{(cx-1)^3}+\frac{1464\,(cx+1)^2bc^5d^3}{(cx-1)^2}+\frac{636\,(cx+1)bc^5d^3}{cx-1}+113\,bc^5d^3}{\frac{(cx+1)^6}{(cx-1)^6}+\frac{6\,(cx+1)^5}{(cx-1)^5}+\frac{15\,(cx+1)^4}{(cx-1)^4}+\frac{20\,(cx+1)^3}{(cx-1)^3}+\frac{15\,(cx+1)^2}{(cx-1)^2}+\frac{6\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

```
[Out] 1/45*(42*b*c^5*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 42*b*c^5*d^3*log(-(c*x +
 1)/(c*x - 1)) + 6*(60*(c*x + 1)^5*b*c^5*d^3/(c*x - 1)^5 + 90*(c*x + 1)^4*b
*c^5*d^3/(c*x - 1)^4 + 140*(c*x + 1)^3*b*c^5*d^3/(c*x - 1)^3 + 105*(c*x + 1
)^2*b*c^5*d^3/(c*x - 1)^2 + 42*(c*x + 1)*b*c^5*d^3/(c*x - 1) + 7*b*c^5*d^3)
*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x -
1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x +
1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1) + (720*(c*x + 1)^5*a*c^5*d^3/
(c*x - 1)^5 + 1080*(c*x + 1)^4*a*c^5*d^3/(c*x - 1)^4 + 1680*(c*x + 1)^3*a*c
^5*d^3/(c*x - 1)^3 + 1260*(c*x + 1)^2*a*c^5*d^3/(c*x - 1)^2 + 504*(c*x + 1)
*a*c^5*d^3/(c*x - 1) + 84*a*c^5*d^3 + 318*(c*x + 1)^5*b*c^5*d^3/(c*x - 1)^5
 + 1119*(c*x + 1)^4*b*c^5*d^3/(c*x - 1)^4 + 1742*(c*x + 1)^3*b*c^5*d^3/(c*x
 - 1)^3 + 1464*(c*x + 1)^2*b*c^5*d^3/(c*x - 1)^2 + 636*(c*x + 1)*b*c^5*d^3/
(c*x - 1) + 113*b*c^5*d^3)/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x -
1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x +
1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 1.07, size = 220, normalized size = 1.12

$$\frac{14\,b\,c^6\,d^3\,\ln\left(x\right)}{15}-\frac{7\,b\,c^6\,d^3\,\ln\left(c^2\,x^2-1\right)}{15}-\frac{3\,a\,c^2\,d^3}{4\,x^4}-\frac{a\,c^3\,d^3}{3\,x^3}-\frac{3\,b\,c^2\,d^3}{20\,x^4}-\frac{11\,b\,c^3\,d^3}{36\,x^3}-\frac{7\,b\,c^4\,d^3}{15\,x^2}-\frac{11\,b\,c^5\,d^3}{12\,x}-\frac{a\,d^3}{6\,x^6}-\frac{3\,a\,c\,d^3}{5\,x^5}-\frac{b\,c\,d^3}{30\,x^5}-\frac{b\,d^3\,\mathrm{atanh}(c\,x)}{6\,x^6}-\frac{11\,b\,c^7\,d^3\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{12\,\sqrt{-c^2}}-\frac{3\,b\,c\,d^3\,\mathrm{atanh}(c\,x)}{5\,x^5}-\frac{3\,b\,c^2\,d^3\,\mathrm{atanh}(c\,x)}{4\,x^4}-\frac{b\,c^3\,d^3\,\mathrm{atanh}(c\,x)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^7,x)
```

```
[Out] (14*b*c^6*d^3*log(x))/15 - (7*b*c^6*d^3*log(c^2*x^2 - 1))/15 - (3*a*c^2*d^3
)/(4*x^4) - (a*c^3*d^3)/(3*x^3) - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)/(
```

```
36*x^3) - (7*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (a*d^3)/(6*x^6)
- (3*a*c*d^3)/(5*x^5) - (b*c*d^3)/(30*x^5) - (b*d^3*atanh(c*x))/(6*x^6) - (
11*b*c^7*d^3*atan((c^2*x)/(-c^2)^(1/2)))/(12*(-c^2)^(1/2)) - (3*b*c*d^3*ata
nh(c*x))/(5*x^5) - (3*b*c^2*d^3*atanh(c*x))/(4*x^4) - (b*c^3*d^3*atanh(c*x)
)/(3*x^3)
```

3.31 $\int x^3(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=224

$$\frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}bc^3d^4x^7+\frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5$$

```
[Out] 11/8*b*d^4*x/c^3+24/35*b*d^4*x^2/c^2+11/24*b*d^4*x^3/c+12/35*b*d^4*x^4+9/40
*b*c*d^4*x^5+2/21*b*c^2*d^4*x^6+1/56*b*c^3*d^4*x^7+1/4*d^4*x^4*(a+b*arctanh
(c*x))+4/5*c*d^4*x^5*(a+b*arctanh(c*x))+c^2*d^4*x^6*(a+b*arctanh(c*x))+4/7*
c^3*d^4*x^7*(a+b*arctanh(c*x))+1/8*c^4*d^4*x^8*(a+b*arctanh(c*x))+769/560*b
*d^4*ln(-c*x+1)/c^4-1/560*b*d^4*ln(c*x+1)/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {45, 6083, 12, 1816, 647, 31}

$$\frac{1}{8}c^4d^4x^8\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{7}c^3d^4x^7\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{769bd^4\log(1-cx)}{560c^4}-\frac{bd^4\log(cx+1)}{560c^4}+\frac{1}{56}bc^3d^4x^7+\frac{11bd^4x}{8c^3}+\frac{2}{21}bc^2d^4x^6+\frac{24bd^4x^2}{35c^2}+\frac{9}{40}bcd^4x^5+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (11*b*d^4*x)/(8*c^3) + (24*b*d^4*x^2)/(35*c^2) + (11*b*d^4*x^3)/(24*c) + (1
2*b*d^4*x^4)/35 + (9*b*c*d^4*x^5)/40 + (2*b*c^2*d^4*x^6)/21 + (b*c^3*d^4*x^
7)/56 + (d^4*x^4*(a + b*ArcTanh[c*x]))/4 + (4*c*d^4*x^5*(a + b*ArcTanh[c*x]
))/5 + c^2*d^4*x^6*(a + b*ArcTanh[c*x]) + (4*c^3*d^4*x^7*(a + b*ArcTanh[c*x
]))/7 + (c^4*d^4*x^8*(a + b*ArcTanh[c*x]))/8 + (769*b*d^4*Log[1 - c*x])/(56
0*c^4) - (b*d^4*Log[1 + c*x])/(560*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^3(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a\right.\\ &= \frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a\right.\\ &= \frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a\right.\\ &= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6\\ &= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6\\ &= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6\end{aligned}$$

Mathematica [A]
time = 0.06, size = 177, normalized size = 0.79

$$\frac{d^4\left(2310bcx+1152bc^2x^2+770bc^3x^3+420ac^4x^4+576bc^4x^4+1344ac^5x^5+378bc^5x^5+1680ac^6x^6+160bc^6x^6+960ac^7x^7+30bc^7x^7+210ac^8x^8+6bc^4x^4(70+224cx+280c^2x^2+160c^3x^3+35c^4x^4)\tanh^{-1}(cx)+2307b\log(1-cx)-3b\log(1+cx)\right)}{1680c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(2310*b*c*x + 1152*b*c^2*x^2 + 770*b*c^3*x^3 + 420*a*c^4*x^4 + 576*b*c
^4*x^4 + 1344*a*c^5*x^5 + 378*b*c^5*x^5 + 1680*a*c^6*x^6 + 160*b*c^6*x^6 +
960*a*c^7*x^7 + 30*b*c^7*x^7 + 210*a*c^8*x^8 + 6*b*c^4*x^4*(70 + 224*c*x +
280*c^2*x^2 + 160*c^3*x^3 + 35*c^4*x^4)*ArcTanh[c*x] + 2307*b*Log[1 - c*x]
- 3*b*Log[1 + c*x]))/(1680*c^4)
```

Maple [A]
time = 0.22, size = 234, normalized size = 1.04

method	result
derivativedivides	$\frac{d^4 a\left(\frac{1}{8}c^8x^8+\frac{4}{7}c^7x^7+c^6x^6+\frac{4}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^4 b \operatorname{arctanh}(cx)c^8x^8}{8}+\frac{4d^4 b \operatorname{arctanh}(cx)c^7x^7}{7}+d^4 b \operatorname{arctanh}(cx)c^6x^6+\frac{4d^4 b \operatorname{arctanh}}{5}$
default	$\frac{d^4 a\left(\frac{1}{8}c^8x^8+\frac{4}{7}c^7x^7+c^6x^6+\frac{4}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^4 b \operatorname{arctanh}(cx)c^8x^8}{8}+\frac{4d^4 b \operatorname{arctanh}(cx)c^7x^7}{7}+d^4 b \operatorname{arctanh}(cx)c^6x^6+\frac{4d^4 b \operatorname{arctanh}}{5}$
risch	$\frac{d^4 b x^4\left(35c^4x^4+160x^3c^3+280c^2x^2+224cx+70\right)\ln(cx+1)}{560}-\frac{d^4c^4 b x^8\ln(-cx+1)}{16}+\frac{d^4c^4 a x^8}{8}-\frac{2d^4c^3 b x^7\ln(-cx+1)}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^4*a*(1/8*c^8*x^8+4/7*c^7*x^7+c^6*x^6+4/5*c^5*x^5+1/4*c^4*x^4)+1/8*
d^4*b*arctanh(c*x)*c^8*x^8+4/7*d^4*b*arctanh(c*x)*c^7*x^7+d^4*b*arctanh(c*x
)*c^6*x^6+4/5*d^4*b*arctanh(c*x)*c^5*x^5+1/4*d^4*b*arctanh(c*x)*c^4*x^4+1/5
6*d^4*b*c^7*x^7+2/21*d^4*b*c^6*x^6+9/40*d^4*b*c^5*x^5+12/35*d^4*b*c^4*x^4+1
1/24*b*c^3*d^4*x^3+24/35*b*c^2*d^4*x^2+11/8*b*c*d^4*x+769/560*d^4*b*ln(c*x-
1)-1/560*d^4*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 373, normalized size = 1.67

$$\frac{1}{8}ac^4d^4x^8+\frac{4}{7}ac^3d^4x^7+ac^2d^4x^6+\frac{4}{5}acd^4x^5+\frac{1}{1680}\left(210x^8\operatorname{artanh}(cx)+c\left(\frac{2(15c^6x^7+21c^4x^5+35c^2x^3+105x)}{c^8}-\frac{105\log(cx+1)}{c^9}+\frac{105\log(cx-1)}{c^9}\right)\right)bc^4d^4+\frac{1}{21}\left(12x^7\operatorname{artanh}(cx)+c\left(\frac{2c^4x^6+3c^2x^4+6x^2}{c^6}+\frac{6\log(c^2x^2-1)}{c^8}\right)\right)bc^3d^4+\frac{1}{4}ad^4x^4+\frac{1}{30}\left(30x^6\operatorname{artanh}(cx)+c\left(\frac{2(3c^4x^5+5c^2x^3+15x)}{c^6}-\frac{15\log(cx+1)}{c^7}+\frac{15\log(cx-1)}{c^7}\right)\right)bc^2d^4+\frac{1}{5}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log(c^2x^2-1)}{c^6}\right)\right)bcd^4+\frac{1}{24}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^4*x^8 + 4/7*a*c^3*d^4*x^7 + a*c^2*d^4*x^6 + 4/5*a*c*d^4*x^5 + 1
/1680*(210*x^8*arctanh(c*x) + c*(2*(15*c^6*x^7 + 21*c^4*x^5 + 35*c^2*x^3 +
105*x)/c^8 - 105*log(c*x + 1)/c^9 + 105*log(c*x - 1)/c^9))*b*c^4*d^4 + 1/21
*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*
x^2 - 1)/c^8))*b*c^3*d^4 + 1/4*a*d^4*x^4 + 1/30*(30*x^6*arctanh(c*x) + c*(2
*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)
/c^7))*b*c^2*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*l
og(c^2*x^2 - 1)/c^6))*b*c*d^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 +
3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^4
```

Fricas [A]
time = 0.37, size = 222, normalized size = 0.99

$$\frac{210\,ac^8d^4x^8+30\,(32\,a+b)c^7d^4x^7+80\,(21\,a+2\,b)c^6d^4x^6+42\,(32\,a+9\,b)c^5d^4x^5+12\,(35\,a+48\,b)c^4d^4x^4+770\,bc^3d^4x^3+1152\,bc^2d^4x^2+2310\,bcd^4x-3\,bd^4\log(cx+1)+2307\,bd^4\log(cx-1)+3\,(35\,bc^8d^4x^8+160\,bc^7d^4x^7+280\,bc^6d^4x^6+224\,bc^5d^4x^5+70\,bc^4d^4x^4)\log\left(-\frac{cx+1}{cx-1}\right)}{1680\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(210*a*c^8*d^4*x^8 + 30*(32*a + b)*c^7*d^4*x^7 + 80*(21*a + 2*b)*c^6
*d^4*x^6 + 42*(32*a + 9*b)*c^5*d^4*x^5 + 12*(35*a + 48*b)*c^4*d^4*x^4 + 770
*b*c^3*d^4*x^3 + 1152*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 3*b*d^4*log(c*x + 1)
 + 2307*b*d^4*log(c*x - 1) + 3*(35*b*c^8*d^4*x^8 + 160*b*c^7*d^4*x^7 + 280*
b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 + 70*b*c^4*d^4*x^4)*log(-(c*x + 1)/(c*x -
 1)))/c^4
```

Sympy [A]
time = 0.65, size = 294, normalized size = 1.31

$$\begin{cases} \frac{ac^4d^4x^8}{8}+\frac{4ac^3d^4x^7}{7}+ac^2d^4x^6+\frac{4acd^4x^5}{5}+\frac{ad^4x^4}{4}+\frac{bc^4d^4x^8\operatorname{atanh}(cx)}{8}+\frac{4bc^3d^4x^7\operatorname{atanh}(cx)}{7}+\frac{bc^3d^4x^7}{56}+bc^2d^4x^6\operatorname{atanh}(cx)+\frac{2bc^2d^4x^6}{21}+\frac{4bcd^4x^5\operatorname{atanh}(cx)}{5}+\frac{9bcd^4x^5}{40}+\frac{bd^4x^4\operatorname{atanh}(cx)}{4}+\frac{12bd^4x^4}{35}+\frac{11bd^4x^3}{24c}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x}{8c^3}+\frac{48bd^4\log\left(x-\frac{1}{c}\right)}{35c^4}-\frac{bd^4\operatorname{atanh}(cx)}{280c^4} & \text{for}\ c\neq 0 \\ \frac{ad^4x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**8/8 + 4*a*c**3*d**4*x**7/7 + a*c**2*d**4*x**6 + 4
*a*c*d**4*x**5/5 + a*d**4*x**4/4 + b*c**4*d**4*x**8*atanh(c*x)/8 + 4*b*c**3
*d**4*x**7*atanh(c*x)/7 + b*c**3*d**4*x**7/56 + b*c**2*d**4*x**6*atanh(c*x)
 + 2*b*c**2*d**4*x**6/21 + 4*b*c*d**4*x**5*atanh(c*x)/5 + 9*b*c*d**4*x**5/4
0 + b*d**4*x**4*atanh(c*x)/4 + 12*b*d**4*x**4/35 + 11*b*d**4*x**3/(24*c) +
24*b*d**4*x**2/(35*c**2) + 11*b*d**4*x/(8*c**3) + 48*b*d**4*log(x - 1/c)/(3
5*c**4) - b*d**4*atanh(c*x)/(280*c**4), Ne(c, 0)), (a*d**4*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(198) = 396.
time = 0.45, size = 817, normalized size = 3.65

$$-\frac{4}{105}\,c\left(\frac{36\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^5}-\frac{12\left(\frac{35\,(cx+1)^7bd^4}{(cx-1)^7}-\frac{70\,(cx+1)^6bd^4}{(cx-1)^6}+\frac{175\,(cx+1)^5bd^4}{(cx-1)^5}-\frac{210\,(cx+1)^4bd^4}{(cx-1)^4}+\frac{168\,(cx+1)^3bd^4}{(cx-1)^3}-\frac{84\,(cx+1)^2bd^4}{(cx-1)^2}+\frac{24\,(cx+1)bd^4}{cx-1}-3\,bd^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^8c^5}{(cx-1)^8}-\frac{8\,(cx+1)^7c^5}{(cx-1)^7}+\frac{28\,(cx+1)^6c^5}{(cx-1)^6}-\frac{56\,(cx+1)^5c^5}{(cx-1)^5}+\frac{70\,(cx+1)^4c^5}{(cx-1)^4}-\frac{56\,(cx+1)^3c^5}{(cx-1)^3}+\frac{28\,(cx+1)^2c^5}{(cx-1)^2}-\frac{8\,(cx+1)c^5}{cx-1}+c^5}-\frac{36\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^5}-\frac{\frac{840\,(cx+1)^7ad^4}{(cx-1)^7}-\frac{1680\,(cx+1)^6ad^4}{(cx-1)^6}+\frac{4200\,(cx+1)^5ad^4}{(cx-1)^5}-\frac{5040\,(cx+1)^4ad^4}{(cx-1)^4}+\frac{4032\,(cx+1)^3ad^4}{(cx-1)^3}-\frac{2016\,(cx+1)^2ad^4}{(cx-1)^2}+\frac{576\,(cx+1)ad^4}{cx-1}-72\,ad^4+\frac{384\,(cx+1)^7bd^4}{(cx-1)^7}-\frac{1830\,(cx+1)^6bd^4}{(cx-1)^6}+\frac{4304\,(cx+1)^5bd^4}{(cx-1)^5}-\frac{6031\,(cx+1)^4bd^4}{(cx-1)^4}+\frac{5228\,(cx+1)^3bd^4}{(cx-1)^3}-\frac{2782\,(cx+1)^2bd^4}{(cx-1)^2}+\frac{836\,(cx+1)bd^4}{cx-1}-109\,bd^4}{\frac{(cx+1)^8c^5}{(cx-1)^8}-\frac{8\,(cx+1)^7c^5}{(cx-1)^7}+\frac{28\,(cx+1)^6c^5}{(cx-1)^6}-\frac{56\,(cx+1)^5c^5}{(cx-1)^5}+\frac{70\,(cx+1)^4c^5}{(cx-1)^4}-\frac{56\,(cx+1)^3c^5}{(cx-1)^3}+\frac{28\,(cx+1)^2c^5}{(cx-1)^2}-\frac{8\,(cx+1)c^5}{cx-1}+c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -4/105*c*(36*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 - 12*(35*(c*x + 1)^7*b
*d^4/(c*x - 1)^7 - 70*(c*x + 1)^6*b*d^4/(c*x - 1)^6 + 175*(c*x + 1)^5*b*d^4
/(c*x - 1)^5 - 210*(c*x + 1)^4*b*d^4/(c*x - 1)^4 + 168*(c*x + 1)^3*b*d^4/(c
*x - 1)^3 - 84*(c*x + 1)^2*b*d^4/(c*x - 1)^2 + 24*(c*x + 1)*b*d^4/(c*x - 1)
 - 3*b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^8*c^5/(c*x - 1)^8 - 8*(c*x
```

```
 + 1)^7*c^5/(c*x - 1)^7 + 28*(c*x + 1)^6*c^5/(c*x - 1)^6 - 56*(c*x + 1)^5*c
^5/(c*x - 1)^5 + 70*(c*x + 1)^4*c^5/(c*x - 1)^4 - 56*(c*x + 1)^3*c^5/(c*x -
 1)^3 + 28*(c*x + 1)^2*c^5/(c*x - 1)^2 - 8*(c*x + 1)*c^5/(c*x - 1) + c^5) -
 36*b*d^4*log(-(c*x + 1)/(c*x - 1))/c^5 - (840*(c*x + 1)^7*a*d^4/(c*x - 1)^
7 - 1680*(c*x + 1)^6*a*d^4/(c*x - 1)^6 + 4200*(c*x + 1)^5*a*d^4/(c*x - 1)^5
 - 5040*(c*x + 1)^4*a*d^4/(c*x - 1)^4 + 4032*(c*x + 1)^3*a*d^4/(c*x - 1)^3
- 2016*(c*x + 1)^2*a*d^4/(c*x - 1)^2 + 576*(c*x + 1)*a*d^4/(c*x - 1) - 72*a
*d^4 + 384*(c*x + 1)^7*b*d^4/(c*x - 1)^7 - 1830*(c*x + 1)^6*b*d^4/(c*x - 1)
^6 + 4304*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 6031*(c*x + 1)^4*b*d^4/(c*x - 1)^
4 + 5228*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 2782*(c*x + 1)^2*b*d^4/(c*x - 1)^2
 + 836*(c*x + 1)*b*d^4/(c*x - 1) - 109*b*d^4)/((c*x + 1)^8*c^5/(c*x - 1)^8
- 8*(c*x + 1)^7*c^5/(c*x - 1)^7 + 28*(c*x + 1)^6*c^5/(c*x - 1)^6 - 56*(c*x
+ 1)^5*c^5/(c*x - 1)^5 + 70*(c*x + 1)^4*c^5/(c*x - 1)^4 - 56*(c*x + 1)^3*c^
5/(c*x - 1)^3 + 28*(c*x + 1)^2*c^5/(c*x - 1)^2 - 8*(c*x + 1)*c^5/(c*x - 1)
+ c^5))
```

Mupad [B]
time = 1.84, size = 337, normalized size = 1.50

$$\frac{a\,d^4\,x^4}{4}+\frac{12\,b\,d^4\,x^4}{35}+a\,c^2\,d^4\,x^6+\frac{4\,a\,c^3\,d^4\,x^7}{7}+\frac{a\,c^4\,d^4\,x^8}{8}+\frac{11\,b\,d^4\,x^3}{24\,c}+\frac{24\,b\,d^4\,x^2}{35\,c^2}+\frac{2\,b\,c^2\,d^4\,x^6}{21}+\frac{b\,c^3\,d^4\,x^7}{56}+\frac{769\,b\,d^4\,\ln\left(c\,x-1\right)}{560\,c^4}-\frac{b\,d^4\,\ln\left(c\,x+1\right)}{560\,c^4}+\frac{b\,d^4\,x^4\,\ln\left(c\,x+1\right)}{8}-\frac{b\,d^4\,x^4\,\ln\left(1-c\,x\right)}{8}+\frac{4\,a\,c\,d^4\,x^5}{5}+\frac{11\,b\,d^4\,x}{8\,c^3}+\frac{9\,b\,c\,d^4\,x^5}{40}+\frac{b\,c^2\,d^4\,x^6\,\ln\left(c\,x+1\right)}{2}-\frac{b\,c^2\,d^4\,x^6\,\ln\left(1-c\,x\right)}{2}+\frac{2\,b\,c^3\,d^4\,x^7\,\ln\left(c\,x+1\right)}{7}-\frac{2\,b\,c^3\,d^4\,x^7\,\ln\left(1-c\,x\right)}{7}+\frac{b\,c^4\,d^4\,x^8\,\ln\left(c\,x+1\right)}{16}-\frac{b\,c^4\,d^4\,x^8\,\ln\left(1-c\,x\right)}{16}+\frac{2\,b\,c\,d^4\,x^5\,\ln\left(c\,x+1\right)}{5}-\frac{2\,b\,c\,d^4\,x^5\,\ln\left(1-c\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] (a*d^4*x^4)/4 + (12*b*d^4*x^4)/35 + a*c^2*d^4*x^6 + (4*a*c^3*d^4*x^7)/7 + (
a*c^4*d^4*x^8)/8 + (11*b*d^4*x^3)/(24*c) + (24*b*d^4*x^2)/(35*c^2) + (2*b*c
^2*d^4*x^6)/21 + (b*c^3*d^4*x^7)/56 + (769*b*d^4*log(c*x - 1))/(560*c^4) -
(b*d^4*log(c*x + 1))/(560*c^4) + (b*d^4*x^4*log(c*x + 1))/8 - (b*d^4*x^4*lo
g(1 - c*x))/8 + (4*a*c*d^4*x^5)/5 + (11*b*d^4*x)/(8*c^3) + (9*b*c*d^4*x^5)/
40 + (b*c^2*d^4*x^6*log(c*x + 1))/2 - (b*c^2*d^4*x^6*log(1 - c*x))/2 + (2*b
*c^3*d^4*x^7*log(c*x + 1))/7 - (2*b*c^3*d^4*x^7*log(1 - c*x))/7 + (b*c^4*d^
4*x^8*log(c*x + 1))/16 - (b*c^4*d^4*x^8*log(1 - c*x))/16 + (2*b*c*d^4*x^5*l
og(c*x + 1))/5 - (2*b*c*d^4*x^5*log(1 - c*x))/5
```

3.32 $\int x^2(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=171

$$\frac{5bd^4x}{3c^2}+\frac{88bd^4x^2}{105c}+\frac{5}{9}bd^4x^3+\frac{47}{140}bcd^4x^4+\frac{2}{15}bc^2d^4x^5+\frac{1}{42}bc^3d^4x^6+\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}-\frac{d^4(1+cx)}{}$$

```
[Out] 5/3*b*d^4*x/c^2+88/105*b*d^4*x^2/c+5/9*b*d^4*x^3+47/140*b*c*d^4*x^4+2/15*b*
c^2*d^4*x^5+1/42*b*c^3*d^4*x^6+1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/c^3-1/3
*d^4*(c*x+1)^6*(a+b*arctanh(c*x))/c^3+1/7*d^4*(c*x+1)^7*(a+b*arctanh(c*x))/
c^3+176/105*b*d^4*ln(-c*x+1)/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {45, 6083, 12, 907}

$$\frac{d^4(cx+1)^7\left(a+b\tanh^{-1}(cx)\right)}{7c^3}-\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{42}bc^3d^4x^6+\frac{176bd^4\log(1-cx)}{105c^3}+\frac{2}{15}bc^2d^4x^5+\frac{5bd^4x}{3c^2}+\frac{47}{140}bcd^4x^4+\frac{88bd^4x^2}{105c}+\frac{5}{9}bd^4x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (5*b*d^4*x)/(3*c^2) + (88*b*d^4*x^2)/(105*c) + (5*b*d^4*x^3)/9 + (47*b*c*d^
4*x^4)/140 + (2*b*c^2*d^4*x^5)/15 + (b*c^3*d^4*x^6)/42 + (d^4*(1 + c*x)^5*(
a + b*ArcTanh[c*x]))/(5*c^3) - (d^4*(1 + c*x)^6*(a + b*ArcTanh[c*x]))/(3*c^
3) + (d^4*(1 + c*x)^7*(a + b*ArcTanh[c*x]))/(7*c^3) + (176*b*d^4*Log[1 - c*
x])/(105*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx=\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}-\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+$$

$$=\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}-\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+$$

$$=\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}-\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+$$

$$=\frac{5bd^4x}{3c^2}+\frac{88bd^4x^2}{105c}+\frac{5}{9}bd^4x^3+\frac{47}{140}bcd^4x^4+\frac{2}{15}bc^2d^4x^5+\frac{1}{42}bc^3d^4x^6$$

Mathematica [A]
time = 0.08, size = 168, normalized size = 0.98

$$\frac{d^4\left(2100bcx+1056bc^2x^2+420ac^3x^3+700bc^3x^3+1260ac^4x^4+423bc^4x^4+1512ac^5x^5+168bc^5x^5+840ac^6x^6+30bc^6x^6+180ac^7x^7+12bc^3x^3(35+105cx+126c^2x^2+70c^3x^3+15c^4x^4)\tanh^{-1}(cx)+2106b\log(1-cx)+6b\log(1+cx)\right)}{1260c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(2100*b*c*x + 1056*b*c^2*x^2 + 420*a*c^3*x^3 + 700*b*c^3*x^3 + 1260*a*
c^4*x^4 + 423*b*c^4*x^4 + 1512*a*c^5*x^5 + 168*b*c^5*x^5 + 840*a*c^6*x^6 +
30*b*c^6*x^6 + 180*a*c^7*x^7 + 12*b*c^3*x^3*(35 + 105*c*x + 126*c^2*x^2 + 7
0*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] + 2106*b*Log[1 - c*x] + 6*b*Log[1 + c*
x]))/(1260*c^3)
```

Maple [A]
time = 0.18, size = 222, normalized size = 1.30

method	result
derivativedivides	$d^4a\left(\frac{1}{7}c^7x^7+\frac{2}{3}c^6x^6+\frac{6}{5}c^5x^5+c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{d^4b\,\mathrm{arctanh}(cx)c^7x^7}{7}+\frac{2d^4b\,\mathrm{arctanh}(cx)c^6x^6}{3}+\frac{6d^4b\,\mathrm{arctanh}(cx)c^5x^5}{5}+d^4b\,\mathrm{arctanh}$

default	$d^4a\left(\frac{1}{7}c^7x^7+\frac{2}{3}c^6x^6+\frac{6}{5}c^5x^5+c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{d^4b\,\mathrm{arctanh}(cx)c^7x^7}{7}+\frac{2d^4b\,\mathrm{arctanh}(cx)c^6x^6}{3}+\frac{6d^4b\,\mathrm{arctanh}(cx)c^5x^5}{5}+d^4b\,\mathrm{arctan}$
risch	$\frac{d^4b\,x^3\left(15c^4x^4+70x^3c^3+126c^2x^2+105cx+35\right)\ln(cx+1)}{210}-\frac{d^4c^4b\,x^7\ln(-cx+1)}{14}+\frac{d^4c^4a\,x^7}{7}-\frac{d^4c^3x^6b\ln(-cx+1)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^4*a*(1/7*c^7*x^7+2/3*c^6*x^6+6/5*c^5*x^5+c^4*x^4+1/3*x^3*c^3)+1/7*
d^4*b*arctanh(c*x)*c^7*x^7+2/3*d^4*b*arctanh(c*x)*c^6*x^6+6/5*d^4*b*arctanh
(c*x)*c^5*x^5+d^4*b*arctanh(c*x)*c^4*x^4+1/3*d^4*b*arctanh(c*x)*c^3*x^3+1/4
2*d^4*b*c^6*x^6+2/15*d^4*b*c^5*x^5+47/140*d^4*b*c^4*x^4+5/9*b*c^3*d^4*x^3+8
8/105*b*c^2*d^4*x^2+5/3*b*c*d^4*x+117/70*d^4*b*ln(c*x-1)+1/210*d^4*b*ln(c*x
+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(151) = 302$.
time = 0.26, size = 339, normalized size = 1.98

$$\frac{1}{7}ac^4d^4x^7+\frac{2}{3}ac^3d^4x^6+\frac{6}{5}ac^2d^4x^5+\frac{1}{84}\left(12x^7\,\mathrm{artanh}(cx)+c\left(\frac{2c^4x^6+3c^2x^4+6x^2}{c^6}+\frac{6\log(c^2x^2-1)}{c^8}\right)\right)bc^4d^4+acd^4x^4+\frac{1}{45}\left(30x^6\,\mathrm{artanh}(cx)+c\left(\frac{2(3c^4x^5+5c^2x^3+15x)}{c^6}-\frac{15\log(cx+1)}{c^7}+\frac{15\log(cx-1)}{c^7}\right)\right)bc^3d^4+\frac{3}{10}\left(4x^5\,\mathrm{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log(c^2x^2-1)}{c^6}\right)\right)bc^2d^4+\frac{1}{3}ad^4x^3+\frac{1}{6}\left(6x^4\,\mathrm{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bcd^4+\frac{1}{6}\left(2x^3\,\mathrm{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^4*x^7 + 2/3*a*c^3*d^4*x^6 + 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7*ar
ctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^
8))*b*c^4*d^4 + a*c*d^4*x^4 + 1/45*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 +
 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*c^3*
d^4 + 3/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
 1)/c^6))*b*c^2*d^4 + 1/3*a*d^4*x^3 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x
^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d^4 + 1/6*(2*
x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d^4
```

Fricas [A]
time = 0.35, size = 208, normalized size = 1.22

$$\frac{180\,ac^7d^4x^7+30\,(28\,a+b)c^6d^4x^6+168\,(9\,a+b)c^5d^4x^5+9\,(140\,a+47\,b)c^4d^4x^4+140\,(3\,a+5\,b)c^3d^4x^3+1056\,bc^2d^4x^2+2100\,bcd^4x+6\,bd^4\log(cx+1)+2106\,bd^4\log(cx-1)+6\,(15\,bc^7d^4x^7+70\,bc^6d^4x^6+126\,bc^5d^4x^5+105\,bc^4d^4x^4+35\,bc^3d^4x^3)\log\left(-\frac{cx+1}{cx-1}\right)}{1260\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1260*(180*a*c^7*d^4*x^7 + 30*(28*a + b)*c^6*d^4*x^6 + 168*(9*a + b)*c^5*d
^4*x^5 + 9*(140*a + 47*b)*c^4*d^4*x^4 + 140*(3*a + 5*b)*c^3*d^4*x^3 + 1056*
b*c^2*d^4*x^2 + 2100*b*c*d^4*x + 6*b*d^4*log(c*x + 1) + 2106*b*d^4*log(c*x
- 1) + 6*(15*b*c^7*d^4*x^7 + 70*b*c^6*d^4*x^6 + 126*b*c^5*d^4*x^5 + 105*b*c
^4*d^4*x^4 + 35*b*c^3*d^4*x^3)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A]
time = 0.55, size = 279, normalized size = 1.63

$$\begin{cases} \frac{ac^4d^4x^7}{7} + \frac{2ac^3d^4x^6}{3} + \frac{6ac^2d^4x^5}{5} + acd^4x^4 + \frac{ad^4x^3}{3} + \frac{bc^4d^4x^7\operatorname{atanh}(cx)}{7} + \frac{2bc^3d^4x^6\operatorname{atanh}(cx)}{3} + \frac{bc^3d^4x^6}{42} + \frac{6bc^2d^4x^5\operatorname{atanh}(cx)}{5} + \frac{2bc^2d^4x^5}{15} + bcd^4x^4\operatorname{atanh}(cx) + \frac{47bcd^4x^4}{140} + \frac{bd^4x^3\operatorname{atanh}(cx)}{3} + \frac{5bd^4x^3}{9} + \frac{88bd^4x^2}{105c} + \frac{5bd^4x}{3c^2} + \frac{176bd^4\log\left(x-\frac{1}{c}\right)}{105c^3} + \frac{bd^4\operatorname{atanh}(cx)}{105c^3} & \text{for } c \neq 0 \\ \frac{ad^4x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**7/7 + 2*a*c**3*d**4*x**6/3 + 6*a*c**2*d**4*x**5/5
 + a*c*d**4*x**4 + a*d**4*x**3/3 + b*c**4*d**4*x**7*atanh(c*x)/7 + 2*b*c**3
*d**4*x**6*atanh(c*x)/3 + b*c**3*d**4*x**6/42 + 6*b*c**2*d**4*x**5*atanh(c*
x)/5 + 2*b*c**2*d**4*x**5/15 + b*c*d**4*x**4*atanh(c*x) + 47*b*c*d**4*x**4/
140 + b*d**4*x**3*atanh(c*x)/3 + 5*b*d**4*x**3/9 + 88*b*d**4*x**2/(105*c) +
 5*b*d**4*x/(3*c**2) + 176*b*d**4*log(x - 1/c)/(105*c**3) + b*d**4*atanh(c*
x)/(105*c**3), Ne(c, 0)), (a*d**4*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(151) = 302.
time = 0.44, size = 723, normalized size = 4.23

$$-\frac{4}{315}\left(\frac{132\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4} - \frac{132\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^4} - \frac{12\left(\frac{105\,(cx+1)^6bd^4}{(cx-1)^6} - \frac{210\,(cx+1)^5bd^4}{(cx-1)^5} + \frac{385\,(cx+1)^4bd^4}{(cx-1)^4} - \frac{385\,(cx+1)^3bd^4}{(cx-1)^3} + \frac{231\,(cx+1)^2bd^4}{(cx-1)^2} - \frac{77\,(cx+1)bd^4}{cx-1} + 11\,bd^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^7c^4}{(cx-1)^7} - \frac{7\,(cx+1)^6c^4}{(cx-1)^6} + \frac{21\,(cx+1)^5c^4}{(cx-1)^5} - \frac{35\,(cx+1)^4c^4}{(cx-1)^4} + \frac{35\,(cx+1)^3c^4}{(cx-1)^3} - \frac{21\,(cx+1)^2c^4}{(cx-1)^2} + \frac{7\,(cx+1)c^4}{cx-1} - c^4} - \frac{\frac{2520\,(cx+1)^6ad^4}{(cx-1)^6} - \frac{5040\,(cx+1)^5ad^4}{(cx-1)^5} + \frac{9240\,(cx+1)^4ad^4}{(cx-1)^4} - \frac{9240\,(cx+1)^3ad^4}{(cx-1)^3} + \frac{5544\,(cx+1)^2ad^4}{(cx-1)^2} - \frac{1848\,(cx+1)ad^4}{cx-1} + 264\,ad^4 + \frac{1128\,(cx+1)^6bd^4}{(cx-1)^6} - \frac{4812\,(cx+1)^5bd^4}{(cx-1)^5} + \frac{9476\,(cx+1)^4bd^4}{(cx-1)^4} - \frac{10631\,(cx+1)^3bd^4}{(cx-1)^3} + \frac{6933\,(cx+1)^2bd^4}{(cx-1)^2} - \frac{2465\,(cx+1)bd^4}{cx-1} + 371\,bd^4}{\frac{(cx+1)^7c^4}{(cx-1)^7} - \frac{7\,(cx+1)^6c^4}{(cx-1)^6} + \frac{21\,(cx+1)^5c^4}{(cx-1)^5} - \frac{35\,(cx+1)^4c^4}{(cx-1)^4} + \frac{35\,(cx+1)^3c^4}{(cx-1)^3} - \frac{21\,(cx+1)^2c^4}{(cx-1)^2} + \frac{7\,(cx+1)c^4}{cx-1} - c^4}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -4/315*(132*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 - 132*b*d^4*log(-(c*x +
 1)/(c*x - 1))/c^4 - 12*(105*(c*x + 1)^6*b*d^4/(c*x - 1)^6 - 210*(c*x + 1)^
5*b*d^4/(c*x - 1)^5 + 385*(c*x + 1)^4*b*d^4/(c*x - 1)^4 - 385*(c*x + 1)^3*b
*d^4/(c*x - 1)^3 + 231*(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 77*(c*x + 1)*b*d^4/(
c*x - 1) + 11*b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^7*c^4/(c*x - 1)^7
 - 7*(c*x + 1)^6*c^4/(c*x - 1)^6 + 21*(c*x + 1)^5*c^4/(c*x - 1)^5 - 35*(c*x
 + 1)^4*c^4/(c*x - 1)^4 + 35*(c*x + 1)^3*c^4/(c*x - 1)^3 - 21*(c*x + 1)^2*c
^4/(c*x - 1)^2 + 7*(c*x + 1)*c^4/(c*x - 1) - c^4) - (2520*(c*x + 1)^6*a*d^4
/(c*x - 1)^6 - 5040*(c*x + 1)^5*a*d^4/(c*x - 1)^5 + 9240*(c*x + 1)^4*a*d^4/
(c*x - 1)^4 - 9240*(c*x + 1)^3*a*d^4/(c*x - 1)^3 + 5544*(c*x + 1)^2*a*d^4/(
c*x - 1)^2 - 1848*(c*x + 1)*a*d^4/(c*x - 1) + 264*a*d^4 + 1128*(c*x + 1)^6*
b*d^4/(c*x - 1)^6 - 4812*(c*x + 1)^5*b*d^4/(c*x - 1)^5 + 9476*(c*x + 1)^4*b
*d^4/(c*x - 1)^4 - 10631*(c*x + 1)^3*b*d^4/(c*x - 1)^3 + 6933*(c*x + 1)^2*b
*d^4/(c*x - 1)^2 - 2465*(c*x + 1)*b*d^4/(c*x - 1) + 371*b*d^4)/((c*x + 1)^7
*c^4/(c*x - 1)^7 - 7*(c*x + 1)^6*c^4/(c*x - 1)^6 + 21*(c*x + 1)^5*c^4/(c*x
- 1)^5 - 35*(c*x + 1)^4*c^4/(c*x - 1)^4 + 35*(c*x + 1)^3*c^4/(c*x - 1)^3 -
21*(c*x + 1)^2*c^4/(c*x - 1)^2 + 7*(c*x + 1)*c^4/(c*x - 1) - c^4))*c
```

Mupad [B]
time = 1.08, size = 196, normalized size = 1.15

$$\frac{\frac{88\,b\,c^2\,d^4\,x^2}{105} - \frac{d^4\,\left(2100\,b\,\mathrm{atanh}(c\,x) - 1056\,b\,\ln(c^2\,x^2-1)\right)}{1260} + \frac{5\,b\,c\,d^4\,x}{3}}{c^3} + \frac{d^4\,\left(420\,a\,x^3 + 700\,b\,x^3 + 420\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{1260} + \frac{c^4\,d^4\,\left(180\,a\,x^7 + 180\,b\,x^7\,\mathrm{atanh}(c\,x)\right)}{1260} + \frac{c\,d^4\,\left(1260\,a\,x^4 + 423\,b\,x^4 + 1260\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{1260} + \frac{c^3\,d^4\,\left(840\,a\,x^6 + 30\,b\,x^6 + 840\,b\,x^6\,\mathrm{atanh}(c\,x)\right)}{1260} + \frac{c^2\,d^4\,\left(1512\,a\,x^5 + 168\,b\,x^5 + 1512\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{1260}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] ((88*b*c^2*d^4*x^2)/105 - (d^4*(2100*b*atanh(c*x) - 1056*b*log(c^2*x^2 - 1)
))/1260 + (5*b*c*d^4*x)/3)/c^3 + (d^4*(420*a*x^3 + 700*b*x^3 + 420*b*x^3*at
anh(c*x)))/1260 + (c^4*d^4*(180*a*x^7 + 180*b*x^7*atanh(c*x)))/1260 + (c*d^
4*(1260*a*x^4 + 423*b*x^4 + 1260*b*x^4*atanh(c*x)))/1260 + (c^3*d^4*(840*a*
x^6 + 30*b*x^6 + 840*b*x^6*atanh(c*x)))/1260 + (c^2*d^4*(1512*a*x^5 + 168*b
*x^5 + 1512*b*x^5*atanh(c*x)))/1260
```

3.33 $\int x(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=153

$$\frac{16bd^4x}{15c}+\frac{4bd^4(1+cx)^2}{15c^2}+\frac{4bd^4(1+cx)^3}{45c^2}+\frac{bd^4(1+cx)^4}{30c^2}+\frac{bd^4(1+cx)^5}{30c^2}-\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1$$

```
[Out] 16/15*b*d^4*x/c+4/15*b*d^4*(c*x+1)^2/c^2+4/45*b*d^4*(c*x+1)^3/c^2+1/30*b*d^
4*(c*x+1)^4/c^2+1/30*b*d^4*(c*x+1)^5/c^2-1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x
))/c^2+1/6*d^4*(c*x+1)^6*(a+b*arctanh(c*x))/c^2+32/15*b*d^4*ln(-c*x+1)/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {45, 6083, 12, 78}

$$\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^4(cx+1)^5}{30c^2}+\frac{bd^4(cx+1)^4}{30c^2}+\frac{4bd^4(cx+1)^3}{45c^2}+\frac{4bd^4(cx+1)^2}{15c^2}+\frac{32bd^4\log(1-cx)}{15c^2}+\frac{16bd^4x}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (16*b*d^4*x)/(15*c) + (4*b*d^4*(1 + c*x)^2)/(15*c^2) + (4*b*d^4*(1 + c*x)^3
)/(45*c^2) + (b*d^4*(1 + c*x)^4)/(30*c^2) + (b*d^4*(1 + c*x)^5)/(30*c^2) -
(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (d^4*(1 + c*x)^6*(a + b*Ar
cTanh[c*x]))/(6*c^2) + (32*b*d^4*Log[1 - c*x])/(15*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}$$
$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}$$
$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}$$
$$= \frac{16bd^4x}{15c}+\frac{4bd^4(1+cx)^2}{15c^2}+\frac{4bd^4(1+cx)^3}{45c^2}+\frac{bd^4(1+cx)^4}{30c^2}+\frac{bd^4(1-}{30}$$

Mathematica [A]
time = 0.05, size = 159, normalized size = 1.04

$$\frac{d^4\left(390bcx+90ac^2x^2+192bc^2x^2+240ac^3x^3+100bc^3x^3+270ac^4x^4+36bc^4x^4+144ac^5x^5+6bc^5x^5+30ac^6x^6+6bc^2x^2(15+40cx+45c^2x^2+24c^3x^3+5c^4x^4)\tanh^{-1}(cx)+387b\log(1-cx)-3b\log(1+cx)\right)}{180c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(390*b*c*x + 90*a*c^2*x^2 + 192*b*c^2*x^2 + 240*a*c^3*x^3 + 100*b*c^3*
x^3 + 270*a*c^4*x^4 + 36*b*c^4*x^4 + 144*a*c^5*x^5 + 6*b*c^5*x^5 + 30*a*c^6
*x^6 + 6*b*c^2*x^2*(15 + 40*c*x + 45*c^2*x^2 + 24*c^3*x^3 + 5*c^4*x^4)*ArcT
anh[c*x] + 387*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(180*c^2)
```

Maple [A]
time = 0.18, size = 212, normalized size = 1.39

method	result
derivativedivides	$\frac{d^4a\left(\frac{1}{6}c^6x^6+\frac{4}{5}c^5x^5+\frac{3}{2}c^4x^4+\frac{4}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d^4b\,\mathrm{arctanh}(cx)c^6x^6}{6}+\frac{4d^4b\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{3d^4b\,\mathrm{arctanh}(cx)c^4x^4}{2}+\frac{4d^4b\,\mathrm{arc}}{}}{c^2}$
default	$\frac{d^4a\left(\frac{1}{6}c^6x^6+\frac{4}{5}c^5x^5+\frac{3}{2}c^4x^4+\frac{4}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d^4b\,\mathrm{arctanh}(cx)c^6x^6}{6}+\frac{4d^4b\,\mathrm{arctanh}(cx)c^5x^5}{5}+\frac{3d^4b\,\mathrm{arctanh}(cx)c^4x^4}{2}+\frac{4d^4b\,\mathrm{arc}}{}}{c^2}$
risch	$\frac{d^4b\,x^2\left(5c^4x^4+24x^3c^3+45c^2x^2+40cx+15\right)\ln(cx+1)}{60}-\frac{d^4c^4x^6b\ln(-cx+1)}{12}+\frac{d^4c^4x^6a}{6}-\frac{2d^4c^3x^5b\ln(-cx+1)}{5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^4*a*(1/6*c^6*x^6+4/5*c^5*x^5+3/2*c^4*x^4+4/3*x^3*c^3+1/2*c^2*x^2)+
1/6*d^4*b*arctanh(c*x)*c^6*x^6+4/5*d^4*b*arctanh(c*x)*c^5*x^5+3/2*d^4*b*arc
tanh(c*x)*c^4*x^4+4/3*d^4*b*arctanh(c*x)*c^3*x^3+1/2*d^4*b*arctanh(c*x)*c^2
*x^2+1/30*d^4*b*c^5*x^5+1/5*d^4*b*c^4*x^4+5/9*b*c^3*d^4*x^3+16/15*b*c^2*d^4
*x^2+13/6*b*c*d^4*x+43/20*d^4*b*ln(c*x-1)-1/60*d^4*b*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(137) = 274$.
time = 0.26, size = 326, normalized size = 2.13

$$\frac{1}{6}ac^4d^4x^6+\frac{4}{5}ac^3d^4x^5+\frac{3}{2}ac^2d^4x^4+\frac{1}{180}\left(30x^6\operatorname{artanh}(cx)+c\left(\frac{2(3c^4x^5+5c^2x^3+15x)}{c^6}-\frac{15\log(cx+1)}{c^7}+\frac{15\log(cx-1)}{c^7}\right)\right)bc^4d^4+\frac{1}{5}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log(c^2x^2-1)}{c^6}\right)\right)bc^3d^4+\frac{4}{3}acd^4x^3+\frac{1}{4}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bc^2d^4+\frac{2}{3}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bcd^4+\frac{1}{2}ad^4x^2+\frac{1}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^4*x^6 + 4/5*a*c^3*d^4*x^5 + 3/2*a*c^2*d^4*x^4 + 1/180*(30*x^6*a
rctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7
 + 15*log(c*x - 1)/c^7))*b*c^4*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4
+ 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^4 + 4/3*a*c*d^4*x^3 + 1/4*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^2*d^4 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^4
```

Fricas [A]
time = 0.38, size = 198, normalized size = 1.29

$$\frac{30ac^6d^4x^6+6(24a+b)c^5d^4x^5+18(15a+2b)c^4d^4x^4+20(12a+5b)c^3d^4x^3+6(15a+32b)c^2d^4x^2+390bcd^4x-3bd^4\log(cx+1)+387bd^4\log(cx-1)+3(5bc^6d^4x^6+24bc^5d^4x^5+45bc^4d^4x^4+40bc^3d^4x^3+15bc^2d^4x^2)\log\left(-\frac{cx+1}{cx-1}\right)}{180c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*d^4*x^6 + 6*(24*a + b)*c^5*d^4*x^5 + 18*(15*a + 2*b)*c^4*d^
4*x^4 + 20*(12*a + 5*b)*c^3*d^4*x^3 + 6*(15*a + 32*b)*c^2*d^4*x^2 + 390*b*c
*d^4*x - 3*b*d^4*log(c*x + 1) + 387*b*d^4*log(c*x - 1) + 3*(5*b*c^6*d^4*x^6
 + 24*b*c^5*d^4*x^5 + 45*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 15*b*c^2*d^4*x^
2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A]
time = 0.51, size = 269, normalized size = 1.76

$$\begin{cases}\frac{ac^4d^4x^6}{6}+\frac{4ac^3d^4x^5}{5}+\frac{3ac^2d^4x^4}{2}+\frac{4acd^4x^3}{3}+\frac{ad^4x^2}{2}+\frac{bc^4d^4x^6\operatorname{atanh}(cx)}{6}+\frac{4bc^3d^4x^5\operatorname{atanh}(cx)}{5}+\frac{bc^3d^4x^5}{30}+\frac{3bc^2d^4x^4\operatorname{atanh}(cx)}{2}+\frac{bc^2d^4x^4}{5}+\frac{4bcd^4x^3\operatorname{atanh}(cx)}{3}+\frac{5bcd^4x^3}{9}+\frac{bd^4x^2\operatorname{atanh}(cx)}{2}+\frac{16bd^4x^2}{15}+\frac{13bd^4x}{6c}+\frac{32bd^4\log\left(x-\frac{1}{c}\right)}{15c^2}-\frac{bd^4\operatorname{atanh}(cx)}{30c^2} & \text{for}\ c\neq 0\\ \frac{ad^4x^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**6/6 + 4*a*c**3*d**4*x**5/5 + 3*a*c**2*d**4*x**4/2
 + 4*a*c*d**4*x**3/3 + a*d**4*x**2/2 + b*c**4*d**4*x**6*atanh(c*x)/6 + 4*b*
c**3*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**5/30 + 3*b*c**2*d**4*x**4*atan
h(c*x)/2 + b*c**2*d**4*x**4/5 + 4*b*c*d**4*x**3*atanh(c*x)/3 + 5*b*c*d**4*x
**3/9 + b*d**4*x**2*atanh(c*x)/2 + 16*b*d**4*x**2/15 + 13*b*d**4*x/(6*c) +
32*b*d**4*log(x - 1/c)/(15*c**2) - b*d**4*atanh(c*x)/(30*c**2), Ne(c, 0)),
(a*d**4*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(137) = 274$.
time = 0.42, size = 621, normalized size = 4.06

$$-\frac{8}{45}\left(\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}-\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^3}-\frac{6\left(\frac{15\,(cx+1)^5bd^4}{(cx-1)^5}-\frac{30\,(cx+1)^4bd^4}{(cx-1)^4}+\frac{40\,(cx+1)^3bd^4}{(cx-1)^3}-\frac{30\,(cx+1)^2bd^4}{(cx-1)^2}+\frac{12\,(cx+1)bd^4}{cx-1}-2\,bd^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^3}{(cx-1)^6}-\frac{6\,(cx+1)^5c^3}{(cx-1)^5}+\frac{15\,(cx+1)^4c^3}{(cx-1)^4}-\frac{20\,(cx+1)^3c^3}{(cx-1)^3}+\frac{15\,(cx+1)^2c^3}{(cx-1)^2}-\frac{6\,(cx+1)c^3}{cx-1}+c^3}-\frac{\frac{180\,(cx+1)^5ad^4}{(cx-1)^5}-\frac{360\,(cx+1)^4ad^4}{(cx-1)^4}+\frac{480\,(cx+1)^3ad^4}{(cx-1)^3}-\frac{360\,(cx+1)^2ad^4}{(cx-1)^2}+\frac{144\,(cx+1)ad^4}{cx-1}-24\,ad^4+\frac{78\,(cx+1)^5bd^4}{(cx-1)^5}-\frac{294\,(cx+1)^4bd^4}{(cx-1)^4}+\frac{472\,(cx+1)^3bd^4}{(cx-1)^3}-\frac{399\,(cx+1)^2bd^4}{(cx-1)^2}+\frac{174\,(cx+1)bd^4}{cx-1}-31\,bd^4}{\frac{(cx+1)^6c^3}{(cx-1)^6}-\frac{6\,(cx+1)^5c^3}{(cx-1)^5}+\frac{15\,(cx+1)^4c^3}{(cx-1)^4}-\frac{20\,(cx+1)^3c^3}{(cx-1)^3}+\frac{15\,(cx+1)^2c^3}{(cx-1)^2}-\frac{6\,(cx+1)c^3}{cx-1}+c^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -8/45*(12*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 12*b*d^4*log(-(c*x + 1)
/(c*x - 1))/c^3 - 6*(15*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 30*(c*x + 1)^4*b*d^
4/(c*x - 1)^4 + 40*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 30*(c*x + 1)^2*b*d^4/(c*
x - 1)^2 + 12*(c*x + 1)*b*d^4/(c*x - 1) - 2*b*d^4)*log(-(c*x + 1)/(c*x - 1)
)/((c*x + 1)^6*c^3/(c*x - 1)^6 - 6*(c*x + 1)^5*c^3/(c*x - 1)^5 + 15*(c*x +
1)^4*c^3/(c*x - 1)^4 - 20*(c*x + 1)^3*c^3/(c*x - 1)^3 + 15*(c*x + 1)^2*c^3/
(c*x - 1)^2 - 6*(c*x + 1)*c^3/(c*x - 1) + c^3) - (180*(c*x + 1)^5*a*d^4/(c*
x - 1)^5 - 360*(c*x + 1)^4*a*d^4/(c*x - 1)^4 + 480*(c*x + 1)^3*a*d^4/(c*x -
 1)^3 - 360*(c*x + 1)^2*a*d^4/(c*x - 1)^2 + 144*(c*x + 1)*a*d^4/(c*x - 1) -
 24*a*d^4 + 78*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 294*(c*x + 1)^4*b*d^4/(c*x -
 1)^4 + 472*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 399*(c*x + 1)^2*b*d^4/(c*x - 1)
^2 + 174*(c*x + 1)*b*d^4/(c*x - 1) - 31*b*d^4)/((c*x + 1)^6*c^3/(c*x - 1)^6
 - 6*(c*x + 1)^5*c^3/(c*x - 1)^5 + 15*(c*x + 1)^4*c^3/(c*x - 1)^4 - 20*(c*x
 + 1)^3*c^3/(c*x - 1)^3 + 15*(c*x + 1)^2*c^3/(c*x - 1)^2 - 6*(c*x + 1)*c^3/
(c*x - 1) + c^3))*c
```

Mupad [B]
time = 1.05, size = 185, normalized size = 1.21

$$\frac{d^4\,(45\,a\,x^2+96\,b\,x^2+45\,b\,x^2\,\mathrm{atanh}(c\,x))}{90}-\frac{\frac{d^4\,(195\,b\,\mathrm{atanh}(c\,x)-96\,b\,\ln(c^2\,x^2-1))}{90}-\frac{13\,b\,c\,d^4\,x}{6}}{c^2}+\frac{c^4\,d^4\,(15\,a\,x^6+15\,b\,x^6\,\mathrm{atanh}(c\,x))}{90}+\frac{c\,d^4\,(120\,a\,x^3+50\,b\,x^3+120\,b\,x^3\,\mathrm{atanh}(c\,x))}{90}+\frac{c^3\,d^4\,(72\,a\,x^5+3\,b\,x^5+72\,b\,x^5\,\mathrm{atanh}(c\,x))}{90}+\frac{c^2\,d^4\,(135\,a\,x^4+18\,b\,x^4+135\,b\,x^4\,\mathrm{atanh}(c\,x))}{90}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] (d^4*(45*a*x^2 + 96*b*x^2 + 45*b*x^2*atanh(c*x)))/90 - ((d^4*(195*b*atanh(c
*x) - 96*b*log(c^2*x^2 - 1)))/90 - (13*b*c*d^4*x)/6)/c^2 + (c^4*d^4*(15*a*x
```

```
^6 + 15*b*x^6*atanh(c*x)))/90 + (c*d^4*(120*a*x^3 + 50*b*x^3 + 120*b*x^3*at
anh(c*x)))/90 + (c^3*d^4*(72*a*x^5 + 3*b*x^5 + 72*b*x^5*atanh(c*x)))/90 + (
c^2*d^4*(135*a*x^4 + 18*b*x^4 + 135*b*x^4*atanh(c*x)))/90
```

3.34 $\int (d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=107

$$\frac{8}{5}bd^4x+\frac{2bd^4(1+cx)^2}{5c}+\frac{2bd^4(1+cx)^3}{15c}+\frac{bd^4(1+cx)^4}{20c}+\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}+\frac{16bd^4\log(1-cx)}{5c}$$

```
[Out] 8/5*b*d^4*x+2/5*b*d^4*(c*x+1)^2/c+2/15*b*d^4*(c*x+1)^3/c+1/20*b*d^4*(c*x+1)
^4/c+1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/c+16/5*b*d^4*ln(-c*x+1)/c
```

Rubi [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6063, 641, 45}

$$\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}+\frac{bd^4(cx+1)^4}{20c}+\frac{2bd^4(cx+1)^3}{15c}+\frac{2bd^4(cx+1)^2}{5c}+\frac{16bd^4\log(1-cx)}{5c}+\frac{8}{5}bd^4x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (8*b*d^4*x)/5 + (2*b*d^4*(1 + c*x)^2)/(5*c) + (2*b*d^4*(1 + c*x)^3)/(15*c)
+ (b*d^4*(1 + c*x)^4)/(20*c) + (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c)
 + (16*b*d^4*Log[1 - c*x])/(5*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)\,dx = \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c} - \frac{b\int \frac{(d+cdx)^5}{1-c^2x^2}\,dx}{5d}$$

$$= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c} - \frac{b\int \frac{(d+cdx)^4}{\frac{1}{d}-\frac{cx}{d}}\,dx}{5d}$$

$$= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c} - \frac{b\int \left(-8d^5+\frac{16d^4}{\frac{1}{d}-\frac{cx}{d}}-4d^4(d+cdx)-\right.}{5}$$

$$= \frac{8}{5}bd^4x + \frac{2bd^4(1+cx)^2}{5c} + \frac{2bd^4(1+cx)^3}{15c} + \frac{bd^4(1+cx)^4}{20c} + \frac{d^4(1+cx)^5}{}$$

Mathematica [A]
time = 0.05, size = 146, normalized size = 1.36

$$\frac{d^4\left(60acx+180bcx+120ac^2x^2+66bc^2x^2+120ac^3x^3+20bc^3x^3+60ac^4x^4+3bc^4x^4+12ac^5x^5+12bcx(5+10cx+10c^2x^2+5c^3x^3+c^4x^4)\tanh^{-1}(cx)+180b\log(1-cx)+6b\log\left(1-c^2x^2\right)\right)}{60c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(60*a*c*x + 180*b*c*x + 120*a*c^2*x^2 + 66*b*c^2*x^2 + 120*a*c^3*x^3 +
 20*b*c^3*x^3 + 60*a*c^4*x^4 + 3*b*c^4*x^4 + 12*a*c^5*x^5 + 12*b*c*x*(5 + 1
0*c*x + 10*c^2*x^2 + 5*c^3*x^3 + c^4*x^4)*ArcTanh[c*x] + 180*b*Log[1 - c*x]
 + 6*b*Log[1 - c^2*x^2]))/(60*c)
```

Maple [A]
time = 0.15, size = 159, normalized size = 1.49

method	result
derivativedivides	$\frac{\frac{d^4(cx+1)^5a}{5}+\frac{d^4b\,\mathrm{arctanh}(cx)c^5x^5}{5}+d^4b\,\mathrm{arctanh}(cx)c^4x^4+2d^4b\,\mathrm{arctanh}(cx)c^3x^3+2d^4b\,\mathrm{arctanh}(cx)c^2x^2+bc\,d^4x\,\mathrm{arctanh}(cx}{c}$
default	$\frac{\frac{d^4(cx+1)^5a}{5}+\frac{d^4b\,\mathrm{arctanh}(cx)c^5x^5}{5}+d^4b\,\mathrm{arctanh}(cx)c^4x^4+2d^4b\,\mathrm{arctanh}(cx)c^3x^3+2d^4b\,\mathrm{arctanh}(cx)c^2x^2+bc\,d^4x\,\mathrm{arctanh}(cx}{c}$
risch	$\frac{d^4(cx+1)^5b\ln(cx+1)}{10c}-\frac{d^4c^4x^5b\ln(-cx+1)}{10}+\frac{d^4c^4x^5a}{5}-\frac{d^4c^3x^4b\ln(-cx+1)}{2}+d^4c^3x^4a+\frac{d^4c^3x^4b}{20}-d^4c^2x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/5*d^4*(c*x+1)^5*a+1/5*d^4*b*arctanh(c*x)*c^5*x^5+d^4*b*arctanh(c*x)*
c^4*x^4+2*d^4*b*arctanh(c*x)*c^3*x^3+2*d^4*b*arctanh(c*x)*c^2*x^2+b*c*d^4*x
*arctanh(c*x)+1/5*b*d^4*arctanh(c*x)+1/20*d^4*b*c^4*x^4+1/3*b*c^3*d^4*x^3+1
1/10*b*c^2*d^4*x^2+3*b*c*d^4*x+16/5*d^4*b*ln(c*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(95) = 190$.
time = 0.27, size = 283, normalized size = 2.64

$$\frac{1}{5}\,ac^4d^4x^5 + ac^3d^4x^4 + \frac{1}{20}\left(4\,x^5\,\operatorname{artanh}\left(cx\right) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2 - 1\right)}{c^6}\right)\right)bc^4d^4 + 2\,ac^2d^4x^3 + \frac{1}{6}\left(6\,x^4\,\operatorname{artanh}\left(cx\right) + c\left(\frac{2\,\left(c^2x^3 + 3\,x\right)}{c^4} - \frac{3\,\log\left(cx + 1\right)}{c^5} + \frac{3\,\log\left(cx - 1\right)}{c^5}\right)\right)bc^3d^4 + \left(2\,x^3\,\operatorname{artanh}\left(cx\right) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2 - 1\right)}{c^4}\right)\right)bc^2d^4 + 2\,acd^4x^2 + \left(2\,x^2\,\operatorname{artanh}\left(cx\right) + c\left(\frac{2\,x}{c^2} - \frac{\log\left(cx + 1\right)}{c^3} + \frac{\log\left(cx - 1\right)}{c^3}\right)\right)bcd^4 + ad^4x + \frac{\left(2\,cx\,\operatorname{artanh}\left(cx\right) + \log\left(-c^2x^2 + 1\right)\right)bd^4}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^4*x^5 + a*c^3*d^4*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4
+ 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^4*d^4 + 2*a*c^2*d^4*x^3 + 1/6*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^3*d^4 + (2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
 1)/c^4))*b*c^2*d^4 + 2*a*c*d^4*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - lo
g(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^4 + a*d^4*x + 1/2*(2*c*x*arctanh(
c*x) + log(-c^2*x^2 + 1))*b*d^4/c
```

Fricas [A]
time = 0.37, size = 177, normalized size = 1.65

$$\frac{12\,ac^5d^4x^5 + 3\,(20\,a + b)c^4d^4x^4 + 20\,(6\,a + b)c^3d^4x^3 + 6\,(20\,a + 11\,b)c^2d^4x^2 + 60\,(a + 3\,b)cd^4x + 6\,bd^4\log\left(cx + 1\right) + 186\,bd^4\log\left(cx - 1\right) + 6\,\left(bc^5d^4x^5 + 5\,bc^4d^4x^4 + 10\,bc^3d^4x^3 + 10\,bc^2d^4x^2 + 5\,bcd^4x\right)\log\left(-\frac{cx+1}{cx-1}\right)}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^4*x^5 + 3*(20*a + b)*c^4*d^4*x^4 + 20*(6*a + b)*c^3*d^4*x^
3 + 6*(20*a + 11*b)*c^2*d^4*x^2 + 60*(a + 3*b)*c*d^4*x + 6*b*d^4*log(c*x +
1) + 186*b*d^4*log(c*x - 1) + 6*(b*c^5*d^4*x^5 + 5*b*c^4*d^4*x^4 + 10*b*c^3
*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(97) = 194$.
time = 0.41, size = 226, normalized size = 2.11

$$\begin{cases} \frac{ac^4d^4x^5}{5} + ac^3d^4x^4 + 2ac^2d^4x^3 + 2acd^4x^2 + ad^4x + \frac{bc^4d^4x^5\operatorname{atanh}\left(cx\right)}{5} + bc^3d^4x^4\operatorname{atanh}\left(cx\right) + \frac{bc^3d^4x^4}{20} + 2bc^2d^4x^3\operatorname{atanh}\left(cx\right) + \frac{bc^2d^4x^3}{3} + 2bcd^4x^2\operatorname{atanh}\left(cx\right) + \frac{11bcd^4x^2}{10} + bd^4x\operatorname{atanh}\left(cx\right) + 3bd^4x + \frac{16bd^4\log\left(x-\frac{1}{c}\right)}{5c} + \frac{bd^4\operatorname{atanh}\left(cx\right)}{5c} & \text{for}\: c \neq 0 \\ ad^4x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**5/5 + a*c**3*d**4*x**4 + 2*a*c**2*d**4*x**3 + 2*a
*c*d**4*x**2 + a*d**4*x + b*c**4*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**4*
atanh(c*x) + b*c**3*d**4*x**4/20 + 2*b*c**2*d**4*x**3*atanh(c*x) + b*c**2*d
**4*x**3/3 + 2*b*c*d**4*x**2*atanh(c*x) + 11*b*c*d**4*x**2/10 + b*d**4*x*at
anh(c*x) + 3*b*d**4*x + 16*b*d**4*log(x - 1/c)/(5*c) + b*d**4*atanh(c*x)/(5
*c), Ne(c, 0)), (a*d**4*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(95) = 190.
time = 0.42, size = 526, normalized size = 4.92

$$-\frac{4}{15}\left(\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{12\left(\frac{5(cx+1)^4bd^4}{(cx-1)^4}-\frac{10(cx+1)^3bd^4}{(cx-1)^3}+\frac{10(cx+1)^2bd^4}{(cx-1)^2}-\frac{5(cx+1)bd^4}{cx-1}+bd^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^2}{(cx-1)^5}-\frac{5(cx+1)^4c^2}{(cx-1)^4}+\frac{10(cx+1)^3c^2}{(cx-1)^3}-\frac{10(cx+1)^2c^2}{(cx-1)^2}+\frac{5(cx+1)c^2}{cx-1}-c^2}-\frac{\frac{120(cx+1)^4ad^4}{(cx-1)^4}-\frac{240(cx+1)^3ad^4}{(cx-1)^3}+\frac{240(cx+1)^2ad^4}{(cx-1)^2}-\frac{120(cx+1)ad^4}{cx-1}+24\,ad^4+\frac{48(cx+1)^4bd^4}{(cx-1)^4}-\frac{156(cx+1)^3bd^4}{(cx-1)^3}+\frac{196(cx+1)^2bd^4}{(cx-1)^2}-\frac{113(cx+1)bd^4}{cx-1}+25\,bd^4}{\frac{(cx+1)^5c^2}{(cx-1)^5}-\frac{5(cx+1)^4c^2}{(cx-1)^4}+\frac{10(cx+1)^3c^2}{(cx-1)^3}-\frac{10(cx+1)^2c^2}{(cx-1)^2}+\frac{5(cx+1)c^2}{cx-1}-c^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -4/15*(12*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 12*b*d^4*log(-(c*x + 1)
/(c*x - 1))/c^2 - 12*(5*(c*x + 1)^4*b*d^4/(c*x - 1)^4 - 10*(c*x + 1)^3*b*d^
4/(c*x - 1)^3 + 10*(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 5*(c*x + 1)*b*d^4/(c*x -
 1) + b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^2/(c*x - 1)^5 - 5*(c*
x + 1)^4*c^2/(c*x - 1)^4 + 10*(c*x + 1)^3*c^2/(c*x - 1)^3 - 10*(c*x + 1)^2*
c^2/(c*x - 1)^2 + 5*(c*x + 1)*c^2/(c*x - 1) - c^2) - (120*(c*x + 1)^4*a*d^4
/(c*x - 1)^4 - 240*(c*x + 1)^3*a*d^4/(c*x - 1)^3 + 240*(c*x + 1)^2*a*d^4/(c
*x - 1)^2 - 120*(c*x + 1)*a*d^4/(c*x - 1) + 24*a*d^4 + 48*(c*x + 1)^4*b*d^4
/(c*x - 1)^4 - 156*(c*x + 1)^3*b*d^4/(c*x - 1)^3 + 196*(c*x + 1)^2*b*d^4/(c
*x - 1)^2 - 113*(c*x + 1)*b*d^4/(c*x - 1) + 25*b*d^4)/((c*x + 1)^5*c^2/(c*x
 - 1)^5 - 5*(c*x + 1)^4*c^2/(c*x - 1)^4 + 10*(c*x + 1)^3*c^2/(c*x - 1)^3 -
10*(c*x + 1)^2*c^2/(c*x - 1)^2 + 5*(c*x + 1)*c^2/(c*x - 1) - c^2))*c
```

Mupad [B]
time = 1.05, size = 168, normalized size = 1.57

$$\frac{d^4\left(60\,a\,x+180\,b\,x+60\,b\,x\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c^4\,d^4\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{60}-\frac{d^4\left(180\,b\,\mathrm{atanh}(c\,x)-96\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60\,c}+\frac{c\,d^4\left(120\,a\,x^2+66\,b\,x^2+120\,b\,x^2\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c^3\,d^4\left(60\,a\,x^4+3\,b\,x^4+60\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c^2\,d^4\left(120\,a\,x^3+20\,b\,x^3+120\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

```
[Out] (d^4*(60*a*x + 180*b*x + 60*b*x*atanh(c*x)))/60 + (c^4*d^4*(12*a*x^5 + 12*b
*x^5*atanh(c*x)))/60 - (d^4*(180*b*atanh(c*x) - 96*b*log(c^2*x^2 - 1)))/(60
*c) + (c*d^4*(120*a*x^2 + 66*b*x^2 + 120*b*x^2*atanh(c*x)))/60 + (c^3*d^4*(
60*a*x^4 + 3*b*x^4 + 60*b*x^4*atanh(c*x)))/60 + (c^2*d^4*(120*a*x^3 + 20*b*
x^3 + 120*b*x^3*atanh(c*x)))/60
```

3.35 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=185

$$4acd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}bc^2d^4x^2+\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\tanh^{-1}(cx)+4bcd^4x\tanh^{-1}(cx)+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 4*a*c*d^4*x+13/4*b*c*d^4*x+2/3*b*c^2*d^4*x^2+1/12*b*c^3*d^4*x^3-13/4*b*d^4*
arctanh(c*x)+4*b*c*d^4*x*arctanh(c*x)+3*c^2*d^4*x^2*(a+b*arctanh(c*x))+4/3*
c^3*d^4*x^3*(a+b*arctanh(c*x))+1/4*c^4*d^4*x^4*(a+b*arctanh(c*x))+a*d^4*ln(
x)+8/3*b*d^4*ln(-c^2*x^2+1)-1/2*b*d^4*polylog(2,-c*x)+1/2*b*d^4*polylog(2,c
*x)
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6021, 266, 6031, 6037, 327, 212, 272, 45, 308}

$$\frac{1}{4}c^4d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+4acd^4x+ad^4\log(x)+\frac{1}{12}bc^3d^4x^3+\frac{2}{3}bc^2d^4x^2+\frac{8}{3}bd^4\log\left(1-c^2x^2\right)-\frac{1}{2}bd^4\text{Li}_2(-cx)+\frac{1}{2}bd^4\text{Li}_2(cx)+\frac{13}{4}bcd^4x-\frac{13}{4}bd^4\tanh^{-1}(cx)+4bcd^4x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 4*a*c*d^4*x + (13*b*c*d^4*x)/4 + (2*b*c^2*d^4*x^2)/3 + (b*c^3*d^4*x^3)/12 -
 (13*b*d^4*ArcTanh[c*x])/4 + 4*b*c*d^4*x*ArcTanh[c*x] + 3*c^2*d^4*x^2*(a +
b*ArcTanh[c*x]) + (4*c^3*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + (c^4*d^4*x^4*(a
+ b*ArcTanh[c*x]))/4 + a*d^4*Log[x] + (8*b*d^4*Log[1 - c^2*x^2])/3 - (b*d^4
*PolyLog[2, -(c*x)])/2 + (b*d^4*PolyLog[2, c*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(4cd^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+6c^2d^4x\left(a+b\right.\right.$$

$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(4cd^4\right)\int \left(a+b\tanh^{-1}(cx)\right)\,dx+\left(6c^2\right.$$

$$= 4acd^4x+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+$$

$$= 4acd^4x+3bcd^4x+4bcd^4x\tanh^{-1}(cx)+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)$$

$$= 4acd^4x+\frac{13}{4}bcd^4x+\frac{1}{12}bc^3d^4x^3-3bd^4\tanh^{-1}(cx)+4bcd^4x\tanh^{-1}($$

$$= 4acd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}bc^2d^4x^2+\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\tanh^{-1}(cx)+4$$

Mathematica [A]
time = 0.08, size = 179, normalized size = 0.97

$$\frac{1}{24}d^4\left(96acx+78bcx+72ac^2x^2+16bc^2x^2+32ac^3x^3+2bc^3x^3+6ac^4x^4+96bcx\tanh^{-1}(cx)+72bc^2x^2\tanh^{-1}(cx)+32bc^3x^3\tanh^{-1}(cx)+6bc^4x^4\tanh^{-1}(cx)+24a\log(x)+39b\log(1-cx)-39b\log(1+cx)+48b\log\left(1-c^2x^2\right)+16b\log\left(-1+c^2x^2\right)-12b\text{PolyLog}(2,-cx)+12b\text{PolyLog}(2,cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^4*(96*a*c*x + 78*b*c*x + 72*a*c^2*x^2 + 16*b*c^2*x^2 + 32*a*c^3*x^3 + 2*
b*c^3*x^3 + 6*a*c^4*x^4 + 96*b*c*x*ArcTanh[c*x] + 72*b*c^2*x^2*ArcTanh[c*x]
 + 32*b*c^3*x^3*ArcTanh[c*x] + 6*b*c^4*x^4*ArcTanh[c*x] + 24*a*Log[x] + 39*
b*Log[1 - c*x] - 39*b*Log[1 + c*x] + 48*b*Log[1 - c^2*x^2] + 16*b*Log[-1 +
c^2*x^2] - 12*b*PolyLog[2, -(c*x)] + 12*b*PolyLog[2, c*x]))/24
```

Maple [A]
time = 0.22, size = 222, normalized size = 1.20

method	result
derivativedivides	$\frac{d^4 a\, c^4 x^4}{4}+\frac{4 d^4 a\, c^3 x^3}{3}+3 d^4 a\, c^2 x^2+4 a c\, d^4 x+d^4 a \ln(cx)+\frac{d^4 b \,\mathrm{arctanh}(cx) c^4 x^4}{4}+\frac{4 d^4 b \,\mathrm{arctanh}(cx)}{3}$
default	$\frac{d^4 a\, c^4 x^4}{4}+\frac{4 d^4 a\, c^3 x^3}{3}+3 d^4 a\, c^2 x^2+4 a c\, d^4 x+d^4 a \ln(cx)+\frac{d^4 b \,\mathrm{arctanh}(cx) c^4 x^4}{4}+\frac{4 d^4 b \,\mathrm{arctanh}(cx)}{3}$
risch	$4 a c\, d^4 x+\frac{13 b c\, d^4 x}{4}+\frac{2 b\, c^2 d^4 x^2}{3}+\frac{b\, c^3 d^4 x^3}{12}+\frac{25 d^4 b \ln(cx+1)}{24}-\frac{58 d^4 b}{9}+\frac{d^4 a\, c^4 x^4}{4}+\frac{4 d^4 a\, c^3 x^3}{3}+3 d^4 a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^4*a*c^4*x^4+4/3*d^4*a*c^3*x^3+3*d^4*a*c^2*x^2+4*a*c*d^4*x+d^4*a*ln(c*
x)+1/4*d^4*b*arctanh(c*x)*c^4*x^4+4/3*d^4*b*arctanh(c*x)*c^3*x^3+3*d^4*b*ar
ctanh(c*x)*c^2*x^2+4*b*c*d^4*x*arctanh(c*x)+d^4*b*arctanh(c*x)*ln(c*x)-1/2*
d^4*b*dilog(c*x)-1/2*d^4*b*dilog(c*x+1)-1/2*d^4*b*ln(c*x)*ln(c*x+1)+1/12*b*
c^3*d^4*x^3+2/3*b*c^2*d^4*x^2+13/4*b*c*d^4*x+103/24*d^4*b*ln(c*x-1)+25/24*d
^4*b*ln(c*x+1)
```

Maxima [A]
time = 0.35, size = 276, normalized size = 1.49

$$\frac{1}{4}\,ac^4d^4x^4 + \frac{4}{3}\,ac^3d^4x^3 + \frac{1}{12}\,bc^3d^4x^3 + 3\,ac^2d^4x^2 + \frac{2}{3}\,bc^2d^4x^2 + 4\,acd^4x + \frac{13}{4}\,bcd^4x + 2\,\left(2\,cx\,\operatorname{artanh}\left(cx\right) + \log\left(-c^2x^2+1\right)\right)bd^4 - \frac{1}{2}\,\left(\log\left(cx\right)\log\left(-cx+1\right) + \operatorname{Li}_2(-cx+1)\right)bd^4 + \frac{1}{2}\,\left(\log\left(cx+1\right)\log\left(-cx\right) + \operatorname{Li}_2(cx+1)\right)bd^4 - \frac{23}{24}\,bd^4\log\left(cx+1\right) + \frac{55}{24}\,bd^4\log\left(cx-1\right) + ad^4\log\left(x\right) + \frac{1}{24}\,\left(3\,bc^4d^4x^4 + 16\,bc^3d^4x^3 + 36\,bc^2d^4x^2\right)\log\left(cx+1\right) - \frac{1}{24}\,\left(3\,bc^4d^4x^4 + 16\,bc^3d^4x^3 + 36\,bc^2d^4x^2\right)\log\left(-cx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^4*x^4 + 4/3*a*c^3*d^4*x^3 + 1/12*b*c^3*d^4*x^3 + 3*a*c^2*d^4*x^
2 + 2/3*b*c^2*d^4*x^2 + 4*a*c*d^4*x + 13/4*b*c*d^4*x + 2*(2*c*x*arctanh(c*x
) + log(-c^2*x^2 + 1))*b*d^4 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1
))*b*d^4 + 1/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^4 - 23/24*b*d^
4*log(c*x + 1) + 55/24*b*d^4*log(c*x - 1) + a*d^4*log(x) + 1/24*(3*b*c^4*d^
4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(c*x + 1) - 1/24*(3*b*c^4*d
^4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(-c*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^4\left(\int 4ac\,dx + \int \frac{a}{x}\,dx + \int 6ac^2x\,dx + \int 4ac^3x^2\,dx + \int ac^4x^3\,dx + \int 4bc\operatorname{atanh}\left(cx\right)dx + \int \frac{b\operatorname{atanh}\left(cx\right)}{x}\,dx + \int 6bc^2x\operatorname{atanh}\left(cx\right)dx + \int 4bc^3x^2\operatorname{atanh}\left(cx\right)dx + \int bc^4x^3\operatorname{atanh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**4*(Integral(4*a*c, x) + Integral(a/x, x) + Integral(6*a*c**2*x, x) + Int
egral(4*a*c**3*x**2, x) + Integral(a*c**4*x**3, x) + Integral(4*b*c*atanh(c
*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(6*b*c**2*x*atanh(c*x), x)
+ Integral(4*b*c**3*x**2*atanh(c*x), x) + Integral(b*c**4*x**3*atanh(c*x),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x, x)
```

3.36 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=178

$$6ac^2d^4x+2bc^2d^4x+\frac{1}{6}bc^3d^4x^2-2bcd^4\tanh^{-1}(cx)+6bc^2d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+2c^3d^4x^2\left(a+b\,\mathrm{t}\right.$$

```
[Out] 6*a*c^2*d^4*x+2*b*c^2*d^4*x+1/6*b*c^3*d^4*x^2-2*b*c*d^4*arctanh(c*x)+6*b*c^
2*d^4*x*arctanh(c*x)-d^4*(a+b*arctanh(c*x))/x+2*c^3*d^4*x^2*(a+b*arctanh(c*
x))+1/3*c^4*d^4*x^3*(a+b*arctanh(c*x))+4*a*c*d^4*ln(x)+b*c*d^4*ln(x)+8/3*b*
c*d^4*ln(-c^2*x^2+1)-2*b*c*d^4*polylog(2,-c*x)+2*b*c*d^4*polylog(2,c*x)
```

Rubi [A]
time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6087, 6021, 266, 6037, 272, 36, 29, 31, 6031, 327, 212, 45}

$$\frac{1}{3}c^4d^4x^3\left(a+b\tanh^{-1}(cx)\right)+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+6ac^2d^4x+4acd^4\log(x)+\frac{1}{6}bc^3d^4x^2+\frac{8}{3}bcd^4\log\left(1-c^2x^2\right)+2bc^2d^4x+6bc^2d^4x\tanh^{-1}(cx)-2bcd^4\text{Li}_2(-cx)+2bcd^4\text{Li}_2(cx)+bcd^4\log(x)-2bcd^4\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] 6*a*c^2*d^4*x + 2*b*c^2*d^4*x + (b*c^3*d^4*x^2)/6 - 2*b*c*d^4*ArcTanh[c*x]
+ 6*b*c^2*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/x + 2*c^3*d^4*x^2
*(a + b*ArcTanh[c*x]) + (c^4*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + 4*a*c*d^4*Lo
g[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 - c^2*x^2])/3 - 2*b*c*d^4*PolyLog[
2, -(c*x)] + 2*b*c*d^4*PolyLog[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(6c^2d^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{4cd^4\left(a+b\,\mathrm{ta}\right.}{x}\right.$$

$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(6c^2d^4\right)$$

$$= 6ac^2d^4x-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^4d$$

$$= 6ac^2d^4x+2bc^2d^4x+6bc^2d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+2$$

$$= 6ac^2d^4x+2bc^2d^4x-2bcd^4\tanh^{-1}(cx)+6bc^2d^4x\tanh^{-1}(cx)-\frac{d^4(a}{}$$

$$= 6ac^2d^4x+2bc^2d^4x+\frac{1}{6}bc^3d^4x^2-2bcd^4\tanh^{-1}(cx)+6bc^2d^4x\tanh^{-1}($$

Mathematica [A]
time = 0.08, size = 194, normalized size = 1.09

$$\frac{d^4\left(-6a+36ac^2x^2+12bc^2x^2+12ac^3x^3+bc^3x^3+2ac^4x^4-6b\tanh^{-1}(cx)+36bc^2x^2\tanh^{-1}(cx)+12bc^3x^3\tanh^{-1}(cx)+2bc^4x^4\tanh^{-1}(cx)+24acx\log(x)+6bcx\log(cx)+6bcx\log(1-cx)-6bcx\log(1+cx)+15bcx\log\left(1-c^2x^2\right)+bcx\log\left(-1+c^2x^2\right)-12bcx\,\mathrm{PolyLog}(2,-cx)+12bcx\,\mathrm{PolyLog}(2,cx)\right)}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^4*(-6*a + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 12*a*c^3*x^3 + b*c^3*x^3 + 2*a*c
^4*x^4 - 6*b*ArcTanh[c*x] + 36*b*c^2*x^2*ArcTanh[c*x] + 12*b*c^3*x^3*ArcTan
h[c*x] + 2*b*c^4*x^4*ArcTanh[c*x] + 24*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 6*
b*c*x*Log[1 - c*x] - 6*b*c*x*Log[1 + c*x] + 15*b*c*x*Log[1 - c^2*x^2] + b*c
*x*Log[-1 + c^2*x^2] - 12*b*c*x*PolyLog[2, -(c*x)] + 12*b*c*x*PolyLog[2, c*
x]))/(6*x)
```

Maple [A]
time = 0.22, size = 223, normalized size = 1.25

method	result
derivativedivides	$c\left(\frac{d^4 a\,c^3 x^3}{3}+2 d^4 a\,c^2 x^2+6 a c\,d^4 x-\frac{d^4 a}{c x}+4 d^4 a\ln\left(c x\right)+\frac{d^4 b\,\mathrm{arctanh}(c x) c^3 x^3}{3}+2 d^4 b\,\mathrm{arctanh}\right.$
default	$c\left(\frac{d^4 a\,c^3 x^3}{3}+2 d^4 a\,c^2 x^2+6 a c\,d^4 x-\frac{d^4 a}{c x}+4 d^4 a\ln\left(c x\right)+\frac{d^4 b\,\mathrm{arctanh}(c x) c^3 x^3}{3}+2 d^4 b\,\mathrm{arctanh}\right.$
risch	$6 a\,c^2 d^4 x+2 b\,c^2 d^4 x+\frac{b\,c^3 d^4 x^2}{6}+\frac{c\,d^4 b\ln(-c x)}{2}+\frac{11 c\,d^4 b\ln(-c x+1)}{3}+2 c\,d^4\,\mathrm{dilog}\left(-c x+1\right) b+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/3*d^4*a*c^3*x^3+2*d^4*a*c^2*x^2+6*a*c*d^4*x-d^4*a/c/x+4*d^4*a*ln(c*x)+
1/3*d^4*b*arctanh(c*x)*c^3*x^3+2*d^4*b*arctanh(c*x)*c^2*x^2+6*b*c*d^4*x*arc
tanh(c*x)-d^4*b*arctanh(c*x)/c/x+4*d^4*b*arctanh(c*x)*ln(c*x)-2*d^4*b*dilog
(c*x)-2*d^4*b*dilog(c*x+1)-2*d^4*b*ln(c*x)*ln(c*x+1)+1/6*b*c^2*d^4*x^2+2*b*
c*d^4*x+d^4*b*ln(c*x)+5/3*d^4*b*ln(c*x+1)+11/3*d^4*b*ln(c*x-1))
```

Maxima [A]
time = 0.35, size = 281, normalized size = 1.58

$$\frac{1}{3}\,ac^4d^4x^3+2\,ac^3d^4x^2+\frac{1}{6}\,bc^3d^4x^2+6\,ac^2d^4x+2\,bc^2d^4x+3\,\left(2\,cx\,\mathrm{artanh}\left(cx\right)+\log\left(-c^2x^2+1\right)\right)bcd^4-2\,\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2(-cx+1)\right)bcd^4+2\,\left(\log\left(cx+1\right)\log\left(-cx\right)+\mathrm{Li}_2(cx+1)\right)bcd^4-\frac{5}{6}\,bcd^4\log\left(cx+1\right)+\frac{7}{6}\,bcd^4\log\left(cx-1\right)+4\,acd^4\log\left(x\right)-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}\left(cx\right)}{x}\right)bd^4-\frac{ad^4}{x}+\frac{1}{6}\left(bc^4d^4x^3+6\,bc^3d^4x^2\right)\log\left(cx+1\right)-\frac{1}{6}\left(bc^4d^4x^3+6\,bc^3d^4x^2\right)\log\left(-cx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^4*x^3 + 2*a*c^3*d^4*x^2 + 1/6*b*c^3*d^4*x^2 + 6*a*c^2*d^4*x + 2
*b*c^2*d^4*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^4 - 2*(log(
c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c*d^4 + 2*(log(c*x + 1)*log(-c*x) +
 dilog(c*x + 1))*b*c*d^4 - 5/6*b*c*d^4*log(c*x + 1) + 7/6*b*c*d^4*log(c*x -
 1) + 4*a*c*d^4*log(x) - 1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c
*x)/x)*b*d^4 - a*d^4/x + 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(c*x + 1)
 - 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(-c*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^4\left(\int 6ac^2\,dx+\int \frac{a}{x^2}\,dx+\int \frac{4ac}{x}\,dx+\int 4ac^3x\,dx+\int ac^4x^2\,dx+\int 6bc^2\operatorname{atanh}(cx)\,dx+\int \frac{b\operatorname{atanh}(cx)}{x^2}\,dx+\int \frac{4bc\operatorname{atanh}(cx)}{x}\,dx+\int 4bc^3x\operatorname{atanh}(cx)\,dx+\int bc^4x^2\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**4*(Integral(6*a*c**2, x) + Integral(a/x**2, x) + Integral(4*a*c/x, x) +
Integral(4*a*c**3*x, x) + Integral(a*c**4*x**2, x) + Integral(6*b*c**2*atan
h(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(4*b*c*atanh(c*x)/x,
x) + Integral(4*b*c**3*x*atanh(c*x), x) + Integral(b*c**4*x**2*atanh(c*x),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^2, x)
```

3.37 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=156

$$-\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^4d^4x^2($$

```
[Out] -1/2*b*c*d^4/x+4*a*c^3*d^4*x+1/2*b*c^3*d^4*x+4*b*c^3*d^4*x*arctanh(c*x)-1/2
*d^4*(a+b*arctanh(c*x))/x^2-4*c*d^4*(a+b*arctanh(c*x))/x+1/2*c^4*d^4*x^2*(a
+b*arctanh(c*x))+6*a*c^2*d^4*ln(x)+4*b*c^2*d^4*ln(x)-3*b*c^2*d^4*polylog(2,
-c*x)+3*b*c^2*d^4*polylog(2,c*x)
```

Rubi [A]
time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6087, 6021, 266, 6037, 331, 212, 272, 36, 29, 31, 6031, 327}

$$\frac{1}{2}c^4d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x}+4ac^3d^4x+6ac^2d^4\log(x)+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-3bc^2d^4\text{Li}_2(-cx)+3bc^2d^4\text{Li}_2(cx)+4bc^2d^4\log(x)-\frac{bcd^4}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^4)/x + 4*a*c^3*d^4*x + (b*c^3*d^4*x)/2 + 4*b*c^3*d^4*x*ArcTanh[
c*x] - (d^4*(a + b*ArcTanh[c*x]))/(2*x^2) - (4*c*d^4*(a + b*ArcTanh[c*x]))/
x + (c^4*d^4*x^2*(a + b*ArcTanh[c*x]))/2 + 6*a*c^2*d^4*Log[x] + 4*b*c^2*d^4
*Log[x] - 3*b*c^2*d^4*PolyLog[2, -(c*x)] + 3*b*c^2*d^4*PolyLog[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx &= \int \left(4c^3d^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}\right.\\ &= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(6c^2d^4\right)\\ &= 4ac^3d^4x-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^4d^4\\ &= -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}\\ &= -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}\\ &= -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 143, normalized size = 0.92

$$\frac{d^4\left(-a-8acx-bcx+8ac^3x^3+bc^3x^3+ac^4x^4-b\tanh^{-1}(cx)-8bcx\tanh^{-1}(cx)+8bc^3x^3\tanh^{-1}(cx)+bc^4x^4\tanh^{-1}(cx)+12ac^2x^2\log(x)+8bc^2x^2\log(cx)-6bc^2x^2\text{PolyLog}(2,-cx)+6bc^2x^2\text{PolyLog}(2,cx)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^4*(-a - 8*a*c*x - b*c*x + 8*a*c^3*x^3 + b*c^3*x^3 + a*c^4*x^4 - b*ArcTan
h[c*x] - 8*b*c*x*ArcTanh[c*x] + 8*b*c^3*x^3*ArcTanh[c*x] + b*c^4*x^4*ArcTan
h[c*x] + 12*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - 6*b*c^2*x^2*PolyLog[2
, -(c*x)] + 6*b*c^2*x^2*PolyLog[2, c*x]))/(2*x^2)
```

Maple [A]
time = 0.22, size = 202, normalized size = 1.29

method	result
derivativedivides	$c^2\left(\frac{d^4ac^2x^2}{2}+4acd^4x+6d^4a\ln(cx)-\frac{d^4a}{2c^2x^2}-\frac{4d^4a}{cx}+\frac{d^4b\,\mathrm{arctanh}(cx)c^2x^2}{2}+4bcd^4x\,\mathrm{arctanh}\,(c\right.$
default	$c^2\left(\frac{d^4ac^2x^2}{2}+4acd^4x+6d^4a\ln(cx)-\frac{d^4a}{2c^2x^2}-\frac{4d^4a}{cx}+\frac{d^4b\,\mathrm{arctanh}(cx)c^2x^2}{2}+4bcd^4x\,\mathrm{arctanh}\,(c\right.$
risch	$-\frac{bcd^4}{2x}+4ac^3d^4x+\frac{bc^3d^4x}{2}-\frac{d^4a}{2x^2}+\frac{bc^4d^4\ln(cx+1)x^2}{4}+2bc^3d^4\ln(cx+1)x-\frac{2bcd^4\ln(cx+1)}{x}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*d^4*a*c^2*x^2+4*a*c*d^4*x+6*d^4*a*ln(c*x)-1/2*d^4*a/c^2/x^2-4*d^4*
a/c/x+1/2*d^4*b*arctanh(c*x)*c^2*x^2+4*b*c*d^4*x*arctanh(c*x)+6*d^4*b*arcta
nh(c*x)*ln(c*x)-1/2*d^4*b*arctanh(c*x)/c^2/x^2-4*d^4*b*arctanh(c*x)/c/x-3*d
^4*b*dilog(c*x)-3*d^4*b*dilog(c*x+1)-3*d^4*b*ln(c*x)*ln(c*x+1)+1/2*b*c*d^4*
x+4*d^4*b*ln(c*x)-1/2*d^4*b/c/x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(146) = 292$.
time = 0.35, size = 293, normalized size = 1.88

$$\frac{1}{4}bc^4d^4x^2\log(cx+1)-\frac{1}{4}bc^4d^4x^2\log(-cx+1)+\frac{1}{2}ac^4d^4x^2+4ac^3d^4x+\frac{1}{2}bc^3d^4x+2(2cx\,\mathrm{artanh}(cx)+\log(-c^2x^2+1))bc^2d^4-3(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1))bc^2d^4+3(\log(cx+1)\log(-cx)+\mathrm{Li}_2(cx+1))bc^2d^4-\frac{1}{4}bc^2d^4\log(cx+1)+\frac{1}{4}bc^2d^4\log(cx-1)+6ac^2d^4\log(x)-2\left(c(\log(c^2x^2-1)-\log(x^2))+\frac{2\,\mathrm{artanh}(cx)}{x}\right)bcd^4+\frac{1}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}(cx)}{x^2}\right)bd^4-\frac{4acd^4}{x}-\frac{ad^4}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^4*d^4*x^2*log(c*x + 1) - 1/4*b*c^4*d^4*x^2*log(-c*x + 1) + 1/2*a*c^
4*d^4*x^2 + 4*a*c^3*d^4*x + 1/2*b*c^3*d^4*x + 2*(2*c*x*arctanh(c*x) + log(-
c^2*x^2 + 1))*b*c^2*d^4 - 3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c^
2*d^4 + 3*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c^2*d^4 - 1/4*b*c^2*d
^4*log(c*x + 1) + 1/4*b*c^2*d^4*log(c*x - 1) + 6*a*c^2*d^4*log(x) - 2*(c*(l
og(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^4 + 1/4*((c*log(c*x +
 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^4 - 4*a*c*d^4/x - 1
/2*a*d^4/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^4\left(\int 4ac^3\,dx + \int \frac{a}{x^3}\,dx + \int \frac{4ac}{x^2}\,dx + \int \frac{6ac^2}{x}\,dx + \int ac^4x\,dx + \int 4bc^3\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x^3}\,dx + \int \frac{4bc\operatorname{atanh}(cx)}{x^2}\,dx + \int \frac{6bc^2\operatorname{atanh}(cx)}{x}\,dx + \int bc^4x\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**4*(Integral(4*a*c**3, x) + Integral(a/x**3, x) + Integral(4*a*c/x**2, x)
 + Integral(6*a*c**2/x, x) + Integral(a*c**4*x, x) + Integral(4*b*c**3*atan
h(c*x), x) + Integral(b*atanh(c*x)/x**3, x) + Integral(4*b*c*atanh(c*x)/x**
2, x) + Integral(6*b*c**2*atanh(c*x)/x, x) + Integral(b*c**4*x*atanh(c*x),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^3, x)
```

3.38 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{bcd^4}{6x^2}-\frac{2bc^2d^4}{x}+ac^4d^4x+2bc^3d^4\tanh^{-1}(cx)+bc^4d^4x\tanh^{-1}(cx)-\frac{d^4(a+b\tanh^{-1}(cx))}{3x^3}-\frac{2cd^4(a+b\tanh^{-1}(c}{x^2}$$

```
[Out] -1/6*b*c*d^4/x^2-2*b*c^2*d^4/x+a*c^4*d^4*x+2*b*c^3*d^4*arctanh(c*x)+b*c^4*d
^4*x*arctanh(c*x)-1/3*d^4*(a+b*arctanh(c*x))/x^3-2*c*d^4*(a+b*arctanh(c*x))
/x^2-6*c^2*d^4*(a+b*arctanh(c*x))/x+4*a*c^3*d^4*ln(x)+19/3*b*c^3*d^4*ln(x)-
8/3*b*c^3*d^4*ln(-c^2*x^2+1)-2*b*c^3*d^4*polylog(2,-c*x)+2*b*c^3*d^4*polylo
g(2,c*x)
```

Rubi [A]
time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6087, 6021, 266, 6037, 272, 46, 331, 212, 36, 29, 31, 6031}

$$-\frac{6c^2d^4(a+b\tanh^{-1}(cx))}{x}-\frac{d^4(a+b\tanh^{-1}(cx))}{3x^3}-\frac{2cd^4(a+b\tanh^{-1}(cx))}{x^2}+ac^4d^4x+4ac^3d^4\log(x)+bc^4d^4x\tanh^{-1}(cx)-2bc^3d^4\text{Li}_2(-cx)+2bc^3d^4\text{Li}_2(cx)+\frac{19}{3}bc^3d^4\log(x)+2bc^3d^4\tanh^{-1}(cx)-\frac{2bc^2d^4}{x}-\frac{8}{3}bc^3d^4\log\left(1-c^2x^2\right)-\frac{bcd^4}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^4)/x^2 - (2*b*c^2*d^4)/x + a*c^4*d^4*x + 2*b*c^3*d^4*ArcTanh[c*
x] + b*c^4*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/(3*x^3) - (2*c*d
^4*(a + b*ArcTanh[c*x]))/x^2 - (6*c^2*d^4*(a + b*ArcTanh[c*x]))/x + 4*a*c^3
*d^4*Log[x] + (19*b*c^3*d^4*Log[x])/3 - (8*b*c^3*d^4*Log[1 - c^2*x^2])/3 -
2*b*c^3*d^4*PolyLog[2, -(c*x)] + 2*b*c^3*d^4*PolyLog[2, c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = \int \left(c^4d^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}+\frac{4cd^4\left(a+b\tan\right.}{x^3}\right.$$
$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(6c^2d^4\right)$$
$$= ac^4d^4x-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{6c^2d^4\left(a\right.}{}$$
$$= -\frac{2bc^2d^4}{x}+ac^4d^4x+bc^4d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{2c}{}$$
$$= -\frac{2bc^2d^4}{x}+ac^4d^4x+2bc^3d^4\tanh^{-1}(cx)+bc^4d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+\right.}{}$$
$$= -\frac{bcd^4}{6x^2}-\frac{2bc^2d^4}{x}+ac^4d^4x+2bc^3d^4\tanh^{-1}(cx)+bc^4d^4x\tanh^{-1}(cx)-$$

Mathematica [A]
time = 0.08, size = 197, normalized size = 1.04

$$\frac{d^4\left(-2a-12acx-bcx-36ac^2x^2-12bc^2x^2+6ac^4x^4-2b\tanh^{-1}(cx)-12bcx\tanh^{-1}(cx)-36bc^2x^2\tanh^{-1}(cx)+6bc^4x^4\tanh^{-1}(cx)+24ac^3x^3\log(x)+38bc^3x^3\log(cx)-6bc^3x^3\log(1-cx)+6bc^3x^3\log(1+cx)-16bc^3x^3\log\left(1-c^2x^2\right)-12bc^3x^3\mathrm{PolyLog}(2,-cx)+12bc^3x^3\mathrm{PolyLog}(2,cx)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] (d^4*(-2*a - 12*a*c*x - b*c*x - 36*a*c^2*x^2 - 12*b*c^2*x^2 + 6*a*c^4*x^4 -
 2*b*ArcTanh[c*x] - 12*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 6*b
*c^4*x^4*ArcTanh[c*x] + 24*a*c^3*x^3*Log[x] + 38*b*c^3*x^3*Log[c*x] - 6*b*c
^3*x^3*Log[1 - c*x] + 6*b*c^3*x^3*Log[1 + c*x] - 16*b*c^3*x^3*Log[1 - c^2*x
^2] - 12*b*c^3*x^3*PolyLog[2, -(c*x)] + 12*b*c^3*x^3*PolyLog[2, c*x]))/(6*x
^3)
```

Maple [A]
time = 0.24, size = 228, normalized size = 1.21

method	result
derivativedivides	$c^3\left(ac\,d^4x-\frac{6d^4a}{cx}+4d^4a\ln(cx)-\frac{d^4a}{3c^3x^3}-\frac{2d^4a}{c^2x^2}+bc\,d^4x\,\mathrm{arctanh}(cx)-\frac{6d^4b\,\mathrm{arctanh}(cx)}{cx}+4\right.$
default	$c^3\left(ac\,d^4x-\frac{6d^4a}{cx}+4d^4a\ln(cx)-\frac{d^4a}{3c^3x^3}-\frac{2d^4a}{c^2x^2}+bc\,d^4x\,\mathrm{arctanh}(cx)-\frac{6d^4b\,\mathrm{arctanh}(cx)}{cx}+4\right.$
risch	$\frac{b\,c^4d^4\ln(cx+1)x}{2}-\frac{3b\,c^2d^4\ln(cx+1)}{x}-\frac{bc\,d^4\ln(cx+1)}{x^2}-\frac{bc\,d^4}{6x^2}-\frac{2b\,c^2d^4}{x}+\frac{c\,d^4b\ln(-cx+1)}{x^2}-\frac{c^4d^4b\ln(-cx+}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a*c*d^4*x-6*d^4*a/c/x+4*d^4*a*ln(c*x)-1/3*d^4*a/c^3/x^3-2*d^4*a/c^2/x^
2+b*c*d^4*x*arctanh(c*x)-6*d^4*b*arctanh(c*x)/c/x+4*d^4*b*arctanh(c*x)*ln(c
*x)-1/3*d^4*b*arctanh(c*x)/c^3/x^3-2*d^4*b*arctanh(c*x)/c^2/x^2-2*d^4*b*dil
og(c*x)-2*d^4*b*dilog(c*x+1)-2*d^4*b*ln(c*x)*ln(c*x+1)-1/6*d^4*b/c^2/x^2-2*
d^4*b/c/x+19/3*d^4*b*ln(c*x)-11/3*d^4*b*ln(c*x-1)-5/3*d^4*b*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^4*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^3*d^4 + 2*b*
c^3*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 4*a*c^3*d^4*log(x)
 - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^4 + ((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^4 - 1/6*(
(c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*d^4
 - 6*a*c^2*d^4/x - 2*a*c*d^4/x^2 - 1/3*a*d^4/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{4}\left(\int ac^{4}\,dx+\int \frac{a}{x^{4}}\,dx+\int \frac{4ac}{x^{3}}\,dx+\int \frac{6ac^{2}}{x^{2}}\,dx+\int \frac{4ac^{3}}{x}\,dx+\int bc^{4}\operatorname{atanh}{\left(cx\right)}\,dx+\int \frac{b\operatorname{atanh}{\left(cx\right)}}{x^{4}}\,dx+\int \frac{4bc\operatorname{atanh}{\left(cx\right)}}{x^{3}}\,dx+\int \frac{6bc^{2}\operatorname{atanh}{\left(cx\right)}}{x^{2}}\,dx+\int \frac{4bc^{3}\operatorname{atanh}{\left(cx\right)}}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] d**4*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(4*a*c/x**3, x) +
 Integral(6*a*c**2/x**2, x) + Integral(4*a*c**3/x, x) + Integral(b*c**4*ata
nh(c*x), x) + Integral(b*atanh(c*x)/x**4, x) + Integral(4*b*c*atanh(c*x)/x*
*3, x) + Integral(6*b*c**2*atanh(c*x)/x**2, x) + Integral(4*b*c**3*atanh(c*
x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^4, x)
```

3.39 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{bcd^4}{12x^3}-\frac{2bc^2d^4}{3x^2}-\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^4\left(a+\right.}{}$$

```
[Out] -1/12*b*c*d^4/x^3-2/3*b*c^2*d^4/x^2-13/4*b*c^3*d^4/x+13/4*b*c^4*d^4*arctanh
(c*x)-1/4*d^4*(a+b*arctanh(c*x))/x^4-4/3*c*d^4*(a+b*arctanh(c*x))/x^3-3*c^2
*d^4*(a+b*arctanh(c*x))/x^2-4*c^3*d^4*(a+b*arctanh(c*x))/x+a*c^4*d^4*ln(x)+
16/3*b*c^4*d^4*ln(x)-8/3*b*c^4*d^4*ln(-c^2*x^2+1)-1/2*b*c^4*d^4*polylog(2,-
c*x)+1/2*b*c^4*d^4*polylog(2,c*x)
```

Rubi [A]

time = 0.17, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6037, 331, 212, 272, 46, 36, 29, 31, 6031}

$$-\frac{4c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{3c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}+ac^4d^4\log(x)-\frac{1}{2}bc^4d^4\text{Li}_2(-cx)+\frac{1}{2}bc^4d^4\text{Li}_2(cx)+\frac{16}{3}bc^4d^4\log(x)+\frac{13}{4}bc^4d^4\tanh^{-1}(cx)-\frac{13bc^3d^4}{4x}-\frac{2bc^2d^4}{3x^2}-\frac{8}{3}bc^4d^4\log\left(1-c^2x^2\right)-\frac{bcd^4}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^4)/x^3 - (2*b*c^2*d^4)/(3*x^2) - (13*b*c^3*d^4)/(4*x) + (13*b*
c^4*d^4*ArcTanh[c*x])/4 - (d^4*(a + b*ArcTanh[c*x]))/(4*x^4) - (4*c*d^4*(a
+ b*ArcTanh[c*x]))/(3*x^3) - (3*c^2*d^4*(a + b*ArcTanh[c*x]))/x^2 - (4*c^3*
d^4*(a + b*ArcTanh[c*x]))/x + a*c^4*d^4*Log[x] + (16*b*c^4*d^4*Log[x])/3 -
(8*b*c^4*d^4*Log[1 - c^2*x^2])/3 - (b*c^4*d^4*PolyLog[2, -(c*x)])/2 + (b*c^
4*d^4*PolyLog[2, c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx &= \int \left(\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}+\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}+\frac{6c^2d^4\left(a+b\right.}{x\ldots}\right.\\
&= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx+\left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\left(6c^2d^4\right.\\
&= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^4\left(a+b\tan\right.}{x^2}\\
&= -\frac{bcd^4}{12x^3}-\frac{3bc^3d^4}{x}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}\\
&= -\frac{bcd^4}{12x^3}-\frac{13bc^3d^4}{4x}+3bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4c}{}\\
&= -\frac{bcd^4}{12x^3}-\frac{2bc^2d^4}{3x^2}-\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-}\right.}{4x^4}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 206, normalized size = 0.99

$$\frac{d^4\left(-6a-32acx-2bcx-72ac^2x^2-16bc^2x^2-96ac^3x^3-78bc^3x^3-6b\tanh^{-1}(cx)-32bcx\tanh^{-1}(cx)-72bc^2x^2\tanh^{-1}(cx)-96bc^3x^3\tanh^{-1}(cx)+24ac^4x^4\log(x)+128bc^4x^4\log(cx)-39bc^4x^4\log(1-cx)+39bc^4x^4\log(1+cx)-64bc^4x^4\log\left(1-c^2x^2\right)-12bc^4x^4\text{PolyLog}(2,-cx)+12bc^4x^4\text{PolyLog}(2,cx)\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] (d^4*(-6*a - 32*a*c*x - 2*b*c*x - 72*a*c^2*x^2 - 16*b*c^2*x^2 - 96*a*c^3*x^
3 - 78*b*c^3*x^3 - 6*b*ArcTanh[c*x] - 32*b*c*x*ArcTanh[c*x] - 72*b*c^2*x^2*
ArcTanh[c*x] - 96*b*c^3*x^3*ArcTanh[c*x] + 24*a*c^4*x^4*Log[x] + 128*b*c^4*
x^4*Log[c*x] - 39*b*c^4*x^4*Log[1 - c*x] + 39*b*c^4*x^4*Log[1 + c*x] - 64*b
*c^4*x^4*Log[1 - c^2*x^2] - 12*b*c^4*x^4*PolyLog[2, -(c*x)] + 12*b*c^4*x^4*
PolyLog[2, c*x]))/(24*x^4)
```

Maple [A]
time = 0.23, size = 248, normalized size = 1.19

method	result
derivativedivides	$c^4\left(-\frac{3d^4a}{c^2x^2}-\frac{d^4a}{4c^4x^4}-\frac{4d^4a}{cx}-\frac{4d^4a}{3c^3x^3}+d^4a\ln(cx)-\frac{3d^4b\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{d^4b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{4d^4b\,\mathrm{arctan}}{cx}\right.$
default	$c^4\left(-\frac{3d^4a}{c^2x^2}-\frac{d^4a}{4c^4x^4}-\frac{4d^4a}{cx}-\frac{4d^4a}{3c^3x^3}+d^4a\ln(cx)-\frac{3d^4b\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{d^4b\,\mathrm{arctanh}(cx)}{4c^4x^4}-\frac{4d^4b\,\mathrm{arctan}}{cx}\right.$
risch	$\frac{2c^3d^4b\ln(-cx+1)}{x}-\frac{bc\,d^4}{12x^3}-\frac{2b\,c^2d^4}{3x^2}-\frac{13b\,c^3d^4}{4x}+\frac{3c^2d^4b\ln(-cx+1)}{2x^2}+\frac{2c\,d^4b\ln(-cx+1)}{3x^3}-\frac{2b\,c^3d^4\ln(cx+1)}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-3*d^4*a/c^2/x^2-1/4*d^4*a/c^4/x^4-4*d^4*a/c/x-4/3*d^4*a/c^3/x^3+d^4*a
*ln(c*x)-3*d^4*b*arctanh(c*x)/c^2/x^2-1/4*d^4*b*arctanh(c*x)/c^4/x^4-4*d^4*
b*arctanh(c*x)/c/x-4/3*d^4*b*arctanh(c*x)/c^3/x^3+d^4*b*arctanh(c*x)*ln(c*x
)-103/24*d^4*b*ln(c*x-1)-25/24*d^4*b*ln(c*x+1)-1/12*d^4*b/c^3/x^3-2/3*d^4*b
/c^2/x^2-13/4*d^4*b/c/x+16/3*d^4*b*ln(c*x)-1/2*d^4*b*dilog(c*x)-1/2*d^4*b*d
ilog(c*x+1)-1/2*d^4*b*ln(c*x)*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^4*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^4*d^4*lo
g(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^4 + 3
/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d
^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/
x^3)*b*c*d^4 - 4*a*c^3*d^4/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x -
1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^4 - 3*a*c^2*d^4/x^2
 - 4/3*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{4}\left(\int \frac{a}{x^{5}}\, dx + \int \frac{4ac}{x^{4}}\, dx + \int \frac{6ac^{2}}{x^{3}}\, dx + \int \frac{4ac^{3}}{x^{2}}\, dx + \int \frac{ac^{4}}{x}\, dx + \int \frac{b \operatorname{atanh}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{4bc \operatorname{atanh}{\left(cx \right)}}{x^{4}}\, dx + \int \frac{6bc^{2} \operatorname{atanh}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{4bc^{3} \operatorname{atanh}{\left(cx \right)}}{x^{2}}\, dx + \int \frac{bc^{4} \operatorname{atanh}{\left(cx \right)}}{x}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] d**4*(Integral(a/x**5, x) + Integral(4*a*c/x**4, x) + Integral(6*a*c**2/x**
3, x) + Integral(4*a*c**3/x**2, x) + Integral(a*c**4/x, x) + Integral(b*ata
nh(c*x)/x**5, x) + Integral(4*b*c*atanh(c*x)/x**4, x) + Integral(6*b*c**2*a
tanh(c*x)/x**3, x) + Integral(4*b*c**3*atanh(c*x)/x**2, x) + Integral(b*c**
4*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^5,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^5, x)
```

3.40 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=109

$$-\frac{bcd^4}{20x^4}-\frac{bc^2d^4}{3x^3}-\frac{11bc^3d^4}{10x^2}-\frac{3bc^4d^4}{x}-\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)$$

```
[Out] -1/20*b*c*d^4/x^4-1/3*b*c^2*d^4/x^3-11/10*b*c^3*d^4/x^2-3*b*c^4*d^4/x-1/5*d
^4*(c*x+1)^5*(a+b*arctanh(c*x))/x^5+16/5*b*c^5*d^4*ln(x)-16/5*b*c^5*d^4*ln(
-c*x+1)
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {37, 6083, 12, 90}

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)-\frac{3bc^4d^4}{x}-\frac{11bc^3d^4}{10x^2}-\frac{bc^2d^4}{3x^3}-\frac{bcd^4}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^4)/x^4 - (b*c^2*d^4)/(3*x^3) - (11*b*c^3*d^4)/(10*x^2) - (3*b*
c^4*d^4)/x - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*x^5) + (16*b*c^5*d^4
*Log[x])/5 - (16*b*c^5*d^4*Log[1 - c*x])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx &= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-(bc)\int \frac{(d+cdx)^4}{5x^5(-1+cx)}\,dx\\ &= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{1}{5}(bc)\int \frac{(d+cdx)^4}{x^5(-1+cx)}\,dx\\ &= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{1}{5}(bc)\int \left(-\frac{d^4}{x^5}-\frac{5cd^4}{x^4}-\frac{11c^2d^4}{x^3}\right.\\ &= -\frac{bcd^4}{20x^4}-\frac{bc^2d^4}{3x^3}-\frac{11bc^3d^4}{10x^2}-\frac{3bc^4d^4}{x}-\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx\right.}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 157, normalized size = 1.44

$$-\frac{d^4\left(12a+60acx+3bcx+120ac^2x^2+20bc^2x^2+120ac^3x^3+66bc^3x^3+60ac^4x^4+180bc^4x^4+12b\left(1+5cx+10c^2x^2+10c^3x^3+5c^4x^4\right)\tanh^{-1}(cx)-192bc^5x^5\log(x)+186bc^5x^5\log(1-cx)+6bc^5x^5\log(1+cx)\right)}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -1/60*(d^4*(12*a + 60*a*c*x + 3*b*c*x + 120*a*c^2*x^2 + 20*b*c^2*x^2 + 120*
a*c^3*x^3 + 66*b*c^3*x^3 + 60*a*c^4*x^4 + 180*b*c^4*x^4 + 12*b*(1 + 5*c*x +
 10*c^2*x^2 + 10*c^3*x^3 + 5*c^4*x^4)*ArcTanh[c*x] - 192*b*c^5*x^5*Log[x] +
 186*b*c^5*x^5*Log[1 - c*x] + 6*b*c^5*x^5*Log[1 + c*x]))/x^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(97) = 194$.
time = 0.17, size = 214, normalized size = 1.96

method	result
derivativedivides	$c^5\left(d^4a\left(-\frac{2}{c^2x^2}-\frac{2}{c^3x^3}-\frac{1}{c^4x^4}-\frac{1}{cx}-\frac{1}{5c^5x^5}\right)-\frac{2d^4b\operatorname{arctanh}(cx)}{c^2x^2}-\frac{2d^4b\operatorname{arctanh}(cx)}{c^3x^3}-\frac{d^4b\operatorname{arctanh}(cx}{c^4x^4}\right.$
default	$c^5\left(d^4a\left(-\frac{2}{c^2x^2}-\frac{2}{c^3x^3}-\frac{1}{c^4x^4}-\frac{1}{cx}-\frac{1}{5c^5x^5}\right)-\frac{2d^4b\operatorname{arctanh}(cx)}{c^2x^2}-\frac{2d^4b\operatorname{arctanh}(cx)}{c^3x^3}-\frac{d^4b\operatorname{arctanh}(cx}{c^4x^4}\right.$
risch	$-\frac{d^4b\left(5c^4x^4+10x^3c^3+10c^2x^2+5cx+1\right)\ln(cx+1)}{10x^5}+\frac{d^4\left(192c^5b\ln(-x)x^5-6b\,c^5\ln(cx+1)x^5-186x^5b\ln(-cx+1)c^5+30\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(d^4*a*(-2/c^2/x^2-2/c^3/x^3-1/c^4/x^4-1/c/x-1/5/c^5/x^5)-2*d^4*b*arcta
nh(c*x)/c^2/x^2-2*d^4*b*arctanh(c*x)/c^3/x^3-d^4*b*arctanh(c*x)/c^4/x^4-d^4
*b*arctanh(c*x)/c/x-1/5*d^4*b*arctanh(c*x)/c^5/x^5-1/20*d^4*b/c^4/x^4-1/3*d
^4*b/c^3/x^3-11/10*d^4*b/c^2/x^2-3*d^4*b/c/x+16/5*d^4*b*ln(c*x)-31/10*d^4*b
*ln(c*x-1)-1/10*d^4*b*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(97) = 194$.
time = 0.26, size = 299, normalized size = 2.74

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)bc^4d^4+\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)bc^3d^4-\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\operatorname{artanh}\left(cx\right)}{x^3}\right)bc^2d^4-\frac{ac^4d^4}{x}+\frac{1}{6}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\operatorname{artanh}\left(cx\right)}{x^4}\right)bcd^4-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\operatorname{artanh}\left(cx\right)}{x^5}\right)bd^4-\frac{2\,ac^3d^4}{x^2}-\frac{2\,ac^2d^4}{x^3}-\frac{acd^4}{x^4}-\frac{ad^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^4*d^4 + ((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*d^4 - ((c
^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*c^2*d
^4 - a*c^4*d^4/x + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2
*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^4 - 1/20*((2*c^4*log(c^2*x^2 -
 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^4 -
 2*a*c^3*d^4/x^2 - 2*a*c^2*d^4/x^3 - a*c*d^4/x^4 - 1/5*a*d^4/x^5
```

Fricas [A]
time = 0.37, size = 191, normalized size = 1.75

$$-\frac{6\,bc^5d^4x^5\log\left(cx+1\right)+186\,bc^5d^4x^5\log\left(cx-1\right)-192\,bc^5d^4x^5\log\left(x\right)+60\,(a+3\,b)c^4d^4x^4+6\,(20\,a+11\,b)c^3d^4x^3+20\,(6\,a+b)c^2d^4x^2+3\,(20\,a+b)cd^4x+12\,ad^4+6\,(5\,bc^4d^4x^4+10\,bc^3d^4x^3+10\,bc^2d^4x^2+5\,bcd^4x+bd^4)\log\left(-\frac{cx+1}{cx-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(6*b*c^5*d^4*x^5*log(c*x + 1) + 186*b*c^5*d^4*x^5*log(c*x - 1) - 192*
b*c^5*d^4*x^5*log(x) + 60*(a + 3*b)*c^4*d^4*x^4 + 6*(20*a + 11*b)*c^3*d^4*x
^3 + 20*(6*a + b)*c^2*d^4*x^2 + 3*(20*a + b)*c*d^4*x + 12*a*d^4 + 6*(5*b*c^
4*d^4*x^4 + 10*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x + b*d^4)*log(
-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(109) = 218$.
time = 0.69, size = 253, normalized size = 2.32

$$\begin{cases} -\frac{ac^4d^4}{x}-\frac{2ac^3d^4}{x^2}-\frac{2ac^2d^4}{x^3}-\frac{acd^4}{x^4}-\frac{ad^4}{5x^5}+\frac{16bc^5d^4\log\left(x\right)}{5}-\frac{16bc^5d^4\log\left(x-\frac{1}{c}\right)}{5}-\frac{bc^5d^4\operatorname{atanh}\left(cx\right)}{5}-\frac{bc^4d^4\operatorname{atanh}\left(cx\right)}{x}-\frac{3bc^4d^4}{x}-\frac{2bc^3d^4\operatorname{atanh}\left(cx\right)}{x^2}-\frac{11bc^3d^4}{10x^2}-\frac{2bc^2d^4\operatorname{atanh}\left(cx\right)}{x^3}-\frac{bc^2d^4}{3x^3}-\frac{bcd^4\operatorname{atanh}\left(cx\right)}{x^4}-\frac{bcd^4}{20x^4}-\frac{bd^4\operatorname{atanh}\left(cx\right)}{5x^5} & \text{for}\: c \neq 0 \\ -\frac{ad^4}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**4*d**4/x - 2*a*c**3*d**4/x**2 - 2*a*c**2*d**4/x**3 - a*c*d
**4/x**4 - a*d**4/(5*x**5) + 16*b*c**5*d**4*log(x)/5 - 16*b*c**5*d**4*log(x
 - 1/c)/5 - b*c**5*d**4*atanh(c*x)/5 - b*c**4*d**4*atanh(c*x)/x - 3*b*c**4*
d**4/x - 2*b*c**3*d**4*atanh(c*x)/x**2 - 11*b*c**3*d**4/(10*x**2) - 2*b*c**
2*d**4*atanh(c*x)/x**3 - b*c**2*d**4/(3*x**3) - b*c*d**4*atanh(c*x)/x**4 -
b*c*d**4/(20*x**4) - b*d**4*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**4/(5*x**
5), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(97) = 194$.
time = 0.41, size = 532, normalized size = 4.88

$$\frac{4}{15}\left(12\,bc^4d^4\log\left(-\frac{cx+1}{cx-1}-1\right)-12\,bc^4d^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{12\left(\frac{5\,(cx+1)^4bc^4d^4}{(cx-1)^4}+\frac{10\,(cx+1)^3bc^4d^4}{(cx-1)^3}+\frac{10\,(cx+1)^2bc^4d^4}{(cx-1)^2}+\frac{5\,(cx+1)bc^4d^4}{cx-1}+bc^4d^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}+\frac{\frac{120\,(cx+1)^4ac^4d^4}{(cx-1)^4}+\frac{240\,(cx+1)^3ac^4d^4}{(cx-1)^3}+\frac{240\,(cx+1)^2ac^4d^4}{(cx-1)^2}+\frac{120\,(cx+1)ac^4d^4}{cx-1}+24\,ac^4d^4+\frac{48\,(cx+1)^4bc^4d^4}{(cx-1)^4}+\frac{156\,(cx+1)^3bc^4d^4}{(cx-1)^3}+\frac{196\,(cx+1)^2bc^4d^4}{(cx-1)^2}+\frac{113\,(cx+1)bc^4d^4}{cx-1}+25\,bc^4d^4}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(12*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 12*b*c^4*d^4*log(-(c*x +
 1)/(c*x - 1)) + 12*(5*(c*x + 1)^4*b*c^4*d^4/(c*x - 1)^4 + 10*(c*x + 1)^3*b
*c^4*d^4/(c*x - 1)^3 + 10*(c*x + 1)^2*b*c^4*d^4/(c*x - 1)^2 + 5*(c*x + 1)*b
*c^4*d^4/(c*x - 1) + b*c^4*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x
 - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x
+ 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (120*(c*x + 1)^4*a*c^4*d^
4/(c*x - 1)^4 + 240*(c*x + 1)^3*a*c^4*d^4/(c*x - 1)^3 + 240*(c*x + 1)^2*a*c
^4*d^4/(c*x - 1)^2 + 120*(c*x + 1)*a*c^4*d^4/(c*x - 1) + 24*a*c^4*d^4 + 48*
(c*x + 1)^4*b*c^4*d^4/(c*x - 1)^4 + 156*(c*x + 1)^3*b*c^4*d^4/(c*x - 1)^3 +
 196*(c*x + 1)^2*b*c^4*d^4/(c*x - 1)^2 + 113*(c*x + 1)*b*c^4*d^4/(c*x - 1)
+ 25*b*c^4*d^4)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(
c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1)
 + 1))*c
```

Mupad [B]
time = 0.96, size = 179, normalized size = 1.64

$$\frac{d^4\,\left(180\,b\,c^5\,\mathrm{atanh}(c\,x)-96\,b\,c^5\,\ln\left(c^2\,x^2-1\right)+192\,b\,c^5\,\ln\left(x\right)\right)}{60}-\frac{\frac{d^4\,\left(12\,a+12\,b\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{d^4\,x\,\left(60\,a\,c+3\,b\,c+60\,b\,c\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{d^4\,x^2\,\left(120\,a\,c^2+20\,b\,c^2+120\,b\,c^2\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{d^4\,x^4\,\left(60\,a\,c^4+180\,b\,c^4+60\,b\,c^4\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{d^4\,x^3\,\left(120\,a\,c^3+66\,b\,c^3+120\,b\,c^3\,\mathrm{atanh}(c\,x)\right)}{60}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^6,x)
```

```
[Out] (d^4*(180*b*c^5*atanh(c*x) - 96*b*c^5*log(c^2*x^2 - 1) + 192*b*c^5*log(x)))
/60 - ((d^4*(12*a + 12*b*atanh(c*x)))/60 + (d^4*x*(60*a*c + 3*b*c + 60*b*c*
atanh(c*x)))/60 + (d^4*x^2*(120*a*c^2 + 20*b*c^2 + 120*b*c^2*atanh(c*x)))/6
0 + (d^4*x^4*(60*a*c^4 + 180*b*c^4 + 60*b*c^4*atanh(c*x)))/60 + (d^4*x^3*(1
20*a*c^3 + 66*b*c^3 + 120*b*c^3*atanh(c*x)))/60)/x^5
```

3.41 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{bcd^4}{30x^5}-\frac{bc^2d^4}{5x^4}-\frac{5bc^3d^4}{9x^3}-\frac{16bc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}$$

```
[Out] -1/30*b*c*d^4/x^5-1/5*b*c^2*d^4/x^4-5/9*b*c^3*d^4/x^3-16/15*b*c^4*d^4/x^2-1
3/6*b*c^5*d^4/x-1/6*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/x^6+1/30*c*d^4*(c*x+1)
^5*(a+b*arctanh(c*x))/x^5+32/15*b*c^6*d^4*ln(x)-32/15*b*c^6*d^4*ln(-c*x+1)
```

Rubi [A]
time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {47, 37, 6083, 12, 153}

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}+\frac{32}{15}bc^6d^4\log(x)-\frac{32}{15}bc^6d^4\log(1-cx)-\frac{13bc^5d^4}{6x}-\frac{16bc^4d^4}{15x^2}-\frac{5bc^3d^4}{9x^3}-\frac{bc^2d^4}{5x^4}-\frac{bcd^4}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d^4)/x^5 - (b*c^2*d^4)/(5*x^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c
^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c
*x]))/(6*x^6) + (c*d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(30*x^5) + (32*b*c
^6*d^4*Log[x])/15 - (32*b*c^6*d^4*Log[1 - c*x])/15
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx &= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}\\ &= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}\\ &= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}\\ &= -\frac{bcd^4}{30x^5}-\frac{bc^2d^4}{5x^4}-\frac{5bc^3d^4}{9x^3}-\frac{16bc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+cx)^5\left(a+b\right.}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 166, normalized size = 1.10

$$-\frac{d^4\left(30a+144acx+6bcx+270ac^2x^2+36bc^2x^2+240ac^3x^3+100bc^3x^3+90ac^4x^4+192bc^4x^4+390bc^5x^5+6b(5+24cx+45c^2x^2+40c^3x^3+15c^4x^4)\tanh^{-1}(cx)-384bc^6x^6\log(x)+387bc^6x^6\log(1-cx)-3bc^6x^6\log(1+cx)\right)}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -1/180*(d^4*(30*a + 144*a*c*x + 6*b*c*x + 270*a*c^2*x^2 + 36*b*c^2*x^2 + 24
0*a*c^3*x^3 + 100*b*c^3*x^3 + 90*a*c^4*x^4 + 192*b*c^4*x^4 + 390*b*c^5*x^5
+ 6*b*(5 + 24*c*x + 45*c^2*x^2 + 40*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] - 38
4*b*c^6*x^6*Log[x] + 387*b*c^6*x^6*Log[1 - c*x] - 3*b*c^6*x^6*Log[1 + c*x])
)/x^6
```

Maple [A]
time = 0.17, size = 226, normalized size = 1.50

method	result
derivativedivides	$c^6\left(d^4a\left(-\frac{4}{3c^3x^3}-\frac{1}{6c^6x^6}-\frac{4}{5c^5x^5}-\frac{1}{2c^2x^2}-\frac{3}{2c^4x^4}\right)-\frac{4d^4b\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{d^4b\operatorname{arctanh}(cx)}{6c^6x^6}-\frac{4d^4b\operatorname{arctan}}{5c^5x^5}\right.$
default	$c^6\left(d^4a\left(-\frac{4}{3c^3x^3}-\frac{1}{6c^6x^6}-\frac{4}{5c^5x^5}-\frac{1}{2c^2x^2}-\frac{3}{2c^4x^4}\right)-\frac{4d^4b\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{d^4b\operatorname{arctanh}(cx)}{6c^6x^6}-\frac{4d^4b\operatorname{arctan}}{5c^5x^5}\right.$
risch	$-\frac{d^4b(15c^4x^4+40x^3c^3+45c^2x^2+24cx+5)\ln(cx+1)}{60x^6}+\frac{d^4(3bc^6\ln(cx+1)x^6+384c^6b\ln(-x)x^6-387x^6b\ln(-cx+1)c^6-39}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(d^4*a*(-4/3/c^3/x^3-1/6/c^6/x^6-4/5/c^5/x^5-1/2/c^2/x^2-3/2/c^4/x^4)-4
/3*d^4*b*arctanh(c*x)/c^3/x^3-1/6*d^4*b*arctanh(c*x)/c^6/x^6-4/5*d^4*b*arct
anh(c*x)/c^5/x^5-1/2*d^4*b*arctanh(c*x)/c^2/x^2-3/2*d^4*b*arctanh(c*x)/c^4/
x^4+1/60*d^4*b*ln(c*x+1)-43/20*d^4*b*ln(c*x-1)-1/30*d^4*b/c^5/x^5-1/5*d^4*b
/c^4/x^4-5/9*d^4*b/c^3/x^3-16/15*d^4*b/c^2/x^2-13/6*d^4*b/c/x+32/15*d^4*b*l
n(c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(133) = 266.
time = 0.27, size = 329, normalized size = 2.18

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}\left(cx\right)}{x^2}\right)bc^4d^4-\frac{2}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\operatorname{artanh}\left(cx\right)}{x^3}\right)bc^3d^4+\frac{1}{4}\left(\left(3c^3\log\left(cx+1\right)-3c^3\log\left(cx-1\right)-\frac{2\left(3c^2x^2+1\right)}{x^3}\right)c-\frac{6\operatorname{artanh}\left(cx\right)}{x^4}\right)bc^2d^4-\frac{1}{5}\left(\left(2c^4\log\left(c^2x^2-1\right)-2c^4\log\left(x^2\right)+\frac{2c^2x^2+1}{x^4}\right)c+\frac{4\operatorname{artanh}\left(cx\right)}{x^5}\right)bcd^4-\frac{ac^4d^4}{2x^2}+\frac{1}{180}\left(\left(15c^5\log\left(cx+1\right)-15c^5\log\left(cx-1\right)-\frac{2\left(15c^4x^4+5c^2x^2+3\right)}{x^5}\right)c-\frac{30\operatorname{artanh}\left(cx\right)}{x^6}\right)bd^4-\frac{4ac^3d^4}{3x^3}-\frac{3ac^2d^4}{2x^4}-\frac{4acd^4}{5x^5}-\frac{ad^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^4*
d^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c^3*d^4 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*
x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^4 - 1/5*((2*c^4*log(c^2*x^2 -
 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^4
 - 1/2*a*c^4*d^4/x^2 + 1/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) -
2*(15*c^4*x^4 + 5*c^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^4 - 4/3*a*
c^3*d^4/x^3 - 3/2*a*c^2*d^4/x^4 - 4/5*a*c*d^4/x^5 - 1/6*a*d^4/x^6
```

Fricas [A]
time = 0.36, size = 208, normalized size = 1.38

$$\frac{3bc^6d^4x^6\log\left(cx+1\right)-387bc^6d^4x^6\log\left(cx-1\right)+384bc^6d^4x^6\log\left(x\right)-390bc^5d^4x^5-6\left(15a+32b\right)c^4d^4x^4-20\left(12a+5b\right)c^3d^4x^3-18\left(15a+2b\right)c^2d^4x^2-6\left(24a+b\right)cd^4x-30ad^4-3\left(15bc^4d^4x^4+40bc^3d^4x^3+45bc^2d^4x^2+24bcd^4x+5bd^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{180x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

[Out] 1/180*(3*b*c^6*d^4*x^6*log(c*x + 1) - 387*b*c^6*d^4*x^6*log(c*x - 1) + 384*
b*c^6*d^4*x^6*log(x) - 390*b*c^5*d^4*x^5 - 6*(15*a + 32*b)*c^4*d^4*x^4 - 20
*(12*a + 5*b)*c^3*d^4*x^3 - 18*(15*a + 2*b)*c^2*d^4*x^2 - 6*(24*a + b)*c*d^
4*x - 30*a*d^4 - 3*(15*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 45*b*c^2*d^4*x^2
+ 24*b*c*d^4*x + 5*b*d^4)*log(-(c*x + 1)/(c*x - 1)))/x^6

Sympy [A]
time = 0.82, size = 291, normalized size = 1.93

$$\begin{cases} - \frac{a c^{4} d^{4}}{2 x^{2}} - \frac{4 a c^{3} d^{4}}{3 x^{3}} - \frac{3 a c^{2} d^{4}}{2 x^{4}} - \frac{4 a c d^{4}}{5 x^{5}} - \frac{a d^{4}}{6 x^{6}} + \frac{32 b c^{6} d^{4} \log{\left(x \right)}}{15} - \frac{32 b c^{6} d^{4} \log{\left(x - \frac{1}{c} \right)}}{15} + \frac{b c^{6} d^{4} \operatorname{atanh}{\left(c x \right)}}{30} - \frac{13 b c^{5} d^{4}}{6 x} - \frac{b c^{4} d^{4} \operatorname{atanh}{\left(c x \right)}}{2 x^{2}} - \frac{16 b c^{4} d^{4}}{15 x^{2}} - \frac{4 b c^{3} d^{4} \operatorname{atanh}{\left(c x \right)}}{3 x^{3}} - \frac{5 b c^{3} d^{4}}{9 x^{3}} - \frac{3 b c^{2} d^{4} \operatorname{atanh}{\left(c x \right)}}{2 x^{4}} - \frac{b c^{2} d^{4}}{5 x^{4}} - \frac{4 b c d^{4} \operatorname{atanh}{\left(c x \right)}}{5 x^{5}} - \frac{b c d^{4}}{30 x^{5}} - \frac{b d^{4} \operatorname{atanh}{\left(c x \right)}}{6 x^{6}} & \text{for}\: c \neq 0 \\- \frac{a d^{4}}{6 x^{6}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**7,x)

[Out] Piecewise((-a*c**4*d**4/(2*x**2) - 4*a*c**3*d**4/(3*x**3) - 3*a*c**2*d**4/(
2*x**4) - 4*a*c*d**4/(5*x**5) - a*d**4/(6*x**6) + 32*b*c**6*d**4*log(x)/15
- 32*b*c**6*d**4*log(x - 1/c)/15 + b*c**6*d**4*atanh(c*x)/30 - 13*b*c**5*d*
*4/(6*x) - b*c**4*d**4*atanh(c*x)/(2*x**2) - 16*b*c**4*d**4/(15*x**2) - 4*b
*c**3*d**4*atanh(c*x)/(3*x**3) - 5*b*c**3*d**4/(9*x**3) - 3*b*c**2*d**4*ata
nh(c*x)/(2*x**4) - b*c**2*d**4/(5*x**4) - 4*b*c*d**4*atanh(c*x)/(5*x**5) -
b*c*d**4/(30*x**5) - b*d**4*atanh(c*x)/(6*x**6), Ne(c, 0)), (-a*d**4/(6*x**
6), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(133) = 266.
time = 0.42, size = 634, normalized size = 4.20

$$\frac{8}{45} \left(12 \, b c^{5} d^{4} \log\left(-\frac{c x + 1}{c x - 1} - 1\right) - 12 \, b c^{5} d^{4} \log\left(-\frac{c x + 1}{c x - 1}\right) + \frac{6 \left(\frac{15 \, {\left(c x + 1\right)}^{5} b c^{5} d^{4}}{{\left(c x - 1\right)}^{5}} + \frac{30 \, {\left(c x + 1\right)}^{4} b c^{5} d^{4}}{{\left(c x - 1\right)}^{4}} + \frac{40 \, {\left(c x + 1\right)}^{3} b c^{5} d^{4}}{{\left(c x - 1\right)}^{3}} + \frac{30 \, {\left(c x + 1\right)}^{2} b c^{5} d^{4}}{{\left(c x - 1\right)}^{2}} + \frac{12 \, {\left(c x + 1\right)} b c^{5} d^{4}}{c x - 1} + 2 \, b c^{5} d^{4}\right)} \log\left(-\frac{c x + 1}{c x - 1}\right)}{\frac{{\left(c x + 1\right)}^{6}}{{\left(c x - 1\right)}^{6}} + \frac{6 \, {\left(c x + 1\right)}^{5}}{{\left(c x - 1\right)}^{5}} + \frac{15 \, {\left(c x + 1\right)}^{4}}{{\left(c x - 1\right)}^{4}} + \frac{20 \, {\left(c x + 1\right)}^{3}}{{\left(c x - 1\right)}^{3}} + \frac{15 \, {\left(c x + 1\right)}^{2}}{{\left(c x - 1\right)}^{2}} + \frac{6 \, {\left(c x + 1\right)}}{c x - 1} + 1} + \frac{\frac{180 \, {\left(c x + 1\right)}^{5} a c^{5} d^{4}}{{\left(c x - 1\right)}^{5}} + \frac{360 \, {\left(c x + 1\right)}^{4} a c^{5} d^{4}}{{\left(c x - 1\right)}^{4}} + \frac{480 \, {\left(c x + 1\right)}^{3} a c^{5} d^{4}}{{\left(c x - 1\right)}^{3}} + \frac{360 \, {\left(c x + 1\right)}^{2} a c^{5} d^{4}}{{\left(c x - 1\right)}^{2}} + \frac{144 \, {\left(c x + 1\right)} a c^{5} d^{4}}{c x - 1} + 24 \, a c^{5} d^{4} + \frac{78 \, {\left(c x + 1\right)}^{5} b c^{5} d^{4}}{{\left(c x - 1\right)}^{5}} + \frac{294 \, {\left(c x + 1\right)}^{4} b c^{5} d^{4}}{{\left(c x - 1\right)}^{4}} + \frac{472 \, {\left(c x + 1\right)}^{3} b c^{5} d^{4}}{{\left(c x - 1\right)}^{3}} + \frac{399 \, {\left(c x + 1\right)}^{2} b c^{5} d^{4}}{{\left(c x - 1\right)}^{2}} + \frac{174 \, {\left(c x + 1\right)} b c^{5} d^{4}}{c x - 1} + 31 \, b c^{5} d^{4}}{\frac{{\left(c x + 1\right)}^{6}}{{\left(c x - 1\right)}^{6}} + \frac{6 \, {\left(c x + 1\right)}^{5}}{{\left(c x - 1\right)}^{5}} + \frac{15 \, {\left(c x + 1\right)}^{4}}{{\left(c x - 1\right)}^{4}} + \frac{20 \, {\left(c x + 1\right)}^{3}}{{\left(c x - 1\right)}^{3}} + \frac{15 \, {\left(c x + 1\right)}^{2}}{{\left(c x - 1\right)}^{2}} + \frac{6 \, {\left(c x + 1\right)}}{c x - 1} + 1}\right)} c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")

[Out] 8/45*(12*b*c^5*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 12*b*c^5*d^4*log(-(c*x +
 1)/(c*x - 1)) + 6*(15*(c*x + 1)^5*b*c^5*d^4/(c*x - 1)^5 + 30*(c*x + 1)^4*b
*c^5*d^4/(c*x - 1)^4 + 40*(c*x + 1)^3*b*c^5*d^4/(c*x - 1)^3 + 30*(c*x + 1)^
2*b*c^5*d^4/(c*x - 1)^2 + 12*(c*x + 1)*b*c^5*d^4/(c*x - 1) + 2*b*c^5*d^4)*l
og(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x - 1)
^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x + 1)
^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1) + (180*(c*x + 1)^5*a*c^5*d^4/(c
x - 1)^5 + 360(c*x + 1)^4*a*c^5*d^4/(c*x - 1)^4 + 480*(c*x + 1)^3*a*c^5*d
^4/(c*x - 1)^3 + 360*(c*x + 1)^2*a*c^5*d^4/(c*x - 1)^2 + 144*(c*x + 1)*a*c^
5*d^4/(c*x - 1) + 24*a*c^5*d^4 + 78*(c*x + 1)^5*b*c^5*d^4/(c*x - 1)^5 + 294
*(c*x + 1)^4*b*c^5*d^4/(c*x - 1)^4 + 472*(c*x + 1)^3*b*c^5*d^4/(c*x - 1)^3
+ 399*(c*x + 1)^2*b*c^5*d^4/(c*x - 1)^2 + 174*(c*x + 1)*b*c^5*d^4/(c*x - 1)
 + 31*b*c^5*d^4)/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x - 1)^5 + 15*

```
(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*x + 1)^2/(c*x
- 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B]
time = 1.24, size = 248, normalized size = 1.64

$$\frac{32\,b\,c^6\,d^4\,\ln\left(x\right)}{15}-\frac{16\,b\,c^6\,d^4\,\ln\left(c^2\,x^2-1\right)}{15}-\frac{3\,a\,c^2\,d^4}{2\,x^4}-\frac{4\,a\,c^3\,d^4}{3\,x^3}-\frac{a\,c^4\,d^4}{2\,x^2}-\frac{b\,c^2\,d^4}{5\,x^4}-\frac{5\,b\,c^3\,d^4}{9\,x^3}-\frac{16\,b\,c^4\,d^4}{15\,x^2}-\frac{13\,b\,c^5\,d^4}{6\,x}-\frac{a\,d^4}{6\,x^6}-\frac{4\,a\,c\,d^4}{5\,x^5}-\frac{b\,c\,d^4}{30\,x^5}-\frac{b\,d^4\,\mathrm{atanh}(c\,x)}{6\,x^6}-\frac{13\,b\,c^7\,d^4\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{6\,\sqrt{-c^2}}-\frac{4\,b\,c\,d^4\,\mathrm{atanh}(c\,x)}{5\,x^5}-\frac{3\,b\,c^2\,d^4\,\mathrm{atanh}(c\,x)}{2\,x^4}-\frac{4\,b\,c^3\,d^4\,\mathrm{atanh}(c\,x)}{3\,x^3}-\frac{b\,c^4\,d^4\,\mathrm{atanh}(c\,x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^7,x)
```

```
[Out] (32*b*c^6*d^4*log(x))/15 - (16*b*c^6*d^4*log(c^2*x^2 - 1))/15 - (3*a*c^2*d^
4)/(2*x^4) - (4*a*c^3*d^4)/(3*x^3) - (a*c^4*d^4)/(2*x^2) - (b*c^2*d^4)/(5*x
^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(6*x
) - (a*d^4)/(6*x^6) - (4*a*c*d^4)/(5*x^5) - (b*c*d^4)/(30*x^5) - (b*d^4*ata
nh(c*x))/(6*x^6) - (13*b*c^7*d^4*atan((c^2*x)/(-c^2)^(1/2)))/(6*(-c^2)^(1/2
)) - (4*b*c*d^4*atanh(c*x))/(5*x^5) - (3*b*c^2*d^4*atanh(c*x))/(2*x^4) - (4
*b*c^3*d^4*atanh(c*x))/(3*x^3) - (b*c^4*d^4*atanh(c*x))/(2*x^2)
```

3.42 $\int \frac{(d+cdx)^4(a+b\tanh^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=229

$$-\frac{bcd^4}{42x^6}-\frac{2bc^2d^4}{15x^5}-\frac{47bc^3d^4}{140x^4}-\frac{5bc^4d^4}{9x^3}-\frac{88bc^5d^4}{105x^2}-\frac{5bc^6d^4}{3x}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-6c^2$$

```
[Out] -1/42*b*c*d^4/x^6-2/15*b*c^2*d^4/x^5-47/140*b*c^3*d^4/x^4-5/9*b*c^4*d^4/x^3
-88/105*b*c^5*d^4/x^2-5/3*b*c^6*d^4/x-1/7*d^4*(a+b*arctanh(c*x))/x^7-2/3*c*
d^4*(a+b*arctanh(c*x))/x^6-6/5*c^2*d^4*(a+b*arctanh(c*x))/x^5-c^3*d^4*(a+b*
arctanh(c*x))/x^4-1/3*c^4*d^4*(a+b*arctanh(c*x))/x^3+176/105*b*c^7*d^4*ln(x
)-117/70*b*c^7*d^4*ln(-c*x+1)-1/210*b*c^7*d^4*ln(c*x+1)
```

Rubi [A]
time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {45, 6083, 12, 1816}

$$-\frac{c^4d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}+\frac{176}{105}bc^7d^4\log(x)-\frac{117}{70}bc^7d^4\log(1-cx)-\frac{1}{210}bc^7d^4\log(cx+1)-\frac{5bc^6d^4}{3x}-\frac{88bc^5d^4}{105x^2}-\frac{5bc^4d^4}{9x^3}-\frac{47bc^3d^4}{140x^4}-\frac{2bc^2d^4}{15x^5}-\frac{bcd^4}{42x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*d^4)/x^6 - (2*b*c^2*d^4)/(15*x^5) - (47*b*c^3*d^4)/(140*x^4) - (
5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/(105*x^2) - (5*b*c^6*d^4)/(3*x) - (d^
4*(a + b*ArcTanh[c*x]))/(7*x^7) - (2*c*d^4*(a + b*ArcTanh[c*x]))/(3*x^6) -
(6*c^2*d^4*(a + b*ArcTanh[c*x]))/(5*x^5) - (c^3*d^4*(a + b*ArcTanh[c*x]))/x
^4 - (c^4*d^4*(a + b*ArcTanh[c*x]))/(3*x^3) + (176*b*c^7*d^4*Log[x])/105 -
(117*b*c^7*d^4*Log[1 - c*x])/70 - (b*c^7*d^4*Log[1 + c*x])/210
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^8}\,dx &= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-\frac{6c^2d^4\left(a+b\tanh\right.}{5x^5} \\
&= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-\frac{6c^2d^4\left(a+b\tanh\right.}{5x^5} \\
&= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-\frac{6c^2d^4\left(a+b\tanh\right.}{5x^5} \\
&= -\frac{bcd^4}{42x^6}-\frac{2bc^2d^4}{15x^5}-\frac{47bc^3d^4}{140x^4}-\frac{5bc^4d^4}{9x^3}-\frac{88bc^5d^4}{105x^2}-\frac{5bc^6d^4}{3x}-\frac{d^4\left(a+b\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 175, normalized size = 0.76

$$-\frac{d^4\left(180a+840acx+30bcx+1512ac^2x^2+168bc^2x^2+1260ac^3x^3+423bc^3x^3+420ac^4x^4+700bc^4x^4+1056bc^5x^5+2100bc^6x^6+12b\left(15+70cx+126c^2x^2+105c^3x^3+35c^4x^4\right)\tanh^{-1}(cx)-2112bc^7x^7\log(x)+2106bc^7x^7\log(1-cx)+6bc^7x^7\log(1+cx)\right)}{1260x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

```
[Out] -1/1260*(d^4*(180*a + 840*a*c*x + 30*b*c*x + 1512*a*c^2*x^2 + 168*b*c^2*x^2
 + 1260*a*c^3*x^3 + 423*b*c^3*x^3 + 420*a*c^4*x^4 + 700*b*c^4*x^4 + 1056*b*
c^5*x^5 + 2100*b*c^6*x^6 + 12*b*(15 + 70*c*x + 126*c^2*x^2 + 105*c^3*x^3 +
35*c^4*x^4)*ArcTanh[c*x] - 2112*b*c^7*x^7*Log[x] + 2106*b*c^7*x^7*Log[1 - c
*x] + 6*b*c^7*x^7*Log[1 + c*x]))/x^7
```

Maple [A]
time = 0.17, size = 238, normalized size = 1.04

method	result
derivativedivides	$c^7\Big(d^4a\big(-\frac{1}{c^4x^4}-\frac{2}{3c^6x^6}-\frac{6}{5c^5x^5}-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}\big)-\frac{d^4b\,\mathrm{arctanh}(cx)}{c^4x^4}-\frac{2d^4b\,\mathrm{arctanh}(cx)}{3c^6x^6}-\frac{6d^4b\,\mathrm{arctanh}}{5c^5x^5}$

default	$c^7\left(d^4a\left(-\frac{1}{c^4x^4}-\frac{2}{3c^6x^6}-\frac{6}{5c^5x^5}-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}\right)-\frac{d^4b\operatorname{arctanh}(cx)}{c^4x^4}-\frac{2d^4b\operatorname{arctanh}(cx)}{3c^6x^6}-\frac{6d^4b\operatorname{arcta}}{5c^5x}\right.$
risch	$-\frac{d^4b(35c^4x^4+105x^3c^3+126c^2x^2+70cx+15)\ln(cx+1)}{210x^7}+\frac{d^4(2112c^7b\ln(-x)x^7-6b\,c^7\ln(cx+1)x^7-2106b\,c^7x^7\ln(-c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(d^4*a*(-1/c^4/x^4-2/3/c^6/x^6-6/5/c^5/x^5-1/3/c^3/x^3-1/7/c^7/x^7)-d^4
*b*arctanh(c*x)/c^4/x^4-2/3*d^4*b*arctanh(c*x)/c^6/x^6-6/5*d^4*b*arctanh(c*
x)/c^5/x^5-1/3*d^4*b*arctanh(c*x)/c^3/x^3-1/7*d^4*b*arctanh(c*x)/c^7/x^7-11
7/70*d^4*b*ln(c*x-1)-1/42*d^4*b/c^6/x^6-2/15*d^4*b/c^5/x^5-47/140*d^4*b/c^4
/x^4-5/9*d^4*b/c^3/x^3-88/105*d^4*b/c^2/x^2-5/3*d^4*b/c/x+176/105*d^4*b*ln(
c*x)-1/210*d^4*b*ln(c*x+1))
```

Maxima [A]
time = 0.26, size = 353, normalized size = 1.54

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\operatorname{artanh}(cx)}{x^3}\right)bc^4d^4+\frac{1}{6}\left(\left(3\,c^3\log(cx+1)-3\,c^3\log(cx-1)-\frac{2\,(3\,c^2x^2+1)}{x^3}\right)c-\frac{6\,\operatorname{artanh}(cx)}{x^4}\right)bc^3d^4-\frac{3}{10}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\operatorname{artanh}(cx)}{x^5}\right)bc^2d^4+\frac{1}{45}\left(\left(15\,c^5\log(cx+1)-15\,c^5\log(cx-1)-\frac{2\,(15\,c^4x^4+5\,c^2x^2+3)}{x^5}\right)c-\frac{30\,\operatorname{artanh}(cx)}{x^6}\right)bcd^4-\frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2-1\right)-6\,c^6\log\left(x^2\right)+\frac{6\,c^4x^4+3\,c^2x^2+2}{x^6}\right)c+\frac{12\,\operatorname{artanh}(cx)}{x^7}\right)bd^4-\frac{ac^4d^4}{3\,x^3}-\frac{ac^3d^4}{x^4}-\frac{6\,ac^2d^4}{5\,x^5}-\frac{2\,acd^4}{3\,x^6}-\frac{ad^4}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^4*d^4 + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^3*d^4 - 3/10*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c^2*d^4 +
 1/45*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x
^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*c*d^4 - 1/84*((6*c^6*log(c^2*x^2 -
1) - 6*c^6*log(x^2) + (6*c^4*x^4 + 3*c^2*x^2 + 2)/x^6)*c + 12*arctanh(c*x)/
x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 - a*c^3*d^4/x^4 - 6/5*a*c^2*d^4/x^5 - 2/3*a*
c*d^4/x^6 - 1/7*a*d^4/x^7
```

Fricas [A]
time = 0.38, size = 218, normalized size = 0.95

$$-\frac{6\,bc^7d^4x^7\log(cx+1)+2106\,bc^7d^4x^7\log(cx-1)-2112\,bc^7d^4x^7\log(x)+2100\,bc^6d^4x^6+1056\,bc^5d^4x^5+140\,(3\,a+5\,b)c^4d^4x^4+9\,(140\,a+47\,b)c^3d^4x^3+168\,(9\,a+b)c^2d^4x^2+30\,(28\,a+b)cd^4x+180\,ad^4+6\,(35\,bc^4d^4x^4+105\,bc^3d^4x^3+126\,bc^2d^4x^2+70\,bcd^4x+15\,bd^4)\log\left(-\frac{cx+1}{cx-1}\right)}{1260\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/1260*(6*b*c^7*d^4*x^7*log(c*x + 1) + 2106*b*c^7*d^4*x^7*log(c*x - 1) - 2
112*b*c^7*d^4*x^7*log(x) + 2100*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^5 + 140*(3
*a + 5*b)*c^4*d^4*x^4 + 9*(140*a + 47*b)*c^3*d^4*x^3 + 168*(9*a + b)*c^2*d^
4*x^2 + 30*(28*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*b*c^4*d^4*x^4 + 105*b*c^3
```

```
*d^4*x^3 + 126*b*c^2*d^4*x^2 + 70*b*c*d^4*x + 15*b*d^4)*log(-(c*x + 1)/(c*x
 - 1)))/x^7
```

Sympy [A]
time = 1.16, size = 301, normalized size = 1.31

$$\begin{cases} -\frac{ac^4d^4}{3x^3} - \frac{ac^3d^4}{x^4} - \frac{6ac^2d^4}{5x^5} - \frac{2acd^4}{3x^6} - \frac{ad^4}{7x^7} + \frac{176bc^7d^4\log(x)}{105} - \frac{176bc^7d^4\log\left(x-\frac{1}{c}\right)}{105} - \frac{bc^7d^4\operatorname{atanh}(cx)}{105} - \frac{5bc^6d^4}{3x} - \frac{88bc^5d^4}{105x^2} - \frac{bc^4d^4\operatorname{atanh}(cx)}{3x^3} - \frac{5bc^4d^4}{9x^3} - \frac{bc^3d^4\operatorname{atanh}(cx)}{x^4} - \frac{47bc^3d^4}{140x^4} - \frac{6bc^2d^4\operatorname{atanh}(cx)}{5x^5} - \frac{2bc^2d^4}{15x^5} - \frac{2bcd^4\operatorname{atanh}(cx)}{3x^6} - \frac{bcd^4}{42x^6} - \frac{bd^4\operatorname{atanh}(cx)}{7x^7} & \text{for}\ c \neq 0 \\ -\frac{ad^4}{7x^7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*c**4*d**4/(3*x**3) - a*c**3*d**4/x**4 - 6*a*c**2*d**4/(5*x**5
) - 2*a*c*d**4/(3*x**6) - a*d**4/(7*x**7) + 176*b*c**7*d**4*log(x)/105 - 17
6*b*c**7*d**4*log(x - 1/c)/105 - b*c**7*d**4*atanh(c*x)/105 - 5*b*c**6*d**4
/(3*x) - 88*b*c**5*d**4/(105*x**2) - b*c**4*d**4*atanh(c*x)/(3*x**3) - 5*b*
c**4*d**4/(9*x**3) - b*c**3*d**4*atanh(c*x)/x**4 - 47*b*c**3*d**4/(140*x**4
) - 6*b*c**2*d**4*atanh(c*x)/(5*x**5) - 2*b*c**2*d**4/(15*x**5) - 2*b*c*d**
4*atanh(c*x)/(3*x**6) - b*c*d**4/(42*x**6) - b*d**4*atanh(c*x)/(7*x**7), Ne
(c, 0)), (-a*d**4/(7*x**7), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(203) = 406.
time = 0.44, size = 735, normalized size = 3.21

$$\frac{4}{315}\left(132bc^6d^4\log\left(-\frac{cx+1}{cx-1}-1\right) - 132bc^6d^4\log\left(-\frac{cx+1}{cx-1}\right) + \frac{12\left(\frac{105(cx+1)^6bc^6d^4}{(cx-1)^6} + \frac{210(cx+1)^5bc^6d^4}{(cx-1)^5} + \frac{385(cx+1)^4bc^6d^4}{(cx-1)^4} + \frac{385(cx+1)^3bc^6d^4}{(cx-1)^3} + \frac{231(cx+1)^2bc^6d^4}{(cx-1)^2} + \frac{77(cx+1)bc^6d^4}{cx-1} + 11bc^6d^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^7}{(cx-1)^7} + \frac{7(cx+1)^6}{(cx-1)^6} + \frac{21(cx+1)^5}{(cx-1)^5} + \frac{35(cx+1)^4}{(cx-1)^4} + \frac{35(cx+1)^3}{(cx-1)^3} + \frac{21(cx+1)^2}{(cx-1)^2} + \frac{7(cx+1)}{cx-1} + 1} + \frac{\frac{2520(cx+1)^6ac^6d^4}{(cx-1)^6} + \frac{5040(cx+1)^5ac^6d^4}{(cx-1)^5} + \frac{9240(cx+1)^4ac^6d^4}{(cx-1)^4} + \frac{9240(cx+1)^3ac^6d^4}{(cx-1)^3} + \frac{5544(cx+1)^2ac^6d^4}{(cx-1)^2} + \frac{1848(cx+1)ac^6d^4}{cx-1} + 264ac^6d^4 + \frac{1128(cx+1)^6bc^6d^4}{(cx-1)^6} + \frac{4812(cx+1)^5bc^6d^4}{(cx-1)^5} + \frac{9476(cx+1)^4bc^6d^4}{(cx-1)^4} + \frac{10631(cx+1)^3bc^6d^4}{(cx-1)^3} + \frac{6933(cx+1)^2bc^6d^4}{(cx-1)^2} + \frac{2465(cx+1)bc^6d^4}{cx-1} + 371bc^6d^4}{\frac{(cx+1)^7}{(cx-1)^7} + \frac{7(cx+1)^6}{(cx-1)^6} + \frac{21(cx+1)^5}{(cx-1)^5} + \frac{35(cx+1)^4}{(cx-1)^4} + \frac{35(cx+1)^3}{(cx-1)^3} + \frac{21(cx+1)^2}{(cx-1)^2} + \frac{7(cx+1)}{cx-1} + 1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 4/315*(132*b*c^6*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 132*b*c^6*d^4*log(-(c*
x + 1)/(c*x - 1)) + 12*(105*(c*x + 1)^6*b*c^6*d^4/(c*x - 1)^6 + 210*(c*x +
1)^5*b*c^6*d^4/(c*x - 1)^5 + 385*(c*x + 1)^4*b*c^6*d^4/(c*x - 1)^4 + 385*(c
*x + 1)^3*b*c^6*d^4/(c*x - 1)^3 + 231*(c*x + 1)^2*b*c^6*d^4/(c*x - 1)^2 + 7
7*(c*x + 1)*b*c^6*d^4/(c*x - 1) + 11*b*c^6*d^4)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^7/(c*x - 1)^7 + 7*(c*x + 1)^6/(c*x - 1)^6 + 21*(c*x + 1)^5/(c*x -
 1)^5 + 35*(c*x + 1)^4/(c*x - 1)^4 + 35*(c*x + 1)^3/(c*x - 1)^3 + 21*(c*x +
 1)^2/(c*x - 1)^2 + 7*(c*x + 1)/(c*x - 1) + 1) + (2520*(c*x + 1)^6*a*c^6*d^
4/(c*x - 1)^6 + 5040*(c*x + 1)^5*a*c^6*d^4/(c*x - 1)^5 + 9240*(c*x + 1)^4*a
*c^6*d^4/(c*x - 1)^4 + 9240*(c*x + 1)^3*a*c^6*d^4/(c*x - 1)^3 + 5544*(c*x +
 1)^2*a*c^6*d^4/(c*x - 1)^2 + 1848*(c*x + 1)*a*c^6*d^4/(c*x - 1) + 264*a*c^
6*d^4 + 1128*(c*x + 1)^6*b*c^6*d^4/(c*x - 1)^6 + 4812*(c*x + 1)^5*b*c^6*d^4
/(c*x - 1)^5 + 9476*(c*x + 1)^4*b*c^6*d^4/(c*x - 1)^4 + 10631*(c*x + 1)^3*b
*c^6*d^4/(c*x - 1)^3 + 6933*(c*x + 1)^2*b*c^6*d^4/(c*x - 1)^2 + 2465*(c*x +
 1)*b*c^6*d^4/(c*x - 1) + 371*b*c^6*d^4)/((c*x + 1)^7/(c*x - 1)^7 + 7*(c*x
+ 1)^6/(c*x - 1)^6 + 21*(c*x + 1)^5/(c*x - 1)^5 + 35*(c*x + 1)^4/(c*x - 1)^
```

```
4 + 35*(c*x + 1)^3/(c*x - 1)^3 + 21*(c*x + 1)^2/(c*x - 1)^2 + 7*(c*x + 1)/(
c*x - 1) + 1))*c
```

Mupad [B]
time = 1.21, size = 260, normalized size = 1.14

$$\frac{176\,b\,c^7\,d^4\,\ln\left(x\right)}{105}-\frac{88\,b\,c^7\,d^4\,\ln\left(c^2\,x^2-1\right)}{105}-\frac{6\,a\,c^2\,d^4}{5\,x^5}-\frac{a\,c^3\,d^4}{x^4}-\frac{a\,c^4\,d^4}{3\,x^3}-\frac{2\,b\,c^2\,d^4}{15\,x^5}-\frac{47\,b\,c^3\,d^4}{140\,x^4}-\frac{5\,b\,c^4\,d^4}{9\,x^3}-\frac{88\,b\,c^5\,d^4}{105\,x^2}-\frac{5\,b\,c^6\,d^4}{3\,x}-\frac{a\,d^4}{7\,x^7}-\frac{2\,a\,c\,d^4}{3\,x^6}-\frac{b\,c\,d^4}{42\,x^6}-\frac{b\,d^4\,\mathrm{atanh}(c\,x)}{7\,x^7}-\frac{5\,b\,c^8\,d^4\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{3\,\sqrt{-c^2}}-\frac{2\,b\,c\,d^4\,\mathrm{atanh}(c\,x)}{3\,x^6}-\frac{6\,b\,c^2\,d^4\,\mathrm{atanh}(c\,x)}{5\,x^5}-\frac{b\,c^3\,d^4\,\mathrm{atanh}(c\,x)}{x^4}-\frac{b\,c^4\,d^4\,\mathrm{atanh}(c\,x)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^8,x)
```

```
[Out] (176*b*c^7*d^4*log(x))/105 - (88*b*c^7*d^4*log(c^2*x^2 - 1))/105 - (6*a*c^2
*d^4)/(5*x^5) - (a*c^3*d^4)/x^4 - (a*c^4*d^4)/(3*x^3) - (2*b*c^2*d^4)/(15*x
^5) - (47*b*c^3*d^4)/(140*x^4) - (5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/(10
5*x^2) - (5*b*c^6*d^4)/(3*x) - (a*d^4)/(7*x^7) - (2*a*c*d^4)/(3*x^6) - (b*c
*d^4)/(42*x^6) - (b*d^4*atanh(c*x))/(7*x^7) - (5*b*c^8*d^4*atan((c^2*x)/(-c
^2)^(1/2)))/(3*(-c^2)^(1/2)) - (2*b*c*d^4*atanh(c*x))/(3*x^6) - (6*b*c^2*d^
4*atanh(c*x))/(5*x^5) - (b*c^3*d^4*atanh(c*x))/x^4 - (b*c^4*d^4*atanh(c*x))
/(3*x^3)
```

3.43 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{d+cdx}\,dx$

Optimal. Leaf size=177

$$\frac{ax}{c^3d}-\frac{bx}{2c^3d}+\frac{bx^2}{6c^2d}+\frac{b\tanh^{-1}(cx)}{2c^4d}+\frac{bx\tanh^{-1}(cx)}{c^3d}-\frac{x^2(a+b\tanh^{-1}(cx))}{2c^2d}+\frac{x^3(a+b\tanh^{-1}(cx))}{3cd}+\frac{(a+b\tanh$$

```
[Out] a*x/c^3/d-1/2*b*x/c^3/d+1/6*b*x^2/c^2/d+1/2*b*arctanh(c*x)/c^4/d+b*x*arctan
h(c*x)/c^3/d-1/2*x^2*(a+b*arctanh(c*x))/c^2/d+1/3*x^3*(a+b*arctanh(c*x))/c/
d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^4/d+2/3*b*ln(-c^2*x^2+1)/c^4/d-1/2*b*p
olylog(2,1-2/(c*x+1))/c^4/d
```

Rubi [A]
time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6077, 6037, 272, 45, 327, 212, 6021, 266, 6055, 2449, 2352}

$$\frac{\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{c^4d}-\frac{x^2(a+b\tanh^{-1}(cx))}{2c^2d}+\frac{x^3(a+b\tanh^{-1}(cx))}{3cd}+\frac{ax}{c^3d}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^4d}+\frac{b\tanh^{-1}(cx)}{2c^4d}-\frac{bx}{2c^3d}+\frac{bx\tanh^{-1}(cx)}{c^3d}+\frac{bx^2}{6c^2d}+\frac{2b\log\left(1-c^2x^2\right)}{3c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (a*x)/(c^3*d) - (b*x)/(2*c^3*d) + (b*x^2)/(6*c^2*d) + (b*ArcTanh[c*x])/(2*c
^4*d) + (b*x*ArcTanh[c*x])/(c^3*d) - (x^2*(a + b*ArcTanh[c*x]))/(2*c^2*d) +
 (x^3*(a + b*ArcTanh[c*x]))/(3*c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)
])/(c^4*d) + (2*b*Log[1 - c^2*x^2])/(3*c^4*d) - (b*PolyLog[2, 1 - 2/(1 + c*
x)])/(2*c^4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(a+b\tanh^{-1}(cx))}{d+cdx}\,dx = -\frac{\int \frac{x^2(a+b\tanh^{-1}(cx))}{d+cdx}\,dx}{c} + \frac{\int x^2(a+b\tanh^{-1}(cx))\,dx}{cd}$$

$$= \frac{x^3(a+b\tanh^{-1}(cx))}{3cd} + \frac{\int \frac{x(a+b\tanh^{-1}(cx))}{d+cdx}\,dx}{c^2} - \frac{b\int \frac{x^3}{1-c^2x^2}\,dx}{3d} - \frac{\int x(a+b\tanh}{c^2d}$$

$$= -\frac{x^2(a+b\tanh^{-1}(cx))}{2c^2d} + \frac{x^3(a+b\tanh^{-1}(cx))}{3cd} - \frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c^3} - \frac{b\text{Subst}}{}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} - \frac{x^2(a+b\tanh^{-1}(cx))}{2c^2d} + \frac{x^3(a+b\tanh^{-1}(cx))}{3cd} + \frac{(a+b\tanh^{-1}(}{c}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} + \frac{bx^2}{6c^2d} + \frac{b\tanh^{-1}(cx)}{2c^4d} + \frac{bx\tanh^{-1}(cx)}{c^3d} - \frac{x^2(a+b\tanh^{-1}(cx))}{2c^2d}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} + \frac{bx^2}{6c^2d} + \frac{b\tanh^{-1}(cx)}{2c^4d} + \frac{bx\tanh^{-1}(cx)}{c^3d} - \frac{x^2(a+b\tanh^{-1}(cx))}{2c^2d}$$

Mathematica [A]
time = 0.24, size = 129, normalized size = 0.73

$$\frac{-b+6acx-3bcx-3ac^2x^2+bc^2x^2+2ac^3x^3+b\tanh^{-1}(cx)\left(3+6cx-3c^2x^2+2c^3x^3+6\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)-6a\log(1+cx)+4b\log\left(1-c^2x^2\right)-3b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{6c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (-b + 6*a*c*x - 3*b*c*x - 3*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + b*ArcTanh
[c*x]*(3 + 6*c*x - 3*c^2*x^2 + 2*c^3*x^3 + 6*Log[1 + E^(-2*ArcTanh[c*x])])
- 6*a*Log[1 + c*x] + 4*b*Log[1 - c^2*x^2] - 3*b*PolyLog[2, -E^(-2*ArcTanh[c
*x])])/(6*c^4*d)
```

Maple [A]
time = 0.30, size = 224, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{a c^3 x^3}{3d}-\frac{a c^2 x^2}{2d}+\frac{acx}{d}-\frac{a\ln(cx+1)}{d}+\frac{b c^3 x^3 \operatorname{arctanh}(cx)}{3d}-\frac{b \operatorname{arctanh}(cx) c^2 x^2}{2d}+\frac{b \operatorname{arctanh}(cx) cx}{d}-\frac{b \operatorname{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\ln\left(-\frac{cx}{2}\right.}{}}{c^4}$
default	$\frac{\frac{a c^3 x^3}{3d}-\frac{a c^2 x^2}{2d}+\frac{acx}{d}-\frac{a\ln(cx+1)}{d}+\frac{b c^3 x^3 \operatorname{arctanh}(cx)}{3d}-\frac{b \operatorname{arctanh}(cx) c^2 x^2}{2d}+\frac{b \operatorname{arctanh}(cx) cx}{d}-\frac{b \operatorname{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\ln\left(-\frac{cx}{2}\right.}{}}{c^4}$
risch	$-\frac{b\ln(cx+1)^2}{4d\,c^4}+\frac{b\left(\frac{1}{3}c^2x^3-\frac{1}{2}c\,x^2+x\right)\ln(cx+1)}{2d\,c^3}-\frac{5a}{6d\,c^4}-\frac{31b}{72d\,c^4}+\frac{x^3a}{3dc}-\frac{x^2a}{2d\,c^2}+\frac{ax}{c^3d}+\frac{\ln(-cx+1)x^2b}{4d\,c^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/3*a/d*c^3*x^3-1/2*a/d*c^2*x^2+a/d*c*x-a/d*ln(c*x+1)+1/3*b/d*c^3*x^
3*arctanh(c*x)-1/2*b/d*arctanh(c*x)*c^2*x^2+b/d*arctanh(c*x)*c*x-b/d*arctan
h(c*x)*ln(c*x+1)-1/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d*ln(-1/2*c*x+1/2
)*ln(1/2*c*x+1/2)+1/2*b/d*dilog(1/2*c*x+1/2)+1/4*b/d*ln(c*x+1)^2+1/6*b/d*c^
2*x^2-1/2*b/d*c*x-2/3*b/d+5/12*b/d*ln(c*x-1)+11/12*b/d*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/72*(2*c^4*(2*(c^2*x^3 + 3*x)/(c^7*d) - 3*log(c*x + 1)/(c^8*d) + 3*log(c*x
 - 1)/(c^8*d)) + 216*c^4*integrate(1/6*x^4*log(c*x + 1)/(c^5*d*x^2 - c^3*d)
, x) - 3*c^3*(x^2/(c^5*d) + log(c^2*x^2 - 1)/(c^7*d)) - 216*c^3*integrate(1
/6*x^3*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x) + 9*c^2*(2*x/(c^5*d) - log(c*x
+ 1)/(c^6*d) + log(c*x - 1)/(c^6*d)) - 216*c*integrate(1/6*x*log(c*x + 1)/(
c^5*d*x^2 - c^3*d), x) - 6*(2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*log(c*x + 1))
*log(-c*x + 1)/(c^4*d) + 18*log(6*c^5*d*x^2 - 6*c^3*d)/(c^4*d) - 216*integr
ate(1/6*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x))*b + 1/6*a*((2*c^2*x^3 - 3*c*x
^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c*d*x + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{cx+1}\,dx + \int \frac{bx^3\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x**3/(c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(c*x + 1), x))/
d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{d+c\,d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

3.44 $\int \frac{x^2(a+b\tanh^{-1}(cx))}{d+cdx}\,dx$

Optimal. Leaf size=145

$$-\frac{ax}{c^2d}+\frac{bx}{2c^2d}-\frac{b\tanh^{-1}(cx)}{2c^3d}-\frac{bx\tanh^{-1}(cx)}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d}-\frac{b\log(1}{2}$$

```
[Out] -a*x/c^2/d+1/2*b*x/c^2/d-1/2*b*arctanh(c*x)/c^3/d-b*x*arctanh(c*x)/c^2/d+1/
2*x^2*(a+b*arctanh(c*x))/c/d-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^3/d-1/2*b*l
n(-c^2*x^2+1)/c^3/d+1/2*b*polylog(2,1-2/(c*x+1))/c^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6077, 6037, 327, 212, 6021, 266, 6055, 2449, 2352}

$$-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{ax}{c^2d}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^3d}-\frac{b\tanh^{-1}(cx)}{2c^3d}+\frac{bx}{2c^2d}-\frac{bx\tanh^{-1}(cx)}{c^2d}-\frac{b\log\left(1-c^2x^2\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] -((a*x)/(c^2*d)) + (b*x)/(2*c^2*d) - (b*ArcTanh[c*x])/(2*c^3*d) - (b*x*ArcT
anh[c*x])/(c^2*d) + (x^2*(a + b*ArcTanh[c*x]))/(2*c*d) - ((a + b*ArcTanh[c*
x])*Log[2/(1 + c*x)])/(c^3*d) - (b*Log[1 - c^2*x^2])/(2*c^3*d) + (b*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*c^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx = -\frac{\int \frac{x(a+b\tanh^{-1}(cx))}{d+cdx}\,dx}{c} + \frac{\int x\left(a+b\tanh^{-1}(cx)\right)\,dx}{cd}$$

$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd} + \frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c^2} - \frac{b\int \frac{x^2}{1-c^2x^2}\,dx}{2d} - \frac{\int \left(a+b\tanh^{-1}\right.}{c^2d}$$

$$= -\frac{ax}{c^2d} + \frac{bx}{2c^2d} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd} - \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d} - \frac{b\int}{}$$

$$= -\frac{ax}{c^2d} + \frac{bx}{2c^2d} - \frac{b\tanh^{-1}(cx)}{2c^3d} - \frac{bx\tanh^{-1}(cx)}{c^2d} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd} - \frac{(a}{}$$

$$= -\frac{ax}{c^2d} + \frac{bx}{2c^2d} - \frac{b\tanh^{-1}(cx)}{2c^3d} - \frac{bx\tanh^{-1}(cx)}{c^2d} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd} - \frac{(a}{}$$

Mathematica [A]

time = 0.15, size = 97, normalized size = 0.67

$$\frac{-2acx+bcx+ac^2x^2+b\tanh^{-1}(cx)\left(-1-2cx+c^2x^2-2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+2a\log(1+cx)-b\log\left(1-c^2x^2\right)+b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (-2*a*c*x + b*c*x + a*c^2*x^2 + b*ArcTanh[c*x]*(-1 - 2*c*x + c^2*x^2 - 2*Lo
g[1 + E^(-2*ArcTanh[c*x])]) + 2*a*Log[1 + c*x] - b*Log[1 - c^2*x^2] + b*Pol
yLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^3*d)
```

Maple [A]

time = 0.28, size = 184, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{a\,c^2x^2}{2d}-\frac{acx}{d}+\frac{a\ln(cx+1)}{d}+\frac{b\,\mathrm{arctanh}(cx)c^2x^2}{2d}-\frac{b\,\mathrm{arctanh}(cx)cx}{d}+\frac{b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right.}{2}}{c^3}$
default	$\frac{\frac{a\,c^2x^2}{2d}-\frac{acx}{d}+\frac{a\ln(cx+1)}{d}+\frac{b\,\mathrm{arctanh}(cx)c^2x^2}{2d}-\frac{b\,\mathrm{arctanh}(cx)cx}{d}+\frac{b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right.}{2}}{c^3}$
risch	$\frac{b\ln(cx+1)^2}{4d\,c^3}+\frac{b\left(\frac{1}{2}c\,x^2-x\right)\ln(cx+1)}{2d\,c^2}-\frac{\ln(-cx+1)x^2b}{4dc}+\frac{\ln(-cx+1)xb}{2d\,c^2}-\frac{b\ln(-cx+1)}{4d\,c^3}+\frac{bx}{2c^2d}+\frac{b}{8d\,c^3}+\frac{b\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/2*a/d*c^2*x^2-a/d*c*x+a/d*ln(c*x+1)+1/2*b/d*arctanh(c*x)*c^2*x^2-b
/d*arctanh(c*x)*c*x+b/d*arctanh(c*x)*ln(c*x+1)+1/2*b/d*ln(-1/2*c*x+1/2)*ln(
```

```
c*x+1)-1/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/2*b/d*dilog(1/2*c*x+1/2)-
1/4*b/d*ln(c*x+1)^2+1/2*b/d*c*x+1/2*b/d-1/4*b/d*ln(c*x-1)-3/4*b/d*ln(c*x+1)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c^3*(x^2/(c^4*d) + log(c^2*x^2 - 1)/(c^6*d)) + 8*c^3*integrate(1/2*x^3
*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x) - c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^
5*d) + log(c*x - 1)/(c^5*d)) - 8*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^4*d*
x^2 - c^2*d), x) + 8*c*integrate(1/2*x*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x)
 - 2*(c^2*x^2 - 2*c*x + 2*log(c*x + 1))*log(-c*x + 1)/(c^3*d) - 2*log(2*c^4
*d*x^2 - 2*c^2*d)/(c^3*d) + 8*integrate(1/2*log(c*x + 1)/(c^4*d*x^2 - c^2*d
), x))*b + 1/2*a*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c*d*x + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{cx+1}\,dx + \int \frac{bx^2 \operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x**2/(c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(c*x + 1), x))/
d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

3.45 $\int \frac{x(a+b\tanh^{-1}(cx))}{d+cdx}\,dx$

Optimal. Leaf size=94

$$\frac{ax}{cd}+\frac{bx\tanh^{-1}(cx)}{cd}+\frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{c^2d}+\frac{b\log(1-c^2x^2)}{2c^2d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2d}$$

```
[Out] a*x/c/d+b*x*arctanh(c*x)/c/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^2/d+1/2*b*l
n(-c^2*x^2+1)/c^2/d-1/2*b*polylog(2,1-2/(c*x+1))/c^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6077, 6021, 266, 6055, 2449, 2352}

$$\frac{\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{c^2d}+\frac{ax}{cd}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{b\log(1-c^2x^2)}{2c^2d}+\frac{bx\tanh^{-1}(cx)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (a*x)/(c*d) + (b*x*ArcTanh[c*x])/(c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c
*x)])/(c^2*d) + (b*Log[1 - c^2*x^2])/(2*c^2*d) - (b*PolyLog[2, 1 - 2/(1 + c
*x)])/(2*c^2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx = -\frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c}+\frac{\int \left(a+b\tanh^{-1}(cx)\right)\,dx}{cd}$$

$$= \frac{ax}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d}+\frac{b\int \tanh^{-1}(cx)\,dx}{cd}-\frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{cd}$$

$$= \frac{ax}{cd}+\frac{bx\tanh^{-1}(cx)}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d}-\frac{b\int \frac{x}{1-c^2x^2}\,dx}{d}-\frac{b\mathrm{Subs}}{}$$

$$= \frac{ax}{cd}+\frac{bx\tanh^{-1}(cx)}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d}+\frac{b\log\left(1-c^2x^2\right)}{2c^2d}-\frac{b\mathrm{Li}}{}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.80

$$\frac{2acx+2b\tanh^{-1}(cx)\left(cx+\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)-2a\log(1+cx)+b\log\left(1-c^2x^2\right)-b\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (2*a*c*x + 2*b*ArcTanh[c*x]*(c*x + Log[1 + E^(-2*ArcTanh[c*x])]) - 2*a*Log[
1 + c*x] + b*Log[1 - c^2*x^2] - b*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^2*
d)
```

Maple [A]
time = 0.28, size = 136, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{acx}{d}-\frac{a\ln(cx+1)}{d}-\frac{b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d}+\frac{b\,\mathrm{arctanh}(cx)cx}{d}+\frac{b\ln(cx+1)^2}{4d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}+}{c^2}$
default	$\frac{\frac{acx}{d}-\frac{a\ln(cx+1)}{d}-\frac{b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d}+\frac{b\,\mathrm{arctanh}(cx)cx}{d}+\frac{b\ln(cx+1)^2}{4d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}+}{c^2}$
risch	$-\frac{b\ln(cx+1)^2}{4d\,c^2}+\frac{bx\ln(cx+1)}{2dc}+\frac{b\ln(cx+1)}{2d\,c^2}-\frac{b\ln(-cx+1)x}{2dc}-\frac{b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d\,c^2}+\frac{b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{2d\,c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a/d*c*x-a/d*ln(c*x+1)-b/d*arctanh(c*x)*ln(c*x+1)+b/d*arctanh(c*x)*c*
x+1/4*b/d*ln(c*x+1)^2-1/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d*ln(-1/2*c*
x+1/2)*ln(1/2*c*x+1/2)+1/2*b/d*dilog(1/2*c*x+1/2)+1/2*b/d*ln((c*x+1)*(c*x-1
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*x/(c^3*d) - log(c*x + 1)/(c^4*d) + log(c*x - 1)/(c^4*d)) + 2*c^
2*integrate(x^2*log(c*x + 1)/(c^3*d*x^2 - c*d), x) - 4*c*integrate(x*log(c*
x + 1)/(c^3*d*x^2 - c*d), x) - 2*(c*x - log(c*x + 1))*log(-c*x + 1)/(c^2*d)
 + log(c^3*d*x^2 - c*d)/(c^2*d) - 2*integrate(log(c*x + 1)/(c^3*d*x^2 - c*d
), x))*b + a*(x/(c*d) - log(c*x + 1)/(c^2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(c*d*x + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{cx+1}\,dx+\int \frac{bx\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x/(c*x + 1), x) + Integral(b*x*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{atanh}(c\,x))}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

```
[Out] int((x*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

3.46 $\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx$

Optimal. Leaf size=51

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2cd}$$

```
[Out] -(a+b*arctanh(c*x))*ln(2/(c*x+1))/c/d+1/2*b*polylog(2,1-2/(c*x+1))/c/d
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6055, 2449, 2352}

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c*d)) + (b*PolyLog[2, 1 - 2/(1 +
 c*x)])/(2*c*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\int\frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+cx}\right)}{cd}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2cd}$$

Mathematica [A]
time = 0.07, size = 52, normalized size = 1.02

$$\frac{-2b\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+2a\log(1+cx)+b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

```
[Out] (-2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a*Log[1 + c*x] + b*Poly
Log[2, -E^(-2*ArcTanh[c*x])])/(2*c*d)
```

Maple [A]
time = 0.31, size = 98, normalized size = 1.92

method	result	siz
risch	$\frac{b\ln(cx+1)^2}{4dc}-\frac{\ln(-cx+1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b}{2dc}+\frac{\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b}{2dc}+\frac{a\ln(-cx-1)}{dc}+\frac{\text{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)b}{2dc}$	96
derivativedivides	$\frac{\frac{a\ln(cx+1)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\ln(cx+1)^2}{4d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}-\frac{b\,\text{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}}{c}$	98
default	$\frac{\frac{a\ln(cx+1)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\ln(cx+1)^2}{4d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}-\frac{b\,\text{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}}{c}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/d*ln(c*x+1)+b/d*arctanh(c*x)*ln(c*x+1)-1/4*b/d*ln(c*x+1)^2+1/2*b/d*l
n(-1/2*c*x+1/2)*ln(c*x+1)-1/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/2*b/d*
dilog(1/2*c*x+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*integrate(x*log(c*x + 1)/(c^2*d*x^2 - d), x) - log(c*x + 1)*log(-c
*x + 1)/(c*d))*b + a*log(c*d*x + d)/(c*d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx+1}\, dx + \int \frac{b\,\operatorname{atanh}(cx)}{cx+1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x + 1), x) + Integral(b*atanh(c*x)/(c*x + 1), x))/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + c*d*x),x)
```

```
[Out] int((a + b*atanh(c*x))/(d + c*d*x), x)
```

3.47 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx$

Optimal. Leaf size=46

$$\frac{(a+b\tanh^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{2d}$$

```
[Out] (a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d-1/2*b*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6079, 2497}

$$\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d}-\frac{b\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

```
[Out] ((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*PolyLog[2, -1 + 2/(1 + c
*x)])/(2*d)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx &= \frac{(a+b\tanh^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{(bc)\int \frac{\log\left(2-\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}\\ &= \frac{(a+b\tanh^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 1.20

$$\frac{2b\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+2a\log(x)-2a\log(1+cx)-b\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

```
[Out] (2*b*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + 2*a*Log[x] - 2*a*Log[1 + c
*x] - b*PolyLog[2, E^(-2*ArcTanh[c*x])])/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(44) = 88$.
time = 0.17, size = 156, normalized size = 3.39

method	result
risch	$-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}+\frac{b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{2d}+\frac{a\ln(-cx)}{d}-\frac{a\ln(-cx-1)}{d}-\frac{b\,\text{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d}+\frac{\text{dilog}(-c}{2d}$
derivativedivides	$\frac{a\ln(cx)}{d}-\frac{a\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx)}{d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}-\frac{b\ln\left(-\frac{cx}{2}+\right.}{2}$
default	$\frac{a\ln(cx)}{d}-\frac{a\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx)}{d}+\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{2d}-\frac{b\ln\left(-\frac{cx}{2}+\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*ln(c*x)-a/d*ln(c*x+1)-b/d*arctanh(c*x)*ln(c*x+1)+b/d*arctanh(c*x)*ln(c*
x)+1/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x
+1)+1/2*b/d*dilog(1/2*c*x+1/2)+1/4*b/d*ln(c*x+1)^2-1/2*b/d*dilog(c*x)-1/2*b
/d*dilog(c*x+1)-1/2*b/d*ln(c*x)*ln(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(c*x + 1)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x +
 1))/(c*d*x^2 + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^2 + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^2+x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{cx^2+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d),x)
[Out] (Integral(a/(c*x**2 + x), x) + Integral(b*atanh(c*x)/(c*x**2 + x), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x\,\left(d+c\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + c*d*x)),x)
[Out] int((a + b*atanh(c*x))/(x*(d + c*d*x)), x)
```

3.48 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx$

Optimal. Leaf size=93

$$-\frac{a+b\tanh^{-1}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log(1-c^2x^2)}{2d}-\frac{c(a+b\tanh^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\text{PolyLog}\left(2,-1+\frac{}{1-}\right.}{2d}$$

```
[Out] (-a-b*arctanh(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(-c^2*x^2+1)/d-c*(a+b*arctanh
(c*x))*ln(2-2/(c*x+1))/d+1/2*b*c*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6081, 6037, 272, 36, 29, 31, 6079, 2497}

$$-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d}-\frac{bc\log(1-c^2x^2)}{2d}+\frac{bc\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 - c^2*x^2])/(2*
d) - (c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c*PolyLog[2, -1 +
 2/(1 + c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx\right)+\frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{(bc)\int \frac{1}{x(1-c^2x^2)}\,dx}{d}+$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}+$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}+$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\right.}{d}$$

Mathematica [A]
time = 0.10, size = 93, normalized size = 1.00

$$\frac{-2\left(a+b\tanh^{-1}(cx)\left(1+cx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+acx\log(x)-acx\log(1+cx)-bcx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)\right)+bcx\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

```
[Out] (-2*(a + b*ArcTanh[c*x]*(1 + c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + a*c*x*Log[
x] - a*c*x*Log[1 + c*x] - b*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + b*c*x*PolyL
og[2, E^(-2*ArcTanh[c*x])])/(2*d*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(91) = 182.
time = 0.21, size = 219, normalized size = 2.35

method	result
risch	$\frac{cb\ln(-cx)}{2d}-\frac{cb\ln(-cx+1)}{2d}+\frac{b\ln(-cx+1)}{2dx}-\frac{c\,\text{dilog}(-cx+1)b}{2d}-\frac{cb\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{2d}+\frac{cb\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}\right.}{2d}$
derivativedivides	$c\left(\frac{a\ln(cx+1)}{d}-\frac{a}{dcx}-\frac{a\ln(cx)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)}{dcx}-\frac{b\,\text{arctanh}(cx)\ln(cx)}{d}+\frac{b\ln(cx)}{d}-\right.$
default	$c\left(\frac{a\ln(cx+1)}{d}-\frac{a}{dcx}-\frac{a\ln(cx)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)}{dcx}-\frac{b\,\text{arctanh}(cx)\ln(cx)}{d}+\frac{b\ln(cx)}{d}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/d*ln(c*x+1)-a/d/c/x-a/d*ln(c*x)+b/d*arctanh(c*x)*ln(c*x+1)-b/d*arctanh
(c*x)/c/x-b/d*arctanh(c*x)*ln(c*x)+b/d*ln(c*x)-1/2*b/d*ln(c*x+1)-1/2*b/d*ln
(c*x-1)+1/2*b/d*dilog(c*x+1)+1/2*b/d*ln(c*x)*ln(c*x+1)+1/2*b/d*dilog(c*x)-1
/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-
1/2*b/d*dilog(1/2*c*x+1/2)-1/4*b/d*ln(c*x+1)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/2*b*integrate((log(c*x + 1)
 - log(-c*x + 1))/(c*d*x^3 + d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^3 + d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^3+x^2}\,dx + \int \frac{b\operatorname{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d),x)
[Out] (Integral(a/(c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c*x**3 + x**2), x)
)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{atanh}(c\,x)}{x^2\,\left(d+c\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)),x)
[Out] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)), x)
```

3.49 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx$

Optimal. Leaf size=146

$$-\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{bc^2\log(x)}{d}+\frac{bc^2\log\left(1-c^2x^2\right)}{2d}+\frac{c^2\left(a+b\tan\right.}{}$$

```
[Out] -1/2*b*c/d/x+1/2*b*c^2*arctanh(c*x)/d+1/2*(-a-b*arctanh(c*x))/d/x^2+c*(a+b*
arctanh(c*x))/d/x-b*c^2*ln(x)/d+1/2*b*c^2*ln(-c^2*x^2+1)/d+c^2*(a+b*arctanh
(c*x))*ln(2-2/(c*x+1))/d-1/2*b*c^2*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6081, 6037, 331, 212, 272, 36, 29, 31, 6079, 2497}

$$\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{bc^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}+\frac{bc^2\log\left(1-c^2x^2\right)}{2d}-\frac{bc^2\log(x)}{d}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{bc}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

```
[Out] -1/2*(b*c)/(d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x
^2) + (c*(a + b*ArcTanh[c*x]))/(d*x) - (b*c^2*Log[x])/d + (b*c^2*Log[1 - c^
2*x^2])/(2*d) + (c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*c^2*
PolyLog[2, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx\right)+\frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2dx^2}+c^2\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx-\frac{c\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d}+\frac{(bc)\int \frac{}{x^2(1\cdots}}{2d\cdots}$$

$$= -\frac{bc}{2dx}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\cdots\right.}{d}$$

$$= -\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{c^2\left(a+b\tan\cdots\right.}{}$$

$$= -\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{c^2\left(a+b\tan\cdots\right.}{}$$

$$= -\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{bc^2\log(x)}{d}+\cdots$$

Mathematica [A]
time = 0.20, size = 133, normalized size = 0.91

$$-\frac{a-2acx+bcx-b\tanh^{-1}(cx)\left(-1+2cx+c^2x^2+2c^2x^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-2ac^2x^2\log(x)+2ac^2x^2\log(1+cx)+2bc^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+bc^2x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

```
[Out] -1/2*(a - 2*a*c*x + b*c*x - b*ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^
2*Log[1 - E^(-2*ArcTanh[c*x])]) - 2*a*c^2*x^2*Log[x] + 2*a*c^2*x^2*Log[1 +
c*x] + 2*b*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + b*c^2*x^2*PolyLog[2, E^(-
2*ArcTanh[c*x])])/(d*x^2)
```

Maple [A]
time = 0.22, size = 260, normalized size = 1.78

method	result
derivativedivides	$c^2\left(-\frac{a\ln(cx+1)}{d}-\frac{a}{2d\,c^2x^2}+\frac{a\ln(cx)}{d}+\frac{a}{dcx}-\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)}{2d\,c^2x^2}+\frac{b\,\text{arctanh}(cx)\ln(cx)}{d}\cdots\right.$
default	$c^2\left(-\frac{a\ln(cx+1)}{d}-\frac{a}{2d\,c^2x^2}+\frac{a\ln(cx)}{d}+\frac{a}{dcx}-\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)}{2d\,c^2x^2}+\frac{b\,\text{arctanh}(cx)\ln(cx)}{d}\cdots\right.$
risch	$-\frac{c^2b\ln(-cx)}{4d}+\frac{c^2b\ln(-cx+1)}{4d}-\frac{cb\ln(-cx+1)}{2dx}-\frac{bc}{2dx}+\frac{b\ln(-cx+1)}{4d\,x^2}+\frac{c^2\,\text{dilog}(-cx+1)b}{2d}-\frac{c^2b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(\cdots}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a/d*ln(c*x+1)-1/2*a/d/c^2/x^2+a/d*ln(c*x)+a/d/c/x-b/d*arctanh(c*x)*ln
(c*x+1)-1/2*b/d*arctanh(c*x)/c^2/x^2+b/d*arctanh(c*x)*ln(c*x)+b/d*arctanh(c
*x)/c/x-1/2*b/d*dilog(c*x)-1/2*b/d*dilog(c*x+1)-1/2*b/d*ln(c*x)*ln(c*x+1)-1
/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+
1/2*b/d*dilog(1/2*c*x+1/2)+1/4*b/d*ln(c*x+1)^2+3/4*b/d*ln(c*x+1)+1/4*b/d*ln
(c*x-1)-1/2*b/d/c/x-b/d*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a + 1/2*
b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^4 + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^4 + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^4+x^3}\,dx + \int \frac{b\operatorname{atanh}(cx)}{cx^4+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c*x**4 + x**3), x)
)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^3), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^3\,\left(d+c\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)), x)
```

3.50 $\int \frac{a+b\tanh^{-1}(cx)}{x^4(d+cdx)}\,dx$

Optimal. Leaf size=185

$$-\frac{bc}{6dx^2}+\frac{bc^2}{2dx}-\frac{bc^3\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{4bc^3\log(x)}{3d}$$

```
[Out] -1/6*b*c/d/x^2+1/2*b*c^2/d/x-1/2*b*c^3*arctanh(c*x)/d+1/3*(-a-b*arctanh(c*x
))/d/x^3+1/2*c*(a+b*arctanh(c*x))/d/x^2-c^2*(a+b*arctanh(c*x))/d/x+4/3*b*c^
3*ln(x)/d-2/3*b*c^3*ln(-c^2*x^2+1)/d-c^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))
/d+1/2*b*c^3*polylog(2,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6081, 6037, 272, 46, 331, 212, 36, 29, 31, 6079, 2497}

$$-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}+\frac{bc^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{2d}+\frac{4bc^3\log(x)}{3d}-\frac{bc^3\tanh^{-1}(cx)}{2d}+\frac{bc^2}{2dx}-\frac{2bc^3\log\left(1-c^2x^2\right)}{3d}-\frac{bc}{6dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

```
[Out] -1/6*(b*c)/(d*x^2) + (b*c^2)/(2*d*x) - (b*c^3*ArcTanh[c*x])/(2*d) - (a + b*
ArcTanh[c*x])/(3*d*x^3) + (c*(a + b*ArcTanh[c*x]))/(2*d*x^2) - (c^2*(a + b*
ArcTanh[c*x]))/(d*x) + (4*b*c^3*Log[x])/(3*d) - (2*b*c^3*Log[1 - c^2*x^2])/
(3*d) - (c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c^3*PolyLog[
2, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^4(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx\right)+\frac{\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{3dx^3}+c^2\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx-\frac{c\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d}+\frac{(bc)\int \frac{}{x^3(}}{3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-c^3\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx+\frac{(bc)\text{Sub}}{}$$

$$= \frac{bc^2}{2dx}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^3(a+}{}$$

$$= -\frac{bc}{6dx^2}+\frac{bc^2}{2dx}-\frac{bc^3\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{c^2}{}$$

$$= -\frac{bc}{6dx^2}+\frac{bc^2}{2dx}-\frac{bc^3\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{c^2}{}$$

$$= -\frac{bc}{6dx^2}+\frac{bc^2}{2dx}-\frac{bc^3\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{c^2}{}$$

Mathematica [A]
time = 0.25, size = 172, normalized size = 0.93

$$\frac{-2a+3acx-bcx-6ac^2x^2+3bc^2x^2+bc^3x^3-b\tanh^{-1}(cx)\left(2-3cx+6c^2x^2+3c^3x^3+6c^3x^3\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-6ac^3x^3\log(x)+6ac^3x^3\log(1+cx)+8bc^3x^3\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+3bc^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

```
[Out] (-2*a + 3*a*c*x - b*c*x - 6*a*c^2*x^2 + 3*b*c^2*x^2 + b*c^3*x^3 - b*ArcTanh
[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[
c*x])]) - 6*a*c^3*x^3*Log[x] + 6*a*c^3*x^3*Log[1 + c*x] + 8*b*c^3*x^3*Log[(
c*x)/Sqrt[1 - c^2*x^2]] + 3*b*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])])/(6*d
*x^3)
```

Maple [A]
time = 0.34, size = 302, normalized size = 1.63

method	result
derivativedivides	$c^3\left(\frac{a\ln(cx+1)}{d}-\frac{a}{3d\,c^3x^3}-\frac{a}{dcx}+\frac{a}{2d\,c^2x^2}-\frac{a\ln(cx)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)}{3d\,c^3x^3}-\frac{b\,\text{arctanh}(}{dcx}\right.$
default	$c^3\left(\frac{a\ln(cx+1)}{d}-\frac{a}{3d\,c^3x^3}-\frac{a}{dcx}+\frac{a}{2d\,c^2x^2}-\frac{a\ln(cx)}{d}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d}-\frac{b\,\text{arctanh}(cx)}{3d\,c^3x^3}-\frac{b\,\text{arctanh}(}{dcx}\right.$
risch	$-\frac{cb\ln(-cx+1)}{4d\,x^2}+\frac{c^3b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d}-\frac{c^3b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{2d}-\frac{bc}{6d\,x^2}+\frac{b\,c^2}{2dx}+\frac{c^2b\ln(-cx+1)}{2dx}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^4/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/d*ln(c*x+1)-1/3*a/d/c^3/x^3-a/d/c/x+1/2*a/d/c^2/x^2-a/d*ln(c*x)+b/d*
arctanh(c*x)*ln(c*x+1)-1/3*b/d*arctanh(c*x)/c^3/x^3-b/d*arctanh(c*x)/c/x+1/
2*b/d*arctanh(c*x)/c^2/x^2-b/d*arctanh(c*x)*ln(c*x)-5/12*b/d*ln(c*x-1)-11/1
2*b/d*ln(c*x+1)-1/6*b/d/c^2/x^2+1/2*b/d/c/x+4/3*b/d*ln(c*x)+1/2*b/d*dilog(c
*x)+1/2*b/d*dilog(c*x+1)+1/2*b/d*ln(c*x)*ln(c*x+1)+1/2*b/d*ln(-1/2*c*x+1/2)
*ln(c*x+1)-1/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/2*b/d*dilog(1/2*c*x+1
/2)-1/4*b/d*ln(c*x+1)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^3
))*a + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^5 + d*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^5 + d*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{cx^5+x^4}\,dx+\int \frac{b\,\text{atanh}\,(cx)}{cx^5+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**4/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**5 + x**4), x) + Integral(b*atanh(c*x)/(c*x**5 + x**4), x)
)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^4\,\left(d+c\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^4*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^4*(d + c*d*x)), x)
```

3.51 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{(d+cdx)^2}\,dx$

Optimal. Leaf size=181

$$-\frac{2ax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{c^4d^2}-\frac{2bx\tanh^{-1}(cx)}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^4d^2(1+cx)}-3$$

```
[Out] -2*a*x/c^3/d^2+1/2*b*x/c^3/d^2+1/2*b/c^4/d^2/(c*x+1)-b*arctanh(c*x)/c^4/d^2
-2*b*x*arctanh(c*x)/c^3/d^2+1/2*x^2*(a+b*arctanh(c*x))/c^2/d^2+(a+b*arctanh
(c*x))/c^4/d^2/(c*x+1)-3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^4/d^2-b*ln(-c^2
*x^2+1)/c^4/d^2+3/2*b*polylog(2,1-2/(c*x+1))/c^4/d^2
```

Rubi [A]

time = 0.16, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6087, 6021, 266, 6037, 327, 212, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{a+b\tanh^{-1}(cx)}{c^4d^2(cx+1)}-\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}-\frac{2ax}{c^3d^2}+\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{b}{2c^4d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{c^4d^2}+\frac{bx}{2c^3d^2}-\frac{2bx\tanh^{-1}(cx)}{c^3d^2}-\frac{b\log\left(1-c^2x^2\right)}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (-2*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(1 + c*x)) - (b*ArcTa
nh[c*x])/(c^4*d^2) - (2*b*x*ArcTanh[c*x])/(c^3*d^2) + (x^2*(a + b*ArcTanh[c
*x]))/(2*c^2*d^2) + (a + b*ArcTanh[c*x])/(c^4*d^2*(1 + c*x)) - (3*(a + b*Ar
cTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^2) - (b*Log[1 - c^2*x^2])/(c^4*d^2) +
(3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
```

```
&& IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx = \int \left(-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2} + \frac{x\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2} - \frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)^2} + \frac{3(\cdots\right.$$

$$= -\frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^3d^2} - \frac{2\int \left(a+b\tanh^{-1}(cx)\right)\,dx}{c^3d^2} + \frac{3\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^3d^2} + \frac{\int x\cdots}{}$$

$$= -\frac{2ax}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2} + \frac{a+b\tanh^{-1}(cx)}{c^4d^2(1+cx)} - \frac{3\left(a+b\tanh^{-1}(cx)\right)\log\cdots}{c^4d^2}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2} + \frac{a+b\tanh^{-1}(cx\cdots}{c^4d^2(1+cx)}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{b\tanh^{-1}(cx)}{2c^4d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2} + \cdots$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(1+cx)} - \frac{b\tanh^{-1}(cx)}{2c^4d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2(a+\cdots}{}$$

$$= -\frac{2ax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(1+cx)} - \frac{b\tanh^{-1}(cx)}{c^4d^2} - \frac{2bx\tanh^{-1}(cx)}{c^3d^2} + \frac{x^2(a+\cdots}{}$$

Mathematica [A]

time = 0.44, size = 142, normalized size = 0.78

$$\frac{-8acx+2ac^2x^2+\frac{4a}{1+cx}+12a\log(1+cx)+b\left(2cx+\cosh\left(2\tanh^{-1}(cx)\right)-4\log\left(1-c^2x^2\right)+6\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2\tanh^{-1}(cx)\left(-1-4cx+c^2x^2+\cosh\left(2\tanh^{-1}(cx)\right)-6\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)\right)-\sinh\left(2\tanh^{-1}(cx)\right)\right)}{4c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (-8*a*c*x + 2*a*c^2*x^2 + (4*a)/(1 + c*x) + 12*a*Log[1 + c*x] + b*(2*c*x +
Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2*ArcTanh[c*x
])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x]] - 6*Log[1
 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4
*c^4*d^2)
```

Maple [A]

time = 0.33, size = 227, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{a\,c^2x^2}{2d^2}-\frac{2acx}{d^2}+\frac{3a\ln(cx+1)}{d^2}+\frac{a}{d^2(cx+1)}+\frac{b\,\text{arctanh}(cx)c^2x^2}{2d^2}-\frac{2b\,\text{arctanh}(cx)cx}{d^2}+\frac{3b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}+\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{3b\ln(\cdots}{}}{}$
default	$\frac{\frac{a\,c^2x^2}{2d^2}-\frac{2acx}{d^2}+\frac{3a\ln(cx+1)}{d^2}+\frac{a}{d^2(cx+1)}+\frac{b\,\text{arctanh}(cx)c^2x^2}{2d^2}-\frac{2b\,\text{arctanh}(cx)cx}{d^2}+\frac{3b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}+\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{3b\ln(\cdots}{}}{}$

risch	$\frac{3b\ln(cx+1)^2}{4c^4d^2}+\left(\frac{b(\frac{1}{2}cx^2-2x)}{2c^3d^2}+\frac{b}{2c^4d^2(cx+1)}\right)\ln(cx+1)+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(cx+1)}-\frac{5b\ln(cx+1)}{4c^4d^2}-\frac{2ax}{c^3d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a/d^2*c^2*x^2-2*a/d^2*c*x+3*a/d^2*ln(c*x+1)+a/d^2/(c*x+1)+1/2*b/
d^2*arctanh(c*x)*c^2*x^2-2*b/d^2*arctanh(c*x)*c*x+3*b/d^2*arctanh(c*x)*ln(c
*x+1)+b/d^2*arctanh(c*x)/(c*x+1)+3/2*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2*b
/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-3/2*b/d^2*dilog(1/2*c*x+1/2)-3/4*b/d^
2*ln(c*x+1)^2+1/2*b/d^2*c*x+1/2*b/d^2-1/2*b/d^2*ln(c*x-1)+1/2*b/d^2/(c*x+1)
-3/2*b/d^2*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c^4*(2/(c^9*d^2*x + c^8*d^2) + 2*(c*x^2 - 2*x)/(c^7*d^2) + 7*log(c*x
+ 1)/(c^8*d^2) + log(c*x - 1)/(c^8*d^2)) + 16*c^4*integrate(1/2*x^4*log(c*x
 + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c^3*(2/(c^8
*d^2*x + c^7*d^2) - 4*x/(c^6*d^2) + 5*log(c*x + 1)/(c^7*d^2) - log(c*x - 1)
/(c^7*d^2)) - 16*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*
x^2 - c^4*d^2*x - c^3*d^2), x) - 7*c^2*(2/(c^7*d^2*x + c^6*d^2) + 3*log(c*x
 + 1)/(c^6*d^2) + log(c*x - 1)/(c^6*d^2)) + 48*c^2*integrate(1/2*x^2*log(c*
x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c*(2/(c^6*
d^2*x + c^5*d^2) + log(c*x + 1)/(c^5*d^2) - log(c*x - 1)/(c^5*d^2)) + 96*c*
integrate(1/2*x*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d
^2), x) - 4*(c^3*x^3 - 3*c^2*x^2 - 4*c*x + 6*(c*x + 1)*log(c*x + 1) + 2)*lo
g(-c*x + 1)/(c^5*d^2*x + c^4*d^2) + 4/(c^5*d^2*x + c^4*d^2) - 2*log(c*x + 1
)/(c^4*d^2) + 2*log(c*x - 1)/(c^4*d^2) + 48*integrate(1/2*log(c*x + 1)/(c^6
*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x))*b + 1/2*a*(2/(c^5*d^2*x
+ c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)/(c^4*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^2x^2+2cx+1}\,dx + \int \frac{bx^3\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(
c**2*x**2 + 2*c*x + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

3.52 $\int \frac{x^2(a+b\tanh^{-1}(cx))}{(d+cdx)^2}\,dx$

Optimal. Leaf size=149

$$\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{b\tanh^{-1}(cx)}{2c^3d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\log}{}$$

```
[Out] a*x/c^2/d^2-1/2*b/c^3/d^2/(c*x+1)+1/2*b*arctanh(c*x)/c^3/d^2+b*x*arctanh(c*
x)/c^2/d^2+(-a-b*arctanh(c*x))/c^3/d^2/(c*x+1)+2*(a+b*arctanh(c*x))*ln(2/(c
*x+1))/c^3/d^2+1/2*b*ln(-c^2*x^2+1)/c^3/d^2-b*polylog(2,1-2/(c*x+1))/c^3/d^
2
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6021, 266, 6063, 641, 46, 213, 6055, 2449, 2352}

$$-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(cx+1)}+\frac{2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{ax}{c^2d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b}{2c^3d^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2c^3d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}+\frac{b\log\left(1-c^2x^2\right)}{2c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (a*x)/(c^2*d^2) - b/(2*c^3*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c^3*d^2) +
(b*x*ArcTanh[c*x])/(c^2*d^2) - (a + b*ArcTanh[c*x])/(c^3*d^2*(1 + c*x)) + (
2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^3*d^2) + (b*Log[1 - c^2*x^2])/(
2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx=\int\left(\frac{a+b\tanh^{-1}(cx)}{c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)^2}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}\right)dx$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)\,dx}{c^2d^2}+\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^2d^2}-\frac{2\int\frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^2d^2}$$

$$=\frac{ax}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\int\frac{1}{(1+cx)(1-c^2x^2)}}{c^2d^2}$$

$$=\frac{ax}{c^2d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}-$$

$$=\frac{ax}{c^2d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+$$

$$=\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(\right.}{c^3d}$$

$$=\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{b\tanh^{-1}(cx)}{2c^3d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+$$

Mathematica [A]
time = 0.37, size = 121, normalized size = 0.81

$$\frac{4acx-\frac{4a}{1+cx}-8a\log(1+cx)+b\left(-\cosh\left(2\tanh^{-1}(cx)\right)+2\log\left(1-c^2x^2\right)-4\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\sinh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(2cx-\cosh\left(2\tanh^{-1}(cx)\right)+4\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+\sinh\left(2\tanh^{-1}(cx)\right)\right)\right)}{4c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (4*a*c*x - (4*a)/(1 + c*x) - 8*a*Log[1 + c*x] + b*(-Cosh[2*ArcTanh[c*x]] +
2*Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*
x]] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*ArcTan
h[c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

Maple [A]
time = 0.31, size = 183, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{acx}{d^2}-\frac{2a\ln(cx+1)}{d^2}-\frac{a}{d^2(cx+1)}+\frac{b\,\mathrm{arctanh}(cx)cx}{d^2}-\frac{2b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^2}-\frac{b\,\mathrm{arctanh}(cx)}{d^2(cx+1)}+\frac{b\ln(cx+1)^2}{2d^2}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{d^2}+}{c^3}$

default	$\frac{\frac{acx}{d^2}-\frac{2a\ln(cx+1)}{d^2}-\frac{a}{d^2(cx+1)}+\frac{b\,\text{arctanh}(cx)cx}{d^2}-\frac{2b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}-\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{b\ln(cx+1)^2}{2d^2}-\frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{d^2}}{c^3}$
risch	$-\frac{b\ln(cx+1)^2}{2c^3d^2}+\left(\frac{bx}{2c^2d^2}-\frac{b}{2c^3d^2(cx+1)}\right)\ln(cx+1)-\frac{b}{2c^3d^2(cx+1)}+\frac{b\ln(cx+1)}{2c^3d^2}+\frac{ax}{c^2d^2}-\frac{a}{d^2c^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/d^2*c*x-2*a/d^2*ln(c*x+1)-a/d^2/(c*x+1)+b/d^2*arctanh(c*x)*c*x-2*b
/d^2*arctanh(c*x)*ln(c*x+1)-b/d^2*arctanh(c*x)/(c*x+1)+1/2*b/d^2*ln(c*x+1)^
2-b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+b
/d^2*dilog(1/2*c*x+1/2)-1/2*b/d^2/(c*x+1)+3/4*b/d^2*ln(c*x+1)+1/4*b/d^2*ln(
c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(c^3*(2/(c^7*d^2*x + c^6*d^2) - 4*x/(c^5*d^2) + 5*log(c*x + 1)/(c^6*d^
2) - log(c*x - 1)/(c^6*d^2)) - 4*c^3*integrate(x^3*log(c*x + 1)/(c^5*d^2*x^
3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) - 2*c^2*(2/(c^6*d^2*x + c^5*d^2)
 + 3*log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 12*c^2*integrate(x^
2*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) + 16*c
*integrate(x*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2)
, x) + 4*(c^2*x^2 + c*x - 2*(c*x + 1)*log(c*x + 1) - 1)*log(-c*x + 1)/(c^4*
d^2*x + c^3*d^2) + 2/(c^4*d^2*x + c^3*d^2) - log(c*x + 1)/(c^3*d^2) + log(c
*x - 1)/(c^3*d^2) + 8*integrate(log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c
^3*d^2*x - c^2*d^2), x))*b - a*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*l
og(c*x + 1)/(c^3*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^2x^2+2cx+1}\,dx + \int \frac{bx^2 \operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(
c**2*x**2 + 2*c*x + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

3.53 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=106

$$\frac{b}{2c^2d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2d^2}$$

```
[Out] 1/2*b/c^2/d^2/(c*x+1)-1/2*b*arctanh(c*x)/c^2/d^2+(a+b*arctanh(c*x))/c^2/d^2
/(c*x+1)-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^2/d^2+1/2*b*polylog(2,1-2/(c*x+
1))/c^2/d^2
```

Rubi [A]

time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6087, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{a+b\tanh^{-1}(cx)}{c^2d^2(cx+1)}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b}{2c^2d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] b/(2*c^2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*c^2*d^2) + (a + b*ArcTanh[c*x
])/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^2*d^2)
+ (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx = \int \left(-\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}\right)\,dx$$

$$= -\frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{cd^2}+\frac{\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{cd^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}-\frac{b\int \frac{1}{(1+cx)(1-c^2x^2)}\,dx}{cd^2}+\frac{b \int}{}$$

$$= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{}{1+}\right.}{c^2d^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c^2d^2}-\frac{b\int\left(\frac{}{2(}\right.}{}$$

$$= \frac{b}{2c^2d^2(1+cx)}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\text{Li}_2\left(1-\right.}{2c^2}$$

$$= \frac{b}{2c^2d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\right.}{c^2d^2}$$

Mathematica [A]

time = 0.23, size = 99, normalized size = 0.93

$$\frac{\frac{4a}{1+cx}+4a\log(1+cx)+b\left(\cosh\left(2\tanh^{-1}(cx)\right)+2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2\tanh^{-1}(cx)\left(\cosh\left(2\tanh^{-1}(cx)\right)-2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)\right)-\sinh\left(2\tanh^{-1}(cx)\right)\right)}{4c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] ((4*a)/(1 + c*x) + 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]] + 2*PolyLog[2
, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*Log[1 +
E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4*c^
2*d^2)
```

Maple [A]

time = 0.29, size = 163, normalized size = 1.54

method	result
derivativedivides	$\frac{\frac{a}{d^2(cx+1)}+\frac{a\ln(cx+1)}{d^2}+\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}+\frac{b}{2d^2(cx+1)}-\frac{b\ln(cx+1)}{4d^2}+\frac{b\ln(cx-1)}{4d^2}-\frac{b\ln(cx+1)^2}{4d^2}+\frac{b\ln\left(-\frac{cx}{2}+\right.}{2c}}{c^2}$
default	$\frac{\frac{a}{d^2(cx+1)}+\frac{a\ln(cx+1)}{d^2}+\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}+\frac{b}{2d^2(cx+1)}-\frac{b\ln(cx+1)}{4d^2}+\frac{b\ln(cx-1)}{4d^2}-\frac{b\ln(cx+1)^2}{4d^2}+\frac{b\ln\left(-\frac{cx}{2}+\right.}{2c}}{c^2}$

risch	$\frac{b\ln(cx+1)^2}{4c^2d^2}+\frac{b\ln(cx+1)}{2c^2d^2(cx+1)}+\frac{b}{2c^2d^2(cx+1)}-\frac{a}{d^2c^2(-cx-1)}+\frac{a\ln(-cx-1)}{d^2c^2}-\frac{b\ln(-cx-1)}{4d^2c^2}-\frac{b\ln(-cx+1)x}{4d^2c(-cx-1)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a/d^2/(c*x+1)+a/d^2*ln(c*x+1)+b/d^2*arctanh(c*x)/(c*x+1)+b/d^2*arcta
nh(c*x)*ln(c*x+1)+1/2*b/d^2/(c*x+1)-1/4*b/d^2*ln(c*x+1)+1/4*b/d^2*ln(c*x-1)
-1/4*b/d^2*ln(c*x+1)^2+1/2*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*b/d^2*ln(-1
/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/2*b/d^2*dilog(1/2*c*x+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*c^2*integrate(x^2*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^2*d^2*
x - c*d^2), x) - c*(2/(c^4*d^2*x + c^3*d^2) + log(c*x + 1)/(c^3*d^2) - log(
c*x - 1)/(c^3*d^2)) + 4*c*integrate(x*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x
^2 - c^2*d^2*x - c*d^2), x) - 4*((c*x + 1)*log(c*x + 1) + 1)*log(-c*x + 1)/
(c^3*d^2*x + c^2*d^2) + 2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) +
log(c*x - 1)/(c^2*d^2) + 4*integrate(log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^
2 - c^2*d^2*x - c*d^2), x))*b + a*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(
c^2*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^2x^2+2cx+1}\,dx+\int \frac{bx\,\mathrm{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x*atanh(c*x)/(c**2*x
**2 + 2*c*x + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)*x/(c*d*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
[Out] int((x*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

3.54 $\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{b}{2cd^2(1+cx)}+\frac{b\tanh^{-1}(cx)}{2cd^2}-\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}$$

```
[Out] -1/2*b/c/d^2/(c*x+1)+1/2*b*arctanh(c*x)/c/d^2+(-a-b*arctanh(c*x))/c/d^2/(c*
x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6063, 641, 46, 213}

$$-\frac{a+b\tanh^{-1}(cx)}{cd^2(cx+1)}-\frac{b}{2cd^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

```
[Out] -1/2*b/(c*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c*d^2) - (a + b*ArcTanh[c*x]
)/(c*d^2*(1 + c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^2}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}+\frac{b\int \frac{1}{(d+cdx)(1-c^2x^2)}\,dx}{d}\\
&= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}+\frac{b\int \frac{1}{\left(\frac{1}{d}-\frac{cx}{d}\right)(d+cdx)^2}\,dx}{d}\\
&= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}+\frac{b\int \left(\frac{1}{2d(1+cx)^2}-\frac{1}{2d(-1+c^2x^2)}\right)dx}{d}\\
&= -\frac{b}{2cd^2(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}-\frac{b\int \frac{1}{-1+c^2x^2}\,dx}{2d^2}\\
&= -\frac{b}{2cd^2(1+cx)}+\frac{b\tanh^{-1}(cx)}{2cd^2}-\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 1.12

$$\frac{-4a-2b-4b\tanh^{-1}(cx)-(b+bcx)\log(1-cx)+b\log(1+cx)+bcx\log(1+cx)}{4cd^2(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
[Out] (-4*a - 2*b - 4*b*ArcTanh[c*x] - (b + b*c*x)*Log[1 - c*x] + b*Log[1 + c*x]
+ b*c*x*Log[1 + c*x])/(4*c*d^2*(1 + c*x))
```

Maple [A]
time = 0.14, size = 73, normalized size = 1.28

method	result	size
derivativedivides	$\frac{-\frac{a}{d^2(cx+1)}-\frac{b\,\mathrm{arctanh}(cx)}{d^2(cx+1)}-\frac{b}{2d^2(cx+1)}+\frac{b\ln(cx+1)}{4d^2}-\frac{b\ln(cx-1)}{4d^2}}{c}$	73
default	$\frac{-\frac{a}{d^2(cx+1)}-\frac{b\,\mathrm{arctanh}(cx)}{d^2(cx+1)}-\frac{b}{2d^2(cx+1)}+\frac{b\ln(cx+1)}{4d^2}-\frac{b\ln(cx-1)}{4d^2}}{c}$	73
risch	$-\frac{b\ln(cx+1)}{2c\,d^2(cx+1)}-\frac{\ln(cx-1)bcx-\ln(-cx-1)bcx+b\ln(cx-1)-b\ln(-cx-1)-2b\ln(-cx+1)+4a+2b}{4d^2(cx+1)c}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a/d^2/(c*x+1)-b/d^2*arctanh(c*x)/(c*x+1)-1/2*b/d^2/(c*x+1)+1/4*b/d^2*
ln(c*x+1)-1/4*b/d^2*ln(c*x-1))
```

Maxima [A]
time = 0.26, size = 96, normalized size = 1.68

$$-\frac{1}{4}\left(c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log\left(cx+1\right)}{c^2d^2}+\frac{\log\left(cx-1\right)}{c^2d^2}\right)+\frac{4\ \mathrm{artanh}\left(cx\right)}{c^2d^2x+cd^2}\right)b-\frac{a}{c^2d^2x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c
^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*b - a/(c^2*d^2*x + c*d^2)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 0.86

$$\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)-4\,a-2\,b}{4\left(c^2d^2x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1)) - 4*a - 2*b)/(c^2*d^2*x + c*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.
time = 0.71, size = 95, normalized size = 1.67

$$\begin{cases}-\frac{2a}{2c^2d^2x+2cd^2}+\frac{bcx\,\mathrm{atanh}\left(cx\right)}{2c^2d^2x+2cd^2}-\frac{b\,\mathrm{atanh}\left(cx\right)}{2c^2d^2x+2cd^2}-\frac{b}{2c^2d^2x+2cd^2} & \text{for}\ c\neq 0\\ \frac{ax}{d^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] Piecewise((-2*a/(2*c**2*d**2*x + 2*c*d**2) + b*c*x*atanh(c*x)/(2*c**2*d**2*
x + 2*c*d**2) - b*atanh(c*x)/(2*c**2*d**2*x + 2*c*d**2) - b/(2*c**2*d**2*x
+ 2*c*d**2), Ne(c, 0)), (a*x/d**2, True))
```

Giac [A]
time = 0.42, size = 63, normalized size = 1.11

$$\frac{1}{4}\,c\left(\frac{(cx-1)b\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2d^2}+\frac{(cx-1)(2\,a+b)}{(cx+1)c^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*c*((c*x - 1)*b*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2) + (c*x - 1
)*(2*a + b)/((c*x + 1)*c^2*d^2))
```

Mupad [B]
time = 1.07, size = 45, normalized size = 0.79

$$-\frac{b\,\mathrm{atanh}(c\,x)-c\,(2\,a\,x+b\,x+b\,x\,\mathrm{atanh}(c\,x))}{2\,x\,c^2\,d^2+2\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + c*d*x)^2,x)
```

```
[Out] -(b*atanh(c*x) - c*(2*a*x + b*x + b*x*atanh(c*x)))/(2*c*d^2 + 2*c^2*d^2*x)
```

3.55 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^2}\,dx$

Optimal. Leaf size=124

$$\frac{b}{2d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{2d^2}+\frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)}+\frac{a\log(x)}{d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2}-\frac{b\text{PolyLog}(2,-cx)}{2d^2}-$$

```
[Out] 1/2*b/d^2/(c*x+1)-1/2*b*arctanh(c*x)/d^2+(a+b*arctanh(c*x))/d^2/(c*x+1)+a*l
n(x)/d^2+(a+b*arctanh(c*x))*ln(2/(c*x+1))/d^2-1/2*b*polylog(2,-c*x)/d^2+1/2
*b*polylog(2,c*x)/d^2-1/2*b*polylog(2,1-2/(c*x+1))/d^2
```

Rubi [A]

time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6087, 6031, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{a+b\tanh^{-1}(cx)}{d^2(cx+1)}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{a\log(x)}{d^2}-\frac{b\text{Li}_2(-cx)}{2d^2}+\frac{b\text{Li}_2(cx)}{2d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^2}+\frac{b}{2d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

```
[Out] b/(2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*d^2) + (a + b*ArcTanh[c*x])/(d^2*
(1 + c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 -
 (b*PolyLog[2, -(c*x)])/(2*d^2) + (b*PolyLog[2, c*x])/(2*d^2) - (b*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c(a+b\tanh^{-1}(cx))}{d^2(1+cx)^2} - \frac{c(a+b\tanh^{-1}(cx))}{d^2(1+cx)}\right) dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{b\text{Li}_2(}{2d}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{b\text{Li}_2(}{2d}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{b\text{Li}_2(}{2d}$$

$$= \frac{b}{2d^2(1+cx)} + \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}}{}$$

$$= \frac{b}{2d^2(1+cx)} - \frac{b\tanh^{-1}(cx)}{2d^2} + \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\text{l}}{d^2}$$

Mathematica [A]
time = 0.26, size = 101, normalized size = 0.81

$$\frac{\frac{4a}{1+cx} + 4a\log(x) - 4a\log(1+cx) + b\left(\cosh\left(2\tanh^{-1}(cx)\right) - 2\text{PolyLog}\left(2, e^{-2\tanh^{-1}(cx)}\right) + 2\tanh^{-1}(cx)\left(\cosh\left(2\tanh^{-1}(cx)\right) + 2\log\left(1-e^{-2\tanh^{-1}(cx)}\right) - \sinh\left(2\tanh^{-1}(cx)\right)\right) - \sinh\left(2\tanh^{-1}(cx)\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

```
[Out] ((4*a)/(1 + c*x) + 4*a*Log[x] - 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]]
- 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]]
+ 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c
*x]]))/(4*d^2)
```

Maple [A]
time = 0.20, size = 221, normalized size = 1.78

method	result
risch	$\frac{a\ln(-cx)}{d^2} - \frac{a}{d^2(-cx-1)} - \frac{a\ln(-cx-1)}{d^2} - \frac{b\ln(-cx-1)}{4d^2} - \frac{b\ln(-cx+1)cx}{4d^2(-cx-1)} + \frac{b\ln(-cx+1)}{4d^2(-cx-1)} + \frac{\text{dilog}(-cx+1)b}{2d^2} -$
derivativedivides	$\frac{a\ln(cx)}{d^2} + \frac{a}{d^2(cx+1)} - \frac{a\ln(cx+1)}{d^2} + \frac{b\,\text{arctanh}(cx)\ln(cx)}{d^2} + \frac{b\,\text{arctanh}(cx)}{d^2(cx+1)} - \frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d^2} - \frac{b\ln(-\frac{cx}{2}}{}$
default	$\frac{a\ln(cx)}{d^2} + \frac{a}{d^2(cx+1)} - \frac{a\ln(cx+1)}{d^2} + \frac{b\,\text{arctanh}(cx)\ln(cx)}{d^2} + \frac{b\,\text{arctanh}(cx)}{d^2(cx+1)} - \frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d^2} - \frac{b\ln(-\frac{cx}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^2*ln(c*x)+a/d^2/(c*x+1)-a/d^2*ln(c*x+1)+b/d^2*arctanh(c*x)*ln(c*x)+b/d^
2*arctanh(c*x)/(c*x+1)-b/d^2*arctanh(c*x)*ln(c*x+1)-1/2*b/d^2*ln(-1/2*c*x+1
/2)*ln(c*x+1)+1/2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/2*b/d^2*dilog(1/
2*c*x+1/2)+1/4*b/d^2*ln(c*x+1)^2+1/4*b/d^2*ln(c*x-1)+1/2*b/d^2/(c*x+1)-1/4*
b/d^2*ln(c*x+1)-1/2*b/d^2*dilog(c*x)-1/2*b/d^2*dilog(c*x+1)-1/2*b/d^2*ln(c*
x)*ln(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/2*b*integrate((lo
g(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^3+2cx^2+x}\,dx + \int \frac{b\operatorname{atanh}(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b*atanh(c*x)/(c**2*x*
*3 + 2*c*x**2 + x), x))/d**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x\,{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + c*d*x)^2),x)
[Out] int((a + b*atanh(c*x))/(x*(d + c*d*x)^2), x)
```

3.56 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^2}\,dx$

Optimal. Leaf size=171

$$-\frac{bc}{2d^2(1+cx)}+\frac{bc\tanh^{-1}(cx)}{2d^2}-\frac{a+b\tanh^{-1}(cx)}{d^2x}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)}-\frac{2ac\log(x)}{d^2}+\frac{bc\log(x)}{d^2}-\frac{2c\left(a+b\,\mathrm{ta}\right.}{}$$

```
[Out] -1/2*b*c/d^2/(c*x+1)+1/2*b*c*arctanh(c*x)/d^2+(-a-b*arctanh(c*x))/d^2/x-c*(
a+b*arctanh(c*x))/d^2/(c*x+1)-2*a*c*ln(x)/d^2+b*c*ln(x)/d^2-2*c*(a+b*arctan
h(c*x))*ln(2/(c*x+1))/d^2-1/2*b*c*ln(-c^2*x^2+1)/d^2+b*c*polylog(2,-c*x)/d^
2-b*c*polylog(2,c*x)/d^2+b*c*polylog(2,1-2/(c*x+1))/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6087, 6037, 272, 36, 29, 31, 6031, 6063, 641, 46, 213, 6055, 2449, 2352}

$$-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{d^2x}-\frac{2c\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2ac\log(x)}{d^2}-\frac{bc\log\left(1-c^2x^2\right)}{2d^2}+\frac{bc\text{Li}_2(-cx)}{d^2}-\frac{bc\text{Li}_2(cx)}{d^2}+\frac{bc\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{d^2}-\frac{bc}{2d^2(cx+1)}+\frac{bc\log(x)}{d^2}+\frac{bc\tanh^{-1}(cx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*(1 + c*x)) + (b*c*ArcTanh[c*x])/(2*d^2) - (a + b*ArcTanh[c*
x])/(d^2*x) - (c*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*a*c*Log[x])/d^2
 + (b*c*Log[x])/d^2 - (2*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 - (b*
c*Log[1 - c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, -(c*x)])/d^2 - (b*c*PolyLog[2
, c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + c*x)])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)^2} + \frac{2c^2\left(a+\right.}{d}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{(2c)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} + \frac{c^2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} + \frac{(2c^2)\int \frac{a+b\tan}{1}}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{d^2}$$

$$= -\frac{bc}{2d^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} + \frac{bc\log(x}{d^2}$$

$$= -\frac{bc}{2d^2(1+cx)} + \frac{bc\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac}{}$$

Mathematica [A]

time = 0.52, size = 140, normalized size = 0.82

$$\frac{-\frac{4a}{x}-\frac{4ac}{1+cx}-8ac\log(x)+8ac\log(1+cx)+bc\left(-\cosh\left(2\tanh^{-1}(cx)\right)+4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+\sinh\left(2\tanh^{-1}(cx)\right)+\tanh^{-1}(cx)\left(-\frac{4}{cx}-2\cosh\left(2\tanh^{-1}(cx)\right)-8\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+2\sinh\left(2\tanh^{-1}(cx)\right)\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

```
[Out] ((-4*a)/x - (4*a*c)/(1 + c*x) - 8*a*c*Log[x] + 8*a*c*Log[1 + c*x] + b*c*(-C
osh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2*A
rcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2*Ar
cTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]])))/(4
*d^2)
```

Maple [A]

time = 0.33, size = 260, normalized size = 1.52

method	result
derivativedivides	$c\Big(-\frac{a}{d^2(cx+1)}+\frac{2a\ln(cx+1)}{d^2}-\frac{a}{d^2cx}-\frac{2a\ln(cx)}{d^2}-\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{2b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}-\frac{b\,\text{arctanh}(cx)}{d^2cx}$
default	$c\Big(-\frac{a}{d^2(cx+1)}+\frac{2a\ln(cx+1)}{d^2}-\frac{a}{d^2cx}-\frac{2a\ln(cx)}{d^2}-\frac{b\,\text{arctanh}(cx)}{d^2(cx+1)}+\frac{2b\,\text{arctanh}(cx)\ln(cx+1)}{d^2}-\frac{b\,\text{arctanh}(cx)}{d^2cx}$
risch	$-\frac{a}{d^2x}-\frac{2ca\ln(-cx)}{d^2}+\frac{ca}{d^2(-cx-1)}+\frac{2ca\ln(-cx-1)}{d^2}+\frac{cb\ln(-cx)}{2d^2}-\frac{cb\ln(-cx+1)}{2d^2}+\frac{b\ln(-cx+1)}{2d^2x}+\frac{bc\ln(-c}{4d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/d^2/(c*x+1)+2*a/d^2*ln(c*x+1)-a/d^2/c/x-2*a/d^2*ln(c*x)-b/d^2*arctanh
(c*x)/(c*x+1)+2*b/d^2*arctanh(c*x)*ln(c*x+1)-b/d^2*arctanh(c*x)/c/x-2*b/d^2
*arctanh(c*x)*ln(c*x)-1/2*b/d^2/(c*x+1)-1/4*b/d^2*ln(c*x+1)-3/4*b/d^2*ln(c*
x-1)+b/d^2*ln(c*x)+b/d^2*dilog(c*x+1)+b/d^2*ln(c*x)*ln(c*x+1)+b/d^2*dilog(c
*x)-b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)
-b/d^2*dilog(1/2*c*x+1/2)-1/2*b/d^2*ln(c*x+1)^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d^2
) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^4 + 2*c*d^2*x
^3 + d^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^4 + 2*c*d^2*x^3 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^4+2cx^3+x^2}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^2x^4+2cx^3+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**2,x)
[Out] (Integral(a/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c**2
*x**4 + 2*c*x**3 + x**2), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^2\,{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^2),x)
[Out] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^2), x)
```

3.57 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{bc}{2d^2x}+\frac{bc^2}{2d^2(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{2d^2x^2}+\frac{2c(a+b\tanh^{-1}(cx))}{d^2x}+\frac{c^2(a+b\tanh^{-1}(cx))}{d^2(1+cx)}+\frac{3ac^2\log(x)}{d^2}-\frac{2bc^2\log}{d^2}$$

```
[Out] -1/2*b*c/d^2/x+1/2*b*c^2/d^2/(c*x+1)+1/2*(-a-b*arctanh(c*x))/d^2/x^2+2*c*(a
+b*arctanh(c*x))/d^2/x+c^2*(a+b*arctanh(c*x))/d^2/(c*x+1)+3*a*c^2*ln(x)/d^2
-2*b*c^2*ln(x)/d^2+3*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/d^2+b*c^2*ln(-c^2
*x^2+1)/d^2-3/2*b*c^2*polylog(2,-c*x)/d^2+3/2*b*c^2*polylog(2,c*x)/d^2-3/2*
b*c^2*polylog(2,1-2/(c*x+1))/d^2
```

Rubi [A]

time = 0.19, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6087, 6037, 331, 212, 272, 36, 29, 31, 6031, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{c^2(a+b\tanh^{-1}(cx))}{d^2(cx+1)}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d^2}-\frac{a+b\tanh^{-1}(cx)}{2d^2x^2}+\frac{2c(a+b\tanh^{-1}(cx))}{d^2x}+\frac{3ac^2\log(x)}{d^2}-\frac{3bc^2\text{Li}_2(-cx)}{2d^2}+\frac{3bc^2\text{Li}_2(cx)}{2d^2}-\frac{3bc^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^2}+\frac{bc^2\log(1-c^2x^2)}{d^2}+\frac{bc^2}{2d^2(cx+1)}-\frac{2bc^2\log(x)}{d^2}-\frac{bc}{2d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*x) + (b*c^2)/(2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])/(2*d^
2*x^2) + (2*c*(a + b*ArcTanh[c*x]))/(d^2*x) + (c^2*(a + b*ArcTanh[c*x]))/(d
^2*(1 + c*x)) + (3*a*c^2*Log[x])/d^2 - (2*b*c^2*Log[x])/d^2 + (3*c^2*(a + b
*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (b*c^2*Log[1 - c^2*x^2])/d^2 - (3*b*
c^2*PolyLog[2, -(c*x)])/(2*d^2) + (3*b*c^2*PolyLog[2, c*x])/(2*d^2) - (3*b*
c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x^3} - \frac{2c\big(a+b\tanh^{-1}(cx)\big)}{d^2x^2} + \frac{3c^2\big(a+b\tanh^{-1}(cx)\big)}{d^2x} - \frac{c^3(a+}{d}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2c)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{(3c^2)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} - \frac{c^3\int \frac{a+b\,\mathrm{ta}}{(1-}}{a}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\big(a+b\tanh^{-1}(cx)\big)}{d^2x} + \frac{c^2\big(a+b\tanh^{-1}(cx)\big)}{d^2(1+cx)} + \frac{3ac^2\log(x}{d^2}$$

$$= -\frac{bc}{2d^2x} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\big(a+b\tanh^{-1}(cx)\big)}{d^2x} + \frac{c^2\big(a+b\tanh^{-1}(cx)\big)}{d^2(1+cx)} + \frac{3c}{}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\big(a+b\tanh^{-1}(cx)\big)}{d^2x} + \frac{c^2(a+b}{d^2(}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2}{2d^2(1+cx)} + \frac{bc^2\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\big(a+b\tanh^{-1}(c}{d^2x}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2}{2d^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\big(a+b\tanh^{-1}(cx)\big)}{d^2x} + \frac{c^2(a+b\tan}{d^2(1+}$$

Mathematica [A]
time = 0.73, size = 189, normalized size = 0.89

$$\frac{-\frac{2a}{x^2} + \frac{8ac}{x} - \frac{2bc}{x} + \frac{4ac^2}{1+cx} + bc^2\cosh\left(2\tanh^{-1}(cx)\right) + 12ac^2\log(x) - 12ac^2\log(1+cx) - 8bc^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) - 6bc^2\mathrm{PolyLog}\left(2, e^{-2\tanh^{-1}(cx)}\right) - bc^2\sinh\left(2\tanh^{-1}(cx)\right) + 2b\tanh^{-1}(cx)\left(c^2 - \frac{1}{x^2} + \frac{4c}{x} + c^2\cosh\left(2\tanh^{-1}(cx)\right) + 6c^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right) - c^2\sinh\left(2\tanh^{-1}(cx)\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

```
[Out] ((-2*a)/x^2 + (8*a*c)/x - (2*b*c)/x + (4*a*c^2)/(1 + c*x) + b*c^2*Cosh[2*Ar
cTanh[c*x]] + 12*a*c^2*Log[x] - 12*a*c^2*Log[1 + c*x] - 8*b*c^2*Log[(c*x)/S
qrt[1 - c^2*x^2]] - 6*b*c^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - b*c^2*Sinh[2*
ArcTanh[c*x]] + 2*b*ArcTanh[c*x]*(c^2 - x^(-2) + (4*c)/x + c^2*Cosh[2*ArcTa
nh[c*x]] + 6*c^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*Sinh[2*ArcTanh[c*x]]))/
(4*d^2)
```

Maple [A]
time = 0.34, size = 303, normalized size = 1.43

method	result
derivativedivides	$c^2\left(\frac{a}{d^2(cx+1)} - \frac{3a\ln(cx+1)}{d^2} - \frac{a}{2d^2c^2x^2} + \frac{2a}{d^2cx} + \frac{3a\ln(cx)}{d^2} + \frac{b\,\mathrm{arctanh}(cx)}{d^2(cx+1)} - \frac{3b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^2} - \right.$
default	$c^2\left(\frac{a}{d^2(cx+1)} - \frac{3a\ln(cx+1)}{d^2} - \frac{a}{2d^2c^2x^2} + \frac{2a}{d^2cx} + \frac{3a\ln(cx)}{d^2} + \frac{b\,\mathrm{arctanh}(cx)}{d^2(cx+1)} - \frac{3b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^2} - \right.$

risch	$\frac{c^2 b \ln(-cx+1)}{4d^2(-cx-1)} - \frac{3c^2 b \ln(\frac{cx}{2}+\frac{1}{2}) \ln(-\frac{cx}{2}+\frac{1}{2})}{2d^2} + \frac{3c^2 b \ln(\frac{cx}{2}+\frac{1}{2}) \ln(-cx+1)}{2d^2} - \frac{bc}{2d^2 x} + \frac{b c^2}{2d^2(cx+1)} + \frac{3c^2 \operatorname{dilog}(-c}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(a/d^2/(c*x+1)-3*a/d^2*ln(c*x+1)-1/2*a/d^2/c^2/x^2+2*a/d^2/c/x+3*a/d^2*
ln(c*x)+b/d^2*arctanh(c*x)/(c*x+1)-3*b/d^2*arctanh(c*x)*ln(c*x+1)-1/2*b/d^2
*arctanh(c*x)/c^2/x^2+2*b/d^2*arctanh(c*x)/c/x+3*b/d^2*arctanh(c*x)*ln(c*x)
-3/2*b/d^2*dilog(c*x)-3/2*b/d^2*dilog(c*x+1)-3/2*b/d^2*ln(c*x)*ln(c*x+1)+3/
4*b/d^2*ln(c*x+1)^2-3/2*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2*b/d^2*ln(-1/2*
c*x+1/2)*ln(1/2*c*x+1/2)+3/2*b/d^2*dilog(1/2*c*x+1/2)+1/2*b/d^2/(c*x+1)+b/d
^2*ln(c*x+1)-1/2*b/d^2/c/x-2*b/d^2*ln(c*x)+b/d^2*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x - 1)
/(c*d^2*x^3 + d^2*x^2)) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c
^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{b\operatorname{atanh}(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c**2
*x**5 + 2*c*x**4 + x**3), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^3\,{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^2), x)
```

3.58 $\int \frac{x^4(a+b\tanh^{-1}(cx))}{(d+cdx)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b}{8c^5d^3(1+cx)^2}+\frac{15b}{8c^5d^3(1+cx)}-\frac{19b\tanh^{-1}(cx)}{8c^5d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x^2(a+b\tanh^{-1}(cx))}{2c^3d^3}-\frac{a}{2}$$

```
[Out] -3*a*x/c^4/d^3+1/2*b*x/c^4/d^3-1/8*b/c^5/d^3/(c*x+1)^2+15/8*b/c^5/d^3/(c*x+
1)-19/8*b*arctanh(c*x)/c^5/d^3-3*b*x*arctanh(c*x)/c^4/d^3+1/2*x^2*(a+b*arct
anh(c*x))/c^3/d^3+1/2*(-a-b*arctanh(c*x))/c^5/d^3/(c*x+1)^2+4*(a+b*arctanh(
c*x))/c^5/d^3/(c*x+1)-6*(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^5/d^3-3/2*b*ln(-
c^2*x^2+1)/c^5/d^3+3*b*polylog(2,1-2/(c*x+1))/c^5/d^3
```

Rubi [A]
time = 0.21, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {6087, 6021, 266, 6037, 327, 212, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{4(a+b\tanh^{-1}(cx))}{c^5d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(cx+1)^2}-\frac{6\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{c^5d^3}+\frac{x^2(a+b\tanh^{-1}(cx))}{2c^3d^3}-\frac{3ax}{c^4d^3}+\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{15b}{8c^5d^3(cx+1)}-\frac{b}{8c^5d^3(cx+1)^2}-\frac{19b\tanh^{-1}(cx)}{8c^5d^3}+\frac{bx}{2c^4d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}-\frac{3b\log(1-c^2x^2)}{2c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (-3*a*x)/(c^4*d^3) + (b*x)/(2*c^4*d^3) - b/(8*c^5*d^3*(1 + c*x)^2) + (15*b)
/(8*c^5*d^3*(1 + c*x)) - (19*b*ArcTanh[c*x])/(8*c^5*d^3) - (3*b*x*ArcTanh[c
*x])/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*c^3*d^3) - (a + b*ArcTanh[c*
x])/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh[c*x]))/(c^5*d^3*(1 + c*x))
- (6*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^5*d^3) - (3*b*Log[1 - c^2*x^
2])/(2*c^5*d^3) + (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\big(a+b\tanh^{-1}(cx)\big)}{(d+cdx)^3}\,dx = \int \bigg(-\frac{3\big(a+b\tanh^{-1}(cx)\big)}{c^4d^3} + \frac{x\big(a+b\tanh^{-1}(cx)\big)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{c^4d^3(1+cx)^3} - 4($$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^4d^3} - \frac{3\int \big(a+b\tanh^{-1}(cx)\big)\,dx}{c^4d^3} - \frac{4\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^4d^3} + 6\int$$

$$= -\frac{3ax}{c^4d^3} + \frac{x^2\big(a+b\tanh^{-1}(cx)\big)}{2c^3d^3} - \frac{a+b\tanh^{-1}(cx)}{2c^5d^3(1+cx)^2} + \frac{4\big(a+b\tanh^{-1}(cx)\big)}{c^5d^3(1+cx)} -$$

$$= -\frac{3ax}{c^4d^3} + \frac{bx}{2c^4d^3} - \frac{3bx\tanh^{-1}(cx)}{c^4d^3} + \frac{x^2\big(a+b\tanh^{-1}(cx)\big)}{2c^3d^3} - \frac{a+b\tanh^{-1}(cx)}{2c^5d^3(1+cx)^2}$$

$$= -\frac{3ax}{c^4d^3} + \frac{bx}{2c^4d^3} - \frac{b\tanh^{-1}(cx)}{2c^5d^3} - \frac{3bx\tanh^{-1}(cx)}{c^4d^3} + \frac{x^2\big(a+b\tanh^{-1}(cx)\big)}{2c^3d^3} -$$

$$= -\frac{3ax}{c^4d^3} + \frac{bx}{2c^4d^3} - \frac{b}{8c^5d^3(1+cx)^2} + \frac{15b}{8c^5d^3(1+cx)} - \frac{b\tanh^{-1}(cx)}{2c^5d^3} - \frac{3bx\tan}{c^4}$$

$$= -\frac{3ax}{c^4d^3} + \frac{bx}{2c^4d^3} - \frac{b}{8c^5d^3(1+cx)^2} + \frac{15b}{8c^5d^3(1+cx)} - \frac{19b\tanh^{-1}(cx)}{8c^5d^3} - \frac{3bx\,t}{}$$

Mathematica [A]
time = 0.51, size = 189, normalized size = 0.83

$$\frac{-96acx+16ac^2x^2-\frac{16a}{(1+cx)^2}+\frac{128a}{1+cx}+192a\log(1+cx)+b\Big(16cx+28\cosh\left(2\tanh^{-1}(cx)\right)-\cosh\left(4\tanh^{-1}(cx)\right)-48\log\left(1-c^2x^2\right)+96\text{PolyLog}\Big(2,-e^{-2\tanh^{-1}(cx)}\Big)-28\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)+4\tanh^{-1}(cx)\Big(-4-24cx+4c^2x^2+14\cosh\left(2\tanh^{-1}(cx)\right)-\cosh\left(4\tanh^{-1}(cx)\right)-48\log\Big(1+e^{-2\tanh^{-1}(cx)}\Big)-14\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)\Big)\Big)}{32c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (-96*a*c*x + 16*a*c^2*x^2 - (16*a)/(1 + c*x)^2 + (128*a)/(1 + c*x) + 192*a*
Log[1 + c*x] + b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] -
 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcT
anh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*c^2*x^2
+ 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh
[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^5*d^3)
```

Maple [A]
time = 0.35, size = 272, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{a\,c^2x^2}{2d^3}-\frac{3acx}{d^3}-\frac{a}{2d^3(cx+1)^2}+\frac{4a}{d^3(cx+1)}+\frac{6a\ln(cx+1)}{d^3}+\frac{b\,\mathrm{arctanh}(cx)c^2x^2}{2d^3}-\frac{3b\,\mathrm{arctanh}(cx)cx}{d^3}-\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}+\frac{4b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}+6}{}$

default	$\frac{a\,c^2x^2}{2d^3}-\frac{3acx}{d^3}-\frac{a}{2d^3(cx+1)^2}+\frac{4a}{d^3(cx+1)}+\frac{6a\ln(cx+1)}{d^3}+\frac{b\,\mathrm{arctanh}(cx)c^2x^2}{2d^3}-\frac{3b\,\mathrm{arctanh}(cx)cx}{d^3}-\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}+\frac{4b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}+\frac{6b}{}$
risch	$\frac{3b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{d^3c^5}-\frac{3b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{d^3c^5}+\frac{3b\ln(-cx+1)x}{2d^3c^4}+\frac{b\ln(-cx+1)}{d^3c^5(-cx-1)}+\frac{3b\ln(-cx+1)}{16d^3c^5(-cx-1)^2}-\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/2*a/d^3*c^2*x^2-3*a/d^3*c*x-1/2*a/d^3/(c*x+1)^2+4*a/d^3/(c*x+1)+6*
a/d^3*ln(c*x+1)+1/2*b/d^3*arctanh(c*x)*c^2*x^2-3*b/d^3*arctanh(c*x)*c*x-1/2
*b/d^3*arctanh(c*x)/(c*x+1)^2+4*b/d^3*arctanh(c*x)/(c*x+1)+6*b/d^3*arctanh(
c*x)*ln(c*x+1)-3/2*b/d^3*ln(c*x+1)^2+3*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-3*b
/d^3*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-3*b/d^3*dilog(1/2*c*x+1/2)+1/2*b/d^3*
c*x+1/2*b/d^3-1/8*b/d^3/(c*x+1)^2+15/8*b/d^3/(c*x+1)-43/16*b/d^3*ln(c*x+1)-
5/16*b/d^3*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(c^5*(2*(9*c*x + 8)/(c^12*d^3*x^2 + 2*c^11*d^3*x + c^10*d^3) + 4*(c*x^
2 - 4*x)/(c^9*d^3) + 31*log(c*x + 1)/(c^10*d^3) + log(c*x - 1)/(c^10*d^3))
+ 32*c^5*integrate(1/2*x^5*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^
5*d^3*x - c^4*d^3), x) + 3*c^4*(2*(7*c*x + 6)/(c^11*d^3*x^2 + 2*c^10*d^3*x
+ c^9*d^3) - 8*x/(c^8*d^3) + 17*log(c*x + 1)/(c^9*d^3) - log(c*x - 1)/(c^9*
d^3)) - 32*c^4*integrate(1/2*x^4*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3
- 2*c^5*d^3*x - c^4*d^3), x) - 15*c^3*(2*(5*c*x + 4)/(c^10*d^3*x^2 + 2*c^9*
d^3*x + c^8*d^3) + 7*log(c*x + 1)/(c^8*d^3) + log(c*x - 1)/(c^8*d^3)) + 192
*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^
3*x - c^4*d^3), x) + 9*c^2*(2*(3*c*x + 2)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*
d^3) + log(c*x + 1)/(c^7*d^3) - log(c*x - 1)/(c^7*d^3)) + 576*c^2*integrate
(1/2*x^2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3)
, x) + 9*c*(2*x/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - log(c*x + 1)/(c^6*d
^3) + log(c*x - 1)/(c^6*d^3)) + 576*c*integrate(1/2*x*log(c*x + 1)/(c^8*d^3
*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3), x) - 8*(c^4*x^4 - 4*c^3*x^3
- 11*c^2*x^2 + 2*c*x + 12*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) + 7)*log(-c*x
+ 1)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + 14*(c*x + 2)/(c^7*d^3*x^2 + 2*
c^6*d^3*x + c^5*d^3) - 7*log(c*x + 1)/(c^5*d^3) + 7*log(c*x - 1)/(c^5*d^3)
+ 192*integrate(1/2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x
 - c^4*d^3), x))*b + 1/2*a*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^
3) + (c*x^2 - 6*x)/(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arctanh(c*x) + a*x^4)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^4\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^4/(c*d*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] int((x^4*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

3.59 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{(d+cdx)^3}\,dx$

Optimal. Leaf size=194

$$\frac{ax}{c^3d^3}+\frac{b}{8c^4d^3(1+cx)^2}-\frac{11b}{8c^4d^3(1+cx)}+\frac{11b\tanh^{-1}(cx)}{8c^4d^3}+\frac{bx\tanh^{-1}(cx)}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2}-\frac{3(a+b\tanh^{-1}(c}{c^4d^3(1+cx)}$$

```
[Out] a*x/c^3/d^3+1/8*b/c^4/d^3/(c*x+1)^2-11/8*b/c^4/d^3/(c*x+1)+11/8*b*arctanh(c
*x)/c^4/d^3+b*x*arctanh(c*x)/c^3/d^3+1/2*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)
^2-3*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)+3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/
c^4/d^3+1/2*b*ln(-c^2*x^2+1)/c^4/d^3-3/2*b*polylog(2,1-2/(c*x+1))/c^4/d^3
```

Rubi [A]
time = 0.19, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6021, 266, 6063, 641, 46, 213, 6055, 2449, 2352}

$$-\frac{3(a+b\tanh^{-1}(cx))}{c^4d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(cx+1)^2}+\frac{3\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{c^4d^3}+\frac{ax}{c^3d^3}-\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{11b}{8c^4d^3(cx+1)}+\frac{b}{8c^4d^3(cx+1)^2}+\frac{11b\tanh^{-1}(cx)}{8c^4d^3}+\frac{bx\tanh^{-1}(cx)}{c^3d^3}+\frac{b\log(1-c^2x^2)}{2c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (a*x)/(c^3*d^3) + b/(8*c^4*d^3*(1 + c*x)^2) - (11*b)/(8*c^4*d^3*(1 + c*x))
+ (11*b*ArcTanh[c*x])/(8*c^4*d^3) + (b*x*ArcTanh[c*x])/(c^3*d^3) + (a + b*A
rcTanh[c*x])/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x]))/(c^4*d^3*(1
 + c*x)) + (3*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^3) + (b*Log[1 -
 c^2*x^2])/(2*c^4*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{c^3d^3} - \frac{a+b\tanh^{-1}(cx)}{c^3d^3(1+cx)^3} + \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)^2} - \frac{3\left(a+b\,\text{ta}\right.}{c^3d^3(\text{]}}\right.$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)\,dx}{c^3d^3} - \frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^3d^3} + \frac{3\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^3d^3} - \frac{3\int \frac{a+b\,\text{t}}{}}{c}$$

$$= \frac{ax}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{}{1-}\right.}{c^4d^3}$$

$$= \frac{ax}{c^3d^3} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)} + \frac{3\left(a+b\,\text{ta}\right.}{}$$

$$= \frac{ax}{c^3d^3} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)} + \frac{3\left(a+b\,\text{ta}\right.}{}$$

$$= \frac{ax}{c^3d^3} + \frac{b}{8c^4d^3(1+cx)^2} - \frac{11b}{8c^4d^3(1+cx)} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2}$$

$$= \frac{ax}{c^3d^3} + \frac{b}{8c^4d^3(1+cx)^2} - \frac{11b}{8c^4d^3(1+cx)} + \frac{11b\tanh^{-1}(cx)}{8c^4d^3} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} +$$

Mathematica [A]
time = 0.44, size = 167, normalized size = 0.86

$$\frac{32acx + \frac{16a}{(1+cx)^2} - \frac{96a}{1+cx} - 96a\log(1+cx) + b\left(-20\cosh\left(2\tanh^{-1}(cx)\right) + \cosh\left(4\tanh^{-1}(cx)\right) + 16\log\left(1-c^2x^2\right) - 48\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 20\sinh\left(2\tanh^{-1}(cx)\right) + 4\tanh^{-1}(cx)\left(8cx - 10\cosh\left(2\tanh^{-1}(cx)\right) + \cosh\left(4\tanh^{-1}(cx)\right) + 24\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + 10\sinh\left(2\tanh^{-1}(cx)\right) - \sinh\left(4\tanh^{-1}(cx)\right)\right) - \sinh\left(4\tanh^{-1}(cx)\right)\right)}{32c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (32*a*c*x + (16*a)/(1 + c*x)^2 - (96*a)/(1 + c*x) - 96*a*Log[1 + c*x] + b*(
-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 - c^2*x^2] - 48*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]
*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24*Log[1 + E^(-2
*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*
ArcTanh[c*x]]))/(32*c^4*d^3)
```

Maple [A]
time = 0.34, size = 228, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{acx}{d^3} - \frac{3a\ln(cx+1)}{d^3} - \frac{3a}{d^3(cx+1)} + \frac{a}{2d^3(cx+1)^2} + \frac{b\,\mathrm{arctanh}(cx)cx}{d^3} - \frac{3b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^3} - \frac{3b\,\mathrm{arctanh}(cx)}{d^3(cx+1)} + \frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2} - \frac{3b\ln\left(-\frac{c}{2}\right.}{}}{}$

default	$\frac{\frac{acx}{d^3}-\frac{3a\ln(cx+1)}{d^3}-\frac{3a}{d^3(cx+1)}+\frac{a}{2d^3(cx+1)^2}+\frac{b\,\mathrm{arctanh}(cx)cx}{d^3}-\frac{3b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^3}-\frac{3b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}+\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}-\frac{3b\ln\left(-\right.}{}}{}$
risch	$-\frac{3b\ln(cx+1)^2}{4c^4d^3}+\left(\frac{bx}{2c^3d^3}+\frac{-3b\,d^3x-\frac{5d^3b}{2c}}{2c^3d^6(cx+1)^2}\right)\ln(cx+1)+\frac{b}{8c^4d^3(cx+1)^2}-\frac{3b}{2c^4d^3(cx+1)}+\frac{b\ln(cx+1)}{2c^4d^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/d^3*c*x-3*a/d^3*ln(c*x+1)-3*a/d^3/(c*x+1)+1/2*a/d^3/(c*x+1)^2+b/d^
3*arctanh(c*x)*c*x-3*b/d^3*arctanh(c*x)*ln(c*x+1)-3*b/d^3*arctanh(c*x)/(c*x
+1)+1/2*b/d^3*arctanh(c*x)/(c*x+1)^2-3/2*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+3
/2*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+3/2*b/d^3*dilog(1/2*c*x+1/2)+3/4*
b/d^3*ln(c*x+1)^2-3/16*b/d^3*ln(c*x-1)+1/8*b/d^3/(c*x+1)^2-11/8*b/d^3/(c*x+
1)+19/16*b/d^3*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(2*c^4*(2*(7*c*x + 6)/(c^10*d^3*x^2 + 2*c^9*d^3*x + c^8*d^3) - 8*x/(c
^7*d^3) + 17*log(c*x + 1)/(c^8*d^3) - log(c*x - 1)/(c^8*d^3)) - 32*c^4*inte
grate(1/2*x^4*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3
*d^3), x) - 6*c^3*(2*(5*c*x + 4)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*d^3) + 7*
log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c^7*d^3)) + 128*c^3*integrate(1/2*x^
3*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) +
288*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4
*d^3*x - c^3*d^3), x) + 9*c*(2*x/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - lo
g(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3)) + 288*c*integrate(1/2*x*log(
c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) + 8*(2*c
^3*x^3 + 4*c^2*x^2 - 4*c*x - 6*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) - 5)*log(
-c*x + 1)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) + 10*(c*x + 2)/(c^6*d^3*x^2
 + 2*c^5*d^3*x + c^4*d^3) - 5*log(c*x + 1)/(c^4*d^3) + 5*log(c*x - 1)/(c^4*
d^3) + 96*integrate(1/2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d
^3*x - c^3*d^3), x))*b - 1/2*a*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^
4*d^3) - 2*x/(c^3*d^3) + 6*log(c*x + 1)/(c^4*d^3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^3\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

3.60 $\int \frac{x^2(a+b\tanh^{-1}(cx))}{(d+cdx)^3} dx$

Optimal. Leaf size=150

$$-\frac{b}{8c^3d^3(1+cx)^2}+\frac{7b}{8c^3d^3(1+cx)}-\frac{7b\tanh^{-1}(cx)}{8c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2(a+b\tanh^{-1}(cx))}{c^3d^3(1+cx)}-\frac{(a+b\tanh^{-1}(\cdots}{c^3\cdots}$$

```
[Out] -1/8*b/c^3/d^3/(c*x+1)^2+7/8*b/c^3/d^3/(c*x+1)-7/8*b*arctanh(c*x)/c^3/d^3+1
/2*(-a-b*arctanh(c*x))/c^3/d^3/(c*x+1)^2+2*(a+b*arctanh(c*x))/c^3/d^3/(c*x+
1)-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^3/d^3+1/2*b*polylog(2,1-2/(c*x+1))/c^
3/d^3
```

Rubi [A]
time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6087, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{2(a+b\tanh^{-1}(cx))}{c^3d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(cx+1)^2}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{c^3d^3}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{7b}{8c^3d^3(cx+1)}-\frac{b}{8c^3d^3(cx+1)^2}-\frac{7b\tanh^{-1}(cx)}{8c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] -1/8*b/(c^3*d^3*(1 + c*x)^2) + (7*b)/(8*c^3*d^3*(1 + c*x)) - (7*b*ArcTanh[c
*x])/(8*c^3*d^3) - (a + b*ArcTanh[c*x])/(2*c^3*d^3*(1 + c*x)^2) + (2*(a + b
*ArcTanh[c*x]))/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)
])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(a+b\tanh^{-1}(cx))}{(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{c^2d^3(1+cx)^3} - \frac{2(a+b\tanh^{-1}(cx))}{c^2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{c^2d^3(1+cx)}\right) dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^2d^3} + \frac{\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^2d^3} - \frac{2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^2d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2} + \frac{2(a+b\tanh^{-1}(cx))}{c^3d^3(1+cx)} - \frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{c^3d^3} -$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2} + \frac{2(a+b\tanh^{-1}(cx))}{c^3d^3(1+cx)} - \frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{c^3d^3} -$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2} + \frac{2(a+b\tanh^{-1}(cx))}{c^3d^3(1+cx)} - \frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{c^3d^3} -$$

$$= -\frac{b}{8c^3d^3(1+cx)^2} + \frac{7b}{8c^3d^3(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2} + \frac{2(a+b\tanh^{-1}(cx))}{c^3d^3(1+cx)}$$

$$= -\frac{b}{8c^3d^3(1+cx)^2} + \frac{7b}{8c^3d^3(1+cx)} - \frac{7b\tanh^{-1}(cx)}{8c^3d^3} - \frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2} + \frac{2(}{}$$

Mathematica [A]

time = 0.29, size = 145, normalized size = 0.97

$$\frac{-\frac{16a}{(1+cx)^2} + \frac{64a}{1+cx} + 32a\log(1+cx) + b\left(12\cosh(2\tanh^{-1}(cx)) - \cosh(4\tanh^{-1}(cx)) + 16\text{PolyLog}\left(2, -e^{-2\tanh^{-1}(cx)}\right) - 12\sinh(2\tanh^{-1}(cx)) + \sinh(4\tanh^{-1}(cx)) + 4\tanh^{-1}(cx)\left(6\cosh(2\tanh^{-1}(cx)) - \cosh(4\tanh^{-1}(cx)) - 8\log\left(1+e^{-2\tanh^{-1}(cx)}\right) - 6\sinh(2\tanh^{-1}(cx)) + \sinh(4\tanh^{-1}(cx))\right)\right)}{32c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] ((-16*a)/(1 + c*x)^2 + (64*a)/(1 + c*x) + 32*a*Log[1 + c*x] + b*(12*Cosh[2*
ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])]
- 12*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2
*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*
Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^3*d^3)
```

Maple [A]

time = 0.32, size = 208, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{a\ln(cx+1)}{d^3} + \frac{2a}{d^3(cx+1)} - \frac{a}{2d^3(cx+1)^2} + \frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d^3} + \frac{2b\,\text{arctanh}(cx)}{d^3(cx+1)} - \frac{b\,\text{arctanh}(cx)}{2d^3(cx+1)^2} + \frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d^3} - \frac{b\ln\left(-\right.}{}}{c^3}$
default	$\frac{\frac{a\ln(cx+1)}{d^3} + \frac{2a}{d^3(cx+1)} - \frac{a}{2d^3(cx+1)^2} + \frac{b\,\text{arctanh}(cx)\ln(cx+1)}{d^3} + \frac{2b\,\text{arctanh}(cx)}{d^3(cx+1)} - \frac{b\,\text{arctanh}(cx)}{2d^3(cx+1)^2} + \frac{b\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{2d^3} - \frac{b\ln\left(-\right.}{}}{c^3}$

risch	$\frac{b\ln(cx+1)^2}{4c^3d^3} + \frac{\left(\frac{bx}{c^2}+\frac{3b}{4c^3}\right)\ln(cx+1)}{d^3(cx+1)^2} - \frac{7b\ln(-cx-1)}{16d^3c^3} - \frac{b\ln(-cx+1)x}{2d^3c^2(-cx-1)} + \frac{b\ln(-cx+1)}{2d^3c^3(-cx-1)} + \frac{b}{8d^3c^3(-cx-1)} - \frac{b\,l}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/d^3*ln(c*x+1)+2*a/d^3/(c*x+1)-1/2*a/d^3/(c*x+1)^2+b/d^3*arctanh(c*
x)*ln(c*x+1)+2*b/d^3*arctanh(c*x)/(c*x+1)-1/2*b/d^3*arctanh(c*x)/(c*x+1)^2+
1/2*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2*c*x+
1/2)-1/2*b/d^3*dilog(1/2*c*x+1/2)-1/4*b/d^3*ln(c*x+1)^2+7/16*b/d^3*ln(c*x-1
)-1/8*b/d^3/(c*x+1)^2+7/8*b/d^3/(c*x+1)-7/16*b/d^3*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(64*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 -
2*c^3*d^3*x - c^2*d^3), x) - 4*c^2*(2*(3*c*x + 2)/(c^7*d^3*x^2 + 2*c^6*d^3*
x + c^5*d^3) + log(c*x + 1)/(c^5*d^3) - log(c*x - 1)/(c^5*d^3)) + 64*c^2*in
tegrate(1/2*x^2*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c
^2*d^3), x) + 7*c*(2*x/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - log(c*x + 1)
/(c^4*d^3) + log(c*x - 1)/(c^4*d^3)) + 96*c*integrate(1/2*x*log(c*x + 1)/(c
^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x) - 8*(4*c*x + 2*(c^2
*x^2 + 2*c*x + 1)*log(c*x + 1) + 3)*log(-c*x + 1)/(c^5*d^3*x^2 + 2*c^4*d^3*
x + c^3*d^3) + 6*(c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*
x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3) + 32*integrate(1/2*log(c*x + 1)
/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x))*b + 1/2*a*((4*c
*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)/(c^3*d^3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^2\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

3.61 $\int \frac{x(a+b\tanh^{-1}(cx))}{(d+cdx)^3}\,dx$

Optimal. Leaf size=77

$$\frac{b}{8c^2d^3(1+cx)^2}-\frac{3b}{8c^2d^3(1+cx)}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}+\frac{x^2(a+b\tanh^{-1}(cx))}{2d^3(1+cx)^2}$$

```
[Out] 1/8*b/c^2/d^3/(c*x+1)^2-3/8*b/c^2/d^3/(c*x+1)-1/8*b*arctanh(c*x)/c^2/d^3+1/
2*x^2*(a+b*arctanh(c*x))/d^3/(c*x+1)^2
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {37, 6083, 12, 90, 213}

$$\frac{x^2(a+b\tanh^{-1}(cx))}{2d^3(cx+1)^2}-\frac{3b}{8c^2d^3(cx+1)}+\frac{b}{8c^2d^3(cx+1)^2}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] b/(8*c^2*d^3*(1 + c*x)^2) - (3*b)/(8*c^2*d^3*(1 + c*x)) - (b*ArcTanh[c*x])/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx &= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-(bc)\int \frac{x^2}{2(1-cx)(d+cdx)^3}\,dx\\
&= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-\frac{1}{2}(bc)\int \frac{x^2}{(1-cx)(d+cdx)^3}\,dx\\
&= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-\frac{1}{2}(bc)\int \left(\frac{1}{2c^2d^3(1+cx)^3}-\frac{3}{4c^2d^3(1+cx)^2}-\frac{}{4c^2d^3(}\right.\\
&= \frac{b}{8c^2d^3(1+cx)^2}-\frac{3b}{8c^2d^3(1+cx)}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}+\frac{b\int \frac{1}{-1+c^2x^2}\,dx}{8cd^3}\\
&= \frac{b}{8c^2d^3(1+cx)^2}-\frac{3b}{8c^2d^3(1+cx)}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 99, normalized size = 1.29

$$-\frac{8a+4b+16acx+6bcx+8(b+2bcx)\tanh^{-1}(cx)+3b(1+cx)^2\log(1-cx)-3b\log(1+cx)-6bcx\log(1+cx)-3bc^2x^2\log(1+cx)}{16c^2d^3(1+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] -1/16*(8*a + 4*b + 16*a*c*x + 6*b*c*x + 8*(b + 2*b*c*x)*ArcTanh[c*x] + 3*b*
(1 + c*x)^2*Log[1 - c*x] - 3*b*Log[1 + c*x] - 6*b*c*x*Log[1 + c*x] - 3*b*c^
2*x^2*Log[1 + c*x])/(c^2*d^3*(1 + c*x)^2)
```

Maple [A]
time = 0.16, size = 114, normalized size = 1.48

method	result

derivativedivides	$\frac{\frac{a\left(-\frac{1}{cx+1}+\frac{1}{2(cx+1)^2}\right)}{d^3}-\frac{b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}+\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}-\frac{3b\ln(cx-1)}{16d^3}+\frac{b}{8d^3(cx+1)^2}-\frac{3b}{8d^3(cx+1)}+\frac{3b\ln(cx+1)}{16d^3}}{c^2}$
default	$\frac{\frac{a\left(-\frac{1}{cx+1}+\frac{1}{2(cx+1)^2}\right)}{d^3}-\frac{b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}+\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}-\frac{3b\ln(cx-1)}{16d^3}+\frac{b}{8d^3(cx+1)^2}-\frac{3b}{8d^3(cx+1)}+\frac{3b\ln(cx+1)}{16d^3}}{c^2}$
risch	$-\frac{b(2cx+1)\ln(cx+1)}{4c^2d^3(cx+1)^2}+\frac{3b\,c^2\ln(-cx-1)x^2-3\ln(cx-1)b\,c^2x^2+6\ln(-cx-1)bcx-6\ln(cx-1)bcx+8bcx\ln(-cx+1)-16cx}{16c^2d^3(cx+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a/d^3*(-1/(c*x+1)+1/2/(c*x+1)^2)-b/d^3*arctanh(c*x)/(c*x+1)+1/2*b/d^
3*arctanh(c*x)/(c*x+1)^2-3/16*b/d^3*ln(c*x-1)+1/8*b/d^3/(c*x+1)^2-3/8*b/d^3
/(c*x+1)+3/16*b/d^3*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(69) = 138$.
time = 0.26, size = 152, normalized size = 1.97

$$-\frac{1}{16}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3}-\frac{3\,\log\left(cx+1\right)}{c^3d^3}+\frac{3\,\log\left(cx-1\right)}{c^3d^3}\right)+\frac{8\,(2\,cx+1)\,\mathrm{artanh}\left(cx\right)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}\right)b-\frac{(2\,cx+1)a}{2\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x +
 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c^4
*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*b - 1/2*(2*c*x + 1)*a/(c^4*d^3*x^2 + 2*c
^3*d^3*x + c^2*d^3)
```

Fricas [A]
time = 0.37, size = 84, normalized size = 1.09

$$-\frac{2\,(8\,a+3\,b)cx-(3\,bc^2x^2-2\,bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(8*a + 3*b)*c*x - (3*b*c^2*x^2 - 2*b*c*x - b)*log(-(c*x + 1)/(c*x
- 1)) + 8*a + 4*b)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(71) = 142$.
time = 0.84, size = 277, normalized size = 3.60

$$\begin{cases} -\frac{8acx}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{4a}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}+\frac{3bc^2x^2\,\mathrm{atanh}\left(cx\right)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{2bcx\,\mathrm{atanh}\left(cx\right)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{3bcx}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{b\,\mathrm{atanh}\left(cx\right)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3}-\frac{2b}{8c^4d^3x^2+16c^3d^3x+8c^2d^3} & \text{for}\: c \neq 0 \\ \frac{ax^2}{2d^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] Piecewise((-8*a*c*x/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 4*a
/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) + 3*b*c**2*x**2*atanh(c*
x)/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 2*b*c*x*atanh(c*x)/(
8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 3*b*c*x/(8*c**4*d**3*x**
2 + 16*c**3*d**3*x + 8*c**2*d**3) - b*atanh(c*x)/(8*c**4*d**3*x**2 + 16*c**
3*d**3*x + 8*c**2*d**3) - 2*b/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d
**3), Ne(c, 0)), (a*x**2/(2*d**3), True))
```

Giac [A]
time = 0.42, size = 114, normalized size = 1.48

$$\frac{1}{32}\,c\left(\frac{2\,(cx-1)^2\left(\frac{2\,(cx+1)b}{cx-1}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^3d^3}+\frac{(cx-1)^2\left(\frac{8\,(cx+1)a}{cx-1}+4\,a+\frac{4\,(cx+1)b}{cx-1}+b\right)}{(cx+1)^2c^3d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/32*c*(2*(c*x - 1)^2*(2*(c*x + 1)*b/(c*x - 1) + b)*log(-(c*x + 1)/(c*x - 1
))/((c*x + 1)^2*c^3*d^3) + (c*x - 1)^2*(8*(c*x + 1)*a/(c*x - 1) + 4*a + 4*(
c*x + 1)*b/(c*x - 1) + b)/((c*x + 1)^2*c^3*d^3))
```

Mupad [B]
time = 1.27, size = 81, normalized size = 1.05

$$\frac{c\,\left(b\,x-2\,b\,x\,\mathrm{atanh}(c\,x)\right)-b\,\mathrm{atanh}(c\,x)+c^2\,\left(4\,a\,x^2+2\,b\,x^2+3\,b\,x^2\,\mathrm{atanh}(c\,x)\right)}{8\,c^4\,d^3\,x^2+16\,c^3\,d^3\,x+8\,c^2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

```
[Out] (c*(b*x - 2*b*x*atanh(c*x)) - b*atanh(c*x) + c^2*(4*a*x^2 + 2*b*x^2 + 3*b*x
^2*atanh(c*x)))/(8*c^2*d^3 + 16*c^3*d^3*x + 8*c^4*d^3*x^2)
```

3.62 $\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=77

$$-\frac{b}{8cd^3(1+cx)^2}-\frac{b}{8cd^3(1+cx)}+\frac{b\tanh^{-1}(cx)}{8cd^3}-\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}$$

```
[Out] -1/8*b/c/d^3/(c*x+1)^2-1/8*b/c/d^3/(c*x+1)+1/8*b*arctanh(c*x)/c/d^3+1/2*(-a
-b*arctanh(c*x))/c/d^3/(c*x+1)^2
```

Rubi [A]

time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6063, 641, 46, 213}

$$-\frac{a+b\tanh^{-1}(cx)}{2cd^3(cx+1)^2}-\frac{b}{8cd^3(cx+1)}-\frac{b}{8cd^3(cx+1)^2}+\frac{b\tanh^{-1}(cx)}{8cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

```
[Out] -1/8*b/(c*d^3*(1 + c*x)^2) - b/(8*c*d^3*(1 + c*x)) + (b*ArcTanh[c*x])/(8*c*
d^3) - (a + b*ArcTanh[c*x])/(2*c*d^3*(1 + c*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^3}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}+\frac{b\int \frac{1}{(d+cdx)^2(1-c^2x^2)}\,dx}{2d}\\
&= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}+\frac{b\int \frac{1}{\left(\frac{1}{d}-\frac{cx}{d}\right)(d+cdx)^3}\,dx}{2d}\\
&= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}+\frac{b\int \left(\frac{1}{2d^2(1+cx)^3}+\frac{1}{4d^2(1+cx)^2}-\frac{1}{4d^2(-1+c^2x^2)}\right)dx}{2d}\\
&= -\frac{b}{8cd^3(1+cx)^2}-\frac{b}{8cd^3(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}-\frac{b\int \frac{1}{-1+c^2x^2}\,dx}{8d^3}\\
&= -\frac{b}{8cd^3(1+cx)^2}-\frac{b}{8cd^3(1+cx)}+\frac{b\tanh^{-1}(cx)}{8cd^3}-\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 1.12

$$\frac{-8a-4b-2bcx-8b\tanh^{-1}(cx)-b(1+cx)^2\log(1-cx)+b\log(1+cx)+2bcx\log(1+cx)+bc^2x^2\log(1+cx)}{16cd^3(1+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
[Out] (-8*a - 4*b - 2*b*c*x - 8*b*ArcTanh[c*x] - b*(1 + c*x)^2*Log[1 - c*x] + b*L
og[1 + c*x] + 2*b*c*x*Log[1 + c*x] + b*c^2*x^2*Log[1 + c*x])/(16*c*d^3*(1 +
 c*x)^2)
```

Maple [A]
time = 0.14, size = 86, normalized size = 1.12

method	result
derivativedivides	$\frac{-\frac{a}{2d^3(cx+1)^2}-\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}-\frac{b}{8d^3(cx+1)^2}-\frac{b}{8d^3(cx+1)}+\frac{b\ln(cx+1)}{16d^3}-\frac{b\ln(cx-1)}{16d^3}}{c}$
default	$\frac{-\frac{a}{2d^3(cx+1)^2}-\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}-\frac{b}{8d^3(cx+1)^2}-\frac{b}{8d^3(cx+1)}+\frac{b\ln(cx+1)}{16d^3}-\frac{b\ln(cx-1)}{16d^3}}{c}$

risch	$-\frac{b\ln(cx+1)}{4c\,d^3(cx+1)^2}-\frac{\ln(cx-1)b\,c^2x^2-b\,c^2\ln(-cx-1)x^2+2\ln(cx-1)bcx-2\ln(-cx-1)bcx+2bcx+b\ln(cx-1)-b\ln(-cx-1}{16d^3(cx+1)^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a/d^3/(c*x+1)^2-1/2*b/d^3*arctanh(c*x)/(c*x+1)^2-1/8*b/d^3/(c*x+1
)^2-1/8*b/d^3/(c*x+1)+1/16*b/d^3*ln(c*x+1)-1/16*b/d^3*ln(c*x-1))
```

Maxima [A]
time = 0.26, size = 134, normalized size = 1.74

$$-\frac{1}{16}\left(c\left(\frac{2\,(cx+2)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}-\frac{\log\left(cx+1\right)}{c^2d^3}+\frac{\log\left(cx-1\right)}{c^2d^3}\right)+\frac{8\ \mathrm{artanh}\left(cx\right)}{c^3d^3x^2+2\,c^2d^3x+cd^3}\right)b-\frac{a}{2\,(c^3d^3x^2+2\,c^2d^3x+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/
(c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*d
^3*x + c*d^3))*b - 1/2*a/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 0.97

$$-\frac{2\,bcx-(bc^2x^2+2\,bcx-3\,b)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\,(c^3d^3x^2+2\,c^2d^3x+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b*c*x - (b*c^2*x^2 + 2*b*c*x - 3*b)*log(-(c*x + 1)/(c*x - 1)) + 8*
a + 4*b)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(66) = 132$.
time = 0.77, size = 224, normalized size = 2.91

$$\begin{cases}-\frac{4a}{8c^3d^3x^2+16c^2d^3x+8cd^3}+\frac{bc^2x^2\,\mathrm{atanh}\,(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3}+\frac{2bcx\,\mathrm{atanh}\,(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3}-\frac{bcx}{8c^3d^3x^2+16c^2d^3x+8cd^3}-\frac{3b\,\mathrm{atanh}\,(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3}-\frac{2b}{8c^3d^3x^2+16c^2d^3x+8cd^3} & \text{for}\ c\neq 0\\ \frac{ax}{d^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] Piecewise((-4*a/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) + b*c**2*x**
2*atanh(c*x)/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) + 2*b*c*x*atanh
```

```
(c*x)/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) - b*c*x/(8*c**3*d**3*x
**2 + 16*c**2*d**3*x + 8*c*d**3) - 3*b*atanh(c*x)/(8*c**3*d**3*x**2 + 16*c*
*2*d**3*x + 8*c*d**3) - 2*b/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3),
 Ne(c, 0)), (a*x/d**3, True))
```

Giac [A]

time = 0.42, size = 118, normalized size = 1.53

$$\frac{1}{32}\,c\left(\frac{2\,(cx-1)^2\left(\frac{2\,(cx+1)b}{cx-1}-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^2d^3}+\frac{(cx-1)^2\left(\frac{8\,(cx+1)a}{cx-1}-4\,a+\frac{4\,(cx+1)b}{cx-1}-b\right)}{(cx+1)^2c^2d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/32*c*(2*(c*x - 1)^2*(2*(c*x + 1)*b/(c*x - 1) - b)*log(-(c*x + 1)/(c*x - 1
))/((c*x + 1)^2*c^2*d^3) + (c*x - 1)^2*(8*(c*x + 1)*a/(c*x - 1) - 4*a + 4*(
c*x + 1)*b/(c*x - 1) - b)/((c*x + 1)^2*c^2*d^3))
```

Mupad [B]

time = 1.09, size = 123, normalized size = 1.60

$$\frac{c^2\left(\frac{a\,x^2}{2}+\frac{b\,x^2}{4}-\frac{b\,x^2\,\ln\left(c^2\,x^2-1\right)}{16}+\frac{b\,x^2\,\ln(c\,x+1)}{8}\right)-\frac{b\,\ln\left(c^2\,x^2-1\right)}{16}-\frac{b\,\mathrm{atanh}(c\,x)}{2}+\frac{b\,\ln(c\,x+1)}{8}+c\left(a\,x+\frac{3\,b\,x}{8}+\frac{b\,x\,\ln(c\,x+1)}{4}-\frac{b\,x\,\ln\left(c^2\,x^2-1\right)}{8}\right)}{c\,d^3\,(c\,x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + c*d*x)^3,x)
```

```
[Out] (c^2*((a*x^2)/2 + (b*x^2)/4 - (b*x^2*log(c^2*x^2 - 1))/16 + (b*x^2*log(c*x
+ 1))/8) - (b*log(c^2*x^2 - 1))/16 - (b*atanh(c*x))/2 + (b*log(c*x + 1))/8
+ c*(a*x + (3*b*x)/8 + (b*x*log(c*x + 1))/4 - (b*x*log(c^2*x^2 - 1))/8))/(c
*d^3*(c*x + 1)^2)
```

3.63 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^3}\,dx$

Optimal. Leaf size=161

$$\frac{b}{8d^3(1+cx)^2}+\frac{5b}{8d^3(1+cx)}-\frac{5b\tanh^{-1}(cx)}{8d^3}+\frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}+\frac{a\log(x)}{d^3}+\frac{(a+b\tanh^{-1}(c}{d^3}$$

```
[Out] 1/8*b/d^3/(c*x+1)^2+5/8*b/d^3/(c*x+1)-5/8*b*arctanh(c*x)/d^3+1/2*(a+b*arcta
nh(c*x))/d^3/(c*x+1)^2+(a+b*arctanh(c*x))/d^3/(c*x+1)+a*ln(x)/d^3+(a+b*arct
anh(c*x))*ln(2/(c*x+1))/d^3-1/2*b*polylog(2,-c*x)/d^3+1/2*b*polylog(2,c*x)/
d^3-1/2*b*polylog(2,1-2/(c*x+1))/d^3
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6087, 6031, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{a+b\tanh^{-1}(cx)}{d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2d^3(cx+1)^2}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{a\log(x)}{d^3}-\frac{b\text{Li}_2(-cx)}{2d^3}+\frac{b\text{Li}_2(cx)}{2d^3}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^3}+\frac{5b}{8d^3(cx+1)}+\frac{b}{8d^3(cx+1)^2}-\frac{5b\tanh^{-1}(cx)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

```
[Out] b/(8*d^3*(1 + c*x)^2) + (5*b)/(8*d^3*(1 + c*x)) - (5*b*ArcTanh[c*x])/(8*d^3
) + (a + b*ArcTanh[c*x])/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])/(d^3*(1
 + c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (
b*PolyLog[2, -(c*x)])/(2*d^3) + (b*PolyLog[2, c*x])/(2*d^3) - (b*PolyLog[2,
 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c(a+b\tanh^{-1}(cx))}{d^3(1+cx)^3} - \frac{c(a+b\tanh^{-1}(cx))}{d^3(1+cx)^2} - \frac{c(a+b\tan}{d^3(1+}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^3} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^3}$$

$$= \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3}$$

$$= \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3}$$

$$= \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} + \frac{(a+b\tanh^{-1}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3}$$

$$= \frac{b}{8d^3(1+cx)^2} + \frac{5b}{8d^3(1+cx)} + \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)} + \frac{a\log(x)}{d^3} +$$

$$= \frac{b}{8d^3(1+cx)^2} + \frac{5b}{8d^3(1+cx)} - \frac{5b\tanh^{-1}(cx)}{8d^3} + \frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2} + \frac{a+b\tanh^{-1}(c}{d^3(1+cx)}$$

Mathematica [A]
time = 0.31, size = 147, normalized size = 0.91

$$\frac{\frac{16a}{(1+cx)^2}+\frac{32a}{1+cx}+32a\log(x)-32a\log(1+cx)+b\left(12\cosh(2\tanh^{-1}(cx))+\cosh(4\tanh^{-1}(cx))-16\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-12\sinh(2\tanh^{-1}(cx))+4\tanh^{-1}(cx)\left(6\cosh(2\tanh^{-1}(cx))+\cosh(4\tanh^{-1}(cx))+8\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-6\sinh(2\tanh^{-1}(cx))-\sinh(4\tanh^{-1}(cx))\right)-\sinh(4\tanh^{-1}(cx))\right)}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

```
[Out] ((16*a)/(1 + c*x)^2 + (32*a)/(1 + c*x) + 32*a*Log[x] - 32*a*Log[1 + c*x] +
b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*PolyLog[2, E^(-2*Arc
Tanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*
x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcT
anh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

Maple [A]
time = 0.23, size = 264, normalized size = 1.64

method	result
derivativedivides	$\frac{a}{2d^3(cx+1)^2}+\frac{a}{d^3(cx+1)}-\frac{a\ln(cx+1)}{d^3}+\frac{a\ln(cx)}{d^3}+\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}+\frac{b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}-\frac{b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^3}+$
default	$\frac{a}{2d^3(cx+1)^2}+\frac{a}{d^3(cx+1)}-\frac{a\ln(cx+1)}{d^3}+\frac{a\ln(cx)}{d^3}+\frac{b\,\mathrm{arctanh}(cx)}{2d^3(cx+1)^2}+\frac{b\,\mathrm{arctanh}(cx)}{d^3(cx+1)}-\frac{b\,\mathrm{arctanh}(cx)\ln(cx+1)}{d^3}+$

risch	$-\frac{5b\ln(-cx-1)}{16d^3}-\frac{b\ln(-cx+1)cx}{4d^3(-cx-1)}+\frac{b\ln(-cx+1)}{4d^3(-cx-1)}-\frac{b}{8d^3(-cx-1)}+\frac{b\ln(-cx+1)x^2c^2}{16d^3(-cx-1)^2}+\frac{b\ln(-cx+1)cx}{8d^3(-cx-1)^2}-\frac{3b}{16c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/d^3/(c*x+1)^2+a/d^3/(c*x+1)-a/d^3*ln(c*x+1)+a/d^3*ln(c*x)+1/2*b/d^3*a
rctanh(c*x)/(c*x+1)^2+b/d^3*arctanh(c*x)/(c*x+1)-b/d^3*arctanh(c*x)*ln(c*x+
1)+b/d^3*arctanh(c*x)*ln(c*x)-1/2*b/d^3*dilog(c*x)-1/2*b/d^3*dilog(c*x+1)-1
/2*b/d^3*ln(c*x)*ln(c*x+1)-1/2*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d^3*l
n(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/2*b/d^3*dilog(1/2*c*x+1/2)+1/4*b/d^3*ln(c
*x+1)^2+1/8*b/d^3/(c*x+1)^2+5/8*b/d^3/(c*x+1)-5/16*b/d^3*ln(c*x+1)+5/16*b/d
^3*ln(c*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 + 2
*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^4
+ 3*c^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^4 + 3*c^2*d^3*x^3 + 3*c*d^3*x^2 +
d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3x^4+3c^2x^3+3cx^2+x}\,dx+\int \frac{b\,\operatorname{atanh}(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b*atanh
(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x\,{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))/(x*(d + c*d*x)^3), x)
```

3.64 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^3}\,dx$

Optimal. Leaf size=218

$$-\frac{bc}{8d^3(1+cx)^2}-\frac{9bc}{8d^3(1+cx)}+\frac{9bc\tanh^{-1}(cx)}{8d^3}-\frac{a+b\tanh^{-1}(cx)}{d^3x}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-\frac{2c\left(a+b\tanh^{-1}(c\right.}{d^3(1+cx)}$$

```
[Out] -1/8*b*c/d^3/(c*x+1)^2-9/8*b*c/d^3/(c*x+1)+9/8*b*c*arctanh(c*x)/d^3+(-a-b*a
rctanh(c*x))/d^3/x-1/2*c*(a+b*arctanh(c*x))/d^3/(c*x+1)^2-2*c*(a+b*arctanh(
c*x))/d^3/(c*x+1)-3*a*c*ln(x)/d^3+b*c*ln(x)/d^3-3*c*(a+b*arctanh(c*x))*ln(2
/(c*x+1))/d^3-1/2*b*c*ln(-c^2*x^2+1)/d^3+3/2*b*c*polylog(2,-c*x)/d^3-3/2*b*
c*polylog(2,c*x)/d^3+3/2*b*c*polylog(2,1-2/(c*x+1))/d^3
```

Rubi [A]
time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6087, 6037, 272, 36, 29, 31, 6031, 6063, 641, 46, 213, 6055, 2449, 2352}

$$-\frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}-\frac{a+b\tanh^{-1}(cx)}{d^3x}-\frac{3c\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{3ac\log(x)}{d^3}-\frac{bc\log\left(1-c^2x^2\right)}{2d^3}+\frac{3bc\text{Li}_2(-cx)}{2d^3}-\frac{3bc\text{Li}_2(cx)}{2d^3}+\frac{3bc\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^3}-\frac{9bc}{8d^3(cx+1)}-\frac{bc}{8d^3(cx+1)^2}+\frac{bc\log(x)}{d^3}+\frac{9bc\tanh^{-1}(cx)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

```
[Out] -1/8*(b*c)/(d^3*(1 + c*x)^2) - (9*b*c)/(8*d^3*(1 + c*x)) + (9*b*c*ArcTanh[c
*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(d^3*x) - (c*(a + b*ArcTanh[c*x]))/(2*d
^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) - (3*a*c*Log[x
])/d^3 + (b*c*Log[x])/d^3 - (3*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3
 - (b*c*Log[1 - c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, -(c*x)])/(2*d^3) - (3
*b*c*PolyLog[2, c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x^2} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^3} + \frac{2c^2\left(a+\right.}{d^3(}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{(3c)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} + \frac{c^2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3} + \frac{(2c^2)\int \frac{a+b\tanh}{(1+c}}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} -$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} -$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} -$$

$$= -\frac{bc}{8d^3(1+cx)^2} - \frac{9bc}{8d^3(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c(a-}{c}$$

$$= -\frac{bc}{8d^3(1+cx)^2} - \frac{9bc}{8d^3(1+cx)} + \frac{9bc\tanh^{-1}(cx)}{8d^3} - \frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c(a+b\tan}{2d^3(1+}$$

Mathematica [A]
time = 0.81, size = 186, normalized size = 0.85

$$\frac{-\frac{32a}{x} - \frac{16ac}{(1+cx)^2} - \frac{64ac}{1+cx} - 96ac\log(x) + 96ac\log(1+cx) + bc\left(-20\cosh\left(2\tanh^{-1}(cx)\right) - \cosh\left(4\tanh^{-1}(cx)\right) + 32\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 48\text{PolyLog}\left(2, e^{-2\tanh^{-1}(cx)}\right) + 20\sinh\left(2\tanh^{-1}(cx)\right) + \sinh\left(4\tanh^{-1}(cx)\right) + 4\tanh^{-1}(cx)\left(-\frac{8}{cx} - 10\cosh\left(2\tanh^{-1}(cx)\right) - \cosh\left(4\tanh^{-1}(cx)\right) - 24\log\left(1-e^{-2\tanh^{-1}(cx)}\right) + 10\sinh\left(2\tanh^{-1}(cx)\right) + \sinh\left(4\tanh^{-1}(cx)\right)\right)\right)}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

```
[Out] ((-32*a)/x - (16*a*c)/(1 + c*x)^2 - (64*a*c)/(1 + c*x) - 96*a*c*Log[x] + 96
*a*c*Log[1 + c*x] + b*c*(-20*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] +
32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 48*PolyLog[2, E^(-2*ArcTanh[c*x])] + 20*S
inh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-8/(c*x) - 10*
Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 24*Log[1 - E^(-2*ArcTanh[c*x]
)] + 10*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*d^3)
```

Maple [A]
time = 0.23, size = 307, normalized size = 1.41

method	result
derivativedivides	$c\left(-\frac{a}{d^3cx} - \frac{3a\ln(cx)}{d^3} - \frac{a}{2d^3(cx+1)^2} - \frac{2a}{d^3(cx+1)} + \frac{3a\ln(cx+1)}{d^3} - \frac{b\,\mathrm{arctanh}(cx)}{d^3cx} - \frac{3b\,\mathrm{arctanh}(cx)\ln(cx)}{d^3} - \right.$
default	$c\left(-\frac{a}{d^3cx} - \frac{3a\ln(cx)}{d^3} - \frac{a}{2d^3(cx+1)^2} - \frac{2a}{d^3(cx+1)} + \frac{3a\ln(cx+1)}{d^3} - \frac{b\,\mathrm{arctanh}(cx)}{d^3cx} - \frac{3b\,\mathrm{arctanh}(cx)\ln(cx)}{d^3} - \right.$

risch	$-\frac{bc\ln(cx+1)}{4d^3(cx+1)^2}-\frac{bc\ln(cx+1)}{d^3(cx+1)}-\frac{cb\ln(-cx+1)}{2d^3(-cx-1)}+\frac{3cb\ln(-cx+1)}{16d^3(-cx-1)^2}-\frac{3cb\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{2d^3}+\frac{3cb\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/d^3/c/x-3*a/d^3*ln(c*x)-1/2*a/d^3/(c*x+1)^2-2*a/d^3/(c*x+1)+3*a/d^3*l
n(c*x+1)-b/d^3*arctanh(c*x)/c/x-3*b/d^3*arctanh(c*x)*ln(c*x)-1/2*b/d^3*arct
anh(c*x)/(c*x+1)^2-2*b/d^3*arctanh(c*x)/(c*x+1)+3*b/d^3*arctanh(c*x)*ln(c*x
+1)-17/16*b/d^3*ln(c*x-1)+b/d^3*ln(c*x)-1/8*b/d^3/(c*x+1)^2-9/8*b/d^3/(c*x+
1)+1/16*b/d^3*ln(c*x+1)+3/2*b/d^3*dilog(c*x)+3/2*b/d^3*dilog(c*x+1)+3/2*b/d
^3*ln(c*x)*ln(c*x+1)-3/2*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+3/2*b/d^3*l
n(-1/2*c*x+1/2)*ln(c*x+1)-3/2*b/d^3*dilog(1/2*c*x+1/2)-3/4*b/d^3*ln(c*x+1)^
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c*l
og(c*x + 1)/d^3 + 6*c*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*
x + 1))/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 +
d^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx+\int \frac{b\,\mathrm{atanh}\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b*at
anh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^2\,{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^3), x)
```

3.65 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^3}\,dx$

Optimal. Leaf size=268

$$-\frac{bc}{2d^3x}+\frac{bc^2}{8d^3(1+cx)^2}+\frac{13bc^2}{8d^3(1+cx)}-\frac{9bc^2\tanh^{-1}(cx)}{8d^3}-\frac{a+b\tanh^{-1}(cx)}{2d^3x^2}+\frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x}+\frac{c^2\left(a+b\right.}{2d^3(}$$

```
[Out] -1/2*b*c/d^3/x+1/8*b*c^2/d^3/(c*x+1)^2+13/8*b*c^2/d^3/(c*x+1)-9/8*b*c^2*arc
tanh(c*x)/d^3+1/2*(-a-b*arctanh(c*x))/d^3/x^2+3*c*(a+b*arctanh(c*x))/d^3/x+
1/2*c^2*(a+b*arctanh(c*x))/d^3/(c*x+1)^2+3*c^2*(a+b*arctanh(c*x))/d^3/(c*x+
1)+6*a*c^2*ln(x)/d^3-3*b*c^2*ln(x)/d^3+6*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1
))/d^3+3/2*b*c^2*ln(-c^2*x^2+1)/d^3-3*b*c^2*polylog(2,-c*x)/d^3+3*b*c^2*pol
ylog(2,c*x)/d^3-3*b*c^2*polylog(2,1-2/(c*x+1))/d^3
```

Rubi [A]
time = 0.23, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6087, 6037, 331, 212, 272, 36, 29, 31, 6031, 6063, 641, 46, 213, 6055, 2449, 2352}

$$\frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}+\frac{6c^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{a+b\tanh^{-1}(cx)}{2d^3x^2}+\frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x}+\frac{6ac^2\log(x)}{d^3}-\frac{3bc^2\text{Li}_2(-cx)}{d^3}+\frac{3bc^2\text{Li}_2(cx)}{d^3}-\frac{3bc^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{d^3}+\frac{3bc^2\log\left(1-c^2x^2\right)}{2d^3}+\frac{13bc^2}{8d^3(cx+1)}+\frac{bc^2}{8d^3(cx+1)^2}-\frac{3bc^2\log(x)}{d^3}-\frac{9bc^2\tanh^{-1}(cx)}{8d^3}-\frac{bc}{2d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

```
[Out] -1/2*(b*c)/(d^3*x) + (b*c^2)/(8*d^3*(1 + c*x)^2) + (13*b*c^2)/(8*d^3*(1 + c
*x)) - (9*b*c^2*ArcTanh[c*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(2*d^3*x^2) +
(3*c*(a + b*ArcTanh[c*x]))/(d^3*x) + (c^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 +
 c*x)^2) + (3*c^2*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) + (6*a*c^2*Log[x])/
d^3 - (3*b*c^2*Log[x])/d^3 + (6*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/
d^3 + (3*b*c^2*Log[1 - c^2*x^2])/(2*d^3) - (3*b*c^2*PolyLog[2, -(c*x)])/d^3
 + (3*b*c^2*PolyLog[2, c*x])/d^3 - (3*b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/d^
3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x^3} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x^2} + \frac{6c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3x} - \frac{c^3\left(a+\right.}{d^3(}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3c)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^3} + \frac{(6c^2)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{c^3\int \frac{a+b\tanh}{(1+c}}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{3c^2\left(a+b\tanh\right.}{d^3(1+}$$

$$= -\frac{bc}{2d^3x} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{3c^2}{}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2\tanh^{-1}(cx)}{2d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh\right.}{2d^3(1}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2}{8d^3(1+cx)^2} + \frac{13bc^2}{8d^3(1+cx)} + \frac{bc^2\tanh^{-1}(cx)}{2d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c(}{}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2}{8d^3(1+cx)^2} + \frac{13bc^2}{8d^3(1+cx)} - \frac{9bc^2\tanh^{-1}(cx)}{8d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c}{}$$

Mathematica [A]
time = 0.93, size = 220, normalized size = 0.82

$$\frac{-\frac{16a}{x^2} + \frac{96ac}{x} + \frac{16ac^2}{(1+cx)^2} + \frac{96ac^2}{1+cx} + 192ac^2\log(x) - 192ac^2\log(1+cx) + bc^2\left(-\frac{16}{cx} + 28\cosh\left(2\tanh^{-1}(cx)\right) + \cosh\left(4\tanh^{-1}(cx)\right) - 96\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) - 96\text{PolyLog}\left(2, e^{-2\tanh^{-1}(cx)}\right) - 28\sinh\left(2\tanh^{-1}(cx)\right) + 4\tanh^{-1}(cx)\left(4 - \frac{4}{c^2x^2} + \frac{24}{cx} + 14\cosh\left(2\tanh^{-1}(cx)\right) + \cosh\left(4\tanh^{-1}(cx)\right) + 48\log\left(1-e^{-2\tanh^{-1}(cx)}\right) - 14\sinh\left(2\tanh^{-1}(cx)\right) - \sinh\left(4\tanh^{-1}(cx)\right)\right) - \sinh\left(4\tanh^{-1}(cx)\right)\right)}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

```
[Out] ((-16*a)/x^2 + (96*a*c)/x + (16*a*c^2)/(1 + c*x)^2 + (96*a*c^2)/(1 + c*x) +
 192*a*c^2*Log[x] - 192*a*c^2*Log[1 + c*x] + b*c^2*(-16/(c*x) + 28*Cosh[2*A
rcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 96*Log[(c*x)/Sqrt[1 - c^2*x^2]] - 96*
PolyLog[2, E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*
(4 - 4/(c^2*x^2) + 24/(c*x) + 14*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]
] + 48*Log[1 - E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcT
anh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

Maple [A]
time = 0.35, size = 350, normalized size = 1.31

method	result
derivativedivides	$c^2\left(-\frac{b\,\mathrm{arctanh}(cx)}{2d^3c^2x^2} + \frac{3b\,\mathrm{arctanh}(cx)}{d^3cx} - \frac{a}{2d^3c^2x^2} - \frac{3b\,\mathrm{dilog}(cx)}{d^3} - \frac{3b\,\mathrm{dilog}(cx+1)}{d^3} - \frac{3b\ln(cx)}{d^3} + \frac{6a\ln(cx)}{d^3} + \right.$

default	$c^2\left(-\frac{b\,\mathrm{arctanh}(cx)}{2d^3c^2x^2}+\frac{3b\,\mathrm{arctanh}(cx)}{d^3cx}-\frac{a}{2d^3c^2x^2}-\frac{3b\,\mathrm{dilog}(cx)}{d^3}-\frac{3b\,\mathrm{dilog}(cx+1)}{d^3}-\frac{3b\ln(cx)}{d^3}+\frac{6a\ln(cx)}{d^3}+\right.$
risch	$-\frac{3cb\ln(-cx+1)}{2d^3x}-\frac{3c^2b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{d^3}+\frac{3c^2b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{d^3}+\frac{3c^2b\ln(-cx+1)}{4d^3(-cx-1)}-\frac{3c^2b\ln(-cx+}{16d^3(-cx-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3*b/d^3*arctanh(c*x)/c/x-1/2*a/d^3/c^2/x^2-3*b/d^3*ln(c*x)+6*a/d^3*ln(
c*x)-3*b/d^3*dilog(c*x)-3*b/d^3*dilog(c*x+1)+1/2*a/d^3/(c*x+1)^2+3*a/d^3/(c
*x+1)-6*a/d^3*ln(c*x+1)+3/2*b/d^3*ln(c*x+1)^2+3*b/d^3*dilog(1/2*c*x+1/2)+15
/16*b/d^3*ln(c*x+1)+33/16*b/d^3*ln(c*x-1)+3*a/d^3/c/x-6*b/d^3*arctanh(c*x)*
ln(c*x+1)-3*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+3*b/d^3*ln(-1/2*c*x+1/2)*ln(1/
2*c*x+1/2)+1/2*b/d^3*arctanh(c*x)/(c*x+1)^2+3*b/d^3*arctanh(c*x)/(c*x+1)+6*
b/d^3*arctanh(c*x)*ln(c*x)-3*b/d^3*ln(c*x)*ln(c*x+1)+1/8*b/d^3/(c*x+1)^2+13
/8*b/d^3/(c*x+1)-1/2*b/d^3*arctanh(c*x)/c^2/x^2-1/2*b/d^3/c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((12*c^3*x^3 + 18*c^2*x^2 + 4*c*x - 1)/(c^2*d^3*x^4 + 2*c*d^3*x^3 + d
^3*x^2) - 12*c^2*log(c*x + 1)/d^3 + 12*c^2*log(x)/d^3) + 1/2*b*integrate((l
og(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 + d
^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 +
d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3x^6+3c^2x^5+3cx^4+x^3}\,dx+\int \frac{b\,\mathrm{atanh}\,(cx)}{c^3x^6+3c^2x^5+3cx^4+x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x) + Integral(b*at
anh(c*x)/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^3\,{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^3), x)
```

3.66 $\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^4}\,dx$

Optimal. Leaf size=80

$$-\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}+\frac{b\tanh^{-1}(cx)}{24c}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}$$

```
[Out] -1/18*b/c/(c*x+1)^3-1/24*b/c/(c*x+1)^2-1/24*b/c/(c*x+1)+1/24*b*arctanh(c*x)
/c+1/3*(-a-b*arctanh(c*x))/c/(c*x+1)^3
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6063, 641, 46, 213}

$$-\frac{a+b\tanh^{-1}(cx)}{3c(cx+1)^3}-\frac{b}{24c(cx+1)}-\frac{b}{24c(cx+1)^2}-\frac{b}{18c(cx+1)^3}+\frac{b\tanh^{-1}(cx)}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

```
[Out] -1/18*b/(c*(1 + c*x)^3) - b/(24*c*(1 + c*x)^2) - b/(24*c*(1 + c*x)) + (b*Ar
cTanh[c*x])/(24*c) - (a + b*ArcTanh[c*x])/(3*c*(1 + c*x)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^4}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \frac{1}{(1+cx)^3\,(1-c^2x^2)}\,dx\\
&= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \frac{1}{(1-cx)(1+cx)^4}\,dx\\
&= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \left(\frac{1}{2(1+cx)^4}+\frac{1}{4(1+cx)^3}+\frac{1}{8(1+cx)^2}-\frac{1}{8\left(-1+c^2x\right.}\right.\\
&= -\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}-\frac{1}{24}b\int \frac{1}{-1+}\\
&= -\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}+\frac{b\tanh^{-1}(cx)}{24c}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.94

$$-\frac{48a+2b(10+9cx+3c^2x^2)+48b\tanh^{-1}(cx)+3b(1+cx)^3\log(1-cx)-3b(1+cx)^3\log(1+cx)}{144c(1+cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

```
[Out] -1/144*(48*a + 2*b*(10 + 9*c*x + 3*c^2*x^2) + 48*b*ArcTanh[c*x] + 3*b*(1 +
c*x)^3*Log[1 - c*x] - 3*b*(1 + c*x)^3*Log[1 + c*x])/(c*(1 + c*x)^3)
```

Maple [A]
time = 0.16, size = 78, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{a}{3(cx+1)^3}-\frac{b\,\mathrm{arctanh}(cx)}{3(cx+1)^3}-\frac{b}{18(cx+1)^3}-\frac{b}{24(cx+1)^2}-\frac{b}{24(cx+1)}+\frac{b\ln(cx+1)}{48}-\frac{b\ln(cx-1)}{48}}{c}$
default	$\frac{-\frac{a}{3(cx+1)^3}-\frac{b\,\mathrm{arctanh}(cx)}{3(cx+1)^3}-\frac{b}{18(cx+1)^3}-\frac{b}{24(cx+1)^2}-\frac{b}{24(cx+1)}+\frac{b\ln(cx+1)}{48}-\frac{b\ln(cx-1)}{48}}{c}$
risch	$-\frac{b\ln(cx+1)}{6c(cx+1)^3}-\frac{3\ln(cx-1)b\,c^3x^3-3\ln(-cx-1)b\,c^3x^3+9\ln(cx-1)b\,c^2x^2-9b\,c^2\ln(-cx-1)x^2+6b\,c^2x^2+9\ln(cx-1)bcx-}{144(cx+1)^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/3*a/(c*x+1)^3-1/3*b/(c*x+1)^3*arctanh(c*x)-1/18*b/(c*x+1)^3-1/24*b/
(c*x+1)^2-1/24*b/(c*x+1)+1/48*b*ln(c*x+1)-1/48*b*ln(c*x-1))
```

Maxima [A]
time = 0.26, size = 132, normalized size = 1.65

$$-\frac{1}{144}\left(c\left(\frac{2\,(3\,c^2x^2+9\,cx+10)}{c^5x^3+3\,c^4x^2+3\,c^3x+c^2}-\frac{3\,\log\left(cx+1\right)}{c^2}+\frac{3\,\log\left(cx-1\right)}{c^2}\right)+\frac{48\,\operatorname{artanh}\left(cx\right)}{c^4x^3+3\,c^3x^2+3\,c^2x+c}\right)b-\frac{a}{3\,(c^4x^3+3\,c^3x^2+3\,c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/144*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
 - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*
c^3*x^2 + 3*c^2*x + c))*b - 1/3*a/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.14

$$-\frac{6\,bc^2x^2+18\,bcx-3\,(bc^3x^3+3\,bc^2x^2+3\,bcx-7\,b)\log\left(-\frac{cx+1}{cx-1}\right)+48\,a+20\,b}{144\,(c^4x^3+3\,c^3x^2+3\,c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/144*(6*b*c^2*x^2 + 18*b*c*x - 3*(b*c^3*x^3 + 3*b*c^2*x^2 + 3*b*c*x - 7*b
)*log(-(c*x + 1)/(c*x - 1)) + 48*a + 20*b)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x +
 c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(65) = 130$.
time = 1.18, size = 294, normalized size = 3.68

$$\begin{cases} -\frac{24a}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{3bc^3x^3\operatorname{atanh}(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{9bc^2x^2\operatorname{atanh}(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c}-\frac{3bc^2x^2}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{9bcx\operatorname{atanh}(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c}-\frac{9bcx}{72c^4x^3+216c^3x^2+216c^2x+72c}-\frac{21b\operatorname{atanh}(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c}-\frac{10b}{72c^4x^3+216c^3x^2+216c^2x+72c} & \text{for } c \neq 0 \\ ax & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*x+1)**4,x)
```

```
[Out] Piecewise((-24*a/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 3*b*c
**3*x**3*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*
b*c**2*x**2*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) -
```

```
3*b*c**2*x**2/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*b*c*x
*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 9*b*c*x/(7
2*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 21*b*atanh(c*x)/(72*c**4
*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 10*b/(72*c**4*x**3 + 216*c**3*
x**2 + 216*c**2*x + 72*c), Ne(c, 0)), (a*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(70) = 140.
time = 0.41, size = 161, normalized size = 2.01

$$\frac{1}{288}\,c\left(\frac{6\,(cx-1)^3\left(\frac{3\,(cx+1)^2b}{(cx-1)^2}-\frac{3\,(cx+1)b}{cx-1}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^3c^2}+\frac{(cx-1)^3\left(\frac{36\,(cx+1)^2a}{(cx-1)^2}-\frac{36\,(cx+1)a}{cx-1}+12\,a+\frac{18\,(cx+1)^2b}{(cx-1)^2}-\frac{9\,(cx+1)b}{cx-1}+2\,b\right)}{(cx+1)^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/288*c*(6*(c*x - 1)^3*(3*(c*x + 1)^2*b/(c*x - 1)^2 - 3*(c*x + 1)*b/(c*x -
1) + b)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (c*x - 1)^3*(36*(c*x
+ 1)^2*a/(c*x - 1)^2 - 36*(c*x + 1)*a/(c*x - 1) + 12*a + 18*(c*x + 1)^2*b/(
c*x - 1)^2 - 9*(c*x + 1)*b/(c*x - 1) + 2*b)/((c*x + 1)^3*c^2))
```

Mupad [B]
time = 1.10, size = 139, normalized size = 1.74

$$\frac{\frac{b\,c^2\,x^3}{8}-\frac{b\,x}{8}-\frac{b\,\mathrm{atanh}(c\,x)}{3\,c}-\frac{12\,a+5\,b}{36\,c}+\frac{b\,c^3\,x^4}{24}+\frac{c\,x^2\,(24\,a+7\,b)}{72}+\frac{b\,c\,x^2\,\mathrm{atanh}(c\,x)}{3}}{-c^5\,x^5-3\,c^4\,x^4-2\,c^3\,x^3+2\,c^2\,x^2+3\,c\,x+1}-\frac{b\,\ln\left(c^2\,x^2-1\right)}{48\,c}+\frac{b\,\ln\left(c\,x+1\right)}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(c*x + 1)^4,x)
```

```
[Out] ((b*c^2*x^3)/8 - (b*x)/8 - (b*atanh(c*x))/(3*c) - (12*a + 5*b)/(36*c) + (b*
c^3*x^4)/24 + (c*x^2*(24*a + 7*b))/72 + (b*c*x^2*atanh(c*x))/3)/(3*c*x + 2*
c^2*x^2 - 2*c^3*x^3 - 3*c^4*x^4 - c^5*x^5 + 1) - (b*log(c^2*x^2 - 1))/(48*c
) + (b*log(c*x + 1))/(24*c)
```

3.67 $\int \frac{\tanh^{-1}(ax)}{cx+acx^2}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c}$$

```
[Out] arctanh(a*x)*ln(2-2/(a*x+1))/c-1/2*polylog(2,-1+2/(a*x+1))/c
```

Rubi [A]
time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1607, 6079, 2497}

$$\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{c}-\frac{\text{Li}_2\left(\frac{2}{ax+1}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - PolyLog[2, -1 + 2/(1 + a*x)]/(2*c)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{cx+acx^2}\,dx = \int \frac{\tanh^{-1}(ax)}{x(c+acx)}\,dx$$
$$= \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c}$$

Mathematica [A]
time = 0.05, size = 39, normalized size = 0.95

$$\frac{\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)}{c} - \frac{\mathrm{PolyLog}\left(2, e^{-2\tanh^{-1}(ax)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])])/c - PolyLog[2, E^(-2*ArcTanh[a*
x])]/(2*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(39) = 78$.
time = 0.15, size = 112, normalized size = 2.73

method	result
risch	$-\frac{\ln(ax+1)^2}{4c} - \frac{\mathrm{dilog}(ax+1)}{2c} + \frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln(-ax+1)}{2c} - \frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2c} + \frac{\mathrm{dilog}(-ax+1)}{2c} - \frac{\mathrm{dilog}\left(-\frac{ax}{2}+\right.}{2c}$
derivativedivides	$\frac{-\frac{a\,\mathrm{arctanh}(ax)\ln(ax+1)}{c} + \frac{a\,\mathrm{arctanh}(ax)\ln(ax)}{c} - \frac{a\left(\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\ln(ax+1)^2}{4} + \frac{\mathrm{dilog}(ax)}{2}\right.}{c}}{a}$
default	$\frac{-\frac{a\,\mathrm{arctanh}(ax)\ln(ax+1)}{c} + \frac{a\,\mathrm{arctanh}(ax)\ln(ax)}{c} - \frac{a\left(\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\ln(ax+1)^2}{4} + \frac{\mathrm{dilog}(ax)}{2}\right.}{c}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-a/c*arctanh(a*x)*ln(a*x+1)+a/c*arctanh(a*x)*ln(a*x)-a/c*(1/2*(ln(a*x+
1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x+1)^2
+1/2*dilog(a*x)+1/2*dilog(a*x+1)+1/2*ln(a*x)*ln(a*x+1)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(38) = 76$.
time = 0.26, size = 120, normalized size = 2.93

$$\frac{1}{4}a\left(\frac{\log\left(ax+1\right)^2}{ac}-\frac{2\left(\log\left(ax+1\right)\log\left(-\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}ax+\frac{1}{2}\right)\right)}{ac}-\frac{2\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)}{ac}+\frac{2\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)}{ac}\right)-\left(\frac{\log\left(ax+1\right)}{c}-\frac{\log\left(x\right)}{c}\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] 1/4*a*(log(a*x + 1)^2/(a*c) - 2*(log(a*x + 1)*log(-1/2*a*x + 1/2) + dilog(1
/2*a*x + 1/2))/(a*c) - 2*(log(a*x + 1)*log(x) + dilog(-a*x))/(a*c) + 2*(log
(-a*x + 1)*log(x) + dilog(a*x))/(a*c)) - (log(a*x + 1)/c - log(x)/c)*arctan
h(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}\left(ax\right)}{ax^2+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(a*c*x**2+c*x),x)
```

```
[Out] Integral(atanh(a*x)/(a*x**2 + x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{a\,c\,x^2+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c*x + a*c*x^2),x)
[Out] int(atanh(a*x)/(c*x + a*c*x^2), x)
```

3.68 $\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=270

$$\frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bdx^3\left(a+b\tan\right.}{6c}$$

```
[Out] 1/2*a*b*d*x/c^3+3/10*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*b^2*d*x^3/c-3/10*b
^2*d*arctanh(c*x)/c^4+1/2*b^2*d*x*arctanh(c*x)/c^3+1/5*b*d*x^2*(a+b*arctanh
(c*x))/c^2+1/6*b*d*x^3*(a+b*arctanh(c*x))/c+1/10*b*d*x^4*(a+b*arctanh(c*x))
-1/20*d*(a+b*arctanh(c*x))^2/c^4+1/4*d*x^4*(a+b*arctanh(c*x))^2+1/5*c*d*x^5
*(a+b*arctanh(c*x))^2-2/5*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+1/3*b^2
*d*ln(-c^2*x^2+1)/c^4-1/5*b^2*d*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A]
time = 0.46, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6087, 6037, 6127, 272, 45, 6021, 266, 6095, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{20c^4}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^4}+\frac{abdx}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{10}bdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{5c^4}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{3b^2dx}{10c^3}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2d\log\left(1-c^2x^2\right)}{3c^4}+\frac{b^2dx^3}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^3) + (3*b^2*d*x)/(10*c^3) + (b^2*d*x^2)/(12*c^2) + (b^2*d*x^
3)/(30*c) - (3*b^2*d*ArcTanh[c*x])/(10*c^4) + (b^2*d*x*ArcTanh[c*x])/(2*c^3
) + (b*d*x^2*(a + b*ArcTanh[c*x]))/(5*c^2) + (b*d*x^3*(a + b*ArcTanh[c*x]))
/(6*c) + (b*d*x^4*(a + b*ArcTanh[c*x]))/10 - (d*(a + b*ArcTanh[c*x])^2)/(20
*c^4) + (d*x^4*(a + b*ArcTanh[c*x])^2)/4 + (c*d*x^5*(a + b*ArcTanh[c*x])^2)
/5 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (b^2*d*Log[1 -
 c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(dx^3\left(a+b\tanh^{-1}(cx)\right)^2+cdx^4\left(a+b\tanh^{-1}(cx)\right)^2\right)\,dx$$

$$=d\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+(cd)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int$$

$$=\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}(2bd)\int$$

$$=\frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{10}bdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\right.$$

$$=\frac{abdx}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{10}bd$$

$$=\frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^3}{30c}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx\right.}{5c^2}$$

$$=\frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(}{2c^3}$$

$$=\frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(}{2c^3}$$

Mathematica [A]
time = 0.50, size = 271, normalized size = 1.00

$$\frac{d\left(-18ab-5b^2+30abcx+18b^2cx+12abc^2x^2+5b^2c^2x^2+10abc^3x^3+2b^2c^3x^3+15a^2c^4x^4+6abc^4x^4+12a^2c^5x^5+3b^2(-9+5c^4x^4+4c^5x^5)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(3ac^4x^4(5+4cx)+b(-9+15cx+6c^2x^2+5c^3x^3+3c^4x^4)-12b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+15ab\log(1-cx)-15ab\log(1+cx)+20b^2\log(1-c^2x^2)+12ab\log(-1+c^2x^2)+12b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(-18*a*b - 5*b^2 + 30*a*b*c*x + 18*b^2*c*x + 12*a*b*c^2*x^2 + 5*b^2*c^2*
x^2 + 10*a*b*c^3*x^3 + 2*b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*
a^2*c^5*x^5 + 3*b^2*(-9 + 5*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 2*b*ArcTa
nh[c*x]*(3*a*c^4*x^4*(5 + 4*c*x) + b*(-9 + 15*c*x + 6*c^2*x^2 + 5*c^3*x^3 +
 3*c^4*x^4) - 12*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15
*a*b*Log[1 + c*x] + 20*b^2*Log[1 - c^2*x^2] + 12*a*b*Log[-1 + c^2*x^2] + 12
*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^4)
```

Maple [A]
time = 0.31, size = 403, normalized size = 1.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d*a^2*(1/5*c^5*x^5+1/4*c^4*x^4)+2/5*d*a*b*arctanh(c*x)*c^5*x^5+9/20*
d*a*b*ln(c*x-1)+1/2*d*a*b*arctanh(c*x)*c^4*x^4-1/20*d*a*b*ln(c*x+1)-9/40*d*
b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/40*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/40*d*
b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+9/20*d*b^2*arctanh(c*x)*ln(c*x-1)-1/20
*d*b^2*arctanh(c*x)*ln(c*x+1)-1/5*d*b^2*dilog(1/2*c*x+1/2)+9/80*d*b^2*ln(c*
x-1)^2+1/80*d*b^2*ln(c*x+1)^2+29/60*d*b^2*ln(c*x-1)+11/60*d*b^2*ln(c*x+1)+1
/5*d*b^2*arctanh(c*x)*c^2*x^2+1/2*d*b^2*arctanh(c*x)*c*x+1/12*d*b^2*c^2*x^2
+3/10*d*b^2*c*x+1/30*d*b^2*c^3*x^3+1/10*d*a*b*c^4*x^4+1/6*d*a*b*c^3*x^3+1/5
*d*a*b*c^2*x^2+1/2*d*a*b*c*x+1/5*d*b^2*arctanh(c*x)^2*c^5*x^5+1/4*d*b^2*arc
tanh(c*x)^2*c^4*x^4+1/10*d*b^2*arctanh(c*x)*c^4*x^4+1/6*d*b^2*arctanh(c*x)*
c^3*x^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctanh(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*(4*x^
5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*
d - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^10 - 15*log(c*x + 1
)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x^2)/c^8 + 2*log(c^2*
x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*x + 1)/(c^6*x^2 - c^
4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)/c^9 + 3*log(c*x - 1
)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900*c^2*(2*x/c^6 - log(
c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1/150*x*log(c*x + 1)/
(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (12*c^5*x^5 - 15*c^4*
x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 + 1)*log(c*x + 1))*log
(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 - 10*log(-c*x + 1) +
2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x + 1) + 1) + 2000*(c*x
 - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 9000*(c*x - 1)^2*(2*log
(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(log(-c*x + 1)^2 - 2*l
og(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4)/c^5 - 540000*integ
rate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2*c*d + 1/12*(6*x^4*arctanh(
c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))
*a*b*d + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
- 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) +
 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2*d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^4 + a^2*d*x^3 + (b^2*c*d*x^4 + b^2*d*x^3)*arctanh(c*x)^2
 + 2*(a*b*c*d*x^4 + a*b*d*x^3)*arctanh(c*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^{2}x^{3}\,dx + \int a^{2}cx^{4}\,dx + \int b^{2}x^{3}\operatorname{atanh}^{2}\left(cx\right)dx + \int 2abx^{3}\operatorname{atanh}\left(cx\right)dx + \int b^{2}cx^{4}\operatorname{atanh}^{2}\left(cx\right)dx + \int 2abcx^{4}\operatorname{atanh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x**3, x) + Integral(a**2*c*x**4, x) + Integral(b**2*x**3*a
tanh(c*x)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(b**2*c*x**
4*atanh(c*x)**2, x) + Integral(2*a*b*c*x**4*atanh(c*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

3.69 $\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=236

$$\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{b^2dx^2}{12c}-\frac{b^2d\tanh^{-1}(cx)}{3c^3}+\frac{b^2dx\tanh^{-1}(cx)}{2c^2}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] 1/2*a*b*d*x/c^2+1/3*b^2*d*x/c^2+1/12*b^2*d*x^2/c-1/3*b^2*d*arctanh(c*x)/c^3
+1/2*b^2*d*x*arctanh(c*x)/c^2+1/3*b*d*x^2*(a+b*arctanh(c*x))/c+1/6*b*d*x^3*
(a+b*arctanh(c*x))+1/12*d*(a+b*arctanh(c*x))^2/c^3+1/3*d*x^3*(a+b*arctanh(c
*x))^2+1/4*c*d*x^4*(a+b*arctanh(c*x))^2-2/3*b*d*(a+b*arctanh(c*x))*ln(2/(-c
*x+1))/c^3+1/3*b^2*d*ln(-c^2*x^2+1)/c^3-1/3*b^2*d*polylog(2,1-2/(-c*x+1))/c
^3
```

Rubi [A]
time = 0.39, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6087, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 272, 45, 6021, 266, 6095}

$$\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{abdx}{2c^2}+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{3c^3}-\frac{b^2d\tanh^{-1}(cx)}{3c^3}+\frac{b^2dx}{3c^2}+\frac{b^2dx\tanh^{-1}(cx)}{2c^2}+\frac{b^2d\log\left(1-c^2x^2\right)}{3c^3}+\frac{b^2dx^2}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^2) + (b^2*d*x)/(3*c^2) + (b^2*d*x^2)/(12*c) - (b^2*d*ArcTanh
[c*x])/(3*c^3) + (b^2*d*x*ArcTanh[c*x])/(2*c^2) + (b*d*x^2*(a + b*ArcTanh[c
*x]))/(3*c) + (b*d*x^3*(a + b*ArcTanh[c*x]))/6 + (d*(a + b*ArcTanh[c*x])^2)
/(12*c^3) + (d*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d*x^4*(a + b*ArcTanh[c*x]
)^2)/4 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*d*Log
[1 - c^2*x^2])/(3*c^3) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(dx^2\left(a+b\tanh^{-1}(cx)\right)^2+cdx^3\left(a+b\tanh^{-1}(cx)\right)^2\right)\,dx$$
$$=d\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+(cd)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$
$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}(2bcd)\int$$
$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}(bd)\int x$$
$$=\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d\left(a+b\tanh\right.}{3c^3}$$
$$=\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right.$$
$$=\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}-\frac{b^2d\tanh^{-1}(cx)}{3c^3}+\frac{b^2dx\tanh^{-1}(cx)}{2c^2}+\frac{bdx^2\left(a+b\,\mathrm{ta}\right.}{3c}$$
$$=\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{b^2dx^2}{12c}-\frac{b^2d\tanh^{-1}(cx)}{3c^3}+\frac{b^2dx\tanh^{-1}(cx)}{2c^2}+\frac{bdx^2}{}$$

Mathematica [A]

time = 0.37, size = 234, normalized size = 0.99

$$\frac{d\left(-b^2+6abcx+4b^2cx+4abc^2x^2+b^2c^2x^2+4a^2c^3x^3+2abc^3x^3+3a^2c^4x^4+b^2(-7+4c^3x^3+3c^4x^4)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(ac^3x^3(4+3cx)+b(-2+3cx+2c^2x^2+c^3x^3)-4b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+3ab\log(1-cx)-3ab\log(1+cx)+4b^2\log\left(1-c^2x^2\right)+4ab\log\left(-1+c^2x^2\right)+4b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(-b^2 + 6*a*b*c*x + 4*b^2*c*x + 4*a*b*c^2*x^2 + b^2*c^2*x^2 + 4*a^2*c^3*
x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-7 + 4*c^3*x^3 + 3*c^4*x^4)*ArcT
anh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^3*x^3*(4 + 3*c*x) + b*(-2 + 3*c*x + 2*c^
2*x^2 + c^3*x^3) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] -
 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 - c^2*x^2] + 4*a*b*Log[-1 + c^2*x^2] + 4*
b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(12*c^3)
```

Maple [A]

time = 0.31, size = 364, normalized size = 1.54

method	result
derivativedivides	$\frac{d\,a^2\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{dab\,\mathrm{arctanh}(cx)c^4x^4}{2}+\frac{2dab\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{7dab\ln(cx-1)}{12}+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^3x^3}{3}+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^4x}{4}}{}$

default	$\frac{d a^2\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+\frac{dab\,\mathrm{arctanh}(cx)c^4x^4}{2}+\frac{2dab\,\mathrm{arctanh}(cx)c^3x^3}{3}+\frac{7dab\ln(cx-1)}{12}+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^3x^3}{3}+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c}{4}}{}$
risch	$-\frac{dcab\ln(-cx+1)x^4}{4}+\frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{b^2d\,x^2}{12c}-\frac{dc\,b^2\ln(-cx+1)x^4}{32}-\frac{7d\,b^2\ln(-cx+1)x^2}{48c}-\frac{7d\,b^2\ln(-cx+1)}{24c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d*a^2*(1/4*c^4*x^4+1/3*x^3*c^3)+7/12*d*a*b*ln(c*x-1)+2/3*d*a*b*arcta
nh(c*x)*c^3*x^3+1/2*d*a*b*arctanh(c*x)*c^4*x^4+1/12*d*a*b*ln(c*x+1)-7/24*d*
b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/24*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/24*d*
b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+7/12*d*b^2*arctanh(c*x)*ln(c*x-1)+1/12
*d*b^2*arctanh(c*x)*ln(c*x+1)-1/3*d*b^2*dilog(1/2*c*x+1/2)+7/48*d*b^2*ln(c*
x-1)^2-1/48*d*b^2*ln(c*x+1)^2+1/2*d*b^2*ln(c*x-1)+1/6*d*b^2*ln(c*x+1)+1/3*d
*b^2*arctanh(c*x)*c^2*x^2+1/2*d*b^2*arctanh(c*x)*c*x+1/12*d*b^2*c^2*x^2+1/3
*d*b^2*c*x+1/3*d*b^2*arctanh(c*x)^2*c^3*x^3+1/6*d*a*b*c^3*x^3+1/3*d*a*b*c^2
*x^2+1/2*d*a*b*c*x+1/4*d*b^2*arctanh(c*x)^2*c^4*x^4+1/6*d*b^2*arctanh(c*x)*
c^3*x^3)
```

Maxima [A]
time = 0.48, size = 402, normalized size = 1.70

$$\frac{1}{4}a^2cdx^4+\frac{1}{3}a^2dx^3+\frac{1}{12}\left(6x^4\,\mathrm{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)abcd+\frac{1}{3}\left(2x^3\,\mathrm{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)abd+\frac{\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d}{3c^3}+\frac{b^2d\log(cx+1)}{6c^3}+\frac{b^2d\log(cx-1)}{2c^3}+\frac{4b^2c^2dx^2+16b^2cdx+(3b^2c^4dx^4+4b^2c^3dx^3+b^2d)\log(cx+1)^2+(3b^2c^4dx^4+4b^2c^3dx^3-7b^2d)\log(-cx+1)^2+4(b^2c^3dx^3+2b^2c^2dx^2+3b^2cdx)\log(cx+1)-2(2b^2c^3dx^3+4b^2c^2dx^2+6b^2cdx+(3b^2c^4dx^4+4b^2c^3dx^3+b^2d)\log(cx+1))\log(-cx+1)}{48c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
+ 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c*d + 1/3*(2*x^3
*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*d + 1/3*(log(c*x +
1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d/c^3 + 1/6*b^2*d*log(c*
x + 1)/c^3 + 1/2*b^2*d*log(c*x - 1)/c^3 + 1/48*(4*b^2*c^2*d*x^2 + 16*b^2*c*
d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x + 1)^2 + (3*b^2*c
^4*d*x^4 + 4*b^2*c^3*d*x^3 - 7*b^2*d)*log(-c*x + 1)^2 + 4*(b^2*c^3*d*x^3 +
2*b^2*c^2*d*x^2 + 3*b^2*c*d*x)*log(c*x + 1) - 2*(2*b^2*c^3*d*x^3 + 4*b^2*c^
2*d*x^2 + 6*b^2*c*d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x
 + 1))*log(-c*x + 1))/c^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^3 + a^2*d*x^2 + (b^2*c*d*x^3 + b^2*d*x^2)*arctanh(c*x)^2
 + 2*(a*b*c*d*x^3 + a*b*d*x^2)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2x^2\,dx+\int a^2cx^3\,dx+\int b^2x^2\operatorname{atanh}^2(cx)\,dx+\int 2abx^2\operatorname{atanh}(cx)\,dx+\int b^2cx^3\operatorname{atanh}^2(cx)\,dx+\int 2abcx^3\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x**2, x) + Integral(a**2*c*x**3, x) + Integral(b**2*x**2*a
tanh(c*x)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(b**2*c*x**
3*atanh(c*x)**2, x) + Integral(2*a*b*c*x**3*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

3.70 $\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=196

$$\frac{abdx}{c}+\frac{b^2dx}{3c}-\frac{b^2d\tanh^{-1}(cx)}{3c^2}+\frac{b^2dx\tanh^{-1}(cx)}{c}+\frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}+\frac{1}{2}dx^2(a$$

```
[Out] a*b*d*x/c+1/3*b^2*d*x/c-1/3*b^2*d*arctanh(c*x)/c^2+b^2*d*x*arctanh(c*x)/c+1
/3*b*d*x^2*(a+b*arctanh(c*x))-1/6*d*(a+b*arctanh(c*x))^2/c^2+1/2*d*x^2*(a+b
*arctanh(c*x))^2+1/3*c*d*x^3*(a+b*arctanh(c*x))^2-2/3*b*d*(a+b*arctanh(c*x)
)*ln(2/(-c*x+1))/c^2+1/2*b^2*d*ln(-c^2*x^2+1)/c^2-1/3*b^2*d*polylog(2,1-2/(
-c*x+1))/c^2
```

Rubi [A]
time = 0.29, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6087, 6037, 6127, 6021, 266, 6095, 327, 212, 6131, 6055, 2449, 2352}

$$-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{abdx}{c}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{3c^2}+\frac{b^2d\log\left(1-c^2x^2\right)}{2c^2}-\frac{b^2d\tanh^{-1}(cx)}{3c^2}+\frac{b^2dx}{3c}+\frac{b^2dx\tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/c + (b^2*d*x)/(3*c) - (b^2*d*ArcTanh[c*x])/(3*c^2) + (b^2*d*x*Arc
Tanh[c*x])/c + (b*d*x^2*(a + b*ArcTanh[c*x]))/3 - (d*(a + b*ArcTanh[c*x])^2
)/(6*c^2) + (d*x^2*(a + b*ArcTanh[c*x])^2)/2 + (c*d*x^3*(a + b*ArcTanh[c*x]
)^2)/3 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (b^2*d*Log
[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx &= \int\left(dx\left(a+b\tanh^{-1}(cx)\right)^2+cdx^2\left(a+b\tanh^{-1}(cx)\right)^2\right)\,dx\\
&= d\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx+(cd)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx\\
&= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2-(bcd)\int \\
&= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}(2bd)\int \\
&= \frac{abdx}{c}+\frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}+\frac{1}{2}dx^2 \\
&= \frac{abdx}{c}+\frac{b^2dx}{3c}+\frac{b^2dx\tanh^{-1}(cx)}{c}+\frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right)- \\
&= \frac{abdx}{c}+\frac{b^2dx}{3c}-\frac{b^2d\tanh^{-1}(cx)}{3c^2}+\frac{b^2dx\tanh^{-1}(cx)}{c}+\frac{1}{3}bdx^2(a+b \\
&= \frac{abdx}{c}+\frac{b^2dx}{3c}-\frac{b^2d\tanh^{-1}(cx)}{3c^2}+\frac{b^2dx\tanh^{-1}(cx)}{c}+\frac{1}{3}bdx^2(a+b
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 201, normalized size = 1.03

$$\frac{d\left(6abcx+2b^2cx+3a^2c^2x^2+2abc^2x^2+2a^2c^3x^3+b^2(-5+3c^2x^2+2c^3x^3)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(ac^2x^2(3+2cx)+b(-1+3cx+c^2x^2)-2b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+3ab\log(1-cx)-3ab\log(1+cx)+3b^2\log\left(1-c^2x^2\right)+2ab\log\left(-1+c^2x^2\right)+2b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]

[Out] (d*(6*a*b*c*x + 2*b^2*c*x + 3*a^2*c^2*x^2 + 2*a*b*c^2*x^2 + 2*a^2*c^3*x^3 + b^2*(-5 + 3*c^2*x^2 + 2*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*x^2*(3 + 2*c*x) + b*(-1 + 3*c*x + c^2*x^2) - 2*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*b^2*Log[1 - c^2*x^2] + 2*a*b*Log[-1 + c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(6*c^2)

Maple [A]
time = 0.32, size = 322, normalized size = 1.64

method	result
derivativedivides	$\frac{d\,a^2\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^3x^3}{3}+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^2x^2}{2}+\frac{d\,b^2\,\mathrm{arctanh}(cx)c^2x^2}{3}+d\,b^2\,\mathrm{arctanh}(cx)cx+\frac{5d\,b^2\,\mathrm{arctanh}}{6}}{}$
default	$\frac{d\,a^2\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^3x^3}{3}+\frac{d\,b^2\,\mathrm{arctanh}(cx)^2c^2x^2}{2}+\frac{d\,b^2\,\mathrm{arctanh}(cx)c^2x^2}{3}+d\,b^2\,\mathrm{arctanh}(cx)cx+\frac{5d\,b^2\,\mathrm{arctanh}}{6}}{}$
risch	$\frac{abdx}{c}-\frac{dcab\ln(-cx+1)x^3}{3}+\frac{b^2dx}{3c}+\frac{d\,b^2\left(2x^3c^3+3c^2x^2-1\right)\ln(cx+1)^2}{24c^2}-\frac{dab\ln(-cx+1)x^2}{2}+\frac{5dab\ln(-cx+1)}{6c^2}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^2*(d*a^2*(1/3*x^3*c^3+1/2*c^2*x^2)+1/3*d*b^2*arctanh(c*x)^2*c^3*x^3+1/2*d*b^2*arctanh(c*x)^2*c^2*x^2+1/3*d*b^2*arctanh(c*x)*c^2*x^2+d*b^2*arctanh(c*x)*c*x+5/6*d*b^2*arctanh(c*x)*ln(c*x-1)-1/6*d*b^2*arctanh(c*x)*ln(c*x+1)+5/24*d*b^2*ln(c*x-1)^2-1/3*d*b^2*dilog(1/2*c*x+1/2)-5/12*d*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/12*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/12*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/24*d*b^2*ln(c*x+1)^2+1/3*d*b^2*c*x+2/3*d*b^2*ln(c*x-1)+1/3*d*b^2*ln(c*x+1)+2/3*d*a*b*arctanh(c*x)*c^3*x^3+d*a*b*arctanh(c*x)*c^2*x^2+1/3*d*a*b*c^2*x^2+d*a*b*c*x+5/6*d*a*b*ln(c*x-1)-1/6*d*a*b*ln(c*x+1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] 1/3*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctanh(c*x)^2 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d - 1/216*(2*c^4*(2*(c^2*x^3 + 3*x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 + log(c*x - 1)/c^5) - 324*c*integrate

```
(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x) - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*
c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 + 1)*log(c*x + 1))*log(-c*x + 1))/
c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 27*(c*x -
1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 54*(c*x - 1)*(log(-c*x + 1
)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*log(9*c^4*x^2 - 9*c^2)/c^3 - 324*integ
rate(1/9*log(c*x + 1)/(c^4*x^2 - c^2), x))*b^2*c*d + 1/2*a^2*d*x^2 + 1/2*(2
*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*
d + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) -
 (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*l
og(c*x - 1))/c^2)*b^2*d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^2 + a^2*d*x + (b^2*c*d*x^2 + b^2*d*x)*arctanh(c*x)^2 + 2
*(a*b*c*d*x^2 + a*b*d*x)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^{2}x\,dx + \int a^{2}cx^{2}\,dx + \int b^{2}x\operatorname{atanh}^{2}{\left(cx \right)}\,dx + \int 2abx\operatorname{atanh}{\left(cx \right)}\,dx + \int b^{2}cx^{2}\operatorname{atanh}^{2}{\left(cx \right)}\,dx + \int 2abcx^{2}\operatorname{atanh}{\left(cx \right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x, x) + Integral(a**2*c*x**2, x) + Integral(b**2*x*atanh(c
*x)**2, x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(b**2*c*x**2*atanh(c
*x)**2, x) + Integral(2*a*b*c*x**2*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int(x*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

3.71 $\int (d+cdx) \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=112

$$abdx+b^2dx\tanh^{-1}(cx)+\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{2bd\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}+\frac{b^2d\log\left(1-c^2\right.}{2c}$$

```
[Out] a*b*d*x+b^2*d*x*arctanh(c*x)+1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))^2/c-2*b*d*(
a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+1/2*b^2*d*ln(-c^2*x^2+1)/c-b^2*d*polylog
(2,1-2/(-c*x+1))/c
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6065, 6021, 266, 1600, 6055, 2449, 2352}

$$\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+abdx+\frac{b^2d\log\left(1-c^2x^2\right)}{2c}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c}+b^2dx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] a*b*d*x + b^2*d*x*ArcTanh[c*x] + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x])^2)/(2*
c) - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b^2*d*Log[1 - c^2*x
^2])/(2*c) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{b\int\left(-d^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2(d^2+\cdots}{\cdots}\right.}{d}$$

$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{(2b)\int\frac{(d^2+cd^2x)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d} + (\cdots$$

$$= abdx + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{(2b)\int\frac{a+b\tanh^{-1}(cx)}{\frac{1}{d^2}-\frac{cx}{d^2}}\,dx}{d} + (b^2\cdots$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd(a+\cdots}{\cdots}$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd(a+\cdots}{\cdots}$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd(a+\cdots}{\cdots}$$

Mathematica [A]

time = 0.18, size = 156, normalized size = 1.39

$$\frac{d\left(2a^2cx+2abcx+a^2c^2x^2+b^2(-3+2cx+c^2x^2)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(cx(2a+b+acx)-2b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+ab\log(1-cx)-ab\log(1+cx)+2ab\log\left(1-c^2x^2\right)+b^2\log\left(1-c^2x^2\right)+2b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-3 + 2*c*x + c^2*x^2)*ArcTan
h[c*x]^2 + 2*b*ArcTanh[c*x]*(c*x*(2*a + b + a*c*x) - 2*b*Log[1 + E^(-2*ArcT
anh[c*x])]) + a*b*Log[1 - c*x] - a*b*Log[1 + c*x] + 2*a*b*Log[1 - c^2*x^2]
+ b^2*Log[1 - c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(108) = 216$.
time = 0.20, size = 273, normalized size = 2.44

method	result
derivativedivides	$\frac{d\,a^2\left(\frac{1}{2}c^2x^2+cx\right)+\frac{d\,b^2\operatorname{arctanh}(cx)^2c^2x^2}{2}+d\,b^2\operatorname{arctanh}(cx)^2cx+d\,b^2\operatorname{arctanh}(cx)cx+\frac{3d\,b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{d\,b^2\operatorname{arctan}}{}}{}$
default	$\frac{d\,a^2\left(\frac{1}{2}c^2x^2+cx\right)+\frac{d\,b^2\operatorname{arctanh}(cx)^2c^2x^2}{2}+d\,b^2\operatorname{arctanh}(cx)^2cx+d\,b^2\operatorname{arctanh}(cx)cx+\frac{3d\,b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{d\,b^2\operatorname{arctan}}{}}{}$
risch	$\frac{d\,b^2\left(c^2x^2+2cx+1\right)\ln(cx+1)^2}{8c}+\left(-\frac{d\,b^2x(cx+2)\ln(-cx+1)}{4}+\frac{db\left(2a\,c^2x^2+4cxa+2bcx+3b\ln(-cx+1)\right)}{4c}\right)\ln\left(cx\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d*a^2*(1/2*c^2*x^2+c*x)+1/2*d*b^2*arctanh(c*x)^2*c^2*x^2+d*b^2*arctanh
(c*x)^2*c*x+d*b^2*arctanh(c*x)*c*x+3/2*d*b^2*arctanh(c*x)*ln(c*x-1)+1/2*d*b
^2*arctanh(c*x)*ln(c*x+1)-1/4*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/4*d*
b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-d*b^2*dilog(1/2*c*x+1/2)-1/8*d*b^2*ln(c*x+1)
^2+1/2*d*b^2*ln(c*x-1)+1/2*d*b^2*ln(c*x+1)-3/4*d*b^2*ln(c*x-1)*ln(1/2*c*x+1
/2)+3/8*d*b^2*ln(c*x-1)^2+d*a*b*arctanh(c*x)*c^2*x^2+2*d*a*b*arctanh(c*x)*c
*x+d*a*b*c*x+3/2*d*a*b*ln(c*x-1)+1/2*d*a*b*ln(c*x+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(105) = 210$.
time = 0.41, size = 290, normalized size = 2.59

$$\frac{1}{2}a^2cdx^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abcd+a^2dx+\frac{(2cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))abd}{c}+\frac{\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d}{c}+\frac{b^2d\log(cx+1)}{2c}+\frac{b^2d\log(cx-1)}{2c}+\frac{4b^2cdx\log(cx+1)+(b^2c^2dx^2+2b^2cdx+b^2d)\log(cx+1)^2+(b^2c^2dx^2+2b^2cdx-3b^2d)\log(-cx+1)^2-2(2b^2cdx+(b^2c^2dx^2+2b^2cdx+b^2d)\log(cx+1))\log(-cx+1)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*d*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
 log(c*x - 1)/c^3))*a*b*c*d + a^2*d*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a*b*d/c + (log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b
^2*d/c + 1/2*b^2*d*log(c*x + 1)/c + 1/2*b^2*d*log(c*x - 1)/c + 1/8*(4*b^2*c
*d*x*log(c*x + 1) + (b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1)^2 +
```

```
(b^2*c^2*d*x^2 + 2*b^2*c*d*x - 3*b^2*d)*log(-c*x + 1)^2 - 2*(2*b^2*c*d*x +
(b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1))*log(-c*x + 1))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c*
d*x + a*b*d)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2\,dx + \int b^2\operatorname{atanh}^2(cx)\,dx + \int 2ab\operatorname{atanh}(cx)\,dx + \int a^2cx\,dx + \int b^2cx\operatorname{atanh}^2(cx)\,dx + \int 2abcx\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*ata
nh(c*x), x) + Integral(a**2*c*x, x) + Integral(b**2*c*x*atanh(c*x)**2, x) +
 Integral(2*a*b*c*x*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + c*d*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + c*d*x), x)
```

3.72 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=191

$$d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-2bd\left(a+b\tan\right.$$

```
[Out] d*(a+b*arctanh(c*x))^2+c*d*x*(a+b*arctanh(c*x))^2-2*d*(a+b*arctanh(c*x))^2*
arctanh(-1+2/(-c*x+1))-2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-b^2*d*polylo
g(2,1-2/(-c*x+1))-b*d*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*d*(a+b*a
rctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*d*polylog(3,1-2/(-c*x+1))-1/2
*b^2*d*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.33, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6033, 6199, 6095, 6205, 6745}

$$-bd\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-d)\text{Li}_2\left(1-\frac{2}{1-cx}\right)+\frac{1}{2}b^2d\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d\text{Li}_3\left(\frac{2}{1-cx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] d*(a + b*ArcTanh[c*x])^2 + c*d*x*(a + b*ArcTanh[c*x])^2 + 2*d*(a + b*ArcTan
h[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 -
c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 - c*x)] - b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)] + b*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x
)] + (b^2*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx &= \int \left(cd\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}\right)dx\\
&= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx+(cd)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx\\
&= cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c\ldots}\right.\\
&= d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\left(a+b\tanh^{-1}(c\ldots\right.\\
&= d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\left(a+b\tanh^{-1}(c\ldots\right.\\
&= d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\left(a+b\tanh^{-1}(c\ldots\right.\\
&= d\left(a+b\tanh^{-1}(cx)\right)^2+cdx\left(a+b\tanh^{-1}(cx)\right)^2+2d\left(a+b\tanh^{-1}(c\ldots\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 228, normalized size = 1.19

$$
d\left(a^2cx+a^2\log(cx)+ab\left(2cx\tanh^{-1}(cx)+\log\left(1-c^2x^2\right)\right)+b^2\left(\tanh^{-1}(cx)\left((-1+cx)\tanh^{-1}(cx)-2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)+ab(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx))+b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\tanh^{-1}(cx)^3-\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] d*(a^2*c*x + a^2*Log[c*x] + a*b*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2]) + b
^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])])
 + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + a*b*(-PolyLog[2, -(c*x)] + PolyLog[2
```

```
, c*x]) + b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 +
E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh
[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTa
nh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*
x])]/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 2.64, size = 3644, normalized size = 19.08

method	result	size
derivativedivides	Expression too large to display	3644
default	Expression too large to display	3644

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*a*b*arctanh(c*x)*c*x+d*a*b*ln(c*x-1)-1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,-(c*x+1)^2/(-c^2*x^2
+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)*l
n(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x)^2*c*x+1/4*I*d*b^2*Pi*c
sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/2*I*d*b^2*Pi*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+d*a*b*ln(c*x+
1)-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2
*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-I*(
c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c
*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1
/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2
+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/
(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1))
)*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2
*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^
```

```
2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctan
h(c*x)^2-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-I*(c*x+1)
/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln
(1+(c*x+1)^2/(-c^2*x^2+1))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))+1/
4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/2
*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1
/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))
-1/2*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(
I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+d*a^2*ln(c*x)+d*b^2*arctanh(c*x)^2-d*b^2*
dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^
(1/2))+1/2*d*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*d*b^2*polylog(3,(c*x+
1)/(-c^2*x^2+1)^(1/2))-2*d*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*d
*b^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+
1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)*ln(1+I*(c*x+1)/(
-c^2*x^2+1)^(1/2))+d*a^2*c*x-d*a*b*dilog(c*x)-d*a*b*dilog(c*x+1)+d*b^2*arct
anh(c*x)^2*ln(c*x)-d*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d*
b^2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-d*b^2*arctanh(c*x)*ln(1+I*(c*
x+1)/(-c^2*x^2+1)^(1/2))-d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+
1))+2*d*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctan
h(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x)^2*ln(1-(c*x+1)
/(-c^2*x^2+1)^(1/2))+2*d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(
1/2))-d*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+1/2*I*d*b^2*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+
1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c*d*x*log(-c*x + 1)^2 + a^2*c*d*x + (2*c*x*arctanh(c*x) + log(-c^2*
x^2 + 1))*a*b*d + a^2*d*log(x) - integrate(-1/4*((b^2*c^2*d*x^2 - b^2*d)*lo
g(c*x + 1)^2 + 4*(a*b*c*d*x - a*b*d)*log(c*x + 1) - 2*(b^2*c^2*d*x^2 + 2*a*
b*c*d*x - 2*a*b*d + (b^2*c^2*d*x^2 - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c
*x^2 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^{2}c\,dx + \int \frac{a^{2}}{x}\,dx + \int b^{2}c\operatorname{atanh}^{2}{\left(cx \right)}\,dx + \int \frac{b^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x}\,dx + \int 2abc\operatorname{atanh}{\left(cx \right)}\,dx + \int \frac{2ab\operatorname{atanh}{\left(cx \right)}}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2*c, x) + Integral(a**2/x, x) + Integral(b**2*c*atanh(c*x)**
2, x) + Integral(b**2*atanh(c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x), x)
 + Integral(2*a*b*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x, x)
```

3.73 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=201

$$cd\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+2bcd\left(a+b\,\mathrm{ta}\right.$$

```
[Out] c*d*(a+b*arctanh(c*x))^2-d*(a+b*arctanh(c*x))^2/x-2*c*d*(a+b*arctanh(c*x))^
2*arctanh(-1+2/(-c*x+1))+2*b*c*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c*d*(
a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*c*d*(a+b*arctanh(c*x))*polylog(
2,-1+2/(-c*x+1))-b^2*c*d*polylog(2,-1+2/(c*x+1))+1/2*b^2*c*d*polylog(3,1-2/
(-c*x+1))-1/2*b^2*c*d*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.36, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6037, 6135, 6079, 2497, 6033, 6199, 6095, 6205, 6745}

$$-bcd\,\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bcd\,\mathrm{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+cd\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2cd\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2+2bcd\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-c)d\,\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{1}{2}b^2cd\,\mathrm{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2cd\,\mathrm{Li}_3\left(\frac{2}{1-cx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] c*d*(a + b*ArcTanh[c*x])^2 - (d*(a + b*ArcTanh[c*x])^2)/x + 2*c*d*(a + b*Ar
cTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 2*b*c*d*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] +
b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d*PolyLog[2
, -1 + 2/(1 + c*x)] + (b^2*c*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*c*d*Po
lyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+(cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c\right.$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2cd\left(a+b\tanh^{-1}(c\right.$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2cd\left(a+b\tanh^{-1}(c\right.$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2cd\left(a+b\tanh^{-1}(c\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 249, normalized size = 1.24

$$-\frac{d\left(a^2-a^2cx\log(x)+ab\left(2\tanh^{-1}(cx)+cx(-2\log(cx)+\log(1-c^2x^2))\right)+b^2\left(\tanh^{-1}(cx)\left((1-cx)\tanh^{-1}(cx)-2cx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+cx\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+abcx(\text{PolyLog}(2,-cx)-\text{PolyLog}(2,cx))-b^2cx\left(\frac{i\pi^3}{24}-\frac{2}{3}\tanh^{-1}(cx)^3-\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] -((d*(a^2 - a^2*c*x*Log[x] + a*b*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1
 - c^2*x^2])) + b^2*(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E
^(-2*ArcTanh[c*x])]) + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + a*b*c*x*(Poly
Log[2, -(c*x)] - PolyLog[2, c*x]) - b^2*c*x*((I/24)*Pi^3 - (2*ArcTanh[c*x]^
3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 -
 E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcT
anh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/
2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2))/x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.75, size = 3070, normalized size = 15.27

method	result	size
derivativedivides	Expression too large to display	3070
default	Expression too large to display	3070

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/8*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))^3*dilog((c*x+1)^2/(-c^2*x^2+1))-d*a*b*ln(c*x-1)-1/8*I*d*b^2*Pi*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*polylog(2,-(c*x+1
)^2/(-c^2*x^2+1))-1/8*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))^3*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*dilog(1+(c*x+1)
^2/(-c^2*x^2+1))-2*d*a*b*arctanh(c*x)/c/x-1/8*I*d*b^2*Pi*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-1/
8*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x
^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*dilog
((c*x+1)^2/(-c^2*x^2+1))-1/8*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*dilog(1+(c*x+1)^2/(-c
^2*x^2+1))+1/4*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-(c*x+1)
^2/(-c^2*x^2+1))+1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)*ln(1-(
c*x+1)^2/(-c^2*x^2+1))-d*a*b*ln(c*x+1)+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(I/(1
+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-
c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)
-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctan
h(c*x)^2+d*a^2*ln(c*x)-d*b^2*arctanh(c*x)^2+1/2*d*b^2*polylog(3,-(c*x+1)^2/
(-c^2*x^2+1))-1/4*d*b^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1
)^2/(-c^2*x^2+1)))^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csgn(
I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*dilog((c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csgn(I/(1+(
```

```
c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^
2*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*d*b^2*Pi*csgn(I*((c*x+
1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*
x^2+1)))^2*dilog((c*x+1)^2/(-c^2*x^2+1))-1/8*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))^2*dilog(1+(c*x+1)^2/(-c^2*x^2+1))-1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*
x^2+1))-1/8*I*d*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*dilog(1+(c*x+1)^2/(-c^2*x^2+
1))+3/2*d*b^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+d*b^2*arctanh(c*x)*
polylog(2,(c*x+1)^2/(-c^2*x^2+1))+d*b^2*arctanh(c*x)^2*ln(1-(c*x+1)^2/(-c^2
*x^2+1))+2*d*a*b*ln(c*x)-1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))-d*b^2*arcta
nh(c*x)^2/c/x-d*a*b*dilog(c*x)-d*a*b*dilog(c*x+1)+d*b^2*arctanh(c*x)^2*ln(c
*x)-d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-d*b^2*arctanh(c*x
)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-1/4*d*b^2*dilog((c*x+1)^2/(-c^2*x^2+1))+1/
4*d*b^2*dilog(1+(c*x+1)^2/(-c^2*x^2+1))-1/2*d*b^2*polylog(3,(c*x+1)^2/(-c^2
*x^2+1))+3/4*d*b^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+2*d
*a*b*arctanh(c*x)*ln(c*x)-d*a*b*ln(c*x)*ln(c*x+1)-d*a^2/c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
[Out] a^2*c*d*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d
 - 1/4*b^2*d*log(-c*x + 1)^2/x - a^2*d/x - integrate(-1/4*((b^2*c^2*d*x^2 -
 b^2*d)*log(c*x + 1)^2 + 4*(a*b*c^2*d*x^2 - a*b*c*d*x)*log(c*x + 1) - 2*(2*
a*b*c^2*d*x^2 - (2*a*b*c*d + b^2*c*d)*x + (b^2*c^2*d*x^2 - b^2*d)*log(c*x +
 1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x^2}\,dx + \int \frac{a^2c}{x}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^2}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^2}\,dx + \int \frac{b^2c\operatorname{atanh}^2(cx)}{x}\,dx + \int \frac{2abc\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(b**2*atanh(c*x
)**2/x**2, x) + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(b**2*c*atanh(
c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^2, x)
```

3.74 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x}+b^2c^2d\,\mathrm{l}$$

```
[Out] -b*c*d*(a+b*arctanh(c*x))/x+3/2*c^2*d*(a+b*arctanh(c*x))^2-1/2*d*(a+b*arcta
nh(c*x))^2/x^2-c*d*(a+b*arctanh(c*x))^2/x+b^2*c^2*d*ln(x)-1/2*b^2*c^2*d*ln(
-c^2*x^2+1)+2*b*c^2*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^2*c^2*d*polylog(
2,-1+2/(c*x+1))
```

Rubi [A]
time = 0.27, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6087, 6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 2497}

$$\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2+2bc^2d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x}-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x}-b^2c^2d\,\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)-\frac{1}{2}b^2c^2d\log\left(1-c^2x^2\right)+b^2c^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*(a + b*ArcTanh[c*x]))/x) + (3*c^2*d*(a + b*ArcTanh[c*x])^2)/2 - (d
*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (c*d*(a + b*ArcTanh[c*x])^2)/x + b^2*c^2
*d*Log[x] - (b^2*c^2*d*Log[1 - c^2*x^2])/2 + 2*b*c^2*d*(a + b*ArcTanh[c*x])
*Log[2 - 2/(1 + c*x)] - b^2*c^2*d*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx+(cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x}+(bcd)\int \frac{a+b\,\mathrm{ta}}{x^2\,(1}$$

$$= c^2d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{cd\left(a+b\tanh^{-1}(c\right.}{x}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh\right.}{2x^2}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh\right.}{2x^2}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh\right.}{2x^2}$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d\left(a+b\tanh\right.}{2x^2}$$

Mathematica [A]
time = 0.16, size = 206, normalized size = 1.36

$$-\frac{d\left(a^2+2a^2cx+2abcx+b^2(1+2cx-3c^2x^2)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(a+2acx+bcx-2bc^2x^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-4abc^2x^2\log(cx)+abc^2x^2\log(1-cx)-abc^2x^2\log(1+cx)-2b^2c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+2abc^2x^2\log\left(1-c^2x^2\right)+2b^2c^2x^2\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(d*(a^2 + 2*a^2*c*x + 2*a*b*c*x + b^2*(1 + 2*c*x - 3*c^2*x^2)*ArcTanh[
c*x]^2 + 2*b*ArcTanh[c*x]*(a + 2*a*c*x + b*c*x - 2*b*c^2*x^2*Log[1 - E^(-2*
ArcTanh[c*x])]) - 4*a*b*c^2*x^2*Log[c*x] + a*b*c^2*x^2*Log[1 - c*x] - a*b*c
^2*x^2*Log[1 + c*x] - 2*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^
2*x^2*Log[1 - c^2*x^2] + 2*b^2*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^
2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(145) = 290$.

time = 0.59, size = 367, normalized size = 2.43

method	result
derivativedivides	$c^2\left(d\,a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{cx}\right)-\frac{3dab\ln(cx-1)}{2}-\frac{dab\,\text{arctanh}(cx)}{c^2x^2}-\frac{d\,b^2\,\text{arctanh}(cx)^2}{cx}-\frac{d\,b^2\,\text{arctanh}(cx)^2}{2c^2x^2}-d\,b^2\right.$
default	$c^2\left(d\,a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{cx}\right)-\frac{3dab\ln(cx-1)}{2}-\frac{dab\,\text{arctanh}(cx)}{c^2x^2}-\frac{d\,b^2\,\text{arctanh}(cx)^2}{cx}-\frac{d\,b^2\,\text{arctanh}(cx)^2}{2c^2x^2}-d\,b^2\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d*a^2*(-1/2/c^2/x^2-1/c/x)-3/2*d*a*b*ln(c*x-1)-d*a*b*arctanh(c*x)/c^2/
x^2-2*d*a*b*arctanh(c*x)/c/x-1/2*d*a*b*ln(c*x+1)+3/4*d*b^2*ln(c*x-1)*ln(1/2
*c*x+1/2)-1/4*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/4*d*b^2*ln(-1/2*c*x+1/2)*l
n(1/2*c*x+1/2)-d*b^2*ln(c*x)*ln(c*x+1)+2*d*b^2*arctanh(c*x)*ln(c*x)-3/2*d*b
^2*arctanh(c*x)*ln(c*x-1)-1/2*d*b^2*arctanh(c*x)*ln(c*x+1)+d*b^2*dilog(1/2*
c*x+1/2)-3/8*d*b^2*ln(c*x-1)^2+1/8*d*b^2*ln(c*x+1)^2-1/2*d*b^2*ln(c*x-1)-1/
2*d*b^2*ln(c*x+1)+d*b^2*ln(c*x)-d*b^2*dilog(c*x+1)-d*b^2*dilog(c*x)-d*a*b/c
/x-1/2*d*b^2*arctanh(c*x)^2/c^2/x^2-d*b^2*arctanh(c*x)/c/x+2*d*a*b*ln(c*x)-
d*b^2*arctanh(c*x)^2/c/x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c*d - 1/4*b^2*c*d
*(log(-c*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x
- 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) + 1/2*((c*log(c*x + 1)
 - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*d + 1/8*((2*(log(c*x -
 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1) +
8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c*arctanh(c*x))*b
^2*d - a^2*c*d/x - 1/2*b^2*d*arctanh(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x^3}\,dx + \int \frac{a^2c}{x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^3}\,dx + \int \frac{b^2c\operatorname{atanh}^2(cx)}{x^2}\,dx + \int \frac{2abc\operatorname{atanh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c/x**2, x) + Integral(b**2*atanh(
c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(b**2*c*ata
nh(c*x)**2/x**2, x) + Integral(2*a*b*c*atanh(c*x)/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^3, x)
```

3.75 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=206

$$-\frac{b^2c^2d}{3x}+\frac{1}{3}b^2c^3d\tanh^{-1}(cx)-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d(}{}$$

```
[Out] -1/3*b^2*c^2*d/x+1/3*b^2*c^3*d*arctanh(c*x)-1/3*b*c*d*(a+b*arctanh(c*x))/x^
2-b*c^2*d*(a+b*arctanh(c*x))/x+5/6*c^3*d*(a+b*arctanh(c*x))^2-1/3*d*(a+b*ar
ctanh(c*x))^2/x^3-1/2*c*d*(a+b*arctanh(c*x))^2/x^2+b^2*c^3*d*ln(x)-1/2*b^2*
c^3*d*ln(-c^2*x^2+1)+2/3*b*c^3*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-1/3*b^2
*c^3*d*polylog(2,-1+2/(c*x+1))
```

Rubi [A]
time = 0.33, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6087, 6037, 6129, 331, 212, 6135, 6079, 2497, 272, 36, 29, 31, 6095}

$$\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{1}{3}b^2c^3d\text{Li}_2\left(\frac{2}{cx+1}-1\right)+b^2c^3d\log(x)+\frac{1}{3}b^2c^3d\tanh^{-1}(cx)-\frac{b^2c^2d}{3x}-\frac{1}{2}b^2c^3d\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d)/x + (b^2*c^3*d*ArcTanh[c*x])/3 - (b*c*d*(a + b*ArcTanh[c*x
]))/(3*x^2) - (b*c^2*d*(a + b*ArcTanh[c*x]))/x + (5*c^3*d*(a + b*ArcTanh[c*
x])^2)/6 - (d*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (c*d*(a + b*ArcTanh[c*x])^2
)/(2*x^2) + b^2*c^3*d*Log[x] - (b^2*c^3*d*Log[1 - c^2*x^2])/2 + (2*b*c^3*d*
(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*d*PolyLog[2, -1 + 2
/(1 + c*x)])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
```

```
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx &= \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\right)dx \\
&= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx+(cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx \\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}(2bcd)\int \frac{a+b}{x^3} \\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}(2bcd)\int \frac{a+b}{} \\
&= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{6}c^3d\left(a+b\tan\right. \\
&= -\frac{b^2c^2d}{3x}-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{6}c^3d\left(\right. \\
&= -\frac{b^2c^2d}{3x}+\frac{1}{3}b^2c^3d\tanh^{-1}(cx)-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\right.}{} \\
&= -\frac{b^2c^2d}{3x}+\frac{1}{3}b^2c^3d\tanh^{-1}(cx)-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.30, size = 246, normalized size = 1.19

$$
-\frac{d\left(2a^2+3a^2cx+2abcx+6abc^2x^2+2b^2c^2x^2+b^2(2+3cx-5c^3x^3)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(a(2+3cx)+bcx(1+3cx-c^2x^2)-2bc^3x^3\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-4abc^3x^3\log(cx)+3abc^3x^3\log(1-cx)-3abc^3x^3\log(1+cx)-6b^2c^3x^3\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+2abc^3x^3\log\left(1-c^2x^2\right)+2b^2c^3x^3\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/6*(d*(2*a^2 + 3*a^2*c*x + 2*a*b*c*x + 6*a*b*c^2*x^2 + 2*b^2*c^2*x^2 + b^
2*(2 + 3*c*x - 5*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*(2 + 3*c*x)
+ b*c*x*(1 + 3*c*x - c^2*x^2) - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) -
 4*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*Log[1 - c*x] - 3*a*b*c^3*x^3*Log[1
+ c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 -
 c^2*x^2] + 2*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(188) = 376.
time = 0.56, size = 407, normalized size = 1.98

method	result

derivativedivides	$c^3\left(d\,a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{3c^3x^3}\right)-\frac{5dab\ln(cx-1)}{6}-\frac{2dab\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{dab\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{d\,b^2\,\mathrm{arctanh}(cx)^2}{2c^2x^2}-\right.$
default	$c^3\left(d\,a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{3c^3x^3}\right)-\frac{5dab\ln(cx-1)}{6}-\frac{2dab\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{dab\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{d\,b^2\,\mathrm{arctanh}(cx)^2}{2c^2x^2}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d*a^2*(-1/2/c^2/x^2-1/3/c^3/x^3)-5/6*d*a*b*ln(c*x-1)-d*a*b*arctanh(c*x
)/c^2/x^2-2/3*d*a*b*arctanh(c*x)/c^3/x^3-1/3*d*a*b/c^2/x^2-1/3*d*b^2*arctan
h(c*x)^2/c^3/x^3-1/3*d*b^2*arctanh(c*x)/c^2/x^2+1/6*d*a*b*ln(c*x+1)+5/12*d*
b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/12*d*
b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/3*d*b^2/c/x-1/3*d*b^2*ln(c*x)*ln(c*x
+1)+2/3*d*b^2*arctanh(c*x)*ln(c*x)-5/6*d*b^2*arctanh(c*x)*ln(c*x-1)+1/6*d*b
^2*arctanh(c*x)*ln(c*x+1)+1/3*d*b^2*dilog(1/2*c*x+1/2)-5/24*d*b^2*ln(c*x-1)
^2-1/24*d*b^2*ln(c*x+1)^2-2/3*d*b^2*ln(c*x-1)-1/3*d*b^2*ln(c*x+1)+d*b^2*ln(
c*x)-1/3*d*b^2*dilog(c*x+1)-1/3*d*b^2*dilog(c*x)-d*a*b/c/x-1/2*d*b^2*arctan
h(c*x)^2/c^2/x^2-d*b^2*arctanh(c*x)/c/x+2/3*d*a*b*ln(c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(187) = 374$.
time = 0.64, size = 417, normalized size = 2.02

$$-\frac{1}{3}\left(\log\left(cx+1\right)\log\left(-\frac{1}{2}\,cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}\,cx+\frac{1}{2}\right)\right)b^2c^3d-\frac{1}{3}\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2(-cx+1)\right)b^2c^3d+\frac{1}{3}\left(\log\left(cx+1\right)\log\left(-cx\right)+\mathrm{Li}_2(cx+1)\right)b^2c^3d-\frac{1}{3}\,b^2c^3d\log\left(cx+1\right)-\frac{2}{3}\,b^2c^3d\log\left(cx-1\right)+b^2c^3d\log\left(x\right)+\frac{1}{2}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\left(cx\right)}{x^2}\right)abcd-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\left(cx\right)}{x^3}\right)abd-\frac{a^2cd}{2\,x^2}-\frac{a^2d}{3\,x^3}-\frac{8\,b^2c^2dx^2-\left(b^2c^3dx^3-3\,b^2cdx-2\,b^2d\right)\log\left(cx+1\right)^2-\left(5\,b^2c^3dx^3-3\,b^2cdx-2\,b^2d\right)\log\left(-cx+1\right)^2+4\left(3\,b^2c^2dx^2+b^2cdx\right)\log\left(cx+1\right)-2\left(6\,b^2c^2dx^2+2\,b^2cdx-\left(b^2c^3dx^3-3\,b^2cdx-2\,b^2d\right)\log\left(cx+1\right)\right)\log\left(-cx+1\right)}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d -
1/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d + 1/3*(log(c*x + 1
)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d - 1/3*b^2*c^3*d*log(c*x + 1) - 2/3*
b^2*c^3*d*log(c*x - 1) + b^2*c^3*d*log(x) + 1/2*((c*log(c*x + 1) - c*log(c*
x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d - 1/3*((c^2*log(c^2*x^2 - 1)
- c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d - 1/2*a^2*c*d/x^2 - 1
/3*a^2*d/x^3 - 1/24*(8*b^2*c^2*d*x^2 - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2
*d)*log(c*x + 1)^2 - (5*b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(-c*x + 1
)^2 + 4*(3*b^2*c^2*d*x^2 + b^2*c*d*x)*log(c*x + 1) - 2*(6*b^2*c^2*d*x^2 + 2
*b^2*c*d*x - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(c*x + 1))*log(-c*x
 + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x^4}\,dx + \int \frac{a^2c}{x^3}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^4}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^4}\,dx + \int \frac{b^2c\operatorname{atanh}^2(cx)}{x^3}\,dx + \int \frac{2abc\operatorname{atanh}(cx)}{x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c/x**3, x) + Integral(b**2*atanh(
c*x)**2/x**4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(b**2*c*ata
nh(c*x)**2/x**3, x) + Integral(2*a*b*c*atanh(c*x)/x**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^2\,\left(d+c\,d\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^4, x)
```

3.76 $\int x^3(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=356

$$\frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh^{-1}(cx)}{5c^4}+\frac{5b^2d^2x\tanh^{-1}(cx)}{6c^3}+\frac{2bd^2x^2\left(a+b\tanh^{-1}\right.}{5c^2}$$

```
[Out] 5/6*a*b*d^2*x/c^3+3/5*b^2*d^2*x/c^3+31/180*b^2*d^2*x^2/c^2+1/15*b^2*d^2*x^3
/c+1/60*b^2*d^2*x^4-3/5*b^2*d^2*arctanh(c*x)/c^4+5/6*b^2*d^2*x*arctanh(c*x)
/c^3+2/5*b*d^2*x^2*(a+b*arctanh(c*x))/c^2+5/18*b*d^2*x^3*(a+b*arctanh(c*x))
/c+1/5*b*d^2*x^4*(a+b*arctanh(c*x))+1/15*b*c*d^2*x^5*(a+b*arctanh(c*x))-1/6
0*d^2*(a+b*arctanh(c*x))^2/c^4+1/4*d^2*x^4*(a+b*arctanh(c*x))^2+2/5*c*d^2*x
^5*(a+b*arctanh(c*x))^2+1/6*c^2*d^2*x^6*(a+b*arctanh(c*x))^2-4/5*b*d^2*(a+b
*arctanh(c*x))*ln(2/(-c*x+1))/c^4+53/90*b^2*d^2*ln(-c^2*x^2+1)/c^4-2/5*b^2*
d^2*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A]
time = 0.72, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6087, 6037, 6127, 272, 45, 6021, 266, 6095, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{60c^4}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^4}+\frac{5abd^2x}{6c^3}+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{15}bcd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}bd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{5bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{18c}-\frac{2b^2d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{5c^4}-\frac{3b^2d^2\tanh^{-1}(cx)}{5c^4}+\frac{3b^2d^2x}{5c^3}+\frac{5b^2d^2x\tanh^{-1}(cx)}{6c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{53b^2d^2\log\left(1-c^2x^2\right)}{90c^4}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (5*a*b*d^2*x)/(6*c^3) + (3*b^2*d^2*x)/(5*c^3) + (31*b^2*d^2*x^2)/(180*c^2)
+ (b^2*d^2*x^3)/(15*c) + (b^2*d^2*x^4)/60 - (3*b^2*d^2*ArcTanh[c*x])/(5*c^4
) + (5*b^2*d^2*x*ArcTanh[c*x])/(6*c^3) + (2*b*d^2*x^2*(a + b*ArcTanh[c*x]))
/(5*c^2) + (5*b*d^2*x^3*(a + b*ArcTanh[c*x]))/(18*c) + (b*d^2*x^4*(a + b*Ar
cTanh[c*x]))/5 + (b*c*d^2*x^5*(a + b*ArcTanh[c*x]))/15 - (d^2*(a + b*ArcTan
h[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 + (2*c*d^2*x^5*(a
+ b*ArcTanh[c*x])^2)/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x])^2)/6 - (4*b*d^2*
(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (53*b^2*d^2*Log[1 - c^2*x^
2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int\Big(d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2$$

$$= d^2\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^4\left(a+b\tanh^{-1}(cx)\right)$$

$$= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}c^2d^2$$

$$= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}c^2d^2$$

$$= \frac{bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{5}bd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{15}bcd^2x$$

$$= \frac{abd^2x}{2c^3}+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{5bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{18c}$$

$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{b^2d^2x^3}{15c}+\frac{b^2d^2x\tanh^{-1}(cx)}{2c^3}+\frac{2bd^2x^2\left(a+b\,\mathrm{t}\right.}{5c^2}$$

$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{7b^2d^2x^2}{60c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh^{-}}{5c^4}$$

$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh}{5c^4}$$

Mathematica [A]
time = 0.70, size = 329, normalized size = 0.92

$$\frac{d^2\left(-108ab-34b^2+150abcx+108b^2cx+72abc^2x^2+31b^2c^2x^2+50abc^3x^3+12b^2c^3x^3+45a^2c^4x^4+36abc^4x^4+3b^2c^4x^4+72a^2c^5x^5+12abc^5x^5+30a^2c^6x^6+3b^2(-49+15c^4x^4+24c^5x^5+10c^6x^6)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(3ac^4x^4(15+24cx+10c^2x^2)+b(-54+75cx+36c^2x^2+25c^3x^3+18c^4x^4+6c^5x^5)-72b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+75ab\log(1-cx)-75ab\log(1+cx)+106b^2\log\left(1-c^2x^2\right)+72ab\log\left(-1+c^2x^2\right)+72b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{180c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-108*a*b - 34*b^2 + 150*a*b*c*x + 108*b^2*c*x + 72*a*b*c^2*x^2 + 31*b
^2*c^2*x^2 + 50*a*b*c^3*x^3 + 12*b^2*c^3*x^3 + 45*a^2*c^4*x^4 + 36*a*b*c^4*
x^4 + 3*b^2*c^4*x^4 + 72*a^2*c^5*x^5 + 12*a*b*c^5*x^5 + 30*a^2*c^6*x^6 + 3*
b^2*(-49 + 15*c^4*x^4 + 24*c^5*x^5 + 10*c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTa
nh[c*x]*(3*a*c^4*x^4*(15 + 24*c*x + 10*c^2*x^2) + b*(-54 + 75*c*x + 36*c^2*
x^2 + 25*c^3*x^3 + 18*c^4*x^4 + 6*c^5*x^5) - 72*b*Log[1 + E^(-2*ArcTanh[c*x
])]) + 75*a*b*Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 106*b^2*Log[1 - c^2*x^2]
 + 72*a*b*Log[-1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(18
0*c^4)
```

Maple [A]
time = 0.44, size = 549, normalized size = 1.54 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^4*(d^2*a^2*(1/6*c^6*x^6+2/5*c^5*x^5+1/4*c^4*x^4)+1/240*b^2*ln(c*x+1)^2*d^2+1/3*d^2*a*b*arctanh(c*x)*c^6*x^6+5/6*a*b*c*d^2*x+5/6*b^2*c*d^2*x*arctanh(c*x)+4/5*d^2*a*b*arctanh(c*x)*c^5*x^5+1/2*d^2*a*b*arctanh(c*x)*c^4*x^4+49/240*b^2*ln(c*x-1)^2*d^2+49/60*d^2*b^2*arctanh(c*x)*ln(c*x-1)-1/60*d^2*b^2*arctanh(c*x)*ln(c*x+1)+1/60*d^2*b^2*c^4*x^4+1/15*d^2*b^2*c^3*x^3+31/180*d^2*b^2*c^2*x^2+3/5*d^2*b^2*c*x+49/60*a*b*ln(c*x-1)*d^2-1/60*a*b*ln(c*x+1)*d^2-49/120*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)*d^2-1/120*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^2+1/120*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)*d^2+1/15*d^2*a*b*c^5*x^5+1/5*d^2*a*b*c^4*x^4+5/18*d^2*a*b*c^3*x^3+2/5*d^2*a*b*c^2*x^2+1/6*d^2*b^2*arctanh(c*x)^2*c^6*x^6+2/5*d^2*b^2*arctanh(c*x)^2*c^5*x^5+1/4*d^2*b^2*arctanh(c*x)^2*c^4*x^4+1/15*d^2*b^2*arctanh(c*x)*c^5*x^5+1/5*d^2*b^2*arctanh(c*x)*c^4*x^4+5/18*d^2*b^2*arctanh(c*x)*c^3*x^3+2/5*d^2*b^2*arctanh(c*x)*c^2*x^2-2/5*d^2*b^2*dilog(1/2*c*x+1/2)+8/9*d^2*b^2*ln(c*x-1)+13/45*d^2*b^2*ln(c*x+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(317) = 634$.
time = 0.49, size = 766, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] 1/6*a^2*c^2*d^2*x^6 + 2/5*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctanh(c*x)^2 + 1/4*a^2*d^2*x^4 + 1/90*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^2 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*d^2 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*d^2 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2*d^2 + 2/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^2/c^4 - 2/45*b^2*d^2*log(c*x + 1)/c^4 + 5/9*b^2*d^2*log(c*x - 1)/c^4 + 1/360*(6*b^2*c^4*d^2*x^4 + 24*b^2*c^3*d^2*x^3 + 32*b^2*c^2*d^2*x^2 + 216*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 + 7*b^2*d^2)*log(c*x + 1)^2 + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 - 17*b^2*d^2)*log(-c*x + 1)^2 + 4*(3*b^2*c^5*d^2*x^5 + 9*b^2*c^4*d^2*x^4 + 5*b^2*c^3*d^2*x^3 + 18*b^2*c^2*d^2*x^2 + 15*b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*c^5*d^2*x^5 + 18*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 36*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 + 7*b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/c^4

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^5 + 2*a^2*c*d^2*x^4 + a^2*d^2*x^3 + (b^2*c^2*d^2*x^5
 + 2*b^2*c*d^2*x^4 + b^2*d^2*x^3)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^5 + 2*a
*b*c*d^2*x^4 + a*b*d^2*x^3)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2x^3\,dx + \int 2a^2cx^4\,dx + \int a^2c^2x^5\,dx + \int b^2x^3\operatorname{atanh}^2(cx)\,dx + \int 2abx^3\operatorname{atanh}(cx)\,dx + \int 2b^2cx^4\operatorname{atanh}^2(cx)\,dx + \int b^2c^2x^5\operatorname{atanh}^2(cx)\,dx + \int 4abcx^4\operatorname{atanh}(cx)\,dx + \int 2abc^2x^5\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x**3, x) + Integral(2*a**2*c*x**4, x) + Integral(a**2*c
**2*x**5, x) + Integral(b**2*x**3*atanh(c*x)**2, x) + Integral(2*a*b*x**3*a
tanh(c*x), x) + Integral(2*b**2*c*x**4*atanh(c*x)**2, x) + Integral(b**2*c*
*2*x**5*atanh(c*x)**2, x) + Integral(4*a*b*c*x**4*atanh(c*x), x) + Integral
(2*a*b*c**2*x**5*atanh(c*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(317) = 634$.
time = 2.25, size = 1135, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/63*(84*((c*x + 1)^5*b^2*d^2/(c*x - 1)^5 + (c*x + 1)^4*b^2*d^2/(c*x - 1)^4
 + (c*x + 1)^3*b^2*d^2/(c*x - 1)^3)*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^
8*c^7/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(c*x
 - 1)^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)^4 -
 56*(c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(c*x +
 1)*c^7/(c*x - 1) + c^7) + 2*(168*(c*x + 1)^5*a*b*d^2/(c*x - 1)^5 + 168*(c*
x + 1)^4*a*b*d^2/(c*x - 1)^4 + 168*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 + 28*(c*
x + 1)^5*b^2*d^2/(c*x - 1)^5 - 35*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 + 28*(c*x
 + 1)^3*b^2*d^2/(c*x - 1)^3 - 28*(c*x + 1)^2*b^2*d^2/(c*x - 1)^2 + 8*(c*x +
 1)*b^2*d^2/(c*x - 1) - b^2*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^8*c^7
```

```
/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(c*x - 1)
^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)^4 - 56*(
c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(c*x + 1)*c
^7/(c*x - 1) + c^7) + (336*(c*x + 1)^5*a^2*d^2/(c*x - 1)^5 + 336*(c*x + 1)^
4*a^2*d^2/(c*x - 1)^4 + 336*(c*x + 1)^3*a^2*d^2/(c*x - 1)^3 + 112*(c*x + 1)
^5*a*b*d^2/(c*x - 1)^5 - 140*(c*x + 1)^4*a*b*d^2/(c*x - 1)^4 + 112*(c*x + 1
)^3*a*b*d^2/(c*x - 1)^3 - 112*(c*x + 1)^2*a*b*d^2/(c*x - 1)^2 + 32*(c*x + 1
)*a*b*d^2/(c*x - 1) - 4*a*b*d^2 - 2*(c*x + 1)^7*b^2*d^2/(c*x - 1)^7 + 15*(c
*x + 1)^6*b^2*d^2/(c*x - 1)^6 - 30*(c*x + 1)^5*b^2*d^2/(c*x - 1)^5 + 34*(c*
x + 1)^4*b^2*d^2/(c*x - 1)^4 - 30*(c*x + 1)^3*b^2*d^2/(c*x - 1)^3 + 15*(c*x
 + 1)^2*b^2*d^2/(c*x - 1)^2 - 2*(c*x + 1)*b^2*d^2/(c*x - 1))/((c*x + 1)^8*c
^7/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(c*x -
1)^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)^4 - 56
*(c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(c*x + 1)
*c^7/(c*x - 1) + c^7) - 2*b^2*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^7 + 2*b^2
*d^2*log(-(c*x + 1)/(c*x - 1))/c^7)*c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

3.77 $\int x^2(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=312

$$\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2d^2x\tanh^{-1}(cx)}{c^2}+\frac{8bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{3}$$

```
[Out] a*b*d^2*x/c^2+19/30*b^2*d^2*x/c^2+1/6*b^2*d^2*x^2/c+1/30*b^2*d^2*x^3-19/30*
b^2*d^2*arctanh(c*x)/c^3+b^2*d^2*x*arctanh(c*x)/c^2+8/15*b*d^2*x^2*(a+b*arc
tanh(c*x))/c+1/3*b*d^2*x^3*(a+b*arctanh(c*x))+1/10*b*c*d^2*x^4*(a+b*arctanh
(c*x))+1/30*d^2*(a+b*arctanh(c*x))^2/c^3+1/3*d^2*x^3*(a+b*arctanh(c*x))^2+1
/2*c*d^2*x^4*(a+b*arctanh(c*x))^2+1/5*c^2*d^2*x^5*(a+b*arctanh(c*x))^2-16/1
5*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^3+2/3*b^2*d^2*ln(-c^2*x^2+1)/c^
3-8/15*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^3
```

Rubi [A]
time = 0.64, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6087, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 272, 45, 6021, 266, 6095, 308}

$$\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^3}+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abd^2x}{c^2}+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{10}bcd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}bd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{8bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}-\frac{8b^2d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{15c^3}-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x\tanh^{-1}(cx)}{c^2}+\frac{2b^2d^2\log\left(1-c^2x^2\right)}{3c^3}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + (b^2*d^2*x^2)/(6*c) + (b^2*d^2*
x^3)/30 - (19*b^2*d^2*ArcTanh[c*x])/(30*c^3) + (b^2*d^2*x*ArcTanh[c*x])/c^2
 + (8*b*d^2*x^2*(a + b*ArcTanh[c*x]))/(15*c) + (b*d^2*x^3*(a + b*ArcTanh[c*
x]))/3 + (b*c*d^2*x^4*(a + b*ArcTanh[c*x]))/10 + (d^2*(a + b*ArcTanh[c*x])^
2)/(30*c^3) + (d^2*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d^2*x^4*(a + b*ArcTan
h[c*x])^2)/2 + (c^2*d^2*x^5*(a + b*ArcTanh[c*x])^2)/5 - (16*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)])/(15*c^3) + (2*b^2*d^2*Log[1 - c^2*x^2])/(3*c^
3) - (8*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x\right.$$

$$=d^2\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2$$

$$=\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}c^2d^2x$$

$$=\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}c^2d^2x$$

$$=\frac{bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{3}bd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{10}bcd^2x^4$$

$$=\frac{abd^2x}{c^2}+\frac{b^2d^2x}{3c^2}+\frac{8bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{3}bd^2x^3\left(a+b\tanh\right.$$

$$=\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{1}{30}b^2d^2x^3-\frac{b^2d^2\tanh^{-1}(cx)}{3c^3}+\frac{b^2d^2x\tanh^{-1}(c}{c^2}$$

$$=\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2}{}$$

$$=\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2}{}$$

Mathematica [A]
time = 0.65, size = 297, normalized size = 0.95

$$\frac{d^2\left(-9ab-5b^2+30abcx+19b^2cx+16abc^2x^2+5b^2c^2x^2+10a^2c^3x^3+10abc^3x^3+b^2c^3x^3+15a^2c^4x^4+3abc^4x^4+6a^2c^5x^5+b^2(-31+10c^3x^3+15c^4x^4+6c^5x^5)\tanh^{-1}(cx)^2+b\tanh^{-1}(cx)\left(2ac^3x^3(10+15cx+6c^2x^2)+b(-19+30cx+16c^2x^2+10c^3x^3+3c^4x^4)-32b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+15ab\log(1-cx)-15ab\log(1+cx)+20b^2\log(1-c^2x^2)+16ab\log(-1+c^2x^2)+16b^2\operatorname{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{30c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-9*a*b - 5*b^2 + 30*a*b*c*x + 19*b^2*c*x + 16*a*b*c^2*x^2 + 5*b^2*c^2
*x^2 + 10*a^2*c^3*x^3 + 10*a*b*c^3*x^3 + b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 3*a
*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-31 + 10*c^3*x^3 + 15*c^4*x^4 + 6*c^5*x^5
)*ArcTanh[c*x]^2 + b*ArcTanh[c*x]*(2*a*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2) +
b*(-19 + 30*c*x + 16*c^2*x^2 + 10*c^3*x^3 + 3*c^4*x^4) - 32*b*Log[1 + E^(-2
*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15*a*b*Log[1 + c*x] + 20*b^2*Log[1
 - c^2*x^2] + 16*a*b*Log[-1 + c^2*x^2] + 16*b^2*PolyLog[2, -E^(-2*ArcTanh[c
*x])]))/(30*c^3)
```

Maple [A]
time = 0.43, size = 501, normalized size = 1.61 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^2*a^2*(1/5*c^5*x^5+1/2*c^4*x^4+1/3*x^3*c^3)-1/120*b^2*ln(c*x+1)^2*
d^2+2/3*d^2*a*b*arctanh(c*x)*c^3*x^3+a*b*c*d^2*x+b^2*c*d^2*x*arctanh(c*x)+2
/5*d^2*a*b*arctanh(c*x)*c^5*x^5+d^2*a*b*arctanh(c*x)*c^4*x^4+31/120*b^2*ln(
c*x-1)^2*d^2+31/30*d^2*b^2*arctanh(c*x)*ln(c*x-1)+1/30*d^2*b^2*arctanh(c*x)
*ln(c*x+1)+1/30*d^2*b^2*c^3*x^3+1/6*d^2*b^2*c^2*x^2+19/30*d^2*b^2*c*x+31/30
*a*b*ln(c*x-1)*d^2+1/30*a*b*ln(c*x+1)*d^2-31/60*b^2*ln(c*x-1)*ln(1/2*c*x+1/
2)*d^2+1/60*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^2-1/60*b^2*ln(-1/2*c*x+1/2)*ln
(1/2*c*x+1/2)*d^2+1/10*d^2*a*b*c^4*x^4+1/3*d^2*a*b*c^3*x^3+8/15*d^2*a*b*c^2
*x^2+1/5*d^2*b^2*arctanh(c*x)^2*c^5*x^5+1/2*d^2*b^2*arctanh(c*x)^2*c^4*x^4+
1/10*d^2*b^2*arctanh(c*x)*c^4*x^4+1/3*d^2*b^2*arctanh(c*x)*c^3*x^3+8/15*d^2
*b^2*arctanh(c*x)*c^2*x^2+1/3*d^2*b^2*arctanh(c*x)^2*c^3*x^3-8/15*d^2*b^2*d
ilog(1/2*c*x+1/2)+59/60*d^2*b^2*ln(c*x-1)+7/20*d^2*b^2*ln(c*x+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(281) = 562$.
time = 0.48, size = 604, normalized size = 1.94

$$\frac{1}{5}a^2c^2d^2x^5+\frac{1}{2}a^2cd^2x^4+\frac{1}{10}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log(c^2x^2-1)}{c^6}\right)\right)abc^2d^2+\frac{1}{3}a^2d^2x^3+\frac{1}{6}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)abcd^2+\frac{1}{3}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)abd^2+\frac{8\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d^2}{15c^3}+\frac{7b^2d^2\log(cx+1)}{20c^3}+\frac{59b^2d^2\log(cx-1)}{60c^3}+\frac{4b^2c^3d^2x^3+20b^2c^2d^2x^2+76b^2cd^2x+(6b^2c^5d^2x^5+15b^2c^4d^2x^4+10b^2c^3d^2x^3+b^2d^2)\log(cx+1)^2+(6b^2c^5d^2x^5+15b^2c^4d^2x^4+10b^2c^3d^2x^3-31b^2d^2)\log(-cx+1)^2+2(3b^2c^4d^2x^4+10b^2c^3d^2x^3+16b^2c^2d^2x^2+30b^2cd^2x)\log(cx+1)-2(3b^2c^4d^2x^4+10b^2c^3d^2x^3+16b^2c^2d^2x^2+30b^2cd^2x+(6b^2c^5d^2x^5+15b^2c^4d^2x^4+10b^2c^3d^2x^3+b^2d^2)\log(cx+1))\log(-cx+1)}{120c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^2*d^2*x^5 + 1/2*a^2*c*d^2*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((c^
2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2*x^3
 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5
+ 3*log(c*x - 1)/c^5))*a*b*c*d^2 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*d^2 + 8/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + di
log(1/2*c*x + 1/2))*b^2*d^2/c^3 + 7/20*b^2*d^2*log(c*x + 1)/c^3 + 59/60*b^2
*d^2*log(c*x - 1)/c^3 + 1/120*(4*b^2*c^3*d^2*x^3 + 20*b^2*c^2*d^2*x^2 + 76*
b^2*c*d^2*x + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3
+ b^2*d^2)*log(c*x + 1)^2 + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^
2*c^3*d^2*x^3 - 31*b^2*d^2)*log(-c*x + 1)^2 + 2*(3*b^2*c^4*d^2*x^4 + 10*b^2
*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x)*log(c*x + 1) - 2*(3*b^2
*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x + (
6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + b^2*d^2)*log(
c*x + 1))*log(-c*x + 1))/c^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^4 + 2*a^2*c*d^2*x^3 + a^2*d^2*x^2 + (b^2*c^2*d^2*x^4
 + 2*b^2*c*d^2*x^3 + b^2*d^2*x^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^4 + 2*a
*b*c*d^2*x^3 + a*b*d^2*x^2)*arctanh(c*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int a^{2}x^{2}\,dx + \int 2a^{2}cx^{3}\,dx + \int a^{2}c^{2}x^{4}\,dx + \int b^{2}x^{2}\operatorname{atanh}^{2}\left(cx\right)dx + \int 2abx^{2}\operatorname{atanh}\left(cx\right)dx + \int 2b^{2}cx^{3}\operatorname{atanh}^{2}\left(cx\right)dx + \int b^{2}c^{2}x^{4}\operatorname{atanh}^{2}\left(cx\right)dx + \int 4abcx^{3}\operatorname{atanh}\left(cx\right)dx + \int 2abc^{2}x^{4}\operatorname{atanh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x**2, x) + Integral(2*a**2*c*x**3, x) + Integral(a**2*c
**2*x**4, x) + Integral(b**2*x**2*atanh(c*x)**2, x) + Integral(2*a*b*x**2*a
tanh(c*x), x) + Integral(2*b**2*c*x**3*atanh(c*x)**2, x) + Integral(b**2*c*
*2*x**4*atanh(c*x)**2, x) + Integral(4*a*b*c*x**3*atanh(c*x), x) + Integral
(2*a*b*c**2*x**4*atanh(c*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

3.78 $\int x(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=280

$$\frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{1}{12}b^2d^2x^2-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}(cx)}{2c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3\left(a\right.$$

```
[Out] 3/2*a*b*d^2*x/c+2/3*b^2*d^2*x/c+1/12*b^2*d^2*x^2-2/3*b^2*d^2*arctanh(c*x)/c
^2+3/2*b^2*d^2*x*arctanh(c*x)/c+2/3*b*d^2*x^2*(a+b*arctanh(c*x))+1/6*b*c*d^
2*x^3*(a+b*arctanh(c*x))-1/12*d^2*(a+b*arctanh(c*x))^2/c^2+1/2*d^2*x^2*(a+b
*arctanh(c*x))^2+2/3*c*d^2*x^3*(a+b*arctanh(c*x))^2+1/4*c^2*d^2*x^4*(a+b*ar
ctanh(c*x))^2-4/3*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2+5/6*b^2*d^2*l
n(-c^2*x^2+1)/c^2-2/3*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^2
```

Rubi [A]
time = 0.47, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6087, 6037, 6127, 6021, 266, 6095, 327, 212, 6131, 6055, 2449, 2352, 272, 45}

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{12c^2}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}bcd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{3abd^2x}{2c}-\frac{2b^2d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{3c^2}+\frac{5b^2d^2\log\left(1-c^2x^2\right)}{6c^2}-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{2b^2d^2x}{3c}+\frac{3b^2d^2x\tanh^{-1}(cx)}{2c}+\frac{1}{12}b^2d^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (3*a*b*d^2*x)/(2*c) + (2*b^2*d^2*x)/(3*c) + (b^2*d^2*x^2)/12 - (2*b^2*d^2*A
rcTanh[c*x])/(3*c^2) + (3*b^2*d^2*x*ArcTanh[c*x])/(2*c) + (2*b*d^2*x^2*(a +
 b*ArcTanh[c*x]))/3 + (b*c*d^2*x^3*(a + b*ArcTanh[c*x]))/6 - (d^2*(a + b*Ar
cTanh[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTanh[c*x])^2)/2 + (2*c*d^2*x^3
*(a + b*ArcTanh[c*x])^2)/3 + (c^2*d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 - (4*b*
d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (5*b^2*d^2*Log[1 - c^2
*x^2])/(6*c^2) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx &= \int\left(d^2x\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^3(\right.\\
&= d^2\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx\\
&= \frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}c^2d^2x^4\\
&= \frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}c^2d^2x^4\\
&= \frac{abd^2x}{c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\\
&= \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{b^2d^2x\tanh^{-1}(cx)}{c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)\\
&= \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}(cx)}{2c}+\frac{2}{3}bd^2\\
&= \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{1}{12}b^2d^2x^2-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}}{2c}
\end{aligned}
$$

Mathematica [A]
time = 0.43, size = 263, normalized size = 0.94

$$
\frac{d^2\left(-b^2+18abcx+8b^2cx+6a^2c^2x^2+8abc^2x^2+b^2c^2x^2+8a^2c^3x^3+2abc^3x^3+3a^2c^4x^4+b^2(-17+6c^2x^2+8c^3x^3+3c^4x^4)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(ac^2x^2(6+8cx+3c^2x^2)+b(-4+9cx+4c^2x^2+c^3x^3)-8b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+9ab\log(1-cx)-9ab\log(1+cx)+10b^2\log\left(1-c^2x^2\right)+8ab\log\left(-1+c^2x^2\right)+8b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{12c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-b^2 + 18*a*b*c*x + 8*b^2*c*x + 6*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + b^2*c
^2*x^2 + 8*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-17 + 6*c^2*x
^2 + 8*c^3*x^3 + 3*c^4*x^4)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*x^2*(6
 + 8*c*x + 3*c^2*x^2) + b*(-4 + 9*c*x + 4*c^2*x^2 + c^3*x^3) - 8*b*Log[1 +
E^(-2*ArcTanh[c*x])]) + 9*a*b*Log[1 - c*x] - 9*a*b*Log[1 + c*x] + 10*b^2*Lo
g[1 - c^2*x^2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^(-2*ArcTanh[
c*x])]))/(12*c^2)
```

Maple [A]
time = 0.47, size = 458, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^2*a^2*(1/4*c^4*x^4+2/3*x^3*c^3+1/2*c^2*x^2)+1/48*b^2*ln(c*x+1)^2*d
^2+4/3*d^2*a*b*arctanh(c*x)*c^3*x^3+3/2*a*b*c*d^2*x+3/2*b^2*c*d^2*x*arctanh
(c*x)+d^2*a*b*arctanh(c*x)*c^2*x^2+1/2*d^2*a*b*arctanh(c*x)*c^4*x^4+1/2*d^2
*b^2*arctanh(c*x)^2*c^2*x^2+17/48*b^2*ln(c*x-1)^2*d^2+17/12*d^2*b^2*arctanh
(c*x)*ln(c*x-1)-1/12*d^2*b^2*arctanh(c*x)*ln(c*x+1)+1/12*d^2*b^2*c^2*x^2+2/
3*d^2*b^2*c*x+17/12*a*b*ln(c*x-1)*d^2-1/12*a*b*ln(c*x+1)*d^2-17/24*b^2*ln(c
*x-1)*ln(1/2*c*x+1/2)*d^2-1/24*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^2+1/24*b^2*
ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)*d^2+1/6*d^2*a*b*c^3*x^3+2/3*d^2*a*b*c^2*x^
2+1/4*d^2*b^2*arctanh(c*x)^2*c^4*x^4+1/6*d^2*b^2*arctanh(c*x)*c^3*x^3+2/3*d
^2*b^2*arctanh(c*x)*c^2*x^2+2/3*d^2*b^2*arctanh(c*x)^2*c^3*x^3-2/3*d^2*b^2*
dilog(1/2*c*x+1/2)+7/6*d^2*b^2*ln(c*x-1)+1/2*d^2*b^2*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(249) = 498$.
time = 0.48, size = 610, normalized size = 2.18

$$\frac{1}{4}a^2c^2d^2x^4+\frac{2}{3}a^2cd^2x^3+\frac{1}{2}b^2d^2x^2\operatorname{artanh}(cx)^2+\frac{1}{12}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)abc^2d^2+\frac{2}{3}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)abcd^2+\frac{1}{2}a^2d^2x^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abd^2+\frac{1}{8}\left(4c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\operatorname{artanh}(cx)-\frac{2(\log(cx-1)-2)\log(cx+1)-\log(cx+1)^2-\log(cx-1)^2-4\log(cx-1)}{c^2}\right)b^2d^2+\frac{2\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d^2}{3c^2}+\frac{2b^2d^2\log(cx-1)}{3c^2}+\frac{4b^2c^2d^2x^2+32b^2cd^2x+(3b^2c^4d^2x^4+8b^2c^3d^2x^3+5b^2d^2)\log(cx+1)^2+(3b^2c^4d^2x^4+8b^2c^3d^2x^3-11b^2d^2)\log(-cx+1)^2+4(b^2c^3d^2x^3+4b^2c^2d^2x^2+3b^2cd^2x)\log(cx+1)-2(2b^2c^3d^2x^3+8b^2c^2d^2x^2+6b^2cd^2x+(3b^2c^4d^2x^4+8b^2c^3d^2x^3+5b^2d^2)\log(cx+1))\log(-cx+1)}{48c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^2*d^2*x^4 + 2/3*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctanh(c*x)^2 +
1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 +
3*log(c*x - 1)/c^5))*a*b*c^2*d^2 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*c*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x)
 + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*d^2 + 1/8*(4*c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x -
1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^
2)*b^2*d^2 + 2/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*
b^2*d^2/c^2 + 2/3*b^2*d^2*log(c*x - 1)/c^2 + 1/48*(4*b^2*c^2*d^2*x^2 + 32*b
^2*c*d^2*x + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x +
1)^2 + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 - 11*b^2*d^2)*log(-c*x + 1)^2
 + 4*(b^2*c^3*d^2*x^3 + 4*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x)*log(c*x + 1) - 2
*(2*b^2*c^3*d^2*x^3 + 8*b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (3*b^2*c^4*d^2*x^
4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^3 + 2*a^2*c*d^2*x^2 + a^2*d^2*x + (b^2*c^2*d^2*x^3 +
 2*b^2*c*d^2*x^2 + b^2*d^2*x)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^3 + 2*a*b*c
*d^2*x^2 + a*b*d^2*x)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2x\,dx+\int 2a^2cx^2\,dx+\int a^2c^2x^3\,dx+\int b^2x\operatorname{atanh}^2(cx)\,dx+\int 2abx\operatorname{atanh}(cx)\,dx+\int 2b^2cx^2\operatorname{atanh}^2(cx)\,dx+\int b^2c^2x^3\operatorname{atanh}^2(cx)\,dx+\int 4abcx^2\operatorname{atanh}(cx)\,dx+\int 2abc^2x^3\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x, x) + Integral(2*a**2*c*x**2, x) + Integral(a**2*c**2
*x**3, x) + Integral(b**2*x*atanh(c*x)**2, x) + Integral(2*a*b*x*atanh(c*x)
, x) + Integral(2*b**2*c*x**2*atanh(c*x)**2, x) + Integral(b**2*c**2*x**3*a
tanh(c*x)**2, x) + Integral(4*a*b*c*x**2*atanh(c*x), x) + Integral(2*a*b*c*
*2*x**3*atanh(c*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. 2(249) = 498.
time = 1.59, size = 761, normalized size = 2.72

$$\frac{2}{45}\left(\frac{30\,(cx+1)^3b^2d^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{\left(\frac{(cx+1)^6c^5}{(cx-1)^6}-\frac{6(cx+1)^5c^5}{(cx-1)^5}+\frac{15(cx+1)^4c^5}{(cx-1)^4}-\frac{20(cx+1)^3c^5}{(cx-1)^3}+\frac{15(cx+1)^2c^5}{(cx-1)^2}-\frac{6(cx+1)c^5}{cx-1}+c^5\right)(cx-1)^3}+\frac{2\left(\frac{60(cx+1)^3abd^2}{(cx-1)^3}+\frac{10(cx+1)^3b^2d^2}{(cx-1)^3}-\frac{15(cx+1)^2b^2d^2}{(cx-1)^2}+\frac{6(cx+1)b^2d^2}{cx-1}-b^2d^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^5}{(cx-1)^6}-\frac{6(cx+1)^5c^5}{(cx-1)^5}+\frac{15(cx+1)^4c^5}{(cx-1)^4}-\frac{20(cx+1)^3c^5}{(cx-1)^3}+\frac{15(cx+1)^2c^5}{(cx-1)^2}-\frac{6(cx+1)c^5}{cx-1}+c^5}+\frac{\frac{120(cx+1)^3a^2d^2}{(cx-1)^3}+\frac{40(cx+1)^3abd^2}{(cx-1)^3}-\frac{60(cx+1)^2abd^2}{(cx-1)^2}+\frac{24(cx+1)abd^2}{cx-1}-4abd^2-\frac{2(cx+1)^5b^2d^2}{(cx-1)^5}+\frac{11(cx+1)^4b^2d^2}{(cx-1)^4}-\frac{18(cx+1)^3b^2d^2}{(cx-1)^3}+\frac{11(cx+1)^2b^2d^2}{(cx-1)^2}-\frac{2(cx+1)b^2d^2}{cx-1}}{\frac{(cx+1)^6c^5}{(cx-1)^6}-\frac{6(cx+1)^5c^5}{(cx-1)^5}+\frac{15(cx+1)^4c^5}{(cx-1)^4}-\frac{20(cx+1)^3c^5}{(cx-1)^3}+\frac{15(cx+1)^2c^5}{(cx-1)^2}-\frac{6(cx+1)c^5}{cx-1}+c^5}-\frac{2b^2d^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^5}+\frac{2b^2d^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^5}\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2/45*(30*(c*x + 1)^3*b^2*d^2*log(-(c*x + 1)/(c*x - 1))^2/(((c*x + 1)^6*c^5/
(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1)^
4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*(c*
x + 1)*c^5/(c*x - 1) + c^5)*(c*x - 1)^3) + 2*(60*(c*x + 1)^3*a*b*d^2/(c*x -
 1)^3 + 10*(c*x + 1)^3*b^2*d^2/(c*x - 1)^3 - 15*(c*x + 1)^2*b^2*d^2/(c*x -
1)^2 + 6*(c*x + 1)*b^2*d^2/(c*x - 1) - b^2*d^2)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^
4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*
x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) + (120*(c*x + 1)^3*a^2*d^2/(c*x
 - 1)^3 + 40*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 - 60*(c*x + 1)^2*a*b*d^2/(c*x
- 1)^2 + 24*(c*x + 1)*a*b*d^2/(c*x - 1) - 4*a*b*d^2 - 2*(c*x + 1)^5*b^2*d^2
/(c*x - 1)^5 + 11*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 - 18*(c*x + 1)^3*b^2*d^2/
(c*x - 1)^3 + 11*(c*x + 1)^2*b^2*d^2/(c*x - 1)^2 - 2*(c*x + 1)*b^2*d^2/(c*x
 - 1))/((c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c
*x + 1)^4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2
*c^5/(c*x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) - 2*b^2*d^2*log(-(c*x +
 1)/(c*x - 1) + 1)/c^5 + 2*b^2*d^2*log(-(c*x + 1)/(c*x - 1))/c^5)*c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
[Out] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

3.79 $\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=175

$$2abd^2x+\frac{1}{3}b^2d^2x-\frac{b^2d^2\tanh^{-1}(cx)}{3c}+2b^2d^2x\tanh^{-1}(cx)+\frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}\right.}{3c}$$

```
[Out] 2*a*b*d^2*x+1/3*b^2*d^2*x-1/3*b^2*d^2*arctanh(c*x)/c+2*b^2*d^2*x*arctanh(c*
x)+1/3*b*c*d^2*x^2*(a+b*arctanh(c*x))+1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))^
2/c-8/3*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+b^2*d^2*ln(-c^2*x^2+1)/c-
4/3*b^2*d^2*polylog(2,1-2/(-c*x+1))/c
```

Rubi [A]
time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6065, 6021, 266, 6037, 327, 212, 1600, 6055, 2449, 2352}

$$\frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c}-\frac{8bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c}+2abd^2x+\frac{b^2d^2\log\left(1-c^2x^2\right)}{c}-\frac{4b^2d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{3c}-\frac{b^2d^2\tanh^{-1}(cx)}{3c}+2b^2d^2x\tanh^{-1}(cx)+\frac{1}{3}b^2d^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] 2*a*b*d^2*x + (b^2*d^2*x)/3 - (b^2*d^2*ArcTanh[c*x])/(3*c) + 2*b^2*d^2*x*Ar
cTanh[c*x] + (b*c*d^2*x^2*(a + b*ArcTanh[c*x]))/3 + (d^2*(1 + c*x)^3*(a + b
*ArcTanh[c*x])^2)/(3*c) - (8*b*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(
3*c) + (b^2*d^2*Log[1 - c^2*x^2])/c - (4*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)
])/(3*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
```

```
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(2b)\int\left(-3d^3\left(a+b\tanh^{-1}(cx)\right)-\right.}{}$$

$$= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(8b)\int\frac{\left(d^3+cd^3x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{3d} +$$

$$= 2abd^2x + \frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx\right.}{3c}$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x - \frac{b^2d^2\tanh^{-1}(cx)}{3c} + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2($$

$$= 2abd^2x + \frac{1}{3}b^2d^2x - \frac{b^2d^2\tanh^{-1}(cx)}{3c} + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2($$

Mathematica [A]
time = 0.37, size = 227, normalized size = 1.30

$$\frac{d^2\left(3a^2cx+6abcx+b^2cx+3a^2c^2x^2+abc^2x^2+a^2c^3x^3+b^2(-7+3cx+3c^2x^2+c^3x^3)\tanh^{-1}(cx)^2+b\tanh^{-1}(cx)\left(2acx(3+3cx+c^2x^2)+b(-1+6cx+c^2x^2)-8b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+3ab\log(1-cx)-3ab\log(1+cx)+3ab\log\left(1-c^2x^2\right)+3b^2\log\left(1-c^2x^2\right)+ab\log\left(-1+c^2x^2\right)+4b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(3*a^2*c*x + 6*a*b*c*x + b^2*c*x + 3*a^2*c^2*x^2 + a*b*c^2*x^2 + a^2*c
^3*x^3 + b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTanh[
c*x]*(2*a*c*x*(3 + 3*c*x + c^2*x^2) + b*(-1 + 6*c*x + c^2*x^2) - 8*b*Log[1
+ E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*a*b*L
og[1 - c^2*x^2] + 3*b^2*Log[1 - c^2*x^2] + a*b*Log[-1 + c^2*x^2] + 4*b^2*Po
lyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(163) = 326$.
time = 0.38, size = 330, normalized size = 1.89

method	result
derivativedivides	$\frac{d^2(cx+1)^3a^2}{3}+\frac{d^2b^2\operatorname{arctanh}(cx)^2c^3x^3}{3}+d^2b^2\operatorname{arctanh}(cx)^2c^2x^2+b^2\operatorname{arctanh}(cx)^2d^2cx+\frac{d^2b^2\operatorname{arctanh}(cx)^2}{3}+\frac{d^2b^2\operatorname{arctanh}(cx)c^2}{3}$

default	$\frac{\frac{d^2(cx+1)^3a^2}{3}+\frac{d^2b^2\operatorname{arctanh}(cx)^2c^3x^3}{3}+d^2b^2\operatorname{arctanh}(cx)^2c^2x^2+b^2\operatorname{arctanh}(cx)^2d^2cx+\frac{d^2b^2\operatorname{arctanh}(cx)^2}{3}+\frac{d^2b^2\operatorname{arctanh}(cx)}{3}}{}$
risch	$-\ln(-cx+1)\,xab\,d^2+\frac{7\ln(-cx+1)ab\,d^2}{3c}+\frac{b\ln(-cx-1)a\,d^2}{3c}-\frac{2b^2(-cx+1)\ln(-cx+1)d^2}{3c}+\frac{4b^2\ln\left(-\frac{cx}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*d^2*(c*x+1)^3*a^2+1/3*d^2*b^2*arctanh(c*x)^2*c^3*x^3+d^2*b^2*arcta
nh(c*x)^2*c^2*x^2+d^2*b^2*arctanh(c*x)^2*c*x+1/3*d^2*b^2*arctanh(c*x)^2+1/3
*d^2*b^2*arctanh(c*x)*c^2*x^2+2*b^2*c*d^2*x*arctanh(c*x)+8/3*d^2*b^2*arctan
h(c*x)*ln(c*x-1)+1/3*d^2*b^2*c*x-1/3*d^2*b^2+7/6*d^2*b^2*ln(c*x-1)+5/6*d^2*
b^2*ln(c*x+1)+2/3*b^2*ln(c*x-1)^2*d^2-4/3*d^2*b^2*dilog(1/2*c*x+1/2)-4/3*b^
2*ln(c*x-1)*ln(1/2*c*x+1/2)*d^2+2/3*d^2*a*b*arctanh(c*x)*c^3*x^3+2*d^2*a*b*
arctanh(c*x)*c^2*x^2+2*d^2*a*b*arctanh(c*x)*c*x+2/3*d^2*a*b*arctanh(c*x)+1/
3*d^2*a*b*c^2*x^2+2*a*b*c*d^2*x+8/3*a*b*ln(c*x-1)*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(160) = 320$.
time = 0.41, size = 464, normalized size = 2.65

$$\frac{1}{3}a^2c^2d^2x^3+\frac{1}{3}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)abc^2d^2+a^2cd^2x^2+\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abcd^2+a^2d^2x+\frac{(2cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))abd^2}{c}+\frac{4\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d^2}{3c}+\frac{5b^2d^2\log(cx+1)}{6c}+\frac{7b^2d^2\log(cx-1)}{6c}+\frac{4b^2cd^2x+(b^2c^3d^2x^3+3b^2c^2d^2x^2+3b^2cd^2x+b^2d^2)\log(cx+1)^2+(b^2c^3d^2x^3+3b^2c^2d^2x^2+3b^2cd^2x-7b^2d^2)\log(-cx+1)^2+2(b^2c^2d^2x^2+6b^2cd^2x)\log(cx+1)-2(b^2c^2d^2x^2+6b^2cd^2x+(b^2c^3d^2x^3+3b^2c^2d^2x^2+3b^2cd^2x+b^2d^2)\log(cx+1))\log(-cx+1)}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*d^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
1)/c^4))*a*b*c^2*d^2 + a^2*c*d^2*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - l
og(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*c*d^2 + a^2*d^2*x + (2*c*x*arctanh
(c*x) + log(-c^2*x^2 + 1))*a*b*d^2/c + 4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2
) + dilog(1/2*c*x + 1/2))*b^2*d^2/c + 5/6*b^2*d^2*log(c*x + 1)/c + 7/6*b^2*
d^2*log(c*x - 1)/c + 1/12*(4*b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2
*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 + (b^2*c^3*d^2*x^3 + 3*b^2*c
^2*d^2*x^2 + 3*b^2*c*d^2*x - 7*b^2*d^2)*log(-c*x + 1)^2 + 2*(b^2*c^2*d^2*x^
2 + 6*b^2*c*d^2*x)*log(c*x + 1) - 2*(b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (b^2
*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1))*l
og(-c*x + 1))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*b
^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x +
 a*b*d^2)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2\,dx + \int b^2\,\mathrm{atanh}^2\,(cx)\,dx + \int 2ab\,\mathrm{atanh}\,(cx)\,dx + \int 2a^2cx\,dx + \int a^2c^2x^2\,dx + \int 2b^2cx\,\mathrm{atanh}^2\,(cx)\,dx + \int b^2c^2x^2\,\mathrm{atanh}^2\,(cx)\,dx + \int 4abcx\,\mathrm{atanh}\,(cx)\,dx + \int 2abc^2x^2\,\mathrm{atanh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*
atanh(c*x), x) + Integral(2*a**2*c*x, x) + Integral(a**2*c**2*x**2, x) + In
tegral(2*b**2*c*x*atanh(c*x)**2, x) + Integral(b**2*c**2*x**2*atanh(c*x)**2
, x) + Integral(4*a*b*c*x*atanh(c*x), x) + Integral(2*a*b*c**2*x**2*atanh(c
*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

3.80 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=278

$$abcd^2x+b^2cd^2x\tanh^{-1}(cx)+\frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)$$

```
[Out] a*b*c*d^2*x+b^2*c*d^2*x*arctanh(c*x)+3/2*d^2*(a+b*arctanh(c*x))^2+2*c*d^2*x
*(a+b*arctanh(c*x))^2+1/2*c^2*d^2*x^2*(a+b*arctanh(c*x))^2-2*d^2*(a+b*arcta
nh(c*x))^2*arctanh(-1+2/(-c*x+1))-4*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))
+1/2*b^2*d^2*ln(-c^2*x^2+1)-2*b^2*d^2*polylog(2,1-2/(-c*x+1))-b*d^2*(a+b*ar
ctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2
/(-c*x+1))+1/2*b^2*d^2*polylog(3,1-2/(-c*x+1))-1/2*b^2*d^2*polylog(3,-1+2/(
-c*x+1))
```

Rubi [A]
time = 0.43, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6033, 6199, 6095, 6205, 6745, 6037, 6127, 266}

$$\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2-bd^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^2\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+abcd^2x+2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2+2d^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}b^2d^2\log\left(1-c^2x^2\right)-2b^2d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)+\frac{1}{2}b^2d^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)+b^2cd^2x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTanh[c*x] + (3*d^2*(a + b*ArcTanh[c*x])^2)/2 +
 2*c*d^2*x*(a + b*ArcTanh[c*x])^2 + (c^2*d^2*x^2*(a + b*ArcTanh[c*x])^2)/2
+ 2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 4*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)] + (b^2*d^2*Log[1 - c^2*x^2])/2 - 2*b^2*d^2*Pol
yLog[2, 1 - 2/(1 - c*x)] - b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)] + b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^2
*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x(a+b\right.$$

$$= d^2 \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + \left(2cd^2\right) \int \left(a+b\tanh^{-1}(cx)\right)^2\,dx + ($$

$$= 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2d^2(a+b$$

$$= 2d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^2d^2x^2(a+b$$

$$= abcd^2x + \frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^2$$

$$= abcd^2x + b^2cd^2x\tanh^{-1}(cx) + \frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x(a+b$$

$$= abcd^2x + b^2cd^2x\tanh^{-1}(cx) + \frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2 + 2cd^2x(a+b$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 324, normalized size = 1.17

$$\frac{1}{2}d^2\left(4a^2cx + a^2c^2x^2 + 2a^2\log(cx) + ab(2cx + 2c^2x^2\tanh^{-1}(cx) + \log(1-cx) - \log(1+cx)) + 4ab\left(2cx\tanh^{-1}(cx) + \log\left(1-c^2x^2\right)\right) + b^2\left(2cx\tanh^{-1}(cx) + \left(-1+c^2x^2\right)\tanh^{-1}(cx)^2 + \log\left(1-c^2x^2\right)\right) + 4b^2\left(\tanh^{-1}(cx)\left((-1+cx)\tanh^{-1}(cx) - 2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right) + 2ab(-\text{PolyLog}(2,-cx) + \text{PolyLog}(2,cx)) + 2b^2\left(\frac{i\pi^3}{24} - \frac{2}{3}\tanh^{-1}(cx)^3 - \tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] (d^2*(4*a^2*c*x + a^2*c^2*x^2 + 2*a^2*Log[c*x] + a*b*(2*c*x + 2*c^2*x^2*Arc
Tanh[c*x] + Log[1 - c*x] - Log[1 + c*x]) + 4*a*b*(2*c*x*ArcTanh[c*x] + Log[
1 - c^2*x^2]) + b^2*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + L
og[1 - c^2*x^2]) + 4*b^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 +
 E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 2*a*b*(-PolyLo
g[2, -(c*x)] + PolyLog[2, c*x]) + 2*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3
 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(
2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[
c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 -
PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.16, size = 1082, normalized size = 3.89

method	result	size

derivativedivides	Expression too large to display	1082
default	Expression too large to display	1082

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*d^2*b^2*arctanh(c*x)^2+1/2*I*d^2*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+a*b*c*d^2*x+b^2*c*d^2*x*arc
tanh(c*x)+d^2*a*b*arctanh(c*x)*c^2*x^2+4*d^2*a*b*arctanh(c*x)*c*x+1/2*I*d^2
*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)
^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+
1)))+1/2*d^2*b^2*arctanh(c*x)^2*c^2*x^2+2*d^2*b^2*arctanh(c*x)^2*c*x+5/2*a*
b*ln(c*x-1)*d^2+3/2*a*b*ln(c*x+1)*d^2-1/2*I*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1))
)^2*arctanh(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-
d^2*a*b*dilog(c*x)-d^2*a*b*dilog(c*x+1)+d^2*b^2*arctanh(c*x)^2*ln(c*x)-d^2*
b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+d^2*b^2*arctanh(c*x)^2*ln(1
-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^2*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2
*x^2+1)^(1/2))+d^2*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^
2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-d^2*b^2*arctanh(c
*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-4*d^2*b^2*arctanh(c*x)*ln(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))-4*d^2*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1
/2))+2*d^2*a^2*c*x+1/2*d^2*a^2*c^2*x^2-2*d^2*b^2*polylog(3,(c*x+1)/(-c^2*x^
2+1)^(1/2))-2*d^2*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*d^2*b^2*po
lylog(3,-(c*x+1)^2/(-c^2*x^2+1))+d^2*b^2*arctanh(c*x)-4*d^2*b^2*dilog(1+I*(
c*x+1)/(-c^2*x^2+1)^(1/2))-4*d^2*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-
d^2*b^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))+d^2*a^2*ln(c*x)+2*d^2*a*b*arctanh(c*x)
*ln(c*x)-d^2*a*b*ln(c*x)*ln(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + 2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a*b*d^2 + a^2*d^2*log(x) + 1/8*(b^2*c^2*d^2*x^2 + 4*b^2*c*d^2*x)*log(
-c*x + 1)^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^
2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*d^2*x^2 + a*b*
c*d^2*x - a*b*d^2)*log(c*x + 1) - (4*a*b*c*d^2*x - 4*a*b*d^2 + (4*a*b*c^3*d
^2 + b^2*c^3*d^2)*x^3 - 4*(a*b*c^2*d^2 - b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*
```

```
x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1))*log(-c*x + 1))
/(c*x^2 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int 2a^2c\,dx+\int \frac{a^2}{x}\,dx+\int a^2c^2x\,dx+\int 2b^2c\operatorname{atanh}^2(cx)\,dx+\int \frac{b^2\operatorname{atanh}^2(cx)}{x}\,dx+\int 4abc\operatorname{atanh}(cx)\,dx+\int \frac{2ab\operatorname{atanh}(cx)}{x}\,dx+\int b^2c^2x\operatorname{atanh}^2(cx)\,dx+\int 2abc^2x\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(2*a**2*c, x) + Integral(a**2/x, x) + Integral(a**2*c**2*x, x
) + Integral(2*b**2*c*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x, x)
 + Integral(4*a*b*c*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x, x) + Inte
gral(b**2*c**2*x*atanh(c*x)**2, x) + Integral(2*a*b*c**2*x*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^2\,{\left(d+c\,d\,x\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x, x)
```

3.81 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=283

$$2cd^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2+4cd^2\left(a+b\tanh^{-1}(cx)\right)^2\tanh$$

```
[Out] 2*c*d^2*(a+b*arctanh(c*x))^2-d^2*(a+b*arctanh(c*x))^2/x+c^2*d^2*x*(a+b*arct
anh(c*x))^2-4*c*d^2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-2*b*c*d^2*(
a+b*arctanh(c*x))*ln(2/(-c*x+1))+2*b*c*d^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1
))-b^2*c*d^2*polylog(2,1-2/(-c*x+1))-2*b*c*d^2*(a+b*arctanh(c*x))*polylog(2
,1-2/(-c*x+1))+2*b*c*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-b^2*c*
d^2*polylog(2,-1+2/(c*x+1))+b^2*c*d^2*polylog(3,1-2/(-c*x+1))-b^2*c*d^2*pol
ylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.46, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6135, 6079, 2497, 6033, 6199, 6095, 6205, 6745}

$$c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2-2bcd^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^2\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+2cd^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+4cd^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-2bcd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-b^2cd^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)-b^2cd^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)+b^2cd^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)-b^2cd^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] 2*c*d^2*(a + b*ArcTanh[c*x])^2 - (d^2*(a + b*ArcTanh[c*x])^2)/x + c^2*d^2*x
*(a + b*ArcTanh[c*x])^2 + 4*c*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 -
 c*x)] - 2*b*c*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] + 2*b*c*d^2*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*d^2*PolyLog[2, 1 - 2/(1 - c*x)]
 - 2*b*c*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 2*b*c*d^2*(
a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^2*PolyLog[2, -1
+ 2/(1 + c*x)] + b^2*c*d^2*PolyLog[3, 1 - 2/(1 - c*x)] - b^2*c*d^2*PolyLog[
3, -1 + 2/(1 - c*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2}{x^2} dx = \int \left(c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{2cd^2\left(a+b\right.}{}\right.$$

$$= d^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} dx + \left(2cd^2\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} dx + \left(\right.$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2 + 4cd^2\left(a+b\,\mathrm{ta}\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\,\mathrm{tanh}\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\,\mathrm{tanh}\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\,\mathrm{tanh}\right.$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^2d^2x\left(a+b\,\mathrm{tanh}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 341, normalized size = 1.20

$$\frac{d^2\left(-12a^2+ib^2c\pi^3x+12a^2c^2x^2-24ab\tanh^{-1}(cx)+24abc^2x^2\tanh^{-1}(cx)-12b^2\tanh^{-1}(cx)^2+12b^2c^2x^2\tanh^{-1}(cx)^2-16b^2cx\tanh^{-1}(cx)^3+24b^2cx\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-24b^2cx\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-24b^2cx\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+24b^2cx\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+24a^2cx\log(x)+24abcx\log(cx)+12b^2cx\left(1+2\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-12b^2cx\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+24b^2cx\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)-24abcx\,\mathrm{PolyLog}(2,-cx)+24abcx\,\mathrm{PolyLog}(2,cx)+12b^2cx\,\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-12b^2cx\,\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)}{12x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-12*a^2 + I*b^2*c*Pi^3*x + 12*a^2*c^2*x^2 - 24*a*b*ArcTanh[c*x] + 24*
a*b*c^2*x^2*ArcTanh[c*x] - 12*b^2*ArcTanh[c*x]^2 + 12*b^2*c^2*x^2*ArcTanh[c
*x]^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 24*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2*A
rcTanh[c*x])] - 24*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b
^2*c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x
]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 24*a*b*c*x*Log[c*x] +
 12*b^2*c*x*(1 + 2*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]*PolyLog[2, E^
(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x)] + 24*a*b*c*x*PolyLog[2, c
*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c*x*PolyLog[3, E
^(2*ArcTanh[c*x])]))/(12*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.00, size = 5974, normalized size = 21.11

method	result	size
derivativedivides	Expression too large to display	5974
default	Expression too large to display	5974

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d^2*x - 1/2*b^2*c^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1), x) +
1/4*b^2*c^2*d^2*integrate(log(c*x + 1)^2/(c^2*x^2), x) + (2*c*x*arctanh(c*x
) + log(-c^2*x^2 + 1))*a*b*c*d^2 + 1/2*(c*x - (c*x - 1)*log(-c*x + 1) - 1)*
b^2*c*d^2 + 1/4*b^2*c*d^2*gamma(3, -log(c*x + 1)) + 1/2*b^2*c*d^2*integrate
(log(c*x + 1)^2/x, x) - b^2*c*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x, x
) + 2*a*b*c*d^2*integrate(log(c*x + 1)/x, x) - 2*a*b*c*d^2*integrate(log(-c
*x + 1)/x, x) - 1/2*b^2*c*d^2*integrate(log(-c*x + 1)/x, x) + 2*a^2*c*d^2*l
og(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d^2 - 1/2*
b^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x^2, x) - a^2*d^2/x + 1/4*(b^2
*c^2*d^2*x^2 - b^2*d^2)*log(-c*x + 1)^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int a^{2}c^{2}\,dx + \int \frac{a^{2}}{x^{2}}\,dx + \int \frac{2a^{2}c}{x}\,dx + \int b^{2}c^{2}\operatorname{atanh}^{2}(cx)\,dx + \int \frac{b^{2}\operatorname{atanh}^{2}(cx)}{x^{2}}\,dx + \int 2abc^{2}\operatorname{atanh}(cx)\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^{2}}\,dx + \int \frac{2b^{2}c\operatorname{atanh}^{2}(cx)}{x}\,dx + \int \frac{4abc\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(2*a**2*c/x
, x) + Integral(b**2*c**2*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x
**2, x) + Integral(2*a*b*c**2*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x*
*2, x) + Integral(2*b**2*c*atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x
)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^2, x)
```

3.82 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=313

$$-\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2$$

```
[Out] -b*c*d^2*(a+b*arctanh(c*x))/x+5/2*c^2*d^2*(a+b*arctanh(c*x))^2-1/2*d^2*(a+b
*arctanh(c*x))^2/x^2-2*c*d^2*(a+b*arctanh(c*x))^2/x-2*c^2*d^2*(a+b*arctanh(
c*x))^2*arctanh(-1+2/(-c*x+1))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(-c^2*x^
2+1)+4*b*c^2*d^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c^2*d^2*(a+b*arctanh(
c*x))*polylog(2,1-2/(-c*x+1))+b*c^2*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2/(
-c*x+1))-2*b^2*c^2*d^2*polylog(2,-1+2/(c*x+1))+1/2*b^2*c^2*d^2*polylog(3,1-
2/(-c*x+1))-1/2*b^2*c^2*d^2*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.48, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6087, 6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 2497, 6033, 6199, 6205, 6745}

$$-bc^2d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^2d^2\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2+2c^2d^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2+4bc^2d^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}-\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}-2b^2c^2d^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{1}{2}b^2c^2d^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2c^2d^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)-\frac{1}{2}b^2c^2d^2\log\left(1-c^2x^2\right)+b^2c^2d^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d^2*(a + b*ArcTanh[c*x]))/x) + (5*c^2*d^2*(a + b*ArcTanh[c*x])^2)/2
- (d^2*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x])^2)/x
 + 2*c^2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + b^2*c^2*d^2*
Log[x] - (b^2*c^2*d^2*Log[1 - c^2*x^2])/2 + 4*b*c^2*d^2*(a + b*ArcTanh[c*x]
)*Log[2 - 2/(1 + c*x)] - b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1
 - c*x)] + b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 2*
b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 + c*x)] + (b^2*c^2*d^2*PolyLog[3, 1 - 2/(1
 - c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{c^2d^2(a+b}{}\right.$$

$$= d^2 \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx + \left(2cd^2\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx + ($$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2c^2d^2\left(a+b\,\mathrm{ta}\right.$$

$$= 2c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{2cd^2\left(a+b\,\mathrm{tanl}\right.}{x}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\,\mathrm{tar}\right.}{2x}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\,\mathrm{tar}\right.}{2x}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\,\mathrm{tar}\right.}{2x}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\,\mathrm{tar}\right.}{2x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 370, normalized size = 1.18

$$-\frac{d^2\left(a^2+4a^2cx-2a^2c^2x^2\log(x)+ab(2\tanh^{-1}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))+b^2\left(2cx\tanh^{-1}(cx)+(1-c^2x^2)\tanh^{-1}(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)\right)+4abcx(2\tanh^{-1}(cx)+cx(-2\log(cx)+\log(1-c^2x^2)))+4b^2cx\left(\tanh^{-1}(cx)\left((1-cx)\tanh^{-1}(cx)-2cx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+cx\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+2abc^2x^2(\mathrm{PolyLog}(2,-cx)-\mathrm{PolyLog}(2,cx))-2b^2c^2x^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\tanh^{-1}(cx)^3-\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(d^2*(a^2 + 4*a^2*c*x - 2*a^2*c^2*x^2*Log[x] + a*b*(2*ArcTanh[c*x] + c
*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])) + b^2*(2*c*x*ArcTanh[c*x] + (
1 - c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + 4*a
*b*c*x*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])) + 4*b^2*c*x*
(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])])
 + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + 2*a*b*c^2*x^2*(PolyLog[2, -(c*x)]
 - PolyLog[2, c*x]) - 2*b^2*c^2*x^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - A
rcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*Ar
cTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]
*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - Poly
Log[3, E^(2*ArcTanh[c*x])]/2)))/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.93, size = 1103, normalized size = 3.52

method	result	size
derivativedivides	Expression too large to display	1103
default	Expression too large to display	1103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-3/2*d^2*b^2*arctanh(c*x)^2+1/2*I*d^2*b^2*arctanh(c*x)^2*Pi*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*d^2*b^2*arctanh
(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-4*d^2*a
*b*arctanh(c*x)/c/x-d^2*a*b*arctanh(c*x)/c^2/x^2-5/2*a*b*ln(c*x-1)*d^2-3/2*
a*b*ln(c*x+1)*d^2-1/2*I*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(
I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1
/2*I*d^2*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*d^2*a^2/c^2/x^2-
2*d^2*b^2*arctanh(c*x)^2/c/x+4*d^2*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+
1)^(1/2))+4*d^2*a*b*ln(c*x)-d^2*a*b*dilog(c*x)-d^2*a*b*dilog(c*x+1)+d^2*b^2
*arctanh(c*x)^2*ln(c*x)-d^2*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)
+d^2*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^2*b^2*arctanh(
c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+d^2*b^2*arctanh(c*x)^2*ln(1+(c*x
+1)/(-c^2*x^2+1)^(1/2))+2*d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2
+1)^(1/2))-d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+d^2*b^2*
ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-4*d^2*b^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2)
)+d^2*b^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+4*d^2*b^2*dilog(1+(c*x+1)/(-c^2*
x^2+1)^(1/2))-2*d^2*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^2*b^2*pol
ylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*d^2*b^2*polylog(3,-(c*x+1)^2/(-c^2*
x^2+1))-d^2*b^2*arctanh(c*x)+d^2*a^2*ln(c*x)-2*d^2*a^2/c/x-1/2*d^2*b^2*arct
anh(c*x)^2/c^2/x^2-d^2*b^2/c/x*arctanh(c*x)-d^2*a*b/c/x+2*d^2*a*b*arctanh(c
*x)*ln(c*x)-d^2*a*b*ln(c*x)*ln(c*x+1))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d^2*log(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c*d^2 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x
)/x^2)*a*b*d^2 - 2*a^2*c*d^2/x - 1/2*a^2*d^2/x^2 - 1/8*(4*b^2*c*d^2*x + b^2
*d^2)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*
x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*
```

```
d^2*x^2)*log(c*x + 1) - (4*a*b*c^3*d^2*x^3 - b^2*c*d^2*x - 4*(a*b*c^2*d^2 +
 b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^
2*d^2)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \frac{a^{2}}{x^{3}}\, dx + \int \frac{2a^{2}c}{x^{2}}\, dx + \int \frac{a^{2}c^{2}}{x}\, dx + \int \frac{b^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{2ab\operatorname{atanh}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{2b^{2}c\operatorname{atanh}^{2}{\left(cx \right)}}{x^{2}}\, dx + \int \frac{b^{2}c^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x}\, dx + \int \frac{4abc\operatorname{atanh}{\left(cx \right)}}{x^{2}}\, dx + \int \frac{2abc^{2}\operatorname{atanh}{\left(cx \right)}}{x}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c/x**2, x) + Integral(a**2*c
**2/x, x) + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x
)/x**3, x) + Integral(2*b**2*c*atanh(c*x)**2/x**2, x) + Integral(b**2*c**2*
atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x)/x**2, x) + Integral(2*a*b
*c**2*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^3, x)
```

3.83 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=244

$$-\frac{b^2c^2d^2}{3x}+\frac{1}{3}b^2c^3d^2\tanh^{-1}(cx)-\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^2(1+cx)^3\left(a+b\tanh\right.}{3x^3}$$

```
[Out] -1/3*b^2*c^2*d^2/x+1/3*b^2*c^3*d^2*arctanh(c*x)-1/3*b*c*d^2*(a+b*arctanh(c*
x))/x^2-2*b*c^2*d^2*(a+b*arctanh(c*x))/x-1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x
))^2/x^3+8/3*a*b*c^3*d^2*ln(x)+2*b^2*c^3*d^2*ln(x)+8/3*b*c^3*d^2*(a+b*arcta
nh(c*x))*ln(2/(-c*x+1))-b^2*c^3*d^2*ln(-c^2*x^2+1)-4/3*b^2*c^3*d^2*polylog(
2,-c*x)+4/3*b^2*c^3*d^2*polylog(2,c*x)+4/3*b^2*c^3*d^2*polylog(2,1-2/(-c*x+
1))
```

Rubi [A]
time = 0.19, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {37, 6085, 6037, 331, 212, 272, 36, 29, 31, 6031, 6055, 2449, 2352}

$$\frac{8}{3}abc^3d^2\log(x)+\frac{8}{3}bc^3d^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{4}{3}b^2c^3d^2\text{Li}_2(-cx)+\frac{4}{3}b^2c^3d^2\text{Li}_2(cx)+\frac{4}{3}b^2c^3d^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)+2b^2c^3d^2\log(x)+\frac{1}{3}b^2c^3d^2\tanh^{-1}(cx)-\frac{b^2c^2d^2}{3x}-b^2c^3d^2\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^2)/x + (b^2*c^3*d^2*ArcTanh[c*x])/3 - (b*c*d^2*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (2*b*c^2*d^2*(a + b*ArcTanh[c*x]))/x - (d^2*(1 + c*x)^3
*(a + b*ArcTanh[c*x])^2)/(3*x^3) + (8*a*b*c^3*d^2*Log[x])/3 + 2*b^2*c^3*d^2
*Log[x] + (8*b*c^3*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/3 - b^2*c^3*d
^2*Log[1 - c^2*x^2] - (4*b^2*c^3*d^2*PolyLog[2, -(c*x)])/3 + (4*b^2*c^3*d^2
*PolyLog[2, c*x])/3 + (4*b^2*c^3*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - (2bc)\int \left(-\frac{d^2\left(a+b\tanh^{-1}(c\right.}{3x^3}\right.$$

$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}\left(2bcd^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x^3}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x} - \frac{d^2(1+cx}{}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x} -$$

$$= -\frac{b^2c^2d^2}{3x} + \frac{1}{3}b^2c^3d^2\tanh^{-1}(cx) - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2}{}$$

$$= -\frac{b^2c^2d^2}{3x} + \frac{1}{3}b^2c^3d^2\tanh^{-1}(cx) - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2}{}$$

Mathematica [A]
time = 0.39, size = 270, normalized size = 1.11

$$-\frac{d^2\left(a^2+3a^2cx+abcx+3a^2c^2x^2+6abc^2x^2+b^2c^2x^2+b^2(1+3cx+3c^2x^2-7c^3x^3)\tanh^{-1}(cx)^2+b\tanh^{-1}(cx)\left(bcx(1+6cx-c^2x^2)+a(2+6cx+6c^2x^2)-8bc^3x^3\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-8abc^3x^3\log(cx)+3abc^3x^3\log(1-cx)-3abc^3x^3\log(1+cx)-6b^2c^3x^3\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+4abc^3x^3\log\left(1-c^2x^2\right)+4b^2c^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d^2*(a^2 + 3*a^2*c*x + a*b*c*x + 3*a^2*c^2*x^2 + 6*a*b*c^2*x^2 + b^2*
c^2*x^2 + b^2*(1 + 3*c*x + 3*c^2*x^2 - 7*c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTan
h[c*x]*(b*c*x*(1 + 6*c*x - c^2*x^2) + a*(2 + 6*c*x + 6*c^2*x^2) - 8*b*c^3*x
^3*Log[1 - E^(-2*ArcTanh[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*L
og[1 - c*x] - 3*a*b*c^3*x^3*Log[1 + c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 -
 c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 - c^2*x^2] + 4*b^2*c^3*x^3*PolyLog[2, E^(-
2*ArcTanh[c*x])]))/x^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(226) = 452$.
time = 0.65, size = 509, normalized size = 2.09

method	result
derivativedivides	$c^3\left(d^2a^2\left(-\frac{1}{c^2x^2}-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+\frac{b^2\ln(cx+1)^2d^2}{12}-\frac{2d^2ab\operatorname{arctanh}(cx)}{3c^3x^3}+\frac{8d^2b^2\operatorname{arctanh}(cx)\ln(cx)}{3}-\frac{b^2\operatorname{arc}}{}\right.$
default	$c^3\left(d^2a^2\left(-\frac{1}{c^2x^2}-\frac{1}{3c^3x^3}-\frac{1}{cx}\right)+\frac{b^2\ln(cx+1)^2d^2}{12}-\frac{2d^2ab\operatorname{arctanh}(cx)}{3c^3x^3}+\frac{8d^2b^2\operatorname{arctanh}(cx)\ln(cx)}{3}-\frac{b^2\operatorname{arc}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d^2*a^2*(-1/c^2/x^2-1/3/c^3/x^3-1/c/x)+1/12*b^2*ln(c*x+1)^2*d^2-2/3*d^
2*a*b*arctanh(c*x)/c^3/x^3-2*d^2*a*b*arctanh(c*x)/c/x-2*d^2*a*b*arctanh(c*x
)/c^2/x^2-1/3*d^2*b^2*arctanh(c*x)*ln(c*x+1)+8/3*d^2*b^2*arctanh(c*x)*ln(c*
x)-4/3*d^2*b^2*ln(c*x)*ln(c*x+1)-1/3*d^2*b^2/c/x-7/12*b^2*ln(c*x-1)^2*d^2-7
/3*d^2*b^2*arctanh(c*x)*ln(c*x-1)-7/3*a*b*ln(c*x-1)*d^2-1/3*a*b*ln(c*x+1)*d
^2+7/6*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)*d^2-1/6*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)
*d^2+1/6*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)*d^2+4/3*d^2*b^2*dilog(1/2*c*x
+1/2)-7/6*d^2*b^2*ln(c*x-1)-5/6*d^2*b^2*ln(c*x+1)-4/3*d^2*b^2*dilog(c*x+1)+
2*d^2*b^2*ln(c*x)-4/3*d^2*b^2*dilog(c*x)-1/3*d^2*a*b/c^2/x^2-1/3*d^2*b^2*ar
ctanh(c*x)^2/c^3/x^3-1/3*d^2*b^2*arctanh(c*x)/c^2/x^2-d^2*b^2*arctanh(c*x)^
2/c/x+8/3*d^2*a*b*ln(c*x)-d^2*b^2*arctanh(c*x)^2/c^2/x^2-2*d^2*b^2/c/x*arct
anh(c*x)-2*d^2*a*b/c/x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(221) = 442$.
time = 0.65, size = 555, normalized size = 2.27

$$-\frac{4}{3}\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2c^3d^2-\frac{4}{3}\left(\log(cx)\log(-cx+1)+\operatorname{Li}_2(-cx+1)\right)b^2c^3d^2+\frac{4}{3}\left(\log(cx+1)\log(-cx)+\operatorname{Li}_2(cx+1)\right)b^2c^3d^2-\frac{5}{6}b^2c^3d^2\log(cx+1)-\frac{7}{6}b^2c^3d^2\log(cx-1)+2b^2c^3d^2\log(x)-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\operatorname{artanh}(cx)}{x}\right)abc^2d^2+\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}(cx)}{x^2}\right)abcd^2-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\operatorname{artanh}(cx)}{x^3}\right)abd^2-\frac{a^2c^2d^2}{x}-\frac{a^2cd^2}{x^2}-\frac{a^2d^2}{3x^3}-\frac{4b^2c^2d^2x^2+\left(b^2c^3d^2x^3+3b^2c^2d^2x^2+3b^2cd^2x+b^2d^2\right)\log(cx+1)^2-\left(7b^2c^3d^2x^3-3b^2c^2d^2x^2-3b^2cd^2x-b^2d^2\right)\log(-cx+1)^2+2\left(6b^2c^2d^2x^2+b^2cd^2x\right)\log(cx+1)-2\left(6b^2c^2d^2x^2+b^2cd^2x+\left(b^2c^3d^2x^3+3b^2c^2d^2x^2+3b^2cd^2x+b^2d^2\right)\log(cx+1)\right)\log(-cx+1)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d^2
- 4/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d^2 + 4/3*(log(c*x
 + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d^2 - 5/6*b^2*c^3*d^2*log(c*x + 1
) - 7/6*b^2*c^3*d^2*log(c*x - 1) + 2*b^2*c^3*d^2*log(x) - (c*(log(c^2*x^2 -
 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c^2*d^2 + ((c*log(c*x + 1) - c*log(
c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d^2 - 1/3*((c^2*log(c^2*x^2 -
 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d^2 - a^2*c^2*d^2/x
 - a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/12*(4*b^2*c^2*d^2*x^2 + (b^2*c^3*d^2
*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 - (7*b^2
*c^3*d^2*x^3 - 3*b^2*c^2*d^2*x^2 - 3*b^2*c*d^2*x - b^2*d^2)*log(-c*x + 1)^2
 + 2*(6*b^2*c^2*d^2*x^2 + b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*c^2*d^2*x^2
+ b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*
d^2)*log(c*x + 1))*log(-c*x + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \frac{a^{2}}{x^{4}}\, dx + \int \frac{2a^{2}c}{x^{3}}\, dx + \int \frac{a^{2}c^{2}}{x^{2}}\, dx + \int \frac{b^{2}\operatorname{atanh}^{2}{(cx)}}{x^{4}}\, dx + \int \frac{2ab\operatorname{atanh}{(cx)}}{x^{4}}\, dx + \int \frac{2b^{2}c\operatorname{atanh}^{2}{(cx)}}{x^{3}}\, dx + \int \frac{b^{2}c^{2}\operatorname{atanh}^{2}{(cx)}}{x^{2}}\, dx + \int \frac{4abc\operatorname{atanh}{(cx)}}{x^{3}}\, dx + \int \frac{2abc^{2}\operatorname{atanh}{(cx)}}{x^{2}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2/x**4, x) + Integral(2*a**2*c/x**3, x) + Integral(a**2*c
**2/x**2, x) + Integral(b**2*atanh(c*x)**2/x**4, x) + Integral(2*a*b*atanh(
c*x)/x**4, x) + Integral(2*b**2*c*atanh(c*x)**2/x**3, x) + Integral(b**2*c*
*2*atanh(c*x)**2/x**2, x) + Integral(4*a*b*c*atanh(c*x)/x**3, x) + Integral
(2*a*b*c**2*atanh(c*x)/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^4, x)
```

3.84 $\int x^3(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=415

$$\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5-\frac{122b^2d^3\tanh^{-1}(cx)}{105c^4}+\frac{3b^2d^3x\tanh^{-1}(cx)}{2c^3}+$$

```
[Out] 3/2*a*b*d^3*x/c^3+122/105*b^2*d^3*x/c^3+7/20*b^2*d^3*x^2/c^2+44/315*b^2*d^3
*x^3/c+1/20*b^2*d^3*x^4+1/105*b^2*c*d^3*x^5-122/105*b^2*d^3*arctanh(c*x)/c^
4+3/2*b^2*d^3*x*arctanh(c*x)/c^3+26/35*b*d^3*x^2*(a+b*arctanh(c*x))/c^2+1/2
*b*d^3*x^3*(a+b*arctanh(c*x))/c+13/35*b*d^3*x^4*(a+b*arctanh(c*x))+1/5*b*c*
d^3*x^5*(a+b*arctanh(c*x))+1/21*b*c^2*d^3*x^6*(a+b*arctanh(c*x))-1/140*d^3*
(a+b*arctanh(c*x))^2/c^4+1/4*d^3*x^4*(a+b*arctanh(c*x))^2+3/5*c*d^3*x^5*(a+
b*arctanh(c*x))^2+1/2*c^2*d^3*x^6*(a+b*arctanh(c*x))^2+1/7*c^3*d^3*x^7*(a+b
*arctanh(c*x))^2-52/35*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+11/10*b^
2*d^3*ln(-c^2*x^2+1)/c^4-26/35*b^2*d^3*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A]
time = 1.02, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6087, 6037, 6127, 272, 45, 6021, 266, 6095, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{140c^4}-\frac{52bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{35c^4}+\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3abd^3x}{2c^3}+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{21}bc^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{26bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{35c^2}+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}bcd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{13}{35}bd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{2c}-\frac{26b^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{35c^4}-\frac{122b^2d^3\tanh^{-1}(cx)}{105c^4}+\frac{122b^2d^3x}{105c^3}+\frac{3b^2d^3x\tanh^{-1}(cx)}{2c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{11b^2d^3\log\left(1-c^2x^2\right)}{10c^4}+\frac{1}{105}b^2cd^3x^5+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (3*a*b*d^3*x)/(2*c^3) + (122*b^2*d^3*x)/(105*c^3) + (7*b^2*d^3*x^2)/(20*c^2
) + (44*b^2*d^3*x^3)/(315*c) + (b^2*d^3*x^4)/20 + (b^2*c*d^3*x^5)/105 - (12
2*b^2*d^3*ArcTanh[c*x])/(105*c^4) + (3*b^2*d^3*x*ArcTanh[c*x])/(2*c^3) + (2
6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/(35*c^2) + (b*d^3*x^3*(a + b*ArcTanh[c*x]
))/(2*c) + (13*b*d^3*x^4*(a + b*ArcTanh[c*x]))/35 + (b*c*d^3*x^5*(a + b*Arc
Tanh[c*x]))/5 + (b*c^2*d^3*x^6*(a + b*ArcTanh[c*x]))/21 - (d^3*(a + b*ArcTa
nh[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c*d^3*x^5*(
a + b*ArcTanh[c*x])^2)/5 + (c^2*d^3*x^6*(a + b*ArcTanh[c*x])^2)/2 + (c^3*d^
3*x^7*(a + b*ArcTanh[c*x])^2)/7 - (52*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 -
 c*x)])/(35*c^4) + (11*b^2*d^3*Log[1 - c^2*x^2])/(10*c^4) - (26*b^2*d^3*Pol
yLog[2, 1 - 2/(1 - c*x)])/(35*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int \left(d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+3c^2d^3\right.$$

$$= d^3\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3cd^3\right)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2$$

$$= \frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^{6}$$

$$= \frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^{6}$$

$$= \frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{3}{10}bd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}bcd^3x^5$$

$$= \frac{abd^3x}{2c^3}+\frac{3bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{2c}+$$

$$= \frac{3abd^3x}{2c^3}+\frac{199b^2d^3x}{210c^3}+\frac{73b^2d^3x^3}{630c}+\frac{1}{105}b^2cd^3x^5+\frac{b^2d^3x\tanh^{-1}(cx)}{2c^3}$$

$$= \frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{11b^2d^3x^2}{60c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2c$$

$$= \frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd$$

$$= \frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd$$

Mathematica [A]
time = 1.53, size = 385, normalized size = 0.93

$$\frac{d^3\left(-1464ab-504b^2+1890abcx+1464b^2cx+936abc^2x^2+441b^2c^2x^2+630abc^3x^3+176b^2c^3x^3+315a^2c^4x^4+468abc^4x^4+63b^2c^4x^4+756a^2c^5x^5+252abc^5x^5+12b^2c^5x^5+630a^2c^6x^6+60abc^6x^6+180a^2c^7x^7+9b^2(-209+35c^4x^4+84c^5x^5+70c^6x^6+20c^7x^7)\tanh^{-1}(cx)^2+6b\tanh^{-1}(cx)\left(3ac^4x^4(35+84cx+70c^2x^2+20c^3x^3)+b(-244+315cx+156c^2x^2+105c^3x^3+78c^4x^4+42c^5x^5+10c^6x^6)-312b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+945ab\log(1-cx)-945ab\log(1+cx)+1386b^2\log\left(1-c^2x^2\right)+936ab\log\left(-1+c^2x^2\right)+936b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{1260c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-1464*a*b - 504*b^2 + 1890*a*b*c*x + 1464*b^2*c*x + 936*a*b*c^2*x^2 +
 441*b^2*c^2*x^2 + 630*a*b*c^3*x^3 + 176*b^2*c^3*x^3 + 315*a^2*c^4*x^4 + 46
8*a*b*c^4*x^4 + 63*b^2*c^4*x^4 + 756*a^2*c^5*x^5 + 252*a*b*c^5*x^5 + 12*b^2
```

```
*c^5*x^5 + 630*a^2*c^6*x^6 + 60*a*b*c^6*x^6 + 180*a^2*c^7*x^7 + 9*b^2*(-209
 + 35*c^4*x^4 + 84*c^5*x^5 + 70*c^6*x^6 + 20*c^7*x^7)*ArcTanh[c*x]^2 + 6*b*
ArcTanh[c*x]*(3*a*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3) + b*(-244
 + 315*c*x + 156*c^2*x^2 + 105*c^3*x^3 + 78*c^4*x^4 + 42*c^5*x^5 + 10*c^6*x
^6) - 312*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 945*a*b*Log[1 - c*x] - 945*a*b*
Log[1 + c*x] + 1386*b^2*Log[1 - c^2*x^2] + 936*a*b*Log[-1 + c^2*x^2] + 936*
b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(1260*c^4)
```

Maple [A]
time = 0.45, size = 638, normalized size = 1.54 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^3*a^2*(1/7*c^7*x^7+1/2*c^6*x^6+3/5*c^5*x^5+1/4*c^4*x^4)+2/7*d^3*a*
b*arctanh(c*x)*c^7*x^7+d^3*a*b*arctanh(c*x)*c^6*x^6+6/5*d^3*a*b*arctanh(c*x
)*c^5*x^5+1/2*d^3*a*b*arctanh(c*x)*c^4*x^4+3/2*a*b*c*d^3*x+3/2*b^2*c*d^3*x*
arctanh(c*x)+109/210*d^3*b^2*ln(c*x+1)+1/7*d^3*b^2*arctanh(c*x)^2*c^7*x^7+1
/2*d^3*b^2*arctanh(c*x)^2*c^6*x^6+13/35*d^3*b^2*arctanh(c*x)*c^4*x^4+1/2*d^
3*b^2*arctanh(c*x)*c^3*x^3+26/35*d^3*b^2*arctanh(c*x)*c^2*x^2+1/21*d^3*b^2*
arctanh(c*x)*c^6*x^6+3/5*d^3*b^2*arctanh(c*x)^2*c^5*x^5+1/4*d^3*b^2*arctanh
(c*x)^2*c^4*x^4-26/35*d^3*b^2*dilog(1/2*c*x+1/2)+209/560*d^3*b^2*ln(c*x-1)^
2+1/560*d^3*b^2*ln(c*x+1)^2+353/210*d^3*b^2*ln(c*x-1)+209/140*d^3*b^2*arcta
nh(c*x)*ln(c*x-1)-1/140*d^3*b^2*arctanh(c*x)*ln(c*x+1)-209/280*d^3*b^2*ln(c
*x-1)*ln(1/2*c*x+1/2)-1/280*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/280*d^3*b^
2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/105*d^3*b^2*c^5*x^5+1/20*d^3*b^2*c^4*x
^4+7/20*d^3*b^2*c^2*x^2+44/315*d^3*b^2*c^3*x^3+209/140*d^3*a*b*ln(c*x-1)-1/
140*d^3*a*b*ln(c*x+1)+1/21*d^3*a*b*c^6*x^6+1/5*d^3*a*b*c^5*x^5+13/35*d^3*a*
b*c^4*x^4+1/2*d^3*a*b*c^3*x^3+26/35*d^3*a*b*c^2*x^2+1/5*d^3*b^2*arctanh(c*x
)*c^5*x^5+122/105*b^2*c*d^3*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(370) = 740$.
time = 0.50, size = 928, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*a^2*c^3*d^3*x^7 + 1/2*a^2*c^2*d^3*x^6 + 3/5*a^2*c*d^3*x^5 + 1/4*b^2*d^3
*x^4*arctanh(c*x)^2 + 1/42*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4
 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*x^4 + 1/
30*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(
c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^3 + 3/10*(4*x^5*arctanh(c*x)
 + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*d^3 + 1/12*(6*
```

```
x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*
x - 1)/c^5))*a*b*d^3 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^
5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*
log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*
b^2*d^3 + 26/35*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b
^2*d^3/c^4 + 13/70*b^2*d^3*log(c*x + 1)/c^4 + 283/210*b^2*d^3*log(c*x - 1)/
c^4 + 1/2520*(24*b^2*c^5*d^3*x^5 + 126*b^2*c^4*d^3*x^4 + 352*b^2*c^3*d^3*x^
3 + 672*b^2*c^2*d^3*x^2 + 2928*b^2*c*d^3*x + 9*(10*b^2*c^7*d^3*x^7 + 35*b^2
*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1)^2 + 9*(10*b^2*
c^7*d^3*x^7 + 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 - 87*b^2*d^3)*log(-c*
x + 1)^2 + 12*(5*b^2*c^6*d^3*x^6 + 21*b^2*c^5*d^3*x^5 + 39*b^2*c^4*d^3*x^4
+ 35*b^2*c^3*d^3*x^3 + 78*b^2*c^2*d^3*x^2 + 105*b^2*c*d^3*x)*log(c*x + 1) -
 6*(10*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 78*b^2*c^4*d^3*x^4 + 70*b^2*c
^3*d^3*x^3 + 156*b^2*c^2*d^3*x^2 + 210*b^2*c*d^3*x + 3*(10*b^2*c^7*d^3*x^7
+ 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1))*log(-
c*x + 1))/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^6 + 3*a^2*c^2*d^3*x^5 + 3*a^2*c*d^3*x^4 + a^2*d^3*x^
3 + (b^2*c^3*d^3*x^6 + 3*b^2*c^2*d^3*x^5 + 3*b^2*c*d^3*x^4 + b^2*d^3*x^3)*a
rctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^6 + 3*a*b*c^2*d^3*x^5 + 3*a*b*c*d^3*x^4 +
a*b*d^3*x^3)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int a^{2}x^{3}\,dx+\int 3a^{2}cx^{4}\,dx+\int 3a^{2}c^{2}x^{5}\,dx+\int a^{2}c^{3}x^{6}\,dx+\int b^{2}x^{3}\operatorname{atanh}^{2}{\left(cx\right)}\,dx+\int 2abx^{3}\operatorname{atanh}{\left(cx\right)}\,dx+\int 3b^{2}cx^{4}\operatorname{atanh}^{2}{\left(cx\right)}\,dx+\int 3b^{2}c^{2}x^{5}\operatorname{atanh}^{2}{\left(cx\right)}\,dx+\int b^{2}c^{3}x^{6}\operatorname{atanh}^{2}{\left(cx\right)}\,dx+\int 6abcx^{4}\operatorname{atanh}{\left(cx\right)}\,dx+\int 6abc^{2}x^{5}\operatorname{atanh}{\left(cx\right)}\,dx+\int 2abc^{3}x^{6}\operatorname{atanh}{\left(cx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x**3, x) + Integral(3*a**2*c*x**4, x) + Integral(3*a**2
*c**2*x**5, x) + Integral(a**2*c**3*x**6, x) + Integral(b**2*x**3*atanh(c*x
)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(3*b**2*c*x**4*atan
h(c*x)**2, x) + Integral(3*b**2*c**2*x**5*atanh(c*x)**2, x) + Integral(b**2
*c**3*x**6*atanh(c*x)**2, x) + Integral(6*a*b*c*x**4*atanh(c*x), x) + Integ
ral(6*a*b*c**2*x**5*atanh(c*x), x) + Integral(2*a*b*c**3*x**6*atanh(c*x), x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^2\,{\left(d+c\,d\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
[Out] int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

3.85 $\int x^2(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=377

$$\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2d^3\tanh^{-1}(cx)}{30c^3}+\frac{11b^2d^3x\tanh^{-1}(cx)}{6c^2}+\frac{14bd^3x^2\left(a-\right.}{}$$

```
[Out] 11/6*a*b*d^3*x/c^2+37/30*b^2*d^3*x/c^2+61/180*b^2*d^3*x^2/c+1/10*b^2*d^3*x^
3+1/60*b^2*c*d^3*x^4-37/30*b^2*d^3*arctanh(c*x)/c^3+11/6*b^2*d^3*x*arctanh(
c*x)/c^2+14/15*b*d^3*x^2*(a+b*arctanh(c*x))/c+11/18*b*d^3*x^3*(a+b*arctanh(
c*x))+3/10*b*c*d^3*x^4*(a+b*arctanh(c*x))+1/15*b*c^2*d^3*x^5*(a+b*arctanh(c
*x))+1/60*d^3*(a+b*arctanh(c*x))^2/c^3+1/3*d^3*x^3*(a+b*arctanh(c*x))^2+3/4
*c*d^3*x^4*(a+b*arctanh(c*x))^2+3/5*c^2*d^3*x^5*(a+b*arctanh(c*x))^2+1/6*c^
3*d^3*x^6*(a+b*arctanh(c*x))^2-28/15*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1)
)/c^3+113/90*b^2*d^3*ln(-c^2*x^2+1)/c^3-14/15*b^2*d^3*polylog(2,1-2/(-c*x+1
))/c^3
```

Rubi [A]
time = 0.87, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6087, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 272, 45, 6021, 266, 6095, 308}

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{60c^3}-\frac{28bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^3}+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{15}bc^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{11abd^3x}{6c^2}+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{10}bcd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{11}{18}bd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{14bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}-\frac{14b^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{15c^3}-\frac{37b^2d^3\tanh^{-1}(cx)}{30c^3}+\frac{37b^2d^3x}{30c^2}+\frac{11b^2d^3x\tanh^{-1}(cx)}{6c^2}+\frac{113b^2d^3\log\left(1-c^2x^2\right)}{90c^3}+\frac{1}{60}b^2cd^3x^4+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (11*a*b*d^3*x)/(6*c^2) + (37*b^2*d^3*x)/(30*c^2) + (61*b^2*d^3*x^2)/(180*c)
 + (b^2*d^3*x^3)/10 + (b^2*c*d^3*x^4)/60 - (37*b^2*d^3*ArcTanh[c*x])/(30*c^
3) + (11*b^2*d^3*x*ArcTanh[c*x])/(6*c^2) + (14*b*d^3*x^2*(a + b*ArcTanh[c*x
]))/(15*c) + (11*b*d^3*x^3*(a + b*ArcTanh[c*x]))/18 + (3*b*c*d^3*x^4*(a + b
*ArcTanh[c*x]))/10 + (b*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/15 + (d^3*(a + b*
ArcTanh[c*x])^2)/(60*c^3) + (d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + (3*c*d^3*x
^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x])^2)/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x])^2)/6 - (28*b*d^3*(a + b*ArcTanh[c*x])*Log[
2/(1 - c*x)])/(15*c^3) + (113*b^2*d^3*Log[1 - c^2*x^2])/(90*c^3) - (14*b^2*
d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+3c^2c\right.$$

$$=d^3\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3cd^3\right)\int x^3\left(a+b\tanh^{-1}(cx)\right)$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3$$

$$=\frac{bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{2}bd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{10}bcd^3x$$

$$=\frac{3abd^3x}{2c^2}+\frac{b^2d^3x}{3c^2}+\frac{14bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{11}{18}bd^3x^3\left(a+b\,\mathrm{t}\right.$$

$$=\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{1}{10}b^2d^3x^3-\frac{b^2d^3\tanh^{-1}(cx)}{3c^3}+\frac{3b^2d^3x\tan}{2c^2}$$

$$=\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{17b^2d^3x^2}{60c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2}{}$$

$$=\frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2}{}$$

Mathematica [A]
time = 0.87, size = 356, normalized size = 0.94

$$\frac{d^3\left(-162ab-64b^2+330abcx+222b^2cx+168abc^2x^2+61b^2c^2x^2+60a^2c^3x^3+110abc^3x^3+18b^2c^3x^3+135a^2c^4x^4+54abc^4x^4+3b^2c^4x^4+108a^2c^5x^5+12abc^5x^5+30a^2c^6x^6+3b^2(-111+20c^3x^3+45c^4x^4+36c^5x^5+10c^6x^6)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(3ac^3x^3(20+45cx+36c^2x^2+10c^3x^3)+b(-111+165cx+84c^2x^2+55c^3x^3+27c^4x^4+6c^5x^5)-168b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+165ab\log(1-cx)-165ab\log(1+cx)+226b^2\log\left(1-c^2x^2\right)+168ab\log\left(-1+c^2x^2\right)+168b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{180c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-162*a*b - 64*b^2 + 330*a*b*c*x + 222*b^2*c*x + 168*a*b*c^2*x^2 + 61*
b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3 + 135*a^2*c
^4*x^4 + 54*a*b*c^4*x^4 + 3*b^2*c^4*x^4 + 108*a^2*c^5*x^5 + 12*a*b*c^5*x^5
+ 30*a^2*c^6*x^6 + 3*b^2*(-111 + 20*c^3*x^3 + 45*c^4*x^4 + 36*c^5*x^5 + 10*
c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*c^3*x^3*(20 + 45*c*x + 36*c
^2*x^2 + 10*c^3*x^3) + b*(-111 + 165*c*x + 84*c^2*x^2 + 55*c^3*x^3 + 27*c^4
*x^4 + 6*c^5*x^5) - 168*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 165*a*b*Log[1 - c
*x] - 165*a*b*Log[1 + c*x] + 226*b^2*Log[1 - c^2*x^2] + 168*a*b*Log[-1 + c^
2*x^2] + 168*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(180*c^3)
```

Maple [A]
time = 0.48, size = 594, normalized size = 1.58 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^3*a^2*(1/6*c^6*x^6+3/5*c^5*x^5+3/4*c^4*x^4+1/3*x^3*c^3)+2/3*d^3*a*
b*arctanh(c*x)*c^3*x^3+1/3*d^3*a*b*arctanh(c*x)*c^6*x^6+6/5*d^3*a*b*arctanh
(c*x)*c^5*x^5+3/2*d^3*a*b*arctanh(c*x)*c^4*x^4+11/6*a*b*c*d^3*x+11/6*b^2*c*
d^3*x*arctanh(c*x)+1/3*d^3*b^2*arctanh(c*x)^2*c^3*x^3+23/36*d^3*b^2*ln(c*x+
1)+1/6*d^3*b^2*arctanh(c*x)^2*c^6*x^6+3/10*d^3*b^2*arctanh(c*x)*c^4*x^4+11/
18*d^3*b^2*arctanh(c*x)*c^3*x^3+14/15*d^3*b^2*arctanh(c*x)*c^2*x^2+3/5*d^3*
b^2*arctanh(c*x)^2*c^5*x^5+3/4*d^3*b^2*arctanh(c*x)^2*c^4*x^4-14/15*d^3*b^2
*dilog(1/2*c*x+1/2)+37/80*d^3*b^2*ln(c*x-1)^2-1/240*d^3*b^2*ln(c*x+1)^2+337
/180*d^3*b^2*ln(c*x-1)+37/20*d^3*b^2*arctanh(c*x)*ln(c*x-1)+1/60*d^3*b^2*ar
ctanh(c*x)*ln(c*x+1)-37/40*d^3*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/120*d^3*b^2*
ln(-1/2*c*x+1/2)*ln(c*x+1)-1/120*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1
/60*d^3*b^2*c^4*x^4+61/180*d^3*b^2*c^2*x^2+1/10*d^3*b^2*c^3*x^3+37/20*d^3*a
*b*ln(c*x-1)+1/60*d^3*a*b*ln(c*x+1)+1/15*d^3*a*b*c^5*x^5+3/10*d^3*a*b*c^4*x
^4+11/18*d^3*a*b*c^3*x^3+14/15*d^3*a*b*c^2*x^2+1/15*d^3*b^2*arctanh(c*x)*c^
5*x^5+37/30*b^2*c*d^3*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(336) = 672$.
time = 0.49, size = 775, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^3*d^3*x^6 + 3/5*a^2*c^2*d^3*x^5 + 3/4*a^2*c*d^3*x^4 + 1/90*(30*x^
6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/
c^7 + 15*log(c*x - 1)/c^7))*a*b*c^3*d^3 + 3/10*(4*x^5*arctanh(c*x) + c*((c^
2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3
 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5
+ 3*log(c*x - 1)/c^5))*a*b*c*d^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*d^3 + 14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + d
ilog(1/2*c*x + 1/2))*b^2*d^3/c^3 + 23/36*b^2*d^3*log(c*x + 1)/c^3 + 337/180
*b^2*d^3*log(c*x - 1)/c^3 + 1/720*(12*b^2*c^4*d^3*x^4 + 72*b^2*c^3*d^3*x^3
+ 244*b^2*c^2*d^3*x^2 + 888*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^
5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*log(c*x + 1)
^2 + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b
^2*c^3*d^3*x^3 - 111*b^2*d^3)*log(-c*x + 1)^2 + 4*(6*b^2*c^5*d^3*x^5 + 27*b
^2*c^4*d^3*x^4 + 55*b^2*c^3*d^3*x^3 + 84*b^2*c^2*d^3*x^2 + 165*b^2*c*d^3*x)
*log(c*x + 1) - 2*(12*b^2*c^5*d^3*x^5 + 54*b^2*c^4*d^3*x^4 + 110*b^2*c^3*d^
```

```
3*x^3 + 168*b^2*c^2*d^3*x^2 + 330*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 + 36*
b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*log(c*
x + 1))*log(-c*x + 1))/c^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^5 + 3*a^2*c^2*d^3*x^4 + 3*a^2*c*d^3*x^3 + a^2*d^3*x^
2 + (b^2*c^3*d^3*x^5 + 3*b^2*c^2*d^3*x^4 + 3*b^2*c*d^3*x^3 + b^2*d^3*x^2)*a
rctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^5 + 3*a*b*c^2*d^3*x^4 + 3*a*b*c*d^3*x^3 +
a*b*d^3*x^2)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int a^{2}x^{2}\,dx+\int 3a^{2}cx^{3}\,dx+\int 3a^{2}c^{2}x^{4}\,dx+\int a^{2}c^{3}x^{5}\,dx+\int b^{2}x^{2}\operatorname{atanh}^{2}(cx)\,dx+\int 2abx^{2}\operatorname{atanh}(cx)\,dx+\int 3b^{2}cx^{3}\operatorname{atanh}^{2}(cx)\,dx+\int 3b^{2}c^{2}x^{4}\operatorname{atanh}^{2}(cx)\,dx+\int b^{2}c^{3}x^{5}\operatorname{atanh}^{2}(cx)\,dx+\int 6abcx^{3}\operatorname{atanh}(cx)\,dx+\int 6abc^{2}x^{4}\operatorname{atanh}(cx)\,dx+\int 2abc^{3}x^{5}\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x**2, x) + Integral(3*a**2*c*x**3, x) + Integral(3*a**2
*c**2*x**4, x) + Integral(a**2*c**3*x**5, x) + Integral(b**2*x**2*atanh(c*x
)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(3*b**2*c*x**3*atan
h(c*x)**2, x) + Integral(3*b**2*c**2*x**4*atanh(c*x)**2, x) + Integral(b**2
*c**3*x**5*atanh(c*x)**2, x) + Integral(6*a*b*c*x**3*atanh(c*x), x) + Integ
ral(6*a*b*c**2*x**4*atanh(c*x), x) + Integral(2*a*b*c**3*x**5*atanh(c*x), x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
[Out] int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

3.86 $\int x(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=286

$$\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{4}b^2d^3x^2+\frac{1}{30}b^2cd^3x^3-\frac{13b^2d^3\tanh^{-1}(cx)}{10c^2}+\frac{5b^2d^3x\tanh^{-1}(cx)}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right.$$

```
[Out] 5/2*a*b*d^3*x/c+13/10*b^2*d^3*x/c+1/4*b^2*d^3*x^2+1/30*b^2*c*d^3*x^3-13/10*
b^2*d^3*arctanh(c*x)/c^2+5/2*b^2*d^3*x*arctanh(c*x)/c+6/5*b*d^3*x^2*(a+b*ar
ctanh(c*x))+1/2*b*c*d^3*x^3*(a+b*arctanh(c*x))+1/10*b*c^2*d^3*x^4*(a+b*arct
anh(c*x))-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/c^2+1/5*d^3*(c*x+1)^5*(a+b
*arctanh(c*x))^2/c^2-12/5*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2+3/2*b
^2*d^3*ln(-c^2*x^2+1)/c^2-6/5*b^2*d^3*polylog(2,1-2/(-c*x+1))/c^2
```

Rubi [A]
time = 0.43, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6087, 6065, 6021, 266, 6037, 327, 212, 272, 45, 1600, 6055, 2449, 2352, 308}

$$\frac{1}{10}bc^2d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}-\frac{12bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{1}{2}bcd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{5abd^3x}{2c}-\frac{6b^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{5c^2}+\frac{3b^2d^3\log\left(1-c^2x^2\right)}{2c^2}-\frac{13b^2d^3\tanh^{-1}(cx)}{10c^2}+\frac{1}{30}b^2cd^3x^3+\frac{13b^2d^3x}{10c}+\frac{5b^2d^3x\tanh^{-1}(cx)}{2c}+\frac{1}{4}b^2d^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (5*a*b*d^3*x)/(2*c) + (13*b^2*d^3*x)/(10*c) + (b^2*d^3*x^2)/4 + (b^2*c*d^3*
x^3)/30 - (13*b^2*d^3*ArcTanh[c*x])/(10*c^2) + (5*b^2*d^3*x*ArcTanh[c*x])/(
2*c) + (6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/5 + (b*c*d^3*x^3*(a + b*ArcTanh[c
*x]))/2 + (b*c^2*d^3*x^4*(a + b*ArcTanh[c*x]))/10 - (d^3*(1 + c*x)^4*(a + b
*ArcTanh[c*x])^2)/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x])^2)/(5*c^2
) - (12*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^2) + (3*b^2*d^3*L
og[1 - c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int x(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(-\frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{(d+cdx)^4\left(a+b\tanh^{-1}(c\right.}{cd}\right.$$

$$=-\frac{\int(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c}+\frac{\int(d+cdx)^4\left(a+b\tanh^{-1}\right.}{cd}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}$$

$$=\frac{5abd^3x}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}bcd^3x^3\left(a+b\tanh^{-1}(cx)\right)$$

$$=\frac{5abd^3x}{2c}+\frac{6b^2d^3x}{5c}+\frac{5b^2d^3x\tanh^{-1}(cx)}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)$$

$$=\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{30}b^2cd^3x^3-\frac{6b^2d^3\tanh^{-1}(cx)}{5c^2}+\frac{5b^2d^3x\tanh}{2c}$$

$$=\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{4}b^2d^3x^2+\frac{1}{30}b^2cd^3x^3-\frac{13b^2d^3\tanh^{-1}(cx)}{10c^2}+$$

Mathematica [**A**]
time = 0.70, size = 325, normalized size = 1.14

$$\frac{d^3\left(-18ab-15b^2+150abcx+78b^2cx+30a^2c^2x^2+72abc^2x^2+15b^2c^2x^2+60a^2c^3x^3+30abc^3x^3+2b^2c^3x^3+45a^2c^4x^4+6abc^4x^4+12a^2c^5x^5+3b^2(-49+10c^2x^2+20c^3x^3+15c^4x^4+4c^5x^5)\tanh^{-1}(cx)^2+6b\tanh^{-1}(cx)\left(ac^2x^2(10+20cx+15c^2x^2+4c^3x^3)+b(-13+25cx+12c^2x^2+5c^3x^3+c^4x^4)-24b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+75ab\log(1-cx)-75ab\log(1+cx)+90b^2\log\left(1-c^2x^2\right)+72ab\log\left(-1+c^2x^2\right)+72b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{60c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-18*a*b - 15*b^2 + 150*a*b*c*x + 78*b^2*c*x + 30*a^2*c^2*x^2 + 72*a*b
*c^2*x^2 + 15*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 30*a*b*c^3*x^3 + 2*b^2*c^3*x^3
 + 45*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*a^2*c^5*x^5 + 3*b^2*(-49 + 10*c^2*x^
2 + 20*c^3*x^3 + 15*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*
(a*c^2*x^2*(10 + 20*c*x + 15*c^2*x^2 + 4*c^3*x^3) + b*(-13 + 25*c*x + 12*c^
2*x^2 + 5*c^3*x^3 + c^4*x^4) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 75*a*b*
Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 90*b^2*Log[1 - c^2*x^2] + 72*a*b*Log[-
1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(258) = 516$.
time = 0.40, size = 546, normalized size = 1.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^3*a^2*(1/5*c^5*x^5+3/4*c^4*x^4+x^3*c^3+1/2*c^2*x^2)+d^3*a*b*arctan
h(c*x)*c^2*x^2+2*d^3*a*b*arctanh(c*x)*c^3*x^3+2/5*d^3*a*b*arctanh(c*x)*c^5*
x^5+3/2*d^3*a*b*arctanh(c*x)*c^4*x^4+5/2*a*b*c*d^3*x+5/2*b^2*c*d^3*x*arctan
h(c*x)+d^3*b^2*arctanh(c*x)^2*c^3*x^3+1/2*d^3*b^2*arctanh(c*x)^2*c^2*x^2+17
/20*d^3*b^2*ln(c*x+1)+1/10*d^3*b^2*arctanh(c*x)*c^4*x^4+1/2*d^3*b^2*arctanh
(c*x)*c^3*x^3+6/5*d^3*b^2*arctanh(c*x)*c^2*x^2+1/5*d^3*b^2*arctanh(c*x)^2*c
^5*x^5+3/4*d^3*b^2*arctanh(c*x)^2*c^4*x^4-6/5*d^3*b^2*dilog(1/2*c*x+1/2)+49
/80*d^3*b^2*ln(c*x-1)^2+1/80*d^3*b^2*ln(c*x+1)^2+43/20*d^3*b^2*ln(c*x-1)+49
/20*d^3*b^2*arctanh(c*x)*ln(c*x-1)-1/20*d^3*b^2*arctanh(c*x)*ln(c*x+1)-49/4
0*d^3*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/40*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)
+1/40*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/4*d^3*b^2*c^2*x^2+1/30*d^3
*b^2*c^3*x^3+49/20*d^3*a*b*ln(c*x-1)-1/20*d^3*a*b*ln(c*x+1)+1/10*d^3*a*b*c^
4*x^4+1/2*d^3*a*b*c^3*x^3+6/5*d^3*a*b*c^2*x^2+13/10*b^2*c*d^3*x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(255) = 510$.
time = 0.49, size = 780, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^3*d^3*x^5 + 3/4*a^2*c^2*d^3*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((
c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^3*d^3 + a^2*c*d^3*x^3
 + 1/2*b^2*d^3*x^2*arctanh(c*x)^2 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^2*d^3 + (2*x^
3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^
3*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x -
 1)/c^3))*a*b*d^3 + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3
)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(
c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2*d^3 + 6/5*(log(c*x + 1)*log(-1/2*c*x
+ 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^3/c^2 + 7/20*b^2*d^3*log(c*x + 1)/c^2
+ 33/20*b^2*d^3*log(c*x - 1)/c^2 + 1/240*(8*b^2*c^3*d^3*x^3 + 60*b^2*c^2*d^
3*x^2 + 312*b^2*c*d^3*x + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 20*b^
2*c^3*d^3*x^3 + 9*b^2*d^3)*log(c*x + 1)^2 + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c
^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 - 39*b^2*d^3)*log(-c*x + 1)^2 + 12*(b^2*c^4
*d^3*x^4 + 5*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 15*b^2*c*d^3*x)*log(c*x
 + 1) - 6*(2*b^2*c^4*d^3*x^4 + 10*b^2*c^3*d^3*x^3 + 24*b^2*c^2*d^3*x^2 + 30
*b^2*c*d^3*x + (4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3
 + 9*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^4 + 3*a^2*c^2*d^3*x^3 + 3*a^2*c*d^3*x^2 + a^2*d^3*x
+ (b^2*c^3*d^3*x^4 + 3*b^2*c^2*d^3*x^3 + 3*b^2*c*d^3*x^2 + b^2*d^3*x)*arcta
nh(c*x)^2 + 2*(a*b*c^3*d^3*x^4 + 3*a*b*c^2*d^3*x^3 + 3*a*b*c*d^3*x^2 + a*b*
d^3*x)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int a^{2}x\,dx + \int 3a^{2}cx^{2}\,dx + \int 3a^{2}c^{2}x^{3}\,dx + \int a^{2}c^{3}x^{4}\,dx + \int b^{2}x\operatorname{atanh}^{2}(cx)\,dx + \int 2abx\operatorname{atanh}(cx)\,dx + \int 3b^{2}cx^{2}\operatorname{atanh}^{2}(cx)\,dx + \int 3b^{2}c^{2}x^{3}\operatorname{atanh}^{2}(cx)\,dx + \int b^{2}c^{3}x^{4}\operatorname{atanh}^{2}(cx)\,dx + \int 6abcx^{2}\operatorname{atanh}(cx)\,dx + \int 6abc^{2}x^{3}\operatorname{atanh}(cx)\,dx + \int 2abc^{3}x^{4}\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x, x) + Integral(3*a**2*c*x**2, x) + Integral(3*a**2*c*
*2*x**3, x) + Integral(a**2*c**3*x**4, x) + Integral(b**2*x*atanh(c*x)**2,
x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(3*b**2*c*x**2*atanh(c*x)**2
, x) + Integral(3*b**2*c**2*x**3*atanh(c*x)**2, x) + Integral(b**2*c**3*x**
4*atanh(c*x)**2, x) + Integral(6*a*b*c*x**2*atanh(c*x), x) + Integral(6*a*b
*c**2*x**3*atanh(c*x), x) + Integral(2*a*b*c**3*x**4*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

3.87 $\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=206

$$\frac{7}{2}abd^3x+b^2d^3x+\frac{1}{12}b^2cd^3x^2-\frac{b^2d^3\tanh^{-1}(cx)}{c}+\frac{7}{2}b^2d^3x\tanh^{-1}(cx)+bcd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}bc^2d^3x^3\left(a\right.$$

```
[Out] 7/2*a*b*d^3*x+b^2*d^3*x+1/12*b^2*c*d^3*x^2-b^2*d^3*arctanh(c*x)/c+7/2*b^2*d
^3*x*arctanh(c*x)+b*c*d^3*x^2*(a+b*arctanh(c*x))+1/6*b*c^2*d^3*x^3*(a+b*arc
tanh(c*x))+1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/c-4*b*d^3*(a+b*arctanh(c*
x))*ln(2/(-c*x+1))/c+11/6*b^2*d^3*ln(-c^2*x^2+1)/c-2*b^2*d^3*polylog(2,1-2/
(-c*x+1))/c
```

Rubi [A]
time = 0.16, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6065, 6021, 266, 6037, 327, 212, 272, 45, 1600, 6055, 2449, 2352}

$$\frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right)+bcd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{4bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{7}{2}abd^3x+\frac{11b^2d^3\log\left(1-c^2x^2\right)}{6c}-\frac{2b^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c}+\frac{1}{12}b^2cd^3x^2-\frac{b^2d^3\tanh^{-1}(cx)}{c}+\frac{7}{2}b^2d^3x\tanh^{-1}(cx)+b^2d^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (7*a*b*d^3*x)/2 + b^2*d^3*x + (b^2*c*d^3*x^2)/12 - (b^2*d^3*ArcTanh[c*x])/c
 + (7*b^2*d^3*x*ArcTanh[c*x])/2 + b*c*d^3*x^2*(a + b*ArcTanh[c*x]) + (b*c^2
*d^3*x^3*(a + b*ArcTanh[c*x]))/6 + (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)
/(4*c) - (4*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (11*b^2*d^3*Lo
g[1 - c^2*x^2])/(6*c) - (2*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{b\int \left(-7d^4\left(a+b\tanh^{-1}(cx)\right) - \right.}{}$$

$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{(4b)\int \frac{\left(d^4+cd^4x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d}$$

$$= \frac{7}{2}abd^3x + bcd^3x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right)$$

$$= \frac{7}{2}abd^3x + b^2d^3x + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x^2\left(a+b\tanh^{-1}(cx)\right) +$$

$$= \frac{7}{2}abd^3x + b^2d^3x - \frac{b^2d^3\tanh^{-1}(cx)}{c} + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x^2($$

$$= \frac{7}{2}abd^3x + b^2d^3x + \frac{1}{12}b^2cd^3x^2 - \frac{b^2d^3\tanh^{-1}(cx)}{c} + \frac{7}{2}b^2d^3x\tanh^{-1}(c$$

Mathematica [A]
time = 0.49, size = 293, normalized size = 1.42

$$\frac{d^3\left(-b^2+12a^2cx+42abcx+12b^2cx+18a^2c^2x^2+12abc^2x^2+b^2c^2x^2+12a^2c^3x^3+2abc^3x^3+3a^2c^4x^4+3b^2(-15+4cx+6c^2x^2+4c^3x^3+c^4x^4)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(3acx(4+6cx+4c^2x^2+c^3x^3)+b(-6+21cx+6c^2x^2+c^3x^3)-24b\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+21ab\log(1-cx)-21ab\log(1+cx)+12ab\log\left(1-c^2x^2\right)+22b^2\log\left(1-c^2x^2\right)+12ab\log\left(-1+c^2x^2\right)+24b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

[Out] (d^3*(-b^2 + 12*a^2*c*x + 42*a*b*c*x + 12*b^2*c*x + 18*a^2*c^2*x^2 + 12*a*b*c^2*x^2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 3*b^2*(-15 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 + c^4*x^4)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3) + b*(-6 + 21*c*x + 6*c^2*x^2 + c^3*x^3) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 21*a*b*Log[1 - c*x] - 21*a*b*Log[1 + c*x] + 12*a*b*Log[1 - c^2*x^2] + 22*b^2*Log[1 - c^2*x^2] + 12*a*b*Log[-1 + c^2*x^2] + 24*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(12*c)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(194) = 388$.
time = 0.36, size = 408, normalized size = 1.98

method	result
derivativedivides	$\frac{d^3(cx+1)^4a^2}{4}+2d^3ab\operatorname{arctanh}(cx)cx+\frac{d^3ab\operatorname{arctanh}(cx)c^4x^4}{2}+3d^3ab\operatorname{arctanh}(cx)c^2x^2+2d^3ab\operatorname{arctanh}(cx)c^3x^3+\frac{7b^2c\,d^3x\operatorname{arcta}}{2}$
default	$\frac{d^3(cx+1)^4a^2}{4}+2d^3ab\operatorname{arctanh}(cx)cx+\frac{d^3ab\operatorname{arctanh}(cx)c^4x^4}{2}+3d^3ab\operatorname{arctanh}(cx)c^2x^2+2d^3ab\operatorname{arctanh}(cx)c^3x^3+\frac{7b^2c\,d^3x\operatorname{arcta}}{2}$
risch	$\frac{b\ln(-cx-1)a\,d^3}{4c}-\ln(-cx+1)\,xab\,d^3+\frac{15\ln(-cx+1)ab\,d^3}{4c}-\frac{b^2(-cx+1)\ln(-cx+1)d^3}{c}-\frac{2b^2\ln(-cx+1)\ln}{c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c*(1/4*d^3*(c*x+1)^4*a^2+2*d^3*a*b*arctanh(c*x)*c*x-13/12*d^3*b^2+3*d^3*a*b*arctanh(c*x)*c^2*x^2+2*d^3*a*b*arctanh(c*x)*c^3*x^3+1/2*d^3*a*b*arctanh(c*x)*c^4*x^4+d^3*b^2*arctanh(c*x)^2*c*x+7/2*a*b*c*d^3*x+7/2*b^2*c*d^3*x*arctanh(c*x)+d^3*b^2*arctanh(c*x)^2*c^3*x^3+3/2*d^3*b^2*arctanh(c*x)^2*c^2*x^2+4/3*d^3*b^2*ln(c*x+1)+1/6*d^3*b^2*arctanh(c*x)*c^3*x^3+d^3*b^2*arctanh(c*x)*c^2*x^2+1/4*d^3*b^2*arctanh(c*x)^2*c^4*x^4+1/4*d^3*b^2*arctanh(c*x)^2-2*d^3*b^2*dilog(1/2*c*x+1/2)+d^3*b^2*ln(c*x-1)^2+7/3*d^3*b^2*ln(c*x-1)+4*d^3*b^2*arctanh(c*x)*ln(c*x-1)-2*d^3*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*d^3*b^2*c^2*x^2+4*d^3*a*b*ln(c*x-1)+1/2*d^3*a*b*arctanh(c*x)+1/6*d^3*a*b*c^3*x^3+d^3*a*b*c^2*x^2+b^2*c*d^3*x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(191) = 382$.
time = 0.42, size = 627, normalized size = 3.04

$$\frac{1}{4}a^2c^3d^3x^4+a^2c^2d^3x^3+\frac{1}{12}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)abc^3d^3+\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)abc^2d^3+\frac{3}{2}a^2cd^3x^2+\frac{3}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abcd^3+a^2d^3x+\frac{(2cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))abd^3}{c}+\frac{2\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d^3}{c}+\frac{4b^2d^3\log(cx+1)}{3c}+\frac{7b^2d^3\log(cx-1)}{3c}+\frac{4b^2c^2d^3x^2+48b^2cd^3x+3(b^2c^4d^3x^4+4b^2c^3d^3x^3+6b^2c^2d^3x^2+4b^2cd^3x+b^2d^3)\log(cx+1)^2+3(b^2c^4d^3x^4+4b^2c^3d^3x^3+6b^2c^2d^3x^2+4b^2cd^3x-15b^2d^3)\log(-cx+1)^2+4(b^2c^3d^3x^3+6b^2c^2d^3x^2+21b^2cd^3x)\log(cx+1)-2(2b^2c^3d^3x^3+12b^2c^2d^3x^2+42b^2cd^3x+3(b^2c^4d^3x^4+4b^2c^3d^3x^3+6b^2c^2d^3x^2+4b^2cd^3x+b^2d^3)\log(cx+1))\log(-cx+1)}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] 1/4*a^2*c^3*d^3*x^4 + a^2*c^2*d^3*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^3*d^3 +

```
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c^2*d^3 + 3/2
*a^2*c*d^3*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
log(c*x - 1)/c^3))*a*b*c*d^3 + a^2*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x
^2 + 1))*a*b*d^3/c + 2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x +
1/2))*b^2*d^3/c + 4/3*b^2*d^3*log(c*x + 1)/c + 7/3*b^2*d^3*log(c*x - 1)/c +
 1/48*(4*b^2*c^2*d^3*x^2 + 48*b^2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*
d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x + b^2*d^3)*log(c*x + 1)^2 + 3*(
b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x - 1
5*b^2*d^3)*log(-c*x + 1)^2 + 4*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 21*b^
2*c*d^3*x)*log(c*x + 1) - 2*(2*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 42*b^
2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*
b^2*c*d^3*x + b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (b
^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)^
2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arcta
nh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int a^2\,dx + \int b^2\operatorname{atanh}^2(cx)\,dx + \int 2ab\operatorname{atanh}(cx)\,dx + \int 3a^2cx\,dx + \int 3a^2c^2x^2\,dx + \int a^2c^3x^3\,dx + \int 3b^2cx\operatorname{atanh}^2(cx)\,dx + \int 3b^2c^2x^2\operatorname{atanh}^2(cx)\,dx + \int b^2c^3x^3\operatorname{atanh}^2(cx)\,dx + \int 6abcx\operatorname{atanh}(cx)\,dx + \int 6abc^2x^2\operatorname{atanh}(cx)\,dx + \int 2abc^3x^3\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*
atanh(c*x), x) + Integral(3*a**2*c*x, x) + Integral(3*a**2*c**2*x**2, x) +
Integral(a**2*c**3*x**3, x) + Integral(3*b**2*c*x*atanh(c*x)**2, x) + Integ
ral(3*b**2*c**2*x**2*atanh(c*x)**2, x) + Integral(b**2*c**3*x**3*atanh(c*x)
**2, x) + Integral(6*a*b*c*x*atanh(c*x), x) + Integral(6*a*b*c**2*x**2*atan
h(c*x), x) + Integral(2*a*b*c**3*x**3*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{atanh}(c\,x))^2\,(d + c\,d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

3.88 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=355

$$3abcd^3x+\frac{1}{3}b^2cd^3x-\frac{1}{3}b^2d^3\tanh^{-1}(cx)+3b^2cd^3x\tanh^{-1}(cx)+\frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{11}{6}d^3\left(a+b\tanh^{-1}\right.$$

```
[Out] 3*a*b*c*d^3*x+1/3*b^2*c*d^3*x-1/3*b^2*d^3*arctanh(c*x)+3*b^2*c*d^3*x*arctan
h(c*x)+1/3*b*c^2*d^3*x^2*(a+b*arctanh(c*x))+11/6*d^3*(a+b*arctanh(c*x))^2+3
*c*d^3*x*(a+b*arctanh(c*x))^2+3/2*c^2*d^3*x^2*(a+b*arctanh(c*x))^2+1/3*c^3*
d^3*x^3*(a+b*arctanh(c*x))^2-2*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+
1))-20/3*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+3/2*b^2*d^3*ln(-c^2*x^2+1)
-10/3*b^2*d^3*polylog(2,1-2/(-c*x+1))-b*d^3*(a+b*arctanh(c*x))*polylog(2,1-
2/(-c*x+1))+b*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*d^3*p
olylog(3,1-2/(-c*x+1))-1/2*b^2*d^3*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.59, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6033, 6199, 6095, 6205, 6745, 6037, 6127, 266, 327, 212}

$$\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)-bd^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^3\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+3abcd^3x+3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{11}{6}d^3\left(a+b\tanh^{-1}(cx)\right)^2+2d^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-\frac{20}{3}bd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}b^2d^3\log\left(1-c^2x^2\right)-\frac{10}{3}b^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)+\frac{1}{2}b^2d^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d^3\text{Li}_3\left(\frac{2}{1-cx}-1\right)+\frac{1}{3}b^2cd^3x-\frac{1}{3}b^2d^3\tanh^{-1}(cx)+3b^2cd^3x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] 3*a*b*c*d^3*x + (b^2*c*d^3*x)/3 - (b^2*d^3*ArcTanh[c*x])/3 + 3*b^2*c*d^3*x*
ArcTanh[c*x] + (b*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/3 + (11*d^3*(a + b*ArcT
anh[c*x])^2)/6 + 3*c*d^3*x*(a + b*ArcTanh[c*x])^2 + (3*c^2*d^3*x^2*(a + b*A
rcTanh[c*x])^2)/2 + (c^3*d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + 2*d^3*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - (20*b*d^3*(a + b*ArcTanh[c*x])*Lo
g[2/(1 - c*x)])/3 + (3*b^2*d^3*Log[1 - c^2*x^2])/2 - (10*b^2*d^3*PolyLog[2,
 1 - 2/(1 - c*x)])/3 - b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x
)] + b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^3*Pol
yLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(3cd^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+3c^2d^3x\left(a+\right.\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx+\left(3cd^3\right)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(\right.$$

$$= 3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{3}c^3d^3x^3\left(\right.$$

$$= 3d^3\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}c^2d^3x^2\left(a+b\right.$$

$$= 3abcd^3x+\frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{11}{6}d^3\left(a+b\tanh^{-1}(cx)\right)^2+$$

$$= 3abcd^3x+\frac{1}{3}b^2cd^3x+3b^2cd^3x\tanh^{-1}(cx)+\frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(c\right.$$

$$= 3abcd^3x+\frac{1}{3}b^2cd^3x-\frac{1}{3}b^2d^3\tanh^{-1}(cx)+3b^2cd^3x\tanh^{-1}(cx)+\frac{1}{3}bc$$

$$= 3abcd^3x+\frac{1}{3}b^2cd^3x-\frac{1}{3}b^2d^3\tanh^{-1}(cx)+3b^2cd^3x\tanh^{-1}(cx)+\frac{1}{3}bc$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.44, size = 448, normalized size = 1.26

$$\frac{1}{24}d^3\left(ib^2\pi^3+72a^2cx+72abcx+8b^2cx+36a^2c^2x^2+8abc^2x^2+8a^2c^3x^3-8b^2\tanh^{-1}(cx)+144abcx\tanh^{-1}(cx)+72b^2cx\tanh^{-1}(cx)+72abc^2x^2\tanh^{-1}(cx)+8b^2c^2x^2\tanh^{-1}(cx)+16abc^3x^3\tanh^{-1}(cx)-116b^2\tanh^{-1}(cx)^2+72b^2cx\tanh^{-1}(cx)^2+36b^2c^2x^2\tanh^{-1}(cx)^2+8b^2c^3x^3\tanh^{-1}(cx)^2-16b^2\tanh^{-1}(cx)^3-160b^2\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-24b^2\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+24b^2\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+24a^2\log(cx)+36ab\log(1-cx)-36ab\log(1+cx)+72ab\log\left(1-c^2x^2\right)+36b^2\log\left(1-c^2x^2\right)+8ab\log\left(-1+c^2x^2\right)+8b^2\left(10+3\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+24b^2\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)-24ab\text{PolyLog}(2,-cx)+24ab\text{PolyLog}(2,cx)+12b^2\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-12b^2\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] (d^3*(I*b^2*Pi^3 + 72*a^2*c*x + 72*a*b*c*x + 8*b^2*c*x + 36*a^2*c^2*x^2 + 8
*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - 8*b^2*ArcTanh[c*x] + 144*a*b*c*x*ArcTanh[c*x
] + 72*b^2*c*x*ArcTanh[c*x] + 72*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*A
```

```
rcTanh[c*x] + 16*a*b*c^3*x^3*ArcTanh[c*x] - 116*b^2*ArcTanh[c*x]^2 + 72*b^2
*c*x*ArcTanh[c*x]^2 + 36*b^2*c^2*x^2*ArcTanh[c*x]^2 + 8*b^2*c^3*x^3*ArcTanh
[c*x]^2 - 16*b^2*ArcTanh[c*x]^3 - 160*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTan
h[c*x])] - 24*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*ArcT
anh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*Log[c*x] + 36*a*b*Log[1 - c
*x] - 36*a*b*Log[1 + c*x] + 72*a*b*Log[1 - c^2*x^2] + 36*b^2*Log[1 - c^2*x^
2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*(10 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-
2*ArcTanh[c*x])] + 24*b^2*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*
a*b*PolyLog[2, -(c*x)] + 24*a*b*PolyLog[2, c*x] + 12*b^2*PolyLog[3, -E^(-2*
ArcTanh[c*x])] - 12*b^2*PolyLog[3, E^(2*ArcTanh[c*x])]))/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.67, size = 1186, normalized size = 3.34

method	result	size
derivativedivides	Expression too large to display	1186
default	Expression too large to display	1186

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1
)^2/(-c^2*x^2+1)))^3+6*d^3*a*b*arctanh(c*x)*c*x+1/2*I*d^3*b^2*arctanh(c*x)^
2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*
csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/3*d^3*b^2+3
*d^3*a*b*arctanh(c*x)*c^2*x^2+2/3*d^3*a*b*arctanh(c*x)*c^3*x^3+3*d^3*b^2*ar
ctanh(c*x)^2*c*x+3*a*b*c*d^3*x+3*b^2*c*d^3*x*arctanh(c*x)+1/3*d^3*b^2*arcta
nh(c*x)^2*c^3*x^3+3/2*d^3*b^2*arctanh(c*x)^2*c^2*x^2+1/3*d^3*b^2*arctanh(c*
x)*c^2*x^2+11/6*d^3*b^2*arctanh(c*x)^2+29/6*d^3*a*b*ln(c*x-1)+11/6*d^3*a*b*
ln(c*x+1)+1/3*d^3*a*b*c^2*x^2+1/3*b^2*c*d^3*x+8/3*b^2*d^3*arctanh(c*x)+2*d^
3*a*b*arctanh(c*x)*ln(c*x)-d^3*a*b*ln(c*x)*ln(c*x+1)+3*d^3*a^2*c*x+1/3*d^3*
a^2*c^3*x^3+3/2*d^3*a^2*c^2*x^2-20/3*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)
^(1/2))-3*d^3*b^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-20/3*d^3*b^2*dilog(1+I*(c*x+
1)/(-c^2*x^2+1)^(1/2))+1/2*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*d^3
*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^3*b^2*polylog(3,-(c*x+1)/(-c
^2*x^2+1)^(1/2))+d^3*a^2*ln(c*x)-1/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c
^2*x^2+1)))^2-1/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-d^3*a
*b*dilog(c*x)-d^3*a*b*dilog(c*x+1)+d^3*b^2*arctanh(c*x)^2*ln(c*x)-20/3*d^3*
b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*d^3*b^2*arctanh(c*
x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(
-c^2*x^2+1)-1)+d^3*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^
3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+d^3*b^2*arctanh(c
*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*arctanh(c*x)*polylog(2,(c*
```

```
x+1)/(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^
2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^3*d^3*x^3 + 3/2*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + 3*(2*c*x*arctan
h(c*x) + log(-c^2*x^2 + 1))*a*b*d^3 + a^2*d^3*log(x) + 1/24*(2*b^2*c^3*d^3*
x^3 + 9*b^2*c^2*d^3*x^2 + 18*b^2*c*d^3*x)*log(-c*x + 1)^2 - integrate(-1/12
*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x
 + 1)^2 + 12*(a*b*c^4*d^3*x^4 + 2*a*b*c^3*d^3*x^3 - 3*a*b*c^2*d^3*x^2 + a*b
*c*d^3*x - a*b*d^3)*log(c*x + 1) - (12*a*b*c*d^3*x - 12*a*b*d^3 + 2*(6*a*b*
c^4*d^3 + b^2*c^4*d^3)*x^4 + 3*(8*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 - 18*(2*
a*b*c^2*d^3 - b^2*c^2*d^3)*x^2 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2
*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^2 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int 3a^{2}c\,dx+\int \frac{a^{2}}{x}\,dx+\int 3a^{2}c^{2}x\,dx+\int a^{2}c^{3}x^{2}\,dx+\int 3b^{2}c\operatorname{atanh}^{2}(cx)\,dx+\int \frac{b^{2}\operatorname{atanh}^{2}(cx)}{x}\,dx+\int 6abc\operatorname{atanh}(cx)\,dx+\int \frac{2ab\operatorname{atanh}(cx)}{x}\,dx+\int 3b^{2}c^{2}x\operatorname{atanh}^{2}(cx)\,dx+\int b^{2}c^{3}x^{2}\operatorname{atanh}^{2}(cx)\,dx+\int 6abc^{2}x\operatorname{atanh}(cx)\,dx+\int 2abc^{3}x^{2}\operatorname{atanh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(3*a**2*c, x) + Integral(a**2/x, x) + Integral(3*a**2*c**2*x,
 x) + Integral(a**2*c**3*x**2, x) + Integral(3*b**2*c*atanh(c*x)**2, x) + I
ntegral(b**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x), x) + Integr
al(2*a*b*atanh(c*x)/x, x) + Integral(3*b**2*c**2*x*atanh(c*x)**2, x) + Inte
```

```
gral(b**2*c**3*x**2*atanh(c*x)**2, x) + Integral(6*a*b*c**2*x*atanh(c*x), x
) + Integral(2*a*b*c**3*x**2*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x,x)
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x, x)
```

3.89 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=361

$$abc^2d^3x+b^2c^2d^3x\tanh^{-1}(cx)+\frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+3c^2d^3x\left(a+b\tanh^{-1}(cx)\right)^2-$$

```
[Out] a*b*c^2*d^3*x+b^2*c^2*d^3*x*arctanh(c*x)+7/2*c*d^3*(a+b*arctanh(c*x))^2-d^3
*(a+b*arctanh(c*x))^2/x+3*c^2*d^3*x*(a+b*arctanh(c*x))^2+1/2*c^3*d^3*x^2*(a
+b*arctanh(c*x))^2-6*c*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-6*b*
c*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+1/2*b^2*c*d^3*ln(-c^2*x^2+1)+2*b*c*
d^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-3*b^2*c*d^3*polylog(2,1-2/(-c*x+1))-
3*b*c*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+3*b*c*d^3*(a+b*arctanh
(c*x))*polylog(2,-1+2/(-c*x+1))-b^2*c*d^3*polylog(2,-1+2/(c*x+1))+3/2*b^2*c
*d^3*polylog(3,1-2/(-c*x+1))-3/2*b^2*c*d^3*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6135, 6079, 2497, 6033, 6199, 6095, 6205, 6745, 6127, 266}

$$\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+abc^2d^3x+3c^2d^3x\left(a+b\tanh^{-1}(cx)\right)^2-3bcd^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bcd^3\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+6cd^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-6bcd^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}b^2cd^3\log\left(1-c^2x^2\right)+b^2c^2d^3x\tanh^{-1}(cx)-3b^2cd^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)-b^2cd^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{3}{2}b^2cd^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b^2cd^3\text{Li}_3\left(\frac{2}{1-cx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] a*b*c^2*d^3*x + b^2*c^2*d^3*x*ArcTanh[c*x] + (7*c*d^3*(a + b*ArcTanh[c*x])^
2)/2 - (d^3*(a + b*ArcTanh[c*x])^2)/x + 3*c^2*d^3*x*(a + b*ArcTanh[c*x])^2
+ (c^3*d^3*x^2*(a + b*ArcTanh[c*x])^2)/2 + 6*c*d^3*(a + b*ArcTanh[c*x])^2*A
rcTanh[1 - 2/(1 - c*x)] - 6*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] +
 (b^2*c*d^3*Log[1 - c^2*x^2])/2 + 2*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2 - 2/
(1 + c*x)] - 3*b^2*c*d^3*PolyLog[2, 1 - 2/(1 - c*x)] - 3*b*c*d^3*(a + b*Arc
Tanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c*d^3*(a + b*ArcTanh[c*x])*Pol
yLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^3*PolyLog[2, -1 + 2/(1 + c*x)] + (3*b^2
*c*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (3*b^2*c*d^3*PolyLog[3, -1 + 2/(1 -
 c*x)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
```

```
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(3c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}+\frac{3cd^3(a+\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\left(3cd^3\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx+$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+3c^2d^3x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^3d^3x^2(a$$

$$= 4cd^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+3c^2d^3x(a+b\,\mathrm{t}$$

$$= abc^2d^3x+\frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+3c^2$$

$$= abc^2d^3x+b^2c^2d^3x\tanh^{-1}(cx)+\frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3(a+}{}$$

$$= abc^2d^3x+b^2c^2d^3x\tanh^{-1}(cx)+\frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3(a+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 479, normalized size = 1.33

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] (d^3*(-8*a^2 + I*b^2*c*Pi^3*x + 24*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + 4*a^2*c^3*
x^3 - 16*a*b*ArcTanh[c*x] + 48*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*Arc
Tanh[c*x] + 8*a*b*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 20*b^2*c*x*
ArcTanh[c*x]^2 + 24*b^2*c^2*x^2*ArcTanh[c*x]^2 + 4*b^2*c^3*x^3*ArcTanh[c*x]
^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 16*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2*ArcT
anh[c*x])] - 48*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^2*
c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]^2
*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 4*
a*b*c*x*Log[1 - c*x] - 4*a*b*c*x*Log[1 + c*x] + 16*a*b*c*x*Log[1 - c^2*x^2]
 + 4*b^2*c*x*Log[1 - c^2*x^2] + 24*b^2*c*x*(1 + ArcTanh[c*x])*PolyLog[2, -E
^(-2*ArcTanh[c*x])] - 8*b^2*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*
x*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x
)] + 24*a*b*c*x*PolyLog[2, c*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x]
)] - 12*b^2*c*x*PolyLog[3, E^(2*ArcTanh[c*x])]))/(8*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.06, size = 1239, normalized size = 3.43

method	result	size
derivativedivides	Expression too large to display	1239
default	Expression too large to display	1239

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(3/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(
I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))+6*d^3*a*b*arctanh(c*x)*c*x+d^3*a*b*arctanh(c*x)*c^2*x^2+3*
d^3*b^2*arctanh(c*x)^2*c*x+a*b*c*d^3*x+b^2*c*d^3*x*arctanh(c*x)+1/2*d^3*b^2
*arctanh(c*x)^2*c^2*x^2+3/2*d^3*b^2*arctanh(c*x)^2+5/2*d^3*a*b*ln(c*x-1)+3/
2*d^3*a*b*ln(c*x+1)+b^2*d^3*arctanh(c*x)-3/2*I*d^3*b^2*arctanh(c*x)^2*Pi*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2-3/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2+6*d^3*a*b*arctanh(c*x)*ln(c*x)-3*d^3*a*b*ln(c*x)*ln(c*x+1)-d^3*b^2*arcta
nh(c*x)^2/c/x-2*d^3*b^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*dilog(1
+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*d^3*a^2*c*x+1/2*d^3*a^2*c^2*x^2-2*d^3*a*b*ar
ctanh(c*x)/c/x+3/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-6*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1
)^(1/2))-d^3*b^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-6*d^3*b^2*dilog(1+I*(c*x+1)/(
-c^2*x^2+1)^(1/2))+3/2*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*d^3*b^2
*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-6*d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x
^2+1)^(1/2))+3*d^3*a^2*ln(c*x)+2*d^3*a*b*ln(c*x)+2*d^3*b^2*arctanh(c*x)*ln(
```

```
1+(c*x+1)/(-c^2*x^2+1)^(1/2))-d^3*a^2/c/x-3*d^3*a*b*dilog(c*x)-3*d^3*a*b*di
log(c*x+1)+3*d^3*b^2*arctanh(c*x)^2*ln(c*x)-6*d^3*b^2*arctanh(c*x)*ln(1+I*(
c*x+1)/(-c^2*x^2+1)^(1/2))-6*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-3*d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*d^3*b^2*a
rctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*d^3*b^2*arctanh(c*x)*polyl
og(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c
^2*x^2+1)^(1/2))+6*d^3*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2
))-3*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^3*d^3*x^2 + 3*a^2*c^2*d^3*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*a*b*c*d^3 + 3*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) +
 2*arctanh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/8*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d
^3*x^2 - 2*b^2*d^3)*log(-c*x + 1)^2/x - integrate(-1/4*((b^2*c^4*d^3*x^4 +
2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 4*(a*b*c^4*d^
3*x^4 - a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 - 3*a*b*c*d^3*x)*log(c*x + 1) -
 (12*a*b*c^2*d^3*x^2 + (4*a*b*c^4*d^3 + b^2*c^4*d^3)*x^4 - 2*(2*a*b*c^3*d^3
 - 3*b^2*c^3*d^3)*x^3 - 2*(6*a*b*c*d^3 + b^2*c*d^3)*x + 2*(b^2*c^4*d^3*x^4
+ 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))
/(c*x^3 - x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int 3a^{2}c^{2}\,dx + \int \frac{a^{2}}{x^{2}}\,dx + \int \frac{3a^{2}c}{x}\,dx + \int a^{2}c^{3}x\,dx + \int 3b^{2}c^{2}\operatorname{atanh}^{2}{\left(cx \right)}\,dx + \int \frac{b^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{2}}\,dx + \int 6abc^{2}\operatorname{atanh}{\left(cx \right)}\,dx + \int \frac{2ab\operatorname{atanh}{\left(cx \right)}}{x^{2}}\,dx + \int \frac{3b^{2}c\operatorname{atanh}^{2}{\left(cx \right)}}{x}\,dx + \int b^{2}c^{3}x\operatorname{atanh}^{2}{\left(cx \right)}\,dx + \int \frac{6abc\operatorname{atanh}{\left(cx \right)}}{x}\,dx + \int 2abc^{3}x\operatorname{atanh}{\left(cx \right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
/x, x) + Integral(a**2*c**3*x, x) + Integral(3*b**2*c**2*atanh(c*x)**2, x)
+ Integral(b**2*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*atanh(c*x), x)
 + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(3*b**2*c*atanh(c*x)**2/x,
x) + Integral(b**2*c**3*x*atanh(c*x)**2, x) + Integral(6*a*b*c*atanh(c*x)/x
, x) + Integral(2*a*b*c**3*x*atanh(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^2, x)
```

3.90 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=385

$$-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^3$$

```
[Out] -b*c*d^3*(a+b*arctanh(c*x))/x+9/2*c^2*d^3*(a+b*arctanh(c*x))^2-1/2*d^3*(a+b
*arctanh(c*x))^2/x^2-3*c*d^3*(a+b*arctanh(c*x))^2/x+c^3*d^3*x*(a+b*arctanh(
c*x))^2-6*c^2*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))+b^2*c^2*d^3*l
n(x)-2*b*c^2*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-1/2*b^2*c^2*d^3*ln(-c^2*
x^2+1)+6*b*c^2*d^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^2*c^2*d^3*polylog(2
,1-2/(-c*x+1))-3*b*c^2*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+3*b*c
^2*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-3*b^2*c^2*d^3*polylog(2,
-1+2/(c*x+1))+3/2*b^2*c^2*d^3*polylog(3,1-2/(-c*x+1))-3/2*b^2*c^2*d^3*polyl
og(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.58, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 2497, 6033, 6199, 6205, 6745}

$$c^3d^3x\left(a+b\tanh^{-1}(cx)\right)^2-3bc^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bc^2d^3\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2+6c^2d^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-2bc^2d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+6bc^2d^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x}-b^2c^2d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)-3b^2c^2d^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{3}{2}b^2c^2d^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b^2c^2d^3\text{Li}_3\left(\frac{2}{1-cx}-1\right)-\frac{1}{2}b^2c^2d^3\log\left(1-c^2x^2\right)+b^2c^2d^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d^3*(a + b*ArcTanh[c*x]))/x) + (9*c^2*d^3*(a + b*ArcTanh[c*x])^2)/2
- (d^3*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/x
 + c^3*d^3*x*(a + b*ArcTanh[c*x])^2 + 6*c^2*d^3*(a + b*ArcTanh[c*x])^2*ArcT
anh[1 - 2/(1 - c*x)] + b^2*c^2*d^3*Log[x] - 2*b*c^2*d^3*(a + b*ArcTanh[c*x]
)*Log[2/(1 - c*x)] - (b^2*c^2*d^3*Log[1 - c^2*x^2])/2 + 6*b*c^2*d^3*(a + b*
ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)
] - 3*b*c^2*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c^2*
d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 3*b^2*c^2*d^3*PolyL
og[2, -1 + 2/(1 + c*x)] + (3*b^2*c^2*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (
3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{3cd^3\left(a+\right.}{}\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx + \left(3cd^3\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx +$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^3d^3x\left(a+b\right.$$

$$= 4c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3cd^3\left(a+b\,\mathrm{ta}\right.}{x}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\,\mathrm{t}\right.}{2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\,\mathrm{t}\right.}{2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\,\mathrm{t}\right.}{2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\,\mathrm{t}\right.}{2}$$

<u>Mathematica</u> [**C**] Result contains complex when optimal does not.
time = 0.76, size = 461, normalized size = 1.20

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-(a^2/x^2) - (6*a^2*c)/x + 2*a^2*c^3*x + 6*a^2*c^2*Log[x] - (a*b*(2*A
rcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])))/x^2 + (b^2*(-
2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + 2*c^2*x^2*Log[(c*x)/Sq
rt[1 - c^2*x^2]]))/x^2 + 2*a*b*c^2*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2])
- (6*a*b*c*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])))/x + 2*b
^2*c^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x
])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + (6*b^2*c*(ArcTanh[c*x]*((-1 + c*
x)*ArcTanh[c*x] + 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*PolyLog[2, E^(-
2*ArcTanh[c*x])]))/x - 6*a*b*c^2*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]) + 6
*b^2*c^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2
*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]
*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*
```

```
x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/
2)))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.84, size = 1276, normalized size = 3.31

method	result	size
derivativedivides	Expression too large to display	1276
default	Expression too large to display	1276

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1
)^2/(-c^2*x^2+1)))+2*d^3*a*b*arctanh(c*x)*c*x-d^3*a*b*arctanh(c*x)/c^2/x^2+
d^3*b^2*arctanh(c*x)^2*c*x-3/2*d^3*b^2*arctanh(c*x)^2-5/2*d^3*a*b*ln(c*x-1)
-3/2*d^3*a*b*ln(c*x+1)-b^2*d^3*arctanh(c*x)-1/2*d^3*a^2/c^2/x^2-3/2*I*d^3*b
^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*I*d^3*b^2*arctanh(c*x)^2*P
i*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(
c*x+1)^2/(-c^2*x^2+1)))^2+6*d^3*a*b*arctanh(c*x)*ln(c*x)-3*d^3*a*b*ln(c*x)*
ln(c*x+1)-3*d^3*b^2*arctanh(c*x)^2/c/x-6*d^3*b^2*dilog((c*x+1)/(-c^2*x^2+1)
^(1/2))+6*d^3*b^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+d^3*a^2*c*x+d^3*b^2*l
n((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+d^3*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-6*
d^3*a*b*arctanh(c*x)/c/x+3/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-2*d^3*b^2*dilog(1-I*(c*x+1)/(
-c^2*x^2+1)^(1/2))-2*d^3*b^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*d^3*
b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^
2+1)^(1/2))-6*d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*d^3*a^2*ln(c
*x)-1/2*d^3*b^2*arctanh(c*x)^2/c^2/x^2-d^3*b^2*arctanh(c*x)/c/x-d^3*a*b/c/x
+6*d^3*a*b*ln(c*x)+6*d^3*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-
3*d^3*a^2/c/x-3*d^3*a*b*dilog(c*x)-3*d^3*a*b*dilog(c*x+1)+3*d^3*b^2*arctanh
(c*x)^2*ln(c*x)-2*d^3*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2
*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3*d^3*b^2*arctanh(
c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*d^3*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(
-c^2*x^2+1)^(1/2))+6*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(
1/2))+3*d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*d^3*b^2*a
rctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3*d^3*b^2*arctanh(c*x)*po
lylog(2,-(c*x+1)^2/(-c^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^3*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b*c^2*d^3 + 3*a^
2*c^2*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a
*b*c*d^3 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/
x^2)*a*b*d^3 - 3*a^2*c*d^3/x - 1/2*a^2*d^3/x^2 + 1/8*(2*b^2*c^3*d^3*x^3 - 6
*b^2*c*d^3*x - b^2*d^3)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^4*d^3*
x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b
*c^3*d^3*x^3 - a*b*c^2*d^3*x^2)*log(c*x + 1) - (2*b^2*c^4*d^3*x^4 + 12*a*b*
c^3*d^3*x^3 - b^2*c*d^3*x - 6*(2*a*b*c^2*d^3 + b^2*c^2*d^3)*x^2 + 2*(b^2*c^
4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(
-c*x + 1))/(c*x^4 - x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int a^{2}c^{3}\,dx + \int \frac{a^{2}}{x^{3}}\,dx + \int \frac{3a^{2}c}{x^{2}}\,dx + \int \frac{3a^{2}c^{2}}{x}\,dx + \int b^{2}c^{3}\operatorname{atanh}^{2}(cx)\,dx + \int \frac{b^{2}\operatorname{atanh}^{2}(cx)}{x^{3}}\,dx + \int 2abc^{3}\operatorname{atanh}(cx)\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^{3}}\,dx + \int \frac{3b^{2}c\operatorname{atanh}^{2}(cx)}{x^{2}}\,dx + \int \frac{3b^{2}c^{2}\operatorname{atanh}^{2}(cx)}{x}\,dx + \int \frac{6abc\operatorname{atanh}(cx)}{x^{2}}\,dx + \int \frac{6abc^{2}\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2*c**3, x) + Integral(a**2/x**3, x) + Integral(3*a**2*c/x
**2, x) + Integral(3*a**2*c**2/x, x) + Integral(b**2*c**3*atanh(c*x)**2, x)
 + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*c**3*atanh(c*x), x
) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(3*b**2*c*atanh(c*x)**2/x*
*2, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(
c*x)/x**2, x) + Integral(6*a*b*c**2*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^3, x)
```

3.91 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=396

$$-\frac{b^2c^2d^3}{3x}+\frac{1}{3}b^2c^3d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{29}{6}c^3d^3\left(a+b\tanh^{-1}(c\right.$$

```
[Out] -1/3*b^2*c^2*d^3/x+1/3*b^2*c^3*d^3*arctanh(c*x)-1/3*b*c*d^3*(a+b*arctanh(c*
x))/x^2-3*b*c^2*d^3*(a+b*arctanh(c*x))/x+29/6*c^3*d^3*(a+b*arctanh(c*x))^2-
1/3*d^3*(a+b*arctanh(c*x))^2/x^3-3/2*c*d^3*(a+b*arctanh(c*x))^2/x^2-3*c^2*d
^3*(a+b*arctanh(c*x))^2/x-2*c^3*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x
+1))+3*b^2*c^3*d^3*ln(x)-3/2*b^2*c^3*d^3*ln(-c^2*x^2+1)+20/3*b*c^3*d^3*(a+b
*arctanh(c*x))*ln(2-2/(c*x+1))-b*c^3*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(
-c*x+1))+b*c^3*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-10/3*b^2*c^3
*d^3*polylog(2,-1+2/(c*x+1))+1/2*b^2*c^3*d^3*polylog(3,1-2/(-c*x+1))-1/2*b^
2*c^3*d^3*polylog(3,-1+2/(-c*x+1))
```

Rubi [A]
time = 0.68, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6087, 6037, 6129, 331, 212, 6135, 6079, 2497, 272, 36, 29, 31, 6095, 6033, 6199, 6205, 6745}

$$-bc^3d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^3d^3\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{29}{6}c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2+2c^3d^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2+\frac{20}{3}bc^3d^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}-\frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{10}{3}b^2c^3d^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)+\frac{1}{2}b^2c^3d^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2c^3d^3\text{Li}_3\left(\frac{2}{1-cx}-1\right)+3b^2c^3d^3\log(x)+\frac{1}{3}b^2c^3d^3\tanh^{-1}(cx)-\frac{b^2c^2d^3}{3x}-\frac{3}{2}b^2c^3d^3\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^3)/x + (b^2*c^3*d^3*ArcTanh[c*x])/3 - (b*c*d^3*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/x + (29*c^3*d^3*(a +
 b*ArcTanh[c*x])^2)/6 - (d^3*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (3*c*d^3*(a
+ b*ArcTanh[c*x])^2)/(2*x^2) - (3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/x + 2*c^3
*d^3*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 3*b^2*c^3*d^3*Log[x]
 - (3*b^2*c^3*d^3*Log[1 - c^2*x^2])/2 + (20*b*c^3*d^3*(a + b*ArcTanh[c*x])*
Log[2 - 2/(1 + c*x)])/3 - b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)] + b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - (
10*b^2*c^3*d^3*PolyLog[2, -1 + 2/(1 + c*x)])/3 + (b^2*c^3*d^3*PolyLog[3, 1
- 2/(1 - c*x)])/2 - (b^2*c^3*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4} + \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{3c^2d^3\left(a+\right.}{}\right.$$

$$= d^3 \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx + \left(3cd^3\right) \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx + (\ldots$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3c^2d^3\left(a+b\,\mathrm{t}\ldots\right.}{x}$$

$$= 3c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - \frac{3cd^3\left(a+b\tanh\ldots\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{29}{6}c^3d^3(a+\ldots$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{29}{6}\ldots$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{1}{3}b^2c^3d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3(\ldots}{}$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{1}{3}b^2c^3d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{3bc^2d^3(\ldots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 569, normalized size = 1.44

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-8*a^2 - 36*a^2*c*x - 8*a*b*c*x - 72*a^2*c^2*x^2 - 72*a*b*c^2*x^2 - 8
*b^2*c^2*x^2 + I*b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTanh[c*x] - 72*a*b*c*x*ArcTan
h[c*x] - 8*b^2*c*x*ArcTanh[c*x] - 144*a*b*c^2*x^2*ArcTanh[c*x] - 72*b^2*c^2
*x^2*ArcTanh[c*x] + 8*b^2*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 36*
b^2*c*x*ArcTanh[c*x]^2 - 72*b^2*c^2*x^2*ArcTanh[c*x]^2 + 116*b^2*c^3*x^3*Ar
cTanh[c*x]^2 - 16*b^2*c^3*x^3*ArcTanh[c*x]^3 + 160*b^2*c^3*x^3*ArcTanh[c*x]
*Log[1 - E^(-2*ArcTanh[c*x])] - 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 + E^(-2
*ArcTanh[c*x])] + 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])]
 + 24*a^2*c^3*x^3*Log[x] + 160*a*b*c^3*x^3*Log[c*x] - 36*a*b*c^3*x^3*Log[1
- c*x] + 36*a*b*c^3*x^3*Log[1 + c*x] + 72*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^
2*x^2]] - 80*a*b*c^3*x^3*Log[1 - c^2*x^2] + 24*b^2*c^3*x^3*ArcTanh[c*x]*Pol
yLog[2, -E^(-2*ArcTanh[c*x])] - 80*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x
])] + 24*b^2*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c
^3*x^3*PolyLog[2, -(c*x)] + 24*a*b*c^3*x^3*PolyLog[2, c*x] + 12*b^2*c^3*x^3
*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c^3*x^3*PolyLog[3, E^(2*ArcTanh[
c*x])]))/(24*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.74, size = 1267, normalized size = 3.20

method	result	size
derivativedivides	Expression too large to display	1267
default	Expression too large to display	1267

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-3*d^3*a*b*arctanh(c*x)/c^2/x^2-1/3*d^3*b^2/(c*x+1-(-c^2*x^2+1)^(1/2))
*(-c^2*x^2+1)^(1/2)+1/3*d^3*b^2/((-c^2*x^2+1)^(1/2)+c*x+1)*(-c^2*x^2+1)^(1/
2)-1/3*d^3*a^2/c^3/x^3-11/6*d^3*b^2*arctanh(c*x)^2-29/6*d^3*a*b*ln(c*x-1)-1
1/6*d^3*a*b*ln(c*x+1)-8/3*b^2*d^3*arctanh(c*x)+1/2*I*d^3*b^2*arctanh(c*x)^2
*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-3/2*d^3*a^2/c^
2/x^2+2*d^3*a*b*arctanh(c*x)*ln(c*x)-d^3*a*b*ln(c*x)*ln(c*x+1)-1/2*I*d^3*b^
2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3*d^3*b^2*arctanh(c*x)^2/c/x-1/
3*d^3*b^2*arctanh(c*x)/c^2/x^2-1/3*d^3*b^2*arctanh(c*x)^2/c^3/x^3-1/3*d^3*a
*b/c^2/x^2-20/3*d^3*b^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+20/3*d^3*b^2*dilo
g(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*d^3*b^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+
3*d^3*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-6*d^3*a*b*arctanh(c*x)/c/x-2/3*d
^3*a*b*arctanh(c*x)/c^3/x^3+1/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*d^3*b^2*polylog(3,-(c*
x+1)^2/(-c^2*x^2+1))-2*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^3*
```

```
b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+d^3*a^2*ln(c*x)-3/2*d^3*b^2*arct
anh(c*x)^2/c^2/x^2-3*d^3*b^2*arctanh(c*x)/c/x-3*d^3*a*b/c/x+20/3*d^3*a*b*ln
(c*x)+20/3*d^3*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3*d^3*a^2/
c/x-1/2*I*d^3*b^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-d^3*a*b*dilog(c
*x)-d^3*a*b*dilog(c*x+1)+d^3*b^2*arctanh(c*x)^2*ln(c*x)-d^3*b^2*arctanh(c*x
)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+d^3*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*
x^2+1)^(1/2))+2*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))
+d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*arctanh(
c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(c*x)*polylog(2,-
(c*x+1)^2/(-c^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] a^2*c^3*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c^2*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c
*x)/x^2)*a*b*c*d^3 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c +
 2*arctanh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 3/2*a^2*c*d^3/x^2 - 1/3*a^
2*d^3/x^3 - 1/24*(18*b^2*c^2*d^3*x^2 + 9*b^2*c*d^3*x + 2*b^2*d^3)*log(-c*x
+ 1)^2/x^3 - integrate(-1/12*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^
2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b*c^4*d^3*x^4 - a*b*c^3*d^3*x^3
)*log(c*x + 1) - (12*a*b*c^4*d^3*x^4 - 9*b^2*c^2*d^3*x^2 - 2*b^2*c*d^3*x -
6*(2*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*
x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int \frac{a^{2}}{x^{4}}\, dx + \int \frac{3a^{2}c}{x^{3}}\, dx + \int \frac{3a^{2}c^{2}}{x^{2}}\, dx + \int \frac{a^{2}c^{3}}{x}\, dx + \int \frac{b^{2}\operatorname{atanh}^{2}(cx)}{x^{4}}\, dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^{4}}\, dx + \int \frac{3b^{2}c\operatorname{atanh}^{2}(cx)}{x^{3}}\, dx + \int \frac{3b^{2}c^{2}\operatorname{atanh}^{2}(cx)}{x^{2}}\, dx + \int \frac{b^{2}c^{3}\operatorname{atanh}^{2}(cx)}{x}\, dx + \int \frac{6abc\operatorname{atanh}(cx)}{x^{3}}\, dx + \int \frac{6abc^{2}\operatorname{atanh}(cx)}{x^{2}}\, dx + \int \frac{2abc^{3}\operatorname{atanh}(cx)}{x}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(a**2/x**4, x) + Integral(3*a**2*c/x**3, x) + Integral(3*a**2
*c**2/x**2, x) + Integral(a**2*c**3/x, x) + Integral(b**2*atanh(c*x)**2/x**
4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(3*b**2*c*atanh(c*x)**
2/x**3, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**2, x) + Integral(b**2*c*
*3*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x)/x**3, x) + Integral(6*
a*b*c**2*atanh(c*x)/x**2, x) + Integral(2*a*b*c**3*atanh(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2\,\left(d+c\,d\,x\right)^3}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^4,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^4, x)
```

3.92 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=271

$$-\frac{b^2c^2d^3}{12x^2}-\frac{b^2c^3d^3}{x}+b^2c^4d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{7bc^3d^3\left(a+b\tanh^{-1}\right.}{2x}$$

```
[Out] -1/12*b^2*c^2*d^3/x^2-b^2*c^3*d^3/x+b^2*c^4*d^3*arctanh(c*x)-1/6*b*c*d^3*(a
+b*arctanh(c*x))/x^3-b*c^2*d^3*(a+b*arctanh(c*x))/x^2-7/2*b*c^3*d^3*(a+b*ar
ctanh(c*x))/x-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/x^4+4*a*b*c^4*d^3*ln(x
)+11/3*b^2*c^4*d^3*ln(x)+4*b*c^4*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-11/6
*b^2*c^4*d^3*ln(-c^2*x^2+1)-2*b^2*c^4*d^3*polylog(2,-c*x)+2*b^2*c^4*d^3*pol
ylog(2,c*x)+2*b^2*c^4*d^3*polylog(2,1-2/(-c*x+1))
```

Rubi [A]
time = 0.23, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {37, 6085, 6037, 272, 46, 331, 212, 36, 29, 31, 6031, 6055, 2449, 2352}

$$4abc^4d^3\log(x)+4bc^4d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{7bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-2b^2c^4d^3\text{Li}_2(-cx)+2b^2c^4d^3\text{Li}_2(cx)+2b^2c^4d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)+\frac{11}{3}b^2c^4d^3\log(x)+b^2c^4d^3\tanh^{-1}(cx)-\frac{b^2c^3d^3}{x}-\frac{b^2c^2d^3}{12x^2}-\frac{11}{6}b^2c^4d^3\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

```
[Out] -1/12*(b^2*c^2*d^3)/x^2 - (b^2*c^3*d^3)/x + b^2*c^4*d^3*ArcTanh[c*x] - (b*c
*d^3*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/x^2 -
 (7*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh
[c*x])^2)/(4*x^4) + 4*a*b*c^4*d^3*Log[x] + (11*b^2*c^4*d^3*Log[x])/3 + 4*b*
c^4*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] - (11*b^2*c^4*d^3*Log[1 - c^2
*x^2])/6 - 2*b^2*c^4*d^3*PolyLog[2, -(c*x)] + 2*b^2*c^4*d^3*PolyLog[2, c*x]
 + 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4} - (2bc)\int\left(-\frac{d^3\left(a+b\tanh^{-1}(c\right.}{4x^4}\right.$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bcd^3\right)\int\frac{a+b\tanh^{-1}(cx)}{x^4}\,d$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{7bc^3d^3\left(a+\right.}{}$$

$$= -\frac{b^2c^3d^3}{x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{7}{}$$

$$= -\frac{b^2c^3d^3}{x} + b^2c^4d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a\right.}{}$$

$$= -\frac{b^2c^2d^3}{12x^2} - \frac{b^2c^3d^3}{x} + b^2c^4d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} -$$

Mathematica [A]
time = 0.48, size = 343, normalized size = 1.27

$$-\frac{d^3\left(3a^2+12a^2cx+2abcx+18a^2c^2x^2+12abc^2x^2+b^2c^2x^2+12a^2c^3x^3+42abc^3x^3+12b^2c^3x^3-b^2c^4x^4+3b^2(1+4cx+6c^2x^2+4c^3x^3-15c^4x^4)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(bcx(1+6cx+21c^2x^2-6c^3x^3)+3a(1+4cx+6c^2x^2+4c^3x^3)-24bc^4x^4\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-48abc^4x^4\log(cx)+21abc^4x^4\log(1-cx)-21abc^4x^4\log(1+cx)-44b^2c^4x^4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+24abc^4x^4\log\left(1-c^2x^2\right)+24b^2c^4x^4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

```
[Out] -1/12*(d^3*(3*a^2 + 12*a^2*c*x + 2*a*b*c*x + 18*a^2*c^2*x^2 + 12*a*b*c^2*x^
2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 42*a*b*c^3*x^3 + 12*b^2*c^3*x^3 - b^2*c^
4*x^4 + 3*b^2*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 - 15*c^4*x^4)*ArcTanh[c*x]
^2 + 2*b*ArcTanh[c*x]*(b*c*x*(1 + 6*c*x + 21*c^2*x^2 - 6*c^3*x^3) + 3*a*(1
+ 4*c*x + 6*c^2*x^2 + 4*c^3*x^3) - 24*b*c^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])
]) - 48*a*b*c^4*x^4*Log[c*x] + 21*a*b*c^4*x^4*Log[1 - c*x] - 21*a*b*c^4*x^4
*Log[1 + c*x] - 44*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 24*a*b*c^4*x^
4*Log[1 - c^2*x^2] + 24*b^2*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(259) = 518$.
time = 0.72, size = 599, normalized size = 2.21

method	result
derivativedivides	$c^4\left(d^3a^2\left(-\frac{1}{cx}-\frac{3}{2c^2x^2}-\frac{1}{c^3x^3}-\frac{1}{4c^4x^4}\right)-\frac{d^3ab\,\mathrm{arctanh}(cx)}{2c^4x^4}-\frac{15d^3b^2\,\mathrm{arctanh}(cx)\ln(cx-1)}{4}-\frac{d^3b^2\,\mathrm{arcta}}{}\right.$
default	$c^4\left(d^3a^2\left(-\frac{1}{cx}-\frac{3}{2c^2x^2}-\frac{1}{c^3x^3}-\frac{1}{4c^4x^4}\right)-\frac{d^3ab\,\mathrm{arctanh}(cx)}{2c^4x^4}-\frac{15d^3b^2\,\mathrm{arctanh}(cx)\ln(cx-1)}{4}-\frac{d^3b^2\,\mathrm{arcta}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x,method=_RETURNVERBOSE)

[Out] c^4*(d^3*a^2*(-1/c/x-3/2/c^2/x^2-1/c^3/x^3-1/4/c^4/x^4)-3*d^3*a*b*arctanh(c*x)/c^2/x^2-1/2*d^3*a*b*arctanh(c*x)/c^4/x^4-4/3*d^3*b^2*ln(c*x+1)+2*d^3*b^2*dilog(1/2*c*x+1/2)-15/16*d^3*b^2*ln(c*x-1)^2+1/16*d^3*b^2*ln(c*x+1)^2-7/3*d^3*b^2*ln(c*x-1)-15/4*d^3*b^2*arctanh(c*x)*ln(c*x-1)-1/4*d^3*b^2*arctanh(c*x)*ln(c*x+1)+15/8*d^3*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/8*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/8*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-15/4*d^3*a*b*ln(c*x-1)-1/4*d^3*a*b*ln(c*x+1)-2*d^3*b^2*dilog(c*x)-2*d^3*b^2*dilog(c*x+1)+11/3*d^3*b^2*ln(c*x)-1/6*d^3*a*b/c^3/x^3-1/4*d^3*b^2*arctanh(c*x)^2/c^4/x^4-1/6*d^3*b^2*arctanh(c*x)/c^3/x^3-1/12*d^3*b^2/c^2/x^2-d^3*b^2/c/x+4*d^3*b^2*arctanh(c*x)*ln(c*x)-2*d^3*b^2*ln(c*x)*ln(c*x+1)-d^3*b^2*arctanh(c*x)^2/c/x-d^3*b^2*arctanh(c*x)/c^2/x^2-d^3*b^2*arctanh(c*x)^2/c^3/x^3-d^3*a*b/c^2/x^2-2*d^3*a*b*arctanh(c*x)/c/x-2*d^3*a*b*arctanh(c*x)/c^3/x^3-3/2*d^3*b^2*arctanh(c*x)^2/c^2/x^2-7/2*d^3*b^2*arctanh(c*x)/c/x-7/2*d^3*a*b/c/x+4*d^3*a*b*ln(c*x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(254) = 508$.
time = 0.66, size = 813, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")

[Out] -2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^4*d^3 - 2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^4*d^3 + 2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^4*d^3 - b^2*c^4*d^3*log(c*x + 1) - 2*b^2*c^4*d^3*log(c*x - 1) + 3*b^2*c^4*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c^3*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^2*d^3 - ((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c*d^3 - a^2*c^3*d^3/x + 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*a*b*d^3 + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2*d^3 - 3/2*a^2*c^2*d^3/x^2 - a^2*c*d^3/x^3 - 1/4*b^2*d^3*arctanh(c*x)^2/x^4 - 1/4*a^2*d^3/x^4 - 1/8*(8*b^2*c^3*d^3*x^2 + (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x + 2*b^2*c*d^3)*log(c*x + 1)^2 - (7*b^2*c^4*d^3*x^3 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c^2*d^3*x - 2*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(3*b^2*c^3*d^3*

```
x^2 + b^2*c^2*d^3*x)*log(c*x + 1) - 2*(6*b^2*c^3*d^3*x^2 + 2*b^2*c^2*d^3*x
+ (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x + 2*b^2*c*d^3)*log
(c*x + 1))*log(-c*x + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int \frac{a^{2}}{x^{5}}\, dx + \int \frac{3a^{2}c}{x^{4}}\, dx + \int \frac{3a^{2}c^{2}}{x^{3}}\, dx + \int \frac{a^{2}c^{3}}{x^{2}}\, dx + \int \frac{b^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{2ab\operatorname{atanh}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{3b^{2}c\operatorname{atanh}^{2}{\left(cx \right)}}{x^{4}}\, dx + \int \frac{3b^{2}c^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{b^{2}c^{3}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{2}}\, dx + \int \frac{6abc\operatorname{atanh}{\left(cx \right)}}{x^{4}}\, dx + \int \frac{6abc^{2}\operatorname{atanh}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{2abc^{3}\operatorname{atanh}{\left(cx \right)}}{x^{2}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**5,x)
```

```
[Out] d**3*(Integral(a**2/x**5, x) + Integral(3*a**2*c/x**4, x) + Integral(3*a**2
*c**2/x**3, x) + Integral(a**2*c**3/x**2, x) + Integral(b**2*atanh(c*x)**2/
x**5, x) + Integral(2*a*b*atanh(c*x)/x**5, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**4, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**3, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c*atanh(c*x)/x**4, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**3, x) + Integral(2*a*b*c**3*atanh(c*x)/x**2, x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^3}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^5,x)
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^5, x)
```

3.93 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^6}\,dx$

Optimal. Leaf size=352

$$-\frac{b^2c^2d^3}{30x^3}-\frac{b^2c^3d^3}{4x^2}-\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3}-\frac{6bc}{}$$

```
[Out] -1/30*b^2*c^2*d^3/x^3-1/4*b^2*c^3*d^3/x^2-13/10*b^2*c^4*d^3/x+13/10*b^2*c^5
*d^3*arctanh(c*x)-1/10*b*c*d^3*(a+b*arctanh(c*x))/x^4-1/2*b*c^2*d^3*(a+b*ar
ctanh(c*x))/x^3-6/5*b*c^3*d^3*(a+b*arctanh(c*x))/x^2-5/2*b*c^4*d^3*(a+b*arc
tanh(c*x))/x-1/5*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/x^5+1/20*c*d^3*(c*x+1)^
4*(a+b*arctanh(c*x))^2/x^4+12/5*a*b*c^5*d^3*ln(x)+3*b^2*c^5*d^3*ln(x)+12/5*
b*c^5*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-3/2*b^2*c^5*d^3*ln(-c^2*x^2+1)-
6/5*b^2*c^5*d^3*polylog(2,-c*x)+6/5*b^2*c^5*d^3*polylog(2,c*x)+6/5*b^2*c^5*
d^3*polylog(2,1-2/(-c*x+1))
```

Rubi [A]

time = 0.27, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {47, 37, 6085, 6037, 331, 212, 272, 46, 36, 29, 31, 6031, 6055, 2449, 2352}

$$\frac{12}{5}abc^5d^3\log(x)+\frac{12}{5}bc^5d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{5bc^4d^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{6bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{5x^2}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5}+\frac{cd^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^2}{20x^4}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{6}{5}b^2c^5d^3\text{Li}_2(-cx)+\frac{6}{5}b^2c^5d^3\text{Li}_2(cx)+\frac{6}{5}b^2c^5d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)+3b^2c^5d^3\log(x)+\frac{13}{10}b^2c^5d^3\tanh^{-1}(cx)-\frac{13b^2c^4d^3}{10x}-\frac{b^2c^3d^3}{4x^2}-\frac{b^2c^2d^3}{30x^3}-\frac{3}{2}b^2c^5d^3\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

```
[Out] -1/30*(b^2*c^2*d^3)/x^3 - (b^2*c^3*d^3)/(4*x^2) - (13*b^2*c^4*d^3)/(10*x) +
 (13*b^2*c^5*d^3*ArcTanh[c*x])/10 - (b*c*d^3*(a + b*ArcTanh[c*x]))/(10*x^4)
 - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/(2*x^3) - (6*b*c^3*d^3*(a + b*ArcTanh[c
*x]))/(5*x^2) - (5*b*c^4*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4
*(a + b*ArcTanh[c*x])^2)/(5*x^5) + (c*d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^
2)/(20*x^4) + (12*a*b*c^5*d^3*Log[x])/5 + 3*b^2*c^5*d^3*Log[x] + (12*b*c^5*
d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/5 - (3*b^2*c^5*d^3*Log[1 - c^2*x
^2])/2 - (6*b^2*c^5*d^3*PolyLog[2, -(c*x)])/5 + (6*b^2*c^5*d^3*PolyLog[2, c
*x])/5 + (6*b^2*c^5*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^6}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} + \frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} + \frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3} - \frac{6bc^3d^3\left(a+b\right.}{5}$$

$$= -\frac{b^2c^2d^3}{30x^3} - \frac{6b^2c^4d^3}{5x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4} - \frac{bc^2d^3\left(a+b\tanh^{-1}(\right.}{2x^3}$$

$$= -\frac{b^2c^2d^3}{30x^3} - \frac{13b^2c^4d^3}{10x} + \frac{6}{5}b^2c^5d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}$$

$$= -\frac{b^2c^2d^3}{30x^3} - \frac{b^2c^3d^3}{4x^2} - \frac{13b^2c^4d^3}{10x} + \frac{13}{10}b^2c^5d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\,\mathrm{t}\right.}{10}$$

Mathematica [A]
time = 0.84, size = 372, normalized size = 1.06

$$-\frac{d^3\left(12a^2+45a^2cx+6abcx+60a^2c^2x^2+30abc^2x^2+2b^2c^2x^2+30a^2c^3x^3+72abc^3x^3+15b^2c^3x^3+150abc^4x^4+78b^2c^4x^4-15b^2c^5x^5+3b^2(4+15cx+20c^2x^2+10c^3x^3-49c^5x^5)\tanh^{-1}(cx)^2+6b\tanh^{-1}(cx)\left(a(4+15cx+20c^2x^2+10c^3x^3)+bcx(1+5cx+12c^2x^2+25c^3x^3-13c^4x^4)-24bc^5x^5\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-144abc^5x^5\log(cx)+75abc^5x^5\log(1-cx)-75abc^5x^5\log(1+cx)-180b^2c^5x^5\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+72abc^5x^5\log\left(1-c^2x^2\right)+72b^2c^5x^5\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

```
[Out] -1/60*(d^3*(12*a^2 + 45*a^2*c*x + 6*a*b*c*x + 60*a^2*c^2*x^2 + 30*a*b*c^2*x
^2 + 2*b^2*c^2*x^2 + 30*a^2*c^3*x^3 + 72*a*b*c^3*x^3 + 15*b^2*c^3*x^3 + 150
*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - 15*b^2*c^5*x^5 + 3*b^2*(4 + 15*c*x + 20*c^2
*x^2 + 10*c^3*x^3 - 49*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*(a*(4 + 1
5*c*x + 20*c^2*x^2 + 10*c^3*x^3) + b*c*x*(1 + 5*c*x + 12*c^2*x^2 + 25*c^3*x
^3 - 13*c^4*x^4) - 24*b*c^5*x^5*Log[1 - E^(-2*ArcTanh[c*x])]) - 144*a*b*c^5
*x^5*Log[c*x] + 75*a*b*c^5*x^5*Log[1 - c*x] - 75*a*b*c^5*x^5*Log[1 + c*x] -
 180*b^2*c^5*x^5*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 72*a*b*c^5*x^5*Log[1 - c^2*
x^2] + 72*b^2*c^5*x^5*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(320) = 640$.
time = 0.73, size = 644, normalized size = 1.83

method	result
derivativedivides	$c^5\left(d^3a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{5c^5x^5}-\frac{3}{4c^4x^4}-\frac{1}{c^3x^3}\right)-\frac{3d^3ab\,\mathrm{arctanh}(cx)}{2c^4x^4}-\frac{d^3b^2\,\mathrm{arctanh}(cx)}{10c^4x^4}-\frac{49d^3b^2\,\mathrm{arctanh}(cx)}{20}\right.$

default	$c^5\left(d^3a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{5c^5x^5}-\frac{3}{4c^4x^4}-\frac{1}{c^3x^3}\right)-\frac{3d^3ab\operatorname{arctanh}(cx)}{2c^4x^4}-\frac{d^3b^2\operatorname{arctanh}(cx)}{10c^4x^4}-\frac{49d^3b^2\operatorname{arctanh}(c}{20}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(d^3*a^2*(-1/2/c^2/x^2-1/5/c^5/x^5-3/4/c^4/x^4-1/c^3/x^3)-2/5*d^3*a*b*a
rctanh(c*x)/c^5/x^5-1/10*d^3*b^2*arctanh(c*x)/c^4/x^4-1/5*d^3*b^2*arctanh(c
*x)^2/c^5/x^5-1/10*d^3*a*b/c^4/x^4-d^3*a*b*arctanh(c*x)/c^2/x^2-3/2*d^3*a*b
*arctanh(c*x)/c^4/x^4-17/20*d^3*b^2*ln(c*x+1)+6/5*d^3*b^2*dilog(1/2*c*x+1/2
)-49/80*d^3*b^2*ln(c*x-1)^2-1/80*d^3*b^2*ln(c*x+1)^2-43/20*d^3*b^2*ln(c*x-1
)-49/20*d^3*b^2*arctanh(c*x)*ln(c*x-1)+1/20*d^3*b^2*arctanh(c*x)*ln(c*x+1)+
49/40*d^3*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/40*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*
x+1)-1/40*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-49/20*d^3*a*b*ln(c*x-1)+
1/20*d^3*a*b*ln(c*x+1)-6/5*d^3*b^2*dilog(c*x)-6/5*d^3*b^2*dilog(c*x+1)+3*d^
3*b^2*ln(c*x)-1/2*d^3*a*b/c^3/x^3-3/4*d^3*b^2*arctanh(c*x)^2/c^4/x^4-1/2*d^
3*b^2*arctanh(c*x)/c^3/x^3-1/30*d^3*b^2/c^3/x^3-1/4*d^3*b^2/c^2/x^2-13/10*d
^3*b^2/c/x+12/5*d^3*b^2*arctanh(c*x)*ln(c*x)-6/5*d^3*b^2*ln(c*x)*ln(c*x+1)-
6/5*d^3*b^2*arctanh(c*x)/c^2/x^2-d^3*b^2*arctanh(c*x)^2/c^3/x^3-6/5*d^3*a*b
/c^2/x^2-2*d^3*a*b*arctanh(c*x)/c^3/x^3-1/2*d^3*b^2*arctanh(c*x)^2/c^2/x^2-
5/2*d^3*b^2*arctanh(c*x)/c/x-5/2*d^3*a*b/c/x+12/5*d^3*a*b*ln(c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(315) = 630$.
time = 0.66, size = 783, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="maxima")
```

```
[Out] -6/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^5*d^3
- 6/5*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^5*d^3 + 6/5*(log(c*x
 + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^5*d^3 - 17/20*b^2*c^5*d^3*log(c*x +
 1) - 43/20*b^2*c^5*d^3*log(c*x - 1) + 3*b^2*c^5*d^3*log(x) + 1/2*((c*log(c
*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^3*d^3 - ((c^2
*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c^2*d
^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)
*c - 6*arctanh(c*x)/x^4)*a*b*c*d^3 - 1/10*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*
log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*d^3 - 1/2*a^2*c
^3*d^3/x^2 - a^2*c^2*d^3/x^3 - 3/4*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/240*
(312*b^2*c^4*d^3*x^4 + 60*b^2*c^3*d^3*x^3 + 8*b^2*c^2*d^3*x^2 - 3*(b^2*c^5*
d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*
d^3)*log(c*x + 1)^2 - 3*(49*b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c
^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*d^3)*log(-c*x + 1)^2 + 12*(25*b^2*c^4*d
```

```
^3*x^4 + 12*b^2*c^3*d^3*x^3 + 5*b^2*c^2*d^3*x^2 + b^2*c*d^3*x)*log(c*x + 1)
 - 6*(50*b^2*c^4*d^3*x^4 + 24*b^2*c^3*d^3*x^3 + 10*b^2*c^2*d^3*x^2 + 2*b^2*
c*d^3*x - (b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b
^2*c*d^3*x - 4*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int \frac{a^{2}}{x^{6}}\, dx + \int \frac{3a^{2}c}{x^{5}}\, dx + \int \frac{3a^{2}c^{2}}{x^{4}}\, dx + \int \frac{a^{2}c^{3}}{x^{3}}\, dx + \int \frac{b^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{6}}\, dx + \int \frac{2ab\operatorname{atanh}{\left(cx \right)}}{x^{6}}\, dx + \int \frac{3b^{2}c\operatorname{atanh}^{2}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{3b^{2}c^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{4}}\, dx + \int \frac{b^{2}c^{3}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{3}}\, dx + \int \frac{6abc\operatorname{atanh}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{6abc^{2}\operatorname{atanh}{\left(cx \right)}}{x^{4}}\, dx + \int \frac{2abc^{3}\operatorname{atanh}{\left(cx \right)}}{x^{3}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**6,x)
```

```
[Out] d**3*(Integral(a**2/x**6, x) + Integral(3*a**2*c/x**5, x) + Integral(3*a**2
*c**2/x**4, x) + Integral(a**2*c**3/x**3, x) + Integral(b**2*atanh(c*x)**2/
x**6, x) + Integral(2*a*b*atanh(c*x)/x**6, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**5, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**4, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**3, x) + Integral(6*a*b*c*atanh(c*x)/x**5, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**4, x) + Integral(2*a*b*c**3*atanh(c*x)/x**3, x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^3}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^6,x)
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^6, x)
```

3.94 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^7}\,dx$

Optimal. Leaf size=479

$$-\frac{b^2c^2d^3}{60x^4}-\frac{b^2c^3d^3}{10x^3}-\frac{61b^2c^4d^3}{180x^2}-\frac{37b^2c^5d^3}{30x}+\frac{37}{30}b^2c^6d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5}-\frac{3bc^2d^3\left(a+b\tanh^{-1}\right.}{10x^4}$$

```
[Out] -1/60*b^2*c^2*d^3/x^4-1/10*b^2*c^3*d^3/x^3-61/180*b^2*c^4*d^3/x^2-37/30*b^2
*c^5*d^3/x+37/30*b^2*c^6*d^3*arctanh(c*x)-1/15*b*c*d^3*(a+b*arctanh(c*x))/x
^5-3/10*b*c^2*d^3*(a+b*arctanh(c*x))/x^4-11/18*b*c^3*d^3*(a+b*arctanh(c*x))
/x^3-14/15*b*c^4*d^3*(a+b*arctanh(c*x))/x^2-11/6*b*c^5*d^3*(a+b*arctanh(c*x
))/x-1/6*d^3*(a+b*arctanh(c*x))^2/x^6-3/5*c*d^3*(a+b*arctanh(c*x))^2/x^5-3/
4*c^2*d^3*(a+b*arctanh(c*x))^2/x^4-1/3*c^3*d^3*(a+b*arctanh(c*x))^2/x^3+28/
15*a*b*c^6*d^3*ln(x)+113/45*b^2*c^6*d^3*ln(x)+37/20*b*c^6*d^3*(a+b*arctanh(
c*x))*ln(2/(-c*x+1))+1/60*b*c^6*d^3*(a+b*arctanh(c*x))*ln(2/(c*x+1))-113/90
*b^2*c^6*d^3*ln(-c^2*x^2+1)-14/15*b^2*c^6*d^3*polylog(2,-c*x)+14/15*b^2*c^6
*d^3*polylog(2,c*x)+37/40*b^2*c^6*d^3*polylog(2,1-2/(-c*x+1))-1/120*b^2*c^6
*d^3*polylog(2,1-2/(c*x+1))
```

Rubi [A]
time = 0.38, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {45, 6085, 6037, 272, 46, 331, 212, 36, 29, 31, 6031, 6055, 2449, 2352}

$$\frac{28}{15}abc^6d^3\log(x)+\frac{37}{20}bc^6d^3\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{60}bc^6d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{11bc^5d^3\left(a+b\tanh^{-1}(cx)\right)}{6x}-\frac{14bc^4d^3\left(a+b\tanh^{-1}(cx)\right)}{15x^2}-\frac{c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{11bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{18x^3}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5}-\frac{14}{15}b^2c^6d^3\text{Li}_2(-cx)+\frac{14}{15}b^2c^6d^3\text{Li}_2(cx)+\frac{37}{40}b^2c^6d^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)-\frac{1}{120}b^2c^6d^3\text{Li}_2\left(1-\frac{2}{cx+1}\right)+\frac{113}{45}b^2c^6d^3\log(x)+\frac{37}{30}b^2c^6d^3\tanh^{-1}(cx)-\frac{37b^2c^5d^3}{30x}-\frac{61b^2c^4d^3}{180x^2}-\frac{b^2c^3d^3}{10x^3}-\frac{b^2c^2d^3}{60x^4}-\frac{113}{90}b^2c^6d^3\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

```
[Out] -1/60*(b^2*c^2*d^3)/x^4 - (b^2*c^3*d^3)/(10*x^3) - (61*b^2*c^4*d^3)/(180*x^
2) - (37*b^2*c^5*d^3)/(30*x) + (37*b^2*c^6*d^3*ArcTanh[c*x])/30 - (b*c*d^3*
(a + b*ArcTanh[c*x]))/(15*x^5) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/(10*x^4
) - (11*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(18*x^3) - (14*b*c^4*d^3*(a + b*Arc
Tanh[c*x]))/(15*x^2) - (11*b*c^5*d^3*(a + b*ArcTanh[c*x]))/(6*x) - (d^3*(a
+ b*ArcTanh[c*x])^2)/(6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/(5*x^5) - (
3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x])^2
)/(3*x^3) + (28*a*b*c^6*d^3*Log[x])/15 + (113*b^2*c^6*d^3*Log[x])/45 + (37*
b*c^6*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/20 + (b*c^6*d^3*(a + b*Arc
Tanh[c*x])*Log[2/(1 + c*x)])/60 - (113*b^2*c^6*d^3*Log[1 - c^2*x^2])/90 - (
14*b^2*c^6*d^3*PolyLog[2, -(c*x)])/15 + (14*b^2*c^6*d^3*PolyLog[2, c*x])/15
 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/40 - (b^2*c^6*d^3*PolyLog[2
, 1 - 2/(1 + c*x)])/120
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^7}\,dx = -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} - \frac{3c^2d^3\left(a+b\right.}{4}$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} - \frac{3c^2d^3\left(a+b\right.}{4}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4} - \frac{11bc^3d^3\left(a\right.}{}$$

$$= -\frac{b^2c^3d^3}{10x^3} - \frac{14b^2c^5d^3}{15x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3\left(a+b\tanh\right.}{10x^4}$$

$$= -\frac{b^2c^3d^3}{10x^3} - \frac{37b^2c^5d^3}{30x} + \frac{14}{15}b^2c^6d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(c\right.}{15x^5}$$

$$= -\frac{b^2c^2d^3}{60x^4} - \frac{b^2c^3d^3}{10x^3} - \frac{61b^2c^4d^3}{180x^2} - \frac{37b^2c^5d^3}{30x} + \frac{37}{30}b^2c^6d^3\tanh^{-1}(cx) -$$

Mathematica [A]
time = 1.19, size = 402, normalized size = 0.84

$$-\frac{d^3\left(30a^2+108a^2cx+12abcx+135a^2c^2x^2+54abc^2x^2+3b^2c^2x^2+60a^2c^3x^3+110abc^3x^3+18b^2c^3x^3+168abc^4x^4+61b^2c^4x^4+330abc^5x^5+222b^2c^5x^5-64b^2c^6x^6+3b^2(10+36cx+45c^2x^2+20c^3x^3-111c^6x^6)\tanh^{-1}(cx)^2+2b\tanh^{-1}(cx)\left(3a(10+36cx+45c^2x^2+20c^3x^3)+bcx(6+27cx+55c^2x^2+84c^3x^3+165c^4x^4-111c^5x^5)-168bc^6x^6\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-336abc^6x^6\log(cx)+165abc^6x^6\log(1-cx)-165abc^6x^6\log(1+cx)-452b^2c^6x^6\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+168abc^6x^6\log(1-c^2x^2)+168b^2c^6x^6\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

```
[Out] -1/180*(d^3*(30*a^2 + 108*a^2*c*x + 12*a*b*c*x + 135*a^2*c^2*x^2 + 54*a*b*c
^2*x^2 + 3*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3
+ 168*a*b*c^4*x^4 + 61*b^2*c^4*x^4 + 330*a*b*c^5*x^5 + 222*b^2*c^5*x^5 - 64
*b^2*c^6*x^6 + 3*b^2*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x^3 - 111*c^6*x^6)*
ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x
^3) + b*c*x*(6 + 27*c*x + 55*c^2*x^2 + 84*c^3*x^3 + 165*c^4*x^4 - 111*c^5*x
^5) - 168*b*c^6*x^6*Log[1 - E^(-2*ArcTanh[c*x])]) - 336*a*b*c^6*x^6*Log[c*x
] + 165*a*b*c^6*x^6*Log[1 - c*x] - 165*a*b*c^6*x^6*Log[1 + c*x] - 452*b^2*c
^6*x^6*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 168*a*b*c^6*x^6*Log[1 - c^2*x^2] + 16
8*b^2*c^6*x^6*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^6
```

Maple [A]
time = 0.74, size = 689, normalized size = 1.44

method	result
derivativedivides	$c^6\left(d^3a^2\left(-\frac{1}{6c^6x^6}-\frac{3}{5c^5x^5}-\frac{1}{3c^3x^3}-\frac{3}{4c^4x^4}\right)-\frac{d^3b^2\operatorname{arctanh}(cx)^2}{6c^6x^6}-\frac{d^3b^2\operatorname{arctanh}(cx)}{15c^5x^5}-\frac{3d^3ab\operatorname{arctanh}(c}{2c^4x^4}\right.$

default	$c^6\Big(d^3a^2\big(-\frac{1}{6c^6x^6}-\frac{3}{5c^5x^5}-\frac{1}{3c^3x^3}-\frac{3}{4c^4x^4}\big)-\frac{d^3b^2\operatorname{arctanh}(cx)^2}{6c^6x^6}-\frac{d^3b^2\operatorname{arctanh}(cx)}{15c^5x^5}-\frac{3d^3ab\operatorname{arctanh}(cx)}{2c^4x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(d^3*a^2*(-1/6/c^6/x^6-3/5/c^5/x^5-1/3/c^3/x^3-3/4/c^4/x^4)-6/5*d^3*a*b
*arctanh(c*x)/c^5/x^5-3/10*d^3*b^2*arctanh(c*x)/c^4/x^4-3/5*d^3*b^2*arctanh
(c*x)^2/c^5/x^5-3/10*d^3*a*b/c^4/x^4-1/6*d^3*b^2*arctanh(c*x)^2/c^6/x^6-1/1
5*d^3*a*b/c^5/x^5-1/15*d^3*b^2*arctanh(c*x)/c^5/x^5-3/2*d^3*a*b*arctanh(c*x
)/c^4/x^4-1/3*d^3*a*b*arctanh(c*x)/c^6/x^6-23/36*d^3*b^2*ln(c*x+1)+14/15*d^
3*b^2*dilog(1/2*c*x+1/2)-37/80*d^3*b^2*ln(c*x-1)^2+1/240*d^3*b^2*ln(c*x+1)^
2-337/180*d^3*b^2*ln(c*x-1)-37/20*d^3*b^2*arctanh(c*x)*ln(c*x-1)-1/60*d^3*b
^2*arctanh(c*x)*ln(c*x+1)+37/40*d^3*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/120*d^3
*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/120*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1
/2)-37/20*d^3*a*b*ln(c*x-1)-1/60*d^3*a*b*ln(c*x+1)-14/15*d^3*b^2*dilog(c*x)
-14/15*d^3*b^2*dilog(c*x+1)+113/45*d^3*b^2*ln(c*x)-11/18*d^3*a*b/c^3/x^3-3/
4*d^3*b^2*arctanh(c*x)^2/c^4/x^4-11/18*d^3*b^2*arctanh(c*x)/c^3/x^3-1/60*d^
3*b^2/c^4/x^4-1/10*d^3*b^2/c^3/x^3-61/180*d^3*b^2/c^2/x^2-37/30*d^3*b^2/c/x
+28/15*d^3*b^2*arctanh(c*x)*ln(c*x)-14/15*d^3*b^2*ln(c*x)*ln(c*x+1)-14/15*d
^3*b^2*arctanh(c*x)/c^2/x^2-1/3*d^3*b^2*arctanh(c*x)^2/c^3/x^3-14/15*d^3*a*
b/c^2/x^2-2/3*d^3*a*b*arctanh(c*x)/c^3/x^3-11/6*d^3*b^2*arctanh(c*x)/c/x-11
/6*d^3*a*b/c/x+28/15*d^3*a*b*ln(c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(427) = 854$.
time = 0.66, size = 961, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="maxima")
```

```
[Out] -14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^6*d^
3 - 14/15*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^6*d^3 + 14/15*(l
og(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^6*d^3 - 23/60*b^2*c^6*d^3*log
(c*x + 1) - 97/60*b^2*c^6*d^3*log(c*x - 1) + 2*b^2*c^6*d^3*log(x) - 1/3*((c
^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c^3
*d^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^
3)*c - 6*arctanh(c*x)/x^4)*a*b*c^2*d^3 - 3/10*((2*c^4*log(c^2*x^2 - 1) - 2*
c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*c*d^3 + 1/9
0*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x^2 +
 3)/x^5)*c - 30*arctanh(c*x)/x^6)*a*b*d^3 + 1/360*((184*c^4*log(x) - (15*c^
4*x^4*log(c*x + 1)^2 + 15*c^4*x^4*log(c*x - 1)^2 + 92*c^4*x^4*log(c*x - 1)
+ 32*c^2*x^2 - 2*(15*c^4*x^4*log(c*x - 1) - 46*c^4*x^4)*log(c*x + 1) + 6)/x
```

```
^4)*c^2 + 4*(15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*
c^2*x^2 + 3)/x^5)*c*arctanh(c*x))*b^2*d^3 - 1/3*a^2*c^3*d^3/x^3 - 3/4*a^2*c
^2*d^3/x^4 - 3/5*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctanh(c*x)^2/x^6 - 1/6*a^2*d
^3/x^6 - 1/240*(296*b^2*c^5*d^3*x^4 + 60*b^2*c^4*d^3*x^3 + 24*b^2*c^3*d^3*x
^2 + (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x + 36*b^2*c
*d^3)*log(c*x + 1)^2 - (101*b^2*c^6*d^3*x^5 - 20*b^2*c^3*d^3*x^2 - 45*b^2*c
^2*d^3*x - 36*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(45*b^2*c^5*d^3*x^4 + 28*b^2*c
^4*d^3*x^3 + 15*b^2*c^3*d^3*x^2 + 9*b^2*c^2*d^3*x)*log(c*x + 1) - 2*(90*b^2
*c^5*d^3*x^4 + 56*b^2*c^4*d^3*x^3 + 30*b^2*c^3*d^3*x^2 + 18*b^2*c^2*d^3*x +
 (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x + 36*b^2*c*d^3
)*log(c*x + 1))*log(-c*x + 1))/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{3}\left(\int \frac{a^{2}}{x^{7}}\, dx + \int \frac{3a^{2}c}{x^{6}}\, dx + \int \frac{3a^{2}c^{2}}{x^{5}}\, dx + \int \frac{a^{2}c^{3}}{x^{4}}\, dx + \int \frac{b^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{7}}\, dx + \int \frac{2ab\operatorname{atanh}{\left(cx \right)}}{x^{7}}\, dx + \int \frac{3b^{2}c\operatorname{atanh}^{2}{\left(cx \right)}}{x^{6}}\, dx + \int \frac{3b^{2}c^{2}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{b^{2}c^{3}\operatorname{atanh}^{2}{\left(cx \right)}}{x^{4}}\, dx + \int \frac{6abc\operatorname{atanh}{\left(cx \right)}}{x^{6}}\, dx + \int \frac{6abc^{2}\operatorname{atanh}{\left(cx \right)}}{x^{5}}\, dx + \int \frac{2abc^{3}\operatorname{atanh}{\left(cx \right)}}{x^{4}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**7,x)
```

```
[Out] d**3*(Integral(a**2/x**7, x) + Integral(3*a**2*c/x**6, x) + Integral(3*a**2
*c**2/x**5, x) + Integral(a**2*c**3/x**4, x) + Integral(b**2*atanh(c*x)**2/
x**7, x) + Integral(2*a*b*atanh(c*x)/x**7, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**6, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**5, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**4, x) + Integral(6*a*b*c*atanh(c*x)/x**6, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**5, x) + Integral(2*a*b*c**3*atanh(c*x)/x**4, x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2\,(d + c\,d\,x)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^7,x)
```

```
[Out] int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^7, x)
```

3.95 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=329

$$-\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a+b\tanh^{-1}(cx)\right)^2}{6c^4d}+\frac{x\left(a+b\tan\right.}{c}$$

```
[Out] -a*b*x/c^3/d+1/3*b^2*x/c^3/d-1/3*b^2*arctanh(c*x)/c^4/d-b^2*x*arctanh(c*x)/
c^3/d+1/3*b*x^2*(a+b*arctanh(c*x))/c^2/d+11/6*(a+b*arctanh(c*x))^2/c^4/d+x*
(a+b*arctanh(c*x))^2/c^3/d-1/2*x^2*(a+b*arctanh(c*x))^2/c^2/d+1/3*x^3*(a+b*
arctanh(c*x))^2/c/d-8/3*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4/d+(a+b*arct
anh(c*x))^2*ln(2/(c*x+1))/c^4/d-1/2*b^2*ln(-c^2*x^2+1)/c^4/d-4/3*b^2*polylo
g(2,1-2/(-c*x+1))/c^4/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^4/d-1
/2*b^2*polylog(3,1-2/(c*x+1))/c^4/d
```

Rubi [A]

time = 0.59, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6077, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 6021, 266, 6095, 6203, 6745}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}+\frac{11\left(a+b\tanh^{-1}(cx)\right)^2}{6c^4d}-\frac{8b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^4d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d}-\frac{abx}{c^3d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d}-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd}-\frac{4b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{3c^4d}-\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}+\frac{b^2x}{3c^3d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}-\frac{b^2\log\left(1-c^2x^2\right)}{2c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] -((a*b*x)/(c^3*d)) + (b^2*x)/(3*c^3*d) - (b^2*ArcTanh[c*x])/(3*c^4*d) - (b^
2*x*ArcTanh[c*x])/(c^3*d) + (b*x^2*(a + b*ArcTanh[c*x]))/(3*c^2*d) + (11*(a
 + b*ArcTanh[c*x])^2)/(6*c^4*d) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d) - (x^2
*(a + b*ArcTanh[c*x])^2)/(2*c^2*d) + (x^3*(a + b*ArcTanh[c*x])^2)/(3*c*d) -
 (8*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^4*d) + ((a + b*ArcTanh[c*
x])^2*Log[2/(1 + c*x)])/(c^4*d) - (b^2*Log[1 - c^2*x^2])/(2*c^4*d) - (4*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^4*d) - (b*(a + b*ArcTanh[c*x])*PolyLog[2
, 1 - 2/(1 + c*x)])/(c^4*d) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
```

```
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c} + \frac{\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$= \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd} + \frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^2} - \frac{(2b)\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{3d} -$$

$$= -\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd} - \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^3} + \int$$

$$= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^4d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d} - \frac{x^2(}{}$$

$$= -\frac{abx}{c^3d} + \frac{b^2x}{3c^3d} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d} + \frac{11\left(a+b\tanh^{-1}(cx)\right)^2}{6c^4d} + \frac{x(a+b\,\mathrm{t}}{}$$

$$= -\frac{abx}{c^3d} + \frac{b^2x}{3c^3d} - \frac{b^2\tanh^{-1}(cx)}{3c^4d} - \frac{b^2x\tanh^{-1}(cx)}{c^3d} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d} +$$

$$= -\frac{abx}{c^3d} + \frac{b^2x}{3c^3d} - \frac{b^2\tanh^{-1}(cx)}{3c^4d} - \frac{b^2x\tanh^{-1}(cx)}{c^3d} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d} +$$

$$= -\frac{abx}{c^3d} + \frac{b^2x}{3c^3d} - \frac{b^2\tanh^{-1}(cx)}{3c^4d} - \frac{b^2x\tanh^{-1}(cx)}{c^3d} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d} +$$

Mathematica [A]
time = 0.51, size = 347, normalized size = 1.05

$$\frac{a^2x}{c^3d} - \frac{a^2x^2}{2c^2d} + \frac{a^2x^3}{3cd} - \frac{a^2\log(1+cx)}{c^4d} + \frac{ab\left(-3cx + 8cx\tanh^{-1}(cx) + (1-c^2x^2)\left(-1+3\tanh^{-1}(cx) - 2cx\tanh^{-1}(cx)\right) + 6\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right) - 8\log\left(\frac{1}{\sqrt{1-c^2x^2}}\right) - 3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\right)}{3c^4d} + \frac{b^2\left(2cx - 6cx\tanh^{-1}(cx) - 2(1-c^2x^2)\tanh^{-1}(cx) - 8\tanh^{-1}(cx)^2 + 8cx\tanh^{-1}(cx)^2 + 3(1-c^2x^2)\tanh^{-1}(cx)^2 - 2cx(1-c^2x^2)\tanh^{-1}(cx)^2 - 16\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + 6\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + 6\log\left(\frac{1}{\sqrt{1-c^2x^2}}\right) + (8-6\tanh^{-1}(cx))\,\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) - 3\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)\right)}{6c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a^2*x)/(c^3*d) - (a^2*x^2)/(2*c^2*d) + (a^2*x^3)/(3*c*d) - (a^2*Log[1 + c*
x])/(c^4*d) + (a*b*(-3*c*x + 8*c*x*ArcTanh[c*x] + (1 - c^2*x^2)*(-1 + 3*Arc
Tanh[c*x] - 2*c*x*ArcTanh[c*x]) + 6*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x]
)] - 8*Log[1/Sqrt[1 - c^2*x^2]] - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c
^4*d) + (b^2*(2*c*x - 6*c*x*ArcTanh[c*x] - 2*(1 - c^2*x^2)*ArcTanh[c*x] - 8
*ArcTanh[c*x]^2 + 8*c*x*ArcTanh[c*x]^2 + 3*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 2
*c*x*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 16*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*Log[1/Sqrt[1 - c^
2*x^2]] + (8 - 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 3*PolyLog
[3, -E^(-2*ArcTanh[c*x])]))/(6*c^4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.39, size = 1200, normalized size = 3.65

method	result	size
derivativedivides	Expression too large to display	1200
default	Expression too large to display	1200

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-a^2/d*ln(c*x+1)+2/3*a*b/d*c^3*x^3*arctanh(c*x)+b^2/d*arctanh(c*x)^2
*c*x+a*b/d*dilog(1/2*c*x+1/2)+1/2*a*b/d*ln(c*x+1)^2+5/6*a*b/d*ln(c*x-1)+11/
6*a*b/d*ln(c*x+1)-8/3*b^2/d*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))
-8/3*b^2/d*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-b^2/d*arctanh(c*
x)^2*ln(c*x+1)+b^2/d*arctanh(c*x)^2*ln(2)+b^2/d*arctanh(c*x)*polylog(2,-(c*
x+1)^2/(-c^2*x^2+1))+2*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-
a*b/d*c*x+1/3*a*b/d*c^2*x^2+1/3*b^2/d*c^3*x^3*arctanh(c*x)^2-b^2/d*arctanh(
c*x)*c*x+1/3*b^2/d*arctanh(c*x)*c^2*x^2-1/2*b^2/d*arctanh(c*x)^2*c^2*x^2-2*
a*b/d*arctanh(c*x)*ln(c*x+1)+a*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-a*b/d*l
n(-1/2*c*x+1/2)*ln(c*x+1)-1/3*b^2/d+b^2/d*ln(1+(c*x+1)^2/(-c^2*x^2+1))-8/3*
b^2/d*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*b^2/d*polylog(3,-(c*x+1)^2/
(-c^2*x^2+1))-8/3*b^2/d*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-4/3*b^2/d*arc
tanh(c*x)+11/6*b^2/d*arctanh(c*x)^2-2/3*b^2/d*arctanh(c*x)^3-1/2*I*b^2/d*ar
ctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^
2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+a^2/d*c*x-1/
2*a^2/d*c^2*x^2+1/3*a^2/d*c^3*x^3+1/3*b^2/d*c*x-1/2*I*b^2/d*arctanh(c*x)^2*
Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/
(-c^2*x^2+1)))^2+I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2
))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/2*I*b^2/d*arctan
h(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-4/3*a*b/d-a*b/d*arctanh(c*x)*c^2*x^2+2*a*b/d
*arctanh(c*x)*c*x+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1
)^2/(c^2*x^2-1))^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*a^2*((2*c^2*x^3 - 3*c*x^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d)) + 1/
24*(2*b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 6*b^2*c*x - 6*b^2*log(c*x + 1))*log(-c*
x + 1)^2/(c^4*d) - integrate(-1/12*(3*(b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x +
```

```
1)^2 + 12*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (3*b^2*c^2*x^2 + 2*(6
*a*b*c^4 + b^2*c^4)*x^4 + 6*b^2*c*x - (12*a*b*c^3 + b^2*c^3)*x^3 + 6*(b^2*c
^4*x^4 - b^2*c^3*x^3 - b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d*x
^2 - c^3*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c*d*x
 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x**3/(c*x + 1), x) + Integral(b**2*x**3*atanh(c*x)**2/(c*x +
 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^2}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

```
[Out] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

3.96 $\int \frac{x^2 (a+b\tanh^{-1}(cx))^2}{d+cdx} \, dx$

Optimal. Leaf size=247

$$\frac{abx}{c^2d}+\frac{b^2x\tanh^{-1}(cx)}{c^2d}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d}-\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd}+\frac{2b\left(a+b\tanh\right.}{}$$

```
[Out] a*b*x/c^2/d+b^2*x*arctanh(c*x)/c^2/d-3/2*(a+b*arctanh(c*x))^2/c^3/d-x*(a+b*
arctanh(c*x))^2/c^2/d+1/2*x^2*(a+b*arctanh(c*x))^2/c/d+2*b*(a+b*arctanh(c*x
))*ln(2/(-c*x+1))/c^3/d-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^3/d+1/2*b^2*ln
(-c^2*x^2+1)/c^3/d+b^2*polylog(2,1-2/(-c*x+1))/c^3/d+b*(a+b*arctanh(c*x))*p
olylog(2,1-2/(c*x+1))/c^3/d+1/2*b^2*polylog(3,1-2/(c*x+1))/c^3/d
```

Rubi [A]

time = 0.39, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {6077, 6037, 6127, 6021, 266, 6095, 6131, 6055, 2449, 2352, 6203, 6745}

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d}+\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d}+\frac{abx}{c^2d}-\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd}+\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^3d}+\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^3d}+\frac{b^2x\tanh^{-1}(cx)}{c^2d}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a*b*x)/(c^2*d) + (b^2*x*ArcTanh[c*x])/(c^2*d) - (3*(a + b*ArcTanh[c*x])^2)
/(2*c^3*d) - (x*(a + b*ArcTanh[c*x])^2)/(c^2*d) + (x^2*(a + b*ArcTanh[c*x])
^2)/(2*c*d) + (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d) - ((a + b
*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d) + (b^2*Log[1 - c^2*x^2])/(2*c^3*
d) + (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^3*d) + (b*(a + b*ArcTanh[c*x])*Po
lyLog[2, 1 - 2/(1 + c*x)])/(c^3*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c
^3*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c} + \frac{\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^2} - \frac{b\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d} - \int$$

$$= -\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log}{c^3d}$$

$$= \frac{abx}{c^2d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x^2\left(a+b\tanh^{-1}(cx\right.}{2cd}$$

$$= \frac{abx}{c^2d} + \frac{b^2x\tanh^{-1}(cx)}{c^2d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + x$$

$$= \frac{abx}{c^2d} + \frac{b^2x\tanh^{-1}(cx)}{c^2d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + x$$

$$= \frac{abx}{c^2d} + \frac{b^2x\tanh^{-1}(cx)}{c^2d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + x$$

Mathematica [A]
time = 0.30, size = 260, normalized size = 1.05

$$\frac{-2a^2cx+2abcx+a^2c^2x^2-2ab\tanh^{-1}(cx)-4abcx\tanh^{-1}(cx)+2b^2cx\tanh^{-1}(cx)+2abc^2x^2\tanh^{-1}(cx)+b^2\tanh^{-1}(cx)^2-2b^2cx\tanh^{-1}(cx)^2+b^2c^2x^2\tanh^{-1}(cx)^2-4ab\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+4b^2\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-2b^2\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+2a^2\log(1+cx)-2ab\log\left(1-c^2x^2\right)+b^2\log\left(1-c^2x^2\right)+2b\left(a-b+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+b^2\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (-2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 - 2*a*b*ArcTanh[c*x] - 4*a*b*c*x*ArcT
anh[c*x] + 2*b^2*c*x*ArcTanh[c*x] + 2*a*b*c^2*x^2*ArcTanh[c*x] + b^2*ArcTan
h[c*x]^2 - 2*b^2*c*x*ArcTanh[c*x]^2 + b^2*c^2*x^2*ArcTanh[c*x]^2 - 4*a*b*Ar
cTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 4*b^2*ArcTanh[c*x]*Log[1 + E^(-2*
ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*
Log[1 + c*x] - 2*a*b*Log[1 - c^2*x^2] + b^2*Log[1 - c^2*x^2] + 2*b*(a - b +
 b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*A
rcTanh[c*x])])/(2*c^3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.85, size = 1099, normalized size = 4.45

method	result	size
derivativedivides	Expression too large to display	1099
default	Expression too large to display	1099

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a^2/d*ln(c*x+1)-b^2/d*arctanh(c*x)^2*c*x-a*b/d*dilog(1/2*c*x+1/2)-1/
2*a*b/d*ln(c*x+1)^2-1/2*a*b/d*ln(c*x-1)-3/2*a*b/d*ln(c*x+1)+2*b^2/d*arctanh
(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*ln(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(c*x+1)-b^2/d*arctanh(c*x)^2*l
n(2)-b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-2*b^2/d*arctanh(
c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+a*b/d*c*x+b^2/d*arctanh(c*x)*c*x+1/2*
b^2/d*arctanh(c*x)^2*c^2*x^2+2*a*b/d*arctanh(c*x)*ln(c*x+1)-a*b/d*ln(-1/2*c
*x+1/2)*ln(1/2*c*x+1/2)+a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-b^2/d*ln(1+(c*x+1)
^2/(-c^2*x^2+1))+2*b^2/d*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*b^2/d*po
lylog(3,-(c*x+1)^2/(-c^2*x^2+1))+2*b^2/d*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/
2))+b^2/d*arctanh(c*x)-3/2*b^2/d*arctanh(c*x)^2+2/3*b^2/d*arctanh(c*x)^3+1/
2*I*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-
a^2/d*c*x+1/2*a^2/d*c^2*x^2-1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)
^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^
2+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-I*b^2/d*arctanh(c*x)^2*Pi*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/2*I*b^2/d
*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arct
anh(c*x)^2+a*b/d+a*b/d*arctanh(c*x)*c^2*x^2-2*a*b/d*arctanh(c*x)*c*x-1/2*I*
b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh
(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d)) + 1/8*(b^2*c^2*x^2
 - 2*b^2*c*x + 2*b^2*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d) - integrate(-1/4
*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2
)*log(c*x + 1) + (2*b^2*c*x - (4*a*b*c^3 + b^2*c^3)*x^3 + (4*a*b*c^2 + b^2*
c^2)*x^2 - 2*(b^2*c^3*x^3 - b^2*c^2*x^2 + b^2*c*x + b^2)*log(c*x + 1))*log(
-c*x + 1))/(c^4*d*x^2 - c^2*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c*d*x
 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x**2/(c*x + 1), x) + Integral(b**2*x**2*atanh(c*x)**2/(c*x +
 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+c\,d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

3.97 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=172

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+\cdots}\right.}{c^2d}$$

```
[Out] (a+b*arctanh(c*x))^2/c^2/d+x*(a+b*arctanh(c*x))^2/c/d-2*b*(a+b*arctanh(c*x)
)*ln(2/(-c*x+1))/c^2/d+(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^2/d-b^2*polylog
(2,1-2/(-c*x+1))/c^2/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^2/d-1/
2*b^2*polylog(3,1-2/(c*x+1))/c^2/d
```

Rubi [A]

time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6077, 6021, 6131, 6055, 2449, 2352, 6095, 6203, 6745}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^2d}-\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/(c^2*d) + (x*(a + b*ArcTanh[c*x])^2)/(c*d) - (2*b*(a
 + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^2*d) + ((a + b*ArcTanh[c*x])^2*Log[
2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^2*d) - (b*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[3, 1 -
 2/(1 + c*x)])/(2*c^2*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c} + \frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d} - \frac{(2b)\int \frac{x\left(a+b\tanh^{-1}(\right.}{1-c^2x^2}}{d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+c\right.}}{c^2d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} - \frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-}\right.}{c^2d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} - \frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-}\right.}{c^2d}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd} - \frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-}\right.}{c^2d}$$

Mathematica [A]
time = 0.27, size = 140, normalized size = 0.81

$$\frac{2b^2\tanh^{-1}(cx)^2\left(-1+cx+\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+4b\tanh^{-1}(cx)\left(acx+(a-b)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)+2a\left(acx-a\log(1+cx)+b\log\left(1-c^2x^2\right)\right)-2b\left(a-b+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-b^2\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (2*b^2*ArcTanh[c*x]^2*(-1 + c*x + Log[1 + E^(-2*ArcTanh[c*x])]) + 4*b*ArcTa
nh[c*x]*(a*c*x + (a - b)*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*a*(a*c*x - a*Log
[1 + c*x] + b*Log[1 - c^2*x^2]) - 2*b*(a - b + b*ArcTanh[c*x])*PolyLog[2, -
E^(-2*ArcTanh[c*x])] - b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2*c^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.82, size = 5134, normalized size = 29.85

method	result	size
derivativedivides	Expression too large to display	5134
default	Expression too large to display	5134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/(c*d) - log(c*x + 1)/(c^2*d)) + 1/4*(b^2*c*x - b^2*log(c*x + 1))*log
(-c*x + 1)^2/(c^2*d) - integrate(-1/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)
^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(c*x + 1) - 2*((2*a*b*c^2 + b^2*c^2)*x^2
- (2*a*b*c - b^2*c)*x + (b^2*c^2*x^2 - 2*b^2*c*x - b^2)*log(c*x + 1))*log(-
c*x + 1))/(c^3*d*x^2 - c*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c*d*x + d),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{cx+1}\,dx + \int \frac{b^2x\operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2abx\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x/(c*x + 1), x) + Integral(b**2*x*atanh(c*x)**2/(c*x + 1), x
) + Integral(2*a*b*x*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{atanh}(c\,x))^2}{d+c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
[Out] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

3.98 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{cd}+\frac{b^2\mathrm{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2cd}$$

```
[Out] -(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c/d+b*(a+b*arctanh(c*x))*polylog(2,1-2/
(c*x+1))/c/d+1/2*b^2*polylog(3,1-2/(c*x+1))/c/d
```

Rubi [A]

time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6055, 6095, 6203, 6745}

$$\frac{b\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{cd}+\frac{b^2\mathrm{Li}_3\left(1-\frac{2}{cx+1}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c*d)) + (b*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 + c*x)])/(c*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2
*c*d)
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{(2b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{cd} - b^2\int$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{cd} + \frac{b^2\mathrm{Li}_3}{}$$

Mathematica [A]
time = 0.13, size = 102, normalized size = 1.21

$$\frac{-4ab\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-2b^2\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+2a^2\log(1+cx)+2b\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+b^2\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

```
[Out] (-4*a*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Lo
g[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*Log[1 + c*x] + 2*b*(a + b*ArcTanh[c*x])*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2
*c*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.43, size = 769, normalized size = 9.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2/d*ln(c*x+1)+b^2/d*arctanh(c*x)^2*ln(c*x+1)-2*b^2/d*arctanh(c*x)^2*
ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3*b^2/d*arctanh(c*x)^3-1/2*I*b^2/d*arctanh
(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I*b^2/d*arctanh(c*x)^2*Pi*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^
```

```
2*x^2+1)))^2-I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))
^3*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*
(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+
1)))*arctanh(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csg
n(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-b^2/d*arctanh(c*x)^2*ln(2)-b^2/d*
arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*b^2/d*polylog(3,-(c*x+1
)^2/(-c^2*x^2+1))+2*a*b/d*arctanh(c*x)*ln(c*x+1)-1/2*a*b/d*ln(c*x+1)^2-a*b/
d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-a*b/d*d
ilog(1/2*c*x+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/(c*d) + a^2*log(c*d*x + d)/(c*d) - int
egrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1
) - 4*(b^2*c*x*log(c*x + 1) + a*b*c*x - a*b)*log(-c*x + 1))/(c^2*d*x^2 - d)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx+1}\,dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2ab \operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c*x + 1), x) +
Integral(2*a*b*atanh(c*x)/(c*x + 1), x))/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(c*d*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2}{d + c\,d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + c*d*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(d + c*d*x), x)
```

3.99 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x(d+cdx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a+b\tanh^{-1}(cx))^2\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b(a+b\tanh^{-1}(cx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d}-\frac{b^2\mathrm{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)}{2d}$$

```
[Out] (a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-b*(a+b*arctanh(c*x))*polylog(2,-1+2/
(c*x+1))/d-1/2*b^2*polylog(3,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6079, 6095, 6203, 6745}

$$-\frac{b\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)(a+b\tanh^{-1}(cx))}{d}+\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))^2}{d}-\frac{b^2\mathrm{Li}_3\left(\frac{2}{cx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

```
[Out] ((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (b*(a + b*ArcTanh[c*x])*P
olyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*PolyLog[3, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{(2bc)\int\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{d}+\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{d}-\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 132, normalized size = 1.71

$$\frac{a^2\log(cx)-a^2\log(1+cx)+ab\left(2\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\tanh^{-1}(cx)^3+\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

```
[Out] (a^2*Log[c*x] - a^2*Log[1 + c*x] + a*b*(2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTan
h[c*x])] - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^2*((I/24)*Pi^3 - (2*ArcTanh
[c*x]^3)/3 + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*Poly
Log[2, E^(2*ArcTanh[c*x])] - PolyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.08, size = 1389, normalized size = 18.04

method	result	size
derivativedivides	Expression too large to display	1389
default	Expression too large to display	1389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d*ln(c*x+1)+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))+a*b/d*dilog(1/2*c*x+1/2)+1/2*a*b/d*ln(c*x+1)^2
```

```
-b^2/d*arctanh(c*x)^2*ln(c*x+1)+b^2/d*arctanh(c*x)^2*ln(2)+2*b^2/d*arctanh(
c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-2*a*b/d*arctanh(c*x)*ln(c*x+1)+a*b/d*
ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*I*b^2
/d*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d*arctanh(c*x)^2*Pi*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))^2-2/3*b^2/d*arctanh(c*x)^3-1/2*I*b^2/d*arctanh(c*x)^2
*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/2*I*b^2/d*arctanh(c*x)
^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/2*I
*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/2*I*b^2/d*arctan
h(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-
1))+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+b^2/d*arctanh(c*x)^2*l
n(c*x)+b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctan
h(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(1+(c*x
+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1
)^(1/2))-b^2/d*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-a*b/d*dilog(c*x)
-2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,-(c*x+1)/(
-c^2*x^2+1)^(1/2))+a^2/d*ln(c*x)+2*a*b/d*arctanh(c*x)*ln(c*x)-a*b/d*ln(c*x)
*ln(c*x+1)+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^
2*x^2-1))^3-a*b/d*dilog(c*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/d - a^2*(log(c*x + 1)/d - log(x)/d) +
 integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x
+ 1) - 2*(2*a*b*c*x - 2*a*b - (b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x +
1))/(c^2*d*x^3 - d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^2 + d*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^2+x}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{cx^2+x}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx^2+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2/(c*x**2 + x),
 x) + Integral(2*a*b*atanh(c*x)/(c*x**2 + x), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atanh}(c\,x))^2}{x\,(d+c\,d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)), x)
```

3.100 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^2(d+cdx)}\,dx$

Optimal. Leaf size=162

$$\frac{c(a+b\tanh^{-1}(cx))^2}{d}-\frac{(a+b\tanh^{-1}(cx))^2}{dx}+\frac{2bc(a+b\tanh^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{c(a+b\tanh^{-1}(cx))^2\log$$

```
[Out] c*(a+b*arctanh(c*x))^2/d-(a+b*arctanh(c*x))^2/d/x+2*b*c*(a+b*arctanh(c*x))*
ln(2-2/(c*x+1))/d-c*(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-b^2*c*polylog(2,
-1+2/(c*x+1))/d+b*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))/d+1/2*b^2*c*
polylog(3,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.28, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6081, 6037, 6135, 6079, 2497, 6095, 6203, 6745}

$$\frac{bc\text{Li}_2\left(\frac{2}{cx+1}-1\right)(a+b\tanh^{-1}(cx))}{d}+\frac{c(a+b\tanh^{-1}(cx))^2}{d}-\frac{(a+b\tanh^{-1}(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d}-\frac{c\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))^2}{d}-\frac{b^2c\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{d}+\frac{b^2c\text{Li}_3\left(\frac{2}{cx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) + (2*b*c*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (c*(a + b*ArcTanh[c*x])^2*Log[2 -
2/(1 + c*x)])/d - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b*c*(a + b*ArcT
anh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c*PolyLog[3, -1 + 2/(1 + c
*x)])/(2*d)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} + \frac{(2bc)\int \frac{a+b\tanh^{-1}}{x(1-c^2}}{d}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{}{1-}\right.}{d}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} + \frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\right.}{d}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} + \frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\right.}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 225, normalized size = 1.39

$$\frac{-\frac{a^2}{x} - a^2c\log(x) + a^2c\log(1+cx) + \frac{ab\left(-2\tanh^{-1}(cx)\left(1+cx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)+2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+cx\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{x} + b^2c\left(-\frac{i\pi^3}{24}+\tanh^{-1}(cx)^2-\frac{\tanh^{-1}(cx)^2}{cx}+\frac{2}{3}\tanh^{-1}(cx)^3+2\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

```
[Out] (-(a^2/x) - a^2*c*Log[x] + a^2*c*Log[1 + c*x] + (a*b*(-2*ArcTanh[c*x]*(1 +
c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]] + c*
x*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x + b^2*c*((-1/24*I)*Pi^3 + ArcTanh[c*x
]^2 - ArcTanh[c*x]^2/(c*x) + (2*ArcTanh[c*x]^3)/3 + 2*ArcTanh[c*x]*Log[1 -
E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - PolyLog
[2, E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + Po
lyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.39, size = 7139, normalized size = 44.07

method	result	size
derivativedivides	Expression too large to display	7139
default	Expression too large to display	7139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/4*(b^2*c*x*log(c*x + 1) -
 b^2)*log(-c*x + 1)^2/(d*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^
2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(b^2*c^2*x^2 + 2*a*b - (2*a*b*c - b^
2*c)*x - (b^2*c^3*x^3 + b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x
 + 1))/(c^2*d*x^4 - d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^3 + d*x^2),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^3+x^2}\,dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{cx^3+x^2}\,dx + \int \frac{2ab \operatorname{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)**2/(c*x**3 +
x**2), x) + Integral(2*a*b*atanh(c*x)/(c*x**3 + x**2), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x^2\,(d+c\,d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)),x)
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)), x)
```

3.101 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3(d+cdx)}\,dx$

Optimal. Leaf size=250

$$-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{b^2c^2\log(x)}{d}-$$

```
[Out] -b*c*(a+b*arctanh(c*x))/d/x-1/2*c^2*(a+b*arctanh(c*x))^2/d-1/2*(a+b*arctanh
(c*x))^2/d/x^2+c*(a+b*arctanh(c*x))^2/d/x+b^2*c^2*ln(x)/d-1/2*b^2*c^2*ln(-c
^2*x^2+1)/d-2*b*c^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d+c^2*(a+b*arctanh(c
*x))^2*ln(2-2/(c*x+1))/d+b^2*c^2*polylog(2,-1+2/(c*x+1))/d-b*c^2*(a+b*arcta
nh(c*x))*polylog(2,-1+2/(c*x+1))/d-1/2*b^2*c^2*polylog(3,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.45, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6081, 6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 2497, 6203, 6745}

$$-\frac{bc^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}+\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{2bc^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{b^2c^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{d}-\frac{b^2c^2\text{Li}_3\left(\frac{2}{cx+1}-1\right)}{2d}-\frac{b^2c^2\log\left(1-c^2x^2\right)}{2d}+\frac{b^2c^2\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/(d*x)) - (c^2*(a + b*ArcTanh[c*x])^2)/(2*d) -
(a + b*ArcTanh[c*x])^2/(2*d*x^2) + (c*(a + b*ArcTanh[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 - c^2*x^2])/(2*d) - (2*b*c^2*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)])/d + (c^2*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 +
c*x)])/d + (b^2*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b*c^2*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*c^2*PolyLog[3, -1 + 2/(1 + c*
x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

```
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx-\frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2\right.}{d}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c(a}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c(a}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c(a}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}+\frac{c(a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.99, size = 317, normalized size = 1.27

$$\frac{-\frac{a^2}{x^2}+\frac{2a^2c}{x}+2a^2c^2\log(x)-2a^2c^2\log(1+cx)+\frac{2ab\left(\tanh^{-1}(cx)\left(-1+2cx+c^2x^2+2c^2x^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-cx\left(1+2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)\right)-c^2x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)}{x^2}+2b^2c^2\left(\frac{i\pi^3}{24}-\frac{\tanh^{-1}(cx)}{cx}-\frac{1}{2}\tanh^{-1}(cx)^2-\frac{\tanh^{-1}(cx)^2}{2c^2x^2}+\frac{\tanh^{-1}(cx)^2}{cx}-\frac{2}{3}\tanh^{-1}(cx)^3-2\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right)+\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

```
[Out] (-(a^2/x^2) + (2*a^2*c)/x + 2*a^2*c^2*Log[x] - 2*a^2*c^2*Log[1 + c*x] + (2*
a*b*(ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*ArcTanh[c
*x])]) - c*x*(1 + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - c^2*x^2*PolyLog[2,
E^(-2*ArcTanh[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTanh[c*x]/(c*x) -
ArcTanh[c*x]^2/2 - ArcTanh[c*x]^2/(2*c^2*x^2) + ArcTanh[c*x]^2/(c*x) - (2*A
rcTanh[c*x]^3)/3 - 2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + ArcTanh[c*
x]^2*Log[1 - E^(2*ArcTanh[c*x])] + Log[(c*x)/Sqrt[1 - c^2*x^2]] + PolyLog[2
, E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - Poly
Log[3, E^(2*ArcTanh[c*x])]/2))/(2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.94, size = 1732, normalized size = 6.93

method	result	size
derivativedivides	Expression too large to display	1732
default	Expression too large to display	1732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a^2/d*ln(c*x+1)+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+a*b/d*dilog(1/2*c*x+1/2)+1/2*a*b/d*ln(c*x
+1)^2+1/2*a*b/d*ln(c*x-1)+3/2*a*b/d*ln(c*x+1)-b^2/d*arctanh(c*x)^2*ln(c*x+1
)+b^2/d*arctanh(c*x)^2*ln(2)+2*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)
^(1/2))-2*a*b/d*arctanh(c*x)*ln(c*x+1)+a*b/d*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/
2)-a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-
c^2*x^2+1)))^2-1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+
1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+2*a*b/
d*arctanh(c*x)/c/x-b^2/d*arctanh(c*x)+3/2*b^2/d*arctanh(c*x)^2-2/3*b^2/d*ar
ctanh(c*x)^3+b^2/d*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*ln((c*x+1)/(-c^2*
x^2+1)^(1/2)-1)-1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))+1/2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+b^2/d*arctanh(c*x)^2/c/x-1/2*I*b^2/d*a
rctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
```

```
/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c
^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/2*I*b^2/d*arctanh(c*x)^2
*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/2*
I*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+b^2/d*arctanh(c*x)^2*ln(c*x)+b
^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*p
olylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^
2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))
-b^2/d*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-a*b/d*dilog(c*x)-2*b^2/d
*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,-(c*x+1)/(-c^2*x^2
+1)^(1/2))+a^2/d*ln(c*x)-2*a*b/d*ln(c*x)-2*b^2/d*arctanh(c*x)*ln(1+(c*x+1)/
(-c^2*x^2+1)^(1/2))+a^2/d/c/x+2*a*b/d*arctanh(c*x)*ln(c*x)-a*b/d*ln(c*x)*ln
(c*x+1)+2*b^2/d*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*dilog(1+(c*x+1)/(
-c^2*x^2+1)^(1/2))-a*b/d*arctanh(c*x)/c^2/x^2+1/2*I*b^2/d*arctanh(c*x)^2*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b^2/d*arc
tanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-a*b/d*dilog(c*x+1)-a*b/d/c/x
-1/2*b^2/d*arctanh(c*x)^2/c^2/x^2-b^2/d*arctanh(c*x)/c/x-1/2*a^2/d/c^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="maxima")
[Out] -1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a^2 - 1/
8*(2*b^2*c^2*x^2*log(c*x + 1) - 2*b^2*c*x + b^2)*log(-c*x + 1)^2/(d*x^2) +
integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x +
 1) - (2*b^2*c^3*x^3 + b^2*c^2*x^2 - 4*a*b + (4*a*b*c - b^2*c)*x - 2*(b^2*c
^4*x^4 + b^2*c^3*x^3 - b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x
^5 - d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^4 + d*x^3),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^4+x^3}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{cx^4+x^3}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx^4+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)**2/(c*x**4 +
x**3), x) + Integral(2*a*b*atanh(c*x)/(c*x**4 + x**3), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2}{x^3\,(d + c\,d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)), x)
```

3.102 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^4(d+cdx)}\,dx$

Optimal. Leaf size=334

$$-\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\left(a+b\tanh^{-1}(cx)\right)^2}{6d}-\frac{\left(a+b\,\mathrm{ta}\right.}{3}$$

```
[Out] -1/3*b^2*c^2/d/x+1/3*b^2*c^3*arctanh(c*x)/d-1/3*b*c*(a+b*arctanh(c*x))/d/x^
2+b*c^2*(a+b*arctanh(c*x))/d/x+5/6*c^3*(a+b*arctanh(c*x))^2/d-1/3*(a+b*arct
anh(c*x))^2/d/x^3+1/2*c*(a+b*arctanh(c*x))^2/d/x^2-c^2*(a+b*arctanh(c*x))^2
/d/x-b^2*c^3*ln(x)/d+1/2*b^2*c^3*ln(-c^2*x^2+1)/d+8/3*b*c^3*(a+b*arctanh(c*
x))*ln(2-2/(c*x+1))/d-c^3*(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-4/3*b^2*c^
3*polylog(2,-1+2/(c*x+1))/d+b*c^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1)
)/d+1/2*b^2*c^3*polylog(3,-1+2/(c*x+1))/d
```

Rubi [A]
time = 0.69, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6081, 6037, 6129, 331, 212, 6135, 6079, 2497, 272, 36, 29, 31, 6095, 6203, 6745}

$$\frac{bc^3\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{5c^3\left(a+b\tanh^{-1}(cx)\right)^2}{6d}-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}+\frac{8bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}-\frac{4b^2c^3\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)}{3d}+\frac{b^2c^3\mathrm{Li}_3\left(\frac{2}{cx+1}-1\right)}{2d}-\frac{b^2c^3\log(x)}{d}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{b^2c^2}{3dx}+\frac{b^2c^3\log\left(1-c^2x^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d*x) + (b^2*c^3*ArcTanh[c*x])/(3*d) - (b*c*(a + b*ArcTanh[c
*x]))/(3*d*x^2) + (b*c^2*(a + b*ArcTanh[c*x]))/(d*x) + (5*c^3*(a + b*ArcTan
h[c*x])^2)/(6*d) - (a + b*ArcTanh[c*x])^2/(3*d*x^3) + (c*(a + b*ArcTanh[c*x
])^2)/(2*d*x^2) - (c^2*(a + b*ArcTanh[c*x])^2)/(d*x) - (b^2*c^3*Log[x])/d +
 (b^2*c^3*Log[1 - c^2*x^2])/(2*d) + (8*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2
/(1 + c*x)])/(3*d) - (c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d -
(4*b^2*c^3*PolyLog[2, -1 + 2/(1 + c*x)])/(3*d) + (b*c^3*(a + b*ArcTanh[c*x]
)*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c^3*PolyLog[3, -1 + 2/(1 + c*x)])/
(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3} + c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d} -$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} - c^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx +$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2} + \frac{c^3\left(a+b\tanh^{-1}(cx)\right)^2}{3d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3} + \frac{c(a-}{}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{5c^3\left(a+b\tanh^{-1}(cx\right.}{6d}$$

$$= -\frac{b^2c^2}{3dx} + \frac{b^2c^3\tanh^{-1}(cx)}{3d} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{5c}{}$$

$$= -\frac{b^2c^2}{3dx} + \frac{b^2c^3\tanh^{-1}(cx)}{3d} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{5c}{}$$

$$= -\frac{b^2c^2}{3dx} + \frac{b^2c^3\tanh^{-1}(cx)}{3d} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{5c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.86, size = 388, normalized size = 1.16

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

```
[Out] ((-8*a^2)/x^3 + (12*a^2*c)/x^2 - (24*a^2*c^2)/x - 24*a^2*c^3*Log[x] + 24*a^
2*c^3*Log[1 + c*x] - (8*a*b*(ArcTanh[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^
3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*(-1 + 3*c*x + c^2*x^2 + 8
*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - 3*c^3*x^3*PolyLog[2, E^(-2*ArcTanh
[c*x])]))/x^3 + b^2*c^3*((-I)*Pi^3 - 8/(c*x) + 8*ArcTanh[c*x] - (8*ArcTanh[
c*x])/(c^2*x^2) + (24*ArcTanh[c*x])/(c*x) + 20*ArcTanh[c*x]^2 - (8*ArcTanh[
c*x]^2)/(c^3*x^3) + (12*ArcTanh[c*x]^2)/(c^2*x^2) - (24*ArcTanh[c*x]^2)/(c*
x) + 16*ArcTanh[c*x]^3 + 64*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*
ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*Log[(c*x)/Sqrt[1 - c^2*x^2]
] - 32*PolyLog[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*Ar
cTanh[c*x])] + 12*PolyLog[3, E^(2*ArcTanh[c*x])]))/(24*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.58, size = 1895, normalized size = 5.67

method	result	size
derivativedivides	Expression too large to display	1895
default	Expression too large to display	1895

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a^2/d*ln(c*x+1)-2/3*a*b/d*arctanh(c*x)/c^3/x^3-a*b/d*dilog(1/2*c*x+1/2
)-1/2*a*b/d*ln(c*x+1)^2-5/6*a*b/d*ln(c*x-1)-11/6*a*b/d*ln(c*x+1)+b^2/d*arct
anh(c*x)^2*ln(c*x+1)-b^2/d*arctanh(c*x)^2*ln(2)-2*b^2/d*arctanh(c*x)^2*ln((
c*x+1)/(-c^2*x^2+1)^(1/2))+2*a*b/d*arctanh(c*x)*ln(c*x+1)-a*b/d*ln(-1/2*c*x
+1/2)*ln(1/2*c*x+1/2)+a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*I*b^2/d*Pi*csgn(
I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-2
*a*b/d*arctanh(c*x)/c/x-1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*c
sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+4/3*b^2/d*arctanh(c*x)-11/6*b^2/d*arcta
nh(c*x)^2+2/3*b^2/d*arctanh(c*x)^3+1/2*I*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2
*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-b^2/d*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))
-b^2/d*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1))
)^2*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/
2*I*b^2/d*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2*I*b^2/d*arctanh(c*x)^2*P
i*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))^2-I*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-b^2/d*arctanh(c*x)^
2/c/x-b^2/d*arctanh(c*x)^2*ln(c*x)-b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*
x^2+1)^(1/2))-2*b^2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-b^
2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*arctanh(c*x)*po
lylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln((c*x+1)^2/(-c^
2*x^2+1)-1)+a*b/d*dilog(c*x)+2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+
2*b^2/d*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-a^2/d*ln(c*x)+8/3*a*b/d*ln(c
*x)+8/3*b^2/d*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-a^2/d/c/x-2*a*b
/d*arctanh(c*x)*ln(c*x)+a*b/d*ln(c*x)*ln(c*x+1)-8/3*b^2/d*dilog((c*x+1)/(-c
^2*x^2+1)^(1/2))+8/3*b^2/d*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+a*b/d*arctan
h(c*x)/c^2/x^2-1/3*b^2/d*arctanh(c*x)/c^2/x^2-1/3*b^2/d*arctanh(c*x)^2/c^3/
x^3+a*b/d*dilog(c*x+1)-1/3*a*b/d/c^2/x^2+a*b/d/c/x+1/2*b^2/d*arctanh(c*x)^2
/c^2/x^2+b^2/d*arctanh(c*x)/c/x+1/2*a^2/d/c^2/x^2-1/3*a^2/d/c^3/x^3-1/2*I*b
```

```
^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(
c*x)^2-1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-1/3*b^
2/d/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)+1/3*b^2/d/((-c^2*x^2+1)^(
1/2)+c*x+1)*(-c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^3
))*a^2 + 1/24*(6*b^2*c^3*x^3*log(c*x + 1) - 6*b^2*c^2*x^2 + 3*b^2*c*x - 2*b
^2)*log(-c*x + 1)^2/(d*x^3) - integrate(-1/12*(3*(b^2*c*x - b^2)*log(c*x +
1)^2 + 12*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b
^2*c^2*x^2 + 12*a*b - 2*(6*a*b*c - b^2*c)*x - 6*(b^2*c^5*x^5 + b^2*c^4*x^4
+ b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x^6 - d*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^5 + d*x^4),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{cx^5+x^4}\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{cx^5+x^4}\,dx + \int \frac{2ab\,\mathrm{atanh}\,(cx)}{cx^5+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**4/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**5 + x**4), x) + Integral(b**2*atanh(c*x)**2/(c*x**5 +
x**4), x) + Integral(2*a*b*atanh(c*x)/(c*x**5 + x**4), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2}{x^4\,(d + c\,d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^4*(d + c*d*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^4*(d + c*d*x)), x)
```

3.103 $\int \frac{x^4(a+b\tanh^{-1}(cx))^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=394

$$-\frac{2abx}{c^4d^2}+\frac{b^2x}{3c^4d^2}-\frac{b^2}{2c^5d^2(1+cx)}+\frac{b^2\tanh^{-1}(cx)}{6c^5d^2}-\frac{2b^2x\tanh^{-1}(cx)}{c^4d^2}+\frac{bx^2(a+b\tanh^{-1}(cx))}{3c^3d^2}-\frac{b(a+b\tanh^{-1}(c}{c^5d^2(1+cx)}$$

```
[Out] -2*a*b*x/c^4/d^2+1/3*b^2*x/c^4/d^2-1/2*b^2/c^5/d^2/(c*x+1)+1/6*b^2*arctanh(
c*x)/c^5/d^2-2*b^2*x*arctanh(c*x)/c^4/d^2+1/3*b*x^2*(a+b*arctanh(c*x))/c^3/
d^2-b*(a+b*arctanh(c*x))/c^5/d^2/(c*x+1)+29/6*(a+b*arctanh(c*x))^2/c^5/d^2+
3*x*(a+b*arctanh(c*x))^2/c^4/d^2-x^2*(a+b*arctanh(c*x))^2/c^3/d^2+1/3*x^3*(
a+b*arctanh(c*x))^2/c^2/d^2-(a+b*arctanh(c*x))^2/c^5/d^2/(c*x+1)-20/3*b*(a+
b*arctanh(c*x))*ln(2/(-c*x+1))/c^5/d^2+4*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))
/c^5/d^2-b^2*ln(-c^2*x^2+1)/c^5/d^2-10/3*b^2*polylog(2,1-2/(-c*x+1))/c^5/d^
2-4*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^5/d^2-2*b^2*polylog(3,1-2
/(c*x+1))/c^5/d^2
```

Rubi [A]
time = 0.62, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 19, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 327, 212, 6065, 6063, 641, 46, 213, 6203, 6745}

$$-\frac{4b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^2(cx+1)}+\frac{29\left(a+b\tanh^{-1}(cx)\right)^2}{6c^5d^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2(cx+1)}-\frac{20b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^5d^2}+\frac{4\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^2}-\frac{2abx}{c^4d^2}+\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c^2d^2}-\frac{10b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{3c^5d^2}-\frac{2b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{c^5d^2}-\frac{b^2}{2c^5d^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{6c^5d^2}+\frac{b^2x}{3c^4d^2}-\frac{2b^2x\tanh^{-1}(cx)}{c^4d^2}-\frac{b^2\log\left(1-c^2x^2\right)}{c^5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (-2*a*b*x)/(c^4*d^2) + (b^2*x)/(3*c^4*d^2) - b^2/(2*c^5*d^2*(1 + c*x)) + (b
^2*ArcTanh[c*x])/(6*c^5*d^2) - (2*b^2*x*ArcTanh[c*x])/(c^4*d^2) + (b*x^2*(a
 + b*ArcTanh[c*x]))/(3*c^3*d^2) - (b*(a + b*ArcTanh[c*x]))/(c^5*d^2*(1 + c*
x)) + (29*(a + b*ArcTanh[c*x])^2)/(6*c^5*d^2) + (3*x*(a + b*ArcTanh[c*x])^2
)/(c^4*d^2) - (x^2*(a + b*ArcTanh[c*x])^2)/(c^3*d^2) + (x^3*(a + b*ArcTanh[
c*x])^2)/(3*c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^5*d^2*(1 + c*x)) - (20*b*(
a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^5*d^2) + (4*(a + b*ArcTanh[c*x])
^2*Log[2/(1 + c*x)])/(c^5*d^2) - (b^2*Log[1 - c^2*x^2])/(c^5*d^2) - (10*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^5*d^2) - (4*b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/(c^5*d^2) - (2*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^
5*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

n + 2, 0])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int \left(\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^4d^2} + \frac{3\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^4d^2} - \frac{4\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^4d^2} -$$

$$= \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c^2d^2} -$$

$$= \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^2} + \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} +$$

$$= -\frac{2abx}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2(1+cx)} + \frac{29\left(a+b\tanh^{-1}(c\right.}{6c^5d^2}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b\left(a+b\tanh\right.}{c^5d^2(1+}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2\tanh^{-1}(cx)}{3c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2}{2c^5d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{3c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2(}{}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2}{2c^5d^2(1+cx)} + \frac{b^2\tanh^{-1}(cx)}{6c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2(}{}$$

Mathematica [A]
time = 1.15, size = 425, normalized size = 1.08

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (36*a^2*c*x - 12*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(1 + c*x) - 48*a^2*
Log[1 + c*x] + b^2*(4*c*x - 4*ArcTanh[c*x] - 24*c*x*ArcTanh[c*x] + 4*c^2*x^
2*ArcTanh[c*x] - 28*ArcTanh[c*x]^2 + 36*c*x*ArcTanh[c*x]^2 - 12*c^2*x^2*Arc
Tanh[c*x]^2 + 4*c^3*x^3*ArcTanh[c*x]^2 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh
[c*x]*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 80*Arc
Tanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 48*ArcTanh[c*x]^2*Log[1 + E^(-2*Ar
cTanh[c*x])] - 12*Log[1 - c^2*x^2] - 8*(-5 + 6*ArcTanh[c*x])*PolyLog[2, -E^
(-2*ArcTanh[c*x])] - 24*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh
```

```
[c*x]] + 6*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcT
anh[c*x]]) + 2*a*b*(-2 - 12*c*x + 2*c^2*x^2 - 3*Cosh[2*ArcTanh[c*x]] + 20*L
og[1 - c^2*x^2] - 24*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*
x]] + 2*ArcTanh[c*x]*(6 + 18*c*x - 6*c^2*x^2 + 2*c^3*x^3 - 3*Cosh[2*ArcTanh
[c*x]] + 24*Log[1 + E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]])))/(12*c^
5*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.34, size = 1347, normalized size = 3.42

method	result	size
derivativedivides	Expression too large to display	1347
default	Expression too large to display	1347

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-a^2/d^
2/(c*x+1)-4*a^2/d^2*ln(c*x+1)+29/6*b^2/d^2*arctanh(c*x)^2-8/3*b^2/d^2*arcta
nh(c*x)^3+2*b^2/d^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))-2*b^2/d^2*polylog(3,-(c*x+
1)^2/(-c^2*x^2+1))-20/3*b^2/d^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*
b^2/d^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/3*b^2/d^2-7/3*a*b/d^2+1/3*a
^2/d^2*c^3*x^3-a^2/d^2*c^2*x^2+3*a^2/d^2*c*x+1/3*b^2/d^2*c*x-2*I*b^2/d^2*ar
ctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^
2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/4*b^2/d^2/
(c*x+1)-7/3*b^2*arctanh(c*x)/d^2+2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*b^2/d^2*arctanh(c*x)/(c*
x+1)*c*x+2/3*a*b/d^2*arctanh(c*x)*c^3*x^3-2*a*b/d^2*arctanh(c*x)*c^2*x^2+6*
a*b/d^2*arctanh(c*x)*c*x-b^2/d^2*arctanh(c*x)^2/(c*x+1)-4*b^2/d^2*arctanh(c
*x)^2*ln(c*x+1)-1/2*b^2/d^2*arctanh(c*x)/(c*x+1)-20/3*b^2/d^2*arctanh(c*x)*
ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*b^2/d^2*arctanh(c*x)*ln(1+I*(c*x+1)
/(-c^2*x^2+1)^(1/2))+4*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+
1))-a*b/d^2/(c*x+1)+4*a*b/d^2*dilog(1/2*c*x+1/2)+2*a*b/d^2*ln(c*x+1)^2+11/6
*a*b/d^2*ln(c*x-1)+29/6*a*b/d^2*ln(c*x+1)+4*b^2/d^2*arctanh(c*x)^2*ln(2)+8*
b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/3*a*b/d^2*c^2*x^2-2
*a*b/d^2*c*x+1/3*b^2/d^2*arctanh(c*x)^2*c^3*x^3-b^2/d^2*arctanh(c*x)^2*c^2*
x^2+3*b^2/d^2*arctanh(c*x)^2*c*x-2*b^2/d^2*arctanh(c*x)*c*x+1/3*b^2/d^2*arc
tanh(c*x)*c^2*x^2+1/4*b^2/d^2/(c*x+1)*c*x-2*a*b/d^2*arctanh(c*x)/(c*x+1)-8*
a*b/d^2*arctanh(c*x)*ln(c*x+1)+4*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-4
*a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c
*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+2*I*b^2/d^2*arcta
nh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+4*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1
)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-2*I*b^2/d^2*arctanh(c
```

```
*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*(3/(c^6*d^2*x + c^5*d^2) - (c^2*x^3 - 3*c*x^2 + 9*x)/(c^4*d^2) + 1
2*log(c*x + 1)/(c^5*d^2)) + 1/12*(b^2*c^4*x^4 - 2*b^2*c^3*x^3 + 6*b^2*c^2*x
^2 + 9*b^2*c*x - 3*b^2 - 12*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(
c^6*d^2*x + c^5*d^2) - integrate(-1/12*(3*(b^2*c^5*x^5 - b^2*c^4*x^4)*log(c
*x + 1)^2 + 12*(a*b*c^5*x^5 - a*b*c^4*x^4)*log(c*x + 1) - 2*(4*b^2*c^3*x^3
+ 15*b^2*c^2*x^2 + (6*a*b*c^5 + b^2*c^5)*x^5 - (6*a*b*c^4 + b^2*c^4)*x^4 +
6*b^2*c*x - 3*b^2 + 3*(b^2*c^5*x^5 - b^2*c^4*x^4 - 4*b^2*c^2*x^2 - 8*b^2*c*
x - 4*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^7*d^2*x^3 + c^6*d^2*x^2 - c^5*d^
2*x - c^4*d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^4}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^4\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^4\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**4*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**4*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=331

$$\frac{abx}{c^3d^2}+\frac{b^2}{2c^4d^2(1+cx)}-\frac{b^2\tanh^{-1}(cx)}{2c^4d^2}+\frac{b^2x\tanh^{-1}(cx)}{c^3d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}-\frac{2x(a+$$

```
[Out] a*b*x/c^3/d^2+1/2*b^2/c^4/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/c^4/d^2+b^2*x*ar
ctanh(c*x)/c^3/d^2+b*(a+b*arctanh(c*x))/c^4/d^2/(c*x+1)-3*(a+b*arctanh(c*x)
)^2/c^4/d^2-2*x*(a+b*arctanh(c*x))^2/c^3/d^2+1/2*x^2*(a+b*arctanh(c*x))^2/c
^2/d^2+(a+b*arctanh(c*x))^2/c^4/d^2/(c*x+1)+4*b*(a+b*arctanh(c*x))*ln(2/(-c
*x+1))/c^4/d^2-3*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^4/d^2+1/2*b^2*ln(-c^2
*x^2+1)/c^4/d^2+2*b^2*polylog(2,1-2/(-c*x+1))/c^4/d^2+3*b*(a+b*arctanh(c*x)
)*polylog(2,1-2/(c*x+1))/c^4/d^2+3/2*b^2*polylog(3,1-2/(c*x+1))/c^4/d^2
```

Rubi [A]
time = 0.45, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 6065, 6063, 641, 46, 213, 6203, 6745}

$$\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2(cx+1)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}+\frac{4b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}-\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}+\frac{abx}{c^3d^2}-\frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{2b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^4d^2}+\frac{3b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{b^2}{2c^4d^2(cx+1)}-\frac{b^2\tanh^{-1}(cx)}{2c^4d^2}+\frac{b^2x\tanh^{-1}(cx)}{c^3d^2}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (a*b*x)/(c^3*d^2) + b^2/(2*c^4*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^4*d
^2) + (b^2*x*ArcTanh[c*x])/(c^3*d^2) + (b*(a + b*ArcTanh[c*x]))/(c^4*d^2*(1
 + c*x)) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^2) - (2*x*(a + b*ArcTanh[c*x])
^2)/(c^3*d^2) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^2*d^2) + (a + b*ArcTanh[c
*x])^2/(c^4*d^2*(1 + c*x)) + (4*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c
^4*d^2) - (3*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^2) + (b^2*Log[
1 - c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^4*d^2) +
 (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^4*d^2) + (3*b^2*
PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int \left(-\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)^2}\right.$$

$$= -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^3d^2}-\frac{2\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^3d^2}+\frac{3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^3d^2}$$

$$= -\frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2(1+cx)}-$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}-\frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}$$

$$= \frac{abx}{c^3d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}-\frac{2x\left(a+b\tanh^{-1}(cx\right.}{c^3d^2}$$

$$= \frac{abx}{c^3d^2}+\frac{b^2x\tanh^{-1}(cx)}{c^3d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}-$$

$$= \frac{abx}{c^3d^2}+\frac{b^2x\tanh^{-1}(cx)}{c^3d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}-$$

$$= \frac{abx}{c^3d^2}+\frac{b^2}{2c^4d^2(1+cx)}+\frac{b^2x\tanh^{-1}(cx)}{c^3d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)}-$$

$$= \frac{abx}{c^3d^2}+\frac{b^2}{2c^4d^2(1+cx)}-\frac{b^2\tanh^{-1}(cx)}{2c^4d^2}+\frac{b^2x\tanh^{-1}(cx)}{c^3d^2}+$$

Mathematica [A]

time = 1.07, size = 354, normalized size = 1.07

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (-8*a^2*c*x + 2*a^2*c^2*x^2 + (4*a^2)/(1 + c*x) + 12*a^2*Log[1 + c*x] + 2*a
*b*(2*c*x + Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2
*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x
]] - 6*Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTan
h[c*x]]) + b^2*(4*c*x*ArcTanh[c*x] + 6*ArcTanh[c*x]^2 - 8*c*x*ArcTanh[c*x]^
2 + 2*c^2*x^2*ArcTanh[c*x]^2 + Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2
*ArcTanh[c*x]] + 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 16*ArcTanh[c*x]*Lo
g[1 + E^(-2*ArcTanh[c*x])] - 12*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])]
 + 2*Log[1 - c^2*x^2] + 4*(-2 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c
*x])] + 6*PolyLog[3, -E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTa
nh[c*x]*Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c
^4*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.82, size = 1239, normalized size = 3.74

method	result	size
derivativedivides	Expression too large to display	1239
default	Expression too large to display	1239

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a^2/d^2/(c*x+1)+3*a^2/d^2*ln(c*x+1)-3*b^2/d^2*arctanh(c*x)^2+2*b^2/d
^2*arctanh(c*x)^3-b^2/d^2*ln(1+(c*x+1)^2/(-c^2*x^2+1))+3/2*b^2/d^2*polylog(
3,-(c*x+1)^2/(-c^2*x^2+1))+4*b^2/d^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+
4*b^2/d^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*I*b^2/d^2*arctanh(c*x)^
2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+a*b
/d^2+1/2*a^2/d^2*c^2*x^2-2*a^2/d^2*c*x+3/2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn
(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/4*b^2/d^2/(c*x+1)+b^2*arctanh(
c*x)/d^2-3/2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3*I*b^2/d^2*arc
tanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2
-1))^2+3/2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*b^2/d^2*arctanh(c*
x)/(c*x+1)*c*x+a*b/d^2*arctanh(c*x)*c^2*x^2-4*a*b/d^2*arctanh(c*x)*c*x+b^2/
d^2*arctanh(c*x)^2/(c*x+1)+3*b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+1/2*b^2/d^2*a
rctanh(c*x)/(c*x+1)+4*b^2/d^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2
))+4*b^2/d^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3*b^2/d^2*arct
anh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+a*b/d^2/(c*x+1)-3*a*b/d^2*dilog
```

```
(1/2*c*x+1/2)-3/2*a*b/d^2*ln(c*x+1)^2-a*b/d^2*ln(c*x-1)-3*a*b/d^2*ln(c*x+1)
-3*b^2/d^2*arctanh(c*x)^2*ln(2)-6*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x
^2+1)^(1/2))+a*b/d^2*c*x+1/2*b^2/d^2*arctanh(c*x)^2*c^2*x^2-2*b^2/d^2*arcta
nh(c*x)^2*c*x+b^2/d^2*arctanh(c*x)*c*x-1/4*b^2/d^2/(c*x+1)*c*x+2*a*b/d^2*ar
ctanh(c*x)/(c*x+1)+6*a*b/d^2*arctanh(c*x)*ln(c*x+1)-3*a*b/d^2*ln(-1/2*c*x+1
/2)*ln(1/2*c*x+1/2)+3*a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2*I*b^2/d^2*arct
anh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/
2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2/(c^5*d^2*x + c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)
/(c^4*d^2)) + 1/8*(b^2*c^3*x^3 - 3*b^2*c^2*x^2 - 4*b^2*c*x + 2*b^2 + 6*(b^2
*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^5*d^2*x + c^4*d^2) - integrate
(-1/4*((b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^
3*x^3)*log(c*x + 1) + (7*b^2*c^2*x^2 - (4*a*b*c^4 + b^2*c^4)*x^4 + 2*b^2*c*
x + 2*(2*a*b*c^3 + b^2*c^3)*x^3 - 2*b^2 - 2*(b^2*c^4*x^4 - b^2*c^3*x^3 + 3*
b^2*c^2*x^2 + 6*b^2*c*x + 3*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^6*d^2*x^3
+ c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**3*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^2}{{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

3.105 $\int \frac{x^2(a+b\tanh^{-1}(cx))^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{b^2}{2c^3d^2(1+cx)}+\frac{b^2\tanh^{-1}(cx)}{2c^3d^2}-\frac{b(a+b\tanh^{-1}(cx))}{c^3d^2(1+cx)}+\frac{3(a+b\tanh^{-1}(cx))^2}{2c^3d^2}+\frac{x(a+b\tanh^{-1}(cx))^2}{c^2d^2}-\frac{(a+$$

```
[Out] -1/2*b^2/c^3/d^2/(c*x+1)+1/2*b^2*arctanh(c*x)/c^3/d^2-b*(a+b*arctanh(c*x))/
c^3/d^2/(c*x+1)+3/2*(a+b*arctanh(c*x))^2/c^3/d^2+x*(a+b*arctanh(c*x))^2/c^2
/d^2-(a+b*arctanh(c*x))^2/c^3/d^2/(c*x+1)-2*b*(a+b*arctanh(c*x))*ln(2/(-c*x
+1))/c^3/d^2+2*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^3/d^2-b^2*polylog(2,1-2
/(-c*x+1))/c^3/d^2-2*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^3/d^2-b^
2*polylog(3,1-2/(c*x+1))/c^3/d^2
```

Rubi [A]
time = 0.36, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6065, 6063, 641, 46, 213, 6095, 6203, 6745}

$$-\frac{2b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(cx+1)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^3d^2}-\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b^2}{2c^3d^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{2c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] -1/2*b^2/(c^3*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c^3*d^2) - (b*(a + b*A
rcTanh[c*x]))/(c^3*d^2*(1 + c*x)) + (3*(a + b*ArcTanh[c*x])^2)/(2*c^3*d^2)
+ (x*(a + b*ArcTanh[c*x])^2)/(c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^3*d^2*(1
 + c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d^2) + (2*(a +
b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d^2) - (b^2*PolyLog[2, 1 - 2/(1 -
c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(
c^3*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
```

```
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)^2} - \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}\right)\,dx$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^2d^2} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^2d^2} - \frac{2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^2d^2}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+c\ldots}\right.}{c^3d^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)} + \frac{2(a+\ldots}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} - \frac{(a\ldots}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} - \frac{(a\ldots}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} - \frac{(a\ldots}{}$$

$$= -\frac{b^2}{2c^3d^2(1+cx)} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2} + \frac{x(a+b\tan\ldots}{c^2\ldots}$$

$$= -\frac{b^2}{2c^3d^2(1+cx)} + \frac{b^2\tanh^{-1}(cx)}{2c^3d^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)} + \frac{3(a+b\tanh^{-1}(cx\ldots}{2c^3d^2}$$

Mathematica [A]
time = 0.57, size = 295, normalized size = 1.13

$$\frac{4a^2cx - \frac{4a^2}{1+cx} - 8a^2\log(1+cx) + b^2\left(-4\tanh^{-1}(cx)^2 + 4cx\tanh^{-1}(cx)^2 - \cosh\left(2\tanh^{-1}(cx)\right) - 2\tanh^{-1}(cx)\cosh\left(2\tanh^{-1}(cx)\right) - 2\tanh^{-1}(cx)^2\cosh\left(2\tanh^{-1}(cx)\right) - 8\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + 8\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + \left(4-8\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) - 4\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right) + \sinh\left(2\tanh^{-1}(cx)\right) + 2\tanh^{-1}(cx)\sinh\left(2\tanh^{-1}(cx)\right) + 2\tanh^{-1}(cx)^2\sinh\left(2\tanh^{-1}(cx)\right)\right) + 2ab\left(-\cosh\left(2\tanh^{-1}(cx)\right) + 2\log\left(1-c^2x^2\right) - 4\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + \sinh\left(2\tanh^{-1}(cx)\right) + 2\tanh^{-1}(cx)\left(2cx - \cosh\left(2\tanh^{-1}(cx)\right) + 4\log\left(1+e^{-2\tanh^{-1}(cx)}\right) + \sinh\left(2\tanh^{-1}(cx)\right)\right)\right)}{4c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (4*a^2*c*x - (4*a^2)/(1 + c*x) - 8*a^2*Log[1 + c*x] + b^2*(-4*ArcTanh[c*x]^
2 + 4*c*x*ArcTanh[c*x]^2 - Cosh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Cosh[2*Arc
Tanh[c*x]] - 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 8*ArcTanh[c*x]*Log[1 +
 E^(-2*ArcTanh[c*x])] + 8*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (4
- 8*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 4*PolyLog[3, -E^(-2*Ar
cTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] +
 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 2*a*b*(-Cosh[2*ArcTanh[c*x]] + 2*
Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]
```

```
] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*ArcTanh[
c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.52, size = 5290, normalized size = 20.35

method	result	size
derivativedivides	Expression too large to display	5290
default	Expression too large to display	5290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*log(c*x + 1)/(c^3*d^2)) + 1
/4*(b^2*c^2*x^2 + b^2*c*x - b^2 - 2*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x
+ 1)^2/(c^4*d^2*x + c^3*d^2) - integrate(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*
log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2)*log(c*x + 1) - 2*((2*a*b*c^3
 + b^2*c^3)*x^3 - 2*(a*b*c^2 - b^2*c^2)*x^2 - b^2 + (b^2*c^3*x^3 - 3*b^2*c^
2*x^2 - 4*b^2*c*x - 2*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d^2*x^3 + c^4*
d^2*x^2 - c^3*d^2*x - c^2*d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^2\,\mathrm{atanh}^2\,(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^2\,\mathrm{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**2*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

3.106 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=188

$$\frac{b^2}{2c^2d^2(1+cx)}-\frac{b^2\tanh^{-1}(cx)}{2c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{(a+b\,\mathrm{ta}}{}$$

```
[Out] 1/2*b^2/c^2/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/c^2/d^2+b*(a+b*arctanh(c*x))/c
^2/d^2/(c*x+1)-1/2*(a+b*arctanh(c*x))^2/c^2/d^2+(a+b*arctanh(c*x))^2/c^2/d^
2/(c*x+1)-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^2/d^2+b*(a+b*arctanh(c*x))*p
olylog(2,1-2/(c*x+1))/c^2/d^2+1/2*b^2*polylog(3,1-2/(c*x+1))/c^2/d^2
```

Rubi [A]
time = 0.25, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6087, 6065, 6063, 641, 46, 213, 6095, 6055, 6203, 6745}

$$\frac{b\,\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}+\frac{b^2\mathrm{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b^2}{2c^2d^2(cx+1)}-\frac{b^2\tanh^{-1}(cx)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] b^2/(2*c^2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^2*d^2) + (b*(a + b*ArcT
anh[c*x]))/(c^2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*c^2*d^2) + (a +
b*ArcTanh[c*x])^2/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(1 +
c*x)])/(c^2*d^2) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int \left(-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{cd^2}+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{cd^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2}-\frac{(2b)\int\left(\frac{a+b\tanh^{-1}(cx)}{2(1+cx)^2}\right.}{c}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{(a+b}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{(a+b}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}-\frac{(a+b}{}$$

$$= \frac{b^2}{2c^2d^2(1+cx)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2}+\frac{\left(a+b\tanh^{-}\right.}{c^2d^2(1+}$$

$$= \frac{b^2}{2c^2d^2(1+cx)}-\frac{b^2\tanh^{-1}(cx)}{2c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}$$

Mathematica [A]
time = 0.33, size = 233, normalized size = 1.24

$$\frac{\frac{4a^2}{1+cx}+4a^2\log(1+cx)+2ab\left(\cosh\left(2\tanh^{-1}(cx)\right)+2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2\tanh^{-1}(cx)\left(\cosh\left(2\tanh^{-1}(cx)\right)-2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)\right)-\sinh\left(2\tanh^{-1}(cx)\right)\right)+b^2\left(\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)^2\cosh\left(2\tanh^{-1}(cx)\right)-4\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+4\tanh^{-1}(cx)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)-2\tanh^{-1}(cx)\sinh\left(2\tanh^{-1}(cx)\right)-2\tanh^{-1}(cx)^2\sinh\left(2\tanh^{-1}(cx)\right)\right)}{4c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] ((4*a^2)/(1 + c*x) + 4*a^2*Log[1 + c*x] + 2*a*b*(Cosh[2*ArcTanh[c*x]] + 2*P
olyLog[2, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*
```

```
Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]
) + b^2*(Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 2*Arc
Tanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c
*x])] + 4*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*PolyLog[3, -E^(
-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*
x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c^2*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.67, size = 946, normalized size = 5.03

method	result	size
derivativedivides	Expression too large to display	946
default	Expression too large to display	946

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+a^2
/d^2/(c*x+1)+a^2/d^2*ln(c*x+1)-1/2*b^2/d^2*arctanh(c*x)^2+2/3*b^2/d^2*arcta
nh(c*x)^3+1/2*b^2/d^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/2*I*b^2/d^2*Pi*c
sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2
/(-c^2*x^2+1)))^2*arctanh(c*x)^2-I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))
^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2+1/2*I*b^2/d^2*Pi*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2*arctanh(c*x)^2-1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*arctanh(c*x)^2+1/2*I*b^2/d^2*Pi*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/4*b^2/d^2/(c*x+1)-1/2
*I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*ar
ctanh(c*x)^2-1/2*b^2/d^2*arctanh(c*x)/(c*x+1)*c*x+b^2/d^2*arctanh(c*x)^2/(c
*x+1)+b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+1/2*b^2/d^2*arctanh(c*x)/(c*x+1)-b^2
/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+a*b/d^2/(c*x+1)-a*b/d^
2*dilog(1/2*c*x+1/2)-1/2*a*b/d^2*ln(c*x+1)^2+1/2*a*b/d^2*ln(c*x-1)-1/2*a*b/
d^2*ln(c*x+1)-b^2/d^2*arctanh(c*x)^2*ln(2)-2*b^2/d^2*arctanh(c*x)^2*ln((c*x
+1)/(-c^2*x^2+1)^(1/2))-1/4*b^2/d^2/(c*x+1)*c*x+2*a*b/d^2*arctanh(c*x)/(c*x
+1)+2*a*b/d^2*arctanh(c*x)*ln(c*x+1)-a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/
2)+a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^2*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(c^2*d^2)) + 1/4*(b^2 + (b^2*c*
x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d^2*x + c^2*d^2) - integrate(-1
/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(
c*x + 1) - 2*(2*a*b*c^2*x^2 + b^2 - (2*a*b*c - b^2*c)*x + (2*b^2*c^2*x^2 +
b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^
2*d^2*x - c*d^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c^2*d^2*x^2
 + 2*c*d^2*x + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
[Out] (Integral(a**2*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x*atanh(c*x)**
2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x*atanh(c*x)/(c**2*x**2 + 2*
c*x + 1), x))/d**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{atanh}(c\,x)\right)}^2}{{\left(d+c\,d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
[Out] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

3.107 $\int \frac{(a+b\tanh^{-1}(cx))^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{b^2}{2cd^2(1+cx)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}$$

```
[Out] -1/2*b^2/c/d^2/(c*x+1)+1/2*b^2*arctanh(c*x)/c/d^2-b*(a+b*arctanh(c*x))/c/d^
2/(c*x+1)+1/2*(a+b*arctanh(c*x))^2/c/d^2-(a+b*arctanh(c*x))^2/c/d^2/(c*x+1)
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6065, 6063, 641, 46, 213, 6095}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{b^2}{2cd^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

```
[Out] -1/2*b^2/(c*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c*d^2) - (b*(a + b*ArcTa
nh[c*x]))/(c*d^2*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(2*c*d^2) - (a + b*Arc
Tanh[c*x])^2/(c*d^2*(1 + c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}+\frac{(2b)\int\left(\frac{a+b\tanh^{-1}(cx)}{2d(1+cx)^2}-\frac{a+b\tanh^{-1}(cx)}{2d(-1+c^2x^2)}\right)dx}{d}\\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}+\frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{d^2}\\
&= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}+\frac{b^2\int\frac{}{(1}}{}\\
&= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}+\frac{b^2\int\frac{}{(1}}{}\\
&= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}+\frac{b^2\int\Big(}{}\\
&= -\frac{b^2}{2cd^2(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{\left(a+b\tanh^{-1}\right.}{cd^2(1+c}\\
&= -\frac{b^2}{2cd^2(1+cx)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}
\end{aligned}
$$

Mathematica [A]

time = 0.07, size = 124, normalized size = 1.16

$$\frac{-4a^2-4ab-2b^2-4b(2a+b)\tanh^{-1}(cx)+2b^2(-1+cx)\tanh^{-1}(cx)^2-b(2a+b)(1+cx)\log(1-cx)+2ab\log(1+cx)+b^2\log(1+cx)+2abcx\log(1+cx)+b^2cx\log(1+cx)}{4cd^2(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

```
[Out] (-4*a^2 - 4*a*b - 2*b^2 - 4*b*(2*a + b)*ArcTanh[c*x] + 2*b^2*(-1 + c*x)*Arc
Tanh[c*x]^2 - b*(2*a + b)*(1 + c*x)*Log[1 - c*x] + 2*a*b*Log[1 + c*x] + b^2
*Log[1 + c*x] + 2*a*b*c*x*Log[1 + c*x] + b^2*c*x*Log[1 + c*x])/(4*c*d^2*(1
+ c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(101) = 202.
time = 0.30, size = 294, normalized size = 2.75

method	result
risch	$\frac{b^2(cx-1)\ln(cx+1)^2}{8d^2(cx+1)c}-\frac{b(bcx\ln(-cx+1)-b\ln(-cx+1)+4a+2b)\ln(cx+1)}{4d^2(cx+1)c}+\frac{b^2cx\ln(-cx+1)^2+4\ln(-cx-1)abcx+2\ln($
derivativedivides	$-\frac{a^2}{d^2(cx+1)}-\frac{b^2\operatorname{arctanh}(cx)^2}{d^2(cx+1)}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2d^2}-\frac{b^2\operatorname{arctanh}(cx)}{d^2(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2d^2}+\frac{b^2\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4d^2}-\frac{b^2\ln(}{8}$
default	$-\frac{a^2}{d^2(cx+1)}-\frac{b^2\operatorname{arctanh}(cx)^2}{d^2(cx+1)}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2d^2}-\frac{b^2\operatorname{arctanh}(cx)}{d^2(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2d^2}+\frac{b^2\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4d^2}-\frac{b^2\ln(}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a^2/d^2/(c*x+1)-b^2/d^2*arctanh(c*x)^2/(c*x+1)-1/2*b^2/d^2*arctanh(c*
x)*ln(c*x-1)-b^2/d^2*arctanh(c*x)/(c*x+1)+1/2*b^2/d^2*arctanh(c*x)*ln(c*x+1
)+1/4*b^2/d^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/8*b^2/d^2*ln(c*x-1)^2-1/4*b^2/d^2
*ln(c*x-1)-1/2*b^2/d^2/(c*x+1)+1/4*b^2/d^2*ln(c*x+1)-1/4*b^2/d^2*ln(-1/2*c*
x+1/2)*ln(1/2*c*x+1/2)+1/4*b^2/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/8*b^2/d^2*l
n(c*x+1)^2-2*a*b/d^2*arctanh(c*x)/(c*x+1)-1/2*a*b/d^2*ln(c*x-1)-a*b/d^2/(c*
x+1)+1/2*a*b/d^2*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(101) = 202.
time = 0.27, size = 277, normalized size = 2.59

$$-\frac{1}{2}\left(c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\log(cx-1)}{c^2d^2}\right)+\frac{4\operatorname{artanh}(cx)}{c^2d^2x+cd^2}\right)ab-\frac{1}{8}\left(4c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\log(cx-1)}{c^2d^2}\right)\operatorname{artanh}(cx)+\frac{((cx+1)\log(cx+1)^2+(cx+1)\log(cx-1)^2-2(cx+(cx+1)\log(cx-1)+1)\log(cx+1)+2(cx+1)\log(cx-1)+4)c^2}{c^4d^2x+c^3d^2}\right)b^2-\frac{b^2\operatorname{artanh}(cx)^2}{c^2d^2x+cd^2}-\frac{a^2}{c^2d^2x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c
^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*a*b - 1/8*(4*c*(2/(c^3*d^2*x
```

```
 + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c^2*d^2))*arctanh(c*x)
 + ((c*x + 1)*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1
)*log(c*x - 1) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*d
^2*x + c^3*d^2))*b^2 - b^2*arctanh(c*x)^2/(c^2*d^2*x + c*d^2) - a^2/(c^2*d^
2*x + c*d^2)
```

Fricas [A]
time = 0.34, size = 101, normalized size = 0.94

$$\frac{(b^2cx-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2-8\,a^2-8\,ab-4\,b^2+2\,((2\,ab+b^2)cx-2\,ab-b^2)\log\left(-\frac{cx+1}{cx-1}\right)}{8\,(c^2d^2x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2*c*x - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 8*a^2 - 8*a*b - 4*b^2 +
2*((2*a*b + b^2)*c*x - 2*a*b - b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^2*d^2*x +
 c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2}{c^2x^2+2cx+1}\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx+\int\frac{2ab\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c
**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**2 + 2*c*x +
1), x))/d**2
```

Giac [A]
time = 0.40, size = 119, normalized size = 1.11

$$\frac{1}{8}\,c\left(\frac{(cx-1)b^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2d^2}+\frac{2\,(2\,ab+b^2)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2d^2}+\frac{2\,(2\,a^2+2\,ab+b^2)(cx-1)}{(cx+1)c^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/8*c*((c*x - 1)*b^2*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2*d^2) + 2*(2
*a*b + b^2)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2) + 2*(2*
a^2 + 2*a*b + b^2)*(c*x - 1)/((c*x + 1)*c^2*d^2))
```

Mupad [B]
time = 1.25, size = 97, normalized size = 0.91

$$\frac{b^2\,\mathrm{atanh}(c\,x)^2+b^2\,\mathrm{atanh}(c\,x)+2\,a\,b\,\mathrm{atanh}(c\,x)}{2\,c\,d^2}-\frac{2\,a^2+4\,a\,b\,\mathrm{atanh}(c\,x)+2\,a\,b+2\,b^2\,\mathrm{atanh}(c\,x)^2+2\,b^2\,\mathrm{atanh}(c\,x)+b^2}{2\,x\,c^2\,d^2+2\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + c*d*x)^2,x)
```

```
[Out] (b^2*atanh(c*x)^2 + b^2*atanh(c*x) + 2*a*b*atanh(c*x))/(2*c*d^2) - (2*b^2*a
tanh(c*x)^2 + 2*a*b + 2*b^2*atanh(c*x) + 2*a^2 + b^2 + 4*a*b*atanh(c*x))/(2
*c*d^2 + 2*c^2*d^2*x)
```

3.108 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x(d+cdx)^2}\,dx$

Optimal. Leaf size=295

$$\frac{b^2}{2d^2(1+cx)}-\frac{b^2\tanh^{-1}(cx)}{2d^2}+\frac{b(a+b\tanh^{-1}(cx))}{d^2(1+cx)}-\frac{(a+b\tanh^{-1}(cx))^2}{2d^2}+\frac{(a+b\tanh^{-1}(cx))^2}{d^2(1+cx)}+\frac{2(a+b\tan$$

```
[Out] 1/2*b^2/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/d^2+b*(a+b*arctanh(c*x))/d^2/(c*x+
1)-1/2*(a+b*arctanh(c*x))^2/d^2+(a+b*arctanh(c*x))^2/d^2/(c*x+1)-2*(a+b*arc
tanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^2+(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/
d^2-b*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2+b*(a+b*arctanh(c*x))*p
olylog(2,-1+2/(-c*x+1))/d^2-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d^2
+1/2*b^2*polylog(3,1-2/(-c*x+1))/d^2-1/2*b^2*polylog(3,-1+2/(-c*x+1))/d^2-1
/2*b^2*polylog(3,1-2/(c*x+1))/d^2
```

Rubi [A]

time = 0.47, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6087, 6033, 6199, 6095, 6205, 6745, 6065, 6063, 641, 46, 213, 6055, 6203}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2}+\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}+\frac{b^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d^2}-\frac{b^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d^2}-\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d^2}+\frac{b^2}{2d^2(cx+1)}-\frac{b^2\tanh^{-1}(cx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

```
[Out] b^2/(2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*d^2) + (b*(a + b*ArcTanh[c*x]
))/(d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*d^2) + (a + b*ArcTanh[c*x])^
2/(d^2*(1 + c*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2
 + ((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)])/d^2 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
2/(1 - c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^
2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) - (b^2*PolyLog[3, -1 + 2/(1 -
 c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^2}\,dx &= \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)^2} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)}\right) dx \\
&= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{d^2} \\
&= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} + \cdots \\
&= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} + \cdots \\
&= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \cdots \\
&= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \cdots \\
&= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \cdots \\
&= \frac{b^2}{2d^2(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \cdots \\
&= \frac{b^2}{2d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{2d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \cdots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.48, size = 254, normalized size = 0.86

$$
\frac{\frac{24a^2}{1+cx} + 24a^2\log(cx) - 24a^2\log(1+cx) + 12ab\left(\cosh\left(2\tanh^{-1}(cx)\right) - 2\text{PolyLog}\left(2, e^{-2\tanh^{-1}(cx)}\right) + 2\tanh^{-1}(cx)\left(\cosh\left(2\tanh^{-1}(cx)\right) + 2\log\left(1-e^{-2\tanh^{-1}(cx)}\right) - \sinh\left(2\tanh^{-1}(cx)\right)\right) - \sinh\left(2\tanh^{-1}(cx)\right)\right) + b^2\left(i\pi^3 - 16\tanh^{-1}(cx)^3 + 6\cosh\left(2\tanh^{-1}(cx)\right) + 12\tanh^{-1}(cx)\cosh\left(2\tanh^{-1}(cx)\right) + 12\tanh^{-1}(cx)^2\cosh\left(2\tanh^{-1}(cx)\right) + 24\tanh^{-1}(cx)^2\log\left(1-e^{2\tanh^{-1}(cx)}\right) + 24\tanh^{-1}(cx)\text{PolyLog}\left(2, e^{2\tanh^{-1}(cx)}\right) - 12\text{PolyLog}\left(3, e^{2\tanh^{-1}(cx)}\right) - 6\sinh\left(2\tanh^{-1}(cx)\right) - 12\tanh^{-1}(cx)\sinh\left(2\tanh^{-1}(cx)\right) - 12\tanh^{-1}(cx)^2\sinh\left(2\tanh^{-1}(cx)\right)\right)}{24d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

```
[Out] ((24*a^2)/(1 + c*x) + 24*a^2*Log[c*x] - 24*a^2*Log[1 + c*x] + 12*a*b*(Cosh[
2*ArcTanh[c*x]] - 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[
2*ArcTanh[c*x]] + 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) -
Sinh[2*ArcTanh[c*x]]) + b^2*(I*Pi^3 - 16*ArcTanh[c*x]^3 + 6*Cosh[2*ArcTanh[
c*x]] + 12*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 12*ArcTanh[c*x]^2*Cosh[2*Arc
Tanh[c*x]] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])] - 6*Si
nh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]
^2*Sinh[2*ArcTanh[c*x]]))/(24*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.37, size = 1566, normalized size = 5.31

method	result	size
derivativedivides	Expression too large to display	1566
default	Expression too large to display	1566

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d^2*ln(c*x)+1/2*I*b^2/d^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+a^2/d^2/(c*x+1)-a^2/d^2*ln(c*x+1)-1/2*b^2/d
^2*arctanh(c*x)^2-2/3*b^2/d^2*arctanh(c*x)^3+1/2*I*b^2/d^2*Pi*arctanh(c*x)^
2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/2*I*b^2/d^2*Pi*arctanh(c*x)^2*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b^2/d^2*Pi*arc
tanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^
2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I*
b^2/d^2*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)
^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+I*
b^2/d^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2+1/2*I*b^2/d^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/2*I*b^2/d^2*Pi*arctanh(c*x)^2*c
sgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2+1/2*I*b^2/d^2*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/
(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1
/2*I*b^2/d^2*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d^2*Pi*arc
tanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(
c*x+1)^2/(-c^2*x^2+1)))^2+1/4*b^2/d^2/(c*x+1)-1/2*b^2/d^2*arctanh(c*x)/(c*x
+1)*c*x+b^2/d^2*arctanh(c*x)^2/(c*x+1)-b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+1/2
*b^2/d^2*arctanh(c*x)/(c*x+1)+a*b/d^2/(c*x+1)+a*b/d^2*dilog(1/2*c*x+1/2)+1/
2*a*b/d^2*ln(c*x+1)^2+1/2*a*b/d^2*ln(c*x-1)-1/2*a*b/d^2*ln(c*x+1)+b^2/d^2*a
rctanh(c*x)^2*ln(2)+2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))
-a*b/d^2*dilog(c*x+1)-a*b/d^2*dilog(c*x)+b^2/d^2*arctanh(c*x)^2*ln(c*x)-b^2
/d^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+2*b^2/d^2*arctanh(c*x)*pol
ylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d^2*arctanh(c*x)*polylog(2,(c*x+1
)/(-c^2*x^2+1)^(1/2))+b^2/d^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2
))+b^2/d^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^2*polylo
g(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(
1/2))+2*a*b/d^2*arctanh(c*x)*ln(c*x)-a*b/d^2*ln(c*x)*ln(c*x+1)-1/4*b^2/d^2/
(c*x+1)*c*x+2*a*b/d^2*arctanh(c*x)/(c*x+1)-2*a*b/d^2*arctanh(c*x)*ln(c*x+1)
+a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^2*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/4*(b^2 - (b^2*c
*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x + d^2) + integrate(1/4*((b
^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - 2*(b^2*c^2*
x^2 - 2*a*b + (2*a*b*c + b^2*c)*x - (b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2)*log
(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^4 + c^2*d^2*x^3 - c*d^2*x^2 - d^2*x),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^3 + 2*c
*d^2*x^2 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^3+2cx^2+x}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{c^2x^3+2cx^2+x}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2
/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**3 + 2*
c*x**2 + x), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x\,(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^2), x)
```

3.109 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^2(d+cdx)^2}\,dx$

Optimal. Leaf size=371

$$-\frac{b^2c}{2d^2(1+cx)}+\frac{b^2c\tanh^{-1}(cx)}{2d^2}-\frac{bc(a+b\tanh^{-1}(cx))}{d^2(1+cx)}+\frac{3c(a+b\tanh^{-1}(cx))^2}{2d^2}-\frac{(a+b\tanh^{-1}(cx))^2}{d^2x}-\frac{c(a+$$

```
[Out] -1/2*b^2*c/d^2/(c*x+1)+1/2*b^2*c*arctanh(c*x)/d^2-b*c*(a+b*arctanh(c*x))/d^
2/(c*x+1)+3/2*c*(a+b*arctanh(c*x))^2/d^2-(a+b*arctanh(c*x))^2/d^2/x-c*(a+b*
arctanh(c*x))^2/d^2/(c*x+1)+4*c*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))
/d^2-2*c*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*ln
(2-2/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2-2*b*
c*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*
polylog(2,1-2/(c*x+1))/d^2-b^2*c*polylog(2,-1+2/(c*x+1))/d^2-b^2*c*polylog(
3,1-2/(-c*x+1))/d^2+b^2*c*polylog(3,-1+2/(-c*x+1))/d^2+b^2*c*polylog(3,1-2/
(c*x+1))/d^2
```

Rubi [A]
time = 0.58, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6087, 6037, 6135, 6079, 2497, 6033, 6199, 6095, 6205, 6745, 6065, 6063, 641, 46, 213, 6055, 6203}

$$\frac{2bc\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2bc\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{2bc\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x}-\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(cx+1)}+\frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2}-\frac{4c\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2c\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}-\frac{b^2c\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{d^2}-\frac{b^2c\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{d^2}+\frac{b^2c\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{d^2}+\frac{b^2c\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{d^2}-\frac{b^2c}{2d^2(cx+1)}+\frac{b^2c\tanh^{-1}(cx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

```
[Out] -1/2*(b^2*c)/(d^2*(1 + c*x)) + (b^2*c*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*A
rcTanh[c*x]))/(d^2*(1 + c*x)) + (3*c*(a + b*ArcTanh[c*x])^2)/(2*d^2) - (a +
 b*ArcTanh[c*x])^2/(d^2*x) - (c*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c*x)) - (
4*c*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (2*c*(a + b*ArcT
anh[c*x])^2*Log[2/(1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(
1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d
^2 - (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (2*b*c
*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2,
-1 + 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/d^2 + (b^2*c*P
olyLog[3, -1 + 2/(1 - c*x)])/d^2 + (b^2*c*PolyLog[3, 1 - 2/(1 + c*x)])/d^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)^2} + \cdots\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} - \frac{(2c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^2} + \frac{(2c^2\cdots}{\cdots}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} - \frac{4c\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\cdots}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} - \frac{4c(a+\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c(a\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c(a\cdots}{\cdots}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c(a\cdots}{\cdots}$$

$$= -\frac{b^2c}{2d^2(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}\cdots\right.}{d^2x}$$

$$= -\frac{b^2c}{2d^2(1+cx)} + \frac{b^2c\tanh^{-1}(cx)}{2d^2} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\cdots\right.}{2d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.44, size = 347, normalized size = 0.94

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

```
[Out] ((-12*a^2)/x - (12*a^2*c)/(1 + c*x) - 24*a^2*c*Log[x] + 24*a^2*c*Log[1 + c*
x] + b^2*c*((-I)*Pi^3 + 12*ArcTanh[c*x]^2 - (12*ArcTanh[c*x]^2)/(c*x) + 16*
ArcTanh[c*x]^3 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x
]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 24*ArcTanh[c*x]*Log[1 - E^(-2*
ArcTanh[c*x])] - 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 12*PolyLog
[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] +
 12*PolyLog[3, E^(2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x
]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 6*a*b*c*(
```

```
-Cosh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2
*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2*
ArcTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]])))/
(12*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.64, size = 7294, normalized size = 19.66

method	result	size
derivativedivides	Expression too large to display	7294
default	Expression too large to display	7294

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d
^2) - 1/4*(2*b^2*c*x + b^2 - 2*(b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c
*x + 1)^2/(c*d^2*x^2 + d^2*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1
)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(2*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 2
*a*b - (2*a*b*c - b^2*c)*x - (2*b^2*c^4*x^4 + 4*b^2*c^3*x^3 + 2*b^2*c^2*x^2
 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^5 + c^2*d^2*x^4 -
 c*d^2*x^3 - d^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^4 + 2*c
*d^2*x^3 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^4+2cx^3+x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{c^2x^4+2cx^3+x^2}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{c^2x^4+2cx^3+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)
**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**
4 + 2*c*x**3 + x**2), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{x^2\,(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^2), x)
```

3.110 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3(d+cdx)^2}\,dx$

Optimal. Leaf size=480

$$\frac{b^2c^2}{2d^2(1+cx)}-\frac{b^2c^2\tanh^{-1}(cx)}{2d^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)}-\frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}-(a$$

```
[Out] 1/2*b^2*c^2/d^2/(c*x+1)-1/2*b^2*c^2*arctanh(c*x)/d^2-b*c*(a+b*arctanh(c*x))
/d^2/x+b*c^2*(a+b*arctanh(c*x))/d^2/(c*x+1)-2*c^2*(a+b*arctanh(c*x))^2/d^2-
1/2*(a+b*arctanh(c*x))^2/d^2/x^2+2*c*(a+b*arctanh(c*x))^2/d^2/x+c^2*(a+b*ar
ctanh(c*x))^2/d^2/(c*x+1)-6*c^2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))
/d^2+b^2*c^2*ln(x)/d^2+3*c^2*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2-1/2*b^2
*c^2*ln(-c^2*x^2+1)/d^2-4*b*c^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d^2-3*b*
c^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2+3*b*c^2*(a+b*arctanh(c*x
))*polylog(2,-1+2/(-c*x+1))/d^2-3*b*c^2*(a+b*arctanh(c*x))*polylog(2,1-2/(c
*x+1))/d^2+2*b^2*c^2*polylog(2,-1+2/(c*x+1))/d^2+3/2*b^2*c^2*polylog(3,1-2/
(-c*x+1))/d^2-3/2*b^2*c^2*polylog(3,-1+2/(-c*x+1))/d^2-3/2*b^2*c^2*polylog(
3,1-2/(c*x+1))/d^2
```

Rubi [A]

time = 0.69, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 22, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6087, 6037, 6129, 272, 36, 29, 31, 6095, 6135, 6079, 2497, 6033, 6199, 6205, 6745, 6065, 6063, 641, 46, 213, 6055, 6203}

$$-\frac{3bc^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{3bc^2\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{3bc^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(cx+1)}-\frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}+\frac{6c^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}-\frac{4bc^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2x^2}+\frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x}+\frac{2b^2c^2\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{d^2}+\frac{3b^2c^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d^2}-\frac{3b^2c^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d^2}-\frac{3b^2c^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d^2}-\frac{b^2c^2\log\left(1-c^2x^2\right)}{2d^2}+\frac{b^2c^2}{2d^2(cx+1)}+\frac{b^2c^2\log(x)}{d^2}-\frac{b^2c^2\tanh^{-1}(cx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

```
[Out] (b^2*c^2)/(2*d^2*(1 + c*x)) - (b^2*c^2*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*
ArcTanh[c*x]))/(d^2*x) + (b*c^2*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*
c^2*(a + b*ArcTanh[c*x])^2)/d^2 - (a + b*ArcTanh[c*x])^2/(2*d^2*x^2) + (2*c
*(a + b*ArcTanh[c*x])^2)/(d^2*x) + (c^2*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c
*x)) + (6*c^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 + (b^2*c
^2*Log[x])/d^2 + (3*c^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b^2
*c^2*Log[1 - c^2*x^2])/(2*d^2) - (4*b*c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
 + c*x)])/d^2 - (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/
d^2 + (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 - (3*
b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 + (2*b^2*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/d^2 + (3*b^2*c^2*PolyLog[3, 1 - 2/(1 - c*x)])/(
2*d^2) - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^2) - (3*b^2*c^2*Poly
Log[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3(d+cdx)^2}\,dx = \int \left(\frac{(a+b\tanh^{-1}(cx))^2}{d^2x^3} - \frac{2c(a+b\tanh^{-1}(cx))^2}{d^2x^2} + \frac{3c^2(a+b\tanh^{-1}(cx))^2}{d^2x}\right.$$

$$= \frac{\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3}\,dx}{d^2} - \frac{(2c)\int \frac{(a+b\tanh^{-1}(cx))^2}{x^2}\,dx}{d^2} + \frac{(3c^2)\int \frac{(a+b\tanh^{-1}(cx))^2}{x}\,dx}{d^2} -$$

$$= -\frac{(a+b\tanh^{-1}(cx))^2}{2d^2x^2} + \frac{2c(a+b\tanh^{-1}(cx))^2}{d^2x} + \frac{c^2(a+b\tanh^{-1}(cx))^2}{d^2(1+cx)} + 6c$$

$$= -\frac{2c^2(a+b\tanh^{-1}(cx))^2}{d^2} - \frac{(a+b\tanh^{-1}(cx))^2}{2d^2x^2} + \frac{2c(a+b\tanh^{-1}(cx))^2}{d^2x} +$$

$$= -\frac{bc(a+b\tanh^{-1}(cx))}{d^2x} + \frac{bc^2(a+b\tanh^{-1}(cx))}{d^2(1+cx)} - \frac{2c^2(a+b\tanh^{-1}(cx))^2}{d^2} -$$

$$= -\frac{bc(a+b\tanh^{-1}(cx))}{d^2x} + \frac{bc^2(a+b\tanh^{-1}(cx))}{d^2(1+cx)} - \frac{2c^2(a+b\tanh^{-1}(cx))^2}{d^2} -$$

$$= -\frac{bc(a+b\tanh^{-1}(cx))}{d^2x} + \frac{bc^2(a+b\tanh^{-1}(cx))}{d^2(1+cx)} - \frac{2c^2(a+b\tanh^{-1}(cx))^2}{d^2} -$$

$$= \frac{b^2c^2}{2d^2(1+cx)} - \frac{bc(a+b\tanh^{-1}(cx))}{d^2x} + \frac{bc^2(a+b\tanh^{-1}(cx))}{d^2(1+cx)} - \frac{2c^2(a+b\tan}{d^2}$$

$$= \frac{b^2c^2}{2d^2(1+cx)} - \frac{b^2c^2\tanh^{-1}(cx)}{2d^2} - \frac{bc(a+b\tanh^{-1}(cx))}{d^2x} + \frac{bc^2(a+b\tanh^{-1}(cx}{d^2(1+cx)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.35, size = 452, normalized size = 0.94

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

```
[Out] ((-4*a^2)/x^2 + (16*a^2*c)/x + (8*a^2*c^2)/(1 + c*x) + 24*a^2*c^2*Log[x] -
24*a^2*c^2*Log[1 + c*x] + b^2*c^2*(I*Pi^3 - (8*ArcTanh[c*x])/(c*x) - 12*Arc
Tanh[c*x]^2 - (4*ArcTanh[c*x]^2)/(c^2*x^2) + (16*ArcTanh[c*x]^2)/(c*x) - 16
*ArcTanh[c*x]^3 + 2*Cosh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*Cosh[2*ArcTanh[c*
x]] + 4*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 32*ArcTanh[c*x]*Log[1 - E^(-2
*ArcTanh[c*x])] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 8*Log[(c*
x)/Sqrt[1 - c^2*x^2]] + 16*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])] - 2*Si
nh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2
*Sinh[2*ArcTanh[c*x]]) + (4*a*b*(-6*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]
 + c*x*(-2 + c*x*Cosh[2*ArcTanh[c*x]] - 8*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]
- c*x*Sinh[2*ArcTanh[c*x]]) + 2*ArcTanh[c*x]*(-1 + 4*c*x + c^2*x^2 + c^2*x^
2*Cosh[2*ArcTanh[c*x]] + 6*c^2*x^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*x^2*S
inh[2*ArcTanh[c*x]])))/x^2)/(8*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.03, size = 1878, normalized size = 3.91

method	result	size
derivativedivides	Expression too large to display	1878
default	Expression too large to display	1878

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3*a^2/d^2*ln(c*x)-a*b/d^2*arctanh(c*x)/c^2/x^2+a^2/d^2/(c*x+1)-3*a^2/d
^2*ln(c*x+1)+2*b^2/d^2*arctanh(c*x)^2-2*b^2/d^2*arctanh(c*x)^3-3/2*I*b^2/d^
2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+3/2*I*b^
2/d^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1
)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))+b^2/d^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+b^2/d^2*ln(1+(c*x+1)/(-c^2*x
^2+1)^(1/2))+1/4*b^2/d^2/(c*x+1)-b^2*arctanh(c*x)/d^2-1/2*b^2/d^2*arctanh(c
*x)/(c*x+1)*c*x+b^2/d^2*arctanh(c*x)^2/(c*x+1)-3*b^2/d^2*arctanh(c*x)^2*ln(
c*x+1)+1/2*b^2/d^2*arctanh(c*x)/(c*x+1)+a*b/d^2/(c*x+1)+3*a*b/d^2*dilog(1/2
*c*x+1/2)+3/2*a*b/d^2*ln(c*x+1)^2+2*a*b/d^2*ln(c*x-1)+2*a*b/d^2*ln(c*x+1)+3
*b^2/d^2*arctanh(c*x)^2*ln(2)+6*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2
+1)^(1/2))-3/2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*I*b^
2/d^2*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/2*I*b^2/d^2*arctanh(c*x)^
2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2+3*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1
```

```
)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+3/2*I*b^2/d^2*arctanh(c*x)^2*Pi*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+3/2*I*b^2/
d^2*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+4*a*b/d^2*arctanh(c*x)/c/x-3*a*b/d
^2*dilog(c*x+1)-3*a*b/d^2*dilog(c*x)+3*b^2/d^2*arctanh(c*x)^2*ln(c*x)-3*b^2
/d^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+6*b^2/d^2*arctanh(c*x)*pol
ylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*b^2/d^2*arctanh(c*x)*polylog(2,(c*x+1
)/(-c^2*x^2+1)^(1/2))+3*b^2/d^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1
/2))+3*b^2/d^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*b^2/d^2*po
lylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*b^2/d^2*polylog(3,(c*x+1)/(-c^2*x^2+
1)^(1/2))+6*a*b/d^2*arctanh(c*x)*ln(c*x)-3*a*b/d^2*ln(c*x)*ln(c*x+1)-1/4*b^
2/d^2/(c*x+1)*c*x+2*a*b/d^2*arctanh(c*x)/(c*x+1)-6*a*b/d^2*arctanh(c*x)*ln(
c*x+1)+3*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-3*a*b/d^2*ln(-1/2*c*x+1/2
)*ln(c*x+1)-a*b/d^2/c/x-1/2*b^2/d^2*arctanh(c*x)^2/c^2/x^2-b^2/d^2*arctanh(
c*x)/c/x+2*a^2/d^2/c/x-4*b^2/d^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+4*b^2/
d^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*a^2/d^2/c^2/x^2+3/2*I*b^2/d^2*arc
tanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1))
)^3+3/2*I*b^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+3/2*I*b
^2/d^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3-4*b^2/d^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d^2*
arctanh(c*x)^2/c/x-4*a*b/d^2*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x -
1)/(c*d^2*x^3 + d^2*x^2)) + 1/8*(6*b^2*c^2*x^2 + 3*b^2*c*x - b^2 - 6*(b^2*c
^3*x^3 + b^2*c^2*x^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x^3 + d^2*x^2) +
 integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x
+ 1) - (6*b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 2*b^2*c^2*x^2 - 4*a*b + (4*a*b*c -
b^2*c)*x - 2*(3*b^2*c^5*x^5 + 6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x + b^2
)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^6 + c^2*d^2*x^5 - c*d^2*x^4 - d^2
*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^5 + 2*c
*d^2*x^4 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)
**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**
5 + 2*c*x**4 + x**3), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{atanh}(c\,x))^2}{x^3\,(d+c\,d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)^2),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)^2), x)
```

3.111 $\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=408

$$\frac{abx}{c^4d^3}-\frac{b^2}{16c^5d^3(1+cx)^2}+\frac{29b^2}{16c^5d^3(1+cx)}-\frac{29b^2\tanh^{-1}(cx)}{16c^5d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b(a+}{4c^5}$$

```
[Out] a*b*x/c^4/d^3-1/16*b^2/c^5/d^3/(c*x+1)^2+29/16*b^2/c^5/d^3/(c*x+1)-29/16*b^
2*arctanh(c*x)/c^5/d^3+b^2*x*arctanh(c*x)/c^4/d^3-1/4*b*(a+b*arctanh(c*x))/
c^5/d^3/(c*x+1)^2+15/4*b*(a+b*arctanh(c*x))/c^5/d^3/(c*x+1)-43/8*(a+b*arcta
nh(c*x))^2/c^5/d^3-3*x*(a+b*arctanh(c*x))^2/c^4/d^3+1/2*x^2*(a+b*arctanh(c*
x))^2/c^3/d^3-1/2*(a+b*arctanh(c*x))^2/c^5/d^3/(c*x+1)^2+4*(a+b*arctanh(c*x
))^2/c^5/d^3/(c*x+1)+6*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^5/d^3-6*(a+b*a
rctanh(c*x))^2*ln(2/(c*x+1))/c^5/d^3+1/2*b^2*ln(-c^2*x^2+1)/c^5/d^3+3*b^2*p
olylog(2,1-2/(-c*x+1))/c^5/d^3+6*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1)
)/c^5/d^3+3*b^2*polylog(3,1-2/(c*x+1))/c^5/d^3
```

Rubi [A]
time = 0.60, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 6065, 6063, 641, 46, 213, 6203, 6745}

$$\frac{6b\,\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(cx+1)^2}+\frac{4\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5d^3(cx+1)^2}-\frac{43\left(a+b\tanh^{-1}(cx)\right)^2}{8c^5d^3}+\frac{6b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}-\frac{6\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3}+\frac{abx}{c^4d^3}-\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3}+\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^5d^3}+\frac{3b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{29b^2}{16c^5d^3(cx+1)}-\frac{b^2}{16c^5d^3(cx+1)^2}-\frac{29b^2\tanh^{-1}(cx)}{16c^5d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (a*b*x)/(c^4*d^3) - b^2/(16*c^5*d^3*(1 + c*x)^2) + (29*b^2)/(16*c^5*d^3*(1
 + c*x)) - (29*b^2*ArcTanh[c*x])/(16*c^5*d^3) + (b^2*x*ArcTanh[c*x])/(c^4*d^
3) - (b*(a + b*ArcTanh[c*x]))/(4*c^5*d^3*(1 + c*x)^2) + (15*b*(a + b*ArcTan
h[c*x]))/(4*c^5*d^3*(1 + c*x)) - (43*(a + b*ArcTanh[c*x])^2)/(8*c^5*d^3) -
(3*x*(a + b*ArcTanh[c*x])^2)/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^
3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh
[c*x])^2)/(c^5*d^3*(1 + c*x)) + (6*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])
/(c^5*d^3) - (6*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^5*d^3) + (b^2*L
og[1 - c^2*x^2])/(2*c^5*d^3) + (3*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^5*d^3
) + (6*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3) + (3*b
^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int \left(-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1+cx)^3}\right.$$

$$=\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{c^4d^3}-\frac{3\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^4d^3}-\frac{4\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^4d^3}$$

$$=-\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5d^3(1+cx)^2}+$$

$$=-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3}-\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3}$$

$$=\frac{abx}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)}-\frac{43\left(a+b\tanh^{-1}(cx\right.}{8c^5d^3}$$

$$=\frac{abx}{c^4d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)}-$$

$$=\frac{abx}{c^4d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)}-$$

$$=\frac{abx}{c^4d^3}-\frac{b^2}{16c^5d^3(1+cx)^2}+\frac{29b^2}{16c^5d^3(1+cx)}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh\right.}{4c^5d^3(1+}$$

$$=\frac{abx}{c^4d^3}-\frac{b^2}{16c^5d^3(1+cx)^2}+\frac{29b^2}{16c^5d^3(1+cx)}-\frac{29b^2\tanh^{-1}(cx)}{16c^5d^3}+\frac{b^2x\tanh^{-1}(}{c^4d^3}$$

Mathematica [A]
time = 1.26, size = 420, normalized size = 1.03

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (-48*a^2*c*x + 8*a^2*c^2*x^2 - (8*a^2)/(1 + c*x)^2 + (64*a^2)/(1 + c*x) + 9
6*a^2*Log[1 + c*x] + a*b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh
[c*x]] - 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sin
h[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*
c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2
*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])) + 16*b^2
*((-3 + 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + (56*Cosh[2*ArcTa
nh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[1 - c^2*x^2] + 192*PolyLog[3, -E^(
```

```
-2*ArcTanh[c*x])] - 56*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcT
anh[c*x]*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 96*Log[
1 + E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])
+ 8*ArcTanh[c*x]^2*(20 - 24*c*x + 4*c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cos
h[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x
]] + Sinh[4*ArcTanh[c*x]]))/64))/(16*c^5*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.52, size = 1428, normalized size = 3.50

method	result	size
derivativedivides	Expression too large to display	1428
default	Expression too large to display	1428

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(6*a^2/d^3*ln(c*x+1)+a*b/d^3*arctanh(c*x)*c^2*x^2+3*I*b^2/d^3*arctanh
(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/64*b^2/d^3/(c*x
+1)^2+7/8*b^2/d^3/(c*x+1)+b^2*arctanh(c*x)/d^3+a*b/d^3*c*x-a*b/d^3*arctanh(
c*x)/(c*x+1)^2+8*a*b/d^3*arctanh(c*x)/(c*x+1)+12*a*b/d^3*arctanh(c*x)*ln(c*
x+1)+6*a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-6*a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2
*c*x+1/2)-1/64*b^2/d^3/(c*x+1)^2*c^2*x^2+1/32*b^2/d^3/(c*x+1)^2*c*x-7/8*b^2
/d^3/(c*x+1)*c*x+1/2*b^2/d^3*arctanh(c*x)^2*c^2*x^2-3*b^2/d^3*arctanh(c*x)^
2*c*x+b^2/d^3*arctanh(c*x)*c*x+1/2*a^2/d^3*c^2*x^2-3*a^2/d^3*c*x-1/4*a*b/d^
3/(c*x+1)^2+15/4*a*b/d^3/(c*x+1)-3*a*b/d^3*ln(c*x+1)^2-6*a*b/d^3*dilog(1/2*
c*x+1/2)-43/8*a*b/d^3*ln(c*x+1)-5/8*a*b/d^3*ln(c*x-1)-6*b^2/d^3*arctanh(c*x
)^2*ln(2)-12*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-6*b^2/d^
3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+6*b^2/d^3*arctanh(c*x)*ln
(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+6*b^2/d^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^
2*x^2+1)^(1/2))-1/2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+4*b^2/d^3*arctanh(c*x)
^2/(c*x+1)+6*b^2/d^3*arctanh(c*x)^2*ln(c*x+1)+7/4*b^2/d^3*arctanh(c*x)/(c*x
+1)-1/16*b^2/d^3*arctanh(c*x)/(c*x+1)^2-3*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(
I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-3*I*b^2/d^3*a
rctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))^2-6*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2+6*b^2/d^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+
6*b^2/d^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^2/d^3*polylog(3,-(c*x+1
)^2/(-c^2*x^2+1))-b^2/d^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))-43/8*b^2/d^3*arctanh
(c*x)^2+4*b^2/d^3*arctanh(c*x)^3-1/2*a^2/d^3/(c*x+1)^2+4*a^2/d^3/(c*x+1)-6*
a*b/d^3*arctanh(c*x)*c*x-7/4*b^2/d^3*arctanh(c*x)/(c*x+1)*c*x-1/16*b^2/d^3*
arctanh(c*x)/(c*x+1)^2*c^2*x^2+1/8*b^2/d^3*arctanh(c*x)/(c*x+1)^2*c*x-3*I*b
```

```
^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3-3*I*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+a*b/
d^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + (c*x^2 - 6*x)/
(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3)) + 1/8*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 -
 11*b^2*c^2*x^2 + 2*b^2*c*x + 7*b^2 + 12*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*lo
g(c*x + 1))*log(-c*x + 1)^2/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - integra
te(-1/4*((b^2*c^5*x^5 - b^2*c^4*x^4)*log(c*x + 1)^2 + 4*(a*b*c^5*x^5 - a*b*
c^4*x^4)*log(c*x + 1) + (15*b^2*c^3*x^3 + 9*b^2*c^2*x^2 - (4*a*b*c^5 + b^2*
c^5)*x^5 + (4*a*b*c^4 + 3*b^2*c^4)*x^4 - 9*b^2*c*x - 7*b^2 - 2*(b^2*c^5*x^5
 - b^2*c^4*x^4 + 6*b^2*c^3*x^3 + 18*b^2*c^2*x^2 + 18*b^2*c*x + 6*b^2)*log(c
*x + 1))*log(-c*x + 1))/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^
3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^4}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^4\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^4\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**4*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
[Out] int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

3.112 $\int \frac{x^3(a+b\tanh^{-1}(cx))^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=337

$$\frac{b^2}{16c^4d^3(1+cx)^2}-\frac{21b^2}{16c^4d^3(1+cx)}+\frac{21b^2\tanh^{-1}(cx)}{16c^4d^3}+\frac{b(a+b\tanh^{-1}(cx))}{4c^4d^3(1+cx)^2}-\frac{11b(a+b\tanh^{-1}(cx))}{4c^4d^3(1+cx)}+\frac{19(a+$$

```
[Out] 1/16*b^2/c^4/d^3/(c*x+1)^2-21/16*b^2/c^4/d^3/(c*x+1)+21/16*b^2*arctanh(c*x)
/c^4/d^3+1/4*b*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)^2-11/4*b*(a+b*arctanh(c*x
))/c^4/d^3/(c*x+1)+19/8*(a+b*arctanh(c*x))^2/c^4/d^3+x*(a+b*arctanh(c*x))^2
/c^3/d^3+1/2*(a+b*arctanh(c*x))^2/c^4/d^3/(c*x+1)^2-3*(a+b*arctanh(c*x))^2/
c^4/d^3/(c*x+1)-2*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4/d^3+3*(a+b*arctan
h(c*x))^2*ln(2/(c*x+1))/c^4/d^3-b^2*polylog(2,1-2/(-c*x+1))/c^4/d^3-3*b*(a+
b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^4/d^3-3/2*b^2*polylog(3,1-2/(c*x+1
))/c^4/d^3
```

Rubi [A]

time = 0.49, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6065, 6063, 641, 46, 213, 6095, 6203, 6745}

$$-\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}-\frac{11b(a+b\tanh^{-1}(cx))}{4c^4d^3(cx+1)}+\frac{b(a+b\tanh^{-1}(cx))}{4c^4d^3(cx+1)^2}-\frac{3(a+b\tanh^{-1}(cx))^2}{c^4d^3(cx+1)}+\frac{(a+b\tanh^{-1}(cx))^2}{2c^4d^3(cx+1)^2}+\frac{19(a+b\tanh^{-1}(cx))^2}{8c^4d^3}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x(a+b\tanh^{-1}(cx))^2}{c^3d^3}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c^4d^3}-\frac{3b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{21b^2}{16c^4d^3(cx+1)}+\frac{b^2}{16c^4d^3(cx+1)^2}+\frac{21b^2\tanh^{-1}(cx)}{16c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] b^2/(16*c^4*d^3*(1 + c*x)^2) - (21*b^2)/(16*c^4*d^3*(1 + c*x)) + (21*b^2*Ar
cTanh[c*x])/(16*c^4*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)^2)
 - (11*b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)) + (19*(a + b*ArcTanh[c
*x])^2)/(8*c^4*d^3) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d^3) + (a + b*ArcTanh
[c*x])^2/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^3*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^4*d^3) + (3*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^3) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^4*d^3) - (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
4*d^3) - (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)^3} + \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)^2} - \frac{3}{}\right.$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^3d^3} - \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{c^3d^3} + \frac{3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^3d^3} - \frac{3}{}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1+cx)} + \frac{3(a}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^4d^3(1+cx)^2} - \frac{3(a+}{c}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)} + \frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3} + \frac{x(}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)} + \frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3} + \frac{x(}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)} + \frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3} + \frac{x(}{}$$

$$= \frac{b^2}{16c^4d^3(1+cx)^2} - \frac{21b^2}{16c^4d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2} - \frac{11b\left(a+b\tanh^{-}\right.}{4c^4d^3(1+c}$$

$$= \frac{b^2}{16c^4d^3(1+cx)^2} - \frac{21b^2}{16c^4d^3(1+cx)} + \frac{21b^2\tanh^{-1}(cx)}{16c^4d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2}$$

Mathematica [A]
time = 1.24, size = 418, normalized size = 1.24

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (64*a^2*c*x + (32*a^2)/(1 + c*x)^2 - (192*a^2)/(1 + c*x) - 192*a^2*Log[1 +
c*x] + 4*a*b*(-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 -
c^2*x^2] - 48*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] +
4*ArcTanh[c*x]*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24
*Log[1 + E^(-2*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*
x]]) - Sinh[4*ArcTanh[c*x]]) + b^2*(-64*ArcTanh[c*x]^2 + 64*c*x*ArcTanh[c*x
]^2 - 40*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*A
rcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*
```

```
Cosh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Cosh[4*ArcTanh[c*x]] - 128*ArcTanh[
c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 192*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTan
h[c*x])] - 64*(-1 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 96*P
olyLog[3, -E^(-2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]
*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - Sinh[4*Arc
Tanh[c*x]] - 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 8*ArcTanh[c*x]^2*Sinh[4*
ArcTanh[c*x]]))/(64*c^4*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.38, size = 5476, normalized size = 16.25

method	result	size
derivativedivides	Expression too large to display	5476
default	Expression too large to display	5476

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - 2*x/(c^3*d^3)
 + 6*log(c*x + 1)/(c^4*d^3)) + 1/8*(2*b^2*c^3*x^3 + 4*b^2*c^2*x^2 - 4*b^2*c
*x - 5*b^2 - 6*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^
2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - integrate(-1/4*((b^2*c^4*x^4 - b^
2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (2
*(2*a*b*c^4 + b^2*c^4)*x^4 - 9*b^2*c*x - 2*(2*a*b*c^3 - 3*b^2*c^3)*x^3 - 5*
b^2 + 2*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 - 9*b^2*c^2*x^2 - 9*b^2*c*x - 3*b^2)*l
og(c*x + 1))*log(-c*x + 1))/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^
3*d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**3*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

3.113 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=265

$$-\frac{b^2}{16c^3d^3(1+cx)^2}+\frac{13b^2}{16c^3d^3(1+cx)}-\frac{13b^2\tanh^{-1}(cx)}{16c^3d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)}-\frac{7(a+$$

```
[Out] -1/16*b^2/c^3/d^3/(c*x+1)^2+13/16*b^2/c^3/d^3/(c*x+1)-13/16*b^2*arctanh(c*x
)/c^3/d^3-1/4*b*(a+b*arctanh(c*x))/c^3/d^3/(c*x+1)^2+7/4*b*(a+b*arctanh(c*x
))/c^3/d^3/(c*x+1)-7/8*(a+b*arctanh(c*x))^2/c^3/d^3-1/2*(a+b*arctanh(c*x))^
2/c^3/d^3/(c*x+1)^2+2*(a+b*arctanh(c*x))^2/c^3/d^3/(c*x+1)-(a+b*arctanh(c*x
))^2*ln(2/(c*x+1))/c^3/d^3+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^3/
d^3+1/2*b^2*polylog(3,1-2/(c*x+1))/c^3/d^3
```

Rubi [A]
time = 0.41, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {6087, 6065, 6063, 641, 46, 213, 6095, 6055, 6203, 6745}

$$\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(cx+1)^2}-\frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3}+\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{13b^2}{16c^3d^3(cx+1)}-\frac{b^2}{16c^3d^3(cx+1)^2}-\frac{13b^2\tanh^{-1}(cx)}{16c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] -1/16*b^2/(c^3*d^3*(1 + c*x)^2) + (13*b^2)/(16*c^3*d^3*(1 + c*x)) - (13*b^2
*ArcTanh[c*x])/(16*c^3*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)
^2) + (7*b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)) - (7*(a + b*ArcTanh[
c*x])^2)/(8*c^3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^3*d^3*(1 + c*x)^2) + (2*
(a + b*ArcTanh[c*x])^2)/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2
/(1 + c*x)])/(c^3*d^3) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)
])/(c^3*d^3) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
```

```
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)^3} - \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)}\right) \cdots$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{c^2d^3} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^2d^3} - \frac{2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^2d^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(1+cx)^2} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{\cdots}{1\cdots}\right.}{c^3d^3} \cdots$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(1+cx)^2} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{\cdots}{1\cdots}\right.}{c^3d^3} \cdots$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)} - \frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3} - \cdots$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)} - \frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3} - \cdots$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)} - \frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3} - \cdots$$

$$= -\frac{b^2}{16c^3d^3(1+cx)^2} + \frac{13b^2}{16c^3d^3(1+cx)} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2} + \frac{7b\left(a+b\tanh\cdots\right.}{4c^3d^3(1+\cdots} \cdots$$

$$= -\frac{b^2}{16c^3d^3(1+cx)^2} + \frac{13b^2}{16c^3d^3(1+cx)} - \frac{13b^2\tanh^{-1}(cx)}{16c^3d^3} - \frac{b\left(a+b\tanh^{-1}(c\cdots\right.}{4c^3d^3(1+cx)^2} \cdots$$

Mathematica [A]
time = 0.92, size = 310, normalized size = 1.17

$$\frac{-\frac{8a^2}{(1+cx)^2} + \frac{32a^2}{1+cx} + 16a^2\log(1+cx) + 16b^2\left(\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right) + \frac{1}{64}\left(-\cosh\left(2\tanh^{-1}(cx)\right) + \sinh\left(2\tanh^{-1}(cx)\right)\right)\left(-24 + \cosh\left(2\tanh^{-1}(cx)\right) + 4\tanh^{-1}(cx)\left(-12 + \cosh\left(2\tanh^{-1}(cx)\right) - \sinh\left(2\tanh^{-1}(cx)\right)\right) - \sinh\left(2\tanh^{-1}(cx)\right) + 8\tanh^{-1}(cx)^2\left(-6 + \cosh\left(2\tanh^{-1}(cx)\right)\left(1 + 8\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right) + \left(-1 + 8\log\left(1+e^{-2\tanh^{-1}(cx)}\right)\right)\sinh\left(2\tanh^{-1}(cx)\right)\right)\right)\right) + ab\left(12\cosh\left(2\tanh^{-1}(cx)\right) - \cosh\left(4\tanh^{-1}(cx)\right) + 16\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) - 12\sinh\left(2\tanh^{-1}(cx)\right) + \sinh\left(4\tanh^{-1}(cx)\right) + 4\tanh^{-1}(cx)\left(6\cosh\left(2\tanh^{-1}(cx)\right) - \cosh\left(4\tanh^{-1}(cx)\right) - 8\log\left(1+e^{-2\tanh^{-1}(cx)}\right) - 6\sinh\left(2\tanh^{-1}(cx)\right) + \sinh\left(4\tanh^{-1}(cx)\right)\right)\right)}{16c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] ((-8*a^2)/(1 + c*x)^2 + (32*a^2)/(1 + c*x) + 16*a^2*Log[1 + c*x] + 16*b^2*(
ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c
*x])]/2 + ((-Cosh[2*ArcTanh[c*x]] + Sinh[2*ArcTanh[c*x]])*(-24 + Cosh[2*Arc
Tanh[c*x]] + 4*ArcTanh[c*x]*(-12 + Cosh[2*ArcTanh[c*x]] - Sinh[2*ArcTanh[c*
x]]) - Sinh[2*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*(-6 + Cosh[2*ArcTanh[c*x]]*(
1 + 8*Log[1 + E^(-2*ArcTanh[c*x])]) + (-1 + 8*Log[1 + E^(-2*ArcTanh[c*x])])
*Sinh[2*ArcTanh[c*x]])))/64) + a*b*(12*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTan
h[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + S
inh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTa
nh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcTanh[c*x]] + Sinh[4
*ArcTanh[c*x]])))/(16*c^3*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.69, size = 1135, normalized size = 4.28

method	result	size
derivativedivides	Expression too large to display	1135
default	Expression too large to display	1135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a^2/d^3*ln(c*x+1)-1/2*I*b^2/d^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arct
anh(c*x)^2*Pi-1/2*I*b^2/d^3*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*Pi-I*b^2/d^3*
csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(
c*x)^2*Pi+1/2*I*b^2/d^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*Pi-1/2*I*b^2/d^3*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*arctanh(c*x)^
2*Pi-1/64*b^2/d^3/(c*x+1)^2+3/8*b^2/d^3/(c*x+1)+1/2*I*b^2/d^3*csgn(I/(1+(c*
x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*Pi-1/2*I*b^2/d^3*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*Pi-a*b/d^3
*arctanh(c*x)/(c*x+1)^2+4*a*b/d^3*arctanh(c*x)/(c*x+1)+2*a*b/d^3*arctanh(c*
x)*ln(c*x+1)+a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-a*b/d^3*ln(-1/2*c*x+1/2)*ln
(1/2*c*x+1/2)-1/64*b^2/d^3/(c*x+1)^2*c^2*x^2+1/32*b^2/d^3/(c*x+1)^2*c*x-3/8
*b^2/d^3/(c*x+1)*c*x-1/4*a*b/d^3/(c*x+1)^2+7/4*a*b/d^3/(c*x+1)-1/2*a*b/d^3*
ln(c*x+1)^2-a*b/d^3*dilog(1/2*c*x+1/2)-7/8*a*b/d^3*ln(c*x+1)+7/8*a*b/d^3*ln
(c*x-1)-b^2/d^3*arctanh(c*x)^2*ln(2)-2*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-
c^2*x^2+1)^(1/2))-b^2/d^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1
/2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+2*b^2/d^3*arctanh(c*x)^2/(c*x+1)+b^2/d^
3*arctanh(c*x)^2*ln(c*x+1)+3/4*b^2/d^3*arctanh(c*x)/(c*x+1)-1/16*b^2/d^3*ar
ctanh(c*x)/(c*x+1)^2+1/2*b^2/d^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-7/8*b^2
```

```
/d^3*arctanh(c*x)^2+2/3*b^2/d^3*arctanh(c*x)^3-1/2*a^2/d^3/(c*x+1)^2+2*a^2/
d^3/(c*x+1)-3/4*b^2/d^3*arctanh(c*x)/(c*x+1)*c*x-1/16*b^2/d^3*arctanh(c*x)/
(c*x+1)^2*c^2*x^2+1/8*b^2/d^3*arctanh(c*x)/(c*x+1)^2*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((4*c*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)
/(c^3*d^3)) + 1/8*(4*b^2*c*x + 3*b^2 + 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*lo
g(c*x + 1))*log(-c*x + 1)^2/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - integra
te(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*
c^2*x^2)*log(c*x + 1) - (4*a*b*c^3*x^3 + 7*b^2*c*x - 4*(a*b*c^2 - b^2*c^2)*
x^2 + 3*b^2 + 2*(2*b^2*c^3*x^3 + 2*b^2*c^2*x^2 + 3*b^2*c*x + b^2)*log(c*x +
 1))*log(-c*x + 1))/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**2*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

3.114 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=157

$$\frac{b^2}{16c^2d^3(1+cx)^2}-\frac{5b^2}{16c^2d^3(1+cx)}+\frac{5b^2\tanh^{-1}(cx)}{16c^2d^3}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)}-\frac{\left(a+b\tan\right.}{8c}$$

```
[Out] 1/16*b^2/c^2/d^3/(c*x+1)^2-5/16*b^2/c^2/d^3/(c*x+1)+5/16*b^2*arctanh(c*x)/c
^2/d^3+1/4*b*(a+b*arctanh(c*x))/c^2/d^3/(c*x+1)^2-3/4*b*(a+b*arctanh(c*x))/
c^2/d^3/(c*x+1)-1/8*(a+b*arctanh(c*x))^2/c^2/d^3+1/2*x^2*(a+b*arctanh(c*x))
^2/d^3/(c*x+1)^2
```

Rubi [A]

time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {37, 6085, 6063, 641, 46, 213, 6095}

$$-\frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(cx+1)^2}-\frac{5b^2}{16c^2d^3(cx+1)}+\frac{b^2}{16c^2d^3(cx+1)^2}+\frac{5b^2\tanh^{-1}(cx)}{16c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] b^2/(16*c^2*d^3*(1 + c*x)^2) - (5*b^2)/(16*c^2*d^3*(1 + c*x)) + (5*b^2*ArcT
anh[c*x])/(16*c^2*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)^2) -
 (3*b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*d^3*(1 + c*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx &= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - (2bc)\int \left(\frac{a+b\tanh^{-1}(cx)}{4c^2d^3(1+cx)^3} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{8c^2d^3(1+cx)^2} \right. \\
&= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{b\int \frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{4cd^3} - \frac{b\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{2cd^3} + \frac{(3b)\int}{} \\
&= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2(a}{} \\
&= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2(a}{} \\
&= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2(a}{} \\
&= \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}\right.}{4c^2d^3(1+cx} \\
&= \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{5b^2\tanh^{-1}(cx)}{16c^2d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 150, normalized size = 0.96

$$
\frac{2(8a^2+4ab+b^2)-2(16a^2+12ab+5b^2)(1+cx)-8b(b(2+3cx)+a(4+8cx))\tanh^{-1}(cx)+4b^2(-1-2cx+3c^2x^2)\tanh^{-1}(cx)^2-b(12a+5b)(1+cx)^2\log(1-cx)+b(12a+5b)(1+cx)^2\log(1+cx)}{32c^2d^3(1+cx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (2*(8*a^2 + 4*a*b + b^2) - 2*(16*a^2 + 12*a*b + 5*b^2)*(1 + c*x) - 8*b*(b*(
2 + 3*c*x) + a*(4 + 8*c*x))*ArcTanh[c*x] + 4*b^2*(-1 - 2*c*x + 3*c^2*x^2)*A
rcTanh[c*x]^2 - b*(12*a + 5*b)*(1 + c*x)^2*Log[1 - c*x] + b*(12*a + 5*b)*(1
 + c*x)^2*Log[1 + c*x])/(32*c^2*d^3*(1 + c*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(143) = 286$.
time = 0.36, size = 391, normalized size = 2.49

method	result
derivativedivides	$\frac{\frac{a^2\left(-\frac{1}{cx+1}+\frac{1}{2(cx+1)^2}\right)}{d^3}-\frac{b^2\,\mathrm{arctanh}(cx)^2}{d^3(cx+1)}+\frac{b^2\,\mathrm{arctanh}(cx)^2}{2d^3(cx+1)^2}+\frac{b^2\,\mathrm{arctanh}(cx)}{4d^3(cx+1)^2}-\frac{3b^2\,\mathrm{arctanh}(cx)}{4d^3(cx+1)}+\frac{3b^2\,\mathrm{arctanh}(cx)\ln(cx+1)}{8d^3}-3b^2}{}$
default	$\frac{\frac{a^2\left(-\frac{1}{cx+1}+\frac{1}{2(cx+1)^2}\right)}{d^3}-\frac{b^2\,\mathrm{arctanh}(cx)^2}{d^3(cx+1)}+\frac{b^2\,\mathrm{arctanh}(cx)^2}{2d^3(cx+1)^2}+\frac{b^2\,\mathrm{arctanh}(cx)}{4d^3(cx+1)^2}-\frac{3b^2\,\mathrm{arctanh}(cx)}{4d^3(cx+1)}+\frac{3b^2\,\mathrm{arctanh}(cx)\ln(cx+1)}{8d^3}-3b^2}{}$

risch	$\frac{b^2(3c^2x^2-2cx-1)\ln(cx+1)^2}{32c^2d^3(cx+1)^2} - \frac{b(3bx^2\ln(-cx+1)c^2-2bcx\ln(-cx+1)+16cxa+6bcx-b\ln(-cx+1)+8a+4b)\ln(cx+1)}{16c^2d^3(cx+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a^2/d^3*(-1/(c*x+1)+1/2/(c*x+1)^2)-b^2/d^3*arctanh(c*x)^2/(c*x+1)+1/
2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+1/4*b^2/d^3*arctanh(c*x)/(c*x+1)^2-3/4*b
^2/d^3*arctanh(c*x)/(c*x+1)+3/8*b^2/d^3*arctanh(c*x)*ln(c*x+1)-3/8*b^2/d^3*
arctanh(c*x)*ln(c*x-1)+3/16*b^2/d^3*ln(c*x-1)*ln(1/2*c*x+1/2)-3/32*b^2/d^3*
ln(c*x-1)^2-3/32*b^2/d^3*ln(c*x+1)^2-3/16*b^2/d^3*ln(-1/2*c*x+1/2)*ln(1/2*c
*x+1/2)+3/16*b^2/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/16*b^2/d^3/(c*x+1)^2-5/16
*b^2/d^3/(c*x+1)+5/32*b^2/d^3*ln(c*x+1)-5/32*b^2/d^3*ln(c*x-1)-2*a*b/d^3*ar
ctanh(c*x)/(c*x+1)+a*b/d^3*arctanh(c*x)/(c*x+1)^2+1/4*a*b/d^3/(c*x+1)^2-3/4
*a*b/d^3/(c*x+1)+3/8*a*b/d^3*ln(c*x+1)-3/8*a*b/d^3*ln(c*x-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(143) = 286$.
time = 0.28, size = 429, normalized size = 2.73

$$-\frac{(2\,cx+1)b^2\,\mathrm{artanh}\,(cx)^2}{2\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)} - \frac{1}{8}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3} - \frac{3\,\log\,(cx+1)}{c^3d^3} + \frac{3\,\log\,(cx-1)}{c^3d^3}\right) + \frac{8\,(2\,cx+1)\,\mathrm{artanh}\,(cx)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}\right)ab - \frac{1}{32}\left(4\,c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3} - \frac{3\,\log\,(cx+1)}{c^3d^3} + \frac{3\,\log\,(cx-1)}{c^3d^3}\right)\mathrm{artanh}\,(cx) + \frac{\left(3\,(c^2x^2+2\,cx+1)\log\,(cx+1)^2+3\,(c^2x^2+2\,cx+1)\log\,(cx-1)^2+10\,cx-(5\,c^2x^2+10\,cx+6\,(c^2x^2+2\,cx+1)\log\,(cx-1)+5)\log\,(cx+1)+5\,(c^2x^2+2\,cx+1)\log\,(cx-1)+8\right)c^2}{c^6d^3x^2+2\,c^5d^3x+c^4d^3}\right)b^2 - \frac{(2\,cx+1)a^2}{2\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x + 1)*b^2*arctanh(c*x)^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) -
 1/8*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x +
1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c^4*
d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*a*b - 1/32*(4*c*(2*(3*c*x + 2)/(c^5*d^3*x
^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^
3*d^3))*arctanh(c*x) + (3*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + 3*(c^2*x^2
 + 2*c*x + 1)*log(c*x - 1)^2 + 10*c*x - (5*c^2*x^2 + 10*c*x + 6*(c^2*x^2 +
2*c*x + 1)*log(c*x - 1) + 5)*log(c*x + 1) + 5*(c^2*x^2 + 2*c*x + 1)*log(c*x
 - 1) + 8)*c^2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3))*b^2 - 1/2*(2*c*x + 1)
*a^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Fricas [**A**]
time = 0.35, size = 164, normalized size = 1.04

$$-\frac{2\,(16\,a^2+12\,ab+5\,b^2)cx-(3\,b^2c^2x^2-2\,b^2cx-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-((12\,ab+5\,b^2)c^2x^2-2\,(4\,ab+b^2)cx-4\,ab-3\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)}{32\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(16*a^2 + 12*a*b + 5*b^2)*c*x - (3*b^2*c^2*x^2 - 2*b^2*c*x - b^2)*
log(-(c*x + 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((12*a*b + 5*b^2)*c
```

```
^2*x^2 - 2*(4*a*b + b^2)*c*x - 4*a*b - 3*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c
^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x\,\mathrm{atanh}^2\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx\,\mathrm{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*
x*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*
x*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [A]
time = 0.41, size = 226, normalized size = 1.44

$$\frac{1}{64}\,c\left(\frac{2\left(\frac{2\,(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^3d^3}+\frac{2\left(\frac{8\,(cx+1)ab}{cx-1}+4\,ab+\frac{4\,(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^3d^3}+\frac{\left(\frac{16\,(cx+1)a^2}{cx-1}+8\,a^2+\frac{16\,(cx+1)ab}{cx-1}+4\,ab+\frac{8\,(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2}{(cx+1)^2c^3d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/64*c*(2*(2*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
 - 1))^2/((c*x + 1)^2*c^3*d^3) + 2*(8*(c*x + 1)*a*b/(c*x - 1) + 4*a*b + 4*(
c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))/((c*x +
 1)^2*c^3*d^3) + (16*(c*x + 1)*a^2/(c*x - 1) + 8*a^2 + 16*(c*x + 1)*a*b/(c*
x - 1) + 4*a*b + 8*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2/((c*x + 1)^2*
c^3*d^3))
```

Mupad [B]
time = 2.69, size = 405, normalized size = 2.58

$$-\frac{16\,a\,b+17\,b^2\,\ln\left(c\,x+1\right)-17\,b^2\,\ln\left(1-c\,x\right)+b^2\ln\left(c\,x+1\right)^2+b^2\ln\left(1-c\,x\right)^2-28\,b^2\,\mathrm{atanh}(c\,x)+16\,a^2+8\,b^2+16\,a\,b\,\ln\left(c\,x+1\right)-16\,a\,b\,\ln\left(1-c\,x\right)-2\,b^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)-24\,a\,b\,\mathrm{atanh}(c\,x)+32\,a^2\,c\,x+10\,b^2\,c\,x+30\,b^2\,c\,x\,\ln\left(c\,x+1\right)-30\,b^2\,c\,x\,\ln\left(1-c\,x\right)-3\,b^2\,c^2\,x^2\ln\left(c\,x+1\right)^2-3\,b^2\,c^2\,x^2\ln\left(1-c\,x\right)^2-28\,b^2\,c^2\,x^2\,\mathrm{atanh}(c\,x)+2\,b^2\,c\,x\ln\left(c\,x+1\right)^2+2\,b^2\,c\,x\ln\left(1-c\,x\right)^2-56\,b^2\,c\,x\,\mathrm{atanh}(c\,x)+9\,b^2\,c^2\,x^2\,\ln\left(c\,x+1\right)-9\,b^2\,c^2\,x^2\,\ln\left(1-c\,x\right)+24\,a\,b\,c\,x+32\,a\,b\,c\,x\,\ln\left(c\,x+1\right)-32\,a\,b\,c\,x\,\ln\left(1-c\,x\right)-4\,b^2\,c\,x\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)-24\,a\,b\,c^2\,x^2\,\mathrm{atanh}(c\,x)-48\,a\,b\,c\,x\,\mathrm{atanh}(c\,x)+6\,b^2\,c^2\,x^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)}{32\,c^2\,d^3\,{\left(c\,x+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

```
[Out] -(16*a*b + 17*b^2*log(c*x + 1) - 17*b^2*log(1 - c*x) + b^2*log(c*x + 1)^2 +
 b^2*log(1 - c*x)^2 - 28*b^2*atanh(c*x) + 16*a^2 + 8*b^2 + 16*a*b*log(c*x +
 1) - 16*a*b*log(1 - c*x) - 2*b^2*log(c*x + 1)*log(1 - c*x) - 24*a*b*atanh(
c*x) + 32*a^2*c*x + 10*b^2*c*x + 30*b^2*c*x*log(c*x + 1) - 30*b^2*c*x*log(1
 - c*x) - 3*b^2*c^2*x^2*log(c*x + 1)^2 - 3*b^2*c^2*x^2*log(1 - c*x)^2 - 28*
b^2*c^2*x^2*atanh(c*x) + 2*b^2*c*x*log(c*x + 1)^2 + 2*b^2*c*x*log(1 - c*x)^
```

```
2 - 56*b^2*c*x*atanh(c*x) + 9*b^2*c^2*x^2*log(c*x + 1) - 9*b^2*c^2*x^2*log(
1 - c*x) + 24*a*b*c*x + 32*a*b*c*x*log(c*x + 1) - 32*a*b*c*x*log(1 - c*x) -
 4*b^2*c*x*log(c*x + 1)*log(1 - c*x) - 24*a*b*c^2*x^2*atanh(c*x) - 48*a*b*c
*x*atanh(c*x) + 6*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x))/(32*c^2*d^3*(c*x +
 1)^2)
```

3.115 $\int \frac{(a+b\tanh^{-1}(cx))^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}+\frac{3b^2\tanh^{-1}(cx)}{16cd^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tan\right.}{8c}$$

```
[Out] -1/16*b^2/c/d^3/(c*x+1)^2-3/16*b^2/c/d^3/(c*x+1)+3/16*b^2*arctanh(c*x)/c/d^
3-1/4*b*(a+b*arctanh(c*x))/c/d^3/(c*x+1)^2-1/4*b*(a+b*arctanh(c*x))/c/d^3/(
c*x+1)+1/8*(a+b*arctanh(c*x))^2/c/d^3-1/2*(a+b*arctanh(c*x))^2/c/d^3/(c*x+1
)^2
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6065, 6063, 641, 46, 213, 6095}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{3b^2}{16cd^3(cx+1)}-\frac{b^2}{16cd^3(cx+1)^2}+\frac{3b^2\tanh^{-1}(cx)}{16cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

```
[Out] -1/16*b^2/(c*d^3*(1 + c*x)^2) - (3*b^2)/(16*c*d^3*(1 + c*x)) + (3*b^2*ArcTa
nh[c*x])/(16*c*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)^2) - (b*(
a + b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(8*c*d^3)
 - (a + b*ArcTanh[c*x])^2/(2*c*d^3*(1 + c*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}+\frac{b\int\left(\frac{a+b\tanh^{-1}(cx)}{2d^2(1+cx)^3}+\frac{a+b\tanh^{-1}(cx)}{4d^2(1+cx)^2}-\frac{a+b\tanh^{-1}(cx)}{4d^2(-1+c^2x^2)}\right)dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}+\frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{4d^3}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{4d^3}+\frac{b\int\frac{a+b\tanh}{(1+c}}{2d}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{(a+b\,\mathrm{t}}{2cd^3}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{(a+b\,\mathrm{t}}{2cd^3}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{(a+b\,\mathrm{t}}{2cd^3}$$

$$= -\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx\right)}{4cd^3(1+cx)}$$

$$= -\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}+\frac{3b^2\tanh^{-1}(cx)}{16cd^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-$$

Mathematica [A]
time = 0.08, size = 183, normalized size = 1.17

$$\frac{-8a^2-4ab-b^2}{16cd^3(1+cx)^2}-\frac{b(4a+3b)}{16cd^3(1+cx)}-\frac{b(4a+2b+bcx)\tanh^{-1}(cx)}{4cd^3(1+cx)^2}+\frac{b^2(-3+2cx+c^2x^2)\tanh^{-1}(cx)^2}{8cd^3(1+cx)^2}+\frac{(-4ab-3b^2)\log(1-cx)}{32cd^3}+\frac{(4ab+3b^2)\log(1+cx)}{32cd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

```
[Out] (-8*a^2 - 4*a*b - b^2)/(16*c*d^3*(1 + c*x)^2) - (b*(4*a + 3*b))/(16*c*d^3*(
1 + c*x)) - (b*(4*a + 2*b + b*c*x)*ArcTanh[c*x])/(4*c*d^3*(1 + c*x)^2) + (b
^2*(-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^2)/(8*c*d^3*(1 + c*x)^2) + ((-4*a*b
- 3*b^2)*Log[1 - c*x])/(32*c*d^3) + ((4*a*b + 3*b^2)*Log[1 + c*x])/(32*c*d^
3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(143) = 286.
time = 0.35, size = 342, normalized size = 2.18

method	result
derivativedivides	$\frac{-\frac{a^2}{2d^3(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)^2}{2d^3(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{8d^3}-\frac{b^2\operatorname{arctanh}(cx)}{4d^3(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)}{4d^3(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{8d^3}+\frac{b^2\ln(cx-}{1}}{}$
default	$\frac{-\frac{a^2}{2d^3(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)^2}{2d^3(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{8d^3}-\frac{b^2\operatorname{arctanh}(cx)}{4d^3(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)}{4d^3(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{8d^3}+\frac{b^2\ln(cx-}{1}}{}$
risch	$\frac{b^2(c^2x^2+2cx-3)\ln(cx+1)^2}{32d^3(cx+1)^2c}-\frac{b(bx^2\ln(-cx+1)c^2+2bcx\ln(-cx+1)+2bcx-3b\ln(-cx+1)+8a+4b)\ln(cx+1)}{16d^3(cx+1)^2c}-\frac{-b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a^2/d^3/(c*x+1)^2-1/2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2-1/8*b^2/d^
3*arctanh(c*x)*ln(c*x-1)-1/4*b^2/d^3*arctanh(c*x)/(c*x+1)^2-1/4*b^2/d^3*arc
tanh(c*x)/(c*x+1)+1/8*b^2/d^3*arctanh(c*x)*ln(c*x+1)+1/16*b^2/d^3*ln(c*x-1)
*ln(1/2*c*x+1/2)-1/32*b^2/d^3*ln(c*x-1)^2-1/16*b^2/d^3*ln(-1/2*c*x+1/2)*ln(
1/2*c*x+1/2)+1/16*b^2/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/32*b^2/d^3*ln(c*x+1)
^2-3/32*b^2/d^3*ln(c*x-1)-1/16*b^2/d^3/(c*x+1)^2-3/16*b^2/d^3/(c*x+1)+3/32*
b^2/d^3*ln(c*x+1)-a*b/d^3*arctanh(c*x)/(c*x+1)^2-1/8*a*b/d^3*ln(c*x-1)-1/4*
a*b/d^3/(c*x+1)^2-1/4*a*b/d^3/(c*x+1)+1/8*a*b/d^3*ln(c*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(143) = 286.
time = 0.29, size = 399, normalized size = 2.54

$$-\frac{1}{8}\left(c\left(\frac{2(cx+2)}{c^4d^3x^2+2c^3d^3x+c^2d^3}-\frac{\log(cx+1)}{c^2d^3}+\frac{\log(cx-1)}{c^2d^3}\right)+\frac{8\operatorname{artanh}(cx)}{c^3d^3x^2+2c^2d^3x+cd^3}\right)ab-\frac{1}{32}\left(4c\left(\frac{2(cx+2)}{c^4d^3x^2+2c^3d^3x+c^2d^3}-\frac{\log(cx+1)}{c^2d^3}+\frac{\log(cx-1)}{c^2d^3}\right)\operatorname{artanh}(cx)+\frac{\left((c^2x^2+2cx+1)\log(cx+1)^2+(c^2x^2+2cx+1)\log(cx-1)^2+6cx-(3c^2x^2+6cx+2(c^2x^2+2cx+1)\log(cx-1)+3)\log(cx+1)+3(c^2x^2+2cx+1)\log(cx-1)+8\right)c^2}{c^5d^3x^2+2c^4d^3x+c^3d^3}\right)b^2-\frac{b^2\operatorname{artanh}(cx)^2}{2(c^3d^3x^2+2c^2d^3x+cd^3)}-\frac{a^2}{2(c^3d^3x^2+2c^2d^3x+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/(
c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*d^
3*x + c*d^3))*a*b - 1/32*(4*c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2
*d^3) - log(c*x + 1)/(c^2*d^3) + log(c*x - 1)/(c^2*d^3))*arctanh(c*x) + ((c
^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 +
 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log
(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*d^3*x^2 + 2*
c^4*d^3*x + c^3*d^3))*b^2 - 1/2*b^2*arctanh(c*x)^2/(c^3*d^3*x^2 + 2*c^2*d^3
*x + c*d^3) - 1/2*a^2/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Fricas [A]
time = 0.39, size = 156, normalized size = 0.99

$$-\frac{2\,(4\,ab+3\,b^2)cx-(b^2c^2x^2+2\,b^2cx-3\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-((4\,ab+3\,b^2)c^2x^2+2\,(4\,ab+b^2)cx-12\,ab-5\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)}{32\,(c^3d^3x^2+2\,c^2d^3x+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(4*a*b + 3*b^2)*c*x - (b^2*c^2*x^2 + 2*b^2*c*x - 3*b^2)*log(-(c*x
+ 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((4*a*b + 3*b^2)*c^2*x^2 + 2*
(4*a*b + b^2)*c*x - 12*a*b - 5*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^3*d^3*x^2
 + 2*c^2*d^3*x + c*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int \frac{b^2\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int \frac{2ab\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*at
anh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*atan
h(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [A]
time = 0.40, size = 232, normalized size = 1.48

$$\frac{1}{64}\,c\left(\frac{2\left(\frac{2\,(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^2d^3}+\frac{2\left(\frac{8\,(cx+1)ab}{cx-1}-4\,ab+\frac{4\,(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^2d^3}+\frac{\left(\frac{16\,(cx+1)a^2}{cx-1}-8\,a^2+\frac{16\,(cx+1)ab}{cx-1}-4\,ab+\frac{8\,(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2}{(cx+1)^2c^2d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/64*c*(2*(2*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
 - 1))^2/((c*x + 1)^2*c^2*d^3) + 2*(8*(c*x + 1)*a*b/(c*x - 1) - 4*a*b + 4*(
c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))/((c*x +
 1)^2*c^2*d^3) + (16*(c*x + 1)*a^2/(c*x - 1) - 8*a^2 + 16*(c*x + 1)*a*b/(c*
x - 1) - 4*a*b + 8*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2/((c*x + 1)^2*
c^2*d^3))
```

Mupad [B]
time = 2.16, size = 373, normalized size = 2.38

$$\frac{11\,b^2\,\ln\left(1-c\,x\right)-11\,b^2\,\ln\left(c\,x+1\right)-16\,a\,b-3\,b^2\ln\left(c\,x+1\right)^2-3\,b^2\ln\left(1-c\,x\right)^2+12\,b^2\,\mathrm{atanh}(c\,x)-16\,a^2-8\,b^2-16\,a\,b\,\ln\left(c\,x+1\right)+16\,a\,b\,\ln\left(1-c\,x\right)+6\,b^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)+8\,a\,b\,\mathrm{atanh}(c\,x)-6\,b^2\,c\,x-10\,b^2\,c\,x\,\ln\left(c\,x+1\right)+10\,b^2\,c\,x\,\ln\left(1-c\,x\right)+b^2\,c^2\,x^2\ln\left(c\,x+1\right)^2+b^2\,c^2\,x^2\ln\left(1-c\,x\right)^2+12\,b^2\,c^2\,x^2\,\mathrm{atanh}(c\,x)+2\,b^2\,c\,x\ln\left(c\,x+1\right)^2+2\,b^2\,c\,x\ln\left(1-c\,x\right)^2+24\,b^2\,c\,x\,\mathrm{atanh}(c\,x)-3\,b^2\,c^2\,x^2\,\ln\left(c\,x+1\right)+3\,b^2\,c^2\,x^2\,\ln\left(1-c\,x\right)-8\,a\,b\,c\,x-4\,b^2\,c\,x\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)+8\,a\,b\,c^2\,x^2\,\mathrm{atanh}(c\,x)+16\,a\,b\,c\,x\,\mathrm{atanh}(c\,x)-2\,b^2\,c^2\,x^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)}{32\,c\,d^3\,{\left(c\,x+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + c*d*x)^3,x)
```

```
[Out] (11*b^2*log(1 - c*x) - 11*b^2*log(c*x + 1) - 16*a*b - 3*b^2*log(c*x + 1)^2
- 3*b^2*log(1 - c*x)^2 + 12*b^2*atanh(c*x) - 16*a^2 - 8*b^2 - 16*a*b*log(c*
x + 1) + 16*a*b*log(1 - c*x) + 6*b^2*log(c*x + 1)*log(1 - c*x) + 8*a*b*atan
h(c*x) - 6*b^2*c*x - 10*b^2*c*x*log(c*x + 1) + 10*b^2*c*x*log(1 - c*x) + b^
2*c^2*x^2*log(c*x + 1)^2 + b^2*c^2*x^2*log(1 - c*x)^2 + 12*b^2*c^2*x^2*atan
h(c*x) + 2*b^2*c*x*log(c*x + 1)^2 + 2*b^2*c*x*log(1 - c*x)^2 + 24*b^2*c*x*a
tanh(c*x) - 3*b^2*c^2*x^2*log(c*x + 1) + 3*b^2*c^2*x^2*log(1 - c*x) - 8*a*b
*c*x - 4*b^2*c*x*log(c*x + 1)*log(1 - c*x) + 8*a*b*c^2*x^2*atanh(c*x) + 16*
a*b*c*x*atanh(c*x) - 2*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x))/(32*c*d^3*(c*
x + 1)^2)
```

3.116 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x(d+cdx)^3}\,dx$

Optimal. Leaf size=362

$$\frac{b^2}{16d^3(1+cx)^2}+\frac{11b^2}{16d^3(1+cx)}-\frac{11b^2\tanh^{-1}(cx)}{16d^3}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2}+\frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)}-\frac{5\left(a+b\tanh^{-1}\right.}{8d^3}$$

```
[Out] 1/16*b^2/d^3/(c*x+1)^2+11/16*b^2/d^3/(c*x+1)-11/16*b^2*arctanh(c*x)/d^3+1/4
*b*(a+b*arctanh(c*x))/d^3/(c*x+1)^2+5/4*b*(a+b*arctanh(c*x))/d^3/(c*x+1)-5/
8*(a+b*arctanh(c*x))^2/d^3+1/2*(a+b*arctanh(c*x))^2/d^3/(c*x+1)^2+(a+b*arct
anh(c*x))^2/d^3/(c*x+1)-2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^3+(
a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^3-b*(a+b*arctanh(c*x))*polylog(2,1-2/(-
c*x+1))/d^3+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^3-b*(a+b*arctan
h(c*x))*polylog(2,1-2/(c*x+1))/d^3+1/2*b^2*polylog(3,1-2/(-c*x+1))/d^3-1/2*
b^2*polylog(3,-1+2/(-c*x+1))/d^3-1/2*b^2*polylog(3,1-2/(c*x+1))/d^3
```

Rubi [A]
time = 0.59, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$,
Rules used = {6087, 6033, 6199, 6095, 6205, 6745, 6065, 6063, 641, 46, 213, 6055, 6203}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{b\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(cx+1)^2}-\frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3}+\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^3}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^3}+\frac{b^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d^3}-\frac{b^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d^3}-\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d^3}+\frac{11b^2}{16d^3(cx+1)}+\frac{b^2}{16d^3(cx+1)^2}-\frac{11b^2\tanh^{-1}(cx)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

```
[Out] b^2/(16*d^3*(1 + c*x)^2) + (11*b^2)/(16*d^3*(1 + c*x)) - (11*b^2*ArcTanh[c*
x])/(16*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) + (5*b*(a + b*A
rcTanh[c*x]))/(4*d^3*(1 + c*x)) - (5*(a + b*ArcTanh[c*x])^2)/(8*d^3) + (a +
 b*ArcTanh[c*x])^2/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])^2/(d^3*(1 + c
*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 + ((a + b*Ar
cTanh[c*x])^2*Log[2/(1 + c*x)])/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1
- 2/(1 - c*x)])/d^3 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])
/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 + (b^2*Poly
Log[3, 1 - 2/(1 - c*x)])/(2*d^3) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^
3) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^3} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^2} - \frac{c}{}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^3} - \frac{c\int \frac{(a+}{}}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1\right.}{d^3}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1\right.}{d^3}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{(a+}{2d}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{(a+}{2d}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{(a+}{2d}$$

$$= \frac{b^2}{16d^3(1+cx)^2} + \frac{11b^2}{16d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)}$$

$$= \frac{b^2}{16d^3(1+cx)^2} + \frac{11b^2}{16d^3(1+cx)} - \frac{11b^2\tanh^{-1}(cx)}{16d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.78, size = 376, normalized size = 1.04

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

```
[Out] ((96*a^2)/(1 + c*x)^2 + (192*a^2)/(1 + c*x) + 192*a^2*Log[c*x] - 192*a^2*Lo
g[1 + c*x] + 12*a*b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*Po
lyLog[2, E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6
*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x]
)] - 6*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]])
 + b^2*((8*I)*Pi^3 - 128*ArcTanh[c*x]^3 + 72*Cosh[2*ArcTanh[c*x]] + 144*Arc
Tanh[c*x]*Cosh[2*ArcTanh[c*x]] + 144*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] +
3*Cosh[4*ArcTanh[c*x]] + 12*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] + 24*ArcTanh[
```

```
c*x]^2*Cosh[4*ArcTanh[c*x]] + 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])
] + 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 96*PolyLog[3, E^(2*Ar
cTanh[c*x])] - 72*Sinh[2*ArcTanh[c*x]] - 144*ArcTanh[c*x]*Sinh[2*ArcTanh[c*
x]] - 144*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - 3*Sinh[4*ArcTanh[c*x]] - 12
*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 24*ArcTanh[c*x]^2*Sinh[4*ArcTanh[c*x]]
))/(192*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.60, size = 1752, normalized size = 4.84

method	result	size
derivativedivides	Expression too large to display	1752
default	Expression too large to display	1752

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d^3*ln(c*x)-a^2/d^3*ln(c*x+1)+1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c
*x+1)^2/(c^2*x^2-1))^3+1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b^2/d^3*Pi*a
rctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^3+1/64*b^2/d^3/(c*x+1)^2+3/8*b^2/d^3/(c*x+1)-1/2*I*b^2/d^3*Pi*arctanh(c*x)
^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/
(-c^2*x^2+1)))^2+1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2*I*b
^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1
)^2/(c^2*x^2-1))+I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1
/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2
/(-c^2*x^2+1)))^2-1/2*I*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+a
*b/d^3*arctanh(c*x)/(c*x+1)^2+2*a*b/d^3*arctanh(c*x)/(c*x+1)-2*a*b/d^3*arct
anh(c*x)*ln(c*x+1)-a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+a*b/d^3*ln(-1/2*c*x+1
/2)*ln(1/2*c*x+1/2)+1/64*b^2/d^3/(c*x+1)^2*c^2*x^2-1/32*b^2/d^3/(c*x+1)^2*c
*x-3/8*b^2/d^3/(c*x+1)*c*x+2*a*b/d^3*arctanh(c*x)*ln(c*x)-a*b/d^3*ln(c*x)*l
n(c*x+1)-2*b^2/d^3*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(
3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/4*a*b/d^3/(c*x+1)^2+5/4*a*b/d^3/(c*x+1)+1/
2*a*b/d^3*ln(c*x+1)^2+a*b/d^3*dilog(1/2*c*x+1/2)-5/8*a*b/d^3*ln(c*x+1)+5/8*
a*b/d^3*ln(c*x-1)+b^2/d^3*arctanh(c*x)^2*ln(2)+2*b^2/d^3*arctanh(c*x)^2*ln(
(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+b^2/d^3*ar
ctanh(c*x)^2/(c*x+1)-b^2/d^3*arctanh(c*x)^2*ln(c*x+1)+3/4*b^2/d^3*arctanh(c
```

```
*x)/(c*x+1)+1/16*b^2/d^3*arctanh(c*x)/(c*x+1)^2+2*b^2/d^3*arctanh(c*x)*poly
log(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^
2*x^2+1)^(1/2))+2*b^2/d^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2)
)+b^2/d^3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-b^2/d^3*arctanh(c
*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2/d^3*arctanh(c*x)^2*ln(c*x)-a*b/d^3*d
ilog(c*x)-a*b/d^3*dilog(c*x+1)-5/8*b^2/d^3*arctanh(c*x)^2-2/3*b^2/d^3*arcta
nh(c*x)^3+1/2*a^2/d^3/(c*x+1)^2+a^2/d^3/(c*x+1)-3/4*b^2/d^3*arctanh(c*x)/(c
*x+1)*c*x+1/16*b^2/d^3*arctanh(c*x)/(c*x+1)^2*c^2*x^2-1/8*b^2/d^3*arctanh(c
*x)/(c*x+1)^2*c*x
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 +
 2*log(x)/d^3) + 1/8*(2*b^2*c*x + 3*b^2 - 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)
*log(c*x + 1))*log(-c*x + 1)^2/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) + integrate(
1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - (2*b
^2*c^3*x^3 + 5*b^2*c^2*x^2 - 4*a*b + (4*a*b*c + 3*b^2*c)*x - 2*(b^2*c^4*x^4
 + 3*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d
^3*x^5 + 2*c^3*d^3*x^4 - 2*c*d^3*x^2 - d^3*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^4 + 3*c
^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b**2
*atanh(c*x)**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(2*a*
b*atanh(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x\,{\left(d+c\,d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^3),x)
[Out] int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^3), x)
```

3.117 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^2(d+cdx)^3}\,dx$

Optimal. Leaf size=448

$$-\frac{b^2c}{16d^3(1+cx)^2}-\frac{19b^2c}{16d^3(1+cx)}+\frac{19b^2c\tanh^{-1}(cx)}{16d^3}-\frac{bc(a+b\tanh^{-1}(cx))}{4d^3(1+cx)^2}-\frac{9bc(a+b\tanh^{-1}(cx))}{4d^3(1+cx)}+\frac{17c(a\ \cdots}{}$$

```
[Out] -1/16*b^2*c/d^3/(c*x+1)^2-19/16*b^2*c/d^3/(c*x+1)+19/16*b^2*c*arctanh(c*x)/
d^3-1/4*b*c*(a+b*arctanh(c*x))/d^3/(c*x+1)^2-9/4*b*c*(a+b*arctanh(c*x))/d^3
/(c*x+1)+17/8*c*(a+b*arctanh(c*x))^2/d^3-(a+b*arctanh(c*x))^2/d^3/x-1/2*c*(
a+b*arctanh(c*x))^2/d^3/(c*x+1)^2-2*c*(a+b*arctanh(c*x))^2/d^3/(c*x+1)+6*c*
(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^3-3*c*(a+b*arctanh(c*x))^2*ln
(2/(c*x+1))/d^3+2*b*c*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d^3+3*b*c*(a+b*arc
tanh(c*x))*polylog(2,1-2/(-c*x+1))/d^3-3*b*c*(a+b*arctanh(c*x))*polylog(2,-
1+2/(-c*x+1))/d^3+3*b*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d^3-b^2*c
*polylog(2,-1+2/(c*x+1))/d^3-3/2*b^2*c*polylog(3,1-2/(-c*x+1))/d^3+3/2*b^2*
c*polylog(3,-1+2/(-c*x+1))/d^3+3/2*b^2*c*polylog(3,1-2/(c*x+1))/d^3
```

Rubi [A]

time = 0.72, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6087, 6037, 6135, 6079, 2497, 6033, 6199, 6095, 6205, 6745, 6065, 6063, 641, 46, 213, 6055, 6203}

$$\frac{3bc\,\text{Li}_2\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))}{d^3}-\frac{3bc\,\text{Li}_2\left(\frac{2}{1-cx}-1\right)(a+b\tanh^{-1}(cx))}{d^3}+\frac{3bc\,\text{Li}_2\left(1-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d^3}-\frac{9bc(a+b\tanh^{-1}(cx))}{4d^3(cx+1)}-\frac{bc(a+b\tanh^{-1}(cx))}{4d^3(cx+1)^2}-\frac{(a+b\tanh^{-1}(cx))^2}{d^3x}-\frac{2c(a+b\tanh^{-1}(cx))^2}{d^3(cx+1)}-\frac{c(a+b\tanh^{-1}(cx))^2}{2d^3(cx+1)^2}+\frac{17c(a+b\tanh^{-1}(cx))^2}{8d^3}-\frac{6c\tanh^{-1}\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))^2}{d^3}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d^3}-\frac{3c\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))^2}{d^3}-\frac{b^2c\,\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{d^3}-\frac{3b^2c\,\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d^3}+\frac{3b^2c\,\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d^3}+\frac{3b^2c\,\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d^3}-\frac{19b^2c}{16d^3(cx+1)}-\frac{b^2c}{16d^3(cx+1)^2}+\frac{19b^2c\tanh^{-1}(cx)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

```
[Out] -1/16*(b^2*c)/(d^3*(1 + c*x)^2) - (19*b^2*c)/(16*d^3*(1 + c*x)) + (19*b^2*c
*ArcTanh[c*x])/(16*d^3) - (b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) -
(9*b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)) + (17*c*(a + b*ArcTanh[c*x])
^2)/(8*d^3) - (a + b*ArcTanh[c*x])^2/(d^3*x) - (c*(a + b*ArcTanh[c*x])^2)/(
2*d^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x])^2)/(d^3*(1 + c*x)) - (6*c*(a
 + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 - (3*c*(a + b*ArcTanh[c*
x])^2*Log[2/(1 + c*x)])/d^3 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*
x)])/d^3 + (3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^3 - (
3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^3 + (3*b*c*(a +
b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 - (b^2*c*PolyLog[2, -1 + 2
/(1 + c*x)])/d^3 - (3*b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^3) + (3*b^2*c
*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*PolyLog[3, 1 - 2/(1 + c*x
)])/(2*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x^2} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^3} + \right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d^3} - \frac{(3c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^3} + \frac{c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{d^3} + \frac{(2}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} - \frac{6c(}{}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - \frac{2c(a-}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} -$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} -$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} -$$

$$= -\frac{b^2c}{16d^3(1+cx)^2} - \frac{19b^2c}{16d^3(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(c\right.}{4d^3(1+cx)}$$

$$= -\frac{b^2c}{16d^3(1+cx)^2} - \frac{19b^2c}{16d^3(1+cx)} + \frac{19b^2c\tanh^{-1}(cx)}{16d^3} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.90, size = 479, normalized size = 1.07

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

```
[Out] ((-64*a^2)/x - (32*a^2*c)/(1 + c*x)^2 - (128*a^2*c)/(1 + c*x) - 192*a^2*c*L
og[x] + 192*a^2*c*Log[1 + c*x] + b^2*c*((-8*I)*Pi^3 + 64*ArcTanh[c*x]^2 - (
64*ArcTanh[c*x]^2)/(c*x) + 128*ArcTanh[c*x]^3 - 40*Cosh[2*ArcTanh[c*x]] - 8
0*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]
] - Cosh[4*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] - 8*ArcTanh[
c*x]^2*Cosh[4*ArcTanh[c*x]] + 128*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])]
 - 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 64*PolyLog[2, E^(-2*Arc
Tanh[c*x])] - 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 96*PolyLog[
```

```
3, E^(2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]*Sinh[2*A
rcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]
] + 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Sinh[4*ArcTanh[c
*x]]) + (4*a*b*(48*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*x*(-20*Cosh[2*Ar
cTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 20*S
inh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]]) - 4*ArcTanh[c*x]*(8 + 10*c*x*Co
sh[2*ArcTanh[c*x]] + c*x*Cosh[4*ArcTanh[c*x]] + 24*c*x*Log[1 - E^(-2*ArcTan
h[c*x])] - 10*c*x*Sinh[2*ArcTanh[c*x]] - c*x*Sinh[4*ArcTanh[c*x]])))/x)/(64
*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.43, size = 7480, normalized size = 16.70

method	result	size
derivativedivides	Expression too large to display	7480
default	Expression too large to display	7480

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c
*log(c*x + 1)/d^3 + 6*c*log(x)/d^3) - 1/8*(6*b^2*c^2*x^2 + 9*b^2*c*x + 2*b^
2 - 6*(b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c*x + 1)^2
/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - integrate(-1/4*((b^2*c*x - b^2)*log(
c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 15*b^2*c^3*x
^3 + 11*b^2*c^2*x^2 + 4*a*b - 2*(2*a*b*c - b^2*c)*x - 2*(3*b^2*c^5*x^5 + 9*
b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*
log(-c*x + 1))/(c^4*d^3*x^6 + 2*c^3*d^3*x^5 - 2*c*d^3*x^3 - d^3*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^5 + 3*c
^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{2ab\,\mathrm{atanh}\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b
**2*atanh(c*x)**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integra
l(2*a*b*atanh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x^2\,{\left(d+c\,d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^3),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^3), x)
```

3.118 $\int \frac{(a+b\tanh^{-1}(cx))^2}{(1+cx)^4}\,dx$

Optimal. Leaf size=176

$$-\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}+\frac{11b^2\tanh^{-1}(cx)}{144c}-\frac{b(a+b\tanh^{-1}(cx))}{9c(1+cx)^3}-\frac{b(a+b\tanh^{-1}(cx))}{12c(1+cx)^2}$$

```
[Out] -1/54*b^2/c/(c*x+1)^3-5/144*b^2/c/(c*x+1)^2-11/144*b^2/c/(c*x+1)+11/144*b^2
*arctanh(c*x)/c-1/9*b*(a+b*arctanh(c*x))/c/(c*x+1)^3-1/12*b*(a+b*arctanh(c*
x))/c/(c*x+1)^2-1/12*b*(a+b*arctanh(c*x))/c/(c*x+1)+1/24*(a+b*arctanh(c*x))
^2/c-1/3*(a+b*arctanh(c*x))^2/c/(c*x+1)^3
```

Rubi [A]
time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {6065, 6063, 641, 46, 213, 6095}

$$-\frac{b(a+b\tanh^{-1}(cx))}{12c(cx+1)}-\frac{b(a+b\tanh^{-1}(cx))}{12c(cx+1)^2}-\frac{b(a+b\tanh^{-1}(cx))}{9c(cx+1)^3}+\frac{(a+b\tanh^{-1}(cx))^2}{24c}-\frac{(a+b\tanh^{-1}(cx))^2}{3c(cx+1)^3}-\frac{11b^2}{144c(cx+1)}-\frac{5b^2}{144c(cx+1)^2}-\frac{b^2}{54c(cx+1)^3}+\frac{11b^2\tanh^{-1}(cx)}{144c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

```
[Out] -1/54*b^2/(c*(1 + c*x)^3) - (5*b^2)/(144*c*(1 + c*x)^2) - (11*b^2)/(144*c*(
1 + c*x)) + (11*b^2*ArcTanh[c*x])/(144*c) - (b*(a + b*ArcTanh[c*x]))/(9*c*(
1 + c*x)^3) - (b*(a + b*ArcTanh[c*x]))/(12*c*(1 + c*x)^2) - (b*(a + b*ArcTa
nh[c*x]))/(12*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(24*c) - (a + b*ArcTanh
[c*x])^2/(3*c*(1 + c*x)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(1+cx)^3}+\frac{1}{3}(2b)\int \left(\frac{a+b\tanh^{-1}(cx)}{2(1+cx)^4}+\frac{a+b\tanh^{-1}(cx)}{4(1+cx)^3}+\frac{a}{}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(1+cx)^3}+\frac{1}{12}b\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx-\frac{1}{12}b\int \frac{a+b\tanh^{-1}(cx}{-1+c^2x^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{(a+b}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{(a+b}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{(a+b}{}$$

$$= -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b(a}{}$$

$$= -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}+\frac{11b^2\tanh^{-1}(cx)}{144c}-\frac{b(a+b}{9c(}$$

Mathematica [A]
time = 0.10, size = 168, normalized size = 0.95

$$-\frac{16(18a^2+6ab+b^2)+6b(12a+5b)(1+cx)+6b(12a+11b)(1+cx)^2+24b(24a+b(10+9cx+3c^2x^2))\tanh^{-1}(cx)-36b^2(-7+3cx+3c^2x^2+c^3x^3)\tanh^{-1}(cx)^2+3b(12a+11b)(1+cx)^3\log(1-cx)-3b(12a+11b)(1+cx)^3\log(1+cx)}{864c(1+cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

```
[Out] -1/864*(16*(18*a^2 + 6*a*b + b^2) + 6*b*(12*a + 5*b)*(1 + c*x) + 6*b*(12*a
+ 11*b)*(1 + c*x)^2 + 24*b*(24*a + b*(10 + 9*c*x + 3*c^2*x^2))*ArcTanh[c*x]
 - 36*b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + 3*b*(12*a + 1
1*b)*(1 + c*x)^3*Log[1 - c*x] - 3*b*(12*a + 11*b)*(1 + c*x)^3*Log[1 + c*x])
/(c*(1 + c*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(158) = 316$.
time = 0.34, size = 321, normalized size = 1.82

method	result
derivativedivides	$-\frac{a^2}{3(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)^2}{3(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)}{9(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)}{12(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)}{12(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{24}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{24}$
default	$-\frac{a^2}{3(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)^2}{3(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)}{9(cx+1)^3}-\frac{b^2\operatorname{arctanh}(cx)}{12(cx+1)^2}-\frac{b^2\operatorname{arctanh}(cx)}{12(cx+1)}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{24}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{24}$
risch	$\frac{b^2(x^3c^3+3c^2x^2+3cx-7)\ln(cx+1)^2}{96(cx+1)^3c}-\frac{b(3x^3b\ln(-cx+1)c^3+9bx^2\ln(-cx+1)c^2+6bc^2x^2+9bcx\ln(-cx+1)+18bcx-21b}{144(cx+1)^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/3*a^2/(c*x+1)^3-1/3*b^2/(c*x+1)^3*arctanh(c*x)^2-1/9*b^2*arctanh(c*
x)/(c*x+1)^3-1/12*b^2*arctanh(c*x)/(c*x+1)^2-1/12*b^2*arctanh(c*x)/(c*x+1)+
1/24*b^2*arctanh(c*x)*ln(c*x+1)-1/24*b^2*arctanh(c*x)*ln(c*x-1)-1/48*b^2*ln
(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)+1/48*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/96*b^2
*ln(c*x+1)^2+1/48*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)-1/96*b^2*ln(c*x-1)^2-1/54*b
^2/(c*x+1)^3-5/144*b^2/(c*x+1)^2-11/144*b^2/(c*x+1)+11/288*b^2*ln(c*x+1)-11
/288*b^2*ln(c*x-1)-2/3*a*b*arctanh(c*x)/(c*x+1)^3-1/9*a*b/(c*x+1)^3-1/12*a*
b/(c*x+1)^2-1/12*a*b/(c*x+1)+1/24*a*b*ln(c*x+1)-1/24*a*b*ln(c*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(158) = 316$.
time = 0.29, size = 445, normalized size = 2.53

$$-\frac{1}{72}\left(c\left(\frac{2(3c^2x^2+9cx+10)}{c^5x^3+3c^4x^2+3c^3x+c^2}-\frac{3\log(cx+1)}{c^2}+\frac{3\log(cx-1)}{c^2}\right)+\frac{48\operatorname{artanh}(cx)}{c^4x^3+3c^3x^2+3c^2x+c}\right)ab-\frac{1}{864}\left(12c\left(\frac{2(3c^2x^2+9cx+10)}{c^5x^3+3c^4x^2+3c^3x+c^2}-\frac{3\log(cx+1)}{c^2}+\frac{3\log(cx-1)}{c^2}\right)\operatorname{artanh}(cx)+\frac{(66c^2x^2+9(c^3x^3+3c^2x^2+3cx+1)\log(cx+1)^2+9(c^3x^3+3c^2x^2+3cx+1)\log(cx-1)^2+162cx-3(11c^3x^3+33c^2x^2+33cx+6(c^3x^3+3c^2x^2+3cx+1)\log(cx-1)+11)\log(cx+1)+33(c^3x^3+3c^2x^2+3cx+1)\log(cx-1)+112)c^2}{c^6x^3+3c^5x^2+3c^4x+c^3}\right)b^2-\frac{b^2\operatorname{artanh}(cx)^2}{3(c^4x^3+3c^3x^2+3c^2x+c)}-\frac{a^2}{3(c^4x^3+3c^3x^2+3c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/72*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
- 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c
^3*x^2 + 3*c^2*x + c))*a*b - 1/864*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x
^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*
arctanh(c*x) + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x +
1)^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11
*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*
x - 1) + 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x -
1) + 112)*c^2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*b^2 - 1/3*b^2*arctanh(
c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^2/(c^4*x^3 + 3*c^3*x^2 +
 3*c^2*x + c)
```

Fricas [A]
time = 0.40, size = 203, normalized size = 1.15

$$-\frac{6\,(12\,ab+11\,b^2)c^2x^2+54\,(4\,ab+3\,b^2)cx-9\,(b^2c^3x^3+3\,b^2c^2x^2+3\,b^2cx-7\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2+288\,a^2+240\,ab+112\,b^2-3\,((12\,ab+11\,b^2)c^3x^3+3\,(12\,ab+7\,b^2)c^2x^2+3\,(12\,ab-b^2)cx-84\,ab-29\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)}{864\,(c^4x^3+3\,c^3x^2+3\,c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/864*(6*(12*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b + 3*b^2)*c*x - 9*(b^2*c^3*x
^3 + 3*b^2*c^2*x^2 + 3*b^2*c*x - 7*b^2)*log(-(c*x + 1)/(c*x - 1))^2 + 288*a
^2 + 240*a*b + 112*b^2 - 3*((12*a*b + 11*b^2)*c^3*x^3 + 3*(12*a*b + 7*b^2)*
c^2*x^2 + 3*(12*a*b - b^2)*c*x - 84*a*b - 29*b^2)*log(-(c*x + 1)/(c*x - 1))
)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atanh}(cx))^2}{(cx+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*x+1)**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(c*x + 1)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) =$ 316.
time = 0.40, size = 333, normalized size = 1.89

$$\frac{1}{1728}\,c\left(\frac{18\left(\frac{3\,(cx+1)^2b^2}{(cx-1)^2}-\frac{3\,(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^3c^2}+\frac{6\left(\frac{36\,(cx+1)^2ab}{(cx-1)^2}-\frac{36\,(cx+1)ab}{cx-1}+12\,ab+\frac{18\,(cx+1)^2b^2}{(cx-1)^2}-\frac{9\,(cx+1)b^2}{cx-1}+2\,b^2\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^3c^2}+\frac{\left(\frac{216\,(cx+1)^2a^2}{(cx-1)^2}-\frac{216\,(cx+1)a^2}{cx-1}+72\,a^2+\frac{216\,(cx+1)^2ab}{(cx-1)^2}-\frac{108\,(cx+1)ab}{cx-1}+24\,ab+\frac{108\,(cx+1)^2b^2}{(cx-1)^2}-\frac{27\,(cx+1)b^2}{cx-1}+4\,b^2\right)(cx-1)^3}{(cx+1)^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/1728*c*(18*(3*(c*x + 1)^2*b^2/(c*x - 1)^2 - 3*(c*x + 1)*b^2/(c*x - 1) + b
^2)*(c*x - 1)^3*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^3*c^2) + 6*(36*(c*x
+ 1)^2*a*b/(c*x - 1)^2 - 36*(c*x + 1)*a*b/(c*x - 1) + 12*a*b + 18*(c*x + 1)
^2*b^2/(c*x - 1)^2 - 9*(c*x + 1)*b^2/(c*x - 1) + 2*b^2)*(c*x - 1)^3*log(-(c
*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (216*(c*x + 1)^2*a^2/(c*x - 1)^2 - 2
16*(c*x + 1)*a^2/(c*x - 1) + 72*a^2 + 216*(c*x + 1)^2*a*b/(c*x - 1)^2 - 108
*(c*x + 1)*a*b/(c*x - 1) + 24*a*b + 108*(c*x + 1)^2*b^2/(c*x - 1)^2 - 27*(c
*x + 1)*b^2/(c*x - 1) + 4*b^2)*(c*x - 1)^3/((c*x + 1)^3*c^2))
```

Mupad [B]
time = 2.30, size = 498, normalized size = 2.83

$$\ln(1-cx)\left(\ln(cx+1)\left(\frac{b^2}{3c\,(2c^3x^3+6c^2x^2+6cx+2)}-\frac{b^2\,(c^3x^3+3c^2x^2+3cx+1)}{24c\,(2c^3x^3+6c^2x^2+6cx+2)}\right)+\frac{b^2}{3c\,(6c^3x^3+18c^2x^2+18cx+6)}+\frac{b\,(6a-b)}{3c\,(6c^3x^3+18c^2x^2+18cx+6)}+\frac{b^2\,(11c^3x^3+45c^2x^2+69cx+51)}{48c\,(6c^3x^3+18c^2x^2+18cx+6)}\right)-\frac{x\,(27b^2+36ab)+x^2\,(11cb^2+12acb)+\frac{8\,(18a^2+15ab+7b^2)}{3c}}{144c^3x^3+432c^2x^2+432cx+144}+\ln(cx+1)^2\left(\frac{b^2}{96c}-\frac{b^2}{12c^2\,\left(3x+3cx^2+\frac{1}{c}+c^2x^3\right)}\right)+\ln(1-cx)^2\left(\frac{b^2}{96c}-\frac{b^2}{3c\,(4c^3x^3+12c^2x^2+12cx+4)}\right)-\frac{\ln(cx+1)\,\left(\frac{7b^2}{96c^2}+\frac{5b^2x^2}{32}+\frac{23b^2x}{96c}+\frac{11b^2cx^3}{288}+\frac{b\,(16a+5b)}{48c^2}\right)}{3x+3cx^2+\frac{1}{c}+c^2x^3}-\frac{b\,\mathrm{atan}(cx\,1i)\,(6a+11b)\,1i}{72c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(c*x + 1)^4,x)
```

```
[Out] log(1 - c*x)*(log(c*x + 1)*(b^2/(3*c*(6*c*x + 6*c^2*x^2 + 2*c^3*x^3 + 2)) -
 (b^2*(3*c*x + 3*c^2*x^2 + c^3*x^3 + 1))/(24*c*(6*c*x + 6*c^2*x^2 + 2*c^3*x
^3 + 2))) + b^2/(3*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6)) + (b*(6*a - b))
/(3*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6)) + (b^2*(69*c*x + 45*c^2*x^2 +
11*c^3*x^3 + 51))/(48*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6))) - (x*(36*a*
b + 27*b^2) + x^2*(11*b^2*c + 12*a*b*c) + (8*(15*a*b + 18*a^2 + 7*b^2))/(3*
c))/(432*c*x + 432*c^2*x^2 + 144*c^3*x^3 + 144) + log(c*x + 1)^2*(b^2/(96*c
) - b^2/(12*c^2*(3*x + 3*c*x^2 + 1/c + c^2*x^3))) + log(1 - c*x)^2*(b^2/(96
*c) - b^2/(3*c*(12*c*x + 12*c^2*x^2 + 4*c^3*x^3 + 4))) - (log(c*x + 1)*((7*
b^2)/(96*c^2) + (5*b^2*x^2)/32 + (23*b^2*x)/(96*c) + (11*b^2*c*x^3)/288 + (
b*(16*a + 5*b))/(48*c^2)))/(3*x + 3*c*x^2 + 1/c + c^2*x^3) - (b*atan(c*x*1i
)*(6*a + 11*b)*1i)/(72*c)
```

3.119 $\int \frac{\tanh^{-1}(ax)^2}{cx-acx^2}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c}-\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{2c}$$

```
[Out] arctanh(a*x)^2*ln(2-2/(-a*x+1))/c+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))/c-1
/2*polylog(3,-1+2/(-a*x+1))/c
```

Rubi [A]
time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1607, 6079, 6095, 6205, 6745}

$$-\frac{\text{Li}_3\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*Log[2 - 2/(1 - a*x)])/c + (ArcTanh[a*x]*PolyLog[2, -1 + 2/(
1 - a*x)])/c - PolyLog[3, -1 + 2/(1 - a*x)]/(2*c)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{cx-acx^2}\,dx &= \int \frac{\tanh^{-1}(ax)^2}{x(c-acx)}\,dx \\
&= \frac{\tanh^{-1}(ax)^2 \log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(2a)\int \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c} \\
&= \frac{\tanh^{-1}(ax)^2 \log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{a\int \frac{\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c} \\
&= \frac{\tanh^{-1}(ax)^2 \log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{\mathrm{Li}_3\left(-1+\frac{2}{1-ax}\right)}{2c}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.88

$$\frac{\tanh^{-1}(ax)^2 \log\left(1-e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{\tanh^{-1}(ax)\mathrm{PolyLog}\left(2, e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{\mathrm{PolyLog}\left(3, e^{2\tanh^{-1}(ax)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])])/c + (ArcTanh[a*x]*PolyLog[2, E
^(2*ArcTanh[a*x])])/c - PolyLog[3, E^(2*ArcTanh[a*x])]/(2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.97, size = 647, normalized size = 9.66

method	result

derivativedivides	$\frac{-\frac{a\,\mathrm{arctanh}(ax)^2\ln(ax-1)}{c}+\frac{a\,\mathrm{arctanh}(ax)^2\ln(ax)}{c}+\frac{2a\left(-\frac{\mathrm{arctanh}(ax)^2\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{2}+\frac{\mathrm{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{2}\right.}{}}{}$
default	$\frac{-\frac{a\,\mathrm{arctanh}(ax)^2\ln(ax-1)}{c}+\frac{a\,\mathrm{arctanh}(ax)^2\ln(ax)}{c}+\frac{2a\left(-\frac{\mathrm{arctanh}(ax)^2\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{2}+\frac{\mathrm{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{2}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-a/c*arctanh(a*x)^2*ln(a*x-1)+a/c*arctanh(a*x)^2*ln(a*x)+2*a/c*(-1/2*a
rctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/2*arctanh(a*x)^2*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))+arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-pol
ylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x
^2+1)^(1/2))+arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-polylog(3,(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*(2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3
+I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-I*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-
a^2*x^2+1)+1))^2-I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/
(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I*((a*x+1)^2/(-a^2*
x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+
1)+1))^2+2*I*Pi+2*ln(2))*arctanh(a*x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(-a*x + 1)^3/c + 1/4*integrate(-(log(a*x + 1)^2 - 2*log(a*x + 1)*l
og(-a*x + 1))/(a*c*x^2 - c*x), x)
```

Fricas [A]
time = 0.35, size = 85, normalized size = 1.27

$$\frac{\log\left(\frac{2\,ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+2\,\mathrm{Li}_2\left(-\frac{2\,ax}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)-2\,\mathrm{polylog}\left(3,-\frac{ax+1}{ax-1}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] 1/4*(log(2*a*x/(a*x - 1))*log(-(a*x + 1)/(a*x - 1))^2 + 2*dilog(-2*a*x/(a*x
 - 1) + 1)*log(-(a*x + 1)/(a*x - 1)) - 2*polylog(3, -(a*x + 1)/(a*x - 1)))/
c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^{2}(ax)}{ax^{2}-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a*c*x**2+c*x),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a*x**2 - x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/(a*c*x^2 - c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{c\,x-a\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(c*x - a*c*x^2),x)
```

```
[Out] int(atanh(a*x)^2/(c*x - a*c*x^2), x)
```

3.120 $\int (1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=306

$$3ab^2x+\frac{b^3x}{4}-\frac{b^3\tanh^{-1}(cx)}{4c}+3b^3x\tanh^{-1}(cx)+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4b\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{21}{4}bx(a$$

```
[Out] 3*a*b^2*x+1/4*b^3*x-1/4*b^3*arctanh(c*x)/c+3*b^3*x*arctanh(c*x)+1/4*b^2*c*x
^2*(a+b*arctanh(c*x))+4*b*(a+b*arctanh(c*x))^2/c+21/4*b*x*(a+b*arctanh(c*x)
)^2+3/2*b*c*x^2*(a+b*arctanh(c*x))^2+1/4*b*c^2*x^3*(a+b*arctanh(c*x))^2+1/4
*(c*x+1)^4*(a+b*arctanh(c*x))^3/c-11*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/
c-6*b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c+3/2*b^3*ln(-c^2*x^2+1)/c-11/2*b
^3*polylog(2,1-2/(-c*x+1))/c-6*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1
))/c+3*b^3*polylog(3,1-2/(-c*x+1))/c
```

Rubi [A]
time = 0.48, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$,
Rules used = {6065, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 327, 212, 1600, 6205, 6745}

$$-\frac{6b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{11b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+3ab^2x+\frac{1}{4}bc^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)^3}{4c}+\frac{21}{4}bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{4b\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{6b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{3b^3\log\left(1-c^2x^2\right)}{2c}-\frac{11b^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{c}+3b^3x\tanh^{-1}(cx)-\frac{b^3\tanh^{-1}(cx)}{4c}+\frac{b^3x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] 3*a*b^2*x + (b^3*x)/4 - (b^3*ArcTanh[c*x])/(4*c) + 3*b^3*x*ArcTanh[c*x] + (
b^2*c*x^2*(a + b*ArcTanh[c*x]))/4 + (4*b*(a + b*ArcTanh[c*x])^2)/c + (21*b*
x*(a + b*ArcTanh[c*x])^2)/4 + (3*b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + (b*c^2
*x^3*(a + b*ArcTanh[c*x])^2)/4 + ((1 + c*x)^4*(a + b*ArcTanh[c*x])^3)/(4*c)
 - (11*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c - (6*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c + (3*b^3*Log[1 - c^2*x^2])/(2*c) - (11*b^3*PolyL
og[2, 1 - 2/(1 - c*x)])/(2*c) - (6*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 -
2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx = \frac{(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^3}{4c} - \frac{1}{4}(3b) \int \left(-7\left(a+b\tanh^{-1}(cx)\right)^2 - 4\right.$$

$$= \frac{(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^3}{4c} + \frac{1}{4}(21b) \int \left(a+b\tanh^{-1}(cx)\right)^2 \, dx - ($$

$$= \frac{21}{4} bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2} bcx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{4} bc^2x^3\left(a+b\right.$$

$$= \frac{21b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} + \frac{21}{4} bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2} bcx^2\left(a+b\tan\right.$$

$$= 3ab^2x + \frac{1}{4} b^2cx^2\left(a+b\tanh^{-1}(cx)\right) + \frac{4b\left(a+b\tanh^{-1}(cx)\right)^2}{c} + \frac{21}{4} bx\left(a\right.$$

$$= 3ab^2x + \frac{b^3x}{4} + 3b^3x\tanh^{-1}(cx) + \frac{1}{4} b^2cx^2\left(a+b\tanh^{-1}(cx)\right) + \frac{4b(a+}{}$$

$$= 3ab^2x + \frac{b^3x}{4} - \frac{b^3\tanh^{-1}(cx)}{4c} + 3b^3x\tanh^{-1}(cx) + \frac{1}{4} b^2cx^2\left(a+b\tanh^{-}\right.$$

$$= 3ab^2x + \frac{b^3x}{4} - \frac{b^3\tanh^{-1}(cx)}{4c} + 3b^3x\tanh^{-1}(cx) + \frac{1}{4} b^2cx^2\left(a+b\tanh^{-}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 644 vs. 2(306) = 612.
time = 0.89, size = 644, normalized size = 2.10

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-2*a*b^2 + 8*a^3*c*x + 42*a^2*b*c*x + 24*a*b^2*c*x + 2*b^3*c*x + 12*a^3*c^
2*x^2 + 12*a^2*b*c^2*x^2 + 2*a*b^2*c^2*x^2 + 8*a^3*c^3*x^3 + 2*a^2*b*c^3*x^
3 + 2*a^3*c^4*x^4 - 24*a*b^2*ArcTanh[c*x] - 2*b^3*ArcTanh[c*x] + 24*a^2*b*c
*x*ArcTanh[c*x] + 84*a*b^2*c*x*ArcTanh[c*x] + 24*b^3*c*x*ArcTanh[c*x] + 36*
a^2*b*c^2*x^2*ArcTanh[c*x] + 24*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^2*x^2*
ArcTanh[c*x] + 24*a^2*b*c^3*x^3*ArcTanh[c*x] + 4*a*b^2*c^3*x^3*ArcTanh[c*x]
 + 6*a^2*b*c^4*x^4*ArcTanh[c*x] - 90*a*b^2*ArcTanh[c*x]^2 - 56*b^3*ArcTanh[
c*x]^2 + 24*a*b^2*c*x*ArcTanh[c*x]^2 + 42*b^3*c*x*ArcTanh[c*x]^2 + 36*a*b^2
*c^2*x^2*ArcTanh[c*x]^2 + 12*b^3*c^2*x^2*ArcTanh[c*x]^2 + 24*a*b^2*c^3*x^3*
ArcTanh[c*x]^2 + 2*b^3*c^3*x^3*ArcTanh[c*x]^2 + 6*a*b^2*c^4*x^4*ArcTanh[c*x
]^2 - 30*b^3*ArcTanh[c*x]^3 + 8*b^3*c*x*ArcTanh[c*x]^3 + 12*b^3*c^2*x^2*Arc
```

```
Tanh[c*x]^3 + 8*b^3*c^3*x^3*ArcTanh[c*x]^3 + 2*b^3*c^4*x^4*ArcTanh[c*x]^3 -
 96*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 88*b^3*ArcTanh[c*x]*L
og[1 + E^(-2*ArcTanh[c*x])] - 48*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c
*x])] + 45*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 44*a*b^2*Log[1 - c^2
*x^2] + 12*b^3*Log[1 - c^2*x^2] + 4*b^2*(12*a + 11*b + 12*b*ArcTanh[c*x])*P
olyLog[2, -E^(-2*ArcTanh[c*x])] + 24*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/
(8*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.49, size = 883, normalized size = 2.89

method	result	size
derivativedivides	Expression too large to display	883
default	Expression too large to display	883

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)^3*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/4*b^3+3/2*a^2*b*c^2*x^2+3/2*b^3*arctanh(c*x)^2*c^2*x^2+3*b^3*arctan
h(c*x)*c*x-6*a*b^2*dilog(1/2*c*x+1/2)-6*b^3*arctanh(c*x)*polylog(2,-(c*x+1)
^2/(-c^2*x^2+1))-6*b^3*ln(2)*arctanh(c*x)^2+3*a*b^2*c^2*x^2*arctanh(c*x)-3*
b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))+3*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3
/4*a^2*b*arctanh(c*x)-11*b^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)-11*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4*a*b^2*arctanh(
c*x)^2+1/4*b^3*arctanh(c*x)^2*c^3*x^3+21/4*b^3*arctanh(c*x)^2*c*x+1/4*b^3*a
rctanh(c*x)^3*c^4*x^4+b^3*arctanh(c*x)^3*c^3*x^3+3/2*b^3*arctanh(c*x)^3*c^2
*x^2+b^3*arctanh(c*x)^3*c*x+1/4*b^3*arctanh(c*x)*c^2*x^2+6*a^2*b*ln(c*x-1)+
6*b^3*arctanh(c*x)^2*ln(c*x-1)+3*a*b^2*ln(c*x-1)^2+7*a*b^2*ln(c*x-1)+4*a*b^
2*ln(c*x+1)+1/4*b^3*c*x-6*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+1/4*a^2*b*c^3*x^3
+21/4*a^2*b*c*x+1/4*a*b^2*c^2*x^2+12*a*b^2*arctanh(c*x)*ln(c*x-1)-13/4*a*b^
2+11/4*b^3*arctanh(c*x)+4*b^3*arctanh(c*x)^2+1/4*b^3*arctanh(c*x)^3-6*I*b^3
*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+6*I*b^3*Pi*csgn(I/(
1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/4*a^2*b*arctanh(c*x)*c^4*x^4+
3*a^2*b*arctanh(c*x)*c^3*x^3+9/2*a^2*b*arctanh(c*x)*c^2*x^2+3*a^2*b*arctanh
(c*x)*c*x+3/4*a*b^2*arctanh(c*x)^2*c^4*x^4+3*a*b^2*arctanh(c*x)^2*c^3*x^3+9
/2*a*b^2*arctanh(c*x)^2*c^2*x^2+3*a*b^2*arctanh(c*x)^2*c*x+1/2*a*b^2*arctan
h(c*x)*c^3*x^3+21/2*a*b^2*arctanh(c*x)*c*x-6*I*b^3*Pi*arctanh(c*x)^2+3*b^2*
c*x*a-11*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-11*b^3*dilog(1-I*(c*x+1)
/(-c^2*x^2+1)^(1/2))+1/4*(c*x+1)^4*a^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 + a^3*c^2*x^3 + 1/8*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3
*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b*c^3 + 3/2*(2*x^3*
arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*c^2 + 3/2*a^3*c*x^
2 + 9/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/
c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/
c - 1/32*((b^3*c^4*x^4 + 4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x - 15*b^3
)*log(-c*x + 1)^3 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^3 + 12*
(3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(b^3*c^4*x^4 +
4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x + b^3)*log(c*x + 1))*log(-c*x + 1
)^2)/c - integrate(-1/16*(2*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*c*x - b^3)
*log(c*x + 1)^3 + 12*(a*b^2*c^4*x^4 + 2*a*b^2*c^3*x^3 - 2*a*b^2*c*x - a*b^2
)*log(c*x + 1)^2 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^3 + 12*(
3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*c*x - b
^3)*log(c*x + 1)^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(6*b^3*c^2*x^2 + (8*a*b^
2*c^4 + b^3*c^4)*x^4 + 4*(4*a*b^2*c^3 + b^3*c^3)*x^3 - 8*a*b^2 + b^3 - 4*(4
*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c^3*x^3 + 3*a^3*c^2*x^2 + 3*a^3*c*x + (b^3*c^3*x^3 + 3*b^3*c^2
*x^2 + 3*b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c^3*x^3 + 3*a*b^2*c
^2*x^2 + 3*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(a^2*b*c^3*x^3 + 3*a^2*b*c
^2*x^2 + 3*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\text{atanh}\,(cx))^3\,(cx+1)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)^3*(b*arctanh(c*x) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{atanh}(c\,x))^3\,(c\,x+1)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(c*x + 1)^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3*(c*x + 1)^3, x)
```

3.121 $\int (1+cx)^2 \left(a+b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=240

$$ab^2x+b^3x\tanh^{-1}(cx)+\frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+3bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{(1+cx)}{}$$

```
[Out] a*b^2*x+b^3*x*arctanh(c*x)+5/2*b*(a+b*arctanh(c*x))^2/c+3*b*x*(a+b*arctanh(
c*x))^2+1/2*b*c*x^2*(a+b*arctanh(c*x))^2+1/3*(c*x+1)^3*(a+b*arctanh(c*x))^3
/c-6*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c-4*b*(a+b*arctanh(c*x))^2*ln(2/
(-c*x+1))/c+1/2*b^3*ln(-c^2*x^2+1)/c-3*b^3*polylog(2,1-2/(-c*x+1))/c-4*b^2*
(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+2*b^3*polylog(3,1-2/(-c*x+1))/
c
```

Rubi [A]
time = 0.33, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6065, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 1600, 6205, 6745}

$$-\frac{4b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{6b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+ab^2x+\frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2+3bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^3}{3c}-\frac{4b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{b^3\log\left(1-c^2x^2\right)}{2c}-\frac{3b^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{c}+\frac{2b^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{c}+b^3x\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] a*b^2*x + b^3*x*ArcTanh[c*x] + (5*b*(a + b*ArcTanh[c*x])^2)/(2*c) + 3*b*x*(
a + b*ArcTanh[c*x])^2 + (b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + ((1 + c*x)^3*(
a + b*ArcTanh[c*x])^3)/(3*c) - (6*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]
)/c - (4*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c + (b^3*Log[1 - c^2*x^
2])/(2*c) - (3*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c - (4*b^2*(a + b*ArcTanh[c
*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (2*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```

```
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)^2 \left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3}{3c} - b\int \Big(-3\left(a+b\tanh^{-1}(cx)\right)^2 - cx(a$$

$$= \frac{(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3}{3c} + (3b)\int \left(a+b\tanh^{-1}(cx)\right)^2 dx - (4b)$$

$$= 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^3 (a+}{3}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-}\right.$$

$$= ab^2x + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2(a+$$

$$= ab^2x + b^3x\tanh^{-1}(cx) + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(c\right.$$

$$= ab^2x + b^3x\tanh^{-1}(cx) + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(c\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 488 vs. 2(240) = 480.
time = 0.78, size = 488, normalized size = 2.03

$$\frac{6a^3cx+18a^2bcx+6ab^2cx+6a^3c^2x^2+3a^2bc^2x^2+2a^3c^3x^3-6ab^2\tanh^{-1}(cx)+18a^2bcx\tanh^{-1}(cx)+36ab^2cx\tanh^{-1}(cx)+6b^3cx\tanh^{-1}(cx)+18a^2bc^2x^2\tanh^{-1}(cx)+6ab^2c^2x^2\tanh^{-1}(cx)+6a^2bc^3x^3\tanh^{-1}(cx)-42ab^2\tanh^{-1}(cx)^2-21b^3\tanh^{-1}(cx)^2+18ab^2cx\tanh^{-1}(cx)^2+18b^3cx\tanh^{-1}(cx)^2+18ab^2c^2x^2\tanh^{-1}(cx)^2+3b^3c^2x^2\tanh^{-1}(cx)^2+6ab^2c^3x^3\tanh^{-1}(cx)^2-14b^3\tanh^{-1}(cx)^3+6b^3cx\tanh^{-1}(cx)^3+6b^3c^2x^2\tanh^{-1}(cx)^3+2b^3c^3x^3\tanh^{-1}(cx)^3-48ab^2\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-36b^3\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-24b^3\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+21a^2b\log(1-cx)+3a^2b\log(1+cx)+18ab^2\log\left(1-c^2x^2\right)+3b^3\log\left(1-c^2x^2\right)+6b^2\left(4a+3b+4b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)+12b^3\mathrm{Li}_3\left(-e^{-2\tanh^{-1}(cx)}\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (6*a^3*c*x + 18*a^2*b*c*x + 6*a*b^2*c*x + 6*a^3*c^2*x^2 + 3*a^2*b*c^2*x^2 +
 2*a^3*c^3*x^3 - 6*a*b^2*ArcTanh[c*x] + 18*a^2*b*c*x*ArcTanh[c*x] + 36*a*b^
2*c*x*ArcTanh[c*x] + 6*b^3*c*x*ArcTanh[c*x] + 18*a^2*b*c^2*x^2*ArcTanh[c*x]
 + 6*a*b^2*c^2*x^2*ArcTanh[c*x] + 6*a^2*b*c^3*x^3*ArcTanh[c*x] - 42*a*b^2*A
rcTanh[c*x]^2 - 21*b^3*ArcTanh[c*x]^2 + 18*a*b^2*c*x*ArcTanh[c*x]^2 + 18*b^
3*c*x*ArcTanh[c*x]^2 + 18*a*b^2*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^2*x^2*ArcT
anh[c*x]^2 + 6*a*b^2*c^3*x^3*ArcTanh[c*x]^2 - 14*b^3*ArcTanh[c*x]^3 + 6*b^3
*c*x*ArcTanh[c*x]^3 + 6*b^3*c^2*x^2*ArcTanh[c*x]^3 + 2*b^3*c^3*x^3*ArcTanh[
c*x]^3 - 48*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 36*b^3*ArcTan
h[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*A
rcTanh[c*x])] + 21*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 18*a*b^2*Log
[1 - c^2*x^2] + 3*b^3*Log[1 - c^2*x^2] + 6*b^2*(4*a + 3*b + 4*b*ArcTanh[c*x
])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 12*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x]
)])/(6*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.61, size = 740, normalized size = 3.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)^2*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*a^2*b*c^2*x^2+1/2*b^3*arctanh(c*x)^2*c^2*x^2+b^3*arctanh(c*x)*c*x+
1/3*(c*x+1)^3*a^3-4*I*b^3*Pi*arctanh(c*x)^2-4*a*b^2*dilog(1/2*c*x+1/2)-4*b^
3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-4*b^3*ln(2)*arctanh(c*x)^
2+a*b^2*c^2*x^2*arctanh(c*x)-b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))+2*b^3*polylog
(3,-(c*x+1)^2/(-c^2*x^2+1))+a^2*b*arctanh(c*x)-6*b^3*arctanh(c*x)*ln(1+I*(c
*x+1)/(-c^2*x^2+1)^(1/2))-6*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1
/2))+a*b^2*arctanh(c*x)^2+3*b^3*arctanh(c*x)^2*c*x+1/3*b^3*arctanh(c*x)^3*c
^3*x^3+b^3*arctanh(c*x)^3*c^2*x^2+b^3*arctanh(c*x)^3*c*x+4*a^2*b*ln(c*x-1)+
4*b^3*arctanh(c*x)^2*ln(c*x-1)+2*a*b^2*ln(c*x-1)^2+7/2*a*b^2*ln(c*x-1)+5/2*
a*b^2*ln(c*x+1)-4*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+3*a^2*b*c*x+8*a*b^2*arcta
nh(c*x)*ln(c*x-1)-a*b^2-4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arc
tanh(c*x)^2+4*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+
b^3*arctanh(c*x)+5/2*b^3*arctanh(c*x)^2+1/3*b^3*arctanh(c*x)^3+a^2*b*arctan
h(c*x)*c^3*x^3+3*a^2*b*arctanh(c*x)*c^2*x^2+3*a^2*b*arctanh(c*x)*c*x+a*b^2*
arctanh(c*x)^2*c^3*x^3+3*a*b^2*arctanh(c*x)^2*c^2*x^2+3*a*b^2*arctanh(c*x)^
```

```
2*c*x+6*a*b^2*arctanh(c*x)*c*x+b^2*c*x*a-6*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-6*b^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*c^2*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c
^4))*a^2*b*c^2 + a^3*c*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x
 + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) +
log(-c^2*x^2 + 1))*a^2*b/c - 1/24*((b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x
 - 7*b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + (6*a*b^2*c^2 + b^3*c^2)*x^
2 + 6*(a*b^2*c + b^3*c)*x + (b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x + b^3)
*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^3*x^3 + b^3*c^2*
x^2 - b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 + a*b^2*c^2*x^2 - a*
b^2*c*x - a*b^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*(6*a*b^2*c^2 + b^3*c
^2)*x^2 + 3*(b^3*c^3*x^3 + b^3*c^2*x^2 - b^3*c*x - b^3)*log(c*x + 1)^2 + 12
*(a*b^2*c + b^3*c)*x + 2*((6*a*b^2*c^3 + b^3*c^3)*x^3 - 6*a*b^2 + b^3 + 3*(
2*a*b^2*c^2 + b^3*c^2)*x^2 - 3*(2*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*
x + 1))/(c*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c^2*x^2 + 2*a^3*c*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^3)*arctanh(
c*x)^3 + a^3 + 3*(a*b^2*c^2*x^2 + 2*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(
a^2*b*c^2*x^2 + 2*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (cx + 1)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)**2*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
[Out] integrate((c*x + 1)^2*(b*arctanh(c*x) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{atanh}(c\,x))^3\,(c\,x+1)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(c*x + 1)^2,x)
[Out] int((a + b*atanh(c*x))^3*(c*x + 1)^2, x)
```

3.122 $\int (1+cx) \left(a+b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=191

$$\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)\log(}{c}$$

```
[Out] 3/2*b*(a+b*arctanh(c*x))^2/c+3/2*b*x*(a+b*arctanh(c*x))^2+1/2*(c*x+1)^2*(a+
b*arctanh(c*x))^3/c-3*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c-3*b*(a+b*arct
anh(c*x))^2*ln(2/(-c*x+1))/c-3/2*b^3*polylog(2,1-2/(-c*x+1))/c-3*b^2*(a+b*a
rctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*polylog(3,1-2/(-c*x+1))/c
```

Rubi [A]
time = 0.23, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6065, 6021, 6131, 6055, 2449, 2352, 1600, 6095, 6205, 6745}

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{3b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{3b^3\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c) + (3*b*x*(a + b*ArcTanh[c*x])^2)/2 + ((1
 + c*x)^2*(a + b*ArcTanh[c*x])^3)/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - (3*b^3*P
olyLog[2, 1 - 2/(1 - c*x)])/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)\left(a+b\tanh^{-1}(cx)\right)^3\,dx = \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{1}{2}(3b)\int\Bigg(-\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{2(1$$

$$= \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} + \frac{1}{2}(3b)\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx - (3b$$

$$= \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - (3b)\int\frac{(a+$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tan$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tan$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tan$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tan$$

Mathematica [**A**]
time = 0.48, size = 334, normalized size = 1.75

$$\frac{4a^3cx+6a^2bcx+2a^3c^2x^2+12a^2bcx\tanh^{-1}(cx)+12ab^2cx\tanh^{-1}(cx)+6a^2bc^2x^2\tanh^{-1}(cx)-18ab^2\tanh^{-1}(cx)^2-6b^3\tanh^{-1}(cx)^2+12ab^2cx\tanh^{-1}(cx)^2+6b^3cx\tanh^{-1}(cx)^2+6ab^2c^2x^2\tanh^{-1}(cx)^2-6b^3\tanh^{-1}(cx)^3+4b^3cx\tanh^{-1}(cx)^3+2b^3c^2x^2\tanh^{-1}(cx)^3-24ab^2\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-12b^3\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-12b^3\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+9a^2b\log(1-cx)+3a^2b\log(1+cx)+6ab^2\log\left(1-c^2x^2\right)+6b^2\left(2a+b+2b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+6b^3\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (4*a^3*c*x + 6*a^2*b*c*x + 2*a^3*c^2*x^2 + 12*a^2*b*c*x*ArcTanh[c*x] + 12*a
*b^2*c*x*ArcTanh[c*x] + 6*a^2*b*c^2*x^2*ArcTanh[c*x] - 18*a*b^2*ArcTanh[c*x
]^2 - 6*b^3*ArcTanh[c*x]^2 + 12*a*b^2*c*x*ArcTanh[c*x]^2 + 6*b^3*c*x*ArcTan
h[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTanh[c*x]^2 - 6*b^3*ArcTanh[c*x]^3 + 4*b^3*c*
x*ArcTanh[c*x]^3 + 2*b^3*c^2*x^2*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log
[1 + E^(-2*ArcTanh[c*x])] - 12*b^3*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])
] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 9*a^2*b*Log[1 - c*
x] + 3*a^2*b*Log[1 + c*x] + 6*a*b^2*Log[1 - c^2*x^2] + 6*b^2*(2*a + b + 2*b
*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*A
rcTanh[c*x])])/(4*c)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.72, size = 6152, normalized size = 32.21

method	result	size
derivativedivides	Expression too large to display	6152
default	Expression too large to display	6152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*c*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a^2*b/c - 1/16*((b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-c*x + 1)^3 - 3
*(2*a*b^2*c^2*x^2 + 2*(2*a*b^2*c + b^3*c)*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^
3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^2*x^2 - b^3)*l
og(c*x + 1)^3 + 6*(a*b^2*c^2*x^2 - a*b^2)*log(c*x + 1)^2 - 3*(2*a*b^2*c^2*x
^2 + (b^3*c^2*x^2 - b^3)*log(c*x + 1)^2 + 2*(2*a*b^2*c + b^3*c)*x + (2*b^3*
c*x - 4*a*b^2 + b^3 + (4*a*b^2*c^2 + b^3*c^2)*x^2)*log(c*x + 1))*log(-c*x +
 1))/(c*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c*x + (b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c*x + a*
b^2)*arctanh(c*x)^2 + 3*(a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atanh}(cx))^3 (cx + 1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)*(b*arctanh(c*x) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atanh}(c\,x))^3\,(c\,x+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(c*x + 1),x)
```

```
[Out] int((a + b*atanh(c*x))^3*(c*x + 1), x)
```

3.123 $\int \frac{(a+b\tanh^{-1}(cx))^3}{1+cx}\,dx$

Optimal. Leaf size=111

$$-\frac{(a+b\tanh^{-1}(cx))^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b(a+b\tanh^{-1}(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c}+\frac{3b^2(a+b\tanh^{-1}(cx))\operatorname{Poly}}{2c}$$

[Out] -(a+b*arctanh(c*x))^3*ln(2/(c*x+1))/c+3/2*b*(a+b*arctanh(c*x))^2*polylog(2,1-2/(c*x+1))/c+3/2*b^2*(a+b*arctanh(c*x))*polylog(3,1-2/(c*x+1))/c+3/4*b^3*polylog(4,1-2/(c*x+1))/c

Rubi [A]
time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6055, 6095, 6203, 6207, 6745}

$$\frac{3b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{3b\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^3}{c}+\frac{3b^3\text{Li}_4\left(1-\frac{2}{cx+1}\right)}{4c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]

[Out] -(((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/c) + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 + c*x)])/(4*c)

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{1+cx}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c}-\left(3b^2\right.\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c}+\frac{3b^2\left(\right.}{}\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c}+\frac{3b^2\left(\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 152, normalized size = 1.37

$$\frac{-12a^2b\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-12ab^2\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-4b^3\tanh^{-1}(cx)^3\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+4a^3\log(1+cx)+6b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+6b^2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+3b^3\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(cx)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

```
[Out] (-12*a^2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 12*a*b^2*ArcTanh[c*x
]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 4*b^3*ArcTanh[c*x]^3*Log[1 + E^(-2*ArcTa
nh[c*x])] + 4*a^3*Log[1 + c*x] + 6*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, -E^(
-2*ArcTanh[c*x])] + 6*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -E^(-2*ArcTanh[c*
x])] + 3*b^3*PolyLog[4, -E^(-2*ArcTanh[c*x])])/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.03, size = 1396, normalized size = 12.58

method	result	size
derivativedivides	Expression too large to display	1396
default	Expression too large to display	1396

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*I*b^3*arctanh(c*x)^3*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi-3/2*I*a*b^2*Pi*csgn(I/
(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))^2*arctanh(c*x)^2-3*I*a*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2+3/2*I*a*b^2*Pi*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2
*arctanh(c*x)^2-3/2*I*a*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)
/(-c^2*x^2+1)^(1/2))^2*arctanh(c*x)^2+1/2*I*b^3*arctanh(c*x)^3*csgn(I/(1+(c
*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi+3/2*I*a*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+
(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-I*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi+1/2*I*b^3*arctanh(c
*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))^2*Pi-1/2*I*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi-3/2*I*a*b^2*Pi*csgn(I*(c*x+1)^2/
(c^2*x^2-1))^3*arctanh(c*x)^2-3/2*I*a*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+3*a^2*b*ln(c*x+1)*arctanh(c*x)+
3/2*a^2*b*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2*a^2*b*ln(-1/2*c*x+1/2)*ln(1/2*c*x+
1/2)+3*a*b^2*ln(c*x+1)*arctanh(c*x)^2-6*a*b^2*arctanh(c*x)^2*ln((c*x+1)/(-c
^2*x^2+1)^(1/2))-3*a*b^2*ln(2)*arctanh(c*x)^2-3*a*b^2*arctanh(c*x)*polylog(
2,-(c*x+1)^2/(-c^2*x^2+1))+a^3*ln(c*x+1)+1/2*b^3*arctanh(c*x)^4-3/4*b^3*pol
ylog(4,-(c*x+1)^2/(-c^2*x^2+1))+b^3*ln(c*x+1)*arctanh(c*x)^3-2*b^3*arctanh(
c*x)^3*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-b^3*ln(2)*arctanh(c*x)^3-3/2*b^3*arct
anh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/2*b^3*arctanh(c*x)*polylog(
3,-(c*x+1)^2/(-c^2*x^2+1))-3/4*a^2*b*ln(c*x+1)^2-3/2*a^2*b*dilog(1/2*c*x+1/
2)+2*a*b^2*arctanh(c*x)^3+3/2*a*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/2*
I*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi-1/2*I*b^3*arctanh(c
*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*log(c*x + 1)*log(-c*x + 1)^3/c + a^3*log(c*x + 1)/c + integrate(1/
8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 +
6*(b^3*c*x*log(c*x + 1) + a*b^2*c*x - a*b^2)*log(-c*x + 1)^2 + 12*(a^2*b*c*
x - a^2*b)*log(c*x + 1) - 3*(4*a^2*b*c*x - 4*a^2*b + (b^3*c*x - b^3)*log(c*
x + 1)^2 + 4*(a*b^2*c*x - a*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*x^2 - 1)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/(c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{cx + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1),x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(c*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^3}{c\,x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1),x)
```

```
[Out] int((a + b*atanh(c*x))^3/(c*x + 1), x)
```

3.124 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{3b^3}{4c(1+cx)}+\frac{3b^3\tanh^{-1}(cx)}{4c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)}+\frac{(a$$

```
[Out] -3/4*b^3/c/(c*x+1)+3/4*b^3*arctanh(c*x)/c-3/2*b^2*(a+b*arctanh(c*x))/c/(c*x
+1)+3/4*b*(a+b*arctanh(c*x))^2/c-3/2*b*(a+b*arctanh(c*x))^2/c/(c*x+1)+1/2*(
a+b*arctanh(c*x))^3/c-(a+b*arctanh(c*x))^3/c/(c*x+1)
```

Rubi [A]

time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6065, 6063, 641, 46, 213, 6095}

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(cx+1)}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(cx+1)}-\frac{3b^3}{4c(cx+1)}+\frac{3b^3\tanh^{-1}(cx)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

```
[Out] (-3*b^3)/(4*c*(1 + c*x)) + (3*b^3*ArcTanh[c*x])/(4*c) - (3*b^2*(a + b*ArcTa
nh[c*x]))/(2*c*(1 + c*x)) + (3*b*(a + b*ArcTanh[c*x])^2)/(4*c) - (3*b*(a +
b*ArcTanh[c*x])^2)/(2*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(2*c) - (a + b*
ArcTanh[c*x])^3/(c*(1 + c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + (3b)\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2\left(-1+c^2x^2\right)}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \frac{1}{2}(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx - \frac{1}{2}(3b)\int \frac{(a+b\,\mathrm{ta}}{-1}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + (3b^2$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \frac{1}{2}(3$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} +$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} +$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} +$$

$$= -\frac{3b^3}{4c(1+cx)} - \frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b(a+b\,\mathrm{ta}}{2c(1}$$

$$= -\frac{3b^3}{4c(1+cx)} + \frac{3b^3\tanh^{-1}(cx)}{4c} - \frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b(a+b\tanh^{-1}(cx}{4c}$$

Mathematica [A]
time = 0.07, size = 198, normalized size = 1.42

$$\frac{-8a^3-12a^2b-12ab^2-6b^3-12b(2a^2+2ab+b^2)\tanh^{-1}(cx)+6b^2(2a+b)(-1+cx)\tanh^{-1}(cx)^2+4b^3(-1+cx)\tanh^{-1}(cx)^3-3b(2a^2+2ab+b^2)(1+cx)\log(1-cx)+6a^2b\log(1+cx)+6ab^2\log(1+cx)+3b^3\log(1+cx)+6a^2bcx\log(1+cx)+6ab^2cx\log(1+cx)+3b^3cx\log(1+cx)}{8c(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

```
[Out] (-8*a^3 - 12*a^2*b - 12*a*b^2 - 6*b^3 - 12*b*(2*a^2 + 2*a*b + b^2)*ArcTanh[
c*x] + 6*b^2*(2*a + b)*(-1 + c*x)*ArcTanh[c*x]^2 + 4*b^3*(-1 + c*x)*ArcTanh
[c*x]^3 - 3*b*(2*a^2 + 2*a*b + b^2)*(1 + c*x)*Log[1 - c*x] + 6*a^2*b*Log[1
+ c*x] + 6*a*b^2*Log[1 + c*x] + 3*b^3*Log[1 + c*x] + 6*a^2*b*c*x*Log[1 + c*
x] + 6*a*b^2*c*x*Log[1 + c*x] + 3*b^3*c*x*Log[1 + c*x])/(8*c*(1 + c*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.84, size = 1811, normalized size = 13.03

method	result

risch	$\frac{b^3(cx-1)\ln(cx+1)^3}{16(cx+1)c}+\frac{3b^2(-bcx\ln(-cx+1)+2cxa+bcx+b\ln(-cx+1)-2a-b)\ln(cx+1)^2}{16(cx+1)c}-\frac{3b\left(-b^2cx\ln(-cx+1)^2+4c\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2*Pi-3
/4*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))*arctanh(c*x)^2*Pi+3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2*P
i-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))^2*arctanh(c*x)^2*Pi+3/4*I*b^3/(c*x+1)*arctanh(c*x)^2*Pi*c*x+3/8*I
*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*
Pi*c*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*Pi-3/4*a^2*b*ln(
c*x-1)+3/4*a^2*b*ln(c*x+1)-3/2*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(
1/2))-3/4*b^3*arctanh(c*x)^2*ln(c*x-1)+3/4*b^3*arctanh(c*x)^2*ln(c*x+1)-3/8
*a*b^2*ln(c*x-1)^2-3/8*a*b^2*ln(c*x+1)^2-3/4*a*b^2*ln(c*x-1)+3/4*a*b^2*ln(c
*x+1)+3/2*a*b^2*arctanh(c*x)*ln(c*x+1)+3/4*a*b^2*ln(c*x-1)*ln(1/2*c*x+1/2)+
3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*
csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*
x)^2*Pi-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^3*arctanh(c*x)^2*Pi*c*x-3/4*I*b^3/(c*x+1)*csgn(I/(1+(c*x+1)^2/(-c^2
*x^2+1)))^2*arctanh(c*x)^2*Pi*c*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x
^2-1))^3*arctanh(c*x)^2*Pi*c*x+3/4*I*b^3/(c*x+1)*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))^3*arctanh(c*x)^2*Pi*c*x+3/4*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/4*a
*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-3/2*a*b^2*arctanh(c*x)*ln(c*x-1)+3/4*
I*b^3/(c*x+1)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*Pi-3/8*I*
b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arct
anh(c*x)^2*Pi-3/4*I*b^3/(c*x+1)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctan
h(c*x)^2*Pi-3/4*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1
)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*Pi*c*x+3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))
*arctanh(c*x)^2*Pi*c*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*arctanh(c*x)^2*Pi*c*x-3/8*I*b^3/(c*x+1)*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))*arctanh(c*x)^2*Pi*c*x-1/(c*x+1)*a^3-3/2/(c*x+1)*a^2*b-3/8*
b^3/(c*x+1)+3/4*I*b^3/(c*x+1)*arctanh(c*x)^2*Pi+1/2*b^3/(c*x+1)*arctanh(c*x
)^3*c*x+3/4*b^3/(c*x+1)*arctanh(c*x)^2*c*x+3/4*b^3/(c*x+1)*arctanh(c*x)*c*x
-3/2*a*b^2/(c*x+1)+3/8*b^3/(c*x+1)*c*x-3*a^2*b*arctanh(c*x)/(c*x+1)-3*a*b^2
```

```
/(c*x+1)*arctanh(c*x)^2-3*a*b^2*arctanh(c*x)/(c*x+1)-3/4*b^3/(c*x+1)*arctan
h(c*x)-1/2*b^3/(c*x+1)*arctanh(c*x)^3-3/4*b^3/(c*x+1)*arctanh(c*x)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(127) = 254$.
time = 0.28, size = 529, normalized size = 3.81

$$-\frac{b^3\operatorname{artanh}(cx)^3}{c^2x+c}-\frac{3}{4}\left(c\left(\frac{2}{c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)+\frac{4\operatorname{artanh}(cx)}{c^2x+c}\right)a^2b-\frac{3}{8}\left(4c\left(\frac{2}{c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)\operatorname{artanh}(cx)+\frac{\left((cx+1)\log(cx+1)^2+(cx+1)\log(cx-1)^2-2\left(cx+(cx+1)\log(cx-1)+1\right)\log(cx+1)+2(cx+1)\log(cx-1)+4\right)c^2}{c^4x+c^3}\right)ab^2-\frac{1}{16}\left(12c\left(\frac{2}{c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)\operatorname{artanh}(cx)^2-\left(\frac{\left((cx+1)\log(cx+1)^3-(cx+1)\log(cx-1)^3-3\left(cx+(cx+1)\log(cx-1)+1\right)\log(cx+1)^2-3(cx+1)\log(cx-1)^2+3\left((cx+1)\log(cx-1)^2+2cx+2(cx+1)\log(cx-1)+2\right)\log(cx+1)-6(cx+1)\log(cx-1)-12\right)c^2}{c^5x+c^4}-\frac{6\left((cx+1)\log(cx+1)^2+(cx+1)\log(cx-1)^2-2\left(cx+(cx+1)\log(cx-1)+1\right)\log(cx+1)+2(cx+1)\log(cx-1)+4\right)c\operatorname{artanh}(cx)}{c^4x+c^3}\right)c\right)b^3-\frac{3ab^2\operatorname{artanh}(cx)^2}{c^2x+c}-\frac{a^3}{c^2x+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="maxima")
```

```
[Out] -b^3*arctanh(c*x)^3/(c^2*x + c) - 3/4*(c*(2/(c^3*x + c^2) - log(c*x + 1)/c^
2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*a^2*b - 3/8*(4*c*(2/(c^
3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c*x + 1)
*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x - 1
) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*x + c^3))*a*b^
2 - 1/16*(12*c*(2/(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arct
anh(c*x)^2 - (((c*x + 1)*log(c*x + 1)^3 - (c*x + 1)*log(c*x - 1)^3 - 3*(c*x
 + (c*x + 1)*log(c*x - 1) + 1)*log(c*x + 1)^2 - 3*(c*x + 1)*log(c*x - 1)^2
+ 3*((c*x + 1)*log(c*x - 1)^2 + 2*c*x + 2*(c*x + 1)*log(c*x - 1) + 2)*log(c
*x + 1) - 6*(c*x + 1)*log(c*x - 1) - 12)*c^2/(c^5*x + c^4) - 6*((c*x + 1)*l
og(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x - 1)
+ 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c*arctanh(c*x)/(c^4*x + c
^3))*c)*b^3 - 3*a*b^2*arctanh(c*x)^2/(c^2*x + c) - a^3/(c^2*x + c)
```

Fricas [A]
time = 0.36, size = 160, normalized size = 1.15

$$\frac{(b^3cx-b^3)\log\left(-\frac{cx+1}{cx-1}\right)^3-16\,a^3-24\,a^2b-24\,ab^2-12\,b^3-3\left(2\,ab^2+b^3-(2\,ab^2+b^3)cx\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-6\left(2\,a^2b+2\,ab^2+b^3-(2\,a^2b+2\,ab^2+b^3)cx\right)\log\left(-\frac{cx+1}{cx-1}\right)}{16\,(c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((b^3*c*x - b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 16*a^3 - 24*a^2*b - 24*
a*b^2 - 12*b^3 - 3*(2*a*b^2 + b^3 - (2*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(c*
x - 1))^2 - 6*(2*a^2*b + 2*a*b^2 + b^3 - (2*a^2*b + 2*a*b^2 + b^3)*c*x)*log
(-(c*x + 1)/(c*x - 1)))/(c^2*x + c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atanh}(cx))^3}{(cx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**2, x)
```

Giac [A]
time = 0.41, size = 172, normalized size = 1.24

$$\frac{1}{16}\left(\frac{(cx-1)b^3\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)c^2}+\frac{3\,(2\,ab^2+b^3)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2}+\frac{6\,(2\,a^2b+2\,ab^2+b^3)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2}+\frac{2\,(4\,a^3+6\,a^2b+6\,ab^2+3\,b^3)(cx-1)}{(cx+1)c^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*((c*x - 1)*b^3*log(-(c*x + 1)/(c*x - 1))^3/((c*x + 1)*c^2) + 3*(2*a*b^
2 + b^3)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2) + 6*(2*a^2*b
 + 2*a*b^2 + b^3)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2) + 2*(
4*a^3 + 6*a^2*b + 6*a*b^2 + 3*b^3)*(c*x - 1)/((c*x + 1)*c^2))*c
```

Mupad [B]
time = 2.30, size = 582, normalized size = 4.19

$$\ln(1-cx)\left(\ln(cx+1)\left(\frac{3b^3x+\frac{3b^2(4a+b)}{c}}{8cx+8}-\frac{3(b^3+ab^2)}{4c}+\frac{6b^3}{c(8cx+8)}\right)+\frac{3b^3x+\frac{3b^2(4a+b)}{c}}{8cx+8}-\ln(cx+1)^2\left(\frac{3b^3}{16c}-\frac{3b^3}{c(8cx+8)}\right)-\frac{3b^3x+\frac{3(-4a^2b+4ab^2+5b^3)}{c}}{8cx+8}+\frac{6b^3}{c(8cx+8)}+\frac{3(8cx+24)(b^3+ab^2)}{4c(8cx+8)}\right)-\ln(1-cx)^2\left(\frac{3b^3}{c(8cx+8)}-\frac{3(b^3+ab^2)}{8c}-\ln(cx+1)\left(\frac{3b^3}{16c}-\frac{3b^3}{c(8cx+8)}\right)+\frac{3b^3(8cx+24)}{16c(8cx+8)}+\frac{3b^2(2a-b)}{c(8cx+8)}\right)-\ln(cx+1)^2\left(\frac{\frac{3b^3x}{16c}+\frac{3b^2(4a+3b)}{16c^2}}{x+\frac{1}{c}}-\frac{3b^2(a+b)}{8c}\right)-\ln(1-cx)^3\left(\frac{b^3}{16c}-\frac{b^3}{c(8cx+8)}\right)+\ln(cx+1)^3\left(\frac{b^3}{16c}-\frac{b^3}{8c^2\left(x+\frac{1}{c}\right)}\right)-\frac{\ln(cx+1)\left(\frac{3b(2a^2+3ab+2b^2)}{4c^2}+\frac{3b^2x(a+b)}{4c}\right)}{x+\frac{1}{c}}-\frac{4a^3+6a^2b+6ab^2+3b^3}{2c(2cx+2)}-\frac{b\,\mathrm{atan}(cx\,1\mathrm{i})\,(2a^2+4ab+3b^2)\,3\mathrm{i}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1)^2,x)
```

```
[Out] log(1 - c*x)*(log(c*x + 1)*((3*b^3*x + (3*b^2*(4*a + b))/c)/(8*c*x + 8) - (
3*(a*b^2 + b^3))/(4*c) + (6*b^3)/(c*(8*c*x + 8))) + (3*b^3*x + (3*b^2*(4*a
+ b))/c)/(8*c*x + 8) - log(c*x + 1)^2*((3*b^3)/(16*c) - (3*b^3)/(c*(8*c*x +
 8))) - (3*b^3*x + (3*(4*a*b^2 - 4*a^2*b + 5*b^3))/c)/(8*c*x + 8) + (6*b^3)
/(c*(8*c*x + 8)) + (3*(8*c*x + 24)*(a*b^2 + b^3))/(4*c*(8*c*x + 8))) - log(
1 - c*x)^2*((3*b^3)/(c*(8*c*x + 8)) - (3*(a*b^2 + b^3))/(8*c) - log(c*x + 1
)*((3*b^3)/(16*c) - (3*b^3)/(c*(8*c*x + 8))) + (3*b^3*(8*c*x + 24))/(16*c*(
8*c*x + 8)) + (3*b^2*(2*a - b))/(c*(8*c*x + 8))) - log(c*x + 1)^2*(((3*b^3*
x)/(16*c) + (3*b^2*(4*a + 3*b))/(16*c^2))/(x + 1/c) - (3*b^2*(a + b))/(8*c)
) - log(1 - c*x)^3*(b^3/(16*c) - b^3/(c*(8*c*x + 8))) + log(c*x + 1)^3*(b^3
/(16*c) - b^3/(8*c^2*(x + 1/c))) - (log(c*x + 1)*((3*b*(3*a*b + 2*a^2 + 2*b
^2))/(4*c^2) + (3*b^2*x*(a + b))/(4*c)))/(x + 1/c) - (6*a*b^2 + 6*a^2*b + 4
*a^3 + 3*b^3)/(2*c*(2*c*x + 2)) - (b*atan(c*x*1i)*(4*a*b + 2*a^2 + 3*b^2)*3
i)/(4*c)
```

3.125 $\int \frac{(a+b\tanh^{-1}(cx))^3}{(1+cx)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}+\frac{21b^3\tanh^{-1}(cx)}{64c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b(a+b\tanh$$

```
[Out] -3/64*b^3/c/(c*x+1)^2-21/64*b^3/c/(c*x+1)+21/64*b^3*arctanh(c*x)/c-3/16*b^2
*(a+b*arctanh(c*x))/c/(c*x+1)^2-9/16*b^2*(a+b*arctanh(c*x))/c/(c*x+1)+9/32*
b*(a+b*arctanh(c*x))^2/c-3/8*b*(a+b*arctanh(c*x))^2/c/(c*x+1)^2-3/8*b*(a+b*
arctanh(c*x))^2/c/(c*x+1)+1/8*(a+b*arctanh(c*x))^3/c-1/2*(a+b*arctanh(c*x))
^3/c/(c*x+1)^2
```

Rubi [A]
time = 0.28, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6065, 6063, 641, 46, 213, 6095}

$$-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)^2}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(cx+1)^2}-\frac{21b^3}{64c(cx+1)}-\frac{3b^3}{64c(cx+1)^2}+\frac{21b^3\tanh^{-1}(cx)}{64c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

```
[Out] (-3*b^3)/(64*c*(1 + c*x)^2) - (21*b^3)/(64*c*(1 + c*x)) + (21*b^3*ArcTanh[c
*x])/(64*c) - (3*b^2*(a + b*ArcTanh[c*x]))/(16*c*(1 + c*x)^2) - (9*b^2*(a +
 b*ArcTanh[c*x]))/(16*c*(1 + c*x)) + (9*b*(a + b*ArcTanh[c*x])^2)/(32*c) -
(3*b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)^2) - (3*b*(a + b*ArcTanh[c*x])^
2)/(8*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(8*c) - (a + b*ArcTanh[c*x])^3/
(2*c*(1 + c*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(1+cx)^2}+\frac{1}{2}(3b)\int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)}{4(1+cx)^2}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(1+cx)^2}+\frac{1}{8}(3b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx-\frac{1}{8}(3b)\int\frac{\left(a+b\,\mathrm{ta}\right.}{-1}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{(}{}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{(}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-$$

$$= -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(c\right.}{16c(1+cx)}$$

$$= -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}+\frac{21b^3\tanh^{-1}(cx)}{64c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-$$

Mathematica [A]
time = 0.12, size = 215, normalized size = 1.03

$$\frac{-2(32a^3+24a^2b+12ab^2+3b^3)-6b(8a^2+12ab+7b^2)(1+cx)-24b(8a^2+4ab(2+cx)+b^2(4+3cx))\tanh^{-1}(cx)+12b^2(-1+cx)(4a(3+cx)+b(5+3cx))\tanh^{-1}(cx)^2+16b^3(-3+2cx+c^2x^2)\tanh^{-1}(cx)^3-3b(8a^2+12ab+7b^2)(1+cx)^2\log(1-cx)+3b(8a^2+12ab+7b^2)(1+cx)^2\log(1+cx)}{128c(1+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

```
[Out] (-2*(32*a^3 + 24*a^2*b + 12*a*b^2 + 3*b^3) - 6*b*(8*a^2 + 12*a*b + 7*b^2)*(
1 + c*x) - 24*b*(8*a^2 + 4*a*b*(2 + c*x) + b^2*(4 + 3*c*x))*ArcTanh[c*x] +
12*b^2*(-1 + c*x)*(4*a*(3 + c*x) + b*(5 + 3*c*x))*ArcTanh[c*x]^2 + 16*b^3*(
-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^3 - 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*
x)^2*Log[1 - c*x] + 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*x)^2*Log[1 + c*x])/
(128*c*(1 + c*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.26, size = 2684, normalized size = 12.90

method	result
risch	$\frac{b^3(c^2x^2+2cx-3)\ln(cx+1)^3}{64(cx+1)^2c}+\frac{3b^2(-2b\,x^2\ln(-cx+1)c^2+4a\,c^2x^2+3b\,c^2x^2-4bcx\ln(-cx+1)+8cxa+2bcx+6b\ln(-cx+1}{128(cx+1)^2c}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(3/16*I*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1
)))^3+3/16*I*b^3/(c*x+1)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^
2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))*Pi*c*x+3/32*I*b^3/(c*x+1)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^2*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))^2*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(I*(
c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*c^2*x^2-3/16*
I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c
*x+1)^2/(c^2*x^2-1))^2*Pi*c^2*x^2+3/32*I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2*Pi*c^2*x^2-3/16*I*b^3/(c*x+1)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))
*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*
Pi*c*x-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2
))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*c*x-3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^
2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi*c*x
+3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(
c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c*x-3/16*a^2*b*ln(c*x
-1)+3/16*a^2*b*ln(c*x+1)-3/8*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/
2))-3/16*b^3*arctanh(c*x)^2*ln(c*x-1)+3/16*b^3*arctanh(c*x)^2*ln(c*x+1)-3/3
2*a*b^2*ln(c*x-1)^2-3/32*a*b^2*ln(c*x+1)^2-9/32*a*b^2*ln(c*x-1)+9/32*a*b^2*
ln(c*x+1)+3/8*a*b^2*arctanh(c*x)*ln(c*x+1)+3/16*a*b^2*ln(c*x-1)*ln(1/2*c*x+
1/2)+3/16*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/16*a*b^2*ln(-1/2*c*x+1/2)*ln(1
/2*c*x+1/2)-3/8*a*b^2*arctanh(c*x)*ln(c*x-1)-3/32*I*b^3/(c*x+1)^2*Pi*arctan
h(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-3/32*I*b^3/(c*x+1)^2*Pi*arctanh(c*
x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/16*I*b^3/
(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/16*I*b^3
/(c*x+1)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*Pi*c^2*x^2-3
/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*c*x-3
/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^3*Pi*c*x-3/8*I*b^3/(c*x+1)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1)))^2*arctanh(c*x)^2*Pi*c*x+3/32*I*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2
```

```
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))+3/16*I*b^3/(c*x+1)^2*csgn(I/(1+(c*
x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^2*arct
anh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^2*
arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*P
i*c^2*x^2+3/8*I*b^3/(c*x+1)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(
c*x)^2*Pi*c*x+3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*c^2*x^2+3/8*I*b^3/(c*x
+1)^2*arctanh(c*x)^2*Pi*c*x-3/32*I*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I/(
1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*
x^2+1)))^2-3/32*I*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-3/16*I*b^3/(c*x+1)^2*Pi*arctanh(c
*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+3/
32*I*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(
c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/16*a*b^2/(c*x+1)^2-3/8
*a^2*b/(c*x+1)^2-27/64*b^3/(c*x+1)^2*arctanh(c*x)-3/8*b^3/(c*x+1)^2*arctanh
(c*x)^3-3/32*b^3/(c*x+1)^2*arctanh(c*x)^2+3/16*I*b^3/(c*x+1)^2*Pi*arctanh(c
*x)^2+1/8*b^3/(c*x+1)^2*arctanh(c*x)^3*c^2*x^2+1/4*b^3/(c*x+1)^2*arctanh(c*
x)^3*c*x+9/32*b^3/(c*x+1)^2*arctanh(c*x)^2*c^2*x^2+9/16*b^3/(c*x+1)^2*arcta
nh(c*x)^2*c*x+21/64*b^3/(c*x+1)^2*arctanh(c*x)*c^2*x^2+3/32*b^3/(c*x+1)^2*a
rctanh(c*x)*c*x-3/8/(c*x+1)*a^2*b-51/256*b^3/(c*x+1)^2-1/2*a^3/(c*x+1)^2-9/
16*a*b^2/(c*x+1)-3/2*a^2*b*arctanh(c*x)/(c*x+1)^2-3/2*a*b^2/(c*x+1)^2*arcta
nh(c*x)^2-3/4*a*b^2*arctanh(c*x)/(c*x+1)^2+45/256*b^3/(c*x+1)^2*c^2*x^2+3/1
28*b^3/(c*x+1)^2*c*x-3/4*a*b^2*arctanh(c*x)/(c*x+1)-3/8*b^3/(c*x+1)*arctanh
(c*x)^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(188) = 376$.
time = 0.47, size = 796, normalized size = 3.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*arctanh(c*x)^3/(c^3*x^2 + 2*c^2*x + c) - 3/16*(c*(2*(c*x + 2)/(c^4
*x^2 + 2*c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*arctanh(c*
x)/(c^3*x^2 + 2*c^2*x + c))*a^2*b - 3/32*(4*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3
*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c^2*x^2 +
 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 + 6*c*x -
 (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1
) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*x^2 + 2*c^4*x + c^3)
)*a*b^2 - 1/128*(24*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3*x + c^2) - log(c*x + 1)
/c^2 + log(c*x - 1)/c^2)*arctanh(c*x)^2 - ((2*(c^2*x^2 + 2*c*x + 1)*log(c*x
 + 1)^3 - 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^3 - 3*(3*c^2*x^2 + 6*c*x + 2
*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1)^2 - 9*(c^2*x^2 + 2*c*
x + 1)*log(c*x - 1)^2 - 42*c*x + 3*(7*c^2*x^2 + 2*(c^2*x^2 + 2*c*x + 1)*log
```

```
(c*x - 1)^2 + 14*c*x + 6*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 7)*log(c*x +
1) - 21*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) - 48)*c^2/(c^6*x^2 + 2*c^5*x + c
^4) - 12*((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(
c*x - 1)^2 + 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x -
 1) + 3)*log(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c*arctanh
(c*x)/(c^5*x^2 + 2*c^4*x + c^3))*c)*b^3 - 3/2*a*b^2*arctanh(c*x)^2/(c^3*x^2
 + 2*c^2*x + c) - 1/2*a^3/(c^3*x^2 + 2*c^2*x + c)
```

Fricas [A]
time = 0.35, size = 250, normalized size = 1.20

$$\frac{2\,(b^3c^2x^2+2\,b^3cx-3\,b^3)\log\left(-\frac{cx+1}{cx-1}\right)^3-64\,a^3-96\,a^2b-96\,ab^2-48\,b^3-6\,(8\,a^2b+12\,ab^2+7\,b^3)cx+3\,((4\,ab^2+3\,b^3)c^2x^2-12\,ab^2-5\,b^3+2\,(4\,ab^2+b^3)cx)\log\left(-\frac{cx+1}{cx-1}\right)^2+3\,((8\,a^2b+12\,ab^2+7\,b^3)c^2x^2-24\,a^2b-20\,ab^2-9\,b^3+2\,(8\,a^2b+4\,ab^2+b^3)cx)\log\left(-\frac{cx+1}{cx-1}\right)}{128\,(c^3x^2+2\,c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(2*(b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 64
*a^3 - 96*a^2*b - 96*a*b^2 - 48*b^3 - 6*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x +
3*((4*a*b^2 + 3*b^3)*c^2*x^2 - 12*a*b^2 - 5*b^3 + 2*(4*a*b^2 + b^3)*c*x)*lo
g(-(c*x + 1)/(c*x - 1))^2 + 3*((8*a^2*b + 12*a*b^2 + 7*b^3)*c^2*x^2 - 24*a^
2*b - 20*a*b^2 - 9*b^3 + 2*(8*a^2*b + 4*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(c
*x - 1)))/(c^3*x^2 + 2*c^2*x + c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atanh}(cx))^3}{(cx+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**3, x)
```

Giac [A]
time = 0.41, size = 362, normalized size = 1.74

$$\frac{1}{256}\left(\frac{4\left(\frac{2\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)^2c^2}+\frac{6\left(\frac{8\,(cx+1)ab^2}{cx-1}-4\,ab^2+\frac{4\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^2}+\frac{6\left(\frac{16\,(cx+1)a^2b}{cx-1}-8\,a^2b+\frac{16\,(cx+1)ab^2}{cx-1}-4\,ab^2+\frac{8\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^2}+\frac{\left(\frac{64\,(cx+1)a^3}{cx-1}-32\,a^3+\frac{96\,(cx+1)a^2b}{cx-1}-24\,a^2b+\frac{96\,(cx+1)ab^2}{cx-1}-12\,ab^2+\frac{48\,(cx+1)b^3}{cx-1}-3\,b^3\right)(cx-1)^2}{(cx+1)^2c^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="giac")
```

```
[Out] 1/256*(4*(2*(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
- 1))^3/((c*x + 1)^2*c^2) + 6*(8*(c*x + 1)*a*b^2/(c*x - 1) - 4*a*b^2 + 4*(c
*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))^2/((c*x
+ 1)^2*c^2) + 6*(16*(c*x + 1)*a^2*b/(c*x - 1) - 8*a^2*b + 16*(c*x + 1)*a*b^
```

```
2/(c*x - 1) - 4*a*b^2 + 8*(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(
c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^2) + (64*(c*x + 1)*a^3/(c*x - 1) - 32*a^
3 + 96*(c*x + 1)*a^2*b/(c*x - 1) - 24*a^2*b + 96*(c*x + 1)*a*b^2/(c*x - 1)
 - 12*a*b^2 + 48*(c*x + 1)*b^3/(c*x - 1) - 3*b^3)*(c*x - 1)^2/((c*x + 1)^2*c
^2))*c
```

Mupad [B]
time = 3.46, size = 930, normalized size = 4.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1)^3,x)
```

```
[Out] (102*b^3*log(1 - c*x) - 102*b^3*log(c*x + 1) - 96*a*b^2 - 96*a^2*b - 15*b^3
*log(c*x + 1)^2 - 6*b^3*log(c*x + 1)^3 - 15*b^3*log(1 - c*x)^2 + 6*b^3*log(
1 - c*x)^3 + 150*b^3*atanh(c*x) - 64*a^3 - 48*b^3 + 144*a*b^2*atanh(c*x) +
48*a^2*b*atanh(c*x) + 30*b^3*log(c*x + 1)*log(1 - c*x) - 132*a*b^2*log(c*x
+ 1) - 96*a^2*b*log(c*x + 1) + 132*a*b^2*log(1 - c*x) + 96*a^2*b*log(1 - c*
x) - 18*b^3*log(c*x + 1)*log(1 - c*x)^2 + 18*b^3*log(c*x + 1)^2*log(1 - c*x
) - 36*a*b^2*log(c*x + 1)^2 - 36*a*b^2*log(1 - c*x)^2 - 42*b^3*c*x - 144*b^
3*c*x*log(c*x + 1) + 144*b^3*c*x*log(1 - c*x) + 9*b^3*c^2*x^2*log(c*x + 1)^
2 + 2*b^3*c^2*x^2*log(c*x + 1)^3 + 9*b^3*c^2*x^2*log(1 - c*x)^2 - 2*b^3*c^2
*x^2*log(1 - c*x)^3 + 150*b^3*c^2*x^2*atanh(c*x) - 72*a*b^2*c*x - 48*a^2*b*
c*x + 6*b^3*c*x*log(c*x + 1)^2 + 4*b^3*c*x*log(c*x + 1)^3 + 6*b^3*c*x*log(1
 - c*x)^2 - 4*b^3*c*x*log(1 - c*x)^3 + 72*a*b^2*log(c*x + 1)*log(1 - c*x) +
 300*b^3*c*x*atanh(c*x) - 54*b^3*c^2*x^2*log(c*x + 1) + 54*b^3*c^2*x^2*log(
1 - c*x) - 12*b^3*c*x*log(c*x + 1)*log(1 - c*x) - 36*a*b^2*c^2*x^2*log(c*x
+ 1) + 36*a*b^2*c^2*x^2*log(1 - c*x) + 6*b^3*c^2*x^2*log(c*x + 1)*log(1 - c
*x)^2 - 6*b^3*c^2*x^2*log(c*x + 1)^2*log(1 - c*x) - 120*a*b^2*c*x*log(c*x +
 1) + 120*a*b^2*c*x*log(1 - c*x) + 12*b^3*c*x*log(c*x + 1)*log(1 - c*x)^2 -
 12*b^3*c*x*log(c*x + 1)^2*log(1 - c*x) + 12*a*b^2*c^2*x^2*log(c*x + 1)^2 +
 12*a*b^2*c^2*x^2*log(1 - c*x)^2 + 144*a*b^2*c^2*x^2*atanh(c*x) + 48*a^2*b*
c^2*x^2*atanh(c*x) + 24*a*b^2*c*x*log(c*x + 1)^2 + 24*a*b^2*c*x*log(1 - c*x
)^2 - 18*b^3*c^2*x^2*log(c*x + 1)*log(1 - c*x) + 288*a*b^2*c*x*atanh(c*x) +
 96*a^2*b*c*x*atanh(c*x) - 24*a*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x) - 48*
a*b^2*c*x*log(c*x + 1)*log(1 - c*x))/(128*c*(c*x + 1)^2)
```

3.126 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^4}\,dx$

Optimal. Leaf size=275

$$-\frac{b^3}{108c(1+cx)^3}-\frac{19b^3}{576c(1+cx)^2}-\frac{85b^3}{576c(1+cx)}+\frac{85b^3\tanh^{-1}(cx)}{576c}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3}-\frac{5b^2\left(a+b\tanh^{-1}(\right.}{48c(1+cx)^2}$$

```
[Out] -1/108*b^3/c/(c*x+1)^3-19/576*b^3/c/(c*x+1)^2-85/576*b^3/c/(c*x+1)+85/576*b
^3*arctanh(c*x)/c-1/18*b^2*(a+b*arctanh(c*x))/c/(c*x+1)^3-5/48*b^2*(a+b*arc
tanh(c*x))/c/(c*x+1)^2-11/48*b^2*(a+b*arctanh(c*x))/c/(c*x+1)+11/96*b*(a+b*
arctanh(c*x))^2/c-1/6*b*(a+b*arctanh(c*x))^2/c/(c*x+1)^3-1/8*b*(a+b*arctanh
(c*x))^2/c/(c*x+1)^2-1/8*b*(a+b*arctanh(c*x))^2/c/(c*x+1)+1/24*(a+b*arctanh
(c*x))^3/c-1/3*(a+b*arctanh(c*x))^3/c/(c*x+1)^3
```

Rubi [A]

time = 0.45, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6065, 6063, 641, 46, 213, 6095}

$$-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)^2}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(cx+1)^3}+\frac{11b\left(a+b\tanh^{-1}(cx)\right)^2}{96c}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(cx+1)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{24c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(cx+1)^3}-\frac{85b^3}{576c(cx+1)}-\frac{19b^3}{576c(cx+1)^2}-\frac{b^3}{108c(cx+1)^3}+\frac{85b^3\tanh^{-1}(cx)}{576c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

```
[Out] -1/108*b^3/(c*(1 + c*x)^3) - (19*b^3)/(576*c*(1 + c*x)^2) - (85*b^3)/(576*c
*(1 + c*x)) + (85*b^3*ArcTanh[c*x])/(576*c) - (b^2*(a + b*ArcTanh[c*x]))/(1
8*c*(1 + c*x)^3) - (5*b^2*(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)^2) - (11*b^
2*(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)) + (11*b*(a + b*ArcTanh[c*x])^2)/(9
6*c) - (b*(a + b*ArcTanh[c*x])^2)/(6*c*(1 + c*x)^3) - (b*(a + b*ArcTanh[c*x
])^2)/(8*c*(1 + c*x)^2) - (b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)) + (a +
 b*ArcTanh[c*x])^3/(24*c) - (a + b*ArcTanh[c*x])^3/(3*c*(1 + c*x)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^4}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(1+cx)^3}+b\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^4}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4(1+cx)^3}+\cdots\right. \\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(1+cx)^3}+\frac{1}{8}b\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx-\frac{1}{8}b\int \frac{\left(a+b\tanh^{-1}(c\cdots\right.}{-1+c^2x^2} \\
&= -\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{(a+\cdots}{\cdots} \\
&= -\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{(a+\cdots}{\cdots} \\
&= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2}-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)}+\frac{1\cdots}{\cdots} \\
&= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2}-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)}+\frac{1\cdots}{\cdots} \\
&= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2}-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)}+\frac{1\cdots}{\cdots} \\
&= -\frac{b^3}{108c(1+cx)^3}-\frac{19b^3}{576c(1+cx)^2}-\frac{85b^3}{576c(1+cx)}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3}-\frac{5b\cdots}{\cdots} \\
&= -\frac{b^3}{108c(1+cx)^3}-\frac{19b^3}{576c(1+cx)^2}-\frac{85b^3}{576c(1+cx)}+\frac{85b^3\tanh^{-1}(cx)}{576c}-\frac{b^2\left(a+\cdots\right.}{18c\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 279, normalized size = 1.01

$$
-\frac{32(36a^3+18a^2b+6ab^2+b^3)+6b(72a^2+60ab+19b^2)(1+cx)+6b(72a^2+132ab+85b^2)(1+cx)^2+24b(144a^2+12ab(10+9cx+3c^2x^2)+b^2(56+81cx+33c^2x^2))\tanh^{-1}(cx)-36b^2(-1+cx)(12a(7+4cx+c^2x^2)+b(29+32cx+11c^2x^2))\tanh^{-1}(cx)^2-144b^3(-7+3cx+3c^2x^2+c^3x^3)\tanh^{-1}(cx)^3+3b(72a^2+132ab+85b^2)(1+cx)^3\log(1-cx)-3b(72a^2+132ab+85b^2)(1+cx)^3\log(1+cx)}{3456c(1+cx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

```
[Out] -1/3456*(32*(36*a^3 + 18*a^2*b + 6*a*b^2 + b^3) + 6*b*(72*a^2 + 60*a*b + 19
*b^2)*(1 + c*x) + 6*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^2 + 24*b*(144*a
^2 + 12*a*b*(10 + 9*c*x + 3*c^2*x^2) + b^2*(56 + 81*c*x + 33*c^2*x^2))*ArcT
anh[c*x] - 36*b^2*(-1 + c*x)*(12*a*(7 + 4*c*x + c^2*x^2) + b*(29 + 32*c*x +
 11*c^2*x^2))*ArcTanh[c*x]^2 - 144*b^3*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*A
rcTanh[c*x]^3 + 3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 - c*x] -
3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 + c*x])/(c*(1 + c*x)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 10.51, size = 3557, normalized size = 12.93

method	result
risch	$\frac{b^3(x^3c^3+3c^2x^2+3cx-7)\ln(cx+1)^3}{192(cx+1)^3c} + \frac{b^2(-6x^3b\ln(-cx+1)c^3+12c^3x^3a+11b\,c^3x^3-18b\,x^2\ln(-cx+1)c^2+36a\,c^2x^2+2}{384(c}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/16*I*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1
)))^3+3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*Pi*c^2*x^2+3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*c*x+1/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2
*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*c^3*x^3-3/16*I*b^3/(c*x+
1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))*Pi*c*x-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*c*x-1/32*I*b^3/(c*x+1)^3*
arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c^3*x^3+1/32*I*b^3/(c*x+1)^3*arctanh(c
*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*Pi*c^3*x^3-1/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(
c*x+1)^2/(c^2*x^2-1))^3*Pi*c^3*x^3+3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn
(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*c^2*x^2-1/16*I*b^3/(c*x+1)^3*arctanh(c*
x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c^3*x^3+1/32*I*b^3/(c*x+1)^3*P
i*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-3/32*I*b^3/
(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^3*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c
^2*x^2-1))^3*Pi*c^2*x^2+3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+
1)^2/(-c^2*x^2+1)))^3*Pi*c*x-3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*c*x-3/32*I*b^3
/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*c*x-3/16*I*b^3
/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c*x+1/16*
I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi*c^3*
x^3-1/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1)))^3*Pi*c^3*x^3-1/32*I*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/32*I*b^3/(c*x+1
)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/16*I*b^3/(c*x+1)^3*
Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/16*a^2*b*ln(c*x-1)
```

```
+1/16*a^2*b*ln(c*x+1)-1/8*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))
-1/16*b^3*arctanh(c*x)^2*ln(c*x-1)+1/16*b^3*arctanh(c*x)^2*ln(c*x+1)-1/32*a
*b^2*ln(c*x-1)^2-1/32*a*b^2*ln(c*x+1)^2-11/96*a*b^2*ln(c*x-1)+11/96*a*b^2*l
n(c*x+1)+1/8*a*b^2*arctanh(c*x)*ln(c*x+1)+1/16*a*b^2*ln(c*x-1)*ln(1/2*c*x+1
/2)-1/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi*c^3*x^3-1/32*I*b^3/(c*x+1)^3*arctanh(c*x
)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*c
^3*x^3-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c^2*x^2+3/
32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2
/(-c^2*x^2+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*c^2*x^2-3/16*I*b^3/(c*x+
1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))*Pi*c^2*x^2-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*c^2*x^2-3/32*I*b^3/(c
*x+1)^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi*c*x+3/32*I*b^3/(c*x+1)^3*arcta
nh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*Pi*c*x+1/16*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/16*a
*b^2*ln(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-1/8*a*b^2*arctanh(c*x)*ln(c*x-1)-1/32
*I*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(
I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/32*I*b^3/(c*x+1)^3*
Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*c^3*
x^3+3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*c^2*x^2+3/16*I*b^3/(c*x+1)^3*arc
tanh(c*x)^2*Pi*c*x-1/16*I*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-
c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/32*I*b^3/(c*x+1)^3*Pi*a
rctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^
2-1))-5/48*a*b^2/(c*x+1)^2-1/8*a^2*b/(c*x+1)^2-1/8*b^3/(c*x+1)^2*arctanh(c*
x)^2-1/3*a^3/(c*x+1)^3-737/6912*b^3/(c*x+1)^3-a^2*b*arctanh(c*x)/(c*x+1)^3-
a*b^2/(c*x+1)^3*arctanh(c*x)^2-1/3*a*b^2*arctanh(c*x)/(c*x+1)^3+575/6912*b^
3/(c*x+1)^3*c^3*x^3+235/2304*b^3/(c*x+1)^3*c^2*x^2-181/2304*b^3/(c*x+1)^3*c
*x-1/8/(c*x+1)*a^2*b-11/48*a*b^2/(c*x+1)-1/18*a*b^2/(c*x+1)^3-7/24*b^3/(c*x
+1)^3*arctanh(c*x)^3-5/96*b^3/(c*x+1)^3*arctanh...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(249) = 498$.

time = 0.32, size = 1085, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*arctanh(c*x)^3/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/48*(c*(2*(3
*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x +
```

```
1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2
*x + c))*a^2*b - 1/288*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x
^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*arctanh(c*x)
 + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 + 9*(c^
3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*x^3 + 3
3*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 11)
*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 112)*c^
2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*a*b^2 - 1/3456*(72*c*(2*(3*c^2*x^2
 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 +
 3*log(c*x - 1)/c^2)*arctanh(c*x)^2 + ((510*c^2*x^2 - 18*(c^3*x^3 + 3*c^2*x
^2 + 3*c*x + 1)*log(c*x + 1)^3 + 18*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c
*x - 1)^3 + 9*(11*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 +
3*c*x + 1)*log(c*x - 1) + 11)*log(c*x + 1)^2 + 99*(c^3*x^3 + 3*c^2*x^2 + 3*
c*x + 1)*log(c*x - 1)^2 + 1134*c*x - 3*(85*c^3*x^3 + 255*c^2*x^2 + 18*(c^3*
x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 255*c*x + 66*(c^3*x^3 + 3*c^2
*x^2 + 3*c*x + 1)*log(c*x - 1) + 85)*log(c*x + 1) + 255*(c^3*x^3 + 3*c^2*x^
2 + 3*c*x + 1)*log(c*x - 1) + 656)*c^2/(c^7*x^3 + 3*c^6*x^2 + 3*c^5*x + c^4
) + 12*(66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 + 9
*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*x^3
 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) +
 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 112
)*c*arctanh(c*x)/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*c)*b^3 - a*b^2*arct
anh(c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^3/(c^4*x^3 + 3*c^3*x
^2 + 3*c^2*x + c)
```

Fricas [A]
time = 0.37, size = 345, normalized size = 1.25

$$-\frac{6\,(72\,a^2b+132\,ab^2+85\,b^3)c^2x^2-18\,(b^3c^3x^3+3\,b^3c^2x^2+3\,b^3cx-7\,b^3)\log\left(-\frac{cx+1}{cx-1}\right)^3+1152\,a^3+1440\,a^2b+1344\,ab^2+656\,b^3+162\,(8\,a^2b+12\,ab^2+7\,b^3)cx-9\,((12\,ab^2+11\,b^3)c^3x^3+3\,(12\,ab^2+7\,b^3)c^2x^2-84\,ab^2-29\,b^3+3\,(12\,ab^2-b^3)cx)\log\left(-\frac{cx+1}{cx-1}\right)^2-3\,((72\,a^2b+132\,ab^2+85\,b^3)c^3x^3+3\,(72\,a^2b+84\,ab^2+41\,b^3)c^2x^2-504\,a^2b-348\,ab^2-139\,b^3+3\,(72\,a^2b-12\,ab^2-23\,b^3)cx)\log\left(-\frac{cx+1}{cx-1}\right)}{3456\,(c^4x^3+3\,c^3x^2+3\,c^2x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3456*(6*(72*a^2*b + 132*a*b^2 + 85*b^3)*c^2*x^2 - 18*(b^3*c^3*x^3 + 3*b^
3*c^2*x^2 + 3*b^3*c*x - 7*b^3)*log(-(c*x + 1)/(c*x - 1))^3 + 1152*a^3 + 144
0*a^2*b + 1344*a*b^2 + 656*b^3 + 162*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x - 9*(
(12*a*b^2 + 11*b^3)*c^3*x^3 + 3*(12*a*b^2 + 7*b^3)*c^2*x^2 - 84*a*b^2 - 29*
b^3 + 3*(12*a*b^2 - b^3)*c*x)*log(-(c*x + 1)/(c*x - 1))^2 - 3*((72*a^2*b +
132*a*b^2 + 85*b^3)*c^3*x^3 + 3*(72*a^2*b + 84*a*b^2 + 41*b^3)*c^2*x^2 - 50
4*a^2*b - 348*a*b^2 - 139*b^3 + 3*(72*a^2*b - 12*a*b^2 - 23*b^3)*c*x)*log(-
(c*x + 1)/(c*x - 1)))/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(cx))^3}{(cx+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(249) = 498.
time = 0.43, size = 555, normalized size = 2.02

$$\frac{1}{6912}\left(\frac{36\left(\frac{3(cx+1)^2b^3}{(cx-1)^2}-\frac{3(cx+1)b^3}{cx-1}+b^3\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)^3c^2}+\frac{18\left(\frac{36(cx+1)^2ab^2}{(cx-1)^2}-\frac{36(cx+1)ab^2}{cx-1}+12ab^2+\frac{18(cx+1)^2b^3}{(cx-1)^2}-\frac{9(cx+1)b^3}{cx-1}+2b^3\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^3c^2}+\frac{6\left(\frac{216(cx+1)^2a^2b}{(cx-1)^2}-\frac{216(cx+1)a^2b}{cx-1}+72a^2b+\frac{216(cx+1)^2ab^2}{(cx-1)^2}-\frac{108(cx+1)ab^2}{cx-1}+24ab^2+\frac{108(cx+1)^2b^3}{(cx-1)^2}-\frac{27(cx+1)b^3}{cx-1}+4b^3\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^3c^2}+\frac{\left(\frac{864(cx+1)^2a^3}{(cx-1)^2}-\frac{864(cx+1)a^3}{cx-1}+288a^3+\frac{1296(cx+1)^2a^2b}{(cx-1)^2}-\frac{648(cx+1)a^2b}{cx-1}+144a^2b+\frac{1296(cx+1)^2ab^2}{(cx-1)^2}-\frac{324(cx+1)ab^2}{cx-1}+48ab^2+\frac{648(cx+1)^2b^3}{(cx-1)^2}-\frac{81(cx+1)b^3}{cx-1}+8b^3\right)(cx-1)^3}{(cx+1)^3c^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/6912*(36*(3*(c*x + 1)^2*b^3/(c*x - 1)^2 - 3*(c*x + 1)*b^3/(c*x - 1) + b^3
)*(c*x - 1)^3*log(-(c*x + 1)/(c*x - 1))^3/((c*x + 1)^3*c^2) + 18*(36*(c*x +
 1)^2*a*b^2/(c*x - 1)^2 - 36*(c*x + 1)*a*b^2/(c*x - 1) + 12*a*b^2 + 18*(c*x
 + 1)^2*b^3/(c*x - 1)^2 - 9*(c*x + 1)*b^3/(c*x - 1) + 2*b^3)*(c*x - 1)^3*lo
g(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^3*c^2) + 6*(216*(c*x + 1)^2*a^2*b/(c*x
 - 1)^2 - 216*(c*x + 1)*a^2*b/(c*x - 1) + 72*a^2*b + 216*(c*x + 1)^2*a*b^2/
(c*x - 1)^2 - 108*(c*x + 1)*a*b^2/(c*x - 1) + 24*a*b^2 + 108*(c*x + 1)^2*b^
3/(c*x - 1)^2 - 27*(c*x + 1)*b^3/(c*x - 1) + 4*b^3)*(c*x - 1)^3*log(-(c*x +
 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (864*(c*x + 1)^2*a^3/(c*x - 1)^2 - 864*(
c*x + 1)*a^3/(c*x - 1) + 288*a^3 + 1296*(c*x + 1)^2*a^2*b/(c*x - 1)^2 - 648
*(c*x + 1)*a^2*b/(c*x - 1) + 144*a^2*b + 1296*(c*x + 1)^2*a*b^2/(c*x - 1)^2
 - 324*(c*x + 1)*a*b^2/(c*x - 1) + 48*a*b^2 + 648*(c*x + 1)^2*b^3/(c*x - 1)
^2 - 81*(c*x + 1)*b^3/(c*x - 1) + 8*b^3)*(c*x - 1)^3/((c*x + 1)^3*c^2))*c
```

Mupad [B]
time = 4.49, size = 1304, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(c*x + 1)^4,x)
```

```
[Out] (1398*b^3*log(1 - c*x) - 1398*b^3*log(c*x + 1) - 1344*a*b^2 - 1440*a^2*b -
261*b^3*log(c*x + 1)^2 - 126*b^3*log(c*x + 1)^3 - 261*b^3*log(1 - c*x)^2 +
126*b^3*log(1 - c*x)^3 + 1962*b^3*atanh(c*x) - 1152*a^3 - 656*b^3 + 1584*a*
b^2*atanh(c*x) + 432*a^2*b*atanh(c*x) + 522*b^3*log(c*x + 1)*log(1 - c*x) -
 1836*a*b^2*log(c*x + 1) - 1728*a^2*b*log(c*x + 1) + 1836*a*b^2*log(1 - c*x
) + 1728*a^2*b*log(1 - c*x) - 378*b^3*log(c*x + 1)*log(1 - c*x)^2 + 378*b^3
*log(c*x + 1)^2*log(1 - c*x) - 510*b^3*c^2*x^2 - 756*a*b^2*log(c*x + 1)^2 -
 756*a*b^2*log(1 - c*x)^2 - 1134*b^3*c*x - 3150*b^3*c*x*log(c*x + 1) + 3150
*b^3*c*x*log(1 - c*x) - 792*a*b^2*c^2*x^2 - 432*a^2*b*c^2*x^2 + 189*b^3*c^2
```

```
*x^2*log(c*x + 1)^2 + 54*b^3*c^2*x^2*log(c*x + 1)^3 + 189*b^3*c^2*x^2*log(1
 - c*x)^2 - 54*b^3*c^2*x^2*log(1 - c*x)^3 + 99*b^3*c^3*x^3*log(c*x + 1)^2 +
 18*b^3*c^3*x^3*log(c*x + 1)^3 + 99*b^3*c^3*x^3*log(1 - c*x)^2 - 18*b^3*c^3
*x^3*log(1 - c*x)^3 + 5886*b^3*c^2*x^2*atanh(c*x) + 1962*b^3*c^3*x^3*atanh(
c*x) - 1944*a*b^2*c*x - 1296*a^2*b*c*x - 27*b^3*c*x*log(c*x + 1)^2 + 54*b^3
*c*x*log(c*x + 1)^3 - 27*b^3*c*x*log(1 - c*x)^2 - 54*b^3*c*x*log(1 - c*x)^3
 + 1512*a*b^2*log(c*x + 1)*log(1 - c*x) + 5886*b^3*c*x*atanh(c*x) - 2574*b^
3*c^2*x^2*log(c*x + 1) + 2574*b^3*c^2*x^2*log(1 - c*x) - 726*b^3*c^3*x^3*lo
g(c*x + 1) + 726*b^3*c^3*x^3*log(1 - c*x) + 54*b^3*c*x*log(c*x + 1)*log(1 -
 c*x) - 1620*a*b^2*c^2*x^2*log(c*x + 1) + 1620*a*b^2*c^2*x^2*log(1 - c*x) -
 396*a*b^2*c^3*x^3*log(c*x + 1) + 396*a*b^2*c^3*x^3*log(1 - c*x) + 162*b^3*
c^2*x^2*log(c*x + 1)*log(1 - c*x)^2 - 162*b^3*c^2*x^2*log(c*x + 1)^2*log(1
- c*x) + 54*b^3*c^3*x^3*log(c*x + 1)*log(1 - c*x)^2 - 54*b^3*c^3*x^3*log(c*
x + 1)^2*log(1 - c*x) - 2484*a*b^2*c*x*log(c*x + 1) + 2484*a*b^2*c*x*log(1
- c*x) + 162*b^3*c*x*log(c*x + 1)*log(1 - c*x)^2 - 162*b^3*c*x*log(c*x + 1)
^2*log(1 - c*x) + 324*a*b^2*c^2*x^2*log(c*x + 1)^2 + 324*a*b^2*c^2*x^2*log(
1 - c*x)^2 + 108*a*b^2*c^3*x^3*log(c*x + 1)^2 + 108*a*b^2*c^3*x^3*log(1 - c
*x)^2 + 4752*a*b^2*c^2*x^2*atanh(c*x) + 1296*a^2*b*c^2*x^2*atanh(c*x) + 158
4*a*b^2*c^3*x^3*atanh(c*x) + 432*a^2*b*c^3*x^3*atanh(c*x) + 324*a*b^2*c*x*l
og(c*x + 1)^2 + 324*a*b^2*c*x*log(1 - c*x)^2 - 378*b^3*c^2*x^2*log(c*x + 1)
*log(1 - c*x) - 198*b^3*c^3*x^3*log(c*x + 1)*log(1 - c*x) + 4752*a*b^2*c*x*
atanh(c*x) + 1296*a^2*b*c*x*atanh(c*x) - 648*a*b^2*c^2*x^2*log(c*x + 1)*log
(1 - c*x) - 216*a*b^2*c^3*x^3*log(c*x + 1)*log(1 - c*x) - 648*a*b^2*c*x*log
(c*x + 1)*log(1 - c*x))/(3456*c*(c*x + 1)^3)
```

3.127 $\int \frac{x^2 \tanh^{-1}(ax)^3}{c+acx} \, dx$

Optimal. Leaf size=309

$$\frac{3\tanh^{-1}(ax)^2}{2a^3c}+\frac{3x\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}-\frac{3\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3c}$$

```
[Out] 3/2*arctanh(a*x)^2/a^3/c+3/2*x*arctanh(a*x)^2/a^2/c-3/2*arctanh(a*x)^3/a^3/
c-x*arctanh(a*x)^3/a^2/c+1/2*x^2*arctanh(a*x)^3/a/c-3*arctanh(a*x)*ln(2/(-a
*x+1))/a^3/c+3*arctanh(a*x)^2*ln(2/(-a*x+1))/a^3/c-arctanh(a*x)^3*ln(2/(a*x
+1))/a^3/c-3/2*polylog(2,1-2/(-a*x+1))/a^3/c+3*arctanh(a*x)*polylog(2,1-2/(
-a*x+1))/a^3/c+3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))/a^3/c-3/2*polylog(
3,1-2/(-a*x+1))/a^3/c+3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a^3/c+3/4*pol
ylog(4,1-2/(a*x+1))/a^3/c
```

Rubi [A]
time = 0.46, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6077, 6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095, 6205, 6745, 6203, 6207}

$$-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^3c}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{3\text{Li}_4\left(1-\frac{2}{ax+1}\right)}{4a^3c}+\frac{3\text{Li}_2\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{2a^3c}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}+\frac{3\text{Li}_3\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{2a^3c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}+\frac{3\tanh^{-1}(ax)^2}{2a^3c}-\frac{\log\left(\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{a^3c}+\frac{3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^3c}-\frac{3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{3x\tanh^{-1}(ax)^2}{2a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (3*ArcTanh[a*x]^2)/(2*a^3*c) + (3*x*ArcTanh[a*x]^2)/(2*a^2*c) - (3*ArcTanh[
a*x]^3)/(2*a^3*c) - (x*ArcTanh[a*x]^3)/(a^2*c) + (x^2*ArcTanh[a*x]^3)/(2*a*
c) - (3*ArcTanh[a*x]*Log[2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]^2*Log[2/(1
 - a*x)])/(a^3*c) - (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^3*c) - (3*PolyLog[
2, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)
])/(a^3*c) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/(2*a^3*c) - (3*
PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(
1 + a*x)])/(2*a^3*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a^3*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \tanh^{-1}(ax)^3}{c+acx}\,dx &= -\frac{\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx}{a} + \frac{\int x\tanh^{-1}(ax)^3\,dx}{ac} \\
&= \frac{x^2\tanh^{-1}(ax)^3}{2ac} + \frac{\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx}{a^2} - \frac{3\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{2c} - \frac{\int \tanh^{-1}(ax)^3\,dx}{a^2c} \\
&= -\frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} - \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^3c} + \frac{3\int \tanh^{-1}(ax)^2\,dx}{2a^2c} \\
&= \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(ax)^3}{2ac} - \frac{\tanh^{-1}(ax}{} \\
&= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(a}{2ac} \\
&= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(a}{2ac} \\
&= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(a}{2ac} \\
&= \frac{3\tanh^{-1}(ax)^2}{2a^3c} + \frac{3x\tanh^{-1}(ax)^2}{2a^2c} - \frac{3\tanh^{-1}(ax)^3}{2a^3c} - \frac{x\tanh^{-1}(ax)^3}{a^2c} + \frac{x^2\tanh^{-1}(a}{2ac}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 172, normalized size = 0.56

$$
\frac{-6\tanh^{-1}(ax)^2 + 6ax\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^3 - 4ax\tanh^{-1}(ax)^3 + 2a^2x^2\tanh^{-1}(ax)^3 - 12\tanh^{-1}(ax)\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + 12\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right) - 4\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + 6\left(-1+\tanh^{-1}(ax)\right)^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 6\left(-1+\tanh^{-1}(ax)\right)\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 3\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)}{4a^3c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]^3 - 4*a*x*ArcTan
h[a*x]^3 + 2*a^2*x^2*ArcTanh[a*x]^3 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh
[a*x])] + 12*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3
*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])^2*PolyLog[2, -E^(-2*A
rcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*
PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a^3*c)
```

Maple [A]
time = 23.01, size = 352, normalized size = 1.14

method	result

derivativedivides	$\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)+3)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4}{2c}-\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}-\frac{3\operatorname{arctanh}(ax)^2\operatorname{polylog}\left(2,-\right.}{2c}$
default	$\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)+3)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4}{2c}-\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}-\frac{3\operatorname{arctanh}(ax)^2\operatorname{polylog}\left(2,-\right.}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/2/c*arctanh(a*x)^2*(a*x*arctanh(a*x)-arctanh(a*x)+3)*(a*x-1)+1/2/c
*arctanh(a*x)^4-1/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)-3/2/c*arcta
nh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2/c*arctanh(a*x)*polylog(3,-
(a*x+1)^2/(-a^2*x^2+1))-3/4/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+3/c*arctan
h(a*x)^2-3/c*arctanh(a*x)*ln((a*x+1)^2/(-a^2*x^2+1)+1)-3/2/c*polylog(2,-(a*
x+1)^2/(-a^2*x^2+1))-2*arctanh(a*x)^3/c+3/c*arctanh(a*x)^2*ln((a*x+1)^2/(-a
^2*x^2+1)+1)+3/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2/c*poly
log(3,-(a*x+1)^2/(-a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(a^2*x^2 - 2*a*x + 2*log(a*x + 1))*log(-a*x + 1)^3/(a^3*c) + 1/8*inte
grate(1/2*(2*(a^3*x^3 - a^2*x^2)*log(a*x + 1)^3 - 6*(a^3*x^3 - a^2*x^2)*log
(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^3 - a^2*x^2 - 2*a*x + 2*(a^3*x^3 - a^2
*x^2 + a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*c*x^2 - a^2*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\operatorname{atanh}^3(ax)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/(a*x + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(c + a*c*x),x)
```

```
[Out] int((x^2*atanh(a*x)^3)/(c + a*c*x), x)
```

3.128 $\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx$

Optimal. Leaf size=205

$$\frac{\tanh^{-1}(ax)^3}{a^2c}+\frac{x\tanh^{-1}(ax)^3}{ac}-\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\tanh^{-1}(ax)\mathrm{PolyLog}(2,\cdots}{a^2c}$$

```
[Out] arctanh(a*x)^3/a^2/c+x*arctanh(a*x)^3/a/c-3*arctanh(a*x)^2*ln(2/(-a*x+1))/a
^2/c+arctanh(a*x)^3*ln(2/(a*x+1))/a^2/c-3*arctanh(a*x)*polylog(2,1-2/(-a*x+
1))/a^2/c-3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))/a^2/c+3/2*polylog(3,1-2
/(-a*x+1))/a^2/c-3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a^2/c-3/4*polylog(
4,1-2/(a*x+1))/a^2/c
```

Rubi [A]
time = 0.28, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6077, 6021, 6131, 6055, 6095, 6205, 6745, 6203, 6207}

$$\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^2c}-\frac{3\text{Li}_4\left(1-\frac{2}{ax+1}\right)}{4a^2c}-\frac{3\text{Li}_2\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2c}-\frac{3\text{Li}_3\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{2a^2c}+\frac{\tanh^{-1}(ax)^3}{a^2c}+\frac{\log\left(\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{a^2c}-\frac{3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2c}+\frac{x\tanh^{-1}(ax)^3}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] ArcTanh[a*x]^3/(a^2*c) + (x*ArcTanh[a*x]^3)/(a*c) - (3*ArcTanh[a*x]^2*Log[2
/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^2*c) - (3*ArcTa
nh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[2,
 1 - 2/(1 + a*x)])/(2*a^2*c) + (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^2*c) -
(3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*a^2*c) - (3*PolyLog[4, 1 -
2/(1 + a*x)])/(4*a^2*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx}{a} + \frac{\int \tanh^{-1}(ax)^3\,dx}{ac}$$

$$= \frac{x\tanh^{-1}(ax)^3}{ac} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - \frac{3\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{c} - \frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1+a}\right.}{1-a^2x^2}}{ac}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2}{1+a}\right.}{2a^2c}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} - \frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} - \frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} - \frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}$$

Mathematica [A]
time = 0.17, size = 126, normalized size = 0.61

$$\frac{-\tanh^{-1}(ax)^3 + ax\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + \tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right) - \frac{3}{2}\left(-2+\tanh^{-1}(ax)\right)\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) - \frac{3}{2}\left(-1+\tanh^{-1}(ax)\right)\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) - \frac{3}{4}\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (-ArcTanh[a*x]^3 + a*x*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcT
anh[a*x])] + ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - (3*(-2 + ArcTanh
[a*x])*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])])/2 - (3*(-1 + ArcTanh[
a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])])/2 - (3*PolyLog[4, -E^(-2*ArcTanh[a*
x])])/4)/(a^2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.64, size = 736, normalized size = 3.59 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/c*arctanh(a*x)^3*a*x-1/c*arctanh(a*x)^3*ln(a*x+1)-3/c*(-2/3*arctan
h(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*arctanh(a*x)^2*polylog(2,-(a*x+
```

```
1)^2/(-a^2*x^2+1))+1/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-1/4*
polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3
*arctanh(a*x)^3+1/6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arc
tanh(a*x)^3-1/6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/3*I*Pi*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3-1/6*
I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arc
tanh(a*x)^3+1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/6*I*Pi*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3-1/3*ln(2)*arctanh
(a*x)^3+arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+arctanh(a*x)*polylog(2,
-(a*x+1)^2/(-a^2*x^2+1))+1/6*arctanh(a*x)^4-1/2*polylog(3,-(a*x+1)^2/(-a^2*
x^2+1))-1/3*arctanh(a*x)^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(a*x - log(a*x + 1))*log(-a*x + 1)^3/(a^2*c) + 1/8*integrate(((a^2*x^2
 - a*x)*log(a*x + 1)^3 - 3*(a^2*x^2 - a*x)*log(a*x + 1)^2*log(-a*x + 1) + 3
*(a^2*x^2 + a*x + (a^2*x^2 - 2*a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^3
*c*x^2 - a*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atanh}^{3}(ax)}{ax+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(x*atanh(a*x)**3/(a*x + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
[Out] integrate(x*arctanh(a*x)^3/(a*c*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\mathrm{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(c + a*c*x),x)
[Out] int((x*atanh(a*x)^3)/(c + a*c*x), x)
```

3.129 $\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx$

Optimal. Leaf size=104

$$-\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\text{Poly}$$

```
[Out] -arctanh(a*x)^3*ln(2/(a*x+1))/a/c+3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))
/a/c+3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a/c+3/4*polylog(4,1-2/(a*x+1))
/a/c
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6055, 6095, 6203, 6207, 6745}

$$\frac{3\text{Li}_4\left(1-\frac{2}{ax+1}\right)}{4ac}+\frac{3\text{Li}_2\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{2ac}+\frac{3\text{Li}_3\left(1-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{2ac}-\frac{\log\left(\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(c + a*c*x),x]
```

```
[Out] -((ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a*c)) + (3*ArcTanh[a*x]^2*PolyLog[2, 1
 - 2/(1 + a*x)])/(2*a*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*
a*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a*c)
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx &= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}\\ &= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac}-\frac{3\int \frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}\\ &= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{Li}_3\left(1-\frac{2}{1+ax}\right)}{2ac}\\ &= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{Li}_3\left(1-\frac{2}{1+ax}\right)}{2ac}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.79

$$\frac{-4\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)}{4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(c + a*c*x),x]
```

```
[Out] (-4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]^2*PolyLog[
2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])]
+ 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 16.34, size = 593, normalized size = 5.70

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c} - \frac{3\left(\frac{2 \operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3} - \frac{\operatorname{arctanh}(ax)^4}{6} - \frac{\left(i\pi \operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right) \operatorname{csgn}\left(\frac{i(ax+1)^2}{a^2x^2-1}\right.\right.}{}\right.}{}$
default	$\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c} - \frac{3\left(\frac{2 \operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3} - \frac{\operatorname{arctanh}(ax)^4}{6} - \frac{\left(i\pi \operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right) \operatorname{csgn}\left(\frac{i(ax+1)^2}{a^2x^2-1}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/c*arctanh(a*x)^3*ln(a*x+1)-3/c*(2/3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*
x^2+1)^(1/2))-1/6*arctanh(a*x)^4-1/6*(I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-
a^2*x^2+1)+1))-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/
2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2
))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+I*P
i*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^3-2*ln(2))*arctanh(a*x)^3+1/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*
x^2+1))-1/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+1/4*polylog(4,-
(a*x+1)^2/(-a^2*x^2+1))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*log(a*x + 1)*log(-a*x + 1)^3/(a*c) + 1/8*integrate((6*a*x*log(a*x + 1)
*log(-a*x + 1)^2 + (a*x - 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*lo
g(-a*x + 1))/(a^2*c*x^2 - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^3(ax)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a*c*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(c + a*c*x),x)
```

```
[Out] int(atanh(a*x)^3/(c + a*c*x), x)
```

3.130 $\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c}-$$

```
[Out] arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3/2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))
/c-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*polylog(4,-1+2/(a*x+1))/c
```

Rubi [A]
time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6079, 6095, 6203, 6207, 6745}

$$-\frac{3\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}-\frac{3\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}-\frac{3\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{2c}+\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx = \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{(3a)\int\frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} + \frac{(3a)\int\frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\right.}{1-a^2x^2}}{c}$$

$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(-1+\right.}{2c}$$

$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(-1+\right.}{2c}$$

Mathematica [A]
time = 0.07, size = 86, normalized size = 0.92

$$\frac{\pi^4 - 32\tanh^{-1}(ax)^4 + 64\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right) + 96\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2, e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\mathrm{PolyLog}\left(3, e^{2\tanh^{-1}(ax)}\right) + 48\mathrm{PolyLog}\left(4, e^{2\tanh^{-1}(ax)}\right)}{64c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

```
[Out] (Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog
[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 23.77, size = 1157, normalized size = 12.44

method	result	size

derivativedivides	Expression too large to display	1157
default	Expression too large to display	1157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*arctanh(a*x)^3*ln(a*x)-1/c*arctanh(a*x)^3*ln(a*x+1)-3/c*(1/6*I*Pi*csgn(
I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3-1/6*I*arctanh(a*x)
^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-2/3*a
rctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*ln(2)*arctanh(a*x)^3-1/6*I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+1/6*I*arctanh(a*x)^3*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-
a^2*x^2+1)+1))^3*arctanh(a*x)^3-1/6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+
1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/6*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+
1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/
(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-arctanh(a*x)^2*polylog(2,-(a*x+
1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/3*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)-1/3*arctanh(a*x)^3*ln(1-(
a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2))+2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^
3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*arctanh(a*x)^4-2*polylog(4,(a*x+1)/(
-a^2*x^2+1)^(1/2))-2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/6*I*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3
-1/3*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^
2*arctanh(a*x)^3+1/6*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))
*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1)
^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x + 1
)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="fricas")
[Out] integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^{3}(ax)}{ax^{2}+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(a*c*x+c),x)
[Out] Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(c + a*c*x)),x)
[Out] int(atanh(a*x)^3/(x*(c + a*c*x)), x)
```

3.131 $\int \frac{\tanh^{-1}(ax)^3}{cx+acx^2}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c}-$$

[Out] arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3/2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*polylog(4,-1+2/(a*x+1))/c

Rubi [A]
time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1607, 6079, 6095, 6203, 6207, 6745}

$$-\frac{3\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}-\frac{3\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}-\frac{3\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{2c}+\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]

[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) - (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{cx+acx^2}\,dx &= \int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx \\
&= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{(3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c} \\
&= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\right.}{1-a^2x^2}}{c} \\
&= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(-1+\right.}{2c} \\
&= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\mathrm{Li}_3\left(-1+\right.}{2c}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 86, normalized size = 0.92

$$\frac{\pi^4-32\tanh^{-1}(ax)^4+64\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)+96\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)+48\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)}{64c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

```
[Out] (Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog
[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 24.27, size = 1164, normalized size = 12.52

method	result	size
derivativedivides	Expression too large to display	1164
default	Expression too large to display	1164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-a/c*arctanh(a*x)^3*ln(a*x+1)+a/c*arctanh(a*x)^3*ln(a*x)-3*a/c*(1/6*I*
Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3-1/6*I*arct
anh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
^3-2/3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*ln(2)*arctanh(a*x)
^3-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+1/6*I*arctanh(a*
x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3-1/6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh
(a*x)^3+1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/6*I*arctanh(a*x)^3*Pi*csgn(
I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a
*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-arctanh(a*x)^2*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+
1)^(1/2))+1/3*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)-1/3*arctanh(a*x)^
3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))+2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arcta
nh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*arctanh(a*x)^4-2*polylog(4,(
a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/6*I*P
i*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctan
h(a*x)^3-1/3*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*
x^2-1))^2*arctanh(a*x)^3+1/6*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^
2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] 1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1)
^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x + 1
)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^{3}(ax)}{ax^{2}+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(a*c*x**2+c*x),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^3}{a\,c\,x^2+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(c*x + a*c*x^2),x)
```

```
[Out] int(atanh(a*x)^3/(c*x + a*c*x^2), x)
```

3.132 $\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)} \, dx$

Optimal. Leaf size=191

$$\frac{a\tanh^{-1}(ax)^3}{c}-\frac{\tanh^{-1}(ax)^3}{cx}+\frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{3a\tanh^{-1}(ax$$

```
[Out] a*arctanh(a*x)^3/c-arctanh(a*x)^3/c/x+3*a*arctanh(a*x)^2*ln(2-2/(a*x+1))/c-
a*arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))
/c+3/2*a*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3/2*a*polylog(3,-1+2/(a*x
+1))/c+3/2*a*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3/4*a*polylog(4,-1+2/(a
*x+1))/c
```

Rubi [A]
time = 0.44, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6081, 6037, 6135, 6079, 6095, 6203, 6745, 6207}

$$-\frac{3a\text{Li}_3\left(\frac{2}{ax+1}-1\right)}{2c}+\frac{3a\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}+\frac{3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}-\frac{3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{c}+\frac{3a\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{2c}+\frac{a\tanh^{-1}(ax)^3}{c}-\frac{\tanh^{-1}(ax)^3}{cx}-\frac{a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}+\frac{3a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

```
[Out] (a*ArcTanh[a*x]^3)/c - ArcTanh[a*x]^3/(c*x) + (3*a*ArcTanh[a*x]^2*Log[2 - 2
/(1 + a*x)])/c - (a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a*ArcTanh[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]^2*PolyLog[2, -1 + 2
/(1 + a*x)])/(2*c) - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*ArcTan
h[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*PolyLog[4, -1 + 2/(1 + a*
x)])/(4*c)
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx &= -\left(a\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx\right) + \frac{\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c} \\
&= -\frac{\tanh^{-1}(ax)^3}{cx} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx}{c} + \frac{(3a^2)\int \frac{\tanh^{-1}}{}}{} \\
&= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{3a\tanh^{-1}(ax)^2\mathrm{Li}_2(}{2c} \\
&= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log(}{c} \\
&= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log(}{c} \\
&= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log(}{c}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 154, normalized size = 0.81

$$\frac{a\left(\frac{i\pi^3}{8}-\frac{\pi^4}{64}-\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{ax}+\frac{1}{2}\tanh^{-1}(ax)^4+3\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)-\frac{3}{2}(-2+\tanh^{-1}(ax))\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}(-1+\tanh^{-1}(ax))\,\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-\frac{3}{4}\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

```
[Out] (a*((I/8)*Pi^3 - Pi^4/64 - ArcTanh[a*x]^3 - ArcTanh[a*x]^3/(a*x) + ArcTanh[
a*x]^4/2 + 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^3*Lo
g[1 - E^(2*ArcTanh[a*x])] - (3*(-2 + ArcTanh[a*x])*ArcTanh[a*x]*PolyLog[2,
E^(2*ArcTanh[a*x])])/2 + (3*(-1 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x
])])/2 - (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 26.78, size = 1339, normalized size = 7.01

method	result	size
derivativedivides	Expression too large to display	1339
default	Expression too large to display	1339

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/c*arctanh(a*x)^3/a/x-1/c*arctanh(a*x)^3*ln(a*x)+1/c*arctanh(a*x)^3*ln
(a*x+1)-3/c*(-1/6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
```

```
(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arcta
nh(a*x)^3+1/6*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
*arctanh(a*x)^3+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,(a*x+1
)/(-a^2*x^2+1)^(1/2))+2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(4,
(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^
(1/2))-2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*
polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*ar
ctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/6*arctanh(a*x)^4+1/3*arc
tanh(a*x)^3-arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*
ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+
1)-1)+1/3*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*arctanh(a*x)^
3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)
^3+1/3*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)^2*arctanh(a*x)^3+1/6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/
6*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/6*I*Pi*csgn(I/((a*x+1)^2/(-
a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^2*arctanh(a*x)^3+1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^3*arctanh(a*x)^3+1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh
(a*x)^3+1/6*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))^3*arctanh(a*x)^3+1/3*ln(2)*arctanh(a*x)^3+2/3*arctanh(a*x)^3*ln((a*x+1)
/(-a^2*x^2+1)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(a*x*log(a*x + 1) - 1)*log(-a*x + 1)^3/(c*x) + 1/8*integrate(((a*x - 1
)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 +
a*x - (a^3*x^3 + a^2*x^2 + a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x
^4 - c*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="fricas")
[Out] integral(arctanh(a*x)^3/(a*c*x^3 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^3+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(a*c*x+c),x)
[Out] Integral(atanh(a*x)**3/(a*x**3 + x**2), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^3}{x^2\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(c + a*c*x)),x)
[Out] int(atanh(a*x)^3/(x^2*(c + a*c*x)), x)
```

3.133 $\int \frac{\tanh^{-1}(ax)^3}{x^3(c+acx)}\,dx$

Optimal. Leaf size=305

$$\frac{3a^2\tanh^{-1}(ax)^2}{2c}-\frac{3a\tanh^{-1}(ax)^2}{2cx}-\frac{a^2\tanh^{-1}(ax)^3}{2c}-\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}+\frac{3a^2\tanh^{-1}(ax)\log(2-}{c}$$

```
[Out] 3/2*a^2*arctanh(a*x)^2/c-3/2*a*arctanh(a*x)^2/c/x-1/2*a^2*arctanh(a*x)^3/c-
1/2*arctanh(a*x)^3/c/x^2+a*arctanh(a*x)^3/c/x+3*a^2*arctanh(a*x)*ln(2-2/(a*
x+1))/c-3*a^2*arctanh(a*x)^2*ln(2-2/(a*x+1))/c+a^2*arctanh(a*x)^3*ln(2-2/(a
*x+1))/c-3/2*a^2*polylog(2,-1+2/(a*x+1))/c+3*a^2*arctanh(a*x)*polylog(2,-1+
2/(a*x+1))/c-3/2*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c+3/2*a^2*polyl
og(3,-1+2/(a*x+1))/c-3/2*a^2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*a^2
*polylog(4,-1+2/(a*x+1))/c
```

Rubi [A]
time = 0.52, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6081, 6037, 6129, 6135, 6079, 2497, 6095, 6203, 6745, 6207}

$$-\frac{3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)}{2c}+\frac{3a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)}{2c}-\frac{3a^2\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{4c}-\frac{3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{2c}+\frac{3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{c}-\frac{3a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{2c}-\frac{a^2\tanh^{-1}(ax)^3}{2c}+\frac{3a^2\tanh^{-1}(ax)^2}{2c}+\frac{a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}-\frac{3a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{c}+\frac{3a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{c}-\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}-\frac{3a\tanh^{-1}(ax)^2}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

```
[Out] (3*a^2*ArcTanh[a*x]^2)/(2*c) - (3*a*ArcTanh[a*x]^2)/(2*c*x) - (a^2*ArcTanh[
a*x]^3)/(2*c) - ArcTanh[a*x]^3/(2*c*x^2) + (a*ArcTanh[a*x]^3)/(c*x) + (3*a^
2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - (3*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1
 + a*x)])/c + (a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a^2*PolyLog[
2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*
x)])/c - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2
*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 +
2/(1 + a*x)])/(2*c) - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3(c+acx)}\,dx = -\left(a\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx\right)+\frac{\int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{2cx^2}+a^2\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx-\frac{a\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c}+\frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx}{2c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}+\frac{a^2\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c}+\frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2}\,d}{2c}$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{2cx}-\frac{a^2\tanh^{-1}(ax)^3}{2c}-\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}+\frac{a^2\tanh^{-1}(ax)^3}{}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c}-\frac{3a\tanh^{-1}(ax)^2}{2cx}-\frac{a^2\tanh^{-1}(ax)^3}{2c}-\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c}-\frac{3a\tanh^{-1}(ax)^2}{2cx}-\frac{a^2\tanh^{-1}(ax)^3}{2c}-\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c}-\frac{3a\tanh^{-1}(ax)^2}{2cx}-\frac{a^2\tanh^{-1}(ax)^3}{2c}-\frac{\tanh^{-1}(ax)^3}{2cx^2}+\frac{a\tanh^{-1}(ax)^3}{cx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.39, size = 222, normalized size = 0.73

$$\frac{a^2\left(-8i\pi^3+\pi^4+96\tanh^{-1}(ax)^2-\frac{96\tanh^{-1}(ax)^2}{ax}+96\tanh^{-1}(ax)^3-\frac{32\tanh^{-1}(ax)^3}{a^2x^2}+\frac{64\tanh^{-1}(ax)^3}{ax}-32\tanh^{-1}(ax)^4+192\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)-192\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)+64\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)-96\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)+96\left(-2+\tanh^{-1}(ax)\right)\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+96\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)+48\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)\right)}{64c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

```
[Out] (a^2*((-8*I)*Pi^3 + Pi^4 + 96*ArcTanh[a*x]^2 - (96*ArcTanh[a*x]^2)/(a*x) +
96*ArcTanh[a*x]^3 - (32*ArcTanh[a*x]^3)/(a^2*x^2) + (64*ArcTanh[a*x]^3)/(a*
x) - 32*ArcTanh[a*x]^4 + 192*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 19
2*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^
(2*ArcTanh[a*x])] - 96*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*(-2 + ArcTanh[a
*x])*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 96*PolyLog[3, E^(2*ArcTa
nh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4,
E^(2*ArcTanh[a*x])]))/(64*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(287) = 574$.
time = 32.82, size = 602, normalized size = 1.97

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)-3ax)(ax-1)}{2c\,a^2x^2}-\frac{\operatorname{arctanh}(ax)^4}{2c}+\frac{\operatorname{arctanh}(ax)^3\ln\left(1+\frac{ax}{\sqrt{-a^2}}\right)}{c}\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)-3ax)(ax-1)}{2c\,a^2x^2}-\frac{\operatorname{arctanh}(ax)^4}{2c}+\frac{\operatorname{arctanh}(ax)^3\ln\left(1+\frac{ax}{\sqrt{-a^2}}\right)}{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/c*arctanh(a*x)^2*(a*x*arctanh(a*x)-arctanh(a*x)-3*a*x)*(a*x-1)/a^
2/x^2-1/2/c*arctanh(a*x)^4+1/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/
2))+3/c*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a
*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,-(a*x+1)/(-a^2*x^2
+1)^(1/2))+1/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(
a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(3,(a*
x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/c*arct
anh(a*x)^2+3/c*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*polylog(2,
-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/
2))+3/c*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)^3/c-3/c*arctan
h(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(2,-(a*x+
1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/c*arcta
nh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(2,(a*x+
1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(2*a^2*x^2*log(a*x + 1) - 2*a*x + 1)*log(-a*x + 1)^3/(c*x^2) - 1/8*int
egrate(-1/2*(2*(a*x - 1)*log(a*x + 1)^3 - 6*(a*x - 1)*log(a*x + 1)^2*log(-a
*x + 1) + 3*(2*a^3*x^3 + a^2*x^2 - a*x - 2*(a^4*x^4 + a^3*x^3 - a*x + 1)*lo
g(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x^5 - c*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^4 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}^{3}(ax)}{ax^{4}+x^{3}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**4 + x**3), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(c + a*c*x)),x)
```

```
[Out] int(atanh(a*x)^3/(x^3*(c + a*c*x)), x)
```

3.134 $\int \frac{x^2 \tanh^{-1}(ax)^4}{c-acx}\,dx$

Optimal. Leaf size=384

$$-\frac{2\tanh^{-1}(ax)^3}{a^3c}-\frac{2x\tanh^{-1}(ax)^3}{a^2c}-\frac{\tanh^{-1}(ax)^4}{2a^3c}-\frac{x\tanh^{-1}(ax)^4}{a^2c}-\frac{x^2\tanh^{-1}(ax)^4}{2ac}+\frac{6\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3c}$$

```
[Out] -2*arctanh(a*x)^3/a^3/c-2*x*arctanh(a*x)^3/a^2/c-1/2*arctanh(a*x)^4/a^3/c-x
*arctanh(a*x)^4/a^2/c-1/2*x^2*arctanh(a*x)^4/a/c+6*arctanh(a*x)^2*ln(2/(-a*
x+1))/a^3/c+4*arctanh(a*x)^3*ln(2/(-a*x+1))/a^3/c+arctanh(a*x)^4*ln(2/(-a*x
+1))/a^3/c+6*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^3/c+6*arctanh(a*x)^2*po
lylog(2,1-2/(-a*x+1))/a^3/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a^3/c-
3*polylog(3,1-2/(-a*x+1))/a^3/c-6*arctanh(a*x)*polylog(3,1-2/(-a*x+1))/a^3/
c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a^3/c+3*polylog(4,1-2/(-a*x+1))/
a^3/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+1))/a^3/c-3/2*polylog(5,1-2/(-a*x+
1))/a^3/c
```

Rubi [A]

time = 0.60, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6077, 6037, 6127, 6021, 6131, 6055, 6095, 6205, 6745, 6209}

$$-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{a^3c}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{a^3c}-\frac{3\text{Li}_5\left(1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^3c}+\frac{6\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^3c}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^3c}+\frac{6\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}-\frac{6\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3c}-\frac{\tanh^{-1}(ax)^4}{2a^3c}-\frac{2\tanh^{-1}(ax)^3}{a^3c}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{a^3c}+\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^3c}+\frac{6\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^3c}-\frac{x\tanh^{-1}(ax)^4}{a^2c}-\frac{2x\tanh^{-1}(ax)^3}{a^2c}-\frac{x^2\tanh^{-1}(ax)^4}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] (-2*ArcTanh[a*x]^3)/(a^3*c) - (2*x*ArcTanh[a*x]^3)/(a^2*c) - ArcTanh[a*x]^4
/(2*a^3*c) - (x*ArcTanh[a*x]^4)/(a^2*c) - (x^2*ArcTanh[a*x]^4)/(2*a*c) + (6
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(a^3*c) + (4*ArcTanh[a*x]^3*Log[2/(1 - a*
x)])/(a^3*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]*
PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(a^3*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c)
 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(a^3*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 -
 2/(1 - a*x)])/(a^3*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^
3*c) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]*PolyLog[4,
 1 - 2/(1 - a*x)])/(a^3*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^3*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\int \frac{x \tanh^{-1}(ax)^4}{c-acx}\,dx}{a} - \frac{\int x \tanh^{-1}(ax)^4\,dx}{ac}$$

$$= -\frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx}{a^2} + \frac{2\int \frac{x^2 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{c} - \frac{\int \tanh^{-1}(ax)^4\,dx}{a^2c}$$

$$= -\frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\tanh^{-1}(ax)^4 \log\left(\frac{2}{1-ax}\right)}{a^3c} - \frac{2\int \tanh^{-1}(ax)^3\,dx}{a^2c}$$

$$= -\frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\tanh^{-1}(ax}{}$$

$$= -\frac{2\tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(}{2ac}$$

$$= -\frac{2\tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(}{2ac}$$

$$= -\frac{2\tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(}{2ac}$$

$$= -\frac{2\tanh^{-1}(ax)^3}{a^3c} - \frac{2x \tanh^{-1}(ax)^3}{a^2c} - \frac{\tanh^{-1}(ax)^4}{2a^3c} - \frac{x \tanh^{-1}(ax)^4}{a^2c} - \frac{x^2 \tanh^{-1}(}{2ac}$$

Mathematica [A]
time = 0.27, size = 233, normalized size = 0.61

$$-\frac{-2\tanh^{-1}(ax)^3 + 2ax\tanh^{-1}(ax)^3 - \tanh^{-1}(ax)^4 + ax\tanh^{-1}(ax)^4 - \frac{1}{2}(1-a^2x^2)\tanh^{-1}(ax)^4 - \frac{2}{5}\tanh^{-1}(ax)^5 - 6\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right) - 4\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)^4\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\left(3+3\tanh^{-1}(ax)+\tanh^{-1}(ax)^2\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 3\left(1+\tanh^{-1}(ax)\right)^2\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 3\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right) + \frac{3}{2}\text{PolyLog}\left(5,-e^{-2\tanh^{-1}(ax)}\right)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -((-2*ArcTanh[a*x]^3 + 2*a*x*ArcTanh[a*x]^3 - ArcTanh[a*x]^4 + a*x*ArcTanh[
a*x]^4 - ((1 - a^2*x^2)*ArcTanh[a*x]^4)/2 - (2*ArcTanh[a*x]^5)/5 - 6*ArcTan
h[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcT
anh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*ArcTanh[a*x]*(
3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*(
1 + ArcTanh[a*x])^2*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*
ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])] + (3*PolyL
og[5, -E^(-2*ArcTanh[a*x])])/2)/(a^3*c))
```

Maple [A]
time = 9.95, size = 442, normalized size = 1.15

method	result
derivativedivides	$-\frac{\frac{\operatorname{arctanh}(ax)^3(ax \operatorname{arctanh}(ax)+3\operatorname{arctanh}(ax)+4)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4 \ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}+\frac{2\operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{c}}{}$
default	$-\frac{\frac{\operatorname{arctanh}(ax)^3(ax \operatorname{arctanh}(ax)+3\operatorname{arctanh}(ax)+4)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4 \ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}+\frac{2\operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^4/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2/c*arctanh(a*x)^3*(a*x*arctanh(a*x)+3*arctanh(a*x)+4)*(a*x-1)+1/
c*arctanh(a*x)^4*ln((a*x+1)^2/(-a^2*x^2+1)+1)+2/c*arctanh(a*x)^3*polylog(2,
-(a*x+1)^2/(-a^2*x^2+1))-3/c*arctanh(a*x)^2*polylog(3,-(a*x+1)^2/(-a^2*x^2+
1))+3/c*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/2/c*polylog(5,-(a
*x+1)^2/(-a^2*x^2+1))-4*arctanh(a*x)^3/c+6/c*arctanh(a*x)^2*ln((a*x+1)^2/(-
a^2*x^2+1)+1)+6/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/c*polyl
og(3,-(a*x+1)^2/(-a^2*x^2+1))-2/c*arctanh(a*x)^4+4/c*arctanh(a*x)^3*ln((a*x
+1)^2/(-a^2*x^2+1)+1)+6/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))
-6/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/c*polylog(4,-(a*x+1)
^2/(-a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/320*(4*log(-a*x + 1)^5 + 5*(2*log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 6*lo
g(-a*x + 1)^2 - 6*log(-a*x + 1) + 3)*(a*x - 1)^2 + 40*(log(-a*x + 1)^4 - 4*
```

```
log(-a*x + 1)^3 + 12*log(-a*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a
^3*c) + 1/16*integrate(-(x^2*log(a*x + 1)^4 - 4*x^2*log(a*x + 1)^3*log(-a*x
 + 1) + 6*x^2*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*x^2*log(a*x + 1)*log(-a*x
+ 1)^3)/(a*c*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
[Out] integral(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2 \operatorname{atanh}^4(ax)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**4/(-a*c*x+c),x)
[Out] -Integral(x**2*atanh(a*x)**4/(a*x - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
[Out] integrate(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^4)/(c - a*c*x),x)
[Out] int((x^2*atanh(a*x)^4)/(c - a*c*x), x)
```

3.135 $\int \frac{x \tanh^{-1}(ax)^4}{c-acx} \, dx$

Optimal. Leaf size=261

$$-\frac{\tanh^{-1}(ax)^4}{a^2c}-\frac{x\tanh^{-1}(ax)^4}{ac}+\frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{6\tanh^{-1}(ax)^2\text{PolyLog}(}{a^2c}$$

```
[Out] -arctanh(a*x)^4/a^2/c-x*arctanh(a*x)^4/a/c+4*arctanh(a*x)^3*ln(2/(-a*x+1))/
a^2/c+arctanh(a*x)^4*ln(2/(-a*x+1))/a^2/c+6*arctanh(a*x)^2*polylog(2,1-2/(-
a*x+1))/a^2/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a^2/c-6*arctanh(a*x)
*polylog(3,1-2/(-a*x+1))/a^2/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a^2
/c+3*polylog(4,1-2/(-a*x+1))/a^2/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+1))/a
^2/c-3/2*polylog(5,1-2/(-a*x+1))/a^2/c
```

Rubi [A]
time = 0.36, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6077, 6021, 6131, 6055, 6095, 6205, 6209, 6745}

$$\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{a^2c}-\frac{3\text{Li}_5\left(1-\frac{2}{1-ax}\right)}{2a^2c}+\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^2c}+\frac{6\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2c}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2c}-\frac{6\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2c}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2c}-\frac{\tanh^{-1}(ax)^4}{a^2c}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{a^2c}+\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^2c}-\frac{x\tanh^{-1}(ax)^4}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -(ArcTanh[a*x]^4/(a^2*c)) - (x*ArcTanh[a*x]^4)/(a*c) + (4*ArcTanh[a*x]^3*Lo
g[2/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^2*c) + (6*Ar
cTanh[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) + (2*ArcTanh[a*x]^3*PolyL
og[2, 1 - 2/(1 - a*x)])/(a^2*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x
)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^2*c) + (3*P
olyLog[4, 1 - 2/(1 - a*x)])/(a^2*c) + (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 -
 a*x)])/(a^2*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^2*c)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6077

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d + e*
x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx}{a} - \frac{\int \tanh^{-1}(ax)^4\,dx}{ac}$$

$$= -\frac{x\tanh^{-1}(ax)^4}{ac} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{4\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{c} - \frac{4\int \frac{\tanh^{-1}(ax)^3\log\left(\frac{}{1-}\right.}{1-a^2x^2}}{ac}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(1-\frac{}{1}\right.}{a^2c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right.}{a^2c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right.}{a^2c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right.}{a^2c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right.}{a^2c}$$

Mathematica [A]
time = 0.17, size = 172, normalized size = 0.66

$$-\frac{-\tanh^{-1}(ax)^4 + ax\tanh^{-1}(ax)^4 - \frac{2}{5}\tanh^{-1}(ax)^5 - 4\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)^4\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)^2(3+\tanh^{-1}(ax))\,\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)(2+\tanh^{-1}(ax))\,\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 3\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right) + \frac{3}{2}\mathrm{PolyLog}\left(5,-e^{-2\tanh^{-1}(ax)}\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -((-ArcTanh[a*x]^4 + a*x*ArcTanh[a*x]^4 - (2*ArcTanh[a*x]^5)/5 - 4*ArcTanh[
a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[
a*x])] + 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, -E^(-2*ArcTanh[a*x]
)] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3
*PolyLog[4, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTan
h[a*x])] + (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a^2*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.06, size = 381, normalized size = 1.46

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^4 ax}{c} - \frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c} + \frac{2\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right) - 3\operatorname{arctanh}(ax)^2\operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right) + 3\operatorname{arct}}{}$

default	$\frac{-\frac{\operatorname{arctanh}(ax)^4 ax}{c}-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{2\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)-3\operatorname{arctanh}(ax)^2\operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right)+3\operatorname{a}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^4/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/c*arctanh(a*x)^4*a*x-1/c*arctanh(a*x)^4*ln(a*x-1)+4/c*(1/2*arctan
h(a*x)^3*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/4*arctanh(a*x)^2*polylog(3,-(
a*x+1)^2/(-a^2*x^2+1))+3/4*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-
3/8*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))+1/4*I*Pi*arctanh(a*x)^4-1/4*I*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^4-1/4*arctanh(a*x)^4+arctanh
(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^
2/(-a^2*x^2+1))-3/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4*pol
ylog(4,-(a*x+1)^2/(-a^2*x^2+1))+1/4*ln(2)*arctanh(a*x)^4+1/4*I*Pi*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*(log(-a*x + 1)^5 + 5*(log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 12*log(-a
*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a^2*c) + 1/16*integrate(-(x*
log(a*x + 1)^4 - 4*x*log(a*x + 1)^3*log(-a*x + 1) + 6*x*log(a*x + 1)^2*log(
-a*x + 1)^2 - 4*x*log(a*x + 1)*log(-a*x + 1)^3)/(a*c*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x\operatorname{atanh}^4(ax)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(x*atanh(a*x)**4/(a*x - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\mathrm{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^4)/(c - a*c*x),x)
```

```
[Out] int((x*atanh(a*x)^4)/(c - a*c*x), x)
```

3.136 $\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx$

Optimal. Leaf size=131

$$\frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{ac}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{ac}+\frac{3\tanh$$

[Out] arctanh(a*x)^4*ln(2/(-a*x+1))/a/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+1))/a/c-3/2*polylog(5,1-2/(-a*x+1))/a/c

Rubi [A]
time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6055, 6095, 6205, 6209, 6745}

$$-\frac{3\text{Li}_5\left(1-\frac{2}{1-ax}\right)}{2ac}+\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{ac}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{ac}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{ac}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{ac}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]^4/(c - a*c*x),x]

[Out] (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x)])/(a*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a*c) + (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 - a*x)])/(a*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a*c)

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx &= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} - \frac{4\int \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c} \\
&= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{6\int \frac{\tanh^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c} \\
&= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} \\
&= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} \\
&= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 112, normalized size = 0.85

$$-\frac{-\frac{2}{5}\tanh^{-1}(ax)^5-\tanh^{-1}(ax)^4\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)+\frac{3}{2}\mathrm{PolyLog}\left(5,-e^{-2\tanh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^4/(c - a*c*x),x]
```

```
[Out] -(((-2*ArcTanh[a*x]^5)/5 - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*
ArcTanh[a*x]^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]^2*PolyLog[
3, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])]
+ (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.14, size = 228, normalized size = 1.74

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^3-i\pi\operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2+i\pi+\ln(2)\right)\operatorname{arctanh}(ax)^4+2\operatorname{arctanh}(ax)^3\operatorname{polylog}}{}}{a}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^3-i\pi\operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2+i\pi+\ln(2)\right)\operatorname{arctanh}(ax)^4+2\operatorname{arctanh}(ax)^3\operatorname{polylog}}{}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/c*arctanh(a*x)^4*ln(a*x-1)+4/c*(1/4*(I*Pi*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))^3-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi+ln(2))*arctanh(a
*x)^4+1/2*arctanh(a*x)^3*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/4*arctanh(a*x
)^2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4*arctanh(a*x)*polylog(4,-(a*x+1)^
2/(-a^2*x^2+1))-3/8*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/(a*c) + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x +
 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log
(-a*x + 1)^3)/(a*c*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/(a*c*x - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(c - a*c*x),x)
```

```
[Out] int(atanh(a*x)^4/(c - a*c*x), x)
```

3.137 $\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{c}$$

```
[Out] arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))
/c-3*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c+3*arctanh(a*x)*polylog(4,-1+
2/(-a*x+1))/c-3/2*polylog(5,-1+2/(-a*x+1))/c
```

Rubi [A]

time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6079, 6095, 6205, 6209, 6745}

$$-\frac{3\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{3\text{Li}_3\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^2}{c}+\frac{3\text{Li}_4\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx = \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(4a)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{(6a)\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\right.}{1-a^2x^2}}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\right.}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\right.}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\right.}{c}$$

Mathematica [A]
time = 0.09, size = 102, normalized size = 0.86

$$\frac{\tanh^{-1}(ax)^4\log\left(1-e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{3\tanh^{-1}(ax)\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\text{PolyLog}\left(5,e^{2\tanh^{-1}(ax)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog[
2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x])
```

```
])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5, E^
(2*ArcTanh[a*x])])/(2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.68, size = 754, normalized size = 6.39

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^4\ln(ax)}{c}-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{-\operatorname{arctanh}(ax)^4\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)+\frac{\left(-2i\pi\operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2+2\cdots\right.}{}}{}$
default	$\frac{\operatorname{arctanh}(ax)^4\ln(ax)}{c}-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{-\operatorname{arctanh}(ax)^4\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)+\frac{\left(-2i\pi\operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2+2\cdots\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*arctanh(a*x)^4*ln(a*x)-1/c*arctanh(a*x)^4*ln(a*x-1)+4/c*(-1/4*arctanh(a
*x)^4*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/8*(-2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2
+1)+1))^2+2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+I*Pi*csgn(I*((a*x+1)^
2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a
^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-I*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3+2*I*Pi+2*ln(2))*arctanh(a*x)^4+1/4*arctanh(a*x)^4*ln(1-(a
*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/
2))-3*arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*p
olylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/4*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))+6*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polyl
og(5,-(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^
3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-a*
x + 1)^3)/(a*c*x^2 - c*x), x)
```

Fricas [A]
time = 0.35, size = 155, normalized size = 1.31

$$\frac{\log\left(\frac{2\,ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+4\,\mathrm{Li}_2\left(-\frac{2\,ax}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3-12\,\log\left(-\frac{ax+1}{ax-1}\right)^2\mathrm{polylog}\left(3,-\frac{ax+1}{ax-1}\right)+24\,\log\left(-\frac{ax+1}{ax-1}\right)\mathrm{polylog}\left(4,-\frac{ax+1}{ax-1}\right)-24\,\mathrm{polylog}\left(5,-\frac{ax+1}{ax-1}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(log(2*a*x/(a*x - 1))*log(-(a*x + 1)/(a*x - 1))^4 + 4*dilog(-2*a*x/(a*
x - 1) + 1)*log(-(a*x + 1)/(a*x - 1))^3 - 12*log(-(a*x + 1)/(a*x - 1))^2*po
lylog(3, -(a*x + 1)/(a*x - 1)) + 24*log(-(a*x + 1)/(a*x - 1))*polylog(4, -(
a*x + 1)/(a*x - 1)) - 24*polylog(5, -(a*x + 1)/(a*x - 1)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^2-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^4}{x\,\left(c-a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(x*(c - a*c*x)),x)
```

```
[Out] int(atanh(a*x)^4/(x*(c - a*c*x)), x)
```

3.138 $\int \frac{\tanh^{-1}(ax)^4}{cx-acx^2}\,dx$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{c}$$

```
[Out] arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))
/c-3*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c+3*arctanh(a*x)*polylog(4,-1+
2/(-a*x+1))/c-3/2*polylog(5,-1+2/(-a*x+1))/c
```

Rubi [A]

time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1607, 6079, 6095, 6205, 6209, 6745}

$$-\frac{3\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{3\text{Li}_3\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^2}{c}+\frac{3\text{Li}_4\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{cx-acx^2}\,dx = \int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(4a)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{(6a)\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\cdots\right.}{1-a^2x^2}}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\cdots\right.}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\cdots\right.}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\cdots\right.}{c}$$

Mathematica [A]
time = 0.02, size = 102, normalized size = 0.86

$$\frac{\tanh^{-1}(ax)^4\log\left(1-e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{3\tanh^{-1}(ax)\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\text{PolyLog}\left(5,e^{2\tanh^{-1}(ax)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog[
2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x])
])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5, E^
(2*ArcTanh[a*x])])/(2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.93, size = 761, normalized size = 6.45

method	result
derivativedivides	$\frac{\frac{a \operatorname{arctanh}(ax)^4 \ln(ax)}{c} - \frac{a \operatorname{arctanh}(ax)^4 \ln(ax-1)}{c} + \frac{4a\left(-\frac{\operatorname{arctanh}(ax)^4 \ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{4} + \frac{\left(-2i\pi \operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2 + 2i\pi \operatorname{csgn}\right.}{}\right.}{}}{}$
default	$\frac{\frac{a \operatorname{arctanh}(ax)^4 \ln(ax)}{c} - \frac{a \operatorname{arctanh}(ax)^4 \ln(ax-1)}{c} + \frac{4a\left(-\frac{\operatorname{arctanh}(ax)^4 \ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{4} + \frac{\left(-2i\pi \operatorname{csgn}\left(\frac{i}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2 + 2i\pi \operatorname{csgn}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/(-a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a/c*arctanh(a*x)^4*ln(a*x)-a/c*arctanh(a*x)^4*ln(a*x-1)+4*a/c*(-1/4*ar
ctanh(a*x)^4*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/8*(-2*I*Pi*csgn(I/((a*x+1)^2/(-
a^2*x^2+1)+1))^2+2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+I*Pi*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-I*Pi*csgn(I*((a*x+1)^2/(-a^
2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
2-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^3+2*I*Pi+2*ln(2))*arctanh(a*x)^4+1/4*arctanh(a*x)^4*
ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))-3*arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh
(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(5,(a*x+1)/(-a^2*x^2+1
)^(1/2))+1/4*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(3,-(a*x+1)
/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
6*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^
3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-a*
x + 1)^3)/(a*c*x^2 - c*x), x)
```

Fricas [A]
time = 0.41, size = 155, normalized size = 1.31

$$\frac{\log\left(\frac{2\,ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+4\,\mathrm{Li}_2\left(-\frac{2\,ax}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}-12\,\log\left(-\frac{ax+1}{ax-1}\right)^{2}\mathrm{polylog}\left(3,-\frac{ax+1}{ax-1}\right)+24\,\log\left(-\frac{ax+1}{ax-1}\right)\mathrm{polylog}\left(4,-\frac{ax+1}{ax-1}\right)-24\,\mathrm{polylog}\left(5,-\frac{ax+1}{ax-1}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] 1/16*(log(2*a*x/(a*x - 1))*log(-(a*x + 1)/(a*x - 1))^4 + 4*dilog(-2*a*x/(a*
x - 1) + 1)*log(-(a*x + 1)/(a*x - 1))^3 - 12*log(-(a*x + 1)/(a*x - 1))^2*po
lylog(3, -(a*x + 1)/(a*x - 1)) + 24*log(-(a*x + 1)/(a*x - 1))*polylog(4, -(
a*x + 1)/(a*x - 1)) - 24*polylog(5, -(a*x + 1)/(a*x - 1)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^{4}(ax)}{ax^{2}-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/(-a*c*x**2+c*x),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^4}{c\,x-a\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(c*x - a*c*x^2),x)
```

```
[Out] int(atanh(a*x)^4/(c*x - a*c*x^2), x)
```

3.139 $\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx$

Optimal. Leaf size=239

$$\frac{a\tanh^{-1}(ax)^4}{c}-\frac{\tanh^{-1}(ax)^4}{cx}+\frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}+\frac{4a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c}+\frac{2a\tanh^{-1}(ax)}{}$$

```
[Out] a*arctanh(a*x)^4/c-arctanh(a*x)^4/c/x+a*arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+4
*a*arctanh(a*x)^3*ln(2-2/(a*x+1))/c+2*a*arctanh(a*x)^3*polylog(2,-1+2/(-a*x
+1))/c-6*a*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3*a*arctanh(a*x)^2*poly
log(3,-1+2/(-a*x+1))/c-6*a*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3*a*arcta
nh(a*x)*polylog(4,-1+2/(-a*x+1))/c-3*a*polylog(4,-1+2/(a*x+1))/c-3/2*a*poly
log(5,-1+2/(-a*x+1))/c
```

Rubi [A]
time = 0.37, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6081, 6037, 6135, 6079, 6095, 6203, 6207, 6745, 6205, 6209}

$$-\frac{3a\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{c}-\frac{3a\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2a\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{6a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{c}-\frac{3a\text{Li}_3\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^2}{c}-\frac{6a\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{c}+\frac{3a\text{Li}_4\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{a\tanh^{-1}(ax)^4}{c}-\frac{\tanh^{-1}(ax)^4}{cx}+\frac{a\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{c}+\frac{4a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

```
[Out] (a*ArcTanh[a*x]^4)/c - ArcTanh[a*x]^4/(c*x) + (a*ArcTanh[a*x]^4*Log[2 - 2/(
1 - a*x)])/c + (4*a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a*ArcTanh[a
*x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/c - (3*a*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (6
*a*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]*PolyLog
[4, -1 + 2/(1 - a*x)])/c - (3*a*PolyLog[4, -1 + 2/(1 + a*x)])/c - (3*a*Poly
Log[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx = a\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx + \frac{\int \frac{\tanh^{-1}(ax)^4}{x^2}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{(4a)\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx}{c} - \frac{(4a^2)\int \frac{\tanh^{-1}(}{}}{}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2a\tanh^{-1}(ax)^3\mathrm{Li}_2(-}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2\right.}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2\right.}{c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 172, normalized size = 0.72

$$-\frac{a\left(-\frac{\pi^4}{16}+\frac{i\pi^5}{160}+\tanh^{-1}(ax)^4+\frac{\tanh^{-1}(ax)^4}{ax}-4\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^4\log\left(1-e^{2\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)^2\left(3+\tanh^{-1}(ax)\right)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)\left(2+\tanh^{-1}(ax)\right)\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-3\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\mathrm{PolyLog}\left(5,e^{2\tanh^{-1}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

```
[Out] -((a*(-1/16*Pi^4 + (I/160)*Pi^5 + ArcTanh[a*x]^4 + ArcTanh[a*x]^4/(a*x) - 4
*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*A
rcTanh[a*x])] - 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, E^(2*ArcTanh
[a*x])] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x])]
- 3*PolyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh
[a*x])] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(237) = 474$.
time = 12.43, size = 573, normalized size = 2.40

method	result
derivativedivides	$a\left(\frac{\operatorname{arctanh}(ax)^4(ax-1)}{cax}+\frac{\operatorname{arctanh}(ax)^4\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}+\frac{4\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{c}\right.$
default	$a\left(\frac{\operatorname{arctanh}(ax)^4(ax-1)}{cax}+\frac{\operatorname{arctanh}(ax)^4\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}+\frac{4\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x^2/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/c*arctanh(a*x)^4/a/x*(a*x-1)+1/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))+4/c*arctanh(a*x)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*a
rctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*poly
log(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*
polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,(a*x+1)
/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2)
)-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))-2/c*arctanh(a*x)^4+4/c*arctanh
(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)^2*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-24/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^
(1/2))+24/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*ln(1-
(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))-24/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*pol
ylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*(a*x*log(-a*x + 1)^5 + 5*log(-a*x + 1)^4)/(c*x) + 1/16*integrate(-(lo
g(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x +
 1)^2 - 4*(a*x + log(a*x + 1))*log(-a*x + 1)^3)/(a*c*x^3 - c*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^3 - c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^{4}(ax)}{ax^{3}-x^{2}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x**2/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**3 - x**2), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^4}{x^2\,\left(c-a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(x^2*(c - a*c*x)),x)
```

```
[Out] int(atanh(a*x)^4/(x^2*(c - a*c*x)), x)
```

3.140 $\int \frac{\tanh^{-1}(ax)^4}{x^3(c-acx)}\,dx$

Optimal. Leaf size=380

$$\frac{2a^2\tanh^{-1}(ax)^3}{c}-\frac{2a\tanh^{-1}(ax)^3}{cx}+\frac{3a^2\tanh^{-1}(ax)^4}{2c}-\frac{\tanh^{-1}(ax)^4}{2cx^2}-\frac{a\tanh^{-1}(ax)^4}{cx}+\frac{a^2\tanh^{-1}(ax)^4\log\left(2\right.}{c}$$

```
[Out] 2*a^2*arctanh(a*x)^3/c-2*a*arctanh(a*x)^3/c/x+3/2*a^2*arctanh(a*x)^4/c-1/2*
arctanh(a*x)^4/c/x^2-a*arctanh(a*x)^4/c/x+a^2*arctanh(a*x)^4*ln(2-2/(-a*x+1
))/c+6*a^2*arctanh(a*x)^2*ln(2-2/(a*x+1))/c+4*a^2*arctanh(a*x)^3*ln(2-2/(a*
x+1))/c+2*a^2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))/c-6*a^2*arctanh(a*x)*
polylog(2,-1+2/(a*x+1))/c-6*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3*
a^2*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c-3*a^2*polylog(3,-1+2/(a*x+1))
/c-6*a^2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3*a^2*arctanh(a*x)*polylog(
4,-1+2/(-a*x+1))/c-3*a^2*polylog(4,-1+2/(a*x+1))/c-3/2*a^2*polylog(5,-1+2/(
-a*x+1))/c
```

Rubi [A]

time = 0.66, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {6081, 6037, 6129, 6135, 6079, 6095, 6203, 6745, 6207, 6205, 6209}

$$-\frac{3a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{Li}_4\left(\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{Li}_5\left(\frac{2}{1-ax}-1\right)}{2c}+\frac{2a^2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^3}{c}-\frac{6a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2}{c}-\frac{3a^2\text{Li}_3\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)^2}{c}-\frac{6a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{c}-\frac{6a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)}{c}+\frac{3a^2\text{Li}_4\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)}{c}+\frac{3a^2\tanh^{-1}(ax)^4}{2c}+\frac{2a^2\tanh^{-1}(ax)^3}{c}+\frac{a^2\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^4}{c}+\frac{4a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3}{c}+\frac{6a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2}{c}-\frac{\tanh^{-1}(ax)^4}{2cx^2}-\frac{a\tanh^{-1}(ax)^4}{cx}-\frac{2a\tanh^{-1}(ax)^3}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

```
[Out] (2*a^2*ArcTanh[a*x]^3)/c - (2*a*ArcTanh[a*x]^3)/(c*x) + (3*a^2*ArcTanh[a*x]
^4)/(2*c) - ArcTanh[a*x]^4/(2*c*x^2) - (a*ArcTanh[a*x]^4)/(c*x) + (a^2*ArcT
anh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (6*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a
*x)])/c + (4*a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a^2*ArcTanh[a*
x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[2, -1 +
2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/c - (
3*a^2*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[3, -1
 + 2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c +
(3*a^2*ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[4, -1
+ 2/(1 + a*x)])/c - (3*a^2*PolyLog[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6081

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^4}{x^3(c-acx)}\,dx &= a\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx + \frac{\int \frac{\tanh^{-1}(ax)^4}{x^3}\,dx}{c} \\
&= -\frac{\tanh^{-1}(ax)^4}{2cx^2} + a^2\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx + \frac{a\int \frac{\tanh^{-1}(ax)^4}{x^2}\,dx}{c} + \frac{(2a)\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)}\,dx}{c} \\
&= -\frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{(2a)\int \frac{\tanh^{-1}(ax)^3}{x^2}\,d}{c} \\
&= -\frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\tanh^{-1}(ax}{} \\
&= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)}{cx} \\
&= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)}{cx} \\
&= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)}{cx} \\
&= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)}{cx}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.02, size = 250, normalized size = 0.66

$$
-\frac{a^2\left(-\frac{i\pi^3}{4}-\frac{\pi^4}{16}+\frac{i\pi^5}{160}+2\tanh^{-1}(ax)^3+\frac{2\tanh^{-1}(ax)^3}{ax}+\frac{1}{2}\tanh^{-1}(ax)^4+\frac{\tanh^{-1}(ax)^4}{2a^2x^2}+\frac{\tanh^{-1}(ax)^4}{ax}-6\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)-4\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^4\log\left(1-e^{2\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\left(3+3\tanh^{-1}(ax)+\tanh^{-1}(ax)^2\right)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+3\left(1+\tanh^{-1}(ax)\right)^2\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-3\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\mathrm{PolyLog}\left(5,e^{2\tanh^{-1}(ax)}\right)\right)}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

```
[Out] -((a^2*((-1/4*I)*Pi^3 - Pi^4/16 + (I/160)*Pi^5 + 2*ArcTanh[a*x]^3 + (2*ArcT
anh[a*x]^3)/(a*x) + ArcTanh[a*x]^4/2 + ArcTanh[a*x]^4/(2*a^2*x^2) + ArcTanh
[a*x]^4/(a*x) - 6*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 4*ArcTanh[a*
x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x]
)] - 2*ArcTanh[a*x]*(3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, E^(2*A
rcTanh[a*x])] + 3*(1 + ArcTanh[a*x])^2*PolyLog[3, E^(2*ArcTanh[a*x])] - 3*P
olyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x]
)] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(374) = 748.
time = 13.82, size = 779, normalized size = 2.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x^3/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2/c*arctanh(a*x)^3*(3*a*x*arctanh
(a*x)+arctanh(a*x)+4*a*x)*(a*x-1)/a^2/x^2-12/c*arctanh(a*x)^2*polylog(3,-(a
*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)
^(1/2))+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x
)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,(a*
x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(
1/2))+4/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)
^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*arctanh(a*x)*polylog(3,-(a*x
+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
+12/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*arctanh(a*x
)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a
^2*x^2+1)^(1/2))+12/c*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6
/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)*polylo
g(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))
-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))-4*arctanh(a*x)^3/c-12/c*polylog
(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-
2/c*arctanh(a*x)^4+24/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*polylog
(4,(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/160*(2*a^2*x^2*log(-a*x + 1)^5 + 5*(2*a*x + 1)*log(-a*x + 1)^4)/(c*x^2)
+ 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(
a*x + 1)^2*log(-a*x + 1)^2 - 2*(2*a^2*x^2 + a*x + 2*log(a*x + 1))*log(-a*x
+ 1)^3)/(a*c*x^4 - c*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^4 - c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^4-x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x**3/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**4 - x**3), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^4}{x^3\,\left(c-a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^4/(x^3*(c - a*c*x)),x)
```

```
[Out] int(atanh(a*x)^4/(x^3*(c - a*c*x)), x)
```

3.141 $\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{x}{(c+acx)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x/(a*c*x+c)/arctanh(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx = \int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x/((c + a*c*x)*ArcTanh[a*x]), x]
```

Maple [A]
time = 18.28, size = 0, normalized size = 0.00

$$\int \frac{x}{(cxa+c)\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c*x+c)/arctanh(a*x),x)
[Out] int(x/(a*c*x+c)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(x/((a*c*x + c)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{ax\operatorname{atanh}(ax)+\operatorname{atanh}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/atanh(a*x),x)
[Out] Integral(x/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(c + a*c*x)),x)
[Out] int(x/(atanh(a*x)*(c + a*c*x)), x)
```

3.142 $\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=18

$$\operatorname{Int}\left(\frac{1}{(c+acx)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(a*c*x+c)/arctanh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a*c*x)*ArcTanh[a*x]),x]
[Out] Defer[Int][1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx = \int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a*c*x)*ArcTanh[a*x]),x]
[Out] Integrate[1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Maple [A]
time = 3.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(cxa+c)\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x+c)/arctanh(a*x),x)
```

```
[Out] int(1/(a*c*x+c)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a*c*x + c)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax \operatorname{atanh}(ax)+\operatorname{atanh}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/atanh(a*x),x)
```

```
[Out] Integral(1/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(c + a*c*x)),x)
[Out] int(1/(atanh(a*x)*(c + a*c*x)), x)
```

3.143 $\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{x(c+acx)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(a*c*x+c)/arctanh(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
[Out] Defer[Int][1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx = \int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
[Out] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Maple [A]
time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(cxa+c)\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c*x+c)/arctanh(a*x),x)
[Out] int(1/x/(a*c*x+c)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(1/((a*c*x^2 + c*x)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^2 \operatorname{atanh}(ax) + x \operatorname{atanh}(ax)}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/atanh(a*x),x)
[Out] Integral(1/(a*x**2*atanh(a*x) + x*atanh(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \operatorname{atanh}(a\,x)\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(c + a*c*x)),x)
```

```
[Out] int(1/(x*atanh(a*x)*(c + a*c*x)), x)
```

3.144 $\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{x}{(c+acx)\tanh^{-1}(ax)^2},x\right)$$

[Out] Unintegrable(x/(a*c*x+c)/arctanh(a*x)^2,x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[x/((c + a*c*x)*ArcTanh[a*x]^2),x]

[Out] Defer[Int][x/((c + a*c*x)*ArcTanh[a*x]^2), x]

Rubi steps

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx=\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2),x]

[Out] Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2), x]

Maple [A]
time = 21.55, size = 0, normalized size = 0.00

$$\int \frac{x}{(cxa+c)\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x^2 - x)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + integrate(-2*(2*a*x
- 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{ax\operatorname{atanh}^{2}(ax)+\operatorname{atanh}^{2}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(x/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\operatorname{atanh}(a x)^2 (c + a c x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(c + a*c*x)),x)
```

```
[Out] int(x/(atanh(a*x)^2*(c + a*c*x)), x)
```

3.145 $\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=18

$$\mathrm{Int}\left(\frac{1}{(c+acx)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Defer[Int][1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx=\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
[Out] Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{1}{(cxa+c)\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x+c)/arctanh(a*x)^2,x)
[Out] int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] 2*(a*x - 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + 2*integrate(-1/(c*log(
a*x + 1) - c*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax\operatorname{atanh}^{2}(ax)+\operatorname{atanh}^{2}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/atanh(a*x)**2,x)
[Out] Integral(1/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,\left(c+a\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(c + a*c*x)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(c + a*c*x)), x)
```

3.146 $\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x(c+acx)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx = \int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(cxa+c)\,\operatorname{arctanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x - 1)/(a*c*x*log(a*x + 1) - a*c*x*log(-a*x + 1)) + 2*integrate(-1/(a*
c*x^2*log(a*x + 1) - a*c*x^2*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a*c*x^2 + c*x)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^2 \operatorname{atanh}^2(ax) + x \operatorname{atanh}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(a*x**2*atanh(a*x)**2 + x*atanh(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \operatorname{atanh}(a x)^2 (c + a c x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^2*(c + a*c*x)),x)
[Out] int(1/(x*atanh(a*x)^2*(c + a*c*x)), x)
```

3.147 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=275

$$\frac{ad^2x}{e^3}-\frac{bdx}{2ce^2}+\frac{bx^2}{6ce}+\frac{bd\tanh^{-1}(cx)}{2c^2e^2}+\frac{bd^2x\tanh^{-1}(cx)}{e^3}-\frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e}+\frac{d^3(a$$

```
[Out] a*d^2*x/e^3-1/2*b*d*x/c/e^2+1/6*b*x^2/c/e+1/2*b*d*arctanh(c*x)/c^2/e^2+b*d^
2*x*arctanh(c*x)/e^3-1/2*d*x^2*(a+b*arctanh(c*x))/e^2+1/3*x^3*(a+b*arctanh(
c*x))/e+d^3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^4-d^3*(a+b*arctanh(c*x))*ln(
2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^4+1/2*b*d^2*ln(-c^2*x^2+1)/c/e^3+1/6*b*ln(-c
^2*x^2+1)/c^3/e-1/2*b*d^3*polylog(2,1-2/(c*x+1))/e^4+1/2*b*d^3*polylog(2,1-
2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^4
```

Rubi [A]
time = 0.20, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6087, 6021, 266, 6037, 327, 212, 272, 45, 6057, 2449, 2352, 2497}

$$\frac{d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^4}-\frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e}+\frac{ad^2x}{e^3}+\frac{bd^2\log\left(1-c^2x^2\right)}{2ce^3}+\frac{bd\tanh^{-1}(cx)}{2c^2e^2}+\frac{b\log\left(1-c^2x^2\right)}{6c^3e}-\frac{bd^3\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2e^4}+\frac{bd^3\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^4}+\frac{bd^2x\tanh^{-1}(cx)}{e^3}-\frac{bdx}{2ce^2}+\frac{bx^2}{6ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (a*d^2*x)/e^3 - (b*d*x)/(2*c*e^2) + (b*x^2)/(6*c*e) + (b*d*ArcTanh[c*x])/(2
*c^2*e^2) + (b*d^2*x*ArcTanh[c*x])/e^3 - (d*x^2*(a + b*ArcTanh[c*x]))/(2*e^
2) + (x^3*(a + b*ArcTanh[c*x]))/(3*e) + (d^3*(a + b*ArcTanh[c*x])*Log[2/(1
+ c*x)])/e^4 - (d^3*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1
+ c*x))])/e^4 + (b*d^2*Log[1 - c^2*x^2])/(2*c*e^3) + (b*Log[1 - c^2*x^2])/(
6*c^3*e) - (b*d^3*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^4) + (b*d^3*PolyLog[2,
1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^3} - \frac{dx\left(a+b\tanh^{-1}(cx)\right)}{e^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{e}\right.$$

$$= \frac{d^2\int\left(a+b\tanh^{-1}(cx)\right)\,dx}{e^3} - \frac{d^3\int\frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{e^3} - \frac{d\int x\left(a+b\tanh^{-1}(cx)\right)}{e^2}$$

$$= \frac{ad^2x}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e} + \frac{d^3\left(a+b\tanh^{-1}(c\right.}{e^4}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tanh^{-1}\right.}{3e}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bd\tanh^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bx^2}{6ce} + \frac{bd\tanh^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}\right.}{2e^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.53, size = 474, normalized size = 1.72

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (-((b*e^3)/c^3) + 6*a*d^2*e*x - (3*b*d*e^2*x)/c - 3*a*d*e^2*x^2 + (b*e^3*x^
2)/c + 2*a*e^3*x^3 + (3*b*d*e^2*ArcTanh[c*x])/c^2 - (3*I)*b*d^3*Pi*ArcTanh[
c*x] + 6*b*d^2*e*x*ArcTanh[c*x] - 3*b*d*e^2*x^2*ArcTanh[c*x] + 2*b*e^3*x^3*
ArcTanh[c*x] - 6*b*d^3*ArcTanh[(c*d)/e]*ArcTanh[c*x] + 3*b*d^3*ArcTanh[c*x]
^2 - (3*b*d^2*e*ArcTanh[c*x]^2)/c + (3*b*d^2*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcT
anh[c*x]^2)/(c*E^ArcTanh[(c*d)/e]) + 6*b*d^3*ArcTanh[c*x]*Log[1 + E^(-2*Arc
Tanh[c*x])] + (3*I)*b*d^3*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 6*b*d^3*ArcTanh[
(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*b*d^3*ArcTan
h[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*a*d^3*Log[d +
e*x] + (3*b*d^2*e*Log[1 - c^2*x^2])/c + (b*e^3*Log[1 - c^2*x^2])/c^3 + ((3*
I)/2)*b*d^3*Pi*Log[1 - c^2*x^2] + 6*b*d^3*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTa
nh[(c*d)/e] + ArcTanh[c*x]]] - 3*b*d^3*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3
*b*d^3*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(6*e^4)
```

Maple [A]
time = 3.74, size = 423, normalized size = 1.54

method	result
derivativedivides	$\frac{\frac{a c^4 d^2 x}{e^3}-\frac{a c^4 d x^2}{2e^2}+\frac{a c^4 x^3}{3e}-\frac{a c^4 d^3 \ln(cex+dc)}{e^4}+\frac{b c^4 \operatorname{arctanh}(cx) d^2 x}{e^3}-\frac{b c^4 \operatorname{arctanh}(cx) d x^2}{2e^2}+\frac{b c^4 \operatorname{arctanh}(cx) x^3}{3e}-\frac{b c^4 \operatorname{arctanh}(cx) d}{e^4}}{}$
default	$\frac{\frac{a c^4 d^2 x}{e^3}-\frac{a c^4 d x^2}{2e^2}+\frac{a c^4 x^3}{3e}-\frac{a c^4 d^3 \ln(cex+dc)}{e^4}+\frac{b c^4 \operatorname{arctanh}(cx) d^2 x}{e^3}-\frac{b c^4 \operatorname{arctanh}(cx) d x^2}{2e^2}+\frac{b c^4 \operatorname{arctanh}(cx) x^3}{3e}-\frac{b c^4 \operatorname{arctanh}(cx) d}{e^4}}{}$
risch	$\frac{b d^3 \operatorname{dilog}\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e^4}-\frac{b d^3 \operatorname{dilog}\left(\frac{(cx+1)e+dc-e}{dc-e}\right)}{2e^4}-\frac{bd\ln(cx+1)x^2}{4e^2}+\frac{bd\ln(cx+1)}{4c^2e^2}+\frac{b\ln(cx+1)x d^2}{2e^3}+ b\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a*c^4/e^3*d^2*x-1/2*a*c^4/e^2*d*x^2+1/3*a*c^4/e*x^3-a*c^4*d^3/e^4*ln
(c*e*x+c*d)+b*c^4*arctanh(c*x)/e^3*d^2*x-1/2*b*c^4*arctanh(c*x)/e^2*d*x^2+1
/3*b*c^4*arctanh(c*x)/e*x^3-b*c^4*arctanh(c*x)*d^3/e^4*ln(c*e*x+c*d)-1/2*b*
c^4/e^4*d^3*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b*c^4/e^4*d^3*dilog((c
*e*x-e)/(-c*d-e))+1/2*b*c^4/e^4*d^3*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/
2*b*c^4/e^4*d^3*dilog((c*e*x+e)/(-c*d+e))-2/3*b*c^3/e^3*d^2-1/2*b*c^3/e^2*d
*x+1/6*b*c^3/e*x^2+1/2*b*c^3/e^3*ln(-c*e*x+e)*d^2-1/4*b*c^2/e^2*ln(-c*e*x+e
)*d+1/6*b*c/e*ln(-c*e*x+e)+1/2*b*c^3/e^3*ln(-c*e*x-e)*d^2+1/4*b*c^2/e^2*ln(
-c*e*x-e)*d+1/6*b*c/e*ln(-c*e*x-e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*(6*d^3*e^(-4)*log(x*e + d) - (2*x^3*e^2 - 3*d*x^2*e + 6*d^2*x)*e^(-3))
*a + 1/2*b*integrate(x^3*(log(c*x + 1) - log(-c*x + 1))/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{atanh}(cx))}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a + b\,\mathrm{atanh}(c\,x)\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atanh(c*x)))/(d + e*x),x)
```

```
[Out] int((x^3*(a + b*atanh(c*x)))/(d + e*x), x)
```

3.148 $\int \frac{x^2(a+b\tanh^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=214

$$-\frac{adx}{e^2}+\frac{bx}{2ce}-\frac{b\tanh^{-1}(cx)}{2c^2e}-\frac{bdx\tanh^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^3}+\frac{d^2(a$$

```
[Out] -a*d*x/e^2+1/2*b*x/c/e-1/2*b*arctanh(c*x)/c^2/e-b*d*x*arctanh(c*x)/e^2+1/2*
x^2*(a+b*arctanh(c*x))/e-d^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^3+d^2*(a+b*
arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3-1/2*b*d*ln(-c^2*x^2+1)/c/
e^2+1/2*b*d^2*polylog(2,1-2/(c*x+1))/e^3-1/2*b*d^2*polylog(2,1-2*c*(e*x+d)/
(c*d+e)/(c*x+1))/e^3
```

Rubi [A]
time = 0.15, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6087, 6021, 266, 6037, 327, 212, 6057, 2449, 2352, 2497}

$$-\frac{d^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{adx}{e^2}-\frac{bd\log\left(1-c^2x^2\right)}{2ce^2}-\frac{b\tanh^{-1}(cx)}{2c^2e}+\frac{bd^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2e^3}-\frac{bd^2\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^3}-\frac{bdx\tanh^{-1}(cx)}{e^2}+\frac{bx}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] -((a*d*x)/e^2) + (b*x)/(2*c*e) - (b*ArcTanh[c*x])/(2*c^2*e) - (b*d*x*ArcTan
h[c*x])/e^2 + (x^2*(a + b*ArcTanh[c*x]))/(2*e) - (d^2*(a + b*ArcTanh[c*x])*
Log[2/(1 + c*x)])/e^3 + (d^2*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d
 + e)*(1 + c*x))])/e^3 - (b*d*Log[1 - c^2*x^2])/(2*c*e^2) + (b*d^2*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*e^3) - (b*d^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d
+ e)*(1 + c*x))])/(2*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)}{e}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^2(d+ex)}\right)$$

$$= -\frac{d\int\left(a+b\tanh^{-1}(cx)\right)\,dx}{e^2}+\frac{d^2\int\frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{e^2}+\frac{\int x\left(a+b\tanh^{-1}(cx)\right)}{e}$$

$$= -\frac{adx}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^3}+$$

$$= -\frac{adx}{e^2}+\frac{bx}{2ce}-\frac{bdx\tanh^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-$$

$$= -\frac{adx}{e^2}+\frac{bx}{2ce}-\frac{b\tanh^{-1}(cx)}{2c^2e}-\frac{bdx\tanh^{-1}(cx)}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2e}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.06, size = 394, normalized size = 1.84

$$\frac{-2adex+\frac{be^2x}{c}+ae^2x^2-\frac{be^2\tanh^{-1}(cx)}{c^2}+ibd^2\pi\tanh^{-1}(cx)-2bdex\tanh^{-1}(cx)+be^2x^2\tanh^{-1}(cx)+2bd^2\tanh^{-1}\left(\frac{cd}{e}\right)\tanh^{-1}(cx)-bd^2\tanh^{-1}(cx)^2+\frac{bde\tanh^{-1}(cx)^2}{c}-\frac{bd\sqrt{1-\frac{c^2d^2}{e^2}}\,ee^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(cx)^2}{c}-2bd^2\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-ibd^2\pi\log\left(1+e^{2\tanh^{-1}(cx)}\right)+2bd^2\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+2bd^2\tanh^{-1}(cx)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+2ad^2\log(d+ex)-\frac{bde\log(1-c^2x^2)}{c}-\frac{1}{2}ibd^2\pi\log\left(1-c^2x^2\right)-2bd^2\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right)+bd^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-bd^2\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (-2*a*d*e*x + (b*e^2*x)/c + a*e^2*x^2 - (b*e^2*ArcTanh[c*x])/c^2 + I*b*d^2*
Pi*ArcTanh[c*x] - 2*b*d*e*x*ArcTanh[c*x] + b*e^2*x^2*ArcTanh[c*x] + 2*b*d^2
*ArcTanh[(c*d)/e]*ArcTanh[c*x] - b*d^2*ArcTanh[c*x]^2 + (b*d*e*ArcTanh[c*x]
^2)/c - (b*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c*d)/e
]) - 2*b*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - I*b*d^2*Pi*Log[1 +
 E^(2*ArcTanh[c*x])] + 2*b*d^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] + 2*b*d^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/
e] + ArcTanh[c*x]))] + 2*a*d^2*Log[d + e*x] - (b*d*e*Log[1 - c^2*x^2])/c -
(I/2)*b*d^2*Pi*Log[1 - c^2*x^2] - 2*b*d^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTa
nh[(c*d)/e] + ArcTanh[c*x]]] + b*d^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] - b*d
^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(2*e^3)
```

Maple [A]
time = 3.73, size = 332, normalized size = 1.55

method	result
derivativedivides	$\frac{-\frac{a\,c^3 dx}{e^2}+\frac{a\,c^3 x^2}{2e}+\frac{a\,c^3 d^2 \ln(cex+dc)}{e^3}-\frac{b\,c^3 \operatorname{arctanh}(cx) dx}{e^2}+\frac{b\,c^3 \operatorname{arctanh}(cx) x^2}{2e}+\frac{b\,c^3 \operatorname{arctanh}(cx) d^2 \ln(cex+dc)}{e^3}+\frac{b\,c^2 d}{2e^2}+\frac{b\,c^2 x}{2e}-}{}$
default	$\frac{-\frac{a\,c^3 dx}{e^2}+\frac{a\,c^3 x^2}{2e}+\frac{a\,c^3 d^2 \ln(cex+dc)}{e^3}-\frac{b\,c^3 \operatorname{arctanh}(cx) dx}{e^2}+\frac{b\,c^3 \operatorname{arctanh}(cx) x^2}{2e}+\frac{b\,c^3 \operatorname{arctanh}(cx) d^2 \ln(cex+dc)}{e^3}+\frac{b\,c^2 d}{2e^2}+\frac{b\,c^2 x}{2e}-}{}$
risch	$\frac{b\ln(-cx+1)dx}{2e^2}-\frac{b\ln(cx+1)xd}{2e^2}-\frac{b\ln(cx+1)d}{2c\,e^2}+\frac{b\,d^2\ln(cx+1)\ln\left(\frac{(cx+1)e+dc-e}{dc-e}\right)}{2e^3}+\frac{b\ln(cx+1)x^2}{4e}-\frac{b\ln(cx+1)}{4c^2 e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-a*c^3/e^2*d*x+1/2*a*c^3/e*x^2+a*c^3*d^2/e^3*ln(c*e*x+c*d)-b*c^3*arc
tanh(c*x)/e^2*d*x+1/2*b*c^3*arctanh(c*x)/e*x^2+b*c^3*arctanh(c*x)*d^2/e^3*l
n(c*e*x+c*d)+1/2*b*c^2/e^2*d+1/2*b*c^2/e*x-1/2*b*c^2/e^2*ln(-c*e*x-e)*d-1/4
*b*c/e*ln(-c*e*x-e)-1/2*b*c^2/e^2*ln(-c*e*x+e)*d+1/4*b*c/e*ln(-c*e*x+e)-1/2
*b*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-1/2*b*c^3/e^3*d^2*dilog
((c*e*x+e)/(-c*d+e))+1/2*b*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))
+1/2*b*c^3/e^3*d^2*dilog((c*e*x-e)/(-c*d-e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^2*e^(-3)*log(x*e + d) + (x^2*e - 2*d*x)*e^(-2))*a + 1/2*b*integrat
e(x^2*(log(c*x + 1) - log(-c*x + 1))/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{atanh}\,(cx))}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x)))/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x)))/(d + e*x), x)
```

3.149 $\int \frac{x(a+b\tanh^{-1}(cx))}{d+ex}\,dx$

Optimal. Leaf size=156

$$\frac{ax}{e}+\frac{bx\tanh^{-1}(cx)}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2}+\frac{b\log\left(1-c^2x\right.}{2ce}$$

```
[Out] a*x/e+b*x*arctanh(c*x)/e+d*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^2-d*(a+b*arct
anh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2+1/2*b*ln(-c^2*x^2+1)/c/e-1/2*
b*d*polylog(2,1-2/(c*x+1))/e^2+1/2*b*d*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x
+1))/e^2
```

Rubi [A]
time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6087, 6021, 266, 6057, 2449, 2352, 2497}

$$\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}+\frac{ax}{e}+\frac{b\log\left(1-c^2x^2\right)}{2ce}-\frac{bd\,\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2e^2}+\frac{bd\,\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^2}+\frac{bx\tanh^{-1}(cx)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (a*x)/e + (b*x*ArcTanh[c*x])/e + (d*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/
e^2 - (d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e
^2 + (b*Log[1 - c^2*x^2])/(2*c*e) - (b*d*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^
2) + (b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{e}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{e(d+ex)}\right)\,dx$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)\,dx}{e}-\frac{d\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{e}$$

$$= \frac{ax}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2}$$

$$= \frac{ax}{e}+\frac{bx\tanh^{-1}(cx)}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{e^2}$$

$$= \frac{ax}{e}+\frac{bx\tanh^{-1}(cx)}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{e^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.66, size = 315, normalized size = 2.02

$$\frac{2aex-2ad\log(d+ex)+\frac{b\left(-icd\pi\tanh^{-1}(cx)+2cex\tanh^{-1}(cx)-2cd\tanh^{-1}\left(\frac{cd}{e}\right)\tanh^{-1}(cx)+cd\tanh^{-1}(cx)^2-e\tanh^{-1}(cx)^2+\sqrt{1-\frac{c^2d^2}{e^2}}\,ee^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(cx)^2+2cd\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+icd\pi\log\left(1+e^{2\tanh^{-1}(cx)}\right)-2cd\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)-2cd\tanh^{-1}(cx)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+e\log(1-c^2x^2)+\frac{1}{2}icd\pi\log(1-c^2x^2)+2cd\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right)-cd\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+cd\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)\right)}{c}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*e*x*ArcT
anh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*Arc
Tanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e]
 + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*
ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTa
nh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*Arc
Tanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2,
 -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))]))/c)/(2*e^2)
```

Maple [A]
time = 3.77, size = 243, normalized size = 1.56

method	result
derivativedivides	$\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+dc)}{e^2}+\frac{b\,c^2\operatorname{arctanh}(cx)x}{e}-\frac{b\,c^2\operatorname{arctanh}(cx)d\ln(cex+dc)}{e^2}+\frac{bc\ln\left(c^2d^2-2cd(cex+dc)-e^2+(cex+dc)^2\right)}{2e}+\frac{b\,c^2d\ln(\cdots}{\cdots}}{c^2}$
default	$\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+dc)}{e^2}+\frac{b\,c^2\operatorname{arctanh}(cx)x}{e}-\frac{b\,c^2\operatorname{arctanh}(cx)d\ln(cex+dc)}{e^2}+\frac{bc\ln\left(c^2d^2-2cd(cex+dc)-e^2+(cex+dc)^2\right)}{2e}+\frac{b\,c^2d\ln(\cdots}{\cdots}}{c^2}$
risch	$-\frac{b\ln(-cx+1)x}{2e}+\frac{b\ln(-cx+1)}{2ce}-\frac{b}{ce}+\frac{bd\operatorname{dilog}\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e^2}+\frac{bd\ln(-cx+1)\ln\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e^2}+\frac{ax}{e}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a*c^2/e*x-a*c^2*d/e^2*ln(c*e*x+c*d)+b*c^2*arctanh(c*x)/e*x-b*c^2*arc
tanh(c*x)*d/e^2*ln(c*e*x+c*d)+1/2*b*c/e*ln(c^2*d^2-2*c*d*(c*e*x+c*d)-e^2+(c
*e*x+c*d)^2)+1/2*b*c^2/e^2*d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/2*b*c^2
/e^2*d*dilog((c*e*x+e)/(-c*d+e))-1/2*b*c^2/e^2*d*ln(c*e*x+c*d)*ln((c*e*x-e)
/(-c*d-e))-1/2*b*c^2/e^2*d*dilog((c*e*x-e)/(-c*d-e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -(d*e^(-2)*log(x*e + d) - x*e^(-1))*a + 1/2*b*integrate(x*(log(c*x + 1) - l
og(-c*x + 1))/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{atanh}\,(cx))}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\mathrm{atanh}(c\,x)\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x)))/(d + e*x),x)
```

```
[Out] int((x*(a + b*atanh(c*x)))/(d + e*x), x)
```

3.150 $\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=114

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2e}-\frac{b\text{PolyLog}(\ldots}{}$$

```
[Out] -(a+b*arctanh(c*x))*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+
e)/(c*x+1))/e+1/2*b*polylog(2,1-2/(c*x+1))/e-1/2*b*polylog(2,1-2*c*(e*x+d)/
(c*d+e)/(c*x+1))/e
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6057, 2449, 2352, 2497}

$$\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e}-\frac{b\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])*Log[(2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(
2*e) - (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{(bc)\int \frac{\mathrm{lo}}{}}{} \\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}-\frac{b\mathrm{Li}_2\Big(1-}{} \\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b\mathrm{Li}_2(1-}{2e}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 244, normalized size = 2.14

$$\frac{a\log(d+ex)}{e}+\frac{b\left(-\frac{1}{4}\left(\pi-2i\tanh^{-1}(cx)\right)^2+\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)^2-i\left(\pi-2i\tanh^{-1}(cx)\right)\log\left(1+e^{2\tanh^{-1}(cx)}\right)+2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+\left(i\pi+2\tanh^{-1}(cx)\right)\log\left(\frac{2}{\sqrt{1-c^2x^2}}\right)+\tanh^{-1}(cx)\left(\log\left(1-c^2x^2\right)+2\log\left(i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right)\right)-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\log\left(2i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right)-\mathrm{PolyLog}\left(2,-e^{2\tanh^{-1}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(-1/4*(Pi - (2*I)*ArcTanh[c*x])^2 + (ArcTanh[(c*d)/
e] + ArcTanh[c*x])^2 - I*(Pi - (2*I)*ArcTanh[c*x])*Log[1 + E^(2*ArcTanh[c*x
])] + 2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[1 - E^(-2*(ArcTanh[(c*d)/e] +
 ArcTanh[c*x]))] + (I*Pi + 2*ArcTanh[c*x])*Log[2/Sqrt[1 - c^2*x^2]] + ArcTa
nh[c*x]*(Log[1 - c^2*x^2] + 2*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]])
 - 2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I)*Sinh[ArcTanh[(c*d)/e] + Ar
cTanh[c*x]]] - PolyLog[2, -E^(2*ArcTanh[c*x])] - PolyLog[2, E^(-2*(ArcTanh[
(c*d)/e] + ArcTanh[c*x]))]))/(2*e)
```

Maple [A]
time = 0.19, size = 158, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{ac\ln(cex+dc)}{e}+\frac{bc\ln(cex+dc)\operatorname{arctanh}(cx)}{e}+\frac{bc\ln(cex+dc)\ln\left(\frac{cex-e}{-dc-e}\right)}{2e}+\frac{bc\operatorname{dilog}\left(\frac{cex-e}{-dc-e}\right)}{2e}-\frac{bc\ln(cex+dc)\ln\left(\frac{cex+e}{-dc+e}\right)}{2e}-\frac{bc\operatorname{dilog}\left(\frac{ce}{-d}\right.}{2e}}{c}$

default	$\frac{\frac{ac\ln(cex+dc)}{e}+\frac{bc\ln(cex+dc)\operatorname{arctanh}(cx)}{e}+\frac{bc\ln(cex+dc)\ln\left(\frac{cex-e}{-dc-e}\right)}{2e}+\frac{bc\operatorname{dilog}\left(\frac{cex-e}{-dc-e}\right)}{2e}-\frac{bc\ln(cex+dc)\ln\left(\frac{cex+e}{-dc+e}\right)}{2e}-\frac{bc\operatorname{dilog}(}{2e}}{c}$
risch	$-\frac{b\operatorname{dilog}\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e}-\frac{b\ln(-cx+1)\ln\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2e}+\frac{a\ln((-cx+1)e-dc-e)}{e}+\frac{b\operatorname{dilog}\left(\frac{(cx+1)e+dc-e}{dc-e}\right.}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c*ln(c*e*x+c*d)/e+b*c*ln(c*e*x+c*d)/e*arctanh(c*x)+1/2*b*c/e*ln(c*e*
x+c*d)*ln((c*e*x-e)/(-c*d-e))+1/2*b*c/e*dilog((c*e*x-e)/(-c*d-e))-1/2*b*c/e
*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-1/2*b*c/e*dilog((c*e*x+e)/(-c*d+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*e^(-1)*log(x*e + d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(x*e
 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atanh}(cx)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))/(d + e*x), x)
```

3.151 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+ex)}\,dx$

Optimal. Leaf size=148

$$\frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-\frac{b\text{PolyLog}(2,-cx)}{2d}+\frac{b\text{PolyI}}{}$$

```
[Out] a*ln(x)/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/d-(a+b*arctanh(c*x))*ln(2*c*(e*x
+d)/(c*d+e)/(c*x+1))/d-1/2*b*polylog(2,-c*x)/d+1/2*b*polylog(2,c*x)/d-1/2*b
*polylog(2,1-2/(c*x+1))/d+1/2*b*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Rubi [A]

time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6087, 6031, 6057, 2449, 2352, 2497}

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{a\log(x)}{d}+\frac{b\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}-\frac{b\text{Li}_2(-cx)}{2d}+\frac{b\text{Li}_2(cx)}{2d}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```

```
[Out] (a*Log[x])/d + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[
c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d - (b*PolyLog[2, -(c*x)]
)/(2*d) + (b*PolyLog[2, c*x])/(2*d) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d)
 + (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{e\left(a+b\tanh^{-1}(cx)\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d}-\frac{e\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d}$$

$$= \frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-$$

$$= \frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-$$

$$= \frac{a\log(x)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 294, normalized size = 1.99

$$\frac{2ad\log(x)-2ad\log(d+ex)+\frac{b\left(-icd\pi\tanh^{-1}(cx)-2cd\tanh^{-1}\left(\frac{cd}{e}\right)\tanh^{-1}(cx)+cd\tanh^{-1}(cx)^2-e\tanh^{-1}(cx)^2+\sqrt{1-\frac{c^2d^2}{e^2}}\,e e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(cx)^2+2cd\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+icd\pi\log\left(1+e^{2\tanh^{-1}(cx)}\right)-2cd\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)-2cd\tanh^{-1}(cx)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+\frac{1}{2}icd\pi\log\left(1-c^2x^2\right)+2cd\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right)-cd\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+cd\,\mathrm{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)\right)}{c}}{2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]

[Out] (2*a*d*Log[x] - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] - 2*c*d*A
rcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2 + (Sqr
t[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*ArcTanh[c
*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 2
*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 2
*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + (I/2)
*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/
e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2,
E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c)/(2*d^2)

Maple [A]
time = 3.75, size = 210, normalized size = 1.42

method	result
risch	$\frac{b\,\mathrm{dilog}\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2d}+\frac{b\ln(-cx+1)\ln\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2d}+\frac{\mathrm{dilog}(-cx+1)b}{2d}-\frac{a\ln((-cx+1)e-dc-e)}{d}+\frac{a\ln(}{}$
derivativedivides	$\frac{a\ln(cx)}{d}-\frac{a\ln(cex+dc)}{d}+\frac{b\,\mathrm{arctanh}(cx)\ln(cx)}{d}-\frac{b\,\mathrm{arctanh}(cx)\ln(cex+dc)}{d}+\frac{b\ln(cex+dc)\ln\left(\frac{cex+e}{-dc+e}\right)}{2d}+\frac{b\,\mathrm{dilo}}{}$
default	$\frac{a\ln(cx)}{d}-\frac{a\ln(cex+dc)}{d}+\frac{b\,\mathrm{arctanh}(cx)\ln(cx)}{d}-\frac{b\,\mathrm{arctanh}(cx)\ln(cex+dc)}{d}+\frac{b\ln(cex+dc)\ln\left(\frac{cex+e}{-dc+e}\right)}{2d}+\frac{b\,\mathrm{dilo}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] a/d*ln(c*x)-a/d*ln(c*e*x+c*d)+b*arctanh(c*x)/d*ln(c*x)-b*arctanh(c*x)/d*ln(
c*e*x+c*d)+1/2*b/d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/2*b/d*dilog((c*e*
x+e)/(-c*d+e))-1/2*b/d*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b/d*dilog((
c*e*x-e)/(-c*d-e))-1/2*b/d*dilog(c*x+1)-1/2*b/d*ln(c*x)*ln(c*x+1)-1/2*b/d*d
ilog(c*x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="maxima")

[Out] -a*(log(x*e + d)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x +
 1))/(x^2*e + d*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx)}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(e*x+d),x)
[Out] Integral((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{atanh}(c\,x)}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x*(d + e*x)),x)
[Out] int((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

3.152 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=200

$$-\frac{a+b\tanh^{-1}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{ae\log(x)}{d^2}-\frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+}{(cd+e)(}\right.}{d^2}$$

```
[Out] (-a-b*arctanh(c*x))/d/x+b*c*ln(x)/d-a*e*ln(x)/d^2-e*(a+b*arctanh(c*x))*ln(2
/(c*x+1))/d^2+e*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2-1/2*
b*c*ln(-c^2*x^2+1)/d+1/2*b*e*polylog(2,-c*x)/d^2-1/2*b*e*polylog(2,c*x)/d^2
+1/2*b*e*polylog(2,1-2/(c*x+1))/d^2-1/2*b*e*polylog(2,1-2*c*(e*x+d)/(c*d+e)
/(c*x+1))/d^2
```

Rubi [A]

time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {6087, 6037, 272, 36, 29, 31, 6031, 6057, 2449, 2352, 2497}

$$-\frac{e\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^2}-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{ae\log(x)}{d^2}-\frac{bc\log\left(1-c^2x^2\right)}{2d}+\frac{be\text{Li}_2(-cx)}{2d^2}-\frac{be\text{Li}_2(cx)}{2d^2}+\frac{be\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^2}-\frac{be\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d^2}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
 b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])*Log[(2*c*(
d + e*x))/((c*d + e)*(1 + c*x))])/d^2 - (b*c*Log[1 - c^2*x^2])/(2*d) + (b*e
*PolyLog[2, -(c*x)])/(2*d^2) - (b*e*PolyLog[2, c*x])/(2*d^2) + (b*e*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*d^2) - (b*e*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.23, size = 360, normalized size = 1.80

$$-\frac{\frac{2ad^2}{x} - ibde\pi\tanh^{-1}(cx) + \frac{2bd^2\tanh^{-1}(cx)}{x} - 2bde\tanh^{-1}\left(\frac{cd}{e}\right)\tanh^{-1}(cx) + bde\tanh^{-1}(cx)^2 - \frac{be^2\tanh^{-1}(cx)^2}{c} + \frac{b\sqrt{1-\frac{c^2d^2}{e^2}}\,e^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(cx)^2}{c} + 2bde\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right) + ibde\pi\log\left(1+e^{2\tanh^{-1}(cx)}\right) - 2bde\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) - 2bde\tanh^{-1}(cx)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) + 2ade\log(x) - 2ade\log(d+ex) - 2bcd^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + \frac{1}{2}ibde\pi\log\left(1-c^2x^2\right) + 2bde\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right) - bde\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) + bde\,\mathrm{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*((2*a*d^2)/x - I*b*d*e*Pi*ArcTanh[c*x] + (2*b*d^2*ArcTanh[c*x])/x - 2*
b*d*e*ArcTanh[(c*d)/e]*ArcTanh[c*x] + b*d*e*ArcTanh[c*x]^2 - (b*e^2*ArcTanh
[c*x]^2)/c + (b*Sqrt[1 - (c^2*d^2)/e^2]*e^2*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c
*d)/e]) + 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*b*d*e*Pi*Lo
g[1 + E^(2*ArcTanh[c*x])] - 2*b*d*e*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh
[(c*d)/e] + ArcTanh[c*x]))] - 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(
c*d)/e] + ArcTanh[c*x]))] + 2*a*d*e*Log[x] - 2*a*d*e*Log[d + e*x] - 2*b*c*d
^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + (I/2)*b*d*e*Pi*Log[1 - c^2*x^2] + 2*b*d*e
*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - b*d*e*Poly
```

```
Log[2, E^(-2*ArcTanh[c*x])] + b*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + Ar
cTanh[c*x]))])/d^3
```

Maple [A]
time = 3.70, size = 317, normalized size = 1.58

method	result
risch	$\frac{cb\ln(-cx)}{2d}-\frac{cb\ln(-cx+1)}{2d}+\frac{b\ln(-cx+1)}{2dx}-\frac{be\,\mathrm{dilog}\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2d^2}-\frac{be\ln(-cx+1)\ln\left(\frac{(-cx+1)e-dc-e}{-dc-e}\right)}{2d^2}-b$
derivativedivides	$c\left(\frac{ae\ln(cex+dc)}{c\,d^2}-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{b\,\mathrm{arctanh}(cx)e\ln(cex+dc)}{c\,d^2}-\frac{b\,\mathrm{arctanh}(cx)}{dcx}-\frac{b\,\mathrm{arctanh}(cx)e\ln(cx)}{c\,d^2}-b\right.$
default	$c\left(\frac{ae\ln(cex+dc)}{c\,d^2}-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{b\,\mathrm{arctanh}(cx)e\ln(cex+dc)}{c\,d^2}-\frac{b\,\mathrm{arctanh}(cx)}{dcx}-\frac{b\,\mathrm{arctanh}(cx)e\ln(cx)}{c\,d^2}-b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c/d^2*e*ln(c*e*x+c*d)-a/d/c/x-a/c/d^2*e*ln(c*x)+b/c*arctanh(c*x)/d^2*e
*ln(c*e*x+c*d)-b*arctanh(c*x)/d/c/x-b/c*arctanh(c*x)/d^2*e*ln(c*x)-1/2*b/d*
ln(c*x+1)-1/2*b/d*ln(c*x-1)+b/d*ln(c*x)+1/2*b/c/d^2*e*dilog(c*x+1)+1/2*b/c/
d^2*e*ln(c*x)*ln(c*x+1)+1/2*b/c/d^2*e*dilog(c*x)+1/2*b/c/d^2*ln(c*e*x+c*d)*
ln((c*e*x-e)/(-c*d-e))*e+1/2*b/c/d^2*dilog((c*e*x-e)/(-c*d-e))*e-1/2*b/c/d^
2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))*e-1/2*b/c/d^2*dilog((c*e*x+e)/(-c*d+
e))*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(x*e + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*b*integrate((log(c*x
+ 1) - log(-c*x + 1))/(x^3*e + d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(x^3*e + d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx)}{x^2(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(e*x+d),x)
[Out] Integral((a + b*atanh(c*x))/(x**2*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^2*(d + e*x)),x)
[Out] int((a + b*atanh(c*x))/(x^2*(d + e*x)), x)
```

3.153 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=261

$$-\frac{bc}{2dx}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x}-\frac{bce\log(x)}{d^2}+\frac{ae^2\log(x)}{d^3}+\frac{e^2\left(a+b\tanh^{-1}(c\right.}{d^3}$$

```
[Out] -1/2*b*c/d/x+1/2*b*c^2*arctanh(c*x)/d+1/2*(-a-b*arctanh(c*x))/d/x^2+e*(a+b*
arctanh(c*x))/d^2/x-b*c*e*ln(x)/d^2+a*e^2*ln(x)/d^3+e^2*(a+b*arctanh(c*x))*
ln(2/(c*x+1))/d^3-e^2*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^
3+1/2*b*c*e*ln(-c^2*x^2+1)/d^2-1/2*b*e^2*polylog(2,-c*x)/d^3+1/2*b*e^2*poly
log(2,c*x)/d^3-1/2*b*e^2*polylog(2,1-2/(c*x+1))/d^3+1/2*b*e^2*polylog(2,1-2
*c*(e*x+d)/(c*d+e)/(c*x+1))/d^3
```

Rubi [A]
time = 0.18, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {6087, 6037, 331, 212, 272, 36, 29, 31, 6031, 6057, 2449, 2352, 2497}

$$\frac{e^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{e^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^3}+\frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{ae^2\log(x)}{d^3}+\frac{bce\log\left(1-c^2x^2\right)}{2d^2}+\frac{bc^2\tanh^{-1}(cx)}{2d}-\frac{be^2\text{Li}_2(-cx)}{2d^3}+\frac{be^2\text{Li}_2(cx)}{2d^3}-\frac{be^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2d^3}+\frac{be^2\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d^3}-\frac{bce\log(x)}{d^2}-\frac{bc}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(b*c)/(d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x
^2) + (e*(a + b*ArcTanh[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x]
)/d^3 + (e^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (e^2*(a + b*ArcTa
nh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d^3 + (b*c*e*Log[1 - c
^2*x^2])/(2*d^2) - (b*e^2*PolyLog[2, -(c*x)])/(2*d^3) + (b*e^2*PolyLog[2, c
*x])/(2*d^3) - (b*e^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3) + (b*e^2*PolyLog
[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx^3} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x^2} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{d^3x} - \frac{e^3\left(a+b\,\mathrm{t}\right.}{d^3(d}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{e^3\int \frac{a+b\tanh^{-1}(cx)}{d+ex}}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)\log}{d^3}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tanh^{-1}\right.}{d^3}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} +$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} - \frac{bce\log(x)}{d^2} +$$

Mathematica [C] Result contains complex when optimal does not.

time = 4.13, size = 435, normalized size = 1.67

$$-\frac{a}{2dx^2}+\frac{ae}{d^2x}+\frac{ae^2\log(x)}{d^3}-\frac{ae^2\log(d+ex)}{d^3}-\frac{b\left(\frac{c^2d^3}{x}+icde^2\pi\tanh^{-1}(cx)-\frac{2cd^2e\tanh^{-1}(cx)}{x}+\frac{cd^3(1-c^2x^2)\tanh^{-1}(cx)}{x^2}+2cde^2\tanh^{-1}\left(\frac{cd}{e}\right)\tanh^{-1}(cx)-cde^2\tanh^{-1}(cx)^2+e^3\tanh^{-1}(cx)^2-\sqrt{1-\frac{c^2d^2}{e^2}}\,e^3e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(cx)^2-2cde^2\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-icde^2\pi\log\left(1+e^{2\tanh^{-1}(cx)}\right)+2cde^2\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+2cde^2\tanh^{-1}(cx)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)+icde^2\pi\log\left(\frac{1}{\sqrt{1-c^2x^2}}\right)+2c^2d^2e\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-2cde^2\tanh^{-1}\left(\frac{cd}{e}\right)\log\left(i\sinh\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)\right)+cde^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-cde^2\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)\right)}{2cd^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*a/(d*x^2) + (a*e)/(d^2*x) + (a*e^2*Log[x])/d^3 - (a*e^2*Log[d + e*x])/
d^3 - (b*((c^2*d^3)/x + I*c*d*e^2*Pi*ArcTanh[c*x] - (2*c*d^2*e*ArcTanh[c*x]
)/x + (c*d^3*(1 - c^2*x^2)*ArcTanh[c*x])/x^2 + 2*c*d*e^2*ArcTanh[(c*d)/e]*A
rcTanh[c*x] - c*d*e^2*ArcTanh[c*x]^2 + e^3*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*
d^2)/e^2]*e^3*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e^2*ArcTanh[c*x]*L
og[1 - E^(-2*ArcTanh[c*x])] - I*c*d*e^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*
c*d*e^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]
+ 2*c*d*e^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]
+ I*c*d*e^2*Pi*Log[1/Sqrt[1 - c^2*x^2]] + 2*c^2*d^2*e*Log[(c*x)/Sqrt[1 - c^
2*x^2]] - 2*c*d*e^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[
c*x]]] + c*d*e^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - c*d*e^2*PolyLog[2, E^(-2
*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(2*c*d^4)
```

Maple [A]

time = 3.80, size = 418, normalized size = 1.60

method	result
derivativedivides	$c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}-\frac{a\,e^2\ln(cex+dc)}{c^2d^3}-\frac{b\,\text{arctanh}(cx)}{2d\,c^2x^2}+\frac{b\,\text{arctanh}(cx)e^2\ln(cx)}{c^2d^3}+\frac{b\,\text{arctan}}{c^2d}\right.$
default	$c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}-\frac{a\,e^2\ln(cex+dc)}{c^2d^3}-\frac{b\,\text{arctanh}(cx)}{2d\,c^2x^2}+\frac{b\,\text{arctanh}(cx)e^2\ln(cx)}{c^2d^3}+\frac{b\,\text{arctan}}{c^2d}\right.$
risch	$-\frac{bc}{2dx}+\frac{c^2b\ln(-cx)}{4d}-\frac{c^2b\ln(-cx+1)}{4d}+\frac{b\ln(-cx+1)}{4d\,x^2}-\frac{cbe\ln(-cx)}{2d^2}+\frac{cbe\ln(-cx+1)}{2d^2}-\frac{be\ln(-cx+1)}{2d^2x}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a/d/c^2/x^2+a/c^2/d^3*e^2*ln(c*x)+a/c^2/d^2*e/x-a/c^2/d^3*e^2*ln(
c*e*x+c*d)-1/2*b*arctanh(c*x)/d/c^2/x^2+b/c^2*arctanh(c*x)/d^3*e^2*ln(c*x)+
b/c^2*arctanh(c*x)/d^2*e/x-b/c^2*arctanh(c*x)/d^3*e^2*ln(c*e*x+c*d)-1/4*b/d
*ln(c*x-1)+1/2*b/c/d^2*ln(c*x-1)*e+1/4*b/d*ln(c*x+1)+1/2*b/c/d^2*ln(c*x+1)*
e-b/c/d^2*e*ln(c*x)-1/2*b/d/c/x-1/2*b/c^2/d^3*e^2*dilog(c*x)-1/2*b/c^2/d^3*
e^2*dilog(c*x+1)-1/2*b/c^2/d^3*e^2*ln(c*x)*ln(c*x+1)-1/2*b/c^2/d^3*e^2*ln(c
*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b/c^2/d^3*e^2*dilog((c*e*x-e)/(-c*d-e)
)+1/2*b/c^2/d^3*e^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/2*b/c^2/d^3*e^2*
dilog((c*e*x+e)/(-c*d+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="maxima")
[Out] -1/2*a*(2*e^2*log(x*e + d)/d^3 - 2*e^2*log(x)/d^3 - (2*x*e - d)/(d^2*x^2))
+ 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(x^4*e + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/(x^4*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}{\left(cx \right)}}{x^{3} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(e*x+d),x)
[Out] Integral((a + b*atanh(c*x))/(x**3*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(x^3*(d + e*x)),x)
[Out] int((a + b*atanh(c*x))/(x^3*(d + e*x)), x)
```

3.154 $\int \frac{x^2(a+b\tanh^{-1}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=385

$$\frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh\right.}{2e}$$

```
[Out] a*b*x/c/e+b^2*x*arctanh(c*x)/c/e-d*(a+b*arctanh(c*x))^2/c/e^2-1/2*(a+b*arct
anh(c*x))^2/c^2/e-d*x*(a+b*arctanh(c*x))^2/e^2+1/2*x^2*(a+b*arctanh(c*x))^2
/e+2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c/e^2-d^2*(a+b*arctanh(c*x))^2*l
n(2/(c*x+1))/e^3+d^2*(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e
^3+1/2*b^2*ln(-c^2*x^2+1)/c^2/e+b^2*d*polylog(2,1-2/(-c*x+1))/c/e^2+b*d^2*(
a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e^3-b*d^2*(a+b*arctanh(c*x))*polyl
og(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3+1/2*b^2*d^2*polylog(3,1-2/(c*x+1))/
e^3-1/2*b^2*d^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3
```

Rubi [A]
time = 0.31, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 6059}

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}+\frac{bd^2\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}-\frac{bd^2\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e^3}-\frac{d^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{e^3}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}+\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{ce^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}+\frac{abx}{ce}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^2e}+\frac{b^2d^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2e^3}-\frac{b^2d^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^3}+\frac{b^2d\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{ce^2}+\frac{b^2x\tanh^{-1}(cx)}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (a*b*x)/(c*e) + (b^2*x*ArcTanh[c*x])/(c*e) - (d*(a + b*ArcTanh[c*x])^2)/(c*
e^2) - (a + b*ArcTanh[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTanh[c*x])^2)/e^2
+ (x^2*(a + b*ArcTanh[c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1
 - c*x)])/(c*e^2) - (d^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e^3 + (d^
2*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^3 +
(b^2*Log[1 - c^2*x^2])/(2*c^2*e) + (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e
^2) + (b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^3 - (b*d^2
*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]
)/e^3 + (b^2*d^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e^3) - (b^2*d^2*PolyLog[3,
 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx = \int \left(-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d^2\left(a+b\tanh^{-1}(cx\right.}{e^2(d+ex)}\right.$$

$$= -\frac{d\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{e^2}+\frac{d^2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{e^2}+\frac{\int x\left(a+b\tanh^{-1}(\right.}{e}$$

$$= -\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{e^3}$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}$$

$$= \frac{abx}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tanh^{-1}(cx)\right.}{e^2}$$

$$= \frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx}{}$$

$$= \frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 12.29, size = 1250, normalized size = 3.25

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (-6*a^2*d*e*x + 3*a^2*e^2*x^2 + 6*a^2*d^2*Log[d + e*x] + (6*a*b*(c*e^2*x +
I*c^2*d^2*Pi*ArcTanh[c*x] - 2*c^2*d*e*x*ArcTanh[c*x] + e^2*(-1 + c^2*x^2)*A
rcTanh[c*x] + 2*c^2*d^2*ArcTanh[(c*d)/e]*ArcTanh[c*x] - c^2*d^2*ArcTanh[c*x
]^2 + c*d*e*ArcTanh[c*x]^2 - (c*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)
/E^ArcTanh[(c*d)/e] - 2*c^2*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] -
 I*c^2*d^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c^2*d^2*ArcTanh[(c*d)/e]*Log[
1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*ArcTanh[c*x]*Log[
1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - c*d*e*Log[1 - c^2*x^2] - (I
/2)*c^2*d^2*Pi*Log[1 - c^2*x^2] - 2*c^2*d^2*ArcTanh[(c*d)/e]*Log[I*Sinh[Arc
Tanh[(c*d)/e] + ArcTanh[c*x]]] + c^2*d^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] -
 c^2*d^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c^2 + (b^2*
(6*c*e^2*x*ArcTanh[c*x] + 6*c*d*e*ArcTanh[c*x]^2 - 6*c^2*d*e*x*ArcTanh[c*x]
^2 + 3*e^2*(-1 + c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*d^2*ArcTanh[c*x]^3 + 2*c*d
*e*ArcTanh[c*x]^3 + 12*c*d*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 6*
c^2*d^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 3*e^2*Log[1 - c^2*x^2
] + 6*c*d*(-e + c*d*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*c^2*
d^2*PolyLog[3, -E^(-2*ArcTanh[c*x])] - (6*c*d*(-(c*d) + e)*(c*d + e)*(-6*c*
d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTan
h[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c
*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*Ar
cTanh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e]
+ ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTa
nh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e
] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*A
rcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x]
)))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c
^2*x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d
)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcT
anh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] + 12*c*d*A
rcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTa
nh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x
]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -(
((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[
(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh
[c*x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(6*c^
2*d^2 - 6*e^2)))/c^2)/(6*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 25.63, size = 1729, normalized size = 4.49

method	result	size
derivativedivides	Expression too large to display	1729
default	Expression too large to display	1729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/2*b^2*c^4/e^3*d^3/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1
)/(-c*d+e))-1/2*b^2*c^3/e^2*d^2/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x
^2+1)/(-c*d+e))-b^2*c^3/e^3*d^2*arctanh(c*x)^2*ln(d*c*(1+(c*x+1)^2/(-c^2*x^
2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))+2*b^2*c^2/e^2*ln(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))*d*arctanh(c*x)+2*b^2*c^2/e^2*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))*d
*arctanh(c*x)+b^2*c^3*arctanh(c*x)^2*d^2/e^3*ln(c*e*x+c*d)-b^2*c^3*d^2/e^3*
arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-a*b*c^2/e^2*ln(-c*e*x-e)*d-
a*b*c^2/e^2*ln(-c*e*x+e)*d-a*b*c^3/e^3*d^2*dilog((c*e*x+e)/(-c*d+e))+a*b*c^
3/e^3*d^2*dilog((c*e*x-e)/(-c*d-e))-a^2*c^3/e^2*d*x+a*b*c^2/e*x+1/2*b^2*c^3
*arctanh(c*x)^2/e*x^2+b^2*c^2*arctanh(c*x)/e*x+1/2*I*b^2*c^3/e^3*d^2*Pi*csg
n(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1
)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-2*a*b*c^3*arctanh(c*x)/e^2*d*x-b^2*c^3*
arctanh(c*x)^2/e^2*d*x+a*b*c^3*arctanh(c*x)/e*x^2+2*a*b*c^3*arctanh(c*x)*d^
2/e^3*ln(c*e*x+c*d)-a*b*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+a*
b*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+b^2*c^3/e^2*d^2/(c*d+e)*
arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b^2*c^4/e^3
*d^3/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b
^2*c^4/e^3*d^3/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1
)/(-c*d+e))+b^2*c^3/e^2*d^2/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(
-c^2*x^2+1)/(-c*d+e))+1/2*I*b^2*c^3/e^3*Pi*d^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^
2+1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))
)*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(
c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/2*a*b*c/e*ln(-c*e*x-e)+1/2*a*b*c/e
*ln(-c*e*x+e)+a*b*c^2/e^2*d+1/2*b^2*c^3*d^2/e^3*polylog(3,-(c*x+1)^2/(-c^2*
x^2+1))-b^2*c^2/e^2*d*arctanh(c*x)^2+2*b^2*c^2/e^2*dilog(1+I*(c*x+1)/(-c^2*
x^2+1)^(1/2))*d+2*b^2*c^2/e^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))*d+a^2*c
^3*d^2/e^3*ln(c*e*x+c*d)+1/2*a^2*c^3/e*x^2-1/2*b^2*c/e*arctanh(c*x)^2-b^2*c
/e*ln(1+(c*x+1)^2/(-c^2*x^2+1))+b^2*c*arctanh(c*x)/e-1/2*I*b^2*c^3/e^3*d^2*
Pi*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1)))*cs
gn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/2*I*b^2*c^3/e^3*Pi*d^2*csgn(I/(1+(c*
x+1)^2/(-c^2*x^2+1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(
-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*x^2*e - 2*b^2*d*x)*e^(-2)*log(-c*x + 1)^2 + 1/2*(2*d^2*e^(-3)*log(
x*e + d) + (x^2*e - 2*d*x)*e^(-2))*a^2 - integrate(-1/4*((b^2*c*x^3*e^2 - b
^2*x^2*e^2)*log(c*x + 1)^2 + 4*(a*b*c*x^3*e^2 - a*b*x^2*e^2)*log(c*x + 1) +
 (2*b^2*c*d^2*x - (4*a*b*c + b^2*c)*x^3*e^2 + (b^2*c*d*e + 4*a*b*e^2)*x^2 -
 2*(b^2*c*x^3*e^2 - b^2*x^2*e^2)*log(c*x + 1))*log(-c*x + 1))/(c*x^2*e^3 +
(c*d*e^2 - e^3)*x - d*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(x*e +
 d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\text{atanh}\,(cx))^2}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(a+b\,\mathrm{atanh}(c\,x))^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x))^2)/(d + e*x),x)
[Out] int((x^2*(a + b*atanh(c*x))^2)/(d + e*x), x)
```

3.155 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=279

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+c\right.}}{e^2}$$

[Out] (a+b*arctanh(c*x))^2/c/e+x*(a+b*arctanh(c*x))^2/e-2*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c/e+d*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e^2-d*(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2-b^2*polylog(2,1-2/(-c*x+1))/c/e-b*d*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e^2+b*d*(a+b*arctanh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2-1/2*b^2*d*polylog(3,1-2/(c*x+1))/e^2+1/2*b^2*d*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2

Rubi [A]

time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6087, 6021, 6131, 6055, 2449, 2352, 6059}

$$-\frac{bd\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{bd\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e^2}+\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{ce}-\frac{b^2d\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2e^2}+\frac{b^2d\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^2}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)}{ce}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]

[Out] (a + b*ArcTanh[c*x])^2/(c*e) + (x*(a + b*ArcTanh[c*x])^2)/e - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c*e) + (d*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e^2 - (d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^2 - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e) - (b*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^2 + (b*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^2 - (b^2*d*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e^2) + (b^2*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^2)

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx=\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{e(d+ex)}\right)dx$$

$$=\frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{e}-\frac{d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{e}$$

$$=\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx\right.}{}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right.}{e^2}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{ce}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{ce}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{ce}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.32, size = 1036, normalized size = 3.71

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (6*a^2*e*x - 6*a^2*d*Log[d + e*x] + (6*a*b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*
e*x*ArcTanh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2
 - e*ArcTanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[
(c*d)/e] + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1
 + E^(2*ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e]
 + ArcTanh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2
*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*Po
lyLog[2, -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))]))/c + (b^2*(-6*e*ArcTanh[c*x]^2 + 6*c*e*x*ArcTanh[c*x]^2 + 8
*c*d*ArcTanh[c*x]^3 - 4*e*ArcTanh[c*x]^3 + (4*Sqrt[1 - (c^2*d^2)/e^2]*e*Arc
Tanh[c*x]^3)/E^ArcTanh[(c*d)/e] - 12*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (6*I)*c*d*Pi*Ar
cTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] + 6*c*d*ArcTanh[c*x]
```

```
^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] - 6*c*d*ArcTanh[c*x]^2
*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[1
+ E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*
(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*
Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])))] - 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x]))
 + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2
*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] + (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c
^2*x^2] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e]
+ ArcTanh[c*x]]] + 6*(e - c*d*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])
] + 6*c*d*ArcTanh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e
))] - 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])]
- 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c
*d*ArcTanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 3*c*d
*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 3*c*d*PolyLog[3, -(((c*d + e)*E^(2*ArcT
anh[c*x]))/(c*d - e))] + 12*c*d*PolyLog[3, -E^(ArcTanh[(c*d)/e] + ArcTanh[c
*x])] + 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 3*c*d*Poly
Log[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c)/(6*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.52, size = 14121, normalized size = 50.61

method	result	size
derivativedivides	Expression too large to display	14121
default	Expression too large to display	14121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x*e^(-1)*log(-c*x + 1)^2 - (d*e^(-2)*log(x*e + d) - x*e^(-1))*a^2 -
 integrate(-1/4*((b^2*c*x^2*e - b^2*x*e)*log(c*x + 1)^2 + 4*(a*b*c*x^2*e -
a*b*x*e)*log(c*x + 1) - 2*((2*a*b*c + b^2*c)*x^2*e + (b^2*c*d - 2*a*b*e)*x
+ (b^2*c*x^2*e - b^2*x*e)*log(c*x + 1))*log(-c*x + 1))/(c*x^2*e^2 + (c*d*e
- e^2)*x - d*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(x*e + d), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{atanh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(e*x+d),x)
[Out] Integral(x*(a + b*atanh(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2*x/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atanh(c*x))^2)/(d + e*x),x)
[Out] int((x*(a + b*atanh(c*x))^2)/(d + e*x), x)
```

3.156 $\int \frac{(a+b\tanh^{-1}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=188

$$-\frac{(a+b\tanh^{-1}(cx))^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{(a+b\tanh^{-1}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b(a+b\tanh^{-1}(cx))\,\text{PolyLog}(2,1$$

```
[Out] -(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(
c*d+e)/(c*x+1))/e+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e-b*(a+b*arct
anh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*b^2*polylog(3,1-2/
(c*x+1))/e-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e
```

Rubi [A]
time = 0.03, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6059}

$$-\frac{b(a+b\tanh^{-1}(cx))\,\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e}+\frac{(a+b\tanh^{-1}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{e}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))^2}{e}-\frac{b^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}+\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/e - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d
+ e*x))/((c*d + e)*(1 + c*x))])/e + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e)
 - (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int\frac{(a+b\tanh^{-1}(cx))^2}{d+ex}\,dx=-\frac{(a+b\tanh^{-1}(cx))^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{(a+b\tanh^{-1}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.85, size = 938, normalized size = 4.99

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] (6*a^2*Log[d + e*x] + 6*a*b*ArcTanh[c*x]*(Log[1 - c^2*x^2] + 2*Log[I*Sinh[A
rcTanh[(c*d)/e] + ArcTanh[c*x]]]) - (6*I)*a*b*((-1/4*I)*(Pi - (2*I)*ArcTanh
[c*x])^2 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x]
)*Log[1 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log
[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*
Log[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I
)*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])
] - I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]) + (b^2*(-8*c*d*
ArcTanh[c*x]^3 + 4*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[
c*x]^3)/E^ArcTanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x
])] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2]
 - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] +
 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*
ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh
[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 12*c*d*ArcTanh[(
c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E
^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1
+ E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] -
 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] - (3*I)*c*d*Pi*A
rcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*S
inh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d*ArcTanh[c*x]*PolyLog[2, -E^(-
2*ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[
c*x]))/(c*d - e))] + 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] +
ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTa
nh[c*x])] + 6*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[
c*x]))] + 3*c*d*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*c*d*PolyLog[3, -(((c*d
 + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[(c*d)
/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x]
)] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(c*d))/(6*
e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 1197, normalized size = 6.37

method	result	size
derivativedivides	Expression too large to display	1197

default	Expression too large to display	1197

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*ln(c*e*x+c*d)/e+b^2*c*ln(c*e*x+c*d)/e*arctanh(c*x)^2-b^2*c/e*arc
tanh(c*x)^2*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))
-1/2*I*b^2*c/e*arctanh(c*x)^2*Pi*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*(
(c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))+1/2*I*b^2*c/e*arctanh(c*x)^2*Pi*csgn(I*(d*c*(1+(c*x+1)^2/(
-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/
2*I*b^2*c/e*arctanh(c*x)^2*Pi*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*
x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(d*c*(1+(c*x+1)^
2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^
2+1)))-1/2*I*b^2*c/e*arctanh(c*x)^2*Pi*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1
))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(d*c*
(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1)))-b^2*c/e*arctanh(c
*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*b^2*c/e*polylog(3,-(c*x+1)^2/(-c
^2*x^2+1))+b^2*c/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)
/(-c*d+e))+b^2*c/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2
+1)/(-c*d+e))-1/2*b^2*c/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-
c*d+e))+b^2*c^2/e*d/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2
+1)/(-c*d+e))+b^2*c^2/e*d/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/
(-c^2*x^2+1)/(-c*d+e))-1/2*b^2*c^2/e*d/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/
(-c^2*x^2+1)/(-c*d+e))+2*a*b*c*ln(c*e*x+c*d)/e*arctanh(c*x)-a*b*c/e*ln(c*e*
x+c*d)*ln((c*e*x+e)/(-c*d+e))-a*b*c/e*dilog((c*e*x+e)/(-c*d+e))+a*b*c/e*ln(
(c*e*x-e)/(-c*d-e))*ln(c*e*x+c*d)+a*b*c/e*dilog((c*e*x-e)/(-c*d-e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*e^(-1)*log(x*e + d) + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^
2/(x*e + d) + a*b*(log(c*x + 1) - log(-c*x + 1))/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^2}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(d + e*x), x)
```

3.157 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x(d+ex)}\,dx$

Optimal. Leaf size=319

$$\frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+e\right.}{(cd+e)(1-}\right.}{d}$$

```
[Out] -2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d+(a+b*arctanh(c*x))^2*ln(2/
(c*x+1))/d-(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d-b*(a+b*ar
ctanh(c*x))*polylog(2,1-2/(-c*x+1))/d+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(
-c*x+1))/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d+b*(a+b*arctanh(c*x
))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d+1/2*b^2*polylog(3,1-2/(-c*x+1
))/d-1/2*b^2*polylog(3,-1+2/(-c*x+1))/d-1/2*b^2*polylog(3,1-2/(c*x+1))/d+1/
2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Rubi [A]
time = 0.32, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6087, 6033, 6199, 6095, 6205, 6745, 6059}

$$\frac{b(a+b\tanh^{-1}(cx))\,\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d}-\frac{(a+b\tanh^{-1}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))}{d}+\frac{b\text{Li}_2\left(\frac{2}{1-cx}-1\right)(a+b\tanh^{-1}(cx))}{d}-\frac{b\text{Li}_2\left(1-\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))}{d}+\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)(a+b\tanh^{-1}(cx))^2}{d}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\tanh^{-1}(cx))^2}{d}+\frac{b^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}+\frac{b^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d}-\frac{b^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d}-\frac{b^2\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d + ((a + b*ArcTanh[c*x
])^2*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + e)*(1 + c*x))])/d - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]
)/d + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d - (b*(a + b*A
rcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + (b^2*PolyLog[3, 1 - 2
/(1 - c*x)])/(2*d) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d) - (b^2*PolyLo
g[3, 1 - 2/(1 + c*x)])/(2*d) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))])/(2*d)
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
```

```
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+ex)}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d} - \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{d}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - $$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - $$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - $$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - $$

Mathematica [C] Result contains complex when optimal does not.
time = 8.02, size = 1034, normalized size = 3.24

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (a^2*Log[x])/d - (a^2*Log[d + e*x])/d + (a*b*((-I)*c*d*Pi*ArcTanh[c*x] - 2*
c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2 +
 (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*ArcT
anh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c*x])
] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))
] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] +
(I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(
c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*d*PolyLo
g[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(c*d^2) + (b^2*(I*c*d*Pi^3
 - 8*c*d*ArcTanh[c*x]^3 - 8*e*ArcTanh[c*x]^3 + 24*c*d*ArcTanh[c*x]^2*Log[1
- E^(2*ArcTanh[c*x])] + 24*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])]
- 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] - (24*(c*d - e)*(c*d + e)*(-6*c*d*A
rcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c
*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]
) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTa
nh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + A
rcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[
```

```
c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]
))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] -
 ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcT
anh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))
/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*
x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e
]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcTanh
[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] + 12*c*d*ArcT
anh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTanh[
c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]*P
olyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -(((c
*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[(c*
d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*
x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(6*c^2*d
^2 - 6*e^2)))/(24*c*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 20.32, size = 1799, normalized size = 5.64

method	result	size
derivativedivides	Expression too large to display	1799
default	Expression too large to display	1799

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d*ln(c*e*x+c*d)-b^2/d*e/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/
(-c^2*x^2+1)/(-c*d+e))-b^2*c/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/
(-c^2*x^2+1)/(-c*d+e))-b^2*c/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)
^2/(-c^2*x^2+1)/(-c*d+e))+1/2*b^2*c/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c
^2*x^2+1)/(-c*d+e))+1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-b^2/d*e/(c*d+e)*arctanh(c*x)*polylog(2,(c*d
+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/2*I*b^2/d*Pi*arct
anh(c*x)^2*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)
-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^
2/(-c^2*x^2+1)-1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c
^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/2*I*b^2/d*Pi*arctanh(c*x)^2*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((
c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d*Pi*arct
anh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1
)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c
```

```
^2*x^2+1)))^2+1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*
x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1
))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+a^2/d*ln(c*x
)-2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,-(c*x+1)/
(-c^2*x^2+1)^(1/2))-a*b/d*ln(c*x)*ln(c*x+1)+a*b/d*dilog((c*e*x+e)/(-c*d+e))
-a*b/d*dilog((c*e*x-e)/(-c*d-e))-a*b/d*dilog(c*x+1)-a*b/d*dilog(c*x)+b^2*ar
ctanh(c*x)^2/d*ln(c*x)-b^2*arctanh(c*x)^2/d*ln(c*e*x+c*d)+b^2*arctanh(c*x)^
2/d*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))-b^2*arc
tanh(c*x)^2/d*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2/d*arctanh(c*x)^2*ln(1-(c*x+1
)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(
1/2))+b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh
(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*b^2/d*e/(c*d+e)*polylog(3,
(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+2*a*b*arctanh(c*x)/d*ln(c*x)-2*a*b
*arctanh(c*x)/d*ln(c*e*x+c*d)+a*b/d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-a*
b/d*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^2*(log(x*e + d)/d - log(x)/d) + integrate(1/4*b^2*(log(c*x + 1) - log(-c
*x + 1))^2/(x^2*e + d*x) + a*b*(log(c*x + 1) - log(-c*x + 1))/(x^2*e + d*x)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}{\left(cx \right)}\right)^{2}}{x \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))^2}{x\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x))^2/(x*(d + e*x)), x)
```

3.158 $\int \frac{(a+b\tanh^{-1}(cx))^2}{x^2(d+ex)}\,dx$

Optimal. Leaf size=412

$$\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2}-\frac{e\left(a+b\tanh^{-1}(cx\right.}{d^2}$$

```
[Out] c*(a+b*arctanh(c*x))^2/d-(a+b*arctanh(c*x))^2/d/x+2*e*(a+b*arctanh(c*x))^2*
arctanh(-1+2/(-c*x+1))/d^2-e*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2+e*(a+b*
arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x)
)*ln(2-2/(c*x+1))/d+b*e*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2-b*e*
(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2+b*e*(a+b*arctanh(c*x))*poly
log(2,1-2/(c*x+1))/d^2-b^2*c*polylog(2,-1+2/(c*x+1))/d-b*e*(a+b*arctanh(c*x
))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2-1/2*b^2*e*polylog(3,1-2/(-c
*x+1))/d^2+1/2*b^2*e*polylog(3,-1+2/(-c*x+1))/d^2+1/2*b^2*e*polylog(3,1-2/(
c*x+1))/d^2-1/2*b^2*e*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2
```

Rubi [A]
time = 0.43, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6087, 6037, 6135, 6079, 2497, 6033, 6199, 6095, 6205, 6745, 6059}

$$\frac{be\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{be\text{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d^2}-\frac{2e\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}-\frac{e\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2e\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d^2}+\frac{b^2e\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d^2}+\frac{b^2e\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d^2}-\frac{b^2e\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d^2}-\frac{b^2c\text{Li}_2\left(\frac{2}{cx+1}-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) - (2*e*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (e*(a + b*ArcTanh[c*x])^2*Lo
g[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d
+ (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2 - (b*e*(a + b*
ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (b*e*(a + b*ArcTanh[c*x])
*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d
- (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 +
c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[
3, -1 + 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
 - (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^2)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(d+ex)}\right) dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}dx}{d} - \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx}{d^2} + \frac{e^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}dx}{d^2}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} - \frac{e\left(a+b\,\mathrm{ta}\right.}{}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(\right.}{d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.76, size = 1188, normalized size = 2.88

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] -(a^2/(d*x)) - (a^2*e*Log[x])/d^2 + (a^2*e*Log[d + e*x])/d^2 + (a*b*(I*c*d*
e*Pi*ArcTanh[c*x] - (2*c*d^2*ArcTanh[c*x])/x + 2*c*d*e*ArcTanh[(c*d)/e]*Arc
Tanh[c*x] - c*d*e*ArcTanh[c*x]^2 + e^2*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*d^2)
/e^2]*e^2*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e*ArcTanh[c*x]*Log[1 -
 E^(-2*ArcTanh[c*x])] - I*c*d*e*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c*d*e*Ar
cTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c*d*e*
ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*
Log[(c*x)/Sqrt[1 - c^2*x^2]] - (I/2)*c*d*e*Pi*Log[1 - c^2*x^2] - 2*c*d*e*Ar
cTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + c*d*e*PolyLog
[2, E^(-2*ArcTanh[c*x])] - c*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTa
nh[c*x]))]))/(c*d^3) + (b^2*((-I)*c*d*e*Pi^3 + 24*c^2*d^2*ArcTanh[c*x]^2 -
(24*c*d^2*ArcTanh[c*x]^2)/x + 8*c*d*e*ArcTanh[c*x]^3 + 8*e^2*ArcTanh[c*x]^3
 + 48*c^2*d^2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*c*d*e*ArcTanh[
c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^2*d^2*PolyLog[2, E^(-2*ArcTanh[c*
x])] - 24*c*d*e*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 12*c*d*e*Poly
Log[3, E^(2*ArcTanh[c*x])]))/(24*c*d^3) + (b^2*(c*d - e)*e*(c*d + e)*(-6*c*
d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTan
```

```
h[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c
*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*Ar
cTanh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e]
+ ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTa
nh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e
] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*A
rcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x]
)))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c
^2*x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d
)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcT
anh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] + 12*c*d*A
rcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTa
nh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x
]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -(
((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[
(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh
[c*x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(d^3*
(6*c^3*d^2 - 6*c*e^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 37.68, size = 26972, normalized size = 65.47

method	result	size
derivativedivides	Expression too large to display	26972
default	Expression too large to display	26972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(e*log(x*e + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/4*b^2*log(-c*x + 1)^2
/(d*x) - integrate(-1/4*((b^2*c*d*x - b^2*d)*log(c*x + 1)^2 + 4*(a*b*c*d*x
- a*b*d)*log(c*x + 1) + 2*(b^2*c*x^2*e + 2*a*b*d - (2*a*b*c*d - b^2*c*d)*x
- (b^2*c*d*x - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c*d*x^4*e - d^2*x^2 + (
c*d^2 - d*e)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(x^3*e + d*x^2), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx))^2}{x^2 (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(e*x+d),x)
[Out] Integral((a + b*atanh(c*x))**2/(x**2*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{atanh}(c\,x))^2}{x^2\,(d + e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(x^2*(d + e*x)),x)
[Out] int((a + b*atanh(c*x))^2/(x^2*(d + e*x)), x)
```

3.159 $\int \frac{\tanh^{-1}(cx)^2}{x(d+ex)}\,dx$

Optimal. Leaf size=275

$$\frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d}-\frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}-\frac{\tanh^{-1}(cx)\mathrm{Poly}}{}$$

```
[Out] -2*arctanh(c*x)^2*arctanh(-1+2/(-c*x+1))/d+arctanh(c*x)^2*ln(2/(c*x+1))/d-a
rctanh(c*x)^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d-arctanh(c*x)*polylog(2,1-2/
(-c*x+1))/d+arctanh(c*x)*polylog(2,-1+2/(-c*x+1))/d-arctanh(c*x)*polylog(2,
1-2/(c*x+1))/d+arctanh(c*x)*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d+1/2*
polylog(3,1-2/(-c*x+1))/d-1/2*polylog(3,-1+2/(-c*x+1))/d-1/2*polylog(3,1-2/
(c*x+1))/d+1/2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Rubi [A]
time = 0.25, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6087, 6033, 6199, 6095, 6205, 6745, 6059}

$$\frac{\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}+\frac{\tanh^{-1}(cx)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d}-\frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2d}-\frac{\text{Li}_3\left(\frac{2}{1-cx}-1\right)}{2d}-\frac{\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2d}-\frac{\text{Li}_2\left(1-\frac{2}{1-cx}\right)\tanh^{-1}(cx)}{d}+\frac{\text{Li}_2\left(\frac{2}{1-cx}-1\right)\tanh^{-1}(cx)}{d}-\frac{\text{Li}_2\left(1-\frac{2}{cx+1}\right)\tanh^{-1}(cx)}{d}+\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\tanh^{-1}(cx)^2}{d}+\frac{\log\left(\frac{2}{cx+1}\right)\tanh^{-1}(cx)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

```
[Out] (2*ArcTanh[c*x]^2*ArcTanh[1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]^2*Log[2/(1 +
c*x)])/d - (ArcTanh[c*x]^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d -
(ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]*PolyLog[2, -1
+ 2/(1 - c*x)])/d - (ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/d + (ArcTanh
[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + PolyLog[3,
 1 - 2/(1 - c*x)]/(2*d) - PolyLog[3, -1 + 2/(1 - c*x)]/(2*d) - PolyLog[3, 1
 - 2/(1 + c*x)]/(2*d) + PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x)
)]/(2*d)
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
```

```
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6087

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(cx)^2}{x(d+ex)}\,dx = \int \left(\frac{\tanh^{-1}(cx)^2}{dx} - \frac{e\tanh^{-1}(cx)^2}{d(d+ex)}\right)dx$$
$$= \frac{\int \frac{\tanh^{-1}(cx)^2}{x}\,dx}{d} - \frac{e\int \frac{\tanh^{-1}(cx)^2}{d+ex}\,dx}{d}$$
$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+}{(cd+e)(1}\right.}{d}$$
$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+}{(cd+e)(1}\right.}{d}$$
$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+}{(cd+e)(1}\right.}{d}$$
$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+}{(cd+e)(1}\right.}{d}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 6.55, size = 733, normalized size = 2.67

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

```
[Out] (I*c*d*Pi^3 - 8*c*d*ArcTanh[c*x]^3 - 8*e*ArcTanh[c*x]^3 + 24*c*d*ArcTanh[c*
x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*ArcT
anh[c*x])] - 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] - (24*(c*d - e)*(c*d + e
)*(-6*c*d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*
e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-A
rcTanh[c*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)
*E^(2*ArcTanh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(
c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh
[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] +
 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcT
anh[c*x])))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sq
rt[1 - c^2*x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcT
anh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*
c*d*ArcTanh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] +
12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c
```

```
*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*Arc
Tanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyL
og[3, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(
ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] +
 ArcTanh[c*x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]
))/(6*c^2*d^2 - 6*e^2))/(24*c*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 49.97, size = 1566, normalized size = 5.69

method	result	size
derivativedivides	Expression too large to display	1566
default	Expression too large to display	1566

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c*x)^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh(c*x)^2/d*ln(c*e*x+c*d)+arctanh(c*x)^2/d*ln(c*x)-2*c*(-1/2*arctanh(
c*x)^2/d/c*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))+
1/2*arctanh(c*x)^2/d/c*ln((c*x+1)^2/(-c^2*x^2+1)-1)-1/2/d/c*arctanh(c*x)^2*
ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-1/d/c*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2
*x^2+1)^(1/2))+1/d/c*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2/d/c*arctanh(
c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-1/d/c*arctanh(c*x)*polylog(2,-(c*x+
1)/(-c^2*x^2+1)^(1/2))+1/d/c*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-1/4*I/d
/c*Pi*arctanh(c*x)^2*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-
c^2*x^2+1)-1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x
^2+1)-1))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/4*I/d/c*Pi*arctanh(c*x)^2*csgn(I*
(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/
(-c^2*x^2+1)))^3-1/4*I/d/c*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+
1)))*csgn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(
1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/4*I/d/c*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))^2+1/4*I/d/c*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-1/4*I/d/c*Pi*arct
anh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-1/4*I/d
/c*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^3+1/4*I/d/c*Pi*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*cs
gn(I*(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1)))*csgn(I*
(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))/(1+(c*x+1)^2/
(-c^2*x^2+1)))+1/2/d/c*e/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^
2*x^2+1)/(-c*d+e))+1/2/d/c*e/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)
^2/(-c^2*x^2+1)/(-c*d+e))-1/4/d/c*e/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c
^2*x^2+1)/(-c*d+e))+1/2/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2
*x^2+1)/(-c*d+e))+1/2/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^
```

```
2*x^2+1)/(-c*d+e))-1/4/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c
*d+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="maxima")
[Out] integrate(arctanh(c*x)^2/((x*e + d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="fricas")
[Out] integral(arctanh(c*x)^2/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(cx)}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c*x)**2/x/(e*x+d),x)
[Out] Integral(atanh(c*x)**2/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="giac")
[Out] integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(c\,x)^2}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c*x)^2/(x*(d + e*x)),x)
[Out] int(atanh(c*x)^2/(x*(d + e*x)), x)
```

3.160 $\int \frac{1}{(d+ex)(a+b\mathbf{ArcTan}(cx))}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{(d+ex)(a+b\mathrm{ArcTan}(cx))},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)(a+b\mathrm{ArcTan}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\,(a+b\tan^{-1}(cx))}\,dx = \int \frac{1}{(d+ex)\,(a+b\tan^{-1}(cx))}\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)(a+b\mathrm{ArcTan}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Maple [A]
time = 2.75, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\arctan(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
[Out] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arctan(c*x) + a)*(x*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*x*e + a*d + (b*x*e + b*d)*arctan(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{atan}(cx))(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
[Out] Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b \operatorname{atan}(c\,x))\,(d + e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*atan(c*x))*(d + e*x)),x)
[Out] int(1/((a + b*atan(c*x))*(d + e*x)), x)
```

3.161 $\int x^4(1-a^2x^2)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=72

$$\frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{35a^5}$$

```
[Out] 1/35*x^2/a^3+1/70*x^4/a-1/42*a*x^6+1/5*x^5*arctanh(a*x)-1/7*a^2*x^7*arctanh
(a*x)+1/35*ln(-a^2*x^2+1)/a^5
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {6161, 6037, 272, 45}

$$\frac{x^2}{35a^3}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{35a^5}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)+\frac{x^4}{70a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int x^4(1-a^2x^2)\tanh^{-1}(ax)\,dx &= -\left(a^2\int x^6\tanh^{-1}(ax)\,dx\right)+\int x^4\tanh^{-1}(ax)\,dx\\ &= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{5}a\int\frac{x^5}{1-a^2x^2}\,dx+\frac{1}{7}a^3\int\frac{x^7}{1-a\\ &= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{10}a\text{Subst}\left(\int\frac{x^2}{1-a^2x}\,dx,x,x^2\right)\\ &= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{10}a\text{Subst}\left(\int\left(-\frac{1}{a^4}-\frac{x}{a^2}-\frac{}{a^4(-}\right.\right.\\ &= \frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log(1-a^2x}{35a^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 1.00

$$\frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{35a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Maple [A]
time = 0.28, size = 70, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}-\frac{a^6x^6}{42}+\frac{a^4x^4}{70}+\frac{a^2x^2}{35}+\frac{\ln(ax-1)}{35}+\frac{\ln(ax+1)}{35}}{a^5}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}-\frac{a^6x^6}{42}+\frac{a^4x^4}{70}+\frac{a^2x^2}{35}+\frac{\ln(ax-1)}{35}+\frac{\ln(ax+1)}{35}}{a^5}$
risch	$\left(-\frac{1}{14}a^2x^7+\frac{1}{10}x^5\right)\ln(ax+1)+\frac{a^2x^7\ln(-ax+1)}{14}-\frac{x^6a}{42}-\frac{x^5\ln(-ax+1)}{10}+\frac{x^4}{70a}+\frac{x^2}{35a^3}+\frac{\ln(a^2x^2}{35a^5}$

meijerg	$-\frac{\frac{x^2a^2\left(4a^4x^4+6a^2x^2+12\right)}{42}-\frac{2x^8a^8\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(-a^2x^2+1\right)}{7}}{4a^5}-\frac{-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\frac{2a^6x^6}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-1/7*arctanh(a*x)*a^7*x^7+1/5*arctanh(a*x)*a^5*x^5-1/42*a^6*x^6+1/70
*a^4*x^4+1/35*a^2*x^2+1/35*ln(a*x-1)+1/35*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 73, normalized size = 1.01

$$-\frac{1}{210}\,a\left(\frac{5\,a^4x^6-3\,a^2x^4-6\,x^2}{a^4}-\frac{6\,\log\left(ax+1\right)}{a^6}-\frac{6\,\log\left(ax-1\right)}{a^6}\right)-\frac{1}{35}\,\left(5\,a^2x^7-7\,x^5\right)\,\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*log(
a*x - 1)/a^6) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)
```

Fricas [A]
time = 0.37, size = 76, normalized size = 1.06

$$-\frac{5\,a^6x^6-3\,a^4x^4-6\,a^2x^2+3\,\left(5\,a^7x^7-7\,a^5x^5\right)\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(a^2x^2-1\right)}{210\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*x^6 - 3*a^4*x^4 - 6*a^2*x^2 + 3*(5*a^7*x^7 - 7*a^5*x^5)*log(-
(a*x + 1)/(a*x - 1)) - 6*log(a^2*x^2 - 1))/a^5
```

Sympy [A]
time = 0.45, size = 71, normalized size = 0.99

$$\begin{cases} -\frac{a^2x^7\,\mathrm{atanh}\left(ax\right)}{7}-\frac{ax^6}{42}+\frac{x^5\,\mathrm{atanh}\left(ax\right)}{5}+\frac{x^4}{70a}+\frac{x^2}{35a^3}+\frac{2\log\left(x-\frac{1}{a}\right)}{35a^5}+\frac{2\,\mathrm{atanh}\left(ax\right)}{35a^5} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**7*atanh(a*x)/7 - a*x**6/42 + x**5*atanh(a*x)/5 + x**4/(
70*a) + x**2/(35*a**3) + 2*log(x - 1/a)/(35*a**5) + 2*atanh(a*x)/(35*a**5),
 Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(60) = 120.

time = 0.40, size = 335, normalized size = 4.65

$$\frac{2}{105}\,a\left(\frac{3\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^{6}}-\frac{3\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^{6}}-\frac{\frac{3\,(ax+1)^{5}}{(ax-1)^{5}}+\frac{36\,(ax+1)^{4}}{(ax-1)^{4}}+\frac{2\,(ax+1)^{3}}{(ax-1)^{3}}+\frac{36\,(ax+1)^{2}}{(ax-1)^{2}}+\frac{3\,(ax+1)}{ax-1}}{a^{6}\left(\frac{ax+1}{ax-1}-1\right)^{6}}-\frac{3\left(\frac{35\,(ax+1)^{5}}{(ax-1)^{5}}+\frac{35\,(ax+1)^{4}}{(ax-1)^{4}}+\frac{70\,(ax+1)^{3}}{(ax-1)^{3}}+\frac{14\,(ax+1)^{2}}{(ax-1)^{2}}+\frac{7\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^{6}\left(\frac{ax+1}{ax-1}-1\right)^{7}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 2/105*a*(3*log(abs(-a*x - 1)/abs(a*x - 1))/a^6 - 3*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^6 - (3*(a*x + 1)^5/(a*x - 1)^5 + 36*(a*x + 1)^4/(a*x - 1)^4 +
2*(a*x + 1)^3/(a*x - 1)^3 + 36*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x + 1)/(a*x -
 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^6) - 3*(35*(a*x + 1)^5/(a*x - 1)^5 + 35
*(a*x + 1)^4/(a*x - 1)^4 + 70*(a*x + 1)^3/(a*x - 1)^3 + 14*(a*x + 1)^2/(a*x
 - 1)^2 + 7*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*
x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x
 - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^7))
```

Mupad [B]

time = 0.99, size = 61, normalized size = 0.85

$$\frac{\frac{\ln\left(a^{2}\,x^{2}-1\right)}{35}+\frac{a^{2}\,x^{2}}{35}+\frac{a^{4}\,x^{4}}{70}}{a^{5}}-\frac{a\,x^{6}}{42}+\frac{x^{5}\,\mathrm{atanh}(a\,x)}{5}-\frac{a^{2}\,x^{7}\,\mathrm{atanh}(a\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] (log(a^2*x^2 - 1)/35 + (a^2*x^2)/35 + (a^4*x^4)/70)/a^5 - (a*x^6)/42 + (x^5
*atanh(a*x))/5 - (a^2*x^7*atanh(a*x))/7
```

3.162 $\int x^3(1-a^2x^2)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=63

$$\frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}-\frac{\tanh^{-1}(ax)}{12a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)$$

```
[Out] 1/12*x/a^3+1/36*x^3/a-1/30*a*x^5-1/12*arctanh(a*x)/a^4+1/4*x^4*arctanh(a*x)
-1/6*a^2*x^6*arctanh(a*x)
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6161, 6037, 308, 212}

$$-\frac{\tanh^{-1}(ax)}{12a^4}+\frac{x}{12a^3}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)+\frac{x^3}{36a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 - ArcTanh[a*x]/(12*a^4) + (x^4*ArcTanh
[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
 + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int x^3(1-a^2x^2)\tanh^{-1}(ax)\,dx &= -\left(a^2\int x^5\tanh^{-1}(ax)\,dx\right)+\int x^3\tanh^{-1}(ax)\,dx\\ &= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{1-a^2x^2}\,dx+\frac{1}{6}a^3\int\frac{x^6}{1-a\\ &= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{1}{4}a\int\left(-\frac{1}{a^4}-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)\\ &= \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{\int\frac{1}{1-a^2x^2}\,dx}{6a^3}\\ &= \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}-\frac{\tanh^{-1}(ax)}{12a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 79, normalized size = 1.25

$$\frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{\log(1-ax)}{24a^4}-\frac{\log(1+ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcT
anh[a*x])/6 + Log[1 - a*x]/(24*a^4) - Log[1 + a*x]/(24*a^4)
```

Maple [A]
time = 0.28, size = 66, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^6x^6}{6}+\frac{a^4x^4\operatorname{arctanh}(ax)}{4}-\frac{a^5x^5}{30}+\frac{a^3x^3}{36}+\frac{ax}{12}+\frac{\ln(ax-1)}{24}-\frac{\ln(ax+1)}{24}}{a^4}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^6x^6}{6}+\frac{a^4x^4\operatorname{arctanh}(ax)}{4}-\frac{a^5x^5}{30}+\frac{a^3x^3}{36}+\frac{ax}{12}+\frac{\ln(ax-1)}{24}-\frac{\ln(ax+1)}{24}}{a^4}$
risch	$\left(-\frac{1}{12}x^6a^2+\frac{1}{8}x^4\right)\ln(ax+1)+\frac{a^2x^6\ln(-ax+1)}{12}-\frac{a\,x^5}{30}-\frac{x^4\ln(-ax+1)}{8}+\frac{x^3}{36a}+\frac{x}{12a^3}+\frac{\ln(-ax+}{24a^4}$

meijerg	$-\frac{i\left(-\frac{2ixa(21a^4x^4+35a^2x^2+105)}{315}-\frac{ixa(-7a^6x^6+7)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{21\sqrt{a^2x^2}}\right)}{4a^4} - \frac{i\left(\frac{ixa(5a^2x^2+15)}{15}+\frac{ixa}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/6*arctanh(a*x)*a^6*x^6+1/4*a^4*x^4*arctanh(a*x)-1/30*a^5*x^5+1/36
*a^3*x^3+1/12*a*x+1/24*ln(a*x-1)-1/24*ln(a*x+1))
```

Maxima [A]
time = 0.27, size = 72, normalized size = 1.14

$$-\frac{1}{360}\,a\left(\frac{2\,(6\,a^4x^5-5\,a^2x^3-15\,x)}{a^4}+\frac{15\,\log\left(ax+1\right)}{a^5}-\frac{15\,\log\left(ax-1\right)}{a^5}\right)-\frac{1}{12}\left(2\,a^2x^6-3\,x^4\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/360*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*l
og(a*x - 1)/a^5) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.97

$$-\frac{12\,a^5x^5-10\,a^3x^3-30\,ax+15\,(2\,a^6x^6-3\,a^4x^4+1)\log\left(-\frac{ax+1}{ax-1}\right)}{360\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/360*(12*a^5*x^5 - 10*a^3*x^3 - 30*a*x + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*l
og(-(a*x + 1)/(a*x - 1)))/a^4
```

Sympy [A]
time = 0.36, size = 54, normalized size = 0.86

$$\begin{cases}-\frac{a^2x^6\operatorname{atanh}\left(ax\right)}{6}-\frac{ax^5}{30}+\frac{x^4\operatorname{atanh}\left(ax\right)}{4}+\frac{x^3}{36a}+\frac{x}{12a^3}-\frac{\operatorname{atanh}\left(ax\right)}{12a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**6*atanh(a*x)/6 - a*x**5/30 + x**4*atanh(a*x)/4 + x**3/(
36*a) + x/(12*a**3) - atanh(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(51) = 102.

time = 0.41, size = 227, normalized size = 3.60

$$-\frac{1}{45}\,a\left(\frac{\frac{45\,(ax+1)^3}{(ax-1)^3}-\frac{25\,(ax+1)^2}{(ax-1)^2}+\frac{35\,(ax+1)}{ax-1}-7}{a^5\left(\frac{ax+1}{ax-1}-1\right)^5}+\frac{30\left(\frac{3\,(ax+1)^4}{(ax-1)^4}+\frac{2\,(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)^2}{(ax-1)^2}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^5\left(\frac{ax+1}{ax-1}-1\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/45*a*((45*(a*x + 1)^3/(a*x - 1)^3 - 25*(a*x + 1)^2/(a*x - 1)^2 + 35*(a*x
 + 1)/(a*x - 1) - 7)/(a^5*((a*x + 1)/(a*x - 1) - 1)^5) + 30*(3*(a*x + 1)^4/
(a*x - 1)^4 + 2*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2)*log(-(
a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/
(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^5*((a*x + 1)/(a*x - 1)
- 1)^6))
```

Mupad [B]

time = 0.95, size = 51, normalized size = 0.81

$$\frac{\frac{a\,x}{12}-\frac{\mathrm{atanh}(a\,x)}{12}+\frac{a^3\,x^3}{36}}{a^4}-\frac{a\,x^5}{30}+\frac{x^4\,\mathrm{atanh}(a\,x)}{4}-\frac{a^2\,x^6\,\mathrm{atanh}(a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] ((a*x)/12 - atanh(a*x)/12 + (a^3*x^3)/36)/a^4 - (a*x^5)/30 + (x^4*atanh(a*x
))/4 - (a^2*x^6*atanh(a*x))/6
```

3.163 $\int x^2(1-a^2x^2)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=62

$$\frac{x^2}{15a}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{15a^3}$$

```
[Out] 1/15*x^2/a-1/20*a*x^4+1/3*x^3*arctanh(a*x)-1/5*a^2*x^5*arctanh(a*x)+1/15*ln
(-a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {6161, 6037, 272, 45}

$$-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{15a^3}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)+\frac{x^2}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int x^2\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx &= -\left(a^2\int x^4\tanh^{-1}(ax)\,dx\right)+\int x^2\tanh^{-1}(ax)\,dx\\ &= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{1-a^2x^2}\,dx+\frac{1}{5}a^3\int\frac{x^5}{1-a\\ &= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{1}{6}a\text{Subst}\left(\int\frac{x}{1-a^2x}\,dx,x,x^2\right)+\\ &= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{1}{6}a\text{Subst}\left(\int\left(-\frac{1}{a^2}-\frac{1}{a^2\left(-1+a\right.}\right.\right.\\ &= \frac{x^2}{15a}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{15a^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.00

$$\frac{x^2}{15a}-\frac{ax^4}{20}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{1}{5}a^2x^5\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

Maple [A]
time = 0.28, size = 62, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^3x^3\operatorname{arctanh}(ax)}{3}-\frac{a^4x^4}{20}+\frac{a^2x^2}{15}+\frac{\ln(ax-1)}{15}+\frac{\ln(ax+1)}{15}}{a^3}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^3x^3\operatorname{arctanh}(ax)}{3}-\frac{a^4x^4}{20}+\frac{a^2x^2}{15}+\frac{\ln(ax-1)}{15}+\frac{\ln(ax+1)}{15}}{a^3}$
risch	$\left(-\frac{1}{10}a^2x^5+\frac{1}{6}x^3\right)\ln\left(ax+1\right)+\frac{a^2x^5\ln(-ax+1)}{10}-\frac{x^4a}{20}-\frac{x^3\ln(-ax+1)}{6}+\frac{x^2}{15a}+\frac{\ln\left(a^2x^2-1\right)}{15a^3}-\frac{}{45}$

meijerg	$\frac{-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\frac{2a^6x^6\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{5\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{5}}{4a^3}+\frac{\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right)-\right.}{3\sqrt{a^2x}}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/5*arctanh(a*x)*a^5*x^5+1/3*a^3*x^3*arctanh(a*x)-1/20*a^4*x^4+1/15
*a^2*x^2+1/15*ln(a*x-1)+1/15*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 65, normalized size = 1.05

$$-\frac{1}{60}\,a\left(\frac{3\,a^2x^4-4\,x^2}{a^2}-\frac{4\,\log\left(ax+1\right)}{a^4}-\frac{4\,\log\left(ax-1\right)}{a^4}\right)-\frac{1}{15}\left(3\,a^2x^5-5\,x^3\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*a*((3*a^2*x^4 - 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)
 - 1/15*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 68, normalized size = 1.10

$$-\frac{3\,a^4x^4-4\,a^2x^2+2\left(3\,a^5x^5-5\,a^3x^3\right)\log\left(-\frac{ax+1}{ax-1}\right)-4\,\log\left(a^2x^2-1\right)}{60\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*x^4 - 4*a^2*x^2 + 2*(3*a^5*x^5 - 5*a^3*x^3)*log(-(a*x + 1)/(a*
x - 1)) - 4*log(a^2*x^2 - 1))/a^3
```

Sympy [A]
time = 0.27, size = 63, normalized size = 1.02

$$\begin{cases} -\frac{a^2x^5\operatorname{atanh}\left(ax\right)}{5}-\frac{ax^4}{20}+\frac{x^3\operatorname{atanh}\left(ax\right)}{3}+\frac{x^2}{15a}+\frac{2\log\left(x-\frac{1}{a}\right)}{15a^3}+\frac{2\operatorname{atanh}\left(ax\right)}{15a^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**5*atanh(a*x)/5 - a*x**4/20 + x**3*atanh(a*x)/3 + x**2/(
15*a) + 2*log(x - 1/a)/(15*a**3) + 2*atanh(a*x)/(15*a**3), Ne(a, 0)), (0, T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(52) = 104.

time = 0.39, size = 268, normalized size = 4.32

$$\frac{2}{15}\,a\left(\frac{\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4}-\frac{\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4}-\frac{\frac{(ax+1)^3}{(ax-1)^3}+\frac{4\,(ax+1)^2}{(ax-1)^2}+\frac{ax+1}{ax-1}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^4}-\frac{\left(\frac{15\,(ax+1)^3}{(ax-1)^3}+\frac{5\,(ax+1)^2}{(ax-1)^2}+\frac{5\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^4\left(\frac{ax+1}{ax-1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 2/15*a*(log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - log(abs(-(a*x + 1)/(a*x - 1)
+ 1))/a^4 - ((a*x + 1)^3/(a*x - 1)^3 + 4*(a*x + 1)^2/(a*x - 1)^2 + (a*x + 1
)/(a*x - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^4) - (15*(a*x + 1)^3/(a*x - 1)^
3 + 5*(a*x + 1)^2/(a*x - 1)^2 + 5*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x +
1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1)
+ 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B]

time = 0.91, size = 53, normalized size = 0.85

$$\frac{\frac{\ln\left(a^2\,x^2-1\right)}{15}+\frac{a^2\,x^2}{15}}{a^3}-\frac{a\,x^4}{20}+\frac{x^3\,\mathrm{atanh}(a\,x)}{3}-\frac{a^2\,x^5\,\mathrm{atanh}(a\,x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] (log(a^2*x^2 - 1)/15 + (a^2*x^2)/15)/a^3 - (a*x^4)/20 + (x^3*atanh(a*x))/3
- (a^2*x^5*atanh(a*x))/5
```

3.164 $\int x(1-a^2x^2)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=40

$$\frac{x}{4a}-\frac{ax^3}{12}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4a^2}$$

```
[Out] 1/4*x/a-1/12*a*x^3-1/4*(-a^2*x^2+1)^2*arctanh(a*x)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6141}

$$-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4a^2}-\frac{ax^3}{12}+\frac{x}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(4*a) - (a*x^3)/12 - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*a^2)
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x(1-a^2x^2)\tanh^{-1}(ax)\,dx&=-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4a^2}+\frac{\int(1-a^2x^2)\,dx}{4a}\\&=\frac{x}{4a}-\frac{ax^3}{12}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.72

$$\frac{x}{4a}-\frac{ax^3}{12}+\frac{1}{2}x^2\tanh^{-1}(ax)-\frac{1}{4}a^2x^4\tanh^{-1}(ax)+\frac{\log(1-ax)}{8a^2}-\frac{\log(1+ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(4*a) - (a*x^3)/12 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTanh[a*x])/4 + Lo
g[1 - a*x]/(8*a^2) - Log[1 + a*x]/(8*a^2)
```

Maple [A]
time = 0.38, size = 48, normalized size = 1.20

method	result
derivativedivides	$\frac{-\frac{a^4x^4\,\mathrm{arctanh}(ax)}{4}+\frac{a^2x^2\,\mathrm{arctanh}(ax)}{2}-\frac{\mathrm{arctanh}(ax)}{4}-\frac{a^3x^3}{12}+\frac{ax}{4}}{a^2}$
default	$\frac{-\frac{a^4x^4\,\mathrm{arctanh}(ax)}{4}+\frac{a^2x^2\,\mathrm{arctanh}(ax)}{2}-\frac{\mathrm{arctanh}(ax)}{4}-\frac{a^3x^3}{12}+\frac{ax}{4}}{a^2}$
risch	$-\frac{(a^2x^2-1)^2\ln(ax+1)}{8a^2}+\frac{a^2x^4\ln(-ax+1)}{8}-\frac{x^3a}{12}-\frac{x^2\ln(-ax+1)}{4}+\frac{x}{4a}+\frac{\ln(ax-1)}{8a^2}$
meijerg	$\frac{i\left(\frac{ixa\left(5a^2x^2+15\right)}{15}+\frac{ixa\left(-5a^4x^4+5\right)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{10\sqrt{a^2x^2}}\right)}{4a^2}+\frac{i(-2ixa+2i(-ax+1)(ax+1)\,\mathrm{arctanh}}{4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*a^4*x^4*arctanh(a*x)+1/2*a^2*x^2*arctanh(a*x)-1/4*arctanh(a*x)-
1/12*a^3*x^3+1/4*a*x)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.92

$$-\frac{(a^2x^2-1)^2\,\mathrm{artanh}\,(ax)}{4\,a^2}-\frac{a^2x^3-3\,x}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*x^2 - 1)^2*arctanh(a*x)/a^2 - 1/12*(a^2*x^3 - 3*x)/a
```

Fricas [A]
time = 0.36, size = 52, normalized size = 1.30

$$-\frac{2\,a^3x^3-6\,ax+3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{24\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/24*(2*a^3*x^3 - 6*a*x + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1)))/a^2
```

Sympy [A]
time = 0.24, size = 46, normalized size = 1.15

$$\begin{cases} -\frac{a^2x^4\operatorname{atanh}(ax)}{4} - \frac{ax^3}{12} + \frac{x^2\operatorname{atanh}(ax)}{2} + \frac{x}{4a} - \frac{\operatorname{atanh}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**4*atanh(a*x)/4 - a*x**3/12 + x**2*atanh(a*x)/2 + x/(4*a
) - atanh(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(33) = 66.
time = 0.40, size = 160, normalized size = 4.00

$$-\frac{1}{3}a\left(\frac{\frac{3(ax+1)}{ax-1}-1}{a^3\left(\frac{ax+1}{ax-1}-1\right)^3}+\frac{6(ax+1)^2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^2a^3\left(\frac{ax+1}{ax-1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/3*a*((3*(a*x + 1)/(a*x - 1) - 1)/(a^3*((a*x + 1)/(a*x - 1) - 1)^3) + 6*(
a*x + 1)^2*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) +
1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x - 1
)^2*a^3*((a*x + 1)/(a*x - 1) - 1)^4))
```

Mupad [B]
time = 0.86, size = 44, normalized size = 1.10

$$\frac{x^2\operatorname{atanh}(a\,x)}{2}-\frac{\frac{\operatorname{atanh}(a\,x)}{4}-\frac{a\,x}{4}}{a^2}-\frac{a\,x^3}{12}-\frac{a^2\,x^4\operatorname{atanh}(a\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(a*x)*(a^2*x^2 - 1),x)
```

```
[Out] (x^2*atanh(a*x))/2 - (atanh(a*x)/4 - (a*x)/4)/a^2 - (a*x^3)/12 - (a^2*x^4*a
tanh(a*x))/4
```

3.165 $\int (1-a^2x^2)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=64

$$\frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{3a}$$

```
[Out] 1/6*(-a^2*x^2+1)/a+2/3*x*arctanh(a*x)+1/3*x*(-a^2*x^2+1)*arctanh(a*x)+1/3*l
n(-a^2*x^2+1)/a
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6089, 6021, 266}

$$\frac{1-a^2x^2}{6a}+\frac{\log(1-a^2x^2)}{3a}+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{2}{3}x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] (1 - a^2*x^2)/(6*a) + (2*x*ArcTanh[a*x])/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x])
/3 + Log[1 - a^2*x^2]/(3*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx &= \frac{1-a^2x^2}{6a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2}{3}\int \tanh^{-1}(ax)\,dx \\
&= \frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)-\frac{1}{3}(2a)\int \frac{x}{1-a^2x^2} \\
&= \frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{3a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.73

$$
-\frac{ax^2}{6}+x\tanh^{-1}(ax)-\frac{1}{3}a^2x^3\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] -1/6*(a*x^2) + x*ArcTanh[a*x] - (a^2*x^3*ArcTanh[a*x])/3 + Log[1 - a^2*x^2]
/(3*a)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{a^3x^3\,\mathrm{arctanh}(ax)}{3}+ax\,\mathrm{arctanh}(ax)-\frac{a^2x^2}{6}+\frac{\ln(ax-1)}{3}+\frac{\ln(ax+1)}{3}}{a}$
default	$\frac{-\frac{a^3x^3\,\mathrm{arctanh}(ax)}{3}+ax\,\mathrm{arctanh}(ax)-\frac{a^2x^2}{6}+\frac{\ln(ax-1)}{3}+\frac{\ln(ax+1)}{3}}{a}$
risch	$\left(-\frac{1}{6}a^2x^3+\frac{1}{2}x\right)\ln\left(ax+1\right)+\frac{a^2x^3\ln(-ax+1)}{6}-\frac{a\,x^2}{6}-\frac{x\ln(-ax+1)}{2}+\frac{\ln\left(a^2x^2-1\right)}{3a}$
meijerg	$-\frac{\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{\sqrt{a^2x^2}}-2\ln\left(-a^2x^2+1\right)}{4a}-\frac{\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right.\right.}{3\sqrt{a^2x^2}}}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3*a^3*x^3*arctanh(a*x)+a*x*arctanh(a*x)-1/6*a^2*x^2+1/3*ln(a*x-1)+1
/3*ln(a*x+1))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.73

$$
-\frac{1}{6}\left(x^2-\frac{2\,\log\left(ax+1\right)}{a^2}-\frac{2\,\log\left(ax-1\right)}{a^2}\right)a-\frac{1}{3}\left(a^2x^3-3\,x\right)\mathrm{artanh}\left(ax\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/6*(x^2 - 2*log(a*x + 1)/a^2 - 2*log(a*x - 1)/a^2)*a - 1/3*(a^2*x^3 - 3*x
)*arctanh(a*x)
```

Fricas [**A**]
time = 0.36, size = 53, normalized size = 0.83

$$-\frac{a^2x^2 + (a^3x^3 - 3\,ax)\log\left(-\frac{ax+1}{ax-1}\right) - 2\,\log\left(a^2x^2 - 1\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*x^2 + (a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 2*log(a^2*x^2
 - 1))/a
```

Sympy [**A**]
time = 0.20, size = 49, normalized size = 0.77

$$\begin{cases} -\frac{a^2x^3\operatorname{atanh}(ax)}{3} - \frac{ax^2}{6} + x\operatorname{atanh}(ax) + \frac{2\log\left(x-\frac{1}{a}\right)}{3a} + \frac{2\operatorname{atanh}(ax)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**3*atanh(a*x)/3 - a*x**2/6 + x*atanh(a*x) + 2*log(x - 1/
a)/(3*a) + 2*atanh(a*x)/(3*a), Ne(a, 0)), (0, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(54) = 108.
time = 0.42, size = 203, normalized size = 3.17

$$\frac{2}{3}\,a\left(\frac{\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2} - \frac{\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2} - \frac{\left(\frac{3\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)^3} - \frac{ax+1}{(ax-1)a^2\left(\frac{ax+1}{ax-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 2/3*a*(log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - log(abs(-(a*x + 1)/(a*x - 1) +
 1))/a^2 - (3*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((
```

```
a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a
*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^3) - (a*x + 1)/((a*x - 1)
*a^2*((a*x + 1)/(a*x - 1) - 1)^2))
```

Mupad [B]
time = 0.84, size = 40, normalized size = 0.62

$$x\,\mathrm{atanh}(a\,x)-\frac{a\,x^2}{6}+\frac{\ln\left(a^2\,x^2-1\right)}{3\,a}-\frac{a^2\,x^3\,\mathrm{atanh}(a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)*(a^2*x^2 - 1),x)
[Out] x*atanh(a*x) - (a*x^2)/6 + log(a^2*x^2 - 1)/(3*a) - (a^2*x^3*atanh(a*x))/3
```

3.166 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=48

$$-\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)$$

```
[Out] -1/2*a*x+1/2*arctanh(a*x)-1/2*a^2*x^2*arctanh(a*x)-1/2*polylog(2,-a*x)+1/2*
polylog(2,a*x)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6161, 6031, 6037, 327, 212}

$$-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}-\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

```
[Out] -1/2*(a*x) + ArcTanh[a*x]/2 - (a^2*x^2*ArcTanh[a*x])/2 - PolyLog[2, -(a*x)]
/2 + PolyLog[2, a*x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x}\,dx &= -\left(a^2\int x\tanh^{-1}(ax)\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x}\,dx\\
&= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}+\frac{1}{2}a^3\int \frac{x^2}{1-a^2x^2}\,dx\\
&= -\frac{ax}{2}-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}+\frac{1}{2}a\int \frac{1}{1-a^2x^2}\,dx\\
&= -\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.25

$$
-\frac{ax}{2}-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{1}{4}\log(1-ax)+\frac{1}{4}\log(1+ax)+\frac{1}{2}(-\text{PolyLog}(2,-ax)+\text{PolyLog}(2,ax))
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

```
[Out] -1/2*(a*x) - (a^2*x^2*ArcTanh[a*x])/2 - Log[1 - a*x]/4 + Log[1 + a*x]/4 + (
-PolyLog[2, -(a*x)] + PolyLog[2, a*x])/2
```

Maple [A]
time = 0.13, size = 69, normalized size = 1.44

method	result
derivativedivides	$-\frac{a^2x^2\,\text{arctanh}(ax)}{2}+\text{arctanh}\,(ax)\ln\,(ax)-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\text{dilog}(ax)}{2}-\frac{\text{dilog}(ax+1)}{2}-$

default	$-\frac{a^{2} x^{2} \operatorname{arctanh}(a x)}{2}+\operatorname{arctanh}(a x) \ln (a x)-\frac{a x}{2}-\frac{\ln (a x-1)}{4}+\frac{\ln (a x+1)}{4}-\frac{\operatorname{dilog}(a x)}{2}-\frac{\operatorname{dilog}(a x+1)}{2}$
risch	$\frac{(-a x+1)^{2} \ln (-a x+1)}{4}-\frac{a x}{2}-\frac{(-a x+1) \ln (-a x+1)}{2}+\frac{\operatorname{dilog}(-a x+1)}{2}-\frac{(a x+1)^{2} \ln (a x+1)}{4}+\frac{(a x+1) \ln (a x+1)}{2}$
meijerg	$-\frac{i\left(\frac{2 i a x \operatorname{polylog}\left(2, \sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-\frac{2 i a x \operatorname{polylog}\left(2,-\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}\right)}{4}-\frac{i(-2 i x a+2 i(-a x+1)(a x+1) \operatorname{arctanh}(a x))}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*a*x-1/4*ln(a*x-1)+1/4*ln
(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(36) = 72$.
time = 0.26, size = 89, normalized size = 1.85

$$-\frac{1}{4}a\left(2x+\frac{2(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax))}{a}-\frac{2(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax))}{a}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)-\frac{1}{2}\left(a^2x^2-\log\left(x^2\right)\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*x + 2*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 2*(log(-a*x + 1)*lo
g(x) + dilog(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) - 1/2*(a^2*x^2 - lo
g(x^2))*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\operatorname{atanh}\left(ax\right)}{x}\right)dx-\int a^2x\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x,x)
```

```
[Out] -Integral(-atanh(a*x)/x, x) - Integral(a**2*x*atanh(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\mathrm{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x,x)
[Out] -int((atanh(a*x)*(a^2*x^2 - 1))/x, x)
```

3.167 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\log(x)-a\log\left(1-a^2x^2\right)$$

[Out] -arctanh(a*x)/x-a^2*x*arctanh(a*x)+a*ln(x)-a*ln(-a^2*x^2+1)

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6161, 6037, 272, 36, 29, 31, 6021, 266}

$$-a\log\left(1-a^2x^2\right)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]

[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^2}\,dx &= -\left(a^2\int \tanh^{-1}(ax)\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\int \frac{1}{x(1-a^2x^2)}\,dx+a^3\int \frac{x}{1-a^2x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(1-a\right.\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx,x,\right.\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\log(x)-a\log\left(1-a^2x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$-\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\log(x)-a\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]
```

Maple [A]
time = 0.12, size = 44, normalized size = 1.16

method	result
derivativedivides	$a\left(-ax\,\mathrm{arctanh}\left(ax\right)-\frac{\mathrm{arctanh}(ax)}{ax}-\ln\left(ax-1\right)+\ln\left(ax\right)-\ln\left(ax+1\right)\right)$
default	$a\left(-ax\,\mathrm{arctanh}\left(ax\right)-\frac{\mathrm{arctanh}(ax)}{ax}-\ln\left(ax-1\right)+\ln\left(ax\right)-\ln\left(ax+1\right)\right)$
risch	$-\frac{(a^2x^2+1)\ln(ax+1)}{2x}+\frac{x^2\ln(-ax+1)a^2+2a\ln(x)x-2a\ln(a^2x^2-1)x+\ln(-ax+1)}{2x}$
meijerg	$\frac{a\left(\frac{2\ln\left(1-\sqrt{a^2x^2}\right)-2\ln\left(1+\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(-a^2x^2+1)+4\ln(x)+4\ln(ia)\right)}{4}+\frac{a\left(\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1\right.\right.}{\sqrt{a^2x^2}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-a*x*arctanh(a*x)-arctanh(a*x)/a/x-ln(a*x-1)+ln(a*x)-ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.95

$$-a(\log\left(ax+1\right)+\log\left(ax-1\right)-\log\left(x\right))-\left(a^2x+\frac{1}{x}\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -a*(log(a*x + 1) + log(a*x - 1) - log(x)) - (a^2*x + 1/x)*arctanh(a*x)
```

Fricas [A]
time = 0.37, size = 51, normalized size = 1.34

$$-\frac{2\,ax\log\left(a^2x^2-1\right)-2\,ax\log\left(x\right)+\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + (a^2*x^2 + 1)*log(-(a*x + 1)/
(a*x - 1)))/x
```

Sympy [A]
time = 0.41, size = 41, normalized size = 1.08

$$\begin{cases} -a^2x\operatorname{atanh}(ax) + a\log(x) - 2a\log\left(x - \frac{1}{a}\right) - 2a\operatorname{atanh}(ax) - \frac{\operatorname{atanh}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**2,x)
```

```
[Out] Piecewise((-a**2*x*atanh(a*x) + a*log(x) - 2*a*log(x - 1/a) - 2*a*atanh(a*x
) - atanh(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(38) = 76.
time = 0.39, size = 145, normalized size = 3.82

$$-a\left(\frac{2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\frac{(ax+1)^2}{(ax-1)^2}-1} + \log\left(\frac{(ax+1)^2}{(ax-1)^2}\right) - \log\left(\left|\frac{(ax+1)^2}{(ax-1)^2}-1\right|\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -a*(2*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a
*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)^2/(
a*x - 1)^2 - 1) + log((a*x + 1)^2/(a*x - 1)^2) - log(abs((a*x + 1)^2/(a*x -
 1)^2 - 1)))
```

Mupad [B]
time = 0.81, size = 37, normalized size = 0.97

$$a\,\ln\left(x\right) - a\,\ln\left(a^2\,x^2-1\right) - \frac{\operatorname{atanh}(a\,x)}{x} - a^2\,x\,\operatorname{atanh}(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^2,x)
```

```
[Out] a*log(x) - a*log(a^2*x^2 - 1) - atanh(a*x)/x - a^2*x*atanh(a*x)
```

3.168 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=56

$$-\frac{a}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\mathrm{PolyLog}(2,-ax)-\frac{1}{2}a^2\mathrm{PolyLog}(2,ax)$$

```
[Out] -1/2*a/x+1/2*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1/2*a^2*polylog(2,-a*x)-
1/2*a^2*polylog(2,a*x)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6161, 6037, 331, 212, 6031}

$$\frac{1}{2}a^2\mathrm{Li}_2(-ax)-\frac{1}{2}a^2\mathrm{Li}_2(ax)+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*a/x + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*PolyLog[2, -(
a*x)])/2 - (a^2*PolyLog[2, a*x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^3}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{x}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\mathrm{Li}_2(-ax)-\frac{1}{2}a^2\mathrm{Li}_2(ax)+\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}\,dx\\
&= -\frac{a}{2x}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\mathrm{Li}_2(-ax)-\frac{1}{2}a^2\mathrm{Li}_2(ax)+\frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\,dx\\
&= -\frac{a}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\mathrm{Li}_2(-ax)-\frac{1}{2}a^2\mathrm{Li}_2(ax)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.21

$$-\frac{a}{2x}-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{1}{4}a^2\log(1-ax)+\frac{1}{4}a^2\log(1+ax)-\frac{1}{2}a^2(-\mathrm{PolyLog}(2,-ax)+\mathrm{PolyLog}(2,ax))$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*a/x - ArcTanh[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x])
/4 - (a^2*(-PolyLog[2, -(a*x)] + PolyLog[2, a*x]))/2
```

Maple [A]
time = 0.16, size = 78, normalized size = 1.39

method	result

derivativedivides	$a^2\left(-\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{1}{2ax}+\frac{\operatorname{dilog}(ax+1)}{2}+\frac{\ln(ax)\ln}{2}\right.$
default	$a^2\left(-\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{1}{2ax}+\frac{\operatorname{dilog}(ax+1)}{2}+\frac{\ln(ax)\ln}{2}\right.$
risch	$-\frac{a}{2x}+\frac{a^2\ln(-ax)}{4}-\frac{a^2\ln(-ax+1)}{4}+\frac{\ln(-ax+1)}{4x^2}-\frac{a^2\operatorname{dilog}(-ax+1)}{2}-\frac{a^2\ln(ax)}{4}+\frac{a^2\ln(ax+1)}{4}-\frac{\ln(ax-}{4x^2}$
meijerg	$\frac{ia^2\left(\frac{2i}{xa}+\frac{2i(-ax+1)(ax+1)\operatorname{arctanh}(ax)}{x^2a^2}\right)}{4}+\frac{ia^2\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-\frac{2iax\operatorname{polylog}\left(2,-\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/a^2/x^2-1/4*ln(a*x-1)+1/4*ln(a*
x+1)-1/2/a/x+1/2*dilog(a*x+1)+1/2*ln(a*x)*ln(a*x+1)+1/2*dilog(a*x))
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.45

$$\frac{1}{4}\left(2\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)a-2\left(\log(-ax+1)\log(x)+\operatorname{Li}_2(ax)\right)a+a\log(ax+1)-a\log(ax-1)-\frac{2}{x}\right)a-\frac{1}{2}\left(a^2\log(x^2)+\frac{1}{x^2}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + di
log(a*x))*a + a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*(a^2*log(x^2)
+ 1/x^2)*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\operatorname{atanh}(ax)}{x^3}\right)dx-\int\frac{a^2\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**3,x)
```

```
[Out] -Integral(-atanh(a*x)/x**3, x) - Integral(a**2*atanh(a*x)/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(44) = 88.
time = 1.31, size = 330, normalized size = 5.89

$$a^{2}\left(\frac{\log\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}\right)}{a}-\frac{\log\left(\left|\frac{(ax+1)^{2}}{(ax-1)^{2}}-1\right|\right)}{a}+\frac{\frac{(ax+1)^{2}}{(ax-1)^{2}}-2}{a\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}-1\right)}-\frac{2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}-1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}+1}\right)}{a\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}-1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] a^2*(log((a*x + 1)^2/(a*x - 1)^2)/a - log(abs((a*x + 1)^2/(a*x - 1)^2 - 1))
/a + ((a*x + 1)^2/(a*x - 1)^2 - 2)/(a*((a*x + 1)^2/(a*x - 1)^2 - 1)) - 2*lo
g(-(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(a*x - 1) + 1)/((a*x +
 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x -
1) - a) - 1)) - 1)/(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(a*x -
 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*
x + 1)*a/(a*x - 1) - a) - 1)) + 1))/(a*((a*x + 1)^2/(a*x - 1)^2 - 1)^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\mathrm{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^3,x)
```

```
[Out] -int((atanh(a*x)*(a^2*x^2 - 1))/x^3, x)
```

3.169 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=58

$$-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log\left(1-a^2x^2\right)$$

```
[Out] -1/6*a/x^2-1/3*arctanh(a*x)/x^3+a^2*arctanh(a*x)/x-2/3*a^3*ln(x)+1/3*a^3*ln
(-a^2*x^2+1)
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6161, 6037, 272, 46, 36, 29, 31}

$$-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{3}a^3\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{3x^3}-\frac{a}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}\,dx-a^3\int \frac{1}{x(1-a^2x^2)}\\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2(1-a^2x)}\,dx,x,x^2\right)-\frac{1}{2}a^3\\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{6}a\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a^2}{x}-\frac{a^4}{-1+a^2x}\right)\,dx,x\right.\\
&= -\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log\left(1-a^2x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.00

$$-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

Maple [A]
time = 0.18, size = 59, normalized size = 1.02

method	result
derivativedivides	$a^{3}\left(\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{3a^{3}x^{3}}+\frac{\ln(ax-1)}{3}+\frac{\ln(ax+1)}{3}-\frac{1}{6a^{2}x^{2}}-\frac{2\ln(ax)}{3}\right)$
default	$a^{3}\left(\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{3a^{3}x^{3}}+\frac{\ln(ax-1)}{3}+\frac{\ln(ax+1)}{3}-\frac{1}{6a^{2}x^{2}}-\frac{2\ln(ax)}{3}\right)$
risch	$\frac{(3a^{2}x^{2}-1)\ln(ax+1)}{6x^{3}}-\frac{4\ln(x)a^{3}x^{3}-2\ln(-a^{2}x^{2}+1)a^{3}x^{3}+3x^{2}\ln(-ax+1)a^{2}+ax-\ln(-ax+1)}{6x^{3}}$
meijerg	$-\frac{a^{3}\left(-\frac{2(10a^{2}x^{2}+30)}{45a^{2}x^{2}}-\frac{2\left(\ln\left(1-\sqrt{a^{2}x^{2}}\right)-\ln\left(1+\sqrt{a^{2}x^{2}}\right)\right)}{3a^{2}x^{2}\sqrt{a^{2}x^{2}}}+\frac{2\ln(-a^{2}x^{2}+1)}{3}+\frac{4}{9}-\frac{4\ln(x)}{3}-\frac{4\ln(ia)}{3}+\frac{2}{a^{2}x^{2}}\right)}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(arctanh(a*x)/a/x-1/3*arctanh(a*x)/a^3/x^3+1/3*ln(a*x-1)+1/3*ln(a*x+1)-
1/6/a^2/x^2-2/3*ln(a*x))
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.91

$$\frac{1}{6}\left(2a^{2}\log\left(a^{2}x^{2}-1\right)-2a^{2}\log\left(x^{2}\right)-\frac{1}{x^{2}}\right)a+\frac{(3a^{2}x^{2}-1)\operatorname{artanh}(ax)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x^2) - 1/x^2)*a + 1/3*(3*a^2*x^2 -
1)*arctanh(a*x)/x^3
```

Fricas [A]
time = 0.37, size = 64, normalized size = 1.10

$$\frac{2a^{3}x^{3}\log\left(a^{2}x^{2}-1\right)-4a^{3}x^{3}\log\left(x\right)-ax+(3a^{2}x^{2}-1)\log\left(-\frac{ax+1}{ax-1}\right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*x^3*log(a^2*x^2 - 1) - 4*a^3*x^3*log(x) - a*x + (3*a^2*x^2 - 1)*
log(-(a*x + 1)/(a*x - 1)))/x^3
```

Sympy [A]
time = 0.39, size = 63, normalized size = 1.09

$$\begin{cases} -\frac{2a^3\log(x)}{3} + \frac{2a^3\log\left(x-\frac{1}{a}\right)}{3} + \frac{2a^3\operatorname{atanh}(ax)}{3} + \frac{a^2\operatorname{atanh}(ax)}{x} - \frac{a}{6x^2} - \frac{\operatorname{atanh}(ax)}{3x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**4,x)
```

```
[Out] Piecewise((-2*a**3*log(x)/3 + 2*a**3*log(x - 1/a)/3 + 2*a**3*atanh(a*x)/3 +
 a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3), Ne(a, 0)), (0, True)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(50) = 100.
time = 0.40, size = 204, normalized size = 3.52

$$\frac{2}{3}\left(a^2\log\left(\frac{|-ax-1|}{|ax-1|}\right) - a^2\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right) + \frac{(ax+1)a^2}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2} - \frac{\left(\frac{3(ax+1)a^2}{ax-1}+a^2\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(a^2*log(abs(-a*x - 1)/abs(a*x - 1)) - a^2*log(abs(-(a*x + 1)/(a*x - 1)
 - 1)) + (a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2) - (3*(a*x +
1)*a^2/(a*x - 1) + a^2)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x
 - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) -
1))/((a*x + 1)/(a*x - 1) + 1)^3)*a
```

Mupad [B]
time = 0.85, size = 49, normalized size = 0.84

$$\frac{a^3\ln\left(a^2x^2-1\right)}{3} - \frac{a}{6x^2} - \frac{\operatorname{atanh}(a\,x)}{3x^3} - \frac{2a^3\ln(x)}{3} + \frac{a^2\operatorname{atanh}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^4,x)
```

```
[Out] (a^3*log(a^2*x^2 - 1))/3 - a/(6*x^2) - atanh(a*x)/(3*x^3) - (2*a^3*log(x))/
3 + (a^2*atanh(a*x))/x
```

3.170 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=42

$$-\frac{a}{12x^3}+\frac{a^3}{4x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}$$

```
[Out] -1/12*a/x^3+1/4*a^3/x-1/4*(-a^2*x^2+1)^2*arctanh(a*x)/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6155, 14}

$$\frac{a^3}{4x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}-\frac{a}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
[Out] -1/12*a/x^3 + a^3/(4*x) - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^5}\,dx &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \frac{1-a^2x^2}{x^4}\,dx\\ &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \left(\frac{1}{x^4}-\frac{a^2}{x^2}\right)dx\\ &= -\frac{a}{12x^3}+\frac{a^3}{4x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 1.69

$$-\frac{a}{12x^3}+\frac{a^3}{4x}-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{2x^2}+\frac{1}{8}a^4\log(1-ax)-\frac{1}{8}a^4\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -1/12*a/x^3 + a^3/(4*x) - ArcTanh[a*x]/(4*x^4) + (a^2*ArcTanh[a*x])/(2*x^2)
 + (a^4*Log[1 - a*x])/8 - (a^4*Log[1 + a*x])/8
```

Maple [A]
time = 0.08, size = 62, normalized size = 1.48

method	result
derivativedivides	$a^4\left(-\frac{\operatorname{arctanh}(ax)}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\frac{1}{4ax}-\frac{1}{12a^3x^3}-\frac{\ln(ax+1)}{8}+\frac{\ln(ax-1)}{8}\right)$
default	$a^4\left(-\frac{\operatorname{arctanh}(ax)}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\frac{1}{4ax}-\frac{1}{12a^3x^3}-\frac{\ln(ax+1)}{8}+\frac{\ln(ax-1)}{8}\right)$
risch	$\frac{(2a^2x^2-1)\ln(ax+1)}{8x^4}-\frac{3\ln(-ax-1)a^4x^4-3x^4\ln(-ax+1)a^4-6a^3x^3+6x^2\ln(-ax+1)a^2+2ax-3\ln(-ax+1)}{24x^4}$
meijerg	$-\frac{ia^4\left(-\frac{i}{3x^3a^3}-\frac{i}{xa}+\frac{4i\left(\frac{3}{8}-\frac{3a^4x^4}{8}\right)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3x^3a^3\sqrt{a^2x^2}}\right)}{4}-\frac{ia^4\left(\frac{2i}{xa}+\frac{2i(-ax+1)(ax+1)\operatorname{arctanh}(ax)}{x^2a^2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*arctanh(a*x)/a^4/x^4+1/2*arctanh(a*x)/a^2/x^2+1/4/a/x-1/12/a^3/x^
3-1/8*ln(a*x+1)+1/8*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 61, normalized size = 1.45

$$-\frac{1}{24}\left(3\,a^3\log\left(ax+1\right)-3\,a^3\log\left(ax-1\right)-\frac{2\left(3\,a^2x^2-1\right)}{x^3}\right)a+\frac{\left(2\,a^2x^2-1\right)\operatorname{artanh}\left(ax\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 - 1)/x^3)*a +
 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/x^4
```

Fricas [A]
time = 0.40, size = 52, normalized size = 1.24

$$\frac{6\,a^3x^3-2\,ax-3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 - 2*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1)))/x^4
```

Sympy [A]
time = 0.31, size = 46, normalized size = 1.10

$$-\frac{a^4 \operatorname{atanh}(ax)}{4}+\frac{a^3}{4x}+\frac{a^2 \operatorname{atanh}(ax)}{2x^2}-\frac{a}{12x^3}-\frac{\operatorname{atanh}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**5,x)
```

```
[Out] -a**4*atanh(a*x)/4 + a**3/(4*x) + a**2*atanh(a*x)/(2*x**2) - a/(12*x**3) -
atanh(a*x)/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(35) = 70.
time = 0.40, size = 160, normalized size = 3.81

$$-\frac{1}{3}a\left(\frac{a^3\left(\frac{3(ax+1)}{ax-1}+1\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}+\frac{6(ax+1)^2a^3\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^2\left(\frac{ax+1}{ax-1}+1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] -1/3*a*(a^3*(3*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)/(a*x - 1) + 1)^3 + 6*(a*
x + 1)^2*a^3*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a)
+ 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x -
 1)^2*((a*x + 1)/(a*x - 1) + 1)^4))
```

Mupad [B]
time = 0.84, size = 61, normalized size = 1.45

$$\frac{a^3}{4\,x}-\frac{\mathrm{atanh}(a\,x)}{4\,x^4}-\frac{a}{12\,x^3}+\frac{a^5\,\mathrm{atan}\left(\frac{a^2\,x}{\sqrt{-a^2}}\right)}{4\,\sqrt{-a^2}}+\frac{a^2\,\mathrm{atanh}(a\,x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^5,x)
```

```
[Out] a^3/(4*x) - atanh(a*x)/(4*x^4) - a/(12*x^3) + (a^5*atan((a^2*x)/(-a^2)^(1/2
)))/(4*(-a^2)^(1/2)) + (a^2*atanh(a*x))/(2*x^2)
```

3.171 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=71

$$-\frac{a}{20x^4}+\frac{a^3}{15x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)+\frac{1}{15}a^5\log\left(1-a^2x^2\right)$$

```
[Out] -1/20*a/x^4+1/15*a^3/x^2-1/5*arctanh(a*x)/x^5+1/3*a^2*arctanh(a*x)/x^3-2/15
*a^5*ln(x)+1/15*a^5*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6161, 6037, 272, 46}

$$-\frac{2}{15}a^5\log(x)+\frac{a^3}{15x^2}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{15}a^5\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{5x^5}-\frac{a}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] -1/20*a/x^4 + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^6}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\right) + \int \frac{\tanh^{-1}(ax)}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5} + \frac{a^2\tanh^{-1}(ax)}{3x^3} + \frac{1}{5}a\int \frac{1}{x^5(1-a^2x^2)}\,dx - \frac{1}{3}a^3\int \frac{1}{x^3(1-a$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5} + \frac{a^2\tanh^{-1}(ax)}{3x^3} + \frac{1}{10}a\text{Subst}\left(\int \frac{1}{x^3(1-a^2x)}\,dx, x, x^2\right) - \frac{1}{6}$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5} + \frac{a^2\tanh^{-1}(ax)}{3x^3} + \frac{1}{10}a\text{Subst}\left(\int \left(\frac{1}{x^3} + \frac{a^2}{x^2} + \frac{a^4}{x} - \frac{a^6}{-1+a^2}\right.\right.$$

$$= -\frac{a}{20x^4} + \frac{a^3}{15x^2} - \frac{\tanh^{-1}(ax)}{5x^5} + \frac{a^2\tanh^{-1}(ax)}{3x^3} - \frac{2}{15}a^5\log(x) + \frac{1}{15}a^5\log(1$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.00

$$-\frac{a}{20x^4} + \frac{a^3}{15x^2} - \frac{\tanh^{-1}(ax)}{5x^5} + \frac{a^2\tanh^{-1}(ax)}{3x^3} - \frac{2}{15}a^5\log(x) + \frac{1}{15}a^5\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
[Out] -1/20*a/x^4 + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Maple [A]
time = 0.21, size = 68, normalized size = 0.96

method	result
derivativedivides	$a^5\left(\frac{\operatorname{arctanh}(ax)}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)}{5a^5x^5} + \frac{\ln(ax+1)}{15} + \frac{\ln(ax-1)}{15} - \frac{1}{20a^4x^4} + \frac{1}{15a^2x^2} - \frac{2\ln(ax)}{15}\right)$
default	$a^5\left(\frac{\operatorname{arctanh}(ax)}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)}{5a^5x^5} + \frac{\ln(ax+1)}{15} + \frac{\ln(ax-1)}{15} - \frac{1}{20a^4x^4} + \frac{1}{15a^2x^2} - \frac{2\ln(ax)}{15}\right)$
risch	$\frac{(5a^2x^2-3)\ln(ax+1)}{30x^5} - \frac{8\ln(x)a^5x^5-4\ln(-a^2x^2+1)a^5x^5-4a^3x^3+10x^2\ln(-ax+1)a^2+3ax-6\ln(-ax+1)}{60x^5}$

meijerg	$\frac{a^5\left(\frac{\frac{4}{25}a^4x^4+\frac{4}{15}a^2x^2+\frac{4}{5}}{a^4x^4}+\frac{\frac{2\ln\left(1-\sqrt{a^2x^2}\right)}{5}-\frac{2\ln\left(1+\sqrt{a^2x^2}\right)}{5}}{a^4x^4\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{5}-\frac{4}{25}+\frac{4\ln(x)}{5}+\frac{4\ln(ia)}{5}-\frac{1}{a^4x^4}-\frac{2}{3a^2x^2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(1/3*arctanh(a*x)/a^3/x^3-1/5*arctanh(a*x)/a^5/x^5+1/15*ln(a*x+1)+1/15*
ln(a*x-1)-1/20/a^4/x^4+1/15/a^2/x^2-2/15*ln(a*x))
```

Maxima [A]
time = 0.26, size = 62, normalized size = 0.87

$$\frac{1}{60}\left(4\,a^4\log\left(a^2x^2-1\right)-4\,a^4\log\left(x^2\right)+\frac{4\,a^2x^2-3}{x^4}\right)a+\frac{\left(5\,a^2x^2-3\right)\operatorname{artanh}\left(ax\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 - 3)/x^4)*a + 1/
15*(5*a^2*x^2 - 3)*arctanh(a*x)/x^5
```

Fricas [A]
time = 0.36, size = 73, normalized size = 1.03

$$\frac{4\,a^5x^5\log\left(a^2x^2-1\right)-8\,a^5x^5\log\left(x\right)+4\,a^3x^3-3\,ax+2\left(5\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(4*a^5*x^5*log(a^2*x^2 - 1) - 8*a^5*x^5*log(x) + 4*a^3*x^3 - 3*a*x + 2
*(5*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

Sympy [A]
time = 0.55, size = 75, normalized size = 1.06

$$\begin{cases}-\frac{2a^5\log\left(x\right)}{15}+\frac{2a^5\log\left(x-\frac{1}{a}\right)}{15}+\frac{2a^5\operatorname{atanh}\left(ax\right)}{15}+\frac{a^3}{15x^2}+\frac{a^2\operatorname{atanh}\left(ax\right)}{3x^3}-\frac{a}{20x^4}-\frac{\operatorname{atanh}\left(ax\right)}{5x^5} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**6,x)
```

```
[Out] Piecewise((-2*a**5*log(x)/15 + 2*a**5*log(x - 1/a)/15 + 2*a**5*atanh(a*x)/1
5 + a**3/(15*x**2) + a**2*atanh(a*x)/(3*x**3) - a/(20*x**4) - atanh(a*x)/(5
*x**5), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(59) = 118$.
time = 0.40, size = 281, normalized size = 3.96

$$\frac{2}{15}\left(a^4\log\left(\frac{|-ax-1|}{|ax-1|}\right) - a^4\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right) + \frac{\frac{(ax+1)^3a^4}{(ax-1)^3} - \frac{4(ax+1)^2a^4}{(ax-1)^2} + \frac{(ax+1)a^4}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^4} - \frac{\left(\frac{15(ax+1)^3a^4}{(ax-1)^3} - \frac{5(ax+1)^2a^4}{(ax-1)^2} + \frac{5(ax+1)a^4}{ax-1} + a^4\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(a^4*log(abs(-a*x - 1)/abs(a*x - 1)) - a^4*log(abs(-(a*x + 1)/(a*x - 1
) - 1)) + ((a*x + 1)^3*a^4/(a*x - 1)^3 - 4*(a*x + 1)^2*a^4/(a*x - 1)^2 + (a
*x + 1)*a^4/(a*x - 1))/((a*x + 1)/(a*x - 1) + 1)^4 - (15*(a*x + 1)^3*a^4/(a
*x - 1)^3 - 5*(a*x + 1)^2*a^4/(a*x - 1)^2 + 5*(a*x + 1)*a^4/(a*x - 1) + a^4
)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a
*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)/(a*x -
1) + 1)^5)*a
```

Mupad [B]
time = 0.88, size = 59, normalized size = 0.83

$$\frac{a^5\,\ln\left(a^2\,x^2-1\right)}{15}-\frac{\frac{\mathrm{atanh}(a\,x)}{5}+\frac{a\,x}{20}-\frac{a^3\,x^3}{15}-\frac{a^2\,x^2\,\mathrm{atanh}(a\,x)}{3}}{x^5}-\frac{2\,a^5\,\ln\left(x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)*(a^2*x^2 - 1))/x^6,x)
```

```
[Out] (a^5*log(a^2*x^2 - 1))/15 - (atanh(a*x)/5 + (a*x)/20 - (a^3*x^3)/15 - (a^2*
x^2*atanh(a*x))/3)/x^5 - (2*a^5*log(x))/15
```

3.172 $\int x^4(1-a^2x^2)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=162

$$\frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{4\tanh^{-1}(ax)}{105a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{35a^5}+\frac{1}{5}$$

[Out] 4/105*x/a^4-2/315*x^3/a^2-1/105*x^5-4/105*arctanh(a*x)/a^5+2/35*x^2*arctanh(a*x)/a^3+1/35*x^4*arctanh(a*x)/a-1/21*a*x^6*arctanh(a*x)+2/35*arctanh(a*x)^2/a^5+1/5*x^5*arctanh(a*x)^2-1/7*a^2*x^7*arctanh(a*x)^2-4/35*arctanh(a*x)*ln(2/(-a*x+1))/a^5-2/35*polylog(2,1-2/(-a*x+1))/a^5

Rubi [A]
time = 0.43, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {6161, 6037, 6127, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{35a^5}+\frac{2\tanh^{-1}(ax)^2}{35a^5}-\frac{4\tanh^{-1}(ax)}{105a^5}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{35a^5}+\frac{4x}{105a^4}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2-\frac{2x^3}{315a^2}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\frac{1}{5}x^5\tanh^{-1}(ax)^2+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{x^5}{105}$$

Antiderivative was successfully verified.

[In] Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]

[Out] (4*x)/(105*a^4) - (2*x^3)/(315*a^2) - x^5/105 - (4*ArcTanh[a*x])/(105*a^5) + (2*x^2*ArcTanh[a*x])/(35*a^3) + (x^4*ArcTanh[a*x])/(35*a) - (a*x^6*ArcTanh[a*x])/21 + (2*ArcTanh[a*x]^2)/(35*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (a^2*x^7*ArcTanh[a*x]^2)/7 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(35*a^5) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a^5)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\left(a^2\int x^6\tanh^{-1}(ax)^2\,dx\right)+\int x^4\tanh^{-1}(ax)^2\,dx$$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2-\frac{1}{5}(2a)\int\frac{x^5\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{7}($$

$$= \frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{2\int x^3\tanh^{-1}(ax)\,dx}{5a}-$$

$$= \frac{x^4\tanh^{-1}(ax)}{10a}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{1}{7}a^2x^7\tanh^{-1}($$

$$= \frac{x^2\tanh^{-1}(ax)}{5a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{5a^5}+\frac{1}{5}$$

$$= \frac{53x}{210a^4}+\frac{11x^3}{630a^2}-\frac{x^5}{105}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\tanh^{-1}$$

$$= \frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{53\tanh^{-1}(ax)}{210a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(a}{35a}$$

$$= \frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{4\tanh^{-1}(ax)}{105a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax}{35a}$$

$$= \frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{4\tanh^{-1}(ax)}{105a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax}{35a}$$

Mathematica [A]
time = 0.68, size = 113, normalized size = 0.70

$$-\frac{-12ax+2a^3x^3+3a^5x^5+9\left(2-7a^5x^5+5a^7x^7\right)\tanh^{-1}(ax)^2+3\tanh^{-1}(ax)\left(4-6a^2x^2-3a^4x^4+5a^6x^6+12\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)-18\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{315a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/315*(-12*a*x + 2*a^3*x^3 + 3*a^5*x^5 + 9*(2 - 7*a^5*x^5 + 5*a^7*x^7)*Arc
Tanh[a*x]^2 + 3*ArcTanh[a*x]*(4 - 6*a^2*x^2 - 3*a^4*x^4 + 5*a^6*x^6 + 12*Lo
g[1 + E^(-2*ArcTanh[a*x])]) - 18*PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^5
```

Maple [A]
time = 0.93, size = 199, normalized size = 1.23

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}+\frac{a^4 x^4 \operatorname{arctanh}(ax)}{35}+\frac{2 a^2 x^2 \operatorname{arctanh}(ax)}{35}+\frac{2 \operatorname{arctanh}(ax) \ln(ax-1)}{35}+2$
default	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}+\frac{a^4 x^4 \operatorname{arctanh}(ax)}{35}+\frac{2 a^2 x^2 \operatorname{arctanh}(ax)}{35}+\frac{2 \operatorname{arctanh}(ax) \ln(ax-1)}{35}+2$
risch	$-\frac{\ln(-ax+1)^2}{70a^5}-\frac{4177\ln(-ax+1)}{29400a^5}+\frac{\ln(-ax+1)^2 x^5}{20}-\frac{\ln(-ax+1) x^5}{50}+\frac{a^2\ln(-ax+1)\ln(ax+1) x^7}{14}+\frac{4x}{105a^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-1/7*arctanh(a*x)^2*a^7*x^7+1/5*arctanh(a*x)^2*a^5*x^5-1/21*arctanh(
a*x)*a^6*x^6+1/35*a^4*x^4*arctanh(a*x)+2/35*a^2*x^2*arctanh(a*x)+2/35*arcta
nh(a*x)*ln(a*x-1)+2/35*arctanh(a*x)*ln(a*x+1)+1/70*ln(a*x-1)^2-2/35*dilog(1
/2*a*x+1/2)-1/35*ln(a*x-1)*ln(1/2*a*x+1/2)-1/70*ln(a*x+1)^2+1/35*(ln(a*x+1)
-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/105*a^5*x^5-2/315*a^3*x^3+4/105*a*x+2/
105*ln(a*x-1)-2/105*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 190, normalized size = 1.17

$$-\frac{1}{630}a^2\left(\frac{6a^5x^5+4a^3x^3-24ax+9\log(ax+1)^2-18\log(ax+1)\log(ax-1)-9\log(ax-1)^2-12\log(ax-1)}{a^7}+\frac{36\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^7}+\frac{12\log(ax+1)}{a^7}\right)-\frac{1}{105}a\left(\frac{5a^4x^6-3a^2x^4-6x^2}{a^4}-\frac{6\log(ax+1)}{a^6}-\frac{6\log(ax-1)}{a^6}\right)\operatorname{artanh}(ax)-\frac{1}{35}\left(5a^2x^7-7x^5\right)\operatorname{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/630*a^2*((6*a^5*x^5 + 4*a^3*x^3 - 24*a*x + 9*log(a*x + 1)^2 - 18*log(a*x
 + 1)*log(a*x - 1) - 9*log(a*x - 1)^2 - 12*log(a*x - 1))/a^7 + 36*(log(a*x
- 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 + 12*log(a*x + 1)/a^7)
 - 1/105*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*lo
g(a*x - 1)/a^6)*arctanh(a*x) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^6 - x^4)*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-x^4 \operatorname{atanh}^2\left(ax\right)\right)\, dx - \int a^2 x^6 \operatorname{atanh}^2\left(ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(-x**4*atanh(a*x)**2, x) - Integral(a**2*x**6*atanh(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x^4*arctanh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int x^4\, \operatorname{atanh}(a\, x)^2\, \left(a^2\, x^2-1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] -int(x^4*atanh(a*x)^2*(a^2*x^2 - 1), x)
```

3.173 $\int x^3(1-a^2x^2)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=116

$$-\frac{x^2}{180a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\,\mathrm{t}$$

```
[Out] -1/180*x^2/a^2-1/60*x^4+1/6*x*arctanh(a*x)/a^3+1/18*x^3*arctanh(a*x)/a-1/15
*a*x^5*arctanh(a*x)-1/12*arctanh(a*x)^2/a^4+1/4*x^4*arctanh(a*x)^2-1/6*a^2*
x^6*arctanh(a*x)^2+7/90*ln(-a^2*x^2+1)/a^4
```

Rubi [A]

time = 0.33, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6161, 6037, 6127, 272, 45, 6021, 266, 6095}

$$-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{x\tanh^{-1}(ax)}{6a^3}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\frac{x^2}{180a^2}+\frac{7\log\left(1-a^2x^2\right)}{90a^4}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(ax)^2+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{x^4}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/180*x^2/a^2 - x^4/60 + (x*ArcTanh[a*x])/(6*a^3) + (x^3*ArcTanh[a*x])/(18
*a) - (a*x^5*ArcTanh[a*x])/15 - ArcTanh[a*x]^2/(12*a^4) + (x^4*ArcTanh[a*x]
^2)/4 - (a^2*x^6*ArcTanh[a*x]^2)/6 + (7*Log[1 - a^2*x^2])/(90*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^3(1-a^2x^2)\tanh^{-1}(ax)^2\,dx = -\left(a^2\int x^5\tanh^{-1}(ax)^2\,dx\right)+\int x^3\tanh^{-1}(ax)^2\,dx$$

$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\frac{1}{2}a\int\frac{x^4\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{3}a^3$$

$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2+\frac{\int x^2\tanh^{-1}(ax)\,dx}{2a}-\frac{\int\frac{x^2\tanh^{-1}}{1-a^2}}{2a}$$

$$= \frac{x^3\tanh^{-1}(ax)}{6a}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}($$

$$= \frac{x\tanh^{-1}(ax)}{2a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{4a^4}+\frac{1}{4}$$

$$= \frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{1}{4}$$

$$= \frac{x^2}{20a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\tan$$

$$= -\frac{x^2}{180a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.76

$$-\frac{a^2x^2+3a^4x^4+2ax(-15-5a^2x^2+6a^4x^4)\tanh^{-1}(ax)+15(1-3a^4x^4+2a^6x^6)\tanh^{-1}(ax)^2-14\log(1-a^2x^2)}{180a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/180*(a^2*x^2 + 3*a^4*x^4 + 2*a*x*(-15 - 5*a^2*x^2 + 6*a^4*x^4)*ArcTanh[a
*x] + 15*(1 - 3*a^4*x^4 + 2*a^6*x^6)*ArcTanh[a*x]^2 - 14*Log[1 - a^2*x^2])/
a^4
```

Maple [A]
time = 0.43, size = 182, normalized size = 1.57

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2a^6x^6}{6}+\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}-\frac{\operatorname{arctanh}(ax)a^5x^5}{15}+\frac{a^3x^3\operatorname{arctanh}(ax)}{18}+\frac{ax\operatorname{arctanh}(ax)}{6}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{12}-\operatorname{arctan}$
default	$-\frac{\operatorname{arctanh}(ax)^2a^6x^6}{6}+\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}-\frac{\operatorname{arctanh}(ax)a^5x^5}{15}+\frac{a^3x^3\operatorname{arctanh}(ax)}{18}+\frac{ax\operatorname{arctanh}(ax)}{6}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{12}-\operatorname{arctan}$
risch	$-\frac{(2a^6x^6-3a^4x^4+1)\ln(ax+1)^2}{48a^4}+\frac{(30a^6x^6\ln(-ax+1)-12a^5x^5-45x^4\ln(-ax+1)a^4+10a^3x^3+30ax+15\ln(-ax+1))}{360a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/6*arctanh(a*x)^2*a^6*x^6+1/4*a^4*x^4*arctanh(a*x)^2-1/15*arctanh(
a*x)*a^5*x^5+1/18*a^3*x^3*arctanh(a*x)+1/6*a*x*arctanh(a*x)+1/12*arctanh(a*
x)*ln(a*x-1)-1/12*arctanh(a*x)*ln(a*x+1)-1/24*ln(a*x-1)*ln(1/2*a*x+1/2)+1/4
8*ln(a*x-1)^2-1/24*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/48*ln(a*x
+1)^2-1/60*a^4*x^4-1/180*a^2*x^2+7/90*ln(a*x-1)+7/90*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 146, normalized size = 1.26

$$-\frac{1}{180}\,a\left(\frac{2\,(6\,a^4x^5-5\,a^2x^3-15\,x)}{a^4}+\frac{15\,\log\left(ax+1\right)}{a^5}-\frac{15\,\log\left(ax-1\right)}{a^5}\right)\operatorname{artanh}\left(ax\right)-\frac{1}{12}\,(2\,a^2x^6-3\,x^4)\operatorname{artanh}\left(ax\right)^2-\frac{12\,a^4x^4+4\,a^2x^2+2\,(15\,\log\left(ax-1\right)-28)\log\left(ax+1\right)-15\,\log\left(ax+1\right)^2-15\,\log\left(ax-1\right)^2-56\,\log\left(ax-1\right)}{720\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/180*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*l
og(a*x - 1)/a^5)*arctanh(a*x) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)^2 - 1
/720*(12*a^4*x^4 + 4*a^2*x^2 + 2*(15*log(a*x - 1) - 28)*log(a*x + 1) - 15*l
og(a*x + 1)^2 - 15*log(a*x - 1)^2 - 56*log(a*x - 1))/a^4
```

Fricas [A]
time = 0.42, size = 109, normalized size = 0.94

$$-\frac{12\,a^4x^4+4\,a^2x^2+15\,(2\,a^6x^6-3\,a^4x^4+1)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(6\,a^5x^5-5\,a^3x^3-15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-56\,\log\left(a^2x^2-1\right)}{720\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/720*(12*a^4*x^4 + 4*a^2*x^2 + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*log(-(a*x +
 1)/(a*x - 1))^2 + 4*(6*a^5*x^5 - 5*a^3*x^3 - 15*a*x)*log(-(a*x + 1)/(a*x -
 1)) - 56*log(a^2*x^2 - 1))/a^4
```

Sympy [A]
time = 0.75, size = 114, normalized size = 0.98

$$\begin{cases} -\frac{a^2x^6\operatorname{atanh}^2\left(ax\right)}{6}-\frac{ax^5\operatorname{atanh}\left(ax\right)}{15}+\frac{x^4\operatorname{atanh}^2\left(ax\right)}{4}-\frac{x^4}{60}+\frac{x^3\operatorname{atanh}\left(ax\right)}{18a}-\frac{x^2}{180a^2}+\frac{x\operatorname{atanh}\left(ax\right)}{6a^3}+\frac{7\log\left(x-\frac{1}{a}\right)}{45a^4}-\frac{\operatorname{atanh}^2\left(ax\right)}{12a^4}+\frac{7\operatorname{atanh}\left(ax\right)}{45a^4} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*x**6*atanh(a*x)**2/6 - a*x**5*atanh(a*x)/15 + x**4*atanh(a
*x)**2/4 - x**4/60 + x**3*atanh(a*x)/(18*a) - x**2/(180*a**2) + x*atanh(a*x
)/(6*a**3) + 7*log(x - 1/a)/(45*a**4) - atanh(a*x)**2/(12*a**4) + 7*atanh(a
*x)/(45*a**4), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(98) = 196$.

time = 0.40, size = 522, normalized size = 4.50

$$-\frac{1}{45}\left(\frac{15\left(\frac{3(ax+1)^4}{(ax-1)^4}+\frac{2(ax+1)^3}{(ax-1)^3}+\frac{3(ax+1)^2}{(ax-1)^2}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^6a^5}{(ax-1)^6}-\frac{6(ax+1)^5a^5}{(ax-1)^5}+\frac{15(ax+1)^4a^5}{(ax-1)^4}-\frac{20(ax+1)^3a^5}{(ax-1)^3}+\frac{15(ax+1)^2a^5}{(ax-1)^2}-\frac{6(ax+1)a^5}{ax-1}+a^5}+\frac{\left(\frac{45(ax+1)^3}{(ax-1)^3}-\frac{25(ax+1)^2}{(ax-1)^2}+\frac{35(ax+1)}{ax-1}-7\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^5a^5}{(ax-1)^5}-\frac{5(ax+1)^4a^5}{(ax-1)^4}+\frac{10(ax+1)^3a^5}{(ax-1)^3}-\frac{10(ax+1)^2a^5}{(ax-1)^2}+\frac{5(ax+1)a^5}{ax-1}-a^5}+\frac{\frac{7(ax+1)^3}{(ax-1)^3}-\frac{2(ax+1)^2}{(ax-1)^2}+\frac{7(ax+1)}{ax-1}}{\frac{(ax+1)^4a^5}{(ax-1)^4}-\frac{4(ax+1)^3a^5}{(ax-1)^3}+\frac{6(ax+1)^2a^5}{(ax-1)^2}-\frac{4(ax+1)a^5}{ax-1}+a^5}+\frac{7\log\left(-\frac{ax+1}{ax-1}+1\right)}{a^5}-\frac{7\log\left(-\frac{ax+1}{ax-1}\right)}{a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/45*(15*(3*(a*x + 1)^4/(a*x - 1)^4 + 2*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x +
 1)^2/(a*x - 1)^2)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^6*a^5/(a*x - 1)^6
 - 6*(a*x + 1)^5*a^5/(a*x - 1)^5 + 15*(a*x + 1)^4*a^5/(a*x - 1)^4 - 20*(a*x
 + 1)^3*a^5/(a*x - 1)^3 + 15*(a*x + 1)^2*a^5/(a*x - 1)^2 - 6*(a*x + 1)*a^5/
(a*x - 1) + a^5) + (45*(a*x + 1)^3/(a*x - 1)^3 - 25*(a*x + 1)^2/(a*x - 1)^2
 + 35*(a*x + 1)/(a*x - 1) - 7)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^5*a^5/(
a*x - 1)^5 - 5*(a*x + 1)^4*a^5/(a*x - 1)^4 + 10*(a*x + 1)^3*a^5/(a*x - 1)^3
 - 10*(a*x + 1)^2*a^5/(a*x - 1)^2 + 5*(a*x + 1)*a^5/(a*x - 1) - a^5) + (7*(
a*x + 1)^3/(a*x - 1)^3 - 2*(a*x + 1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1))
/((a*x + 1)^4*a^5/(a*x - 1)^4 - 4*(a*x + 1)^3*a^5/(a*x - 1)^3 + 6*(a*x + 1)
^2*a^5/(a*x - 1)^2 - 4*(a*x + 1)*a^5/(a*x - 1) + a^5) + 7*log(-(a*x + 1)/(a
*x - 1) + 1)/a^5 - 7*log(-(a*x + 1)/(a*x - 1))/a^5)*a
```

Mupad [B]
time = 0.98, size = 101, normalized size = 0.87

$$-\frac{a^2\,x^2-14\,\ln\left(a^2\,x^2-1\right)+3\,a^4\,x^4+15\,\mathrm{atanh}\left(a\,x\right)^2-10\,a^3\,x^3\,\mathrm{atanh}\left(a\,x\right)+12\,a^5\,x^5\,\mathrm{atanh}\left(a\,x\right)-30\,a\,x\,\mathrm{atanh}\left(a\,x\right)-45\,a^4\,x^4\,\mathrm{atanh}\left(a\,x\right)^2+30\,a^6\,x^6\,\mathrm{atanh}\left(a\,x\right)^2}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] -(a^2*x^2 - 14*log(a^2*x^2 - 1) + 3*a^4*x^4 + 15*atanh(a*x)^2 - 10*a^3*x^3*
atanh(a*x) + 12*a^5*x^5*atanh(a*x) - 30*a*x*atanh(a*x) - 45*a^4*x^4*atanh(a
*x)^2 + 30*a^6*x^6*atanh(a*x)^2)/(180*a^4)
```

3.174 $\int x^2(1-a^2x^2)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=138

$$\frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{30a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{15a^3}+\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{1}{5}a^2x^5\tanh$$

```
[Out] 1/30*x/a^2-1/30*x^3-1/30*arctanh(a*x)/a^3+2/15*x^2*arctanh(a*x)/a-1/10*a*x^
4*arctanh(a*x)+2/15*arctanh(a*x)^2/a^3+1/3*x^3*arctanh(a*x)^2-1/5*a^2*x^5*a
rctanh(a*x)^2-4/15*arctanh(a*x)*ln(2/(-a*x+1))/a^3-2/15*polylog(2,1-2/(-a*x
+1))/a^3
```

Rubi [A]
time = 0.30, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6161, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 308}

$$-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{15a^3}+\frac{2\tanh^{-1}(ax)^2}{15a^3}-\frac{\tanh^{-1}(ax)}{30a^3}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{x}{30a^2}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{1}{3}x^3\tanh^{-1}(ax)^2+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{x^3}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] x/(30*a^2) - x^3/30 - ArcTanh[a*x]/(30*a^3) + (2*x^2*ArcTanh[a*x])/(15*a) -
 (a*x^4*ArcTanh[a*x])/10 + (2*ArcTanh[a*x]^2)/(15*a^3) + (x^3*ArcTanh[a*x]^
2)/3 - (a^2*x^5*ArcTanh[a*x]^2)/5 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a
^3) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(15*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
```

```
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int x^2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\left(a^2\int x^4\tanh^{-1}(ax)^2\,dx\right)+\int x^2\tanh^{-1}(ax)^2\,dx$$
$$=\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2-\frac{1}{3}(2a)\int\frac{x^3\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{5}($$
$$=\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{2\int x\tanh^{-1}(ax)\,dx}{3a}-\frac{2\int\frac{x\tanh^{-1}}{1-a}}{3a}$$
$$=\frac{x^2\tanh^{-1}(ax)}{3a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\tanh^{-1}(ax)^2-$$
$$=\frac{x}{3a^2}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{15a^3}+\frac{1}{3}x^3\tanh^{-}$$
$$=\frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh}{1}$$
$$=\frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{30a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh}{1}$$
$$=\frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{30a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh}{1}$$

Mathematica [A]
time = 0.18, size = 95, normalized size = 0.69

$$-\frac{-ax+a^3x^3+2(2-5a^3x^3+3a^5x^5)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(1-4a^2x^2+3a^4x^4+8\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)-4\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{30a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/30*(-(a*x) + a^3*x^3 + 2*(2 - 5*a^3*x^3 + 3*a^5*x^5)*ArcTanh[a*x]^2 + Ar
cTanh[a*x]*(1 - 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^(-2*ArcTanh[a*x])]) - 4
*PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^3
```

Maple [A]
time = 1.02, size = 179, normalized size = 1.30

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}-\frac{a^4 x^4 \operatorname{arctanh}(ax)}{10}+\frac{2a^2 x^2 \operatorname{arctanh}(ax)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(ax+}{15}$
default	$-\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}-\frac{a^4 x^4 \operatorname{arctanh}(ax)}{10}+\frac{2a^2 x^2 \operatorname{arctanh}(ax)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(ax+}{15}$
risch	$-\frac{443}{3375a^3}-\frac{x^3}{30}-\frac{\left(\left(-\frac{1}{9}+\frac{\ln(ax+1)}{3}\right)(ax+1)^3+\left(\frac{1}{2}-\ln(ax+1)\right)(ax+1)^2+(-1+\ln(ax+1))(ax+1)\right)\ln(-ax+1)}{2a^3}+\frac{(a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/5*arctanh(a*x)^2*a^5*x^5+1/3*arctanh(a*x)^2*a^3*x^3-1/10*a^4*x^4*
arctanh(a*x)+2/15*a^2*x^2*arctanh(a*x)+2/15*arctanh(a*x)*ln(a*x-1)+2/15*arc
tanh(a*x)*ln(a*x+1)+1/30*ln(a*x-1)^2-2/15*dilog(1/2*a*x+1/2)-1/15*ln(a*x-1)
*ln(1/2*a*x+1/2)+1/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/30*ln(
a*x+1)^2-1/30*a^3*x^3+1/30*a*x+1/60*ln(a*x-1)-1/60*ln(a*x+1))
```

Maxima [A]

time = 0.26, size = 173, normalized size = 1.25

$$-\frac{1}{60}a^2\left(\frac{2a^3x^3-2ax+2\log(ax+1)^2-4\log(ax+1)\log(ax-1)-2\log(ax-1)^2-\log(ax-1)}{a^5}+\frac{8\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^5}+\frac{\log(ax+1)}{a^5}\right)-\frac{1}{30}a\left(\frac{3a^2x^4-4x^2}{a^2}-\frac{4\log(ax+1)}{a^4}-\frac{4\log(ax-1)}{a^4}\right)\operatorname{artanh}(ax)-\frac{1}{15}\left(3a^2x^5-5x^3\right)\operatorname{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/60*a^2*((2*a^3*x^3 - 2*a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x -
 1) - 2*log(a*x - 1)^2 - log(a*x - 1))/a^5 + 8*(log(a*x - 1)*log(1/2*a*x +
1/2) + dilog(-1/2*a*x + 1/2))/a^5 + log(a*x + 1)/a^5) - 1/30*a*((3*a^2*x^4
- 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)*arctanh(a*x) - 1/15
*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)^2
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^4 - x^2)*arctanh(a*x)^2, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-x^2\operatorname{atanh}^2(ax)\right)dx-\int a^2x^4\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(-x**2*atanh(a*x)**2, x) - Integral(a**2*x**4*atanh(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x^2*arctanh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int x^2\,\mathrm{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] -int(x^2*atanh(a*x)^2*(a^2*x^2 - 1), x)
```

3.175 $\int x(1-a^2x^2)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=95

$$\frac{1-a^2x^2}{12a^2}+\frac{x\tanh^{-1}(ax)}{3a}+\frac{x(1-a^2x^2)\tanh^{-1}(ax)}{6a}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4a^2}+\frac{\log(1-a^2x^2)}{6a^2}$$

```
[Out] 1/12*(-a^2*x^2+1)/a^2+1/3*x*arctanh(a*x)/a+1/6*x*(-a^2*x^2+1)*arctanh(a*x)/
a-1/4*(-a^2*x^2+1)^2*arctanh(a*x)^2/a^2+1/6*ln(-a^2*x^2+1)/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6141, 6089, 6021, 266}

$$\frac{1-a^2x^2}{12a^2}+\frac{\log(1-a^2x^2)}{6a^2}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4a^2}+\frac{x(1-a^2x^2)\tanh^{-1}(ax)}{6a}+\frac{x\tanh^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (1 - a^2*x^2)/(12*a^2) + (x*ArcTanh[a*x])/(3*a) + (x*(1 - a^2*x^2)*ArcTanh[
a*x])/(6*a) - ((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/(4*a^2) + Log[1 - a^2*x^2]/(
6*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x(1-a^2x^2)\tanh^{-1}(ax)^2\,dx = -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\int(1-a^2x^2)\tanh^{-1}(ax)\,dx}{2a}$$
$$= \frac{1-a^2x^2}{12a^2} + \frac{x(1-a^2x^2)\tanh^{-1}(ax)}{6a} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\int\tanh^{-}}{3}$$
$$= \frac{1-a^2x^2}{12a^2} + \frac{x\tanh^{-1}(ax)}{3a} + \frac{x(1-a^2x^2)\tanh^{-1}(ax)}{6a} - \frac{(1-a^2x^2)^2\tanh^{-1}(}{4a^2}$$
$$= \frac{1-a^2x^2}{12a^2} + \frac{x\tanh^{-1}(ax)}{3a} + \frac{x(1-a^2x^2)\tanh^{-1}(ax)}{6a} - \frac{(1-a^2x^2)^2\tanh^{-1}(}{4a^2}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.69

$$\frac{-a^2x^2+(6ax-2a^3x^3)\tanh^{-1}(ax)-3(-1+a^2x^2)^2\tanh^{-1}(ax)^2+2\log(1-a^2x^2)}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (-(a^2*x^2) + (6*a*x - 2*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTanh
[a*x]^2 + 2*Log[1 - a^2*x^2])/(12*a^2)
```

Maple [A]
time = 0.37, size = 86, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}+\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{4}-\frac{a^3x^3\operatorname{arctanh}(ax)}{6}+\frac{ax\operatorname{arctanh}(ax)}{2}-\frac{a^2x^2}{12}+\frac{\ln(ax-1)}{6}+\frac{\ln(ax+1)}{6}}{a^2}$
default	$\frac{-\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}+\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{4}-\frac{a^3x^3\operatorname{arctanh}(ax)}{6}+\frac{ax\operatorname{arctanh}(ax)}{2}-\frac{a^2x^2}{12}+\frac{\ln(ax-1)}{6}+\frac{\ln(ax+1)}{6}}{a^2}$
risch	$-\frac{(a^2x^2-1)^2\ln(ax+1)^2}{16a^2}+\frac{(3x^4\ln(-ax+1)a^4-2a^3x^3-6x^2\ln(-ax+1)a^2+6ax+3\ln(-ax+1))\ln(ax+1)}{24a^2}-\frac{\ln(-ax+}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*a^4*x^4*arctanh(a*x)^2+1/2*a^2*x^2*arctanh(a*x)^2-1/4*arctanh(a
*x)^2-1/6*a^3*x^3*arctanh(a*x)+1/2*a*x*arctanh(a*x)-1/12*a^2*x^2+1/6*ln(a*x
-1)+1/6*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 74, normalized size = 0.78

$$-\frac{\left(a^2x^2-1\right)^2\operatorname{artanh}\left(ax\right)^2}{4\,a^2}-\frac{\left(x^2-\frac{2\,\log\left(ax+1\right)}{a^2}-\frac{2\,\log\left(ax-1\right)}{a^2}\right)a+2\,\left(a^2x^3-3\,x\right)\operatorname{artanh}\left(ax\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*x^2 - 1)^2*arctanh(a*x)^2/a^2 - 1/12*((x^2 - 2*log(a*x + 1)/a^2 -
 2*log(a*x - 1)/a^2)*a + 2*(a^2*x^3 - 3*x)*arctanh(a*x))/a
```

Fricas [A]
time = 0.34, size = 91, normalized size = 0.96

$$-\frac{4\,a^2x^2+3\,\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,\left(a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-8\,\log\left(a^2x^2-1\right)}{48\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
+ 4*(a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 8*log(a^2*x^2 - 1))/a^2
```

Sympy [A]
time = 0.29, size = 88, normalized size = 0.93

$$\begin{cases} -\frac{a^2x^4\operatorname{atanh}^2\left(ax\right)}{4}-\frac{ax^3\operatorname{atanh}\left(ax\right)}{6}+\frac{x^2\operatorname{atanh}^2\left(ax\right)}{2}-\frac{x^2}{12}+\frac{x\operatorname{atanh}\left(ax\right)}{2a}+\frac{\log\left(x-\frac{1}{a}\right)}{3a^2}-\frac{\operatorname{atanh}^2\left(ax\right)}{4a^2}+\frac{\operatorname{atanh}\left(ax\right)}{3a^2} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*x**4*atanh(a*x)**2/4 - a*x**3*atanh(a*x)/6 + x**2*atanh(a*
x)**2/2 - x**2/12 + x*atanh(a*x)/(2*a) + log(x - 1/a)/(3*a**2) - atanh(a*x)
**2/(4*a**2) + atanh(a*x)/(3*a**2), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(82) = 164.
time = 0.40, size = 305, normalized size = 3.21

$$-\frac{1}{3}\,a\left(\frac{\left(\frac{3\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^3a^3}{(ax-1)^3}-\frac{3\,(ax+1)^2a^3}{(ax-1)^2}+\frac{3\,(ax+1)a^3}{ax-1}-a^3}+\frac{3\,(ax+1)^2\log\left(-\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^4a^3}{(ax-1)^4}-\frac{4\,(ax+1)^3a^3}{(ax-1)^3}+\frac{6\,(ax+1)^2a^3}{(ax-1)^2}-\frac{4\,(ax+1)a^3}{ax-1}+a^3\right)(ax-1)^2}+\frac{ax+1}{\left(\frac{(ax+1)^2a^3}{(ax-1)^2}-\frac{2\,(ax+1)a^3}{ax-1}+a^3\right)(ax-1)}+\frac{\log\left(-\frac{ax+1}{ax-1}+1\right)}{a^3}-\frac{\log\left(-\frac{ax+1}{ax-1}\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/3*a*((3*(a*x + 1)/(a*x - 1) - 1)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^3*
a^3/(a*x - 1)^3 - 3*(a*x + 1)^2*a^3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x - 1)
 - a^3) + 3*(a*x + 1)^2*log(-(a*x + 1)/(a*x - 1))^2/(((a*x + 1)^4*a^3/(a*x
- 1)^4 - 4*(a*x + 1)^3*a^3/(a*x - 1)^3 + 6*(a*x + 1)^2*a^3/(a*x - 1)^2 - 4*
(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)^2) + (a*x + 1)/(((a*x + 1)^2*a^3/(
a*x - 1)^2 - 2*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)) + log(-(a*x + 1)/(
a*x - 1) + 1)/a^3 - log(-(a*x + 1)/(a*x - 1))/a^3)
```

Mupad [B]
time = 0.90, size = 77, normalized size = 0.81

$$\frac{x^2\,\mathrm{atanh}(a\,x)^2}{2}-\frac{\mathrm{atanh}(a\,x)^2}{4\,a^2}-\frac{x^2}{12}+\frac{\ln\left(a^2\,x^2-1\right)}{6\,a^2}+\frac{x\,\mathrm{atanh}(a\,x)}{2\,a}-\frac{a\,x^3\,\mathrm{atanh}(a\,x)}{6}-\frac{a^2\,x^4\,\mathrm{atanh}(a\,x)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

```
[Out] (x^2*atanh(a*x)^2)/2 - atanh(a*x)^2/(4*a^2) - x^2/12 + log(a^2*x^2 - 1)/(6*
a^2) + (x*atanh(a*x))/(2*a) - (a*x^3*atanh(a*x))/6 - (a^2*x^4*atanh(a*x)^2)
/4
```

3.176 $\int (1-a^2x^2)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=115

$$-\frac{x}{3}+\frac{(1-a^2x^2)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)^2-\frac{4\tanh^{-1}(ax)\,\mathrm{lo}}{3a}$$

```
[Out] -1/3*x+1/3*(-a^2*x^2+1)*arctanh(a*x)/a+2/3*arctanh(a*x)^2/a+2/3*x*arctanh(a
*x)^2+1/3*x*(-a^2*x^2+1)*arctanh(a*x)^2-4/3*arctanh(a*x)*ln(2/(-a*x+1))/a-2
/3*polylog(2,1-2/(-a*x+1))/a
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6091, 6021, 6131, 6055, 2449, 2352, 8}

$$\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)^2+\frac{(1-a^2x^2)\tanh^{-1}(ax)}{3a}-\frac{2\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{2\tanh^{-1}(ax)^2}{3a}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{3a}-\frac{x}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/3*x + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*a) + (2*ArcTanh[a*x]^2)/(3*a) + (2
*x*ArcTanh[a*x]^2)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/3 - (4*ArcTanh[a*x]
*Log[2/(1 - a*x)])/(3*a) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a} + \frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2 - \frac{\int 1\,dx}{3} + \frac{2}{3}\int \tanh^{-1}$$

$$= -\frac{x}{3} + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^2 + \frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)$$

$$= -\frac{x}{3} + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a} + \frac{2\tanh^{-1}(ax)^2}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^2 + \frac{1}{3}x(1-$$

$$= -\frac{x}{3} + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a} + \frac{2\tanh^{-1}(ax)^2}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^2 + \frac{1}{3}x(1-$$

$$= -\frac{x}{3} + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a} + \frac{2\tanh^{-1}(ax)^2}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^2 + \frac{1}{3}x(1-$$

$$= -\frac{x}{3} + \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a} + \frac{2\tanh^{-1}(ax)^2}{3a} + \frac{2}{3}x\tanh^{-1}(ax)^2 + \frac{1}{3}x(1-$$

Mathematica [A]

time = 0.09, size = 71, normalized size = 0.62

$$-\frac{ax+(-1+ax)^2(2+ax)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(-1+a^2x^2+4\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)-2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -1/3*(a*x + (-1 + a*x)^2*(2 + a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-1 + a^2*
x^2 + 4*Log[1 + E^(-2*ArcTanh[a*x])]) - 2*PolyLog[2, -E^(-2*ArcTanh[a*x])])
/a
```

Maple [A]
time = 0.55, size = 154, normalized size = 1.34

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2a^3x^3}{3}+\operatorname{arctanh}(ax)^2ax-\frac{a^2x^2\operatorname{arctanh}(ax)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax+1)}{3}-\frac{ax}{3}-\frac{\ln(ax-1)}{6}+}{a}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2a^3x^3}{3}+\operatorname{arctanh}(ax)^2ax-\frac{a^2x^2\operatorname{arctanh}(ax)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax+1)}{3}-\frac{ax}{3}-\frac{\ln(ax-1)}{6}+}{a}$
risch	$-\frac{x}{3}-\frac{2\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{3a}-\frac{5\ln(ax-1)}{9a}+\frac{a^2\ln(-ax+1)\ln(ax+1)x^3}{6}-\frac{(-1+\ln(ax+1))(ax+1)\ln(-ax+1)}{2a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3*arctanh(a*x)^2*a^3*x^3+arctanh(a*x)^2*a*x-1/3*a^2*x^2*arctanh(a*x
)+2/3*arctanh(a*x)*ln(a*x-1)+2/3*arctanh(a*x)*ln(a*x+1)-1/3*a*x-1/6*ln(a*x-
1)+1/6*ln(a*x+1)-2/3*dilog(1/2*a*x+1/2)-1/3*ln(a*x-1)*ln(1/2*a*x+1/2)+1/6*l
n(a*x-1)^2+1/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/6*ln(a*x+1)^2
)
```

Maxima [A]
time = 0.26, size = 144, normalized size = 1.25

$$-\frac{1}{6}a^2\left(\frac{2ax+\log(ax+1)^2-2\log(ax+1)\log(ax-1)-\log(ax-1)^2+\log(ax-1)}{a^3}+\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^3}-\frac{\log(ax+1)}{a^3}\right)-\frac{1}{3}\left(x^2-\frac{2\log(ax+1)}{a^2}-\frac{2\log(ax-1)}{a^2}\right)a\operatorname{artanh}(ax)-\frac{1}{3}\left(a^2x^3-3x\right)\operatorname{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a^2*((2*a*x + log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x -
 1)^2 + log(a*x - 1))/a^3 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2
*a*x + 1/2))/a^3 - log(a*x + 1)/a^3) - 1/3*(x^2 - 2*log(a*x + 1)/a^2 - 2*lo
g(a*x - 1)/a^2)*a*arctanh(a*x) - 1/3*(a^2*x^3 - 3*x)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int a^{2}x^{2}\operatorname{atanh}^{2}(ax)\,dx-\int\left(-\operatorname{atanh}^{2}(ax)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2,x)
[Out] -Integral(a**2*x**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2*(a^2*x^2 - 1),x)
[Out] -int(atanh(a*x)^2*(a^2*x^2 - 1), x)
```

3.177 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=146

$$-ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}\log\left(1-a^2x\right.$$

```
[Out] -a*x*arctanh(a*x)+1/2*arctanh(a*x)^2-1/2*a^2*x^2*arctanh(a*x)^2-2*arctanh(a
*x)^2*arctanh(-1+2/(-a*x+1))-1/2*ln(-a^2*x^2+1)-arctanh(a*x)*polylog(2,1-2/
(-a*x+1))+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*polylog(3,1-2/(-a*x+1))
-1/2*polylog(3,-1+2/(-a*x+1))
```

Rubi [A]
time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6161, 6033, 6199, 6095, 6205, 6745, 6037, 6127, 6021, 266}

$$-\frac{1}{2}\log\left(1-a^2x^2\right)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+\frac{1}{2}\text{Li}_3\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{Li}_3\left(\frac{2}{1-ax}-1\right)-\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)+\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)+2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2+\frac{1}{2}\tanh^{-1}(ax)^2-ax\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

```
[Out] -(a*x*ArcTanh[a*x]) + ArcTanh[a*x]^2/2 - (a^2*x^2*ArcTanh[a*x]^2)/2 + 2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 - ArcTanh[a*x]*Po
lyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + Pol
yLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x}\,dx &= -\left(a^2\int x\tanh^{-1}(ax)^2\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\\ &= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-(4a)\int \frac{\text{tanh}}{}\\ &= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-a\int \tanh^{-1}\\ &= -ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\,\text{ta}\\ &= -ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\,\text{ta}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 145, normalized size = 0.99

$$-ax\tanh^{-1}(ax)-\frac{1}{2}(-1+a^2x^2)\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}\log\left(1-a^2x^2\right)+\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{-1-ax}{-1+ax}\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{1+ax}{-1+ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{-1-ax}{-1+ax}\right)+\frac{1}{2}\text{PolyLog}\left(3,\frac{1+ax}{-1+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

```
[Out] -(a*x*ArcTanh[a*x]) - ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/2 + 2*ArcTanh[a*x]^2*
ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 + ArcTanh[a*x]*PolyLog[2, (-1
 - a*x)/(-1 + a*x)] - ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a*x)] - PolyL
og[3, (-1 - a*x)/(-1 + a*x)]/2 + PolyLog[3, (1 + a*x)/(-1 + a*x)]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 18.24, size = 663, normalized size = 4.54

method	result
derivativedivides	$-\frac{a^2x^2\,\text{arctanh}(ax)^2}{2}+\text{arctanh}\left(ax\right)^2\ln\left(ax\right)-\text{arctanh}\left(ax\right)^2\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)+\text{arctanh}\,($

default	$-\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}+\operatorname{arctanh}(ax)^2\ln(ax)-\operatorname{arctanh}(ax)^2\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)+\operatorname{arctanh}(ax$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*x^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1
)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arcta
nh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2
+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a*x+1)^
2/(-a^2*x^2+1))-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)
^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*cs
gn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-(a*x+1)*arctanh(a*x)+1/2*arctanh(a*x)^
2+ln((a*x+1)^2/(-a^2*x^2+1)+1)+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-
a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/2*I*arctanh(a*x)^2*Pi*csgn(I*
((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x
+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*x^2*log(-a*x + 1)^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1
)*log(a*x + 1)^2 - (a^3*x^3 + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))
*log(-a*x + 1))/(a*x^2 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\operatorname{atanh}^2(ax)}{x}\right)dx-\int a^2x\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x,x)
[Out] -Integral(-atanh(a*x)**2/x, x) - Integral(a**2*x*atanh(a*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x,x)
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x, x)
```

3.178 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\tanh^{-1}(ax)^2}{x}-a^2x\tanh^{-1}(ax)^2+2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)+a\text{PolyLo}$$

```
[Out] -arctanh(a*x)^2/x-a^2*x*arctanh(a*x)^2+2*a*arctanh(a*x)*ln(2/(-a*x+1))+2*a*
arctanh(a*x)*ln(2-2/(a*x+1))+a*polylog(2,1-2/(-a*x+1))-a*polylog(2,-1+2/(a*
x+1))
```

Rubi [A]
time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6161, 6037, 6135, 6079, 2497, 6021, 6131, 6055, 2449, 2352}

$$-a^2x\tanh^{-1}(ax)^2+a\text{Li}_2\left(1-\frac{2}{1-ax}\right)-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -(ArcTanh[a*x]^2/x) - a^2*x*ArcTanh[a*x]^2 + 2*a*ArcTanh[a*x]*Log[2/(1 - a*
x)] + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + a*PolyLog[2, 1 - 2/(1 - a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
```

```
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^2}\,dx &= -\left(a^2\int \tanh^{-1}(ax)^2\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx \\
&= -\frac{\tanh^{-1}(ax)^2}{x}-a^2x\tanh^{-1}(ax)^2+(2a)\int \frac{\tanh^{-1}(ax)}{x\,(1-a^2x^2)}\,dx+\left(2a^3\right)\int \frac{x\tan}{1} \\
&= -\frac{\tanh^{-1}(ax)^2}{x}-a^2x\tanh^{-1}(ax)^2+(2a)\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx+\left(2a^2\right)\int \frac{\tanh}{1-} \\
&= -\frac{\tanh^{-1}(ax)^2}{x}-a^2x\tanh^{-1}(ax)^2+2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)+2a\tanh \\
&= -\frac{\tanh^{-1}(ax)^2}{x}-a^2x\tanh^{-1}(ax)^2+2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)+2a\tanh \\
&= -\frac{\tanh^{-1}(ax)^2}{x}-a^2x\tanh^{-1}(ax)^2+2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)+2a\tanh
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 102, normalized size = 1.10

$$-a\tanh^{-1}(ax)\left(-\tanh^{-1}(ax)+ax\tanh^{-1}(ax)-2\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)-a\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+a\left(\tanh^{-1}(ax)\left(\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{ax}+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -(a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcTan
h[a*x])])) - a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + a*(ArcTanh[a*x]*(ArcTanh[
a*x] - ArcTanh[a*x]/(a*x) + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2, E^
(-2*ArcTanh[a*x])])
```

Maple [A]
time = 0.33, size = 156, normalized size = 1.68

method	result
derivativedivides	$a\Big(-\operatorname{arctanh}(ax)^2\,ax-\frac{\operatorname{arctanh}(ax)^2}{ax}+2\operatorname{arctanh}(ax)\ln(ax)-2\operatorname{arctanh}(ax)\ln(ax-1)$
default	$a\Big(-\operatorname{arctanh}(ax)^2\,ax-\frac{\operatorname{arctanh}(ax)^2}{ax}+2\operatorname{arctanh}(ax)\ln(ax)-2\operatorname{arctanh}(ax)\ln(ax-1)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arctanh(a*x)^2*a*x-arctanh(a*x)^2/a/x+2*arctanh(a*x)*ln(a*x)-2*arctanh(
a*x)*ln(a*x-1)-2*arctanh(a*x)*ln(a*x+1)-dilog(a*x+1)-ln(a*x)*ln(a*x+1)-dilo
g(a*x)+2*dilog(1/2*a*x+1/2)+ln(a*x-1)*ln(1/2*a*x+1/2)-1/2*ln(a*x-1)^2-(ln(a
*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/2*ln(a*x+1)^2)
```

Maxima [A]
time = 0.28, size = 152, normalized size = 1.63

$$\frac{1}{2}a^2\left(\frac{\log(ax+1)^2-2\log(ax+1)\log(ax-1)-\log(ax-1)^2}{a}+\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}-\frac{2\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)}{a}+\frac{2\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)}{a}\right)-2a(\log(ax+1)+\log(ax-1)-\log(x))\,\mathrm{artanh}(ax)-\left(a^2x+\frac{1}{x}\right)\mathrm{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
+ 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 2*(log(a*
x + 1)*log(x) + dilog(-a*x))/a + 2*(log(-a*x + 1)*log(x) + dilog(a*x))/a) -
 2*a*(log(a*x + 1) + log(a*x - 1) - log(x))*arctanh(a*x) - (a^2*x + 1/x)*ar
ctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int a^2\,\mathrm{atanh}^2(ax)\,dx-\int\left(-\frac{\mathrm{atanh}^2(ax)}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**2,x)
```

```
[Out] -Integral(a**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^2,x)
```

```
[Out] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^2, x)
```

3.179 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=172

$$-\frac{a\tanh^{-1}(ax)}{x}+\frac{1}{2}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)+a^2\log(x)-\frac{1}{2}a^2\,\mathrm{lo}$$

```
[Out] -a*arctanh(a*x)/x+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+2*a^2*arcta
nh(a*x)^2*arctanh(-1+2/(-a*x+1))+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+a^2*arcta
nh(a*x)*polylog(2,1-2/(-a*x+1))-a^2*arctanh(a*x)*polylog(2,-1+2/(-a*x+1))-1
/2*a^2*polylog(3,1-2/(-a*x+1))+1/2*a^2*polylog(3,-1+2/(-a*x+1))
```

Rubi [A]
time = 0.25, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6161, 6037, 6129, 272, 36, 29, 31, 6095, 6033, 6199, 6205, 6745}

$$-\frac{1}{2}a^2\mathrm{Li}_3\left(1-\frac{2}{1-ax}\right)+\frac{1}{2}a^2\mathrm{Li}_3\left(\frac{2}{1-ax}-1\right)+a^2\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)-a^2\mathrm{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{2}a^2\tanh^{-1}(ax)^2-2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-\frac{\tanh^{-1}(ax)^2}{2x^2}-\frac{a\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) - 2
*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^
2*x^2])/2 + a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - a^2*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 - a*x)] - (a^2*PolyLog[3, 1 - 2/(1 - a*x)])/2 + (a^2*
PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^3}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx \\
&= -\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)+a\int \frac{\tanh^{-1}(a
x}{x^2\,(1-a^2x
\\
&= -\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)+a\int \frac{\tanh^{-1}(ax}{x^2}
\\
&= -\frac{a\tanh^{-1}(ax)}{x}+\frac{1}{2}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh \\
&= -\frac{a\tanh^{-1}(ax)}{x}+\frac{1}{2}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh \\
&= -\frac{a\tanh^{-1}(ax)}{x}+\frac{1}{2}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh \\
&= -\frac{a\tanh^{-1}(ax)}{x}+\frac{1}{2}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 174, normalized size = 1.01

$$-\frac{a\tanh^{-1}(ax)}{x}+\frac{(-1+a^2x^2)\tanh^{-1}(ax)^2}{2x^2}-2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)-a^2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,\frac{-1-ax}{-1+ax}\right)+a^2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,\frac{1+ax}{-1+ax}\right)+\frac{1}{2}a^2\mathrm{PolyLog}\left(3,\frac{-1-ax}{-1+ax}\right)-\frac{1}{2}a^2\mathrm{PolyLog}\left(3,\frac{1+ax}{-1+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) - 2*a^2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/
```

```
2 - a^2*ArcTanh[a*x]*PolyLog[2, (-1 - a*x)/(-1 + a*x)] + a^2*ArcTanh[a*x]*P
olyLog[2, (1 + a*x)/(-1 + a*x)] + (a^2*PolyLog[3, (-1 - a*x)/(-1 + a*x)])/2
 - (a^2*PolyLog[3, (1 + a*x)/(-1 + a*x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.02, size = 736, normalized size = 4.28

method	result
derivativedivides	$a^2\left(-\operatorname{arctanh}(ax)^2\ln(ax)-\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{i\pi\,\mathrm{csgn}\left(i\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)\right)\mathrm{csgn}\left(\frac{i\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2\operatorname{arctanh}}{2}\right.$
default	$a^2\left(-\operatorname{arctanh}(ax)^2\ln(ax)-\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{i\pi\,\mathrm{csgn}\left(i\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)\right)\mathrm{csgn}\left(\frac{i\left(\frac{(ax+1)^2}{-a^2x^2+1}-1\right)}{\frac{(ax+1)^2}{-a^2x^2+1}+1}\right)^2\operatorname{arctanh}}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^2/a^2/x^2-1/2*I*arctanh(a*x)^
2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*
csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+1/2*I*Pi*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh
(a*x)^2+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+1/2*arctanh(a*x)^2+ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))-1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*(-(-a^2*x^2+1)^(1/2)+a*x+1)/a/x*arctanh(
a*x)-1/2*arctanh(a*x)*((-a^2*x^2+1)^(1/2)+a*x+1)/a/x+arctanh(a*x)^2*ln((a*x
+1)^2/(-a^2*x^2+1)-1)-arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arc
tanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))-arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*
x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(
1/2))+arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*polylog(3,-(a*x+1
)^2/(-a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*log(-a*x + 1)^2/x^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1)*lo
g(a*x + 1)^2 - (a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))*log(-a*
x + 1))/(a*x^4 - x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\operatorname{atanh}^{2}(ax)}{x^{3}}\right)dx-\int\frac{a^{2}\operatorname{atanh}^{2}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**3,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**3, x) - Integral(a**2*atanh(a*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\operatorname{atanh}(a\,x)^{2}\,\left(a^{2}\,x^{2}-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^3,x)
```

```
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x^3, x)
```

3.180 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^4} dx$

Optimal. Leaf size=116

$$-\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{4}{3}a^3\tanh^{-1}(ax)\log$$

```
[Out] -1/3*a^2/x+1/3*a^3*arctanh(a*x)-1/3*a*arctanh(a*x)/x^2-2/3*a^3*arctanh(a*x)
^2-1/3*arctanh(a*x)^2/x^3+a^2*arctanh(a*x)^2/x-4/3*a^3*arctanh(a*x)*ln(2-2/
(a*x+1))+2/3*a^3*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.23, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6161, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$\frac{2}{3}a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{4}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{a^2}{3x}+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{\tanh^{-1}(ax)^2}{3x^3}-\frac{a\tanh^{-1}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -1/3*a^2/x + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (a^2*ArcTanh[a*x]^2)/x - (4*a^3*Arc
Tanh[a*x]*Log[2 - 2/(1 + a*x)])/3 + (2*a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^4}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}+\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx-(2a^3)\int \frac{\text{ta}}{x(}$$
$$= -a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}+\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3}\,d$$
$$= -\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}-2a^3$$
$$= -\frac{a^2}{3x}-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}$$
$$= -\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}$$

Mathematica [A]
time = 0.21, size = 93, normalized size = 0.80

$$\frac{-a^2x^2-(-1+ax)^2(1+2ax)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(-ax+a^3x^3-4a^3x^3\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)+2a^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] (-(a^2*x^2) - (-1 + a*x)^2*(1 + 2*a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-(a*x
) + a^3*x^3 - 4*a^3*x^3*Log[1 - E^(-2*ArcTanh[a*x])]) + 2*a^3*x^3*PolyLog[2
, E^(-2*ArcTanh[a*x])])/(3*x^3)
```

Maple [A]
time = 0.15, size = 198, normalized size = 1.71

method	result
derivativedivides	$a^3\left(-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}+\frac{\operatorname{arctanh}(ax)^2}{ax}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax+1)}{3}-\frac{\operatorname{arctanh}(ax)}{3a^2x^2}-\frac{4\,\text{ar}}{}\right.$
default	$a^3\left(-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}+\frac{\operatorname{arctanh}(ax)^2}{ax}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax+1)}{3}-\frac{\operatorname{arctanh}(ax)}{3a^2x^2}-\frac{4\,\text{ar}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3*arctanh(a*x)^2/a^3/x^3+arctanh(a*x)^2/a/x+2/3*arctanh(a*x)*ln(a*x
-1)+2/3*arctanh(a*x)*ln(a*x+1)-1/3*arctanh(a*x)/a^2/x^2-4/3*arctanh(a*x)*ln
(a*x)-1/6*ln(a*x-1)-1/3/a/x+1/6*ln(a*x+1)+2/3*dilog(a*x)+2/3*dilog(a*x+1)+2
```

```
/3*ln(a*x)*ln(a*x+1)-2/3*dilog(1/2*a*x+1/2)-1/3*ln(a*x-1)*ln(1/2*a*x+1/2)+1
/6*ln(a*x-1)^2+1/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/6*ln(a*x+
1)^2)
```

Maxima [A]
time = 0.26, size = 188, normalized size = 1.62

$$-\frac{1}{6}\left(4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a-4\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a+4\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)a-a\log\left(ax+1\right)+a\log\left(ax-1\right)+\frac{ax\log\left(ax+1\right)^2-2\,ax\log\left(ax+1\right)\log\left(ax-1\right)-ax\log\left(ax-1\right)^2+2}{x}\right)a^2+\frac{1}{3}\left(2\,a^2\log\left(a^2x^2-1\right)-2\,a^2\log\left(x^2\right)-\frac{1}{x^2}\right)a\,\mathrm{artanh}\left(ax\right)+\frac{\left(3\,a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*
a - a*log(a*x + 1) + a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*x +
 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 + 2)/x)*a^2 + 1/3*(2*a^2*log(a^2*x^2
- 1) - 2*a^2*log(x^2) - 1/x^2)*a*arctanh(a*x) + 1/3*(3*a^2*x^2 - 1)*arctanh
(a*x)^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\mathrm{atanh}^2\left(ax\right)}{x^4}\right)dx-\int\frac{a^2\,\mathrm{atanh}^2\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**4,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**4, x) - Integral(a**2*atanh(a*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\mathrm{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^4,x)
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x^4, x)
```

3.181 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2}{12x^2}-\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)+\frac{1}{6}a^4\log(1-a^2x^2)$$

```
[Out] -1/12*a^2/x^2-1/6*a*arctanh(a*x)/x^3+1/2*a^3*arctanh(a*x)/x-1/4*(-a^2*x^2+1
)^2*arctanh(a*x)^2/x^4-1/3*a^4*ln(x)+1/6*a^4*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6155, 6161, 6037, 272, 46, 36, 29, 31}

$$-\frac{1}{3}a^4\log(x)+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{a^2}{12x^2}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^4\log(1-a^2x^2)-\frac{a\tanh^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] -1/12*a^2/x^2 - (a*ArcTanh[a*x])/(6*x^3) + (a^3*ArcTanh[a*x])/(2*x) - ((1 -
 a^2*x^2)^2*ArcTanh[a*x]^2)/(4*x^4) - (a^4*Log[x])/3 + (a^4*Log[1 - a^2*x^2
])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^5}\,dx = -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{2}a\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx$$
$$= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{2}a\int \frac{\tanh^{-1}(ax)}{x^4}\,dx - \frac{1}{2}a^3\int \frac{\tanh^{-1}(ax)}{x^2}\,dx$$
$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{a^3\tanh^{-1}(ax)}{2x} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{}{x^3(1}$$
$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{a^3\tanh^{-1}(ax)}{2x} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2\text{Subst}($$
$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{a^3\tanh^{-1}(ax)}{2x} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2\text{Subst}($$
$$= -\frac{a^2}{12x^2} - \frac{a\tanh^{-1}(ax)}{6x^3} + \frac{a^3\tanh^{-1}(ax)}{2x} - \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{4x^4} - \frac{1}{3}a^4$$

Mathematica [**A**]
time = 0.03, size = 82, normalized size = 0.92

$$\frac{-a^2x^2+(-2ax+6a^3x^3)\tanh^{-1}(ax)-3(-1+a^2x^2)^2\tanh^{-1}(ax)^2-4a^4x^4\log(x)+2a^4x^4\log(1-a^2x^2)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] (-(a^2*x^2) + (-2*a*x + 6*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^2 - 4*a^4*x^4*Log[x] + 2*a^4*x^4*Log[1 - a^2*x^2])/(12*x^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(77) = 154.
time = 0.14, size = 172, normalized size = 1.93

method	result
derivativedivides	$a^4\left(-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)}{6a^3x^3}+\frac{\text{arcta}}{2}\right.$
default	$a^4\left(-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)}{6a^3x^3}+\frac{\text{arcta}}{2}\right.$
risch	$-\frac{(a^4x^4-2a^2x^2+1)\ln(ax+1)^2}{16x^4}+\frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-2ax+3\ln(-ax+1))\ln(ax+1)}{24x^4}-\frac{3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*arctanh(a*x)^2/a^4/x^4+1/2*arctanh(a*x)^2/a^2/x^2+1/4*arctanh(a*x
)*ln(a*x-1)-1/4*arctanh(a*x)*ln(a*x+1)-1/6*arctanh(a*x)/a^3/x^3+1/2*arctanh
```

```
(a*x)/a/x-1/8*ln(a*x-1)*ln(1/2*a*x+1/2)+1/16*ln(a*x-1)^2-1/8*(ln(a*x+1)-ln(
1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/16*ln(a*x+1)^2+1/6*ln(a*x-1)+1/6*ln(a*x+1)
-1/12/a^2/x^2-1/3*ln(a*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(76) = 152$.
time = 0.27, size = 164, normalized size = 1.84

$$-\frac{1}{48}\left(16\,a^2\log\left(x\right)-\frac{3\,a^2x^2\log\left(ax+1\right)^2+3\,a^2x^2\log\left(ax-1\right)^2+8\,a^2x^2\log\left(ax-1\right)-2\left(3\,a^2x^2\log\left(ax-1\right)-4\,a^2x^2\right)\log\left(ax+1\right)-4}{x^2}\right)a^2-\frac{1}{12}\left(3\,a^3\log\left(ax+1\right)-3\,a^3\log\left(ax-1\right)-\frac{2\left(3\,a^2x^2-1\right)}{x^3}\right)a\,\mathrm{artanh}\left(ax\right)+\frac{\left(2\,a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/48*(16*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^2
 + 8*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 4*a^2*x^2)*log(a*x
+ 1) - 4)/x^2)*a^2 - 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a
^2*x^2 - 1)/x^3)*a*arctanh(a*x) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/x^4
```

Fricas [A]
time = 0.37, size = 108, normalized size = 1.21

$$\frac{8\,a^4x^4\log\left(a^2x^2-1\right)-16\,a^4x^4\log\left(x\right)-4\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^3x^3-ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(8*a^4*x^4*log(a^2*x^2 - 1) - 16*a^4*x^4*log(x) - 4*a^2*x^2 - 3*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 - a*x)*log(-
(a*x + 1)/(a*x - 1)))/x^4
```

Sympy [A]
time = 0.76, size = 102, normalized size = 1.15

$$\begin{cases}-\frac{a^4\log\left(x\right)}{3}+\frac{a^4\log\left(x-\frac{1}{a}\right)}{3}-\frac{a^4\,\mathrm{atanh}^2\left(ax\right)}{4}+\frac{a^4\,\mathrm{atanh}\left(ax\right)}{3}+\frac{a^3\,\mathrm{atanh}\left(ax\right)}{2x}+\frac{a^2\,\mathrm{atanh}^2\left(ax\right)}{2x^2}-\frac{a^2}{12x^2}-\frac{a\,\mathrm{atanh}\left(ax\right)}{6x^3}-\frac{\mathrm{atanh}^2\left(ax\right)}{4x^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**5,x)
```

```
[Out] Piecewise((-a**4*log(x)/3 + a**4*log(x - 1/a)/3 - a**4*atanh(a*x)**2/4 + a*
*4*atanh(a*x)/3 + a**3*atanh(a*x)/(2*x) + a**2*atanh(a*x)**2/(2*x**2) - a**
2/(12*x**2) - a*atanh(a*x)/(6*x**3) - atanh(a*x)**2/(4*x**4), Ne(a, 0)), (0
, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(76) = 152.

time = 0.42, size = 282, normalized size = 3.17

$$-\frac{1}{3}\left(a^{3}\log\left(-\frac{ax+1}{ax-1}-1\right)-a^{3}\log\left(-\frac{ax+1}{ax-1}\right)+\frac{3\,(ax+1)^{2}a^{3}\log\left(-\frac{ax+1}{ax-1}\right)^{2}}{(ax-1)^{2}\left(\frac{(ax+1)^{4}}{(ax-1)^{4}}+\frac{4\,(ax+1)^{3}}{(ax-1)^{3}}+\frac{6\,(ax+1)^{2}}{(ax-1)^{2}}+\frac{4\,(ax+1)}{ax-1}+1\right)}-\frac{(ax+1)a^{3}}{(ax-1)\left(\frac{(ax+1)^{2}}{(ax-1)^{2}}+\frac{2\,(ax+1)}{ax-1}+1\right)}+\frac{\left(\frac{3\,(ax+1)a^{3}}{ax-1}+a^{3}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^{3}}{(ax-1)^{3}}+\frac{3\,(ax+1)^{2}}{(ax-1)^{2}}+\frac{3\,(ax+1)}{ax-1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/3*(a^3*log(-(a*x + 1)/(a*x - 1) - 1) - a^3*log(-(a*x + 1)/(a*x - 1)) + 3
*(a*x + 1)^2*a^3*log(-(a*x + 1)/(a*x - 1))^2/((a*x - 1)^2*((a*x + 1)^4/(a*x
 - 1)^4 + 4*(a*x + 1)^3/(a*x - 1)^3 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x +
1)/(a*x - 1) + 1)) - (a*x + 1)*a^3/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2 + 2*
(a*x + 1)/(a*x - 1) + 1)) + (3*(a*x + 1)*a^3/(a*x - 1) + a^3)*log(-(a*x + 1
)/(a*x - 1))/((a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x
+ 1)/(a*x - 1) + 1))*a
```

Mupad [B]

time = 1.36, size = 246, normalized size = 2.76

$$\ln(1-a\,x)^2\left(\frac{\frac{a^2\,x^2}{2}-\frac{1}{4}}{4\,x^4}-\frac{a^4}{16}\right)-\ln(1-a\,x)\left(\ln(a\,x+1)\left(\frac{\frac{a^2\,x^2}{2}-\frac{1}{4}}{2\,x^4}-\frac{a^4}{8}\right)+\frac{3\,a^5\,x-2\,a^4}{24\,a^3\,x^3}-\frac{3\,x\,a^5+2\,a^4}{24\,a^3\,x^3}-\frac{a\,(22\,a^3\,x^3-12\,a^2\,x^2+6\,a\,x-4)}{96\,x^3}+\frac{a\,(44\,a^3\,x^3+24\,a^2\,x^2+12\,a\,x+8)}{192\,x^3}\right)-\frac{a^4\,\ln(x)}{3}+\ln(a\,x+1)^2\left(\frac{\frac{a^2\,x^2}{8}-\frac{1}{16}}{x^4}-\frac{a^4}{16}\right)+\frac{a^4\,\ln(a^2\,x^2-1)}{6}-\frac{a^2}{12\,x^2}+\frac{a\,\ln(a\,x+1)\left(\frac{a^2\,x^2}{4}-\frac{1}{12}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^5,x)
```

```
[Out] log(1 - a*x)^2*(((a^2*x^2)/2 - 1/4)/(4*x^4) - a^4/16) - log(1 - a*x)*(log(a
*x + 1)*(((a^2*x^2)/2 - 1/4)/(2*x^4) - a^4/8) + (3*a^5*x - 2*a^4)/(24*a^3*x
^3) - (3*a^5*x + 2*a^4)/(24*a^3*x^3) - (a*(6*a*x - 12*a^2*x^2 + 22*a^3*x^3
- 4))/(96*x^3) + (a*(12*a*x + 24*a^2*x^2 + 44*a^3*x^3 + 8))/(192*x^3)) - (a
^4*log(x))/3 + log(a*x + 1)^2*(((a^2*x^2)/8 - 1/16)/x^4 - a^4/16) + (a^4*lo
g(a^2*x^2 - 1))/6 - a^2/(12*x^2) + (a*log(a*x + 1)*((a^2*x^2)/4 - 1/12))/x^
3
```

3.182 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^6}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2}{30x^3}+\frac{a^4}{30x}-\frac{1}{30}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}$$

```
[Out] -1/30*a^2/x^3+1/30*a^4/x-1/30*a^5*arctanh(a*x)-1/10*a*arctanh(a*x)/x^4+2/15
*a^3*arctanh(a*x)/x^2-2/15*a^5*arctanh(a*x)^2-1/5*arctanh(a*x)^2/x^5+1/3*a^
2*arctanh(a*x)^2/x^3-4/15*a^5*arctanh(a*x)*ln(2-2/(a*x+1))+2/15*a^5*polylog
(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.32, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6161, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$\frac{2}{15}a^5\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{1}{30}a^5\tanh^{-1}(ax)-\frac{4}{15}a^5\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)+\frac{a^4}{30x}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{a^2}{30x^3}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}-\frac{\tanh^{-1}(ax)^2}{5x^5}-\frac{a\tanh^{-1}(ax)}{10x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] -1/30*a^2/x^3 + a^4/(30*x) - (a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x])/(10*x
^4) + (2*a^3*ArcTanh[a*x])/(15*x^2) - (2*a^5*ArcTanh[a*x]^2)/15 - ArcTanh[a
*x]^2/(5*x^5) + (a^2*ArcTanh[a*x]^2)/(3*x^3) - (4*a^5*ArcTanh[a*x]*Log[2 -
2/(1 + a*x)])/15 + (2*a^5*PolyLog[2, -1 + 2/(1 + a*x)])/15
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^6}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\right) + \int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{a^2\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5(1-a^2x^2)}\,dx - \frac{1}{3}(2a^3)\int$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{a^2\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5}\,dx + \frac{1}{5}(2a^3)\int$$

$$= -\frac{a\tanh^{-1}(ax)}{10x^4} + \frac{a^3\tanh^{-1}(ax)}{3x^2} - \frac{1}{3}a^5\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{5x^5} +$$

$$= -\frac{a^2}{30x^3} + \frac{a^4}{3x} - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{2a^3\tanh^{-1}(ax)}{15x^2} - \frac{2}{15}a^5\tanh^{-1}(ax)^2 -$$

$$= -\frac{a^2}{30x^3} + \frac{a^4}{30x} - \frac{1}{3}a^5\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{2a^3\tanh^{-1}(ax)}{15x^2} - \frac{2}{15}a^5$$

$$= -\frac{a^2}{30x^3} + \frac{a^4}{30x} - \frac{1}{30}a^5\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{2a^3\tanh^{-1}(ax)}{15x^2} - \frac{2}{15}a$$

Mathematica [A]
time = 0.34, size = 114, normalized size = 0.80

$$\frac{a^2x^2(-1+a^2x^2) - 2(3-5a^2x^2+2a^5x^5)\tanh^{-1}(ax)^2 - ax\tanh^{-1}(ax)\left(3-4a^2x^2+a^4x^4+8a^4x^4\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right) + 4a^5x^5\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] (a^2*x^2*(-1 + a^2*x^2) - 2*(3 - 5*a^2*x^2 + 2*a^5*x^5)*ArcTanh[a*x]^2 - a*
x*ArcTanh[a*x]*(3 - 4*a^2*x^2 + a^4*x^4 + 8*a^4*x^4*Log[1 - E^(-2*ArcTanh[a
*x])]) + 4*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*x^5)
```

Maple [A]
time = 0.14, size = 219, normalized size = 1.53

method	result
derivativedivides	$a^5\left(\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)^2}{5a^5x^5} - \frac{\operatorname{arctanh}(ax)}{10a^4x^4} + \frac{2\operatorname{arctanh}(ax)}{15a^2x^2} - \frac{4\operatorname{arctanh}(ax)\ln(ax)}{15} + \frac{2\operatorname{arctanh}(ax)\ln(a}{15}\right.$
default	$a^5\left(\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)^2}{5a^5x^5} - \frac{\operatorname{arctanh}(ax)}{10a^4x^4} + \frac{2\operatorname{arctanh}(ax)}{15a^2x^2} - \frac{4\operatorname{arctanh}(ax)\ln(ax)}{15} + \frac{2\operatorname{arctanh}(ax)\ln(a}{15}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(1/3*arctanh(a*x)^2/a^3/x^3-1/5*arctanh(a*x)^2/a^5/x^5-1/10*arctanh(a*x
)/a^4/x^4+2/15*arctanh(a*x)/a^2/x^2-4/15*arctanh(a*x)*ln(a*x)+2/15*arctanh(
a*x)*ln(a*x-1)+2/15*arctanh(a*x)*ln(a*x+1)-1/30/a^3/x^3+1/30/a/x+1/60*ln(a*
x-1)-1/60*ln(a*x+1)+2/15*dilog(a*x)+2/15*dilog(a*x+1)+2/15*ln(a*x)*ln(a*x+1
)+1/30*ln(a*x-1)^2-2/15*dilog(1/2*a*x+1/2)-1/15*ln(a*x-1)*ln(1/2*a*x+1/2)+1
/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/30*ln(a*x+1)^2)
```

Maxima [A]
time = 0.25, size = 228, normalized size = 1.59

$$-\frac{1}{60}\left(8\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^3-8\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a^3+8\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)a^3+a^3\log\left(ax+1\right)-a^3\log\left(ax-1\right)+\frac{2\left(a^3x^3\log\left(ax+1\right)^2-2a^3x^3\log\left(ax+1\right)\log\left(ax-1\right)-a^3x^3\log\left(ax-1\right)^2-a^2x^2+1\right)}{x^3}\right)a^2+\frac{1}{30}\left(4a^4\log\left(a^2x^2-1\right)-4a^4\log\left(x^2\right)+\frac{4a^2x^2-3}{x^4}\right)a\,\mathrm{artanh}\left(ax\right)+\frac{\left(5a^2x^2-3\right)\mathrm{artanh}\left(ax\right)^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 8*
(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 8*(log(-a*x + 1)*log(x) + dilog(a
*x))*a^3 + a^3*log(a*x + 1) - a^3*log(a*x - 1) + 2*(a^3*x^3*log(a*x + 1)^2
- 2*a^3*x^3*log(a*x + 1)*log(a*x - 1) - a^3*x^3*log(a*x - 1)^2 - a^2*x^2 +
1)/x^3)*a^2 + 1/30*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 -
3)/x^4)*a*arctanh(a*x) + 1/15*(5*a^2*x^2 - 3)*arctanh(a*x)^2/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\operatorname{atanh}^{2}\left(ax\right)}{x^{6}}\right)dx-\int\frac{a^{2}\operatorname{atanh}^{2}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**6,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**6, x) - Integral(a**2*atanh(a*x)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^6,x)
```

```
[Out] -int((atanh(a*x)^2*(a^2*x^2 - 1))/x^6, x)
```

3.183 $\int (1-a^2x^2)\tanh^{-1}(ax)^3\,dx$

Optimal. Leaf size=157

$$-x\tanh^{-1}(ax)+\frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a}+\frac{2\tanh^{-1}(ax)^3}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)^3-\frac{2\,\mathrm{t}}{}$$

```
[Out] -x*arctanh(a*x)+1/2*(-a^2*x^2+1)*arctanh(a*x)^2/a+2/3*arctanh(a*x)^3/a+2/3*
x*arctanh(a*x)^3+1/3*x*(-a^2*x^2+1)*arctanh(a*x)^3-2*arctanh(a*x)^2*ln(2/(-
a*x+1))/a-1/2*ln(-a^2*x^2+1)/a-2*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+pol
ylog(3,1-2/(-a*x+1))/a
```

Rubi [A]
time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6091, 6021, 6131, 6055, 6095, 6205, 6745, 266}

$$-\frac{\log(1-a^2x^2)}{2a}+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)^3+\frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a}+\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{a}-\frac{2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{2\tanh^{-1}(ax)^3}{3a}-x\tanh^{-1}(ax)-\frac{2\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

```
[Out] -(x*ArcTanh[a*x]) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(2*a) + (2*ArcTanh[a*x]^
3)/(3*a) + (2*x*ArcTanh[a*x]^3)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^3)/3 - (2
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - Log[1 - a^2*x^2]/(2*a) - (2*ArcTanh[a
*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/a
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1-a^2x^2)\tanh^{-1}(ax)^3\,dx = \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a} + \frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)^3 + \frac{2}{3}\int \tanh^{-1}(ax)^3\,d$$

$$= -x\tanh^{-1}(ax) + \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a} + \frac{2}{3}x\tanh^{-1}(ax)^3 + \frac{1}{3}x(1-a^2x$$

$$= -x\tanh^{-1}(ax) + \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}($$

$$= -x\tanh^{-1}(ax) + \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}($$

$$= -x\tanh^{-1}(ax) + \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}($$

$$= -x\tanh^{-1}(ax) + \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{2a} + \frac{2\tanh^{-1}(ax)^3}{3a} + \frac{2}{3}x\tanh^{-1}($$

Mathematica [**A**]
time = 0.21, size = 134, normalized size = 0.85

$$-\frac{6ax\tanh^{-1}(ax) - 3\tanh^{-1}(ax)^2 + 3a^2x^2\tanh^{-1}(ax)^2 + 4\tanh^{-1}(ax)^3 - 6ax\tanh^{-1}(ax)^3 + 2a^3x^3\tanh^{-1}(ax)^3 + 12\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + 3\log(1-a^2x^2) - 12\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) - 6\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

```
[Out] -1/6*(6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 + 4*
ArcTanh[a*x]^3 - 6*a*x*ArcTanh[a*x]^3 + 2*a^3*x^3*ArcTanh[a*x]^3 + 12*ArcTa
nh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + 3*Log[1 - a^2*x^2] - 12*ArcTanh[a*
x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6*PolyLog[3, -E^(-2*ArcTanh[a*x])])/a
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 24.34, size = 749, normalized size = 4.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3*arctanh(a*x)^3*a^3*x^3+arctanh(a*x)^3*a*x-1/2*a^2*x^2*arctanh(a*x
)^2+arctanh(a*x)^2*ln(a*x-1)+arctanh(a*x)^2*ln(a*x+1)-2*arctanh(a*x)^2*ln((
a*x+1)/(-a^2*x^2+1)^(1/2))-1/6*arctanh(a*x)*(3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1))^3*arctanh(a*x)+6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))*arctanh(a*x)-3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)-3*I*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)+3*I*Pi*csgn(I*(
a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)+3*I
```

```
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)
+3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)+6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+
1)+1))^3*arctanh(a*x)-6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a
*x)+6*I*Pi*arctanh(a*x)+12*ln(2)*arctanh(a*x)-4*arctanh(a*x)^2-3*arctanh(a*
x)+6*a*x+6)+ln((a*x+1)^2/(-a^2*x^2+1)+1)-2*arctanh(a*x)*polylog(2,-(a*x+1)^
2/(-a^2*x^2+1))+polylog(3,-(a*x+1)^2/(-a^2*x^2+1)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*a^3*x^3 - 3*a^2*x^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*x + 1)
)*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x
 + 1) - 6)*(a*x - 1)/a + 1/864*(4*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2 +
6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^
2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 - 3*log(-a*x +
1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/3*(3*(a^3*x^3 -
 a^2*x^2 - a*x + 1)*log(a*x + 1)^3 + (2*a^3*x^3 - 3*a^2*x^2 - 9*(a^3*x^3 -
a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*
x + 1))*log(-a*x + 1))/(a*x - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int a^{2}x^{2}\operatorname{atanh}^{3}(ax)\,dx-\int\left(-\operatorname{atanh}^{3}(ax)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**3,x)
```

```
[Out] -Integral(a**2*x**2*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3*(a^2*x^2 - 1),x)
```

```
[Out] -int(atanh(a*x)^3*(a^2*x^2 - 1), x)
```

3.184 $\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx$

Optimal. Leaf size=193

$$\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log$$

```
[Out] -1/2*arctanh(1/2*x*2^(1/2))*ln(-4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))+arctanh(1/
2*x*2^(1/2))*ln(2*2^(1/2)/(x+2^(1/2)))-1/2*arctanh(1/2*x*2^(1/2))*ln(4*(1+x
)/(2+2^(1/2))/(x+2^(1/2)))+1/4*polylog(2,1+4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))
-1/2*polylog(2,1-2*2^(1/2)/(x+2^(1/2)))+1/4*polylog(2,1-4*(1+x)/(2+2^(1/2))
/(x+2^(1/2)))
```

Rubi [A]
time = 0.17, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6139, 6057, 2449, 2352, 2497}

$$-\frac{1}{2}\text{Li}_2\left(1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{4}\text{Li}_2\left(\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)+\frac{1}{4}\text{Li}_2\left(1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)+\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

```
[Out] ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Log
[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/2 - (ArcTanh[x/Sqrt[2]]*Log[(
4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/2 - PolyLog[2, 1 - (2*Sqrt[2])/(
Sqrt[2] + x)]/2 + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]
/4 + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/4
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = \int \left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx\right)-\frac{1}{2}\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+\right.}\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+\right.}\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+\right.}\right.$$

Mathematica [A]
time = 0.19, size = 232, normalized size = 1.20

$$\frac{1}{4}\left(-4\sinh^{-1}(1)\tanh^{-1}(x)+4\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1+e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)+2\sinh^{-1}(1)\log\left(1+\left(-3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1+\left(-3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)-2\sinh^{-1}(1)\log\left(1-\left(3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-\left(3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)-2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)+\text{PolyLog}\left(2,\left(3-2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)+\text{PolyLog}\left(2,\left(3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

```
[Out] (-4*ArcSinh[1]*ArcTanh[x] + 4*ArcTanh[x/Sqrt[2]]*Log[1 + E^(-2*ArcTanh[x/Sq
rt[2]])] + 2*ArcSinh[1]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])]
- 2*ArcTanh[x/Sqrt[2]]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] -
 2*ArcSinh[1]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*ArcTanh
[x/Sqrt[2]]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*PolyLog[2
, -E^(-2*ArcTanh[x/Sqrt[2]])] + PolyLog[2, (3 - 2*Sqrt[2])/E^(2*ArcTanh[x/S
qrt[2]])] + PolyLog[2, (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])])/4
```

Maple [A]
time = 0.41, size = 251, normalized size = 1.30

method	result
derivativedivides	$-\frac{\ln(x^2-1)\,\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)}{2}-\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln(x^2-1)}{4}+\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{-2+\sqrt{2}}\right)}{4}+$
default	$-\frac{\ln(x^2-1)\,\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)}{2}-\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln(x^2-1)}{4}+\frac{\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{-2+\sqrt{2}}\right)}{4}+$
risch	$\frac{\ln\left(\frac{\sqrt{2}+x\sqrt{2}}{2+\sqrt{2}}\right)\ln\left(1-\frac{x\sqrt{2}}{2}\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}+x\sqrt{2}}{2+\sqrt{2}}\right)\ln\left(\frac{2-x\sqrt{2}}{2+\sqrt{2}}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{2-x\sqrt{2}}{2+\sqrt{2}}\right)}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x^2-1)*arctanh(1/2*x*2^(1/2))-1/4*ln(1/2*x*2^(1/2)-1)*ln(x^2-1)+1/4
*ln(1/2*x*2^(1/2)-1)*ln((2^(1/2)-x*2^(1/2))/(-2+2^(1/2)))+1/4*ln(1/2*x*2^(1
/2)-1)*ln((2^(1/2)+x*2^(1/2))/(2+2^(1/2)))+1/4*dilog((2^(1/2)-x*2^(1/2))/(-
2+2^(1/2)))+1/4*dilog((2^(1/2)+x*2^(1/2))/(2+2^(1/2)))+1/4*ln(1/2*x*2^(1/2)
+1)*ln(x^2-1)-1/4*ln(1/2*x*2^(1/2)+1)*ln((2^(1/2)-x*2^(1/2))/(2+2^(1/2)))-1
/4*ln(1/2*x*2^(1/2)+1)*ln((2^(1/2)+x*2^(1/2))/(-2+2^(1/2)))-1/4*dilog((2^(1
/2)-x*2^(1/2))/(2+2^(1/2)))-1/4*dilog((2^(1/2)+x*2^(1/2))/(-2+2^(1/2)))
```

Maxima [A]
time = 0.47, size = 277, normalized size = 1.44

$$-\frac{1}{2}\,\operatorname{artanh}\left(\frac{1}{2}\,\sqrt{2}\,x\right)\log\left(x^2-1\right)-\frac{1}{4}\,\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{8}\,\sqrt{2}\left(\sqrt{2}\,\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\sqrt{2}\left(\left(\log\left(2\,x+2\,\sqrt{2}\right)-\log\left(2\,x-2\,\sqrt{2}\right)\right)\log\left(x^2-1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\log\left(x-\sqrt{2}\right)\log\left(\frac{x-\sqrt{2}}{\sqrt{2}+1}+1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}-1}+1\right)+\log\left(x-\sqrt{2}\right)\log\left(\frac{x-\sqrt{2}}{\sqrt{2}-1}+1\right)-\mathrm{Li}_2\left(\frac{x+\sqrt{2}}{\sqrt{2}+1}\right)+\mathrm{Li}_2\left(-\frac{x-\sqrt{2}}{\sqrt{2}+1}\right)-\mathrm{Li}_2\left(\frac{x+\sqrt{2}}{\sqrt{2}-1}\right)+\mathrm{Li}_2\left(-\frac{x-\sqrt{2}}{\sqrt{2}-1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*arctanh(1/2*sqrt(2)*x)*log(x^2 - 1) - 1/4*log(x^2 - 1)*log((x - sqrt(2
))/(x + sqrt(2))) + 1/8*sqrt(2)*(sqrt(2)*log(x^2 - 1)*log((x - sqrt(2))/(x
+ sqrt(2))) + sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(x^
2 - 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - s
qrt(2))*log((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + s
qrt(2))/(sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) -
1) + 1) - dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2
) + 1)) - dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2
) - 1))))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \operatorname{atanh}\left(\frac{\sqrt{2}\, x}{2}\right)}{x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1/2*x*2**(1/2))/(-x**2+1),x)
```

```
[Out] -Integral(x*atanh(sqrt(2)*x/2)/(x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x \operatorname{atanh}\left(\frac{\sqrt{2}\, x}{2}\right)}{x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh((2^(1/2)*x)/2))/(x^2 - 1),x)
[Out] -int((x*atanh((2^(1/2)*x)/2))/(x^2 - 1), x)
```

3.185 $\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x(1-a^2x^2)}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
[Out] Defer[Int][(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)}\,dx = \int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
[Out] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Maple [A]
time = 23.29, size = 0, normalized size = 0.00

$$\int \frac{x(-a^2x^2+1)}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)/arctanh(a*x),x)
[Out] int(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*x/arctanh(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*x^3 - x)/arctanh(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x}{\operatorname{atanh}(ax)}\right)dx-\int\frac{a^2x^3}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)/atanh(a*x),x)
[Out] -Integral(-x/atanh(a*x), x) - Integral(a**2*x**3/atanh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*x/arctanh(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$-\int\frac{x\left(a^2\,x^2-1\right)}{\operatorname{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a^2*x^2 - 1))/atanh(a*x),x)
[Out] -int((x*(a^2*x^2 - 1))/atanh(a*x), x)
```

3.186 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x],x]
```

```
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Maple [A]
time = 32.66, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x),x)
[Out] int((-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)/arctanh(a*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2}{\operatorname{atanh}(ax)}\,dx - \int \left(-\frac{1}{\operatorname{atanh}(ax)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x),x)
[Out] -Integral(a**2*x**2/atanh(a*x), x) - Integral(-1/atanh(a*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{a^2\,x^2-1}{\operatorname{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/atanh(a*x),x)
[Out] -int((a^2*x^2 - 1)/atanh(a*x), x)
```

3.187 $\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1-a^2x^2}{x\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/x/arctanh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
[Out] Defer[Int][(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx = \int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
[Out] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Maple [A]
time = 29.81, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{x\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/x/arctanh(a*x),x)
[Out] int((-a^2*x^2+1)/x/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{1}{x\operatorname{atanh}(ax)}\right)dx-\int\frac{a^2x}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/x/atanh(a*x),x)
[Out] -Integral(-1/(x*atanh(a*x)), x) - Integral(a**2*x/atanh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$-\int\frac{a^2\,x^2-1}{x\,\operatorname{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x*atanh(a*x)),x)
[Out] -int((a^2*x^2 - 1)/(x*atanh(a*x)), x)
```

3.188 $\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} dx = \int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Maple [A]
time = 31.84, size = 0, normalized size = 0.00

$$\int \frac{x(-a^2x^2+1)}{\operatorname{arctanh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^5 - 2*a^2*x^3 + x)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrate
(-2*(5*a^4*x^4 - 6*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^3 - x)/arctanh(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x}{\operatorname{atanh}^{2}(ax)}\right)dx-\int\frac{a^{2}x^{3}}{\operatorname{atanh}^{2}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(-x/atanh(a*x)**2, x) - Integral(a**2*x**3/atanh(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x/arctanh(a*x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{x\,\left(a^2\,x^2-1\right)}{{\mathrm{atanh}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a^2*x^2 - 1))/atanh(a*x)^2,x)
[Out] -int((x*(a^2*x^2 - 1))/atanh(a*x)^2, x)
```

3.189 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Maple [A]
time = 34.07, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrate
(-8*(a^3*x^3 - a*x)/(log(a*x + 1) - log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2}{\operatorname{atanh}^2(ax)}\,dx - \int \left(-\frac{1}{\operatorname{atanh}^2(ax)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x)**2, x) - Integral(-1/atanh(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{a^2\,x^2-1}{\mathrm{atanh}\left(a\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/atanh(a*x)^2,x)
```

```
[Out] -int((a^2*x^2 - 1)/atanh(a*x)^2, x)
```

3.190 $\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1-a^2x^2}{x\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
[Out] Defer[Int][(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx = \int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
[Out] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 31.36, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{x\,\mathrm{arctanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) - integ
rate(-2*(3*a^4*x^4 - 2*a^2*x^2 - 1)/(a*x^2*log(a*x + 1) - a*x^2*log(-a*x +
1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{1}{x\operatorname{atanh}^{2}(ax)}\right)dx-\int\frac{a^{2}x}{\operatorname{atanh}^{2}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/x/atanh(a*x)**2,x)
```

```
[Out] -Integral(-1/(x*atanh(a*x)**2), x) - Integral(a**2*x/atanh(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{a^2\,x^2-1}{x\,\mathrm{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x*atanh(a*x)^2),x)
[Out] -int((a^2*x^2 - 1)/(x*atanh(a*x)^2), x)
```

3.191 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3}\,dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
```

```
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Maple [A]
time = 25.57, size = 0, normalized size = 0.00

$$\int \frac{-a^2x^2+1}{\operatorname{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 - 2*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) + 2*(a
^5*x^5 - 2*a^3*x^3 + a*x)*log(-a*x + 1) + 1)/(a*log(a*x + 1)^2 - 2*a*log(a*
x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2) + integrate(-4*(5*a^4*x^4 - 6*a^2
*x^2 + 1)/(log(a*x + 1) - log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^{2}x^{2}}{\operatorname{atanh}^{3}(ax)}\,dx-\int \left(-\frac{1}{\operatorname{atanh}^{3}(ax)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x)**3, x) - Integral(-1/atanh(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$-\int \frac{a^2\,x^2-1}{\mathrm{atanh}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/atanh(a*x)^3,x)
```

```
[Out] -int((a^2*x^2 - 1)/atanh(a*x)^3, x)
```

3.192 $\int x^4(1-a^2x^2)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=96

$$\frac{4x^2}{315a^3}+\frac{2x^4}{315a}-\frac{11ax^6}{378}+\frac{a^3x^8}{72}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{1}{9}a^4x^9\tanh^{-1}(ax)+\frac{4\log(1-a^2x^2)}{315a^5}$$

```
[Out] 4/315*x^2/a^3+2/315*x^4/a-11/378*a*x^6+1/72*a^3*x^8+1/5*x^5*arctanh(a*x)-2/
7*a^2*x^7*arctanh(a*x)+1/9*a^4*x^9*arctanh(a*x)+4/315*ln(-a^2*x^2+1)/a^5
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6159, 6037, 272, 45}

$$\frac{1}{9}a^4x^9\tanh^{-1}(ax)+\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{4\log(1-a^2x^2)}{315a^5}-\frac{11ax^6}{378}+\frac{1}{5}x^5\tanh^{-1}(ax)+\frac{2x^4}{315a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^4(1-a^2x^2)^2\tanh^{-1}(ax)\,dx = \int \left(x^4\tanh^{-1}(ax)-2a^2x^6\tanh^{-1}(ax)+a^4x^8\tanh^{-1}(ax)\right)\,dx$$
$$= -\left((2a^2)\int x^6\tanh^{-1}(ax)\,dx\right)+a^4\int x^8\tanh^{-1}(ax)\,dx+\int x^4\tanh^{-1}($$
$$= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{1}{9}a^4x^9\tanh^{-1}(ax)-\frac{1}{5}a\int\frac{x^5}{1-a^2}$$
$$= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{1}{9}a^4x^9\tanh^{-1}(ax)-\frac{1}{10}a\text{Subst}\Bigg($$
$$= \frac{1}{5}x^5\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{1}{9}a^4x^9\tanh^{-1}(ax)-\frac{1}{10}a\text{Subst}\Bigg($$
$$= \frac{4x^2}{315a^3}+\frac{2x^4}{315a}-\frac{11ax^6}{378}+\frac{a^3x^8}{72}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)-$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 1.00

$$\frac{4x^2}{315a^3}+\frac{2x^4}{315a}-\frac{11ax^6}{378}+\frac{a^3x^8}{72}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)+\frac{1}{9}a^4x^9\tanh^{-1}(ax)+\frac{4\log(1-a^2x^2)}{315a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

Maple [A]
time = 0.56, size = 90, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^9x^9}{9}-\frac{2\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^8x^8}{72}-\frac{11a^6x^6}{378}+\frac{2a^4x^4}{315}+\frac{4a^2x^2}{315}+\frac{4\ln(ax-1)}{315}+\frac{4\ln(ax+1)}{315}}{a^5}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^9x^9}{9}-\frac{2\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^8x^8}{72}-\frac{11a^6x^6}{378}+\frac{2a^4x^4}{315}+\frac{4a^2x^2}{315}+\frac{4\ln(ax-1)}{315}+\frac{4\ln(ax+1)}{315}}{a^5}$

risch	$\left(\frac{1}{18}a^4x^9 - \frac{1}{7}a^2x^7 + \frac{1}{10}x^5\right)\ln(ax+1) - \frac{a^4x^9\ln(-ax+1)}{18} + \frac{a^3x^8}{72} + \frac{a^2x^7\ln(-ax+1)}{7} - \frac{11x^6a}{378} - \frac{\ln(-$
meijerg	$-\frac{-\frac{x^2a^2(15a^6x^6+20a^4x^4+30a^2x^2+60)}{270} + \frac{2x^{10}a^{10}\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{9\sqrt{a^2x^2}} - \frac{2\ln(-a^2x^2+1)}{9}}{4a^5} - \frac{a^2x^2(4a^4x^4}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/9*arctanh(a*x)*a^9*x^9-2/7*arctanh(a*x)*a^7*x^7+1/5*arctanh(a*x)*a
^5*x^5+1/72*a^8*x^8-11/378*a^6*x^6+2/315*a^4*x^4+4/315*a^2*x^2+4/315*ln(a*x
-1)+4/315*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 89, normalized size = 0.93

$$\frac{1}{7560}a\left(\frac{105\,a^6x^8 - 220\,a^4x^6 + 48\,a^2x^4 + 96\,x^2}{a^4} + \frac{96\,\log(ax+1)}{a^6} + \frac{96\,\log(ax-1)}{a^6}\right) + \frac{1}{315}\left(35\,a^4x^9 - 90\,a^2x^7 + 63\,x^5\right)\text{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x^4 + 96*x^2)/a^4 + 96*log(a*
x + 1)/a^6 + 96*log(a*x - 1)/a^6) + 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5
)*arctanh(a*x)
```

Fricas [A]
time = 0.46, size = 92, normalized size = 0.96

$$\frac{105\,a^8x^8 - 220\,a^6x^6 + 48\,a^4x^4 + 96\,a^2x^2 + 12\left(35\,a^9x^9 - 90\,a^7x^7 + 63\,a^5x^5\right)\log\left(-\frac{ax+1}{ax-1}\right) + 96\,\log\left(a^2x^2-1\right)}{7560\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*x^8 - 220*a^6*x^6 + 48*a^4*x^4 + 96*a^2*x^2 + 12*(35*a^9*x^
9 - 90*a^7*x^7 + 63*a^5*x^5)*log(-(a*x + 1)/(a*x - 1)) + 96*log(a^2*x^2 - 1
))/a^5
```

Sympy [A]
time = 0.63, size = 100, normalized size = 1.04

$$\begin{cases} \frac{a^4x^9\,\text{atanh}\,(ax)}{9} + \frac{a^3x^8}{72} - \frac{2a^2x^7\,\text{atanh}\,(ax)}{7} - \frac{11ax^6}{378} + \frac{x^5\,\text{atanh}\,(ax)}{5} + \frac{2x^4}{315a} + \frac{4x^2}{315a^3} + \frac{8\log\left(x-\frac{1}{a}\right)}{315a^5} + \frac{8\,\text{atanh}\,(ax)}{315a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**9*atanh(a*x)/9 + a**3*x**8/72 - 2*a**2*x**7*atanh(a*x)/7
 - 11*a*x**6/378 + x**5*atanh(a*x)/5 + 2*x**4/(315*a) + 4*x**2/(315*a**3) +
 8*log(x - 1/a)/(315*a**5) + 8*atanh(a*x)/(315*a**5), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(80) = 160.
time = 0.40, size = 383, normalized size = 3.99

$$\frac{4}{945}\,a\left(\frac{6\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6}-\frac{6\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6}-\frac{\frac{6\,(ax+1)^7}{(ax-1)^7}-\frac{45\,(ax+1)^6}{(ax-1)^6}-\frac{274\,(ax+1)^5}{(ax-1)^5}-\frac{214\,(ax+1)^4}{(ax-1)^4}-\frac{274\,(ax+1)^3}{(ax-1)^3}-\frac{45\,(ax+1)^2}{(ax-1)^2}+\frac{6\,(ax+1)}{ax-1}}{a^6\left(\frac{ax+1}{ax-1}-1\right)^8}+\frac{6\left(\frac{210\,(ax+1)^6}{(ax-1)^6}+\frac{315\,(ax+1)^5}{(ax-1)^5}+\frac{441\,(ax+1)^4}{(ax-1)^4}+\frac{126\,(ax+1)^3}{(ax-1)^3}+\frac{36\,(ax+1)^2}{(ax-1)^2}-\frac{9\,(ax+1)}{ax-1}+1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/945*a*(6*log(abs(-a*x - 1)/abs(a*x - 1))/a^6 - 6*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^6 - (6*(a*x + 1)^7/(a*x - 1)^7 - 45*(a*x + 1)^6/(a*x - 1)^6 -
274*(a*x + 1)^5/(a*x - 1)^5 - 214*(a*x + 1)^4/(a*x - 1)^4 - 274*(a*x + 1)^3
/(a*x - 1)^3 - 45*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1))/(a^6*((a
*x + 1)/(a*x - 1) - 1)^8) + 6*(210*(a*x + 1)^6/(a*x - 1)^6 + 315*(a*x + 1)^
5/(a*x - 1)^5 + 441*(a*x + 1)^4/(a*x - 1)^4 + 126*(a*x + 1)^3/(a*x - 1)^3 +
 36*(a*x + 1)^2/(a*x - 1)^2 - 9*(a*x + 1)/(a*x - 1) + 1)*log(-(a*((a*x + 1)
/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) +
1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^9))
```

Mupad [B]
time = 1.01, size = 106, normalized size = 1.10

$$\frac{4\,\ln\left(a^2\,x^2-1\right)}{315\,a^5}-\frac{11\,a\,x^6}{378}+\ln\left(a\,x+1\right)\,\left(\frac{a^4\,x^9}{18}-\frac{a^2\,x^7}{7}+\frac{x^5}{10}\right)-\ln\left(1-a\,x\right)\,\left(\frac{a^4\,x^9}{18}-\frac{a^2\,x^7}{7}+\frac{x^5}{10}\right)+\frac{2\,x^4}{315\,a}+\frac{4\,x^2}{315\,a^3}+\frac{a^3\,x^8}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] (4*log(a^2*x^2 - 1))/(315*a^5) - (11*a*x^6)/378 + log(a*x + 1)*(x^5/10 - (a
^2*x^7)/7 + (a^4*x^9)/18) - log(1 - a*x)*(x^5/10 - (a^2*x^7)/7 + (a^4*x^9)/
18) + (2*x^4)/(315*a) + (4*x^2)/(315*a^3) + (a^3*x^8)/72
```

3.193 $\int x^3(1-a^2x^2)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=87

$$\frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}-\frac{\tanh^{-1}(ax)}{24a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\tanh^{-1}(ax)$$

```
[Out] 1/24*x/a^3+1/72*x^3/a-1/24*a*x^5+1/56*a^3*x^7-1/24*arctanh(a*x)/a^4+1/4*x^4
*arctanh(a*x)-1/3*a^2*x^6*arctanh(a*x)+1/8*a^4*x^8*arctanh(a*x)
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6159, 6037, 308, 212}

$$\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{24a^4}+\frac{a^3x^7}{56}+\frac{x}{24a^3}-\frac{1}{3}a^2x^6\tanh^{-1}(ax)-\frac{ax^5}{24}+\frac{1}{4}x^4\tanh^{-1}(ax)+\frac{x^3}{72a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 - ArcTanh[a*x]/(24*a^4)
 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])
/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^3(1-a^2x^2)^2\tanh^{-1}(ax)\,dx = \int \left(x^3\tanh^{-1}(ax)-2a^2x^5\tanh^{-1}(ax)+a^4x^7\tanh^{-1}(ax)\right)\,dx$$
$$= -\left((2a^2)\int x^5\tanh^{-1}(ax)\,dx\right)+a^4\int x^7\tanh^{-1}(ax)\,dx+\int x^3\tanh^{-1}($$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{1-a^2}$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{4}a\int\left(-\frac{1}{a^4}\right.$$
$$= \frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a$$
$$= \frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}-\frac{\tanh^{-1}(ax)}{24a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tan$$

Mathematica [A]
time = 0.02, size = 103, normalized size = 1.18

$$\frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{1}{8}a^4x^8\tanh^{-1}(ax)+\frac{\log(1-ax)}{48a^4}-\frac{\log(1+ax)}{48a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 + (x^4*ArcTanh[a*x])/4
- (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])/8 + Log[1 - a*x]/(48*a^
4) - Log[1 + a*x]/(48*a^4)
```

Maple [A]
time = 0.90, size = 76, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^8x^8}{8}-\frac{\operatorname{arctanh}(ax)a^6x^6}{3}+\frac{a^4x^4\operatorname{arctanh}(ax)}{4}-\frac{\operatorname{arctanh}(ax)}{24}+\frac{a^7x^7}{56}-\frac{a^5x^5}{24}+\frac{a^3x^3}{72}+\frac{ax}{24}}{a^4}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^8x^8}{8}-\frac{\operatorname{arctanh}(ax)a^6x^6}{3}+\frac{a^4x^4\operatorname{arctanh}(ax)}{4}-\frac{\operatorname{arctanh}(ax)}{24}+\frac{a^7x^7}{56}-\frac{a^5x^5}{24}+\frac{a^3x^3}{72}+\frac{ax}{24}}{a^4}$
risch	$\left(\frac{1}{16}a^4x^8-\frac{1}{6}x^6a^2+\frac{1}{8}x^4\right)\ln(ax+1)-\frac{a^4x^8\ln(-ax+1)}{16}+\frac{a^3x^7}{56}+\frac{a^2x^6\ln(-ax+1)}{6}-\frac{a\,x^5}{24}-\frac{x^4\ln($

meijerg	$-\frac{i\left(\frac{ixa(45a^6x^6+63a^4x^4+105a^2x^2+315)}{630}+\frac{ixa(-9a^8x^8+9)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{36\sqrt{a^2x^2}}\right)}{4a^4} - \frac{i\left(-\frac{2ixa(21a^4x^4}{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)

[Out] 1/a^4*(1/8*arctanh(a*x)*a^8*x^8-1/3*arctanh(a*x)*a^6*x^6+1/4*a^4*x^4*arctanh(a*x)-1/24*arctanh(a*x)+1/56*a^7*x^7-1/24*a^5*x^5+1/72*a^3*x^3+1/24*a*x)

Maxima [A]
time = 0.26, size = 88, normalized size = 1.01

$$\frac{1}{1008}\,a\left(\frac{2\,(9\,a^6x^7-21\,a^4x^5+7\,a^2x^3+21\,x)}{a^4}-\frac{21\,\log{(ax+1)}}{a^5}+\frac{21\,\log{(ax-1)}}{a^5}\right)+\frac{1}{24}\,(3\,a^4x^8-8\,a^2x^6+6\,x^4)\,\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")

[Out] 1/1008*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x + 1)/a^5 + 21*log(a*x - 1)/a^5) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6*x^4)*arctanh(a*x)

Fricas [A]
time = 0.35, size = 77, normalized size = 0.89

$$\frac{18\,a^7x^7-42\,a^5x^5+14\,a^3x^3+42\,ax+21\,(3\,a^8x^8-8\,a^6x^6+6\,a^4x^4-1)\log\left(-\frac{ax+1}{ax-1}\right)}{1008\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")

[Out] 1/1008*(18*a^7*x^7 - 42*a^5*x^5 + 14*a^3*x^3 + 42*a*x + 21*(3*a^8*x^8 - 8*a^6*x^6 + 6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^4

Sympy [A]
time = 0.53, size = 76, normalized size = 0.87

$$\begin{cases} \frac{a^4x^8\,\mathrm{atanh}\,(ax)}{8}+\frac{a^3x^7}{56}-\frac{a^2x^6\,\mathrm{atanh}\,(ax)}{3}-\frac{ax^5}{24}+\frac{x^4\,\mathrm{atanh}\,(ax)}{4}+\frac{x^3}{72a}+\frac{x}{24a^3}-\frac{\mathrm{atanh}\,(ax)}{24a^4} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x),x)

```
[Out] Piecewise((a**4*x**8*atanh(a*x)/8 + a**3*x**7/56 - a**2*x**6*atanh(a*x)/3 -
 a*x**5/24 + x**4*atanh(a*x)/4 + x**3/(72*a) + x/(24*a**3) - atanh(a*x)/(24
*a**4), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(71) = 142.
time = 0.41, size = 240, normalized size = 2.76

$$\frac{4}{63}\,a\left(\frac{\frac{28\,(ax+1)^4}{(ax-1)^4}-\frac{7\,(ax+1)^3}{(ax-1)^3}+\frac{21\,(ax+1)^2}{(ax-1)^2}-\frac{7\,(ax+1)}{ax-1}+1}{a^5\left(\frac{ax+1}{ax-1}-1\right)^7}+\frac{84\left(\frac{(ax+1)^5}{(ax-1)^5}+\frac{(ax+1)^4}{(ax-1)^4}+\frac{(ax+1)^3}{(ax-1)^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^5\left(\frac{ax+1}{ax-1}-1\right)^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/63*a*((28*(a*x + 1)^4/(a*x - 1)^4 - 7*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x +
 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x - 1) + 1)/(a^5*((a*x + 1)/(a*x - 1) -
1)^7) + 84*((a*x + 1)^5/(a*x - 1)^5 + (a*x + 1)^4/(a*x - 1)^4 + (a*x + 1)^3
/(a*x - 1)^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a)
 + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^5*(
(a*x + 1)/(a*x - 1) - 1)^8))
```

Mupad [B]
time = 1.21, size = 101, normalized size = 1.16

$$\frac{x}{24\,a^3}-\frac{a\,x^5}{24}+\ln\left(a\,x+1\right)\,\left(\frac{a^4\,x^8}{16}-\frac{a^2\,x^6}{6}+\frac{x^4}{8}\right)-\ln\left(1-a\,x\right)\,\left(\frac{a^4\,x^8}{16}-\frac{a^2\,x^6}{6}+\frac{x^4}{8}\right)+\frac{x^3}{72\,a}+\frac{a^3\,x^7}{56}+\frac{\mathrm{atan}(a\,x\,1\mathrm{i})\;1\mathrm{i}}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] x/(24*a^3) - (a*x^5)/24 + (atan(a*x*1i)*1i)/(24*a^4) + log(a*x + 1)*(x^4/8
- (a^2*x^6)/6 + (a^4*x^8)/16) - log(1 - a*x)*(x^4/8 - (a^2*x^6)/6 + (a^4*x^
8)/16) + x^3/(72*a) + (a^3*x^7)/56
```

3.194 $\int x^2(1-a^2x^2)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=86

$$\frac{4x^2}{105a}-\frac{9ax^4}{140}+\frac{a^3x^6}{42}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{1}{7}a^4x^7\tanh^{-1}(ax)+\frac{4\log(1-a^2x^2)}{105a^3}$$

```
[Out] 4/105*x^2/a-9/140*a*x^4+1/42*a^3*x^6+1/3*x^3*arctanh(a*x)-2/5*a^2*x^5*arcta
nh(a*x)+1/7*a^4*x^7*arctanh(a*x)+4/105*ln(-a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6159, 6037, 272, 45}

$$\frac{1}{7}a^4x^7\tanh^{-1}(ax)+\frac{a^3x^6}{42}-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{4\log(1-a^2x^2)}{105a^3}-\frac{9ax^4}{140}+\frac{1}{3}x^3\tanh^{-1}(ax)+\frac{4x^2}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int x^2(1-a^2x^2)^2\tanh^{-1}(ax)\,dx &= \int \left(x^2\tanh^{-1}(ax)-2a^2x^4\tanh^{-1}(ax)+a^4x^6\tanh^{-1}(ax)\right)\,dx\\ &= -\left((2a^2)\int x^4\tanh^{-1}(ax)\,dx\right)+a^4\int x^6\tanh^{-1}(ax)\,dx+\int x^2\tanh^{-1}(\\ &= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{1}{7}a^4x^7\tanh^{-1}(ax)-\frac{1}{3}a\int \frac{x^3}{1-a^2}\\ &= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{1}{7}a^4x^7\tanh^{-1}(ax)-\frac{1}{6}a\text{Subst}\Big(\int\\ &= \frac{1}{3}x^3\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{1}{7}a^4x^7\tanh^{-1}(ax)-\frac{1}{6}a\text{Subst}\Big(\int\\ &= \frac{4x^2}{105a}-\frac{9ax^4}{140}+\frac{a^3x^6}{42}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{1}{7}a^4x^7\text{ta}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 86, normalized size = 1.00

$$\frac{4x^2}{105a}-\frac{9ax^4}{140}+\frac{a^3x^6}{42}+\frac{1}{3}x^3\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)+\frac{1}{7}a^4x^7\tanh^{-1}(ax)+\frac{4\log(1-a^2x^2)}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

Maple [A]
time = 0.40, size = 82, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^7x^7}{7}-\frac{2\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^3x^3\operatorname{arctanh}(ax)}{3}+\frac{a^6x^6}{42}-\frac{9a^4x^4}{140}+\frac{4a^2x^2}{105}+\frac{4\ln(ax-1)}{105}+\frac{4\ln(ax+1)}{105}}{a^3}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^7x^7}{7}-\frac{2\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^3x^3\operatorname{arctanh}(ax)}{3}+\frac{a^6x^6}{42}-\frac{9a^4x^4}{140}+\frac{4a^2x^2}{105}+\frac{4\ln(ax-1)}{105}+\frac{4\ln(ax+1)}{105}}{a^3}$

risch	$\left(\frac{1}{14}a^4x^7 - \frac{1}{5}a^2x^5 + \frac{1}{6}x^3\right)\ln(ax+1) - \frac{a^4x^7\ln(-ax+1)}{14} + \frac{a^3x^6}{42} + \frac{a^2x^5\ln(-ax+1)}{5} - \frac{9x^4a}{140} - \ln(-a$
meijerg	$\frac{\frac{a^2x^2(4a^4x^4+6a^2x^2+12)}{42} - \frac{2a^8x^8\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{7\sqrt{a^2x^2}} + \frac{2\ln(-a^2x^2+1)}{7}}{4a^3} + \frac{-\frac{a^2x^2(3a^2x^2+6)}{15} + \frac{2a^6x^6(\ln}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/7*arctanh(a*x)*a^7*x^7-2/5*arctanh(a*x)*a^5*x^5+1/3*a^3*x^3*arctan
h(a*x)+1/42*a^6*x^6-9/140*a^4*x^4+4/105*a^2*x^2+4/105*ln(a*x-1)+4/105*ln(a*
x+1))
```

Maxima [A]
time = 0.26, size = 81, normalized size = 0.94

$$\frac{1}{420}\,a\left(\frac{10\,a^4x^6 - 27\,a^2x^4 + 16\,x^2}{a^2} + \frac{16\,\log(ax+1)}{a^4} + \frac{16\,\log(ax-1)}{a^4}\right) + \frac{1}{105}\left(15\,a^4x^7 - 42\,a^2x^5 + 35\,x^3\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 + 16*
log(a*x - 1)/a^4) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^3)*arctanh(a*x)
```

Fricas [A]
time = 0.35, size = 84, normalized size = 0.98

$$\frac{10\,a^6x^6 - 27\,a^4x^4 + 16\,a^2x^2 + 2\left(15\,a^7x^7 - 42\,a^5x^5 + 35\,a^3x^3\right)\log\left(-\frac{ax+1}{ax-1}\right) + 16\,\log\left(a^2x^2 - 1\right)}{420\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*x^6 - 27*a^4*x^4 + 16*a^2*x^2 + 2*(15*a^7*x^7 - 42*a^5*x^5 +
35*a^3*x^3)*log(-(a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^3
```

Sympy [A]
time = 0.44, size = 90, normalized size = 1.05

$$\begin{cases} \frac{a^4x^7\operatorname{atanh}(ax)}{7} + \frac{a^3x^6}{42} - \frac{2a^2x^5\operatorname{atanh}(ax)}{5} - \frac{9ax^4}{140} + \frac{x^3\operatorname{atanh}(ax)}{3} + \frac{4x^2}{105a} + \frac{8\log\left(x-\frac{1}{a}\right)}{105a^3} + \frac{8\operatorname{atanh}(ax)}{105a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**7*atanh(a*x)/7 + a**3*x**6/42 - 2*a**2*x**5*atanh(a*x)/5
 - 9*a*x**4/140 + x**3*atanh(a*x)/3 + 4*x**2/(105*a) + 8*log(x - 1/a)/(105*
a**3) + 8*atanh(a*x)/(105*a**3), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(72) = 144$.

time = 0.40, size = 319, normalized size = 3.71

$$\frac{4}{105}\,a\left(\frac{2\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4}-\frac{2\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4}-\frac{\frac{2\,(ax+1)^5}{(ax-1)^5}-\frac{11\,(ax+1)^4}{(ax-1)^4}-\frac{22\,(ax+1)^3}{(ax-1)^3}-\frac{11\,(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^6}+\frac{2\left(\frac{70\,(ax+1)^4}{(ax-1)^4}+\frac{35\,(ax+1)^3}{(ax-1)^3}+\frac{21\,(ax+1)^2}{(ax-1)^2}-\frac{7\,(ax+1)}{ax-1}+1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^4\left(\frac{ax+1}{ax-1}-1\right)^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/105*a*(2*log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - 2*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^4 - (2*(a*x + 1)^5/(a*x - 1)^5 - 11*(a*x + 1)^4/(a*x - 1)^4 -
22*(a*x + 1)^3/(a*x - 1)^3 - 11*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x
- 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^6) + 2*(70*(a*x + 1)^4/(a*x - 1)^4 + 3
5*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x -
 1) + 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)
/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x
+ 1)/(a*x - 1) - 1)^7))
```

Mupad [B]

time = 0.95, size = 71, normalized size = 0.83

$$\frac{x^3\,\mathrm{atanh}(a\,x)}{3}-\frac{9\,a\,x^4}{140}+\frac{4\,\ln\left(a^2\,x^2-1\right)}{105\,a^3}+\frac{4\,x^2}{105\,a}+\frac{a^3\,x^6}{42}-\frac{2\,a^2\,x^5\,\mathrm{atanh}(a\,x)}{5}+\frac{a^4\,x^7\,\mathrm{atanh}(a\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] (x^3*atanh(a*x))/3 - (9*a*x^4)/140 + (4*log(a^2*x^2 - 1))/(105*a^3) + (4*x^
2)/(105*a) + (a^3*x^6)/42 - (2*a^2*x^5*atanh(a*x))/5 + (a^4*x^7*atanh(a*x))
/7
```

3.195 $\int x(1-a^2x^2)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=50

$$\frac{x}{6a}-\frac{ax^3}{9}+\frac{a^3x^5}{30}-\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{6a^2}$$

```
[Out] 1/6*x/a-1/9*a*x^3+1/30*a^3*x^5-1/6*(-a^2*x^2+1)^3*arctanh(a*x)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6141, 200}

$$\frac{a^3x^5}{30}-\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{6a^2}-\frac{ax^3}{9}+\frac{x}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 - ((1 - a^2*x^2)^3*ArcTanh[a*x])/(6*a^2)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x(1-a^2x^2)^2\tanh^{-1}(ax)\,dx &= -\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{6a^2}+\frac{\int (1-a^2x^2)^2\,dx}{6a}\\ &= -\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{6a^2}+\frac{\int (1-2a^2x^2+a^4x^4)\,dx}{6a}\\ &= \frac{x}{6a}-\frac{ax^3}{9}+\frac{a^3x^5}{30}-\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{6a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 1.86

$$\frac{x}{6a}-\frac{ax^3}{9}+\frac{a^3x^5}{30}+\frac{1}{2}x^2\tanh^{-1}(ax)-\frac{1}{2}a^2x^4\tanh^{-1}(ax)+\frac{1}{6}a^4x^6\tanh^{-1}(ax)+\frac{\log(1-ax)}{12a^2}-\frac{\log(1+ax)}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTan
h[a*x])/2 + (a^4*x^6*ArcTanh[a*x])/6 + Log[1 - a*x]/(12*a^2) - Log[1 + a*x]
/(12*a^2)
```

Maple [A]
time = 0.57, size = 68, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^6x^6}{6}-\frac{a^4x^4\operatorname{arctanh}(ax)}{2}+\frac{a^2x^2\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{9}+\frac{ax}{6}}{a^2}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^6x^6}{6}-\frac{a^4x^4\operatorname{arctanh}(ax)}{2}+\frac{a^2x^2\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{9}+\frac{ax}{6}}{a^2}$
risch	$\frac{(a^2x^2-1)^3\ln(ax+1)}{12a^2}-\frac{a^4x^6\ln(-ax+1)}{12}+\frac{a^3x^5}{30}+\frac{a^2x^4\ln(-ax+1)}{4}-\frac{x^3a}{9}-\frac{x^2\ln(-ax+1)}{4}+\frac{x}{6a}+\frac{\ln(-ax}{12a}$
meijerg	$\frac{i\left(-\frac{2ixa(21a^4x^4+35a^2x^2+105)}{315}-\frac{ixa(-7a^6x^6+7)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{21\sqrt{a^2x^2}}\right)}{4a^2}+\frac{i\left(\frac{ixa(5a^2x^2+15)}{15}+\frac{ixa(}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/6*arctanh(a*x)*a^6*x^6-1/2*a^4*x^4*arctanh(a*x)+1/2*a^2*x^2*arctan
h(a*x)-1/6*arctanh(a*x)+1/30*a^5*x^5-1/9*a^3*x^3+1/6*a*x)
```

Maxima [A]
time = 0.25, size = 46, normalized size = 0.92

$$\frac{(a^2x^2-1)^3\operatorname{artanh}(ax)}{6a^2}+\frac{3a^4x^5-10a^2x^3+15x}{90a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 1)^3*arctanh(a*x)/a^2 + 1/90*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)
/a
```

Fricas [A]
time = 0.38, size = 68, normalized size = 1.36

$$\frac{6\,a^5x^5 - 20\,a^3x^3 + 30\,ax + 15\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log\left(-\frac{ax+1}{ax-1}\right)}{180\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/180*(6*a^5*x^5 - 20*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^
2 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^2
```

Sympy [A]
time = 0.32, size = 68, normalized size = 1.36

$$\begin{cases} \frac{a^4x^6\,\mathrm{atanh}\,(ax)}{6} + \frac{a^3x^5}{30} - \frac{a^2x^4\,\mathrm{atanh}\,(ax)}{2} - \frac{ax^3}{9} + \frac{x^2\,\mathrm{atanh}\,(ax)}{2} + \frac{x}{6a} - \frac{\mathrm{atanh}\,(ax)}{6a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**6*atanh(a*x)/6 + a**3*x**5/30 - a**2*x**4*atanh(a*x)/2 -
 a*x**3/9 + x**2*atanh(a*x)/2 + x/(6*a) - atanh(a*x)/(6*a**2), Ne(a, 0)), (
0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(41) = 82.
time = 0.39, size = 176, normalized size = 3.52

$$\frac{8}{45}\,a\left(\frac{\frac{10\,(ax+1)^2}{(ax-1)^2} - \frac{5\,(ax+1)}{ax-1} + 1}{a^3\left(\frac{ax+1}{ax-1} - 1\right)^5} + \frac{30\,(ax+1)^3\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^3a^3\left(\frac{ax+1}{ax-1} - 1\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 8/45*a*((10*(a*x + 1)^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)/(a^3*((a*x
 + 1)/(a*x - 1) - 1)^5) + 30*(a*x + 1)^3*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/
(a*x - 1) - a) - 1))/((a*x - 1)^3*a^3*((a*x + 1)/(a*x - 1) - 1)^6))
```

Mupad [B]
time = 0.92, size = 64, normalized size = 1.28

$$\frac{x^2\,\mathrm{atanh}(a\,x)}{2}-\frac{\frac{\mathrm{atanh}(a\,x)}{6}-\frac{a\,x}{6}}{a^2}-\frac{a\,x^3}{9}+\frac{a^3\,x^5}{30}-\frac{a^2\,x^4\,\mathrm{atanh}(a\,x)}{2}+\frac{a^4\,x^6\,\mathrm{atanh}(a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] (x^2*atanh(a*x))/2 - (atanh(a*x)/6 - (a*x)/6)/a^2 - (a*x^3)/9 + (a^3*x^5)/3
0 - (a^2*x^4*atanh(a*x))/2 + (a^4*x^6*atanh(a*x))/6
```

3.196 $\int (1-a^2x^2)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=104

$$\frac{2(1-a^2x^2)}{15a}+\frac{(1-a^2x^2)^2}{20a}+\frac{8}{15}x\tanh^{-1}(ax)+\frac{4}{15}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{1}{5}x(1-a^2x^2)^2\tanh^{-1}(ax)+\frac{4\log(}{}$$

```
[Out] 2/15*(-a^2*x^2+1)/a+1/20*(-a^2*x^2+1)^2/a+8/15*x*arctanh(a*x)+4/15*x*(-a^2*
x^2+1)*arctanh(a*x)+1/5*x*(-a^2*x^2+1)^2*arctanh(a*x)+4/15*ln(-a^2*x^2+1)/a
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6089, 6021, 266}

$$\frac{(1-a^2x^2)^2}{20a}+\frac{2(1-a^2x^2)}{15a}+\frac{4\log(1-a^2x^2)}{15a}+\frac{1}{5}x(1-a^2x^2)^2\tanh^{-1}(ax)+\frac{4}{15}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{8}{15}x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (2*(1 - a^2*x^2))/(15*a) + (1 - a^2*x^2)^2/(20*a) + (8*x*ArcTanh[a*x])/15 +
 (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 +
 (4*Log[1 - a^2*x^2])/(15*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax) + \frac{4}{5}\int \left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx$$

$$= \frac{2(1-a^2x^2)}{15a} + \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{1}{5}x\left(1-a^2x^2\right)^2$$

$$= \frac{2(1-a^2x^2)}{15a} + \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{8}{15}x\tanh^{-1}(ax) + \frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

$$= \frac{2(1-a^2x^2)}{15a} + \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{8}{15}x\tanh^{-1}(ax) + \frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax$$

Mathematica [A]

time = 0.02, size = 71, normalized size = 0.68

$$-\frac{7ax^2}{30} + \frac{a^3x^4}{20} + x\tanh^{-1}(ax) - \frac{2}{3}a^2x^3\tanh^{-1}(ax) + \frac{1}{5}a^4x^5\tanh^{-1}(ax) + \frac{4\log\left(1-a^2x^2\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (-7*a*x^2)/30 + (a^3*x^4)/20 + x*ArcTanh[a*x] - (2*a^2*x^3*ArcTanh[a*x])/3
+ (a^4*x^5*ArcTanh[a*x])/5 + (4*Log[1 - a^2*x^2])/(15*a)
```

Maple [A]

time = 0.27, size = 69, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^5x^5}{5} - \frac{2a^3x^3\operatorname{arctanh}(ax)}{3} + ax\operatorname{arctanh}(ax) + \frac{a^4x^4}{20} - \frac{7a^2x^2}{30} + \frac{4\ln(ax-1)}{15} + \frac{4\ln(ax+1)}{15}}{a}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^5x^5}{5} - \frac{2a^3x^3\operatorname{arctanh}(ax)}{3} + ax\operatorname{arctanh}(ax) + \frac{a^4x^4}{20} - \frac{7a^2x^2}{30} + \frac{4\ln(ax-1)}{15} + \frac{4\ln(ax+1)}{15}}{a}$
risch	$\left(\frac{1}{10}a^4x^5 - \frac{1}{3}a^2x^3 + \frac{1}{2}x\right)\ln(ax+1) - \frac{a^4x^5\ln(-ax+1)}{10} + \frac{a^3x^4}{20} + \frac{a^2x^3\ln(-ax+1)}{3} - \frac{7ax^2}{30} - \frac{x\ln(-$
meijerg	$-\frac{\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{\sqrt{a^2x^2}} - 2\ln\left(-a^2x^2+1\right)}{4a} - \frac{-\frac{a^2x^2\left(3a^2x^2+6\right)}{15} + \frac{2a^6x^6\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(\right.\right.}{5\sqrt{a^2x^2}}}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/5*arctanh(a*x)*a^5*x^5-2/3*a^3*x^3*arctanh(a*x)+a*x*arctanh(a*x)+1/2
0*a^4*x^4-7/30*a^2*x^2+4/15*ln(a*x-1)+4/15*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 66, normalized size = 0.63

$$\frac{1}{60}\left(3\,a^2x^4-14\,x^2+\frac{16\,\log\left(ax+1\right)}{a^2}+\frac{16\,\log\left(ax-1\right)}{a^2}\right)a+\frac{1}{15}\left(3\,a^4x^5-10\,a^2x^3+15\,x\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*(3*a^2*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 1
/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)
```

Fricas [A]
time = 0.34, size = 72, normalized size = 0.69

$$\frac{3\,a^4x^4-14\,a^2x^2+2\left(3\,a^5x^5-10\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+16\,\log\left(a^2x^2-1\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*x^4 - 14*a^2*x^2 + 2*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*
x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a
```

Sympy [A]
time = 0.28, size = 75, normalized size = 0.72

$$\begin{cases}\frac{a^4x^5\operatorname{atanh}\left(ax\right)}{5}+\frac{a^3x^4}{20}-\frac{2a^2x^3\operatorname{atanh}\left(ax\right)}{3}-\frac{7ax^2}{30}+x\operatorname{atanh}\left(ax\right)+\frac{8\log\left(x-\frac{1}{a}\right)}{15a}+\frac{8\operatorname{atanh}\left(ax\right)}{15a} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**5*atanh(a*x)/5 + a**3*x**4/20 - 2*a**2*x**3*atanh(a*x)/3
 - 7*a*x**2/30 + x*atanh(a*x) + 8*log(x - 1/a)/(15*a) + 8*atanh(a*x)/(15*a)
, Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(88) = 176.
time = 0.40, size = 255, normalized size = 2.45

$$\frac{4}{15}\,a\left(\frac{2\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2}-\frac{2\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2}-\frac{\frac{2\,(ax+1)^3}{(ax-1)^3}-\frac{7\,(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}}{a^2\left(\frac{ax+1}{ax-1}-1\right)^4}+\frac{2\left(\frac{10\,(ax+1)^2}{(ax-1)^2}-\frac{5\,(ax+1)}{ax-1}+1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 4/15*a*(2*log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - 2*log(abs(-(a*x + 1)/(a*x -
 1) + 1))/a^2 - (2*(a*x + 1)^3/(a*x - 1)^3 - 7*(a*x + 1)^2/(a*x - 1)^2 + 2*
(a*x + 1)/(a*x - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^4) + 2*(10*(a*x + 1)^2/
(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/
(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B]
time = 0.91, size = 60, normalized size = 0.58

$$x\,\mathrm{atanh}(a\,x)-\frac{7\,a\,x^2}{30}+\frac{4\,\ln\left(a^2\,x^2-1\right)}{15\,a}+\frac{a^3\,x^4}{20}-\frac{2\,a^2\,x^3\,\mathrm{atanh}(a\,x)}{3}+\frac{a^4\,x^5\,\mathrm{atanh}(a\,x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

```
[Out] x*atanh(a*x) - (7*a*x^2)/30 + (4*log(a^2*x^2 - 1))/(15*a) + (a^3*x^4)/20 -
(2*a^2*x^3*atanh(a*x))/3 + (a^4*x^5*atanh(a*x))/5
```

3.197 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x} \, dx$

Optimal. Leaf size=70

$$-\frac{3ax}{4}+\frac{a^3x^3}{12}+\frac{3}{4}\tanh^{-1}(ax)-a^2x^2\tanh^{-1}(ax)+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)$$

```
[Out] -3/4*a*x+1/12*a^3*x^3+3/4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+1/4*a^4*x^4*arc
tanh(a*x)-1/2*polylog(2,-a*x)+1/2*polylog(2,a*x)
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6159, 6031, 6037, 327, 212, 308}

$$\frac{1}{4}a^4x^4\tanh^{-1}(ax)+\frac{a^3x^3}{12}-a^2x^2\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}-\frac{3ax}{4}+\frac{3}{4}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

```
[Out] (-3*a*x)/4 + (a^3*x^3)/12 + (3*ArcTanh[a*x])/4 - a^2*x^2*ArcTanh[a*x] + (a^
4*x^4*ArcTanh[a*x])/4 - PolyLog[2, -(a*x)]/2 + PolyLog[2, a*x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
 + 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
 + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{x}\,dx &= \int\left(\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+a^4x^3\tanh^{-1}(ax)\right)dx\\
&= -\left((2a^2)\int x\tanh^{-1}(ax)\,dx\right)+a^4\int x^3\tanh^{-1}(ax)\,dx+\int\frac{\tanh^{-1}(ax)}{x}\,d\\
&= -a^2x^2\tanh^{-1}(ax)+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}+a^3\int\frac{}{1-}\\
&= -ax-a^2x^2\tanh^{-1}(ax)+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-\frac{\text{Li}_2(-ax)}{2}+\frac{\text{Li}_2(ax)}{2}+a\int\\
&= -\frac{3ax}{4}+\frac{a^3x^3}{12}+\tanh^{-1}(ax)-a^2x^2\tanh^{-1}(ax)+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-\frac{\text{Li}_2}{}\\
&= -\frac{3ax}{4}+\frac{a^3x^3}{12}+\frac{3}{4}\tanh^{-1}(ax)-a^2x^2\tanh^{-1}(ax)+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-\frac{\text{L}}{}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 1.17

$$-\frac{3ax}{4}+\frac{a^3x^3}{12}-a^2x^2\tanh^{-1}(ax)+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-\frac{3}{8}\log(1-ax)+\frac{3}{8}\log(1+ax)+\frac{1}{2}(-\text{PolyLog}(2,-ax)+\text{PolyLog}(2,ax))$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

[Out] (-3*a*x)/4 + (a^3*x^3)/12 - a^2*x^2*ArcTanh[a*x] + (a^4*x^4*ArcTanh[a*x])/4 - (3*Log[1 - a*x])/8 + (3*Log[1 + a*x])/8 + (-PolyLog[2, -(a*x)] + PolyLog [2, a*x])/2

Maple [A]
time = 0.30, size = 89, normalized size = 1.27

method	result
derivativedivides	$\frac{a^4x^4\operatorname{arctanh}(ax)}{4} - a^2x^2\operatorname{arctanh}(ax) + \operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(ax)\,\mathrm{l}}{}$
default	$\frac{a^4x^4\operatorname{arctanh}(ax)}{4} - a^2x^2\operatorname{arctanh}(ax) + \operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(ax)\,\mathrm{l}}{}$
risch	$\frac{(ax+1)^4\ln(ax+1)}{8} + \frac{a^3x^3}{12} - \frac{3ax}{4} - \frac{(ax+1)^3\ln(ax+1)}{2} + \frac{(ax+1)^2\ln(ax+1)}{4} + \frac{(ax+1)\ln(ax+1)}{2} - \frac{\operatorname{dilog}(ax+1)}{2}$
meijerg	$-\frac{i\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}} - \frac{2iax\operatorname{polylog}\left(2,-\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4} - \frac{i\left(\frac{ixa\left(5a^2x^2+15\right)}{15} + \frac{ixa\left(-5a^4x^4+5\right)\left(\ln\left(1-\sqrt{a^2x^2}\right.\right.}{10\sqrt{a^2x^2}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x,x,method=_RETURNVERBOSE)

[Out] 1/4*a^4*x^4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)+1/12*a^3*x^3-3/4*a*x-3/8*ln(a*x-1)+3/8*ln(a*x+1)

Maxima [A]
time = 0.28, size = 106, normalized size = 1.51

$$\frac{1}{24}\left(2a^2x^3 - 18x - \frac{12(\log(ax+1)\log(x) + \mathrm{Li}_2(-ax))}{a} + \frac{12(\log(-ax+1)\log(x) + \mathrm{Li}_2(ax))}{a} + \frac{9\log(ax+1)}{a} - \frac{9\log(ax-1)}{a}\right)a + \frac{1}{4}\left(a^4x^4 - 4a^2x^2 + 2\log(x^2)\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="maxima")

[Out] 1/24*(2*a^2*x^3 - 18*x - 12*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 12*(log(-a*x + 1)*log(x) + dilog(a*x))/a + 9*log(a*x + 1)/a - 9*log(a*x - 1)/a)*a + 1/4*(a^4*x^4 - 4*a^2*x^2 + 2*log(x^2))*arctanh(a*x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="fricas")

[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x,x)
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x,x)
[Out] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x, x)
```

3.198 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=64

$$\frac{a^3x^2}{6} - \frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) + a\log(x) - \frac{4}{3}a\log\left(1-a^2x^2\right)$$

[Out] 1/6*a^3*x^2-arctanh(a*x)/x-2*a^2*x*arctanh(a*x)+1/3*a^4*x^3*arctanh(a*x)+a*ln(x)-4/3*a*ln(-a^2*x^2+1)

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6159, 6021, 266, 6037, 272, 36, 29, 31, 45}

$$\frac{1}{3}a^4x^3\tanh^{-1}(ax) + \frac{a^3x^2}{6} - \frac{4}{3}a\log\left(1-a^2x^2\right) - 2a^2x\tanh^{-1}(ax) + a\log(x) - \frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]

[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x])/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^2}\,dx = \int \left(-2a^2\tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)}{x^2} + a^4x^2\tanh^{-1}(ax)\right)dx$$
$$= -\left((2a^2)\int \tanh^{-1}(ax)\,dx\right) + a^4\int x^2\tanh^{-1}(ax)\,dx + \int \frac{\tanh^{-1}(ax)}{x^2}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) + a\int \frac{1}{x(1-a^2x^2)}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) - a\log(1-a^2x^2) + $$
$$= -\frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) - a\log(1-a^2x^2) + $$
$$= \frac{a^3x^2}{6} - \frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) + a\log(x) - \frac{4}{3}a$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 1.00

$$\frac{a^3x^2}{6} - \frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) + a\log(x) - \frac{4}{3}a\log(1-a^2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

```
[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x]
)/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

Maple [A]
time = 0.22, size = 64, normalized size = 1.00

method	result
derivativedivides	$a\left(\frac{a^3x^3\,\mathrm{arctanh}(ax)}{3} - 2ax\,\mathrm{arctanh}(ax) - \frac{\mathrm{arctanh}(ax)}{ax} + \frac{a^2x^2}{6} - \frac{4\ln(ax+1)}{3} - \frac{4\ln(ax-1)}{3} + \ln(ax)\right)$
default	$a\left(\frac{a^3x^3\,\mathrm{arctanh}(ax)}{3} - 2ax\,\mathrm{arctanh}(ax) - \frac{\mathrm{arctanh}(ax)}{ax} + \frac{a^2x^2}{6} - \frac{4\ln(ax+1)}{3} - \frac{4\ln(ax-1)}{3} + \ln(ax)\right)$
risch	$\frac{(a^4x^4-6a^2x^2-3)\ln(ax+1)}{6x} + \frac{-x^4\ln(-ax+1)a^4+a^3x^3+6x^2\ln(-ax+1)a^2+6a\ln(x)x-8a\ln(a^2x^2-1)x+3\ln(-ax+1)}{6x}$
meijerg	$\frac{a\left(\frac{2\ln\left(1-\sqrt{a^2x^2}\right)-2\ln\left(1+\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(-a^2x^2+1)+4\ln(x)+4\ln(ia)\right)}{4} + \frac{a\left(\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right)\right.}{3\sqrt{a^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*a^3*x^3*arctanh(a*x)-2*a*x*arctanh(a*x)-arctanh(a*x)/a/x+1/6*a^2*x^2
-4/3*ln(a*x+1)-4/3*ln(a*x-1)+ln(a*x))
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.89

$$\frac{1}{6}\left(a^{2}x^{2}-8\,\log\left(ax+1\right)-8\,\log\left(ax-1\right)+6\,\log\left(x\right)\right)a+\frac{1}{3}\left(a^{4}x^{3}-6\,a^{2}x-\frac{3}{x}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 8*log(a*x + 1) - 8*log(a*x - 1) + 6*log(x))*a + 1/3*(a^4*x^3
 - 6*a^2*x - 3/x)*arctanh(a*x)
```

Fricas [A]
time = 0.37, size = 66, normalized size = 1.03

$$\frac{a^{3}x^{3}-8\,ax\log\left(a^{2}x^{2}-1\right)+6\,ax\log\left(x\right)+\left(a^{4}x^{4}-6\,a^{2}x^{2}-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3 - 8*a*x*log(a^2*x^2 - 1) + 6*a*x*log(x) + (a^4*x^4 - 6*a^2*x^2
 - 3)*log(-(a*x + 1)/(a*x - 1)))/x
```

Sympy [A]
time = 0.67, size = 68, normalized size = 1.06

$$\begin{cases}\frac{a^{4}x^{3}\operatorname{atanh}\left(ax\right)}{3}+\frac{a^{3}x^{2}}{6}-2a^{2}x\operatorname{atanh}\left(ax\right)+a\log\left(x\right)-\frac{8a\log\left(x-\frac{1}{a}\right)}{3}-\frac{8a\operatorname{atanh}\left(ax\right)}{3}-\frac{\operatorname{atanh}\left(ax\right)}{x} & \text{for}\: a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**2,x)
```

```
[Out] Piecewise((a**4*x**3*atanh(a*x)/3 + a**3*x**2/6 - 2*a**2*x*atanh(a*x) + a*l
og(x) - 8*a*log(x - 1/a)/3 - 8*a*atanh(a*x)/3 - atanh(a*x)/x, Ne(a, 0)), (0
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(58) = 116.
time = 0.41, size = 249, normalized size = 3.89

$$\frac{1}{3}\left(\left(\frac{3}{\frac{ax+1}{ax-1}+1}-\frac{\frac{3\,(ax+1)^{2}}{(ax-1)^{2}}-\frac{12\,(ax+1)}{ax-1}+5}{\left(\frac{ax+1}{ax-1}-1\right)^{3}}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)+\frac{2\,(ax+1)}{(ax-1)\left(\frac{ax+1}{ax-1}-1\right)^{2}}-8\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)+5\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)+3\,\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*((3/((a*x + 1)/(a*x - 1) + 1) - (3*(a*x + 1)^2/(a*x - 1)^2 - 12*(a*x +
1)/(a*x - 1) + 5)/((a*x + 1)/(a*x - 1) - 1)^3)*log(-(a*((a*x + 1)/(a*x - 1)
 + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x +
 1)*a/(a*x - 1) - a) - 1)) + 2*(a*x + 1)/((a*x - 1)*((a*x + 1)/(a*x - 1) -
1)^2) - 8*log(abs(-a*x - 1)/abs(a*x - 1)) + 5*log(abs(-(a*x + 1)/(a*x - 1)
+ 1)) + 3*log(abs(-(a*x + 1)/(a*x - 1) - 1)))*a
```

Mupad [B]
time = 0.91, size = 57, normalized size = 0.89

$$a\,\ln\left(x\right)-\frac{4\,a\,\ln\left(a^2\,x^2-1\right)}{3}-\frac{\mathrm{atanh}(a\,x)}{x}+\frac{a^3\,x^2}{6}-2\,a^2\,x\,\mathrm{atanh}(a\,x)+\frac{a^4\,x^3\,\mathrm{atanh}(a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^2,x)
```

```
[Out] a*log(x) - (4*a*log(a^2*x^2 - 1))/3 - atanh(a*x)/x + (a^3*x^2)/6 - 2*a^2*x*
atanh(a*x) + (a^4*x^3*atanh(a*x))/3
```

3.199 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=62

$$-\frac{a}{2x}+\frac{a^3x}{2}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\text{PolyLog}(2,-ax)-a^2\text{PolyLog}(2,ax)$$

```
[Out] -1/2*a/x+1/2*a^3*x-1/2*arctanh(a*x)/x^2+1/2*a^4*x^2*arctanh(a*x)+a^2*polylo
g(2,-a*x)-a^2*polylog(2,a*x)
```

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6159, 6037, 331, 212, 6031, 327}

$$\frac{1}{2}a^4x^2\tanh^{-1}(ax)+\frac{a^3x}{2}+a^2\text{Li}_2(-ax)-a^2\text{Li}_2(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*a/x + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 + a^
2*PolyLog[2, -(a*x)] - a^2*PolyLog[2, a*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^3}\,dx &= \int \left(\frac{\tanh^{-1}(ax)}{x^3}-\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)\right)dx \\
&= -\left((2a^2)\int \frac{\tanh^{-1}(ax)}{x}\,dx\right)+a^4\int x\tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx \\
&= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\mathrm{Li}_2(-ax)-a^2\mathrm{Li}_2(ax)+\frac{1}{2}a\int \frac{}{x^2\,(1} \\
&= -\frac{a}{2x}+\frac{a^3x}{2}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\mathrm{Li}_2(-ax)-a^2\mathrm{Li}_2(ax)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.98

$$-\frac{a}{2x}+\frac{a^3x}{2}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)-a^2(-\mathrm{PolyLog}(2,-ax)+\mathrm{PolyLog}(2,ax))$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*a/x + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 - a^
2*(-PolyLog[2, -(a*x)] + PolyLog[2, a*x])
```

Maple [A]
time = 0.29, size = 73, normalized size = 1.18

method	result
derivativedivides	$a^2\left(\frac{a^2x^2 \operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}-2\operatorname{arctanh}(ax)\ln(ax)+\operatorname{dilog}(ax+1)+\ln(ax)\ln(ax\right.$
default	$a^2\left(\frac{a^2x^2 \operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}-2\operatorname{arctanh}(ax)\ln(ax)+\operatorname{dilog}(ax+1)+\ln(ax)\ln(ax\right.$
risch	$\frac{a^4\ln(ax+1)x^2}{4}+\frac{a^3x}{2}+a^2\operatorname{dilog}(ax+1)-\frac{a}{2x}-\frac{a^2\ln(ax)}{4}-\frac{\ln(ax+1)}{4x^2}-\frac{a^4\ln(-ax+1)x^2}{4}-a^2\operatorname{dilo}$
meijerg	$\frac{ia^2\left(\frac{2i}{xa}+\frac{2i(-ax+1)(ax+1)\operatorname{arctanh}(ax)}{x^2a^2}\right)}{4}+\frac{ia^2(-2ixa+2i(-ax+1)(ax+1)\operatorname{arctanh}(ax))}{4}+\frac{ia^2\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x}\right.}{\sqrt{a^2x^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*a^2*x^2*arctanh(a*x)-1/2*arctanh(a*x)/a^2/x^2-2*arctanh(a*x)*ln(a*
x)+dilog(a*x+1)+ln(a*x)*ln(a*x+1)+dilog(a*x)+1/2*a*x-1/2/a/x)
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.32

$$\frac{1}{2}\left(2\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a-2\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)a+\frac{a^2x^2-1}{x}\right)a+\frac{1}{2}\left(a^4x^2-2\,a^2\log\left(x^2\right)-\frac{1}{x^2}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + di
log(a*x))*a + (a^2*x^2 - 1)/x)*a + 1/2*(a^4*x^2 - 2*a^2*log(x^2) - 1/x^2)*a
rctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2\,(ax+1)^2\operatorname{atanh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^3,x)
```

```
[Out] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^3, x)
```

3.200 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=68

$$-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)-\frac{5}{3}a^3\log(x)+\frac{4}{3}a^3\log\left(1-a^2x^2\right)$$

```
[Out] -1/6*a/x^2-1/3*arctanh(a*x)/x^3+2*a^2*arctanh(a*x)/x+a^4*x*arctanh(a*x)-5/3
*a^3*ln(x)+4/3*a^3*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6159, 6021, 266, 6037, 272, 46, 36, 29, 31}

$$a^4x\tanh^{-1}(ax)-\frac{5}{3}a^3\log(x)+\frac{2a^2\tanh^{-1}(ax)}{x}+\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{3x^3}-\frac{a}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^4} \, dx = \int \left(a^4 \tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)}{x^4} - \frac{2a^2 \tanh^{-1}(ax)}{x^2} \right) dx$$
$$= -\left((2a^2) \int \frac{\tanh^{-1}(ax)}{x^2} \, dx \right) + a^4 \int \tanh^{-1}(ax) \, dx + \int \frac{\tanh^{-1}(ax)}{x^4} \, dx$$
$$= -\frac{\tanh^{-1}(ax)}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)}{x} + a^4 x \tanh^{-1}(ax) + \frac{1}{3} a \int \frac{1}{x^3(1-a^2x^2)} \, d$$
$$= -\frac{\tanh^{-1}(ax)}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)}{x} + a^4 x \tanh^{-1}(ax) + \frac{1}{2} a^3 \log(1-a^2x^2) +$$
$$= -\frac{\tanh^{-1}(ax)}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)}{x} + a^4 x \tanh^{-1}(ax) + \frac{1}{2} a^3 \log(1-a^2x^2) +$$
$$= -\frac{a}{6x^2} - \frac{\tanh^{-1}(ax)}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)}{x} + a^4 x \tanh^{-1}(ax) - \frac{5}{3} a^3 \log(x) + \frac{4}{3}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.00

$$-\frac{a}{6x^2} - \frac{\tanh^{-1}(ax)}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)}{x} + a^4 x \tanh^{-1}(ax) - \frac{5}{3} a^3 \log(x) + \frac{4}{3} a^3 \log(1-a^2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Maple [A]
time = 0.28, size = 67, normalized size = 0.99

method	result
derivativedivides	$a^3 \left(ax \operatorname{arctanh}(ax) - \frac{\operatorname{arctanh}(ax)}{3a^3x^3} + \frac{2 \operatorname{arctanh}(ax)}{ax} + \frac{4 \ln(ax-1)}{3} + \frac{4 \ln(ax+1)}{3} - \frac{1}{6a^2x^2} - \frac{5 \ln(ax)}{3} \right)$
default	$a^3 \left(ax \operatorname{arctanh}(ax) - \frac{\operatorname{arctanh}(ax)}{3a^3x^3} + \frac{2 \operatorname{arctanh}(ax)}{ax} + \frac{4 \ln(ax-1)}{3} + \frac{4 \ln(ax+1)}{3} - \frac{1}{6a^2x^2} - \frac{5 \ln(ax)}{3} \right)$
risch	$\frac{(3a^4x^4+6a^2x^2-1)\ln(ax+1)}{6x^3} - \frac{3x^4 \ln(-ax+1)a^4+10\ln(x)a^3x^3-8\ln(-a^2x^2+1)a^3x^3+6x^2\ln(-ax+1)a^2+ax-\ln(-}{6x^3}$
meijerg	$-\frac{a^3 \left(-\frac{2(10a^2x^2+30)}{45a^2x^2} - \frac{2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3a^2x^2\sqrt{a^2x^2}} + \frac{2\ln(-a^2x^2+1)}{3} + \frac{4}{9} - \frac{4\ln(x)}{3} - \frac{4\ln(ia)}{3} + \frac{2}{a^2x^2} \right)}{4} - $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(a*x*arctanh(a*x)-1/3*arctanh(a*x)/a^3/x^3+2*arctanh(a*x)/a/x+4/3*ln(a*
x-1)+4/3*ln(a*x+1)-1/6/a^2/x^2-5/3*ln(a*x))
```

Maxima [A]
time = 0.27, size = 66, normalized size = 0.97

$$\frac{1}{6}\left(8\,a^2\log\left(ax+1\right)+8\,a^2\log\left(ax-1\right)-10\,a^2\log\left(x\right)-\frac{1}{x^2}\right)a+\frac{1}{3}\left(3\,a^4x+\frac{6\,a^2x^2-1}{x^3}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(8*a^2*log(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a + 1
/3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.06

$$\frac{8\,a^3x^3\log\left(a^2x^2-1\right)-10\,a^3x^3\log\left(x\right)-ax+\left(3\,a^4x^4+6\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(8*a^3*x^3*log(a^2*x^2 - 1) - 10*a^3*x^3*log(x) - a*x + (3*a^4*x^4 + 6*
a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1)))/x^3
```

Sympy [A]
time = 0.47, size = 75, normalized size = 1.10

$$\begin{cases} a^4x\operatorname{atanh}\left(ax\right)-\frac{5a^3\log\left(x\right)}{3}+\frac{8a^3\log\left(x-\frac{1}{a}\right)}{3}+\frac{8a^3\operatorname{atanh}\left(ax\right)}{3}+\frac{2a^2\operatorname{atanh}\left(ax\right)}{x}-\frac{a}{6x^2}-\frac{\operatorname{atanh}\left(ax\right)}{3x^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**4,x)
```

```
[Out] Piecewise((a**4*x*atanh(a*x) - 5*a**3*log(x)/3 + 8*a**3*log(x - 1/a)/3 + 8*
a**3*atanh(a*x)/3 + 2*a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3),
 Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(60) = 120.
time = 0.39, size = 274, normalized size = 4.03

$$\frac{1}{3}\left(8\,a^2\log\left(\frac{|-ax-1|}{|ax-1|}\right)-3\,a^2\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)-5\,a^2\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\left(\frac{3\,a^2}{\frac{ax+1}{ax-1}-1}-\frac{\frac{3\,(ax+1)^2a^2}{(ax-1)^2}+\frac{12\,(ax+1)a^2}{ax-1}+5\,a^2}{\left(\frac{ax+1}{ax-1}+1\right)^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)+\frac{2\,(ax+1)a^2}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(8*a^2*log(abs(-a*x - 1)/abs(a*x - 1)) - 3*a^2*log(abs(-(a*x + 1)/(a*x
- 1) + 1)) - 5*a^2*log(abs(-(a*x + 1)/(a*x - 1) - 1)) + (3*a^2/((a*x + 1)/(
a*x - 1) - 1) - (3*(a*x + 1)^2*a^2/(a*x - 1)^2 + 12*(a*x + 1)*a^2/(a*x - 1)
 + 5*a^2)/((a*x + 1)/(a*x - 1) + 1)^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((
a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a
*x - 1) - a) - 1)) + 2*(a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2
))*a
```

Mupad [B]
time = 0.88, size = 59, normalized size = 0.87

$$\frac{4\,a^3\,\ln\left(a^2\,x^2-1\right)}{3}-\frac{a}{6\,x^2}-\frac{\mathrm{atanh}(a\,x)}{3\,x^3}-\frac{5\,a^3\,\ln\left(x\right)}{3}+a^4\,x\,\mathrm{atanh}(a\,x)+\frac{2\,a^2\,\mathrm{atanh}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^4,x)
```

```
[Out] (4*a^3*log(a^2*x^2 - 1))/3 - a/(6*x^2) - atanh(a*x)/(3*x^3) - (5*a^3*log(x)
)/3 + a^4*x*atanh(a*x) + (2*a^2*atanh(a*x))/x
```

3.201 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^5} dx$

Optimal. Leaf size=77

$$-\frac{a}{12x^3}+\frac{3a^3}{4x}-\frac{3}{4}a^4\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\text{PolyLog}(2,-ax)+\frac{1}{2}a^4\text{PolyLog}(2,ax)$$

```
[Out] -1/12*a/x^3+3/4*a^3/x-3/4*a^4*arctanh(a*x)-1/4*arctanh(a*x)/x^4+a^2*arctanh
(a*x)/x^2-1/2*a^4*polylog(2,-a*x)+1/2*a^4*polylog(2,a*x)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6159, 6037, 331, 212, 6031}

$$-\frac{1}{2}a^4\text{Li}_2(-ax)+\frac{1}{2}a^4\text{Li}_2(ax)-\frac{3}{4}a^4\tanh^{-1}(ax)+\frac{3a^3}{4x}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{\tanh^{-1}(ax)}{4x^4}-\frac{a}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

```
[Out] -1/12*a/x^3 + (3*a^3)/(4*x) - (3*a^4*ArcTanh[a*x])/4 - ArcTanh[a*x]/(4*x^4)
 + (a^2*ArcTanh[a*x])/x^2 - (a^4*PolyLog[2, -(a*x)])/2 + (a^4*PolyLog[2, a*
x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{x^5}\,dx &= \int \left(\frac{\tanh^{-1}(ax)}{x^5}-\frac{2a^2\tanh^{-1}(ax)}{x^3}+\frac{a^4\tanh^{-1}(ax)}{x}\right)dx\\ &= -\left((2a^2)\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\right)+a^4\int \frac{\tanh^{-1}(ax)}{x}\,dx+\int \frac{\tanh^{-1}(ax)}{x^5}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-ax)+\frac{1}{2}a^4\mathrm{Li}_2(ax)+\frac{1}{4}a\int \frac{}{x^4(}\\ &= -\frac{a}{12x^3}+\frac{a^3}{x}-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-ax)+\frac{1}{2}a^4\mathrm{Li}_2(ax\\ &= -\frac{a}{12x^3}+\frac{3a^3}{4x}-a^4\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2(-\\ &= -\frac{a}{12x^3}+\frac{3a^3}{4x}-\frac{3}{4}a^4\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}-\frac{1}{2}a^4\mathrm{Li}_2\end{aligned}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 1.16

$$-\frac{a}{12x^3}+\frac{3a^3}{4x}-\frac{\tanh^{-1}(ax)}{4x^4}+\frac{a^2\tanh^{-1}(ax)}{x^2}+\frac{3}{8}a^4\log(1-ax)-\frac{3}{8}a^4\log(1+ax)+\frac{1}{2}a^4(-\mathrm{PolyLog}(2,-ax)+\mathrm{PolyLog}(2,ax))$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

```
[Out] -1/12*a/x^3 + (3*a^3)/(4*x) - ArcTanh[a*x]/(4*x^4) + (a^2*ArcTanh[a*x])/x^2
 + (3*a^4*Log[1 - a*x])/8 - (3*a^4*Log[1 + a*x])/8 + (a^4*(-PolyLog[2, -(a*
x)] + PolyLog[2, a*x]))/2
```

Maple [A]
time = 0.30, size = 96, normalized size = 1.25

method	result
derivativedivides	$a^4\left(\operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{arctanh}(ax)}{4a^4x^4} + \frac{\operatorname{arctanh}(ax)}{a^2x^2} - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(ax)\ln(ax+1)}{2} -\right.$
default	$a^4\left(\operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{arctanh}(ax)}{4a^4x^4} + \frac{\operatorname{arctanh}(ax)}{a^2x^2} - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(ax)\ln(ax+1)}{2} -\right.$
risch	$-\frac{a}{12x^3} + \frac{3a^3}{4x} + \frac{3a^4\ln(ax)}{8} - \frac{3a^4\ln(ax+1)}{8} - \frac{\ln(ax+1)}{8x^4} - \frac{a^4\operatorname{dilog}(ax+1)}{2} + \frac{a^2\ln(ax+1)}{2x^2} - \frac{3a^4\ln(-ax)}{8} +$
meijerg	$-\frac{ia^4\left(-\frac{i}{3x^3a^3} - \frac{i}{xa} + \frac{4i\left(\frac{3}{8} - \frac{3a^4x^4}{8}\right)\left(\ln\left(1-\sqrt{a^2x^2}\right) - \ln\left(1+\sqrt{a^2x^2}\right)\right)}{3x^3a^3\sqrt{a^2x^2}}\right)}{4} - \frac{ia^4\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}} - \frac{2iax}{\right.}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(arctanh(a*x)*ln(a*x)-1/4*arctanh(a*x)/a^4/x^4+arctanh(a*x)/a^2/x^2-1/2
*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)-1/12/a^3/x^3+3/4/a/x-3/8
*ln(a*x+1)+3/8*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 112, normalized size = 1.45

$$-\frac{1}{24}\left(12\left(\log(ax+1)\log(x) + \mathrm{Li}_2(-ax)\right)a^3 - 12\left(\log(-ax+1)\log(x) + \mathrm{Li}_2(ax)\right)a^3 + 9a^3\log(ax+1) - 9a^3\log(ax-1) - \frac{2\left(9a^2x^2-1\right)}{x^3}\right)a + \frac{1}{4}\left(2a^4\log\left(x^2\right) + \frac{4a^2x^2-1}{x^4}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 - 12*(log(-a*x + 1)*log(x
) + dilog(a*x))*a^3 + 9*a^3*log(a*x + 1) - 9*a^3*log(a*x - 1) - 2*(9*a^2*x^
2 - 1)/x^3)*a + 1/4*(2*a^4*log(x^2) + (4*a^2*x^2 - 1)/x^4)*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2\,(ax+1)^2\operatorname{atanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**5,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^5,x)
```

```
[Out] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^5, x)
```

3.202 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^6} dx$

Optimal. Leaf size=83

$$-\frac{a}{20x^4}+\frac{7a^3}{30x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{8}{15}a^5\log(x)-\frac{4}{15}a^5\log\left(1-a^2x^2\right)$$

```
[Out] -1/20*a/x^4+7/30*a^3/x^2-1/5*arctanh(a*x)/x^5+2/3*a^2*arctanh(a*x)/x^3-a^4*
arctanh(a*x)/x+8/15*a^5*ln(x)-4/15*a^5*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6159, 6037, 272, 46, 36, 29, 31}

$$\frac{8}{15}a^5\log(x)-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{7a^3}{30x^2}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{4}{15}a^5\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)}{5x^5}-\frac{a}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

```
[Out] -1/20*a/x^4 + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{x^6}\,dx &= \int \left(\frac{\tanh^{-1}(ax)}{x^6}-\frac{2a^2\tanh^{-1}(ax)}{x^4}+\frac{a^4\tanh^{-1}(ax)}{x^2}\right)dx \\
&= -\left((2a^2)\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\right)+a^4\int \frac{\tanh^{-1}(ax)}{x^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^6}\,dx \\
&= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{1}{5}a\int \frac{1}{x^5(1-a^2x^2)}\,dx \\
&= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{1}{10}a\text{Subst}\left(\int \frac{1}{x^3(1-\right. \\
&= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{1}{10}a\text{Subst}\left(\int\left(\frac{1}{x^3}+\right.\right. \\
&= -\frac{a}{20x^4}+\frac{7a^3}{30x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{8}{15}a^5
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 1.00

$$
-\frac{a}{20x^4}+\frac{7a^3}{30x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{8}{15}a^5\log(x)-\frac{4}{15}a^5\log\left(1-a^2x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

```
[Out] -1/20*a/x^4 + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

Maple [A]
time = 0.35, size = 80, normalized size = 0.96

method	result
derivativedivides	$a^5\left(-\frac{\operatorname{arctanh}(ax)}{ax}+\frac{2\operatorname{arctanh}(ax)}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{5a^5x^5}-\frac{4\ln(ax+1)}{15}-\frac{1}{20a^4x^4}+\frac{7}{30a^2x^2}+\frac{8\ln(ax)}{15}-\frac{4\ln(ax-}{15}\right.$
default	$a^5\left(-\frac{\operatorname{arctanh}(ax)}{ax}+\frac{2\operatorname{arctanh}(ax)}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{5a^5x^5}-\frac{4\ln(ax+1)}{15}-\frac{1}{20a^4x^4}+\frac{7}{30a^2x^2}+\frac{8\ln(ax)}{15}-\frac{4\ln(ax-}{15}\right.$
risch	$-\frac{(15a^4x^4-10a^2x^2+3)\ln(ax+1)}{30x^5}+\frac{32\ln(x)a^5x^5-16\ln(a^2x^2-1)a^5x^5+30x^4\ln(-ax+1)a^4+14a^3x^3-20x^2\ln(-ax+1)}{60x^5}$
meijerg	$\frac{a^5\left(\frac{\frac{4}{25}a^4x^4+\frac{4}{15}a^2x^2+\frac{4}{5}}{a^4x^4}+\frac{\frac{2\ln\left(1-\sqrt{a^2x^2}\right)}{5}-\frac{2\ln\left(1+\sqrt{a^2x^2}\right)}{5}}{a^4x^4\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{5}-\frac{4}{25}+\frac{4\ln(x)}{5}+\frac{4\ln(ia)}{5}-\frac{1}{a^4x^4}-\frac{2}{3a^2x^2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-arctanh(a*x)/a/x+2/3*arctanh(a*x)/a^3/x^3-1/5*arctanh(a*x)/a^5/x^5-4/
15*ln(a*x+1)-1/20/a^4/x^4+7/30/a^2/x^2+8/15*ln(a*x)-4/15*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 71, normalized size = 0.86

$$-\frac{1}{60}\left(16\,a^4\log\left(a^2x^2-1\right)-16\,a^4\log\left(x^2\right)-\frac{14\,a^2x^2-3}{x^4}\right)a-\frac{\left(15\,a^4x^4-10\,a^2x^2+3\right)\operatorname{artanh}\left(ax\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2) - (14*a^2*x^2 - 3)/x^4)*a
- 1/15*(15*a^4*x^4 - 10*a^2*x^2 + 3)*arctanh(a*x)/x^5
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.98

$$-\frac{16\,a^5x^5\log\left(a^2x^2-1\right)-32\,a^5x^5\log\left(x\right)-14\,a^3x^3+3\,ax+2\left(15\,a^4x^4-10\,a^2x^2+3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(16*a^5*x^5*log(a^2*x^2 - 1) - 32*a^5*x^5*log(x) - 14*a^3*x^3 + 3*a*x
 + 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

Sympy [A]
time = 0.56, size = 88, normalized size = 1.06

$$\begin{cases} \frac{8a^5 \log(x)}{15} - \frac{8a^5 \log\left(x - \frac{1}{a}\right)}{15} - \frac{8a^5 \operatorname{atanh}(ax)}{15} - \frac{a^4 \operatorname{atanh}(ax)}{x} + \frac{7a^3}{30x^2} + \frac{2a^2 \operatorname{atanh}(ax)}{3x^3} - \frac{a}{20x^4} - \frac{\operatorname{atanh}(ax)}{5x^5} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**6,x)
```

```
[Out] Piecewise((8*a**5*log(x)/15 - 8*a**5*log(x - 1/a)/15 - 8*a**5*atanh(a*x)/15
 - a**4*atanh(a*x)/x + 7*a**3/(30*x**2) + 2*a**2*atanh(a*x)/(3*x**3) - a/(2
0*x**4) - atanh(a*x)/(5*x**5), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(71) = 142.
time = 0.41, size = 265, normalized size = 3.19

$$-\frac{4}{15}\left(2a^4\log\left(\frac{|-ax-1|}{|ax-1|}\right) - 2a^4\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right) + \frac{\frac{2(ax+1)^3a^4}{(ax-1)^3} + \frac{7(ax+1)^2a^4}{(ax-1)^2} + \frac{2(ax+1)a^4}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^4} - \frac{2\left(\frac{10(ax+1)^2a^4}{(ax-1)^2} + \frac{5(ax+1)a^4}{ax-1} + a^4\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -4/15*(2*a^4*log(abs(-a*x - 1)/abs(a*x - 1)) - 2*a^4*log(abs(-(a*x + 1)/(a*
x - 1) - 1)) + (2*(a*x + 1)^3*a^4/(a*x - 1)^3 + 7*(a*x + 1)^2*a^4/(a*x - 1)
^2 + 2*(a*x + 1)*a^4/(a*x - 1))/((a*x + 1)/(a*x - 1) + 1)^4 - 2*(10*(a*x +
1)^2*a^4/(a*x - 1)^2 + 5*(a*x + 1)*a^4/(a*x - 1) + a^4)*log(-(a*((a*x + 1)/
(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1
)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)/(a*x - 1) + 1)^5)*a
```

Mupad [B]
time = 0.89, size = 70, normalized size = 0.84

$$\frac{8\,a^5\,\ln(x)}{15} - \frac{a}{20\,x^4} - \frac{\mathrm{atanh}(a\,x)}{5\,x^5} - \frac{4\,a^5\,\ln\left(a^2\,x^2-1\right)}{15} + \frac{7\,a^3}{30\,x^2} + \frac{2\,a^2\,\mathrm{atanh}(a\,x)}{3\,x^3} - \frac{a^4\,\mathrm{atanh}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^6,x)
```

```
[Out] (8*a^5*log(x))/15 - a/(20*x^4) - atanh(a*x)/(5*x^5) - (4*a^5*log(a^2*x^2 -
1))/15 + (7*a^3)/(30*x^2) + (2*a^2*atanh(a*x))/(3*x^3) - (a^4*atanh(a*x))/x
```

3.203 $\int x^4(1-a^2x^2)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=202

$$\frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{29\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+\frac{4x^4\tanh^{-1}(ax)}{315a}-\frac{11}{189}ax^6\tanh^{-1}(ax)+\frac{1}{36}$$

```
[Out] 29/3780*x/a^4-67/11340*x^3/a^2-23/3780*x^5+1/252*a^2*x^7-29/3780*arctanh(a*
x)/a^5+8/315*x^2*arctanh(a*x)/a^3+4/315*x^4*arctanh(a*x)/a-11/189*a*x^6*arc
tanh(a*x)+1/36*a^3*x^8*arctanh(a*x)+8/315*arctanh(a*x)^2/a^5+1/5*x^5*arctan
h(a*x)^2-2/7*a^2*x^7*arctanh(a*x)^2+1/9*a^4*x^9*arctanh(a*x)^2-16/315*arcta
nh(a*x)*ln(2/(-a*x+1))/a^5-8/315*polylog(2,1-2/(-a*x+1))/a^5
```

Rubi [A]
time = 0.69, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 59, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {6159, 6037, 6127, 308, 212, 327, 6131, 6055, 2449, 2352}

$$-\frac{8\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{315a^5}+\frac{8\tanh^{-1}(ax)^2}{315a^5}-\frac{29\tanh^{-1}(ax)}{3780a^5}-\frac{16\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{315a^5}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+\frac{29x}{3780a^4}+\frac{1}{36}a^3x^8\tanh^{-1}(ax)+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+\frac{a^2x^7}{252}-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2-\frac{67x^3}{11340a^2}-\frac{11}{189}ax^6\tanh^{-1}(ax)+\frac{1}{5}x^5\tanh^{-1}(ax)^2+\frac{4x^4\tanh^{-1}(ax)}{315a}-\frac{23x^5}{3780}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (29*x)/(3780*a^4) - (67*x^3)/(11340*a^2) - (23*x^5)/3780 + (a^2*x^7)/252 -
(29*ArcTanh[a*x])/(3780*a^5) + (8*x^2*ArcTanh[a*x])/(315*a^3) + (4*x^4*ArcT
anh[a*x])/(315*a) - (11*a*x^6*ArcTanh[a*x])/189 + (a^3*x^8*ArcTanh[a*x])/36
 + (8*ArcTanh[a*x]^2)/(315*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (2*a^2*x^7*ArcTa
nh[a*x]^2)/7 + (a^4*x^9*ArcTanh[a*x]^2)/9 - (16*ArcTanh[a*x]*Log[2/(1 - a*x
)])/(315*a^5) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(315*a^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx=\int\left(x^4\tanh^{-1}(ax)^2-2a^2x^6\tanh^{-1}(ax)^2+a^4x^8\tanh^{-1}(ax)^2\right)\,dx$$

$$=-\left(\left(2a^2\right)\int x^6\tanh^{-1}(ax)^2\,dx\right)+a^4\int x^8\tanh^{-1}(ax)^2\,dx+\int x^4$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2-\frac{1}{5}(2a)\int$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+$$

$$=\frac{x^4\tanh^{-1}(ax)}{10a}-\frac{2}{21}ax^6\tanh^{-1}(ax)+\frac{1}{36}a^3x^8\tanh^{-1}(ax)+\frac{1}{5}x^5$$

$$=\frac{x^2\tanh^{-1}(ax)}{5a^3}-\frac{3x^4\tanh^{-1}(ax)}{70a}-\frac{11}{189}ax^6\tanh^{-1}(ax)+\frac{1}{36}a^3x^8$$

$$=\frac{293x}{1260a^4}+\frac{41x^3}{3780a^2}-\frac{17x^5}{1260}+\frac{a^2x^7}{252}-\frac{3x^2\tanh^{-1}(ax)}{35a^3}+\frac{4x^4\tanh^{-1}(ax)}{315a}$$

$$=-\frac{601x}{3780a^4}-\frac{277x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{293\tanh^{-1}(ax)}{1260a^5}+$$

$$=\frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}+\frac{601\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}$$

$$=\frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{29\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+$$

$$=\frac{29x}{3780a^4}-\frac{67x^3}{11340a^2}-\frac{23x^5}{3780}+\frac{a^2x^7}{252}-\frac{29\tanh^{-1}(ax)}{3780a^5}+\frac{8x^2\tanh^{-1}(ax)}{315a^3}+$$

Mathematica [A]
time = 1.41, size = 138, normalized size = 0.68

$$\frac{ax(87-67a^2x^2-69a^4x^4+45a^6x^6)+36(-8+63a^5x^5-90a^7x^7+35a^9x^9)\tanh^{-1}(ax)^2+3\tanh^{-1}(ax)\left(-29+96a^2x^2+48a^4x^4-220a^6x^6+105a^8x^8-192\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)+288\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{11340a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(87 - 67*a^2*x^2 - 69*a^4*x^4 + 45*a^6*x^6) + 36*(-8 + 63*a^5*x^5 - 90
*a^7*x^7 + 35*a^9*x^9)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-29 + 96*a^2*x^2 +
48*a^4*x^4 - 220*a^6*x^6 + 105*a^8*x^8 - 192*Log[1 + E^(-2*ArcTanh[a*x])])
+ 288*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(11340*a^5)
```

Maple [A]
time = 3.37, size = 233, normalized size = 1.15

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2 a^9 x^9}{9}-\frac{2\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax) a^8 x^8}{36}-\frac{11\operatorname{arctanh}(ax) a^6 x^6}{189}+\frac{4a^4x^4\operatorname{arctanh}(ax)}{315}+\frac{8a^2}{}$
default	$\frac{\operatorname{arctanh}(ax)^2 a^9 x^9}{9}-\frac{2\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax) a^8 x^8}{36}-\frac{11\operatorname{arctanh}(ax) a^6 x^6}{189}+\frac{4a^4x^4\operatorname{arctanh}(ax)}{315}+\frac{8a^2}{}$
risch	$-\frac{2\ln(-ax+1)^2}{315a^5}-\frac{78647\ln(-ax+1)}{396900a^5}+\frac{\ln(-ax+1)^2 x^5}{20}-\frac{\ln(-ax+1)x^5}{50}+\frac{a^2\ln(-ax+1)\ln(ax+1)x^7}{7}-\frac{a^4\ln(-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/9*arctanh(a*x)^2*a^9*x^9-2/7*arctanh(a*x)^2*a^7*x^7+1/5*arctanh(a*
x)^2*a^5*x^5+1/36*arctanh(a*x)*a^8*x^8-11/189*arctanh(a*x)*a^6*x^6+4/315*a^
4*x^4*arctanh(a*x)+8/315*a^2*x^2*arctanh(a*x)+8/315*arctanh(a*x)*ln(a*x-1)+
8/315*arctanh(a*x)*ln(a*x+1)+1/252*a^7*x^7-23/3780*a^5*x^5-67/11340*a^3*x^3
+29/3780*a*x+29/7560*ln(a*x-1)-29/7560*ln(a*x+1)+2/315*ln(a*x-1)^2-8/315*di
log(1/2*a*x+1/2)-4/315*ln(a*x-1)*ln(1/2*a*x+1/2)-2/315*ln(a*x+1)^2+4/315*(l
n(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2))
```

Maxima [A]
time = 0.26, size = 214, normalized size = 1.06

$$\frac{1}{22680}a^2\left(\frac{90\,a^7x^7-138\,a^5x^5-134\,a^3x^3+174\,ax-144\,\log(ax+1)^2+288\,\log(ax+1)\log(ax-1)+144\,\log(ax-1)^2+87\,\log(ax-1)}{a^7}-\frac{576\left(\log(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a^7}-\frac{87\,\log(ax+1)}{a^7}\right)+\frac{1}{3780}\,a\left(\frac{105\,a^6x^8-220\,a^4x^6+48\,a^2x^4+96\,x^2}{a^4}+\frac{96\,\log(ax+1)}{a^6}+\frac{96\,\log(ax-1)}{a^6}\right)\operatorname{artanh}(ax)+\frac{1}{315}\left(35\,a^4x^9-90\,a^2x^7+63\,x^5\right)\operatorname{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/22680*a^2*((90*a^7*x^7 - 138*a^5*x^5 - 134*a^3*x^3 + 174*a*x - 144*log(a*
x + 1)^2 + 288*log(a*x + 1)*log(a*x - 1) + 144*log(a*x - 1)^2 + 87*log(a*x
- 1))/a^7 - 576*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a
^7 - 87*log(a*x + 1)/a^7) + 1/3780*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x
^4 + 96*x^2)/a^4 + 96*log(a*x + 1)/a^6 + 96*log(a*x - 1)/a^6)*arctanh(a*x)
+ 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral((a^4*x^8 - 2*a^2*x^6 + x^4)*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(ax-1)^2\,(ax+1)^2\operatorname{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
[Out] Integral(x**4*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*x^4*arctanh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\mathrm{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
[Out] int(x^4*atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

3.204 $\int x^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=156

$$-\frac{5x^2}{504a^2}-\frac{x^4}{84}+\frac{a^2x^6}{168}+\frac{x\tanh^{-1}(ax)}{12a^3}+\frac{x^3\tanh^{-1}(ax)}{36a}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{28}a^3x^7\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{24a^4}$$

```
[Out] -5/504*x^2/a^2-1/84*x^4+1/168*a^2*x^6+1/12*x*arctanh(a*x)/a^3+1/36*x^3*arct
anh(a*x)/a-1/12*a*x^5*arctanh(a*x)+1/28*a^3*x^7*arctanh(a*x)-1/24*arctanh(a
*x)^2/a^4+1/4*x^4*arctanh(a*x)^2-1/3*a^2*x^6*arctanh(a*x)^2+1/8*a^4*x^8*arc
tanh(a*x)^2+2/63*ln(-a^2*x^2+1)/a^4
```

Rubi [A]
time = 0.57, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6159, 6037, 6127, 272, 45, 6021, 266, 6095}

$$\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{24a^4}+\frac{1}{28}a^3x^7\tanh^{-1}(ax)+\frac{x\tanh^{-1}(ax)}{12a^3}+\frac{a^2x^6}{168}-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2-\frac{5x^2}{504a^2}+\frac{2\log\left(1-a^2x^2\right)}{63a^4}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(ax)^2+\frac{x^3\tanh^{-1}(ax)}{36a}-\frac{x^4}{84}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-5*x^2)/(504*a^2) - x^4/84 + (a^2*x^6)/168 + (x*ArcTanh[a*x])/(12*a^3) + (
x^3*ArcTanh[a*x])/(36*a) - (a*x^5*ArcTanh[a*x])/12 + (a^3*x^7*ArcTanh[a*x])
/28 - ArcTanh[a*x]^2/(24*a^4) + (x^4*ArcTanh[a*x]^2)/4 - (a^2*x^6*ArcTanh[a
*x]^2)/3 + (a^4*x^8*ArcTanh[a*x]^2)/8 + (2*Log[1 - a^2*x^2])/(63*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^3(1-a^2x^2)^2\tanh^{-1}(ax)^2\,dx = \int \left(x^3\tanh^{-1}(ax)^2 - 2a^2x^5\tanh^{-1}(ax)^2 + a^4x^7\tanh^{-1}(ax)^2\right)\,dx$$
$$= -\left((2a^2)\int x^5\tanh^{-1}(ax)^2\,dx\right) + a^4\int x^7\tanh^{-1}(ax)^2\,dx + \int x^3\tanh$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2 - \frac{1}{3}a^2x^6\tanh^{-1}(ax)^2 + \frac{1}{8}a^4x^8\tanh^{-1}(ax)^2 - \frac{1}{2}a\int \frac{x^4}{}$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2 - \frac{1}{3}a^2x^6\tanh^{-1}(ax)^2 + \frac{1}{8}a^4x^8\tanh^{-1}(ax)^2 + \frac{\int x^2\tan}{}$$
$$= \frac{x^3\tanh^{-1}(ax)}{6a} - \frac{2}{15}ax^5\tanh^{-1}(ax) + \frac{1}{28}a^3x^7\tanh^{-1}(ax) + \frac{1}{4}x^4\tanh^{-1}$$
$$= \frac{x\tanh^{-1}(ax)}{2a^3} - \frac{x^3\tanh^{-1}(ax)}{18a} - \frac{1}{12}ax^5\tanh^{-1}(ax) + \frac{1}{28}a^3x^7\tanh^{-1}(a$$
$$= -\frac{x\tanh^{-1}(ax)}{6a^3} + \frac{x^3\tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5\tanh^{-1}(ax) + \frac{1}{28}a^3x^7\tanh^{-1}$$
$$= \frac{29x^2}{840a^2} - \frac{41x^4}{1680} + \frac{a^2x^6}{168} + \frac{x\tanh^{-1}(ax)}{12a^3} + \frac{x^3\tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5\tanh^{-}$$
$$= -\frac{13x^2}{252a^2} - \frac{x^4}{84} + \frac{a^2x^6}{168} + \frac{x\tanh^{-1}(ax)}{12a^3} + \frac{x^3\tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5\tanh^{-}$$
$$= -\frac{5x^2}{504a^2} - \frac{x^4}{84} + \frac{a^2x^6}{168} + \frac{x\tanh^{-1}(ax)}{12a^3} + \frac{x^3\tanh^{-1}(ax)}{36a} - \frac{1}{12}ax^5\tanh^{-}$$

Mathematica [A]
time = 0.03, size = 108, normalized size = 0.69

$$\frac{-5a^2x^2 - 6a^4x^4 + 3a^6x^6 + 2ax(21 + 7a^2x^2 - 21a^4x^4 + 9a^6x^6)\tanh^{-1}(ax) + 21(-1 + a^2x^2)^3(1 + 3a^2x^2)\tanh^{-1}(ax)^2 + 16\log(1 - a^2x^2)}{504a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-5*a^2*x^2 - 6*a^4*x^4 + 3*a^6*x^6 + 2*a*x*(21 + 7*a^2*x^2 - 21*a^4*x^4 +
9*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a^2*x^2)^3*(1 + 3*a^2*x^2)*ArcTanh[a*x]^
2 + 16*Log[1 - a^2*x^2])/(504*a^4)
```

Maple [A]
time = 1.05, size = 140, normalized size = 0.90

method	result

derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2a^8x^8}{8}-\frac{\operatorname{arctanh}(ax)^2a^6x^6}{3}+\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}-\frac{\operatorname{arctanh}(ax)^2}{24}+\frac{\operatorname{arctanh}(ax)a^7x^7}{28}-\frac{\operatorname{arctanh}(ax)a^5x^5}{12}+\frac{a^3x^3\operatorname{arctanh}(a}{36}}{a^4}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2a^8x^8}{8}-\frac{\operatorname{arctanh}(ax)^2a^6x^6}{3}+\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}-\frac{\operatorname{arctanh}(ax)^2}{24}+\frac{\operatorname{arctanh}(ax)a^7x^7}{28}-\frac{\operatorname{arctanh}(ax)a^5x^5}{12}+\frac{a^3x^3\operatorname{arctanh}(a}{36}}{a^4}$
risch	$\frac{(3a^8x^8-8a^6x^6+6a^4x^4-1)\ln(ax+1)^2}{96a^4}-\frac{(63a^8x^8\ln(-ax+1)-18a^7x^7-168a^6x^6\ln(-ax+1)+42a^5x^5+126x^4\ln(-ax+1}{1008a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/8*arctanh(a*x)^2*a^8*x^8-1/3*arctanh(a*x)^2*a^6*x^6+1/4*a^4*x^4*ar
ctanh(a*x)^2-1/24*arctanh(a*x)^2+1/28*arctanh(a*x)*a^7*x^7-1/12*arctanh(a*x
)*a^5*x^5+1/36*a^3*x^3*arctanh(a*x)+1/12*a*x*arctanh(a*x)+1/168*a^6*x^6-1/8
4*a^4*x^4-5/504*a^2*x^2+2/63*ln(a*x-1)+2/63*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 170, normalized size = 1.09

$$\frac{1}{504}\,a\left(\frac{2\,(9\,a^6x^7-21\,a^4x^5+7\,a^2x^3+21\,x)}{a^4}-\frac{21\,\log(ax+1)}{a^5}+\frac{21\,\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)+\frac{1}{24}\,(3\,a^4x^8-8\,a^2x^6+6\,x^4)\operatorname{artanh}(ax)^2+\frac{12\,a^6x^6-24\,a^4x^4-20\,a^2x^2-2\,(21\,\log(ax-1)-32)\log(ax+1)+21\,\log(ax+1)^2+21\,\log(ax-1)^2+64\,\log(ax-1)}{2016\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/504*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x + 1
)/a^5 + 21*log(a*x - 1)/a^5)*arctanh(a*x) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6
*x^4)*arctanh(a*x)^2 + 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 - 2*(21
*log(a*x - 1) - 32)*log(a*x + 1) + 21*log(a*x + 1)^2 + 21*log(a*x - 1)^2 +
64*log(a*x - 1))/a^4
```

Fricas [A]
time = 0.37, size = 133, normalized size = 0.85

$$\frac{12\,a^6x^6-24\,a^4x^4-20\,a^2x^2+21\,(3\,a^8x^8-8\,a^6x^6+6\,a^4x^4-1)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(9\,a^7x^7-21\,a^5x^5+7\,a^3x^3+21\,ax)\log\left(-\frac{ax+1}{ax-1}\right)+64\,\log(a^2x^2-1)}{2016\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 + 21*(3*a^8*x^8 - 8*a^6*x^6 +
6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(9*a^7*x^7 - 21*a^5*x^5 + 7*
a^3*x^3 + 21*a*x)*log(-(a*x + 1)/(a*x - 1)) + 64*log(a^2*x^2 - 1))/a^4
```

Sympy [A]
time = 0.75, size = 153, normalized size = 0.98

$$\begin{cases}\frac{a^4x^8\operatorname{atanh}^2(ax)}{8}+\frac{a^3x^7\operatorname{atanh}(ax)}{28}-\frac{a^2x^6\operatorname{atanh}^2(ax)}{3}+\frac{a^2x^6}{168}-\frac{ax^5\operatorname{atanh}(ax)}{12}+\frac{x^4\operatorname{atanh}^2(ax)}{4}-\frac{x^4}{84}+\frac{x^3\operatorname{atanh}(ax)}{36a}-\frac{5x^2}{504a^2}+\frac{x\operatorname{atanh}(ax)}{12a^3}+\frac{4\log\left(x-\frac{1}{a}\right)}{63a^4}-\frac{\operatorname{atanh}^2(ax)}{24a^4}+\frac{4\operatorname{atanh}(ax)}{63a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*x**8*atanh(a*x)**2/8 + a**3*x**7*atanh(a*x)/28 - a**2*x**6*
atanh(a*x)**2/3 + a**2*x**6/168 - a*x**5*atanh(a*x)/12 + x**4*atanh(a*x)**2
/4 - x**4/84 + x**3*atanh(a*x)/(36*a) - 5*x**2/(504*a**2) + x*atanh(a*x)/(1
2*a**3) + 4*log(x - 1/a)/(63*a**4) - atanh(a*x)**2/(24*a**4) + 4*atanh(a*x)
/(63*a**4), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(132) = 264$.
time = 0.42, size = 683, normalized size = 4.38

$$\frac{2}{63}\left(\frac{84\left(\frac{(ax+1)^5}{(ax-1)^5}+\frac{(ax+1)^4}{(ax-1)^4}+\frac{(ax+1)^3}{(ax-1)^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^8a^5}{(ax-1)^8}-\frac{8(ax+1)^7a^5}{(ax-1)^7}+\frac{28(ax+1)^6a^5}{(ax-1)^6}-\frac{56(ax+1)^5a^5}{(ax-1)^5}+\frac{70(ax+1)^4a^5}{(ax-1)^4}-\frac{56(ax+1)^3a^5}{(ax-1)^3}+\frac{28(ax+1)^2a^5}{(ax-1)^2}-\frac{8(ax+1)a^5}{ax-1}+a^5}+\frac{2\left(\frac{28(ax+1)^4}{(ax-1)^4}-\frac{7(ax+1)^3}{(ax-1)^3}+\frac{21(ax+1)^2}{(ax-1)^2}-\frac{7(ax+1)}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^7a^5}{(ax-1)^7}-\frac{7(ax+1)^6a^5}{(ax-1)^6}+\frac{21(ax+1)^5a^5}{(ax-1)^5}-\frac{35(ax+1)^4a^5}{(ax-1)^4}+\frac{35(ax+1)^3a^5}{(ax-1)^3}-\frac{21(ax+1)^2a^5}{(ax-1)^2}+\frac{7(ax+1)a^5}{ax-1}-a^5}-\frac{\frac{2(ax+1)^5}{(ax-1)^5}-\frac{11(ax+1)^4}{(ax-1)^4}+\frac{6(ax+1)^3}{(ax-1)^3}-\frac{11(ax+1)^2}{(ax-1)^2}+\frac{2(ax+1)}{ax-1}}{\frac{(ax+1)^6a^5}{(ax-1)^6}-\frac{6(ax+1)^5a^5}{(ax-1)^5}+\frac{15(ax+1)^4a^5}{(ax-1)^4}-\frac{20(ax+1)^3a^5}{(ax-1)^3}+\frac{15(ax+1)^2a^5}{(ax-1)^2}-\frac{6(ax+1)a^5}{ax-1}+a^5}-\frac{2\log\left(-\frac{ax+1}{ax-1}+1\right)}{a^5}+\frac{2\log\left(-\frac{ax+1}{ax-1}\right)}{a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] 2/63*(84*((a*x + 1)^5/(a*x - 1)^5 + (a*x + 1)^4/(a*x - 1)^4 + (a*x + 1)^3/(
a*x - 1)^3)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^8*a^5/(a*x - 1)^8 - 8*(a
*x + 1)^7*a^5/(a*x - 1)^7 + 28*(a*x + 1)^6*a^5/(a*x - 1)^6 - 56*(a*x + 1)^5
*a^5/(a*x - 1)^5 + 70*(a*x + 1)^4*a^5/(a*x - 1)^4 - 56*(a*x + 1)^3*a^5/(a*x
 - 1)^3 + 28*(a*x + 1)^2*a^5/(a*x - 1)^2 - 8*(a*x + 1)*a^5/(a*x - 1) + a^5)
 + 2*(28*(a*x + 1)^4/(a*x - 1)^4 - 7*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)
^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x - 1) + 1)*log(-(a*x + 1)/(a*x - 1))/((a*x
 + 1)^7*a^5/(a*x - 1)^7 - 7*(a*x + 1)^6*a^5/(a*x - 1)^6 + 21*(a*x + 1)^5*a^
5/(a*x - 1)^5 - 35*(a*x + 1)^4*a^5/(a*x - 1)^4 + 35*(a*x + 1)^3*a^5/(a*x -
1)^3 - 21*(a*x + 1)^2*a^5/(a*x - 1)^2 + 7*(a*x + 1)*a^5/(a*x - 1) - a^5) -
(2*(a*x + 1)^5/(a*x - 1)^5 - 11*(a*x + 1)^4/(a*x - 1)^4 + 6*(a*x + 1)^3/(a*
x - 1)^3 - 11*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1))/((a*x + 1)^6
*a^5/(a*x - 1)^6 - 6*(a*x + 1)^5*a^5/(a*x - 1)^5 + 15*(a*x + 1)^4*a^5/(a*x
- 1)^4 - 20*(a*x + 1)^3*a^5/(a*x - 1)^3 + 15*(a*x + 1)^2*a^5/(a*x - 1)^2 -
6*(a*x + 1)*a^5/(a*x - 1) + a^5) - 2*log(-(a*x + 1)/(a*x - 1) + 1)/a^5 + 2*
log(-(a*x + 1)/(a*x - 1))/a^5)*a
```

Mupad [B]
time = 1.22, size = 221, normalized size = 1.42

$$\frac{2\ln\left(a^2x^2-1\right)}{63a^4}-\ln\left(1-ax\right)^2\left(\frac{1}{96a^4}-\frac{x^4}{16}+\frac{a^2x^6}{12}-\frac{a^4x^8}{32}\right)-\frac{x^4}{84}-\ln\left(ax+1\right)^2\left(\frac{1}{96a^4}-\frac{x^4}{16}+\frac{a^2x^6}{12}-\frac{a^4x^8}{32}\right)-\ln\left(1-ax\right)\left(\frac{x}{24a^3}-\ln\left(ax+1\right)\left(\frac{1}{48a^4}-\frac{x^4}{8}+\frac{a^2x^6}{6}-\frac{a^4x^8}{16}\right)-\frac{ax^5}{24}+\frac{x^3}{72a}+\frac{a^3x^7}{56}\right)-\frac{5x^2}{504a^2}+\frac{a^2x^6}{168}+a\ln\left(ax+1\right)\left(\frac{x}{24a^4}-\frac{x^5}{24}+\frac{x^3}{72a^2}+\frac{a^2x^7}{56}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] (2*log(a^2*x^2 - 1))/(63*a^4) - log(1 - a*x)^2*(1/(96*a^4) - x^4/16 + (a^2*
x^6)/12 - (a^4*x^8)/32) - x^4/84 - log(a*x + 1)^2*(1/(96*a^4) - x^4/16 + (a
```

```
^2*x^6)/12 - (a^4*x^8)/32) - log(1 - a*x)*(x/(24*a^3) - log(a*x + 1)*(1/(48
*a^4) - x^4/8 + (a^2*x^6)/6 - (a^4*x^8)/16) - (a*x^5)/24 + x^3/(72*a) + (a^
3*x^7)/56) - (5*x^2)/(504*a^2) + (a^2*x^6)/168 + a*log(a*x + 1)*(x/(24*a^4)
 - x^5/24 + x^3/(72*a^2) + (a^2*x^7)/56)
```

3.205 $\int x^2(1-a^2x^2)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=178

$$-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)+\frac{8\tanh^{-1}(a$$

```
[Out] -1/210*x/a^2-17/630*x^3+1/105*a^2*x^5+1/210*arctanh(a*x)/a^3+8/105*x^2*arct
anh(a*x)/a-9/70*a*x^4*arctanh(a*x)+1/21*a^3*x^6*arctanh(a*x)+8/105*arctanh(
a*x)^2/a^3+1/3*x^3*arctanh(a*x)^2-2/5*a^2*x^5*arctanh(a*x)^2+1/7*a^4*x^7*ar
ctanh(a*x)^2-16/105*arctanh(a*x)*ln(2/(-a*x+1))/a^3-8/105*polylog(2,1-2/(-a
*x+1))/a^3
```

Rubi [A]
time = 0.53, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {6159, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352, 308}

$$\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2-\frac{8\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{105a^3}+\frac{1}{21}a^3x^6\tanh^{-1}(ax)+\frac{8\tanh^{-1}(ax)^2}{105a^3}+\frac{\tanh^{-1}(ax)}{210a^3}-\frac{16\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{105a^3}+\frac{a^2x^5}{105}-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2-\frac{x}{210a^2}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{3}x^3\tanh^{-1}(ax)^2+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{17x^3}{630}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] -1/210*x/a^2 - (17*x^3)/630 + (a^2*x^5)/105 + ArcTanh[a*x]/(210*a^3) + (8*x
^2*ArcTanh[a*x])/(105*a) - (9*a*x^4*ArcTanh[a*x])/70 + (a^3*x^6*ArcTanh[a*x
])/21 + (8*ArcTanh[a*x]^2)/(105*a^3) + (x^3*ArcTanh[a*x]^2)/3 - (2*a^2*x^5*
ArcTanh[a*x]^2)/5 + (a^4*x^7*ArcTanh[a*x]^2)/7 - (16*ArcTanh[a*x]*Log[2/(1
- a*x)])/(105*a^3) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(105*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int x^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx=\int\left(x^2\tanh^{-1}(ax)^2-2a^2x^4\tanh^{-1}(ax)^2+a^4x^6\tanh^{-1}(ax)^2\right)\,dx$$

$$=-\left(\left(2a^2\right)\int x^4\tanh^{-1}(ax)^2\,dx\right)+a^4\int x^6\tanh^{-1}(ax)^2\,dx+\int x^2\tanh$$

$$=\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2-\frac{1}{3}(2a)\int$$

$$=\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2+\frac{2\int x\tan}{}$$

$$=\frac{x^2\tanh^{-1}(ax)}{3a}-\frac{1}{5}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{3a^3}$$

$$=\frac{x}{3a^2}-\frac{x^2\tanh^{-1}(ax)}{15a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)-\frac{\tanh}{}$$

$$=-\frac{23x}{105a^2}-\frac{16x^3}{315}+\frac{a^2x^5}{105}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh$$

$$=-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{23\tanh^{-1}(ax)}{105a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh$$

$$=-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh$$

$$=-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh$$

Mathematica [A]
time = 0.80, size = 121, normalized size = 0.68

$$\frac{ax(-3-17a^2x^2+6a^4x^4)+6(-8+35a^3x^3-42a^5x^5+15a^7x^7)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(3+48a^2x^2-81a^4x^4+30a^6x^6-96\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)+48\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{630a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(-3 - 17*a^2*x^2 + 6*a^4*x^4) + 6*(-8 + 35*a^3*x^3 - 42*a^5*x^5 + 15*a
^7*x^7)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(3 + 48*a^2*x^2 - 81*a^4*x^4 + 30*a^6
```

```
*x^6 - 96*Log[1 + E^(-2*ArcTanh[a*x])]) + 48*PolyLog[2, -E^(-2*ArcTanh[a*x]
)])/(630*a^3)
```

Maple [A]
time = 1.35, size = 213, normalized size = 1.20

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}-\frac{2\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}-\frac{9a^4x^4\operatorname{arctanh}(ax)}{70}+\frac{8a^2x^2\operatorname{arctanh}(ax)}{105}+8\,\mathrm{arctan}$
default	$\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}-\frac{2\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}-\frac{9a^4x^4\operatorname{arctanh}(ax)}{70}+\frac{8a^2x^2\operatorname{arctanh}(ax)}{105}+8\,\mathrm{arctan}$
risch	$-\frac{177151}{2315250a^3}-\frac{17x^3}{630}-\frac{\left(\left(-\frac{1}{9}+\frac{\ln(ax+1)}{3}\right)(ax+1)^3+\left(\frac{1}{2}-\ln(ax+1)\right)(ax+1)^2+(-1+\ln(ax+1))(ax+1)\right)\ln(-ax+1)}{2a^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/7*arctanh(a*x)^2*a^7*x^7-2/5*arctanh(a*x)^2*a^5*x^5+1/3*arctanh(a*
x)^2*a^3*x^3+1/21*arctanh(a*x)*a^6*x^6-9/70*a^4*x^4*arctanh(a*x)+8/105*a^2*
x^2*arctanh(a*x)+8/105*arctanh(a*x)*ln(a*x-1)+8/105*arctanh(a*x)*ln(a*x+1)+
1/105*a^5*x^5-17/630*a^3*x^3-1/210*a*x-1/420*ln(a*x-1)+1/420*ln(a*x+1)+2/10
5*ln(a*x-1)^2-8/105*dilog(1/2*a*x+1/2)-4/105*ln(a*x-1)*ln(1/2*a*x+1/2)+4/10
5*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-2/105*ln(a*x+1)^2)
```

Maxima [A]
time = 0.27, size = 198, normalized size = 1.11

$$\frac{1}{1260}a^2\left(\frac{12a^5x^5-34a^3x^3-6ax-24\log(ax+1)^2+48\log(ax+1)\log(ax-1)+24\log(ax-1)^2-3\log(ax-1)}{a^5}-\frac{96\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^5}+\frac{3\log(ax+1)}{a^5}\right)+\frac{1}{210}a\left(\frac{10a^4x^6-27a^2x^4+16x^2}{a^2}+\frac{16\log(ax+1)}{a^4}+\frac{16\log(ax-1)}{a^4}\right)\operatorname{artanh}(ax)+\frac{1}{105}\left(15a^4x^7-42a^2x^5+35x^3\right)\operatorname{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/1260*a^2*((12*a^5*x^5 - 34*a^3*x^3 - 6*a*x - 24*log(a*x + 1)^2 + 48*log(a
*x + 1)*log(a*x - 1) + 24*log(a*x - 1)^2 - 3*log(a*x - 1))/a^5 - 96*(log(a*
x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 + 3*log(a*x + 1)/a^5
) + 1/210*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 +
 16*log(a*x - 1)/a^4)*arctanh(a*x) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^
3)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^6 - 2*a^2*x^4 + x^2)*arctanh(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral(x**2*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x^2*arctanh(a*x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] int(x^2*atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

3.206 $\int x(1-a^2x^2)^2 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=138

$$\frac{2(1-a^2x^2)}{45a^2}+\frac{(1-a^2x^2)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x(1-a^2x^2)\tanh^{-1}(ax)}{45a}+\frac{x(1-a^2x^2)^2\tanh^{-1}(ax)}{15a}-\frac{(1-a^2x^2)}{}$$

```
[Out] 2/45*(-a^2*x^2+1)/a^2+1/60*(-a^2*x^2+1)^2/a^2+8/45*x*arctanh(a*x)/a+4/45*x*
(-a^2*x^2+1)*arctanh(a*x)/a+1/15*x*(-a^2*x^2+1)^2*arctanh(a*x)/a-1/6*(-a^2*
x^2+1)^3*arctanh(a*x)^2/a^2+4/45*ln(-a^2*x^2+1)/a^2
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6141, 6089, 6021, 266}

$$\frac{(1-a^2x^2)^2}{60a^2}+\frac{2(1-a^2x^2)}{45a^2}+\frac{4\log(1-a^2x^2)}{45a^2}-\frac{(1-a^2x^2)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{x(1-a^2x^2)^2\tanh^{-1}(ax)}{15a}+\frac{4x(1-a^2x^2)\tanh^{-1}(ax)}{45a}+\frac{8x\tanh^{-1}(ax)}{45a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (2*(1 - a^2*x^2))/(45*a^2) + (1 - a^2*x^2)^2/(60*a^2) + (8*x*ArcTanh[a*x])/
(45*a) + (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/(45*a) + (x*(1 - a^2*x^2)^2*ArcTa
nh[a*x])/(15*a) - ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(6*a^2) + (4*Log[1 - a^2
*x^2])/(45*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx = -\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{\int\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx}{3a}$$
$$= \frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{4}{\cdots}$$
$$= \frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x\left(1-a^2x^2\right)^2\tanh\cdots}{15a}$$
$$= \frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}\cdots$$
$$= \frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}\cdots$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 0.59

$$\frac{-14a^2x^2+3a^4x^4+4ax\left(15-10a^2x^2+3a^4x^4\right)\tanh^{-1}(ax)+30\left(-1+a^2x^2\right)^3\tanh^{-1}(ax)^2+16\log\left(1-a^2x^2\right)}{180a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-14*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 10*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x]
 + 30*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2 + 16*Log[1 - a^2*x^2])/(180*a^2)
```

Maple [A]
time = 0.62, size = 120, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2a^6x^6}{6}-\frac{a^4x^4\operatorname{arctanh}(ax)^2}{2}+\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{6}+\frac{\operatorname{arctanh}(ax)a^5x^5}{15}-\frac{2a^3x^3\operatorname{arctanh}(ax)}{9}+\frac{ax\operatorname{arctanh}(\cdots}{3}}{a^2}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2a^6x^6}{6}-\frac{a^4x^4\operatorname{arctanh}(ax)^2}{2}+\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{6}+\frac{\operatorname{arctanh}(ax)a^5x^5}{15}-\frac{2a^3x^3\operatorname{arctanh}(ax)}{9}+\frac{ax\operatorname{arctanh}(\cdots}{3}}{a^2}$

risch	$\frac{(a^2x^2-1)^3 \ln(ax+1)^2}{24a^2} - \frac{(15a^6x^6 \ln(-ax+1) - 6a^5x^5 - 45x^4 \ln(-ax+1)a^4 + 20a^3x^3 + 45x^2 \ln(-ax+1)a^2 - 30ax - 15 \ln(-}{180a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/a^2*(1/6*arctanh(a*x)^2*a^6*x^6-1/2*a^4*x^4*arctanh(a*x)^2+1/2*a^2*x^2*arctanh(a*x)^2-1/6*arctanh(a*x)^2+1/15*arctanh(a*x)*a^5*x^5-2/9*a^3*x^3*arctanh(a*x)+1/3*a*x*arctanh(a*x)+1/60*a^4*x^4-7/90*a^2*x^2+4/45*ln(a*x-1)+4/45*ln(a*x+1))

Maxima [A]
time = 0.26, size = 93, normalized size = 0.67

$$\frac{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}{6a^2} + \frac{\left(3a^2x^4 - 14x^2 + \frac{16\log(ax+1)}{a^2} + \frac{16\log(ax-1)}{a^2}\right)a + 4(3a^4x^5 - 10a^2x^3 + 15x)\operatorname{artanh}(ax)}{180a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")

[Out] 1/6*(a^2*x^2 - 1)^3*arctanh(a*x)^2/a^2 + 1/180*((3*a^2*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 4*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x))/a

Fricas [A]
time = 0.35, size = 116, normalized size = 0.84

$$\frac{6a^4x^4 - 28a^2x^2 + 15(a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 4(3a^5x^5 - 10a^3x^3 + 15ax)\log\left(-\frac{ax+1}{ax-1}\right) + 32\log(a^2x^2-1)}{360a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")

[Out] 1/360*(6*a^4*x^4 - 28*a^2*x^2 + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*x + 1)/(a*x - 1)) + 32*log(a^2*x^2 - 1))/a^2

Sympy [A]
time = 0.51, size = 133, normalized size = 0.96

$$\begin{cases} \frac{a^4x^6 \operatorname{atanh}^2(ax)}{6} + \frac{a^3x^5 \operatorname{atanh}(ax)}{15} - \frac{a^2x^4 \operatorname{atanh}^2(ax)}{2} + \frac{a^2x^4}{60} - \frac{2ax^3 \operatorname{atanh}(ax)}{9} + \frac{x^2 \operatorname{atanh}^2(ax)}{2} - \frac{7x^2}{90} + \frac{x \operatorname{atanh}(ax)}{3a} + \frac{8\log\left(x-\frac{1}{a}\right)}{45a^2} - \frac{\operatorname{atanh}^2(ax)}{6a^2} + \frac{8\operatorname{atanh}(ax)}{45a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-a**2*x**2+1)**2*atanh(a*x)**2,x)

[Out] Piecewise((a**4*x**6*atanh(a*x)**2/6 + a**3*x**5*atanh(a*x)/15 - a**2*x**4*atanh(a*x)**2/2 + a**2*x**4/60 - 2*a*x**3*atanh(a*x)/9 + x**2*atanh(a*x)**2

```
/2 - 7*x**2/90 + x*atanh(a*x)/(3*a) + 8*log(x - 1/a)/(45*a**2) - atanh(a*x)
**2/(6*a**2) + 8*atanh(a*x)/(45*a**2), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(119) = 238.

time = 0.41, size = 473, normalized size = 3.43

$$\frac{4}{45}\,a\left(\frac{2\left(\frac{10\,(ax+1)^2}{(ax-1)^2}-\frac{5\,(ax+1)}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^5a^3}{(ax-1)^5}-\frac{5\,(ax+1)^4a^3}{(ax-1)^4}+\frac{10\,(ax+1)^3a^3}{(ax-1)^3}-\frac{10\,(ax+1)^2a^3}{(ax-1)^2}+\frac{5\,(ax+1)a^3}{ax-1}-a^3}+\frac{30\,(ax+1)^3\log\left(-\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^6a^3}{(ax-1)^6}-\frac{6\,(ax+1)^5a^3}{(ax-1)^5}+\frac{15\,(ax+1)^4a^3}{(ax-1)^4}-\frac{20\,(ax+1)^3a^3}{(ax-1)^3}+\frac{15\,(ax+1)^2a^3}{(ax-1)^2}-\frac{6\,(ax+1)a^3}{ax-1}+a^3\right)(ax-1)^3}-\frac{\frac{2\,(ax+1)^3}{(ax-1)^3}-\frac{7\,(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}}{\frac{(ax+1)^4a^3}{(ax-1)^4}-\frac{4\,(ax+1)^3a^3}{(ax-1)^3}+\frac{6\,(ax+1)^2a^3}{(ax-1)^2}-\frac{4\,(ax+1)a^3}{ax-1}+a^3}-\frac{2\,\log\left(-\frac{ax+1}{ax-1}+1\right)}{a^3}+\frac{2\,\log\left(-\frac{ax+1}{ax-1}\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] 4/45*a*(2*(10*(a*x + 1)^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)*log(-(a*
x + 1)/(a*x - 1))/((a*x + 1)^5*a^3/(a*x - 1)^5 - 5*(a*x + 1)^4*a^3/(a*x - 1
)^4 + 10*(a*x + 1)^3*a^3/(a*x - 1)^3 - 10*(a*x + 1)^2*a^3/(a*x - 1)^2 + 5*(
a*x + 1)*a^3/(a*x - 1) - a^3) + 30*(a*x + 1)^3*log(-(a*x + 1)/(a*x - 1))^2/
(((a*x + 1)^6*a^3/(a*x - 1)^6 - 6*(a*x + 1)^5*a^3/(a*x - 1)^5 + 15*(a*x + 1
)^4*a^3/(a*x - 1)^4 - 20*(a*x + 1)^3*a^3/(a*x - 1)^3 + 15*(a*x + 1)^2*a^3/(
a*x - 1)^2 - 6*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)^3) - (2*(a*x + 1)^3
/(a*x - 1)^3 - 7*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1))/((a*x + 1
)^4*a^3/(a*x - 1)^4 - 4*(a*x + 1)^3*a^3/(a*x - 1)^3 + 6*(a*x + 1)^2*a^3/(a*
x - 1)^2 - 4*(a*x + 1)*a^3/(a*x - 1) + a^3) - 2*log(-(a*x + 1)/(a*x - 1) +
1)/a^3 + 2*log(-(a*x + 1)/(a*x - 1))/a^3)
```

Mupad [B]

time = 1.00, size = 111, normalized size = 0.80

$$\frac{x^2\,\mathrm{atanh}(a\,x)^2}{2}-\frac{\mathrm{atanh}(a\,x)^2}{6\,a^2}-\frac{7\,x^2}{90}+\frac{4\,\ln\left(a^2\,x^2-1\right)}{45\,a^2}+\frac{a^2\,x^4}{60}+\frac{x\,\mathrm{atanh}(a\,x)}{3\,a}-\frac{2\,a\,x^3\,\mathrm{atanh}(a\,x)}{9}+\frac{a^3\,x^5\,\mathrm{atanh}(a\,x)}{15}-\frac{a^2\,x^4\,\mathrm{atanh}(a\,x)^2}{2}+\frac{a^4\,x^6\,\mathrm{atanh}(a\,x)^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] (x^2*atanh(a*x)^2)/2 - atanh(a*x)^2/(6*a^2) - (7*x^2)/90 + (4*log(a^2*x^2 -
 1))/(45*a^2) + (a^2*x^4)/60 + (x*atanh(a*x))/(3*a) - (2*a*x^3*atanh(a*x))/
9 + (a^3*x^5*atanh(a*x))/15 - (a^2*x^4*atanh(a*x)^2)/2 + (a^4*x^6*atanh(a*x
)^2)/6
```

3.207 $\int (1-a^2x^2)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=171

$$-\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a}+\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{10a}+\frac{8\tanh^{-1}(ax)^2}{15a}+\frac{8}{15}x\tanh^{-1}(ax)^2+\frac{4}{15}x(1-$$

```
[Out] -11/30*x+1/30*a^2*x^3+4/15*(-a^2*x^2+1)*arctanh(a*x)/a+1/10*(-a^2*x^2+1)^2*
arctanh(a*x)/a+8/15*arctanh(a*x)^2/a+8/15*x*arctanh(a*x)^2+4/15*x*(-a^2*x^2
+1)*arctanh(a*x)^2+1/5*x*(-a^2*x^2+1)^2*arctanh(a*x)^2-16/15*arctanh(a*x)*l
n(2/(-a*x+1))/a-8/15*polylog(2,1-2/(-a*x+1))/a
```

Rubi [A]
time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6091, 6021, 6131, 6055, 2449, 2352, 8}

$$\frac{a^2x^3}{30}+\frac{1}{5}x(1-a^2x^2)^2\tanh^{-1}(ax)^2+\frac{4}{15}x(1-a^2x^2)\tanh^{-1}(ax)^2+\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{10a}+\frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a}-\frac{8\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{15a}+\frac{8}{15}x\tanh^{-1}(ax)^2+\frac{8\tanh^{-1}(ax)^2}{15a}-\frac{16\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{15a}-\frac{11x}{30}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-11*x)/30 + (a^2*x^3)/30 + (4*(1 - a^2*x^2)*ArcTanh[a*x])/(15*a) + ((1 - a
^2*x^2)^2*ArcTanh[a*x])/(10*a) + (8*ArcTanh[a*x]^2)/(15*a) + (8*x*ArcTanh[a
*x]^2)/15 + (4*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/15 + (x*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/5 - (16*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a) - (8*PolyLog[2, 1
 - 2/(1 - a*x)])/(15*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 \, dx = \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{10a} + \frac{1}{5}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 - \frac{1}{10}\int \left(1-a^2x^2\right) c$$

$$= -\frac{x}{10} + \frac{a^2x^3}{30} + \frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{4}{15}x(1$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8}{15}x\,t$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tan}{}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tan}{}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tan}{}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4(1-a^2x^2)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tan}{}$$

Mathematica [A]
time = 0.46, size = 99, normalized size = 0.58

$$\frac{ax(-11+a^2x^2)+2(-1+ax)^3(8+9ax+3a^2x^2)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(11-14a^2x^2+3a^4x^4-32\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)+16\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(-11 + a^2*x^2) + 2*(-1 + a*x)^3*(8 + 9*a*x + 3*a^2*x^2)*ArcTanh[a*x]^
2 + ArcTanh[a*x]*(11 - 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^(-2*ArcTanh[a*
x])]) + 16*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(30*a)
```

Maple [A]
time = 1.01, size = 188, normalized size = 1.10

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2 a^5x^5}{5} - \frac{2\operatorname{arctanh}(ax)^2 a^3x^3}{3} + \operatorname{arctanh}(ax)^2 ax + \frac{a^4x^4\operatorname{arctanh}(ax)}{10} - \frac{7a^2x^2\operatorname{arctanh}(ax)}{15} + \frac{8\operatorname{arctanh}(ax)\ln(ax-1)}{15} + \frac{8\,\mathrm{ar}}{}$
default	$\frac{\operatorname{arctanh}(ax)^2 a^5x^5}{5} - \frac{2\operatorname{arctanh}(ax)^2 a^3x^3}{3} + \operatorname{arctanh}(ax)^2 ax + \frac{a^4x^4\operatorname{arctanh}(ax)}{10} - \frac{7a^2x^2\operatorname{arctanh}(ax)}{15} + \frac{8\operatorname{arctanh}(ax)\ln(ax-1)}{15} + \frac{8\,\mathrm{ar}}{}$
risch	$-\frac{11x}{30} - \frac{94\ln(ax-1)}{225a} - \frac{a^4\ln(-ax+1)\ln(ax+1)x^5}{10} + \frac{a^2x^3}{30} + \frac{a^2\ln(-ax+1)\ln(ax+1)x^3}{3} - \frac{(-1+\ln(ax+1))(ax+}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/5*arctanh(a*x)^2*a^5*x^5-2/3*arctanh(a*x)^2*a^3*x^3+arctanh(a*x)^2*a
*x+1/10*a^4*x^4*arctanh(a*x)-7/15*a^2*x^2*arctanh(a*x)+8/15*arctanh(a*x)*ln
(a*x-1)+8/15*arctanh(a*x)*ln(a*x+1)+1/30*a^3*x^3-11/30*a*x-11/60*ln(a*x-1)+
11/60*ln(a*x+1)+2/15*ln(a*x-1)^2-8/15*dilog(1/2*a*x+1/2)-4/15*ln(a*x-1)*ln(
1/2*a*x+1/2)+4/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-2/15*ln(a*x+
1)^2)
```

Maxima [A]
time = 0.26, size = 175, normalized size = 1.02

$$\frac{1}{60}\,a^2\left(\frac{2\,a^3x^3-22\,ax-8\,\log\left(ax+1\right)^2+16\,\log\left(ax+1\right)\log\left(ax-1\right)+8\,\log\left(ax-1\right)^2-11\,\log\left(ax-1\right)}{a^3}-\frac{32\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a^3}+\frac{11\,\log\left(ax+1\right)}{a^3}\right)+\frac{1}{30}\left(3\,a^2x^4-14\,x^2+\frac{16\,\log\left(ax+1\right)}{a^2}+\frac{16\,\log\left(ax-1\right)}{a^2}\right)a\,\mathrm{artanh}\left(ax\right)+\frac{1}{15}\left(3\,a^4x^5-10\,a^2x^3+15\,x\right)\mathrm{artanh}\left(ax\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/60*a^2*((2*a^3*x^3 - 22*a*x - 8*log(a*x + 1)^2 + 16*log(a*x + 1)*log(a*x
 - 1) + 8*log(a*x - 1)^2 - 11*log(a*x - 1))/a^3 - 32*(log(a*x - 1)*log(1/2*a
*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 + 11*log(a*x + 1)/a^3) + 1/30*(3*a^2
*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a*arctanh(a*x) +
 1/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax-1\right)^2\left(ax+1\right)^2\,\mathrm{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

```
[Out] int(atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

3.208 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x} \, dx$

Optimal. Leaf size=186

$$\frac{a^2x^2}{12}-\frac{3}{2}ax\tanh^{-1}(ax)+\frac{1}{6}a^3x^3\tanh^{-1}(ax)+\frac{3}{4}\tanh^{-1}(ax)^2-a^2x^2\tanh^{-1}(ax)^2+\frac{1}{4}a^4x^4\tanh^{-1}(ax)^2+2\tanh$$

```
[Out] 1/12*a^2*x^2-3/2*a*x*arctanh(a*x)+1/6*a^3*x^3*arctanh(a*x)+3/4*arctanh(a*x)
^2-a^2*x^2*arctanh(a*x)^2+1/4*a^4*x^4*arctanh(a*x)^2-2*arctanh(a*x)^2*arcta
nh(-1+2/(-a*x+1))-2/3*ln(-a^2*x^2+1)-arctanh(a*x)*polylog(2,1-2/(-a*x+1))+a
rctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*polylog(3,1-2/(-a*x+1))-1/2*polylo
g(3,-1+2/(-a*x+1))
```

Rubi [A]
time = 0.36, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$,
Rules used = {6159, 6033, 6199, 6095, 6205, 6745, 6037, 6127, 6021, 266, 272, 45}

$$\frac{1}{4}a^4x^4\tanh^{-1}(ax)^2+\frac{1}{6}a^3x^3\tanh^{-1}(ax)+\frac{a^2x^2}{12}-\frac{2}{3}\log\left(1-a^2x^2\right)-a^2x^2\tanh^{-1}(ax)^2+\frac{1}{2}\text{Li}_3\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{Li}_3\left(\frac{2}{1-ax}-1\right)-\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)+\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)-\frac{3}{2}ax\tanh^{-1}(ax)+\frac{3}{4}\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

```
[Out] (a^2*x^2)/12 - (3*a*x*ArcTanh[a*x])/2 + (a^3*x^3*ArcTanh[a*x])/6 + (3*ArcTa
nh[a*x]^2)/4 - a^2*x^2*ArcTanh[a*x]^2 + (a^4*x^4*ArcTanh[a*x]^2)/4 + 2*ArcT
anh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 - ArcTanh[a*x]
*PolyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] +
PolyLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + a^4x^3\tanh^{-1}(ax)^2\right)dx$$

$$= -\left((2a^2)\int x\tanh^{-1}(ax)^2\,dx\right) + a^4\int x^3\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax}{x}$$

$$= -a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{}{1}\right.$$

$$= -a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1 - \frac{}{1}\right.$$

$$= -2ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2$$

$$= -\frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax$$

$$= -\frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax$$

$$= \frac{a^2x^2}{12} - \frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh$$

Mathematica [A]

time = 0.05, size = 191, normalized size = 1.03

$$\frac{a^2x^2}{12} - 2ax\tanh^{-1}(ax) + \frac{1}{6}ax(3+a^2x^2)\tanh^{-1}(ax) - (-1+a^2x^2)\tanh^{-1}(ax)^2 + \frac{1}{4}(-1+a^4x^4)\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) - \frac{2}{3}\log(1-a^2x^2) + \tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{-1-ax}{-1+ax}\right) - \tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{1+ax}{-1+ax}\right) - \frac{1}{2}\text{PolyLog}\left(3,\frac{-1-ax}{-1+ax}\right) + \frac{1}{2}\text{PolyLog}\left(3,\frac{1+ax}{-1+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

```
[Out] (a^2*x^2)/12 - 2*a*x*ArcTanh[a*x] + (a*x*(3 + a^2*x^2)*ArcTanh[a*x])/6 - (-
1 + a^2*x^2)*ArcTanh[a*x]^2 + ((-1 + a^4*x^4)*ArcTanh[a*x]^2)/4 + 2*ArcTanh
[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 + ArcTanh[a*x]*Po
lyLog[2, (-1 - a*x)/(-1 + a*x)] - ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a
*x)] - PolyLog[3, (-1 - a*x)/(-1 + a*x)]/2 + PolyLog[3, (1 + a*x)/(-1 + a*x
)]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 21.44, size = 733, normalized size = 3.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^4*x^4*arctanh(a*x)^2-a^2*x^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-ar
ctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polyl
og(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(
1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,-(a
*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1
/2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+
1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^2+1/2*(a*x-3)*(a*x+1)*arctanh(a*x)+1/6*(a^2*x^2-4*a*x+7)*(a*x+1)*a
rctanh(a*x)+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^3+1/6*a*x-1/6+4/3*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/4*a
rctanh(a*x)^2+1/12*(a*x-1)^2+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^
2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2
+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-(a*x+1)*arctanh(a*x)-1/2*I*arctanh(a*x)^
2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/16*(a^4*x^4 - 4*a^2*x^2)*log(-a*x + 1)^2 - 1/4*integrate(-1/2*(2*(a^5*x^5
 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (a^5*x^5 - 4
*a^3*x^3 + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
+ 1))*log(-a*x + 1))/(a*x^2 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^2\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x, x)
```

3.209 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=156

$$\frac{a^2x}{3}-\frac{1}{3}a\tanh^{-1}(ax)+\frac{1}{3}a^3x^2\tanh^{-1}(ax)-\frac{2}{3}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}-2a^2x\tanh^{-1}(ax)^2+\frac{1}{3}a^4x^3\tanh^{-1}(a$$

```
[Out] 1/3*a^2*x-1/3*a*arctanh(a*x)+1/3*a^3*x^2*arctanh(a*x)-2/3*a*arctanh(a*x)^2-
arctanh(a*x)^2/x-2*a^2*x*arctanh(a*x)^2+1/3*a^4*x^3*arctanh(a*x)^2+10/3*a*a
rctanh(a*x)*ln(2/(-a*x+1))+2*a*arctanh(a*x)*ln(2-2/(a*x+1))+5/3*a*polylog(2
,1-2/(-a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A]

time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6159, 6021, 6131, 6055, 2449, 2352, 6037, 6135, 6079, 2497, 6127, 327, 212}

$$\frac{1}{3}a^4x^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3x^2\tanh^{-1}(ax)+\frac{a^2x}{3}-2a^2x\tanh^{-1}(ax)^2+\frac{5}{3}a\text{Li}_2\left(1-\frac{2}{1-ax}\right)-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{3}a\tanh^{-1}(ax)^2-\frac{1}{3}a\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{x}+\frac{10}{3}a\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] (a^2*x)/3 - (a*ArcTanh[a*x])/3 + (a^3*x^2*ArcTanh[a*x])/3 - (2*a*ArcTanh[a*
x]^2)/3 - ArcTanh[a*x]^2/x - 2*a^2*x*ArcTanh[a*x]^2 + (a^4*x^3*ArcTanh[a*x]
^2)/3 + (10*a*ArcTanh[a*x]*Log[2/(1 - a*x)])/3 + 2*a*ArcTanh[a*x]*Log[2 - 2
/(1 + a*x)] + (5*a*PolyLog[2, 1 - 2/(1 - a*x)])/3 - a*PolyLog[2, -1 + 2/(1
+ a*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^2}\,dx = \int \left(-2a^2\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{x^2} + a^4x^2\tanh^{-1}(ax)^2\right)dx$$

$$= -\left((2a^2)\int \tanh^{-1}(ax)^2\,dx\right) + a^4\int x^2\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2}$$

$$= -\frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3\tanh^{-1}(ax)^2 + (2a)\int \frac{\tanh^{-}}{x\,(1-}$$

$$= -a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3\tanh^{-1}(ax)^2$$

$$= \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2$$

$$= \frac{a^2x}{3} + \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}$$

$$= \frac{a^2x}{3} - \frac{1}{3}a\tanh^{-1}(ax) + \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(a}{x}$$

$$= \frac{a^2x}{3} - \frac{1}{3}a\tanh^{-1}(ax) + \frac{1}{3}a^3x^2\tanh^{-1}(ax) - \frac{2}{3}a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(a}{x}$$

Mathematica [A]
time = 0.30, size = 149, normalized size = 0.96

$$\frac{a^2x^2 - ax\tanh^{-1}(ax) + a^3x^3\tanh^{-1}(ax) - 3\tanh^{-1}(ax)^2 + 8ax\tanh^{-1}(ax)^2 - 6a^2x^2\tanh^{-1}(ax)^2 + a^4x^4\tanh^{-1}(ax)^2 + 6ax\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right) + 10ax\tanh^{-1}(ax)\log\left(1+e^{-2\tanh^{-1}(ax)}\right) - 5ax\,\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) - 3ax\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] (a^2*x^2 - a*x*ArcTanh[a*x] + a^3*x^3*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + 8*a
*x*ArcTanh[a*x]^2 - 6*a^2*x^2*ArcTanh[a*x]^2 + a^4*x^4*ArcTanh[a*x]^2 + 6*a
*x*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] + 10*a*x*ArcTanh[a*x]*Log[1 +
E^(-2*ArcTanh[a*x])] - 5*a*x*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 3*a*x*PolyL
og[2, E^(-2*ArcTanh[a*x])])/(3*x)
```

Maple [A]
time = 0.58, size = 203, normalized size = 1.30

method	result
derivativedivides	$a\left(\frac{\operatorname{arctanh}(ax)^2 a^3x^3}{3} - 2\operatorname{arctanh}(ax)^2 ax - \frac{\operatorname{arctanh}(ax)^2}{ax} + \frac{a^2x^2\operatorname{arctanh}(ax)}{3} - \frac{8\operatorname{arctanh}(ax)\ln(ax+1}{3}\right.$
default	$a\left(\frac{\operatorname{arctanh}(ax)^2 a^3x^3}{3} - 2\operatorname{arctanh}(ax)^2 ax - \frac{\operatorname{arctanh}(ax)^2}{ax} + \frac{a^2x^2\operatorname{arctanh}(ax)}{3} - \frac{8\operatorname{arctanh}(ax)\ln(ax+1}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*arctanh(a*x)^2*a^3*x^3-2*arctanh(a*x)^2*a*x-arctanh(a*x)^2/a/x+1/3*a
^2*x^2*arctanh(a*x)-8/3*arctanh(a*x)*ln(a*x+1)+2*arctanh(a*x)*ln(a*x)-8/3*a
rctanh(a*x)*ln(a*x-1)+1/3*a*x+1/6*ln(a*x-1)-1/6*ln(a*x+1)-dilog(a*x)-dilog(
a*x+1)-ln(a*x)*ln(a*x+1)-2/3*ln(a*x-1)^2+8/3*dilog(1/2*a*x+1/2)+4/3*ln(a*x-
1)*ln(1/2*a*x+1/2)-4/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+2/3*ln(
a*x+1)^2)
```

Maxima [A]
time = 0.27, size = 200, normalized size = 1.28

$$\frac{1}{6}\,a^2\left(\frac{2\,\left(ax+2\,\log\left(ax+1\right)^2-4\,\log\left(ax+1\right)\log\left(ax-1\right)-2\,\log\left(ax-1\right)^2\right)}{a}+\frac{16\,\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a}-\frac{6\,\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)}{a}+\frac{6\,\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(ax\right)\right)}{a}-\frac{\log\left(ax+1\right)}{a}+\frac{\log\left(ax-1\right)}{a}\right)+\frac{1}{3}\,\left(a^2x^2-8\,\log\left(ax+1\right)-8\,\log\left(ax-1\right)+6\,\log\left(x\right)\right)a\,\mathrm{artanh}\left(ax\right)+\frac{1}{3}\,\left(a^4x^3-6\,a^2x-\frac{3}{x}\right)\mathrm{artanh}\left(ax\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*(2*(a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x - 1) - 2*log(a*
x - 1)^2)/a + 16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/
a - 6*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 6*(log(-a*x + 1)*log(x) + dil
og(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) + 1/3*(a^2*x^2 - 8*log(a*x +
1) - 8*log(a*x - 1) + 6*log(x))*a*arctanh(a*x) + 1/3*(a^4*x^3 - 6*a^2*x - 3
/x)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^2,x)
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^2, x)
```

3.210 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^3} \, dx$

Optimal. Leaf size=162

$$-\frac{a\tanh^{-1}(ax)}{x}+a^3x\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)^2-4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)$$

```
[Out] -a*arctanh(a*x)/x+a^3*x*arctanh(a*x)-1/2*arctanh(a*x)^2/x^2+1/2*a^4*x^2*arc
tanh(a*x)^2+4*a^2*arctanh(a*x)^2*arctanh(-1+2/(-a*x+1))+a^2*ln(x)+2*a^2*arc
tanh(a*x)*polylog(2,1-2/(-a*x+1))-2*a^2*arctanh(a*x)*polylog(2,-1+2/(-a*x+1
))-a^2*polylog(3,1-2/(-a*x+1))+a^2*polylog(3,-1+2/(-a*x+1))
```

Rubi [A]
time = 0.32, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6159, 6037, 6129, 272, 36, 29, 31, 6095, 6033, 6199, 6205, 6745, 6127, 6021, 266}

$$\frac{1}{2}a^4x^2\tanh^{-1}(ax)^2+a^3x\tanh^{-1}(ax)-a^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)+a^2\text{Li}_3\left(\frac{2}{1-ax}-1\right)+2a^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)-2a^2\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)+a^2\log(x)-4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-\frac{\tanh^{-1}(ax)^2}{2x^2}-\frac{a\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] - ArcTanh[a*x]^2/(2*x^2) + (a^4*
x^2*ArcTanh[a*x]^2)/2 - 4*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2
*Log[x] + 2*a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - 2*a^2*ArcTanh[a*
x]*PolyLog[2, -1 + 2/(1 - a*x)] - a^2*PolyLog[3, 1 - 2/(1 - a*x)] + a^2*Pol
yLog[3, -1 + 2/(1 - a*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^3}\,dx &= \int \left(\frac{\tanh^{-1}(ax)^2}{x^3} - \frac{2a^2\tanh^{-1}(ax)^2}{x} + a^4x\tanh^{-1}(ax)^2\right)dx \\
&= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\right) + a^4\int x\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3} \\
&= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-}\right. \\
&= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-}\right. \\
&= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - \\
&= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - \\
&= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - \\
&= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 -
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 183, normalized size = 1.13

$$-\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) + \frac{1}{2}a^2(-1+a^2x^2)\tanh^{-1}(ax)^2 + \frac{(-1+a^2x^2)\tanh^{-1}(ax)^2}{2x^2} - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a^2\log(x) - 2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{-1-ax}{-1+ax}\right) + 2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{1+ax}{-1+ax}\right) + a^2\text{PolyLog}\left(3,\frac{-1-ax}{-1+ax}\right) - a^2\text{PolyLog}\left(3,\frac{1+ax}{-1+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] + (a^2*(-1 + a^2*x^2)*ArcTanh[a*
x]^2)/2 + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) - 4*a^2*ArcTanh[a*x]^2*Ar
cTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - 2*a^2*ArcTanh[a*x]*PolyLog[2, (-1 - a
*x)/(-1 + a*x)] + 2*a^2*ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a*x)] + a^2
*PolyLog[3, (-1 - a*x)/(-1 + a*x)] - a^2*PolyLog[3, (1 + a*x)/(-1 + a*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 21.69, size = 779, normalized size = 4.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*a^2*x^2*arctanh(a*x)^2-2*arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^2
/a^2/x^2+2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-2*arctanh(a*x)^2*ln(
1-(a*x+1)/(-a^2*x^2+1)^(1/2))-4*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)
```

```
^(1/2))+4*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)^2*ln(1+(a*x+
1)/(-a^2*x^2+1)^(1/2))-4*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)+4*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1
)^2/(-a^2*x^2+1))-polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+I*Pi*csgn(I*((a*x+1)^2
/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^2*arctanh(a*x)^2+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+ln(1+(a*x+1)/(-a^2*x
^2+1)^(1/2))-ln((a*x+1)^2/(-a^2*x^2+1)+1)-1/2*arctanh(a*x)*((-a^2*x^2+1)^(1
/2)+a*x+1)/a/x+(a*x+1)*arctanh(a*x)-1/2*(a*x+1-(-a^2*x^2+1)^(1/2))/a/x*arct
anh(a*x)+I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^
2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-I*arctanh(a*x)^2*Pi*cs
gn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-I*arctanh(a*x
)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*x^2*log(-a*x + 1) - a*((a*x^2 + 2*x)/a^2 + 2*log(a*x - 1)/a^3))*a^
4 - 1/2*a^4*integrate(x*log(a*x + 1)*log(-a*x + 1), x) + 1/4*a^3*integrate(
a*x*log(a*x + 1)^2, x) + 1/4*a^3*integrate(log(a*x + 1)^2/(a^3*x^3), x) + 1
/4*(a*x - (a*x - 1)*log(-a*x + 1) - 1)*a^2 - 1/2*a^2*integrate(log(a*x + 1)
^2/x, x) + a^2*integrate(log(a*x + 1)*log(-a*x + 1)/x, x) - 1/4*a^2*integra
te(log(-a*x + 1)/x, x) - 1/4*(a*(log(a*x - 1) - log(x)) - log(-a*x + 1)/x)*
a + 1/8*(a^4*x^4 - 1)*log(-a*x + 1)^2/x^2 - 1/2*integrate(log(a*x + 1)*log(
-a*x + 1)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^3,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^3, x)
```

3.211 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=167

$$-\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{2a^2\tanh^{-1}(ax)^2}{x}+a^4x\tanh^{-1}(ax)^2-$$

```
[Out] -1/3*a^2/x+1/3*a^3*arctanh(a*x)-1/3*a*arctanh(a*x)/x^2-2/3*a^3*arctanh(a*x)
^2-1/3*arctanh(a*x)^2/x^3+2*a^2*arctanh(a*x)^2/x+a^4*x*arctanh(a*x)^2-2*a^3
*arctanh(a*x)*ln(2/(-a*x+1))-10/3*a^3*arctanh(a*x)*ln(2-2/(a*x+1))-a^3*poly
log(2,1-2/(-a*x+1))+5/3*a^3*polylog(2,-1+2/(a*x+1))
```

Rubi [A]

time = 0.31, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6159, 6021, 6131, 6055, 2449, 2352, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$a^4x\tanh^{-1}(ax)^2-a^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)+\frac{5}{3}a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)-2a^3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)-\frac{10}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{a^2}{3x}+\frac{2a^2\tanh^{-1}(ax)^2}{x}-\frac{\tanh^{-1}(ax)^2}{3x^3}-\frac{a\tanh^{-1}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -1/3*a^2/x + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (2*a^2*ArcTanh[a*x]^2)/x + a^4*x*Ar
cTanh[a*x]^2 - 2*a^3*ArcTanh[a*x]*Log[2/(1 - a*x)] - (10*a^3*ArcTanh[a*x]*L
og[2 - 2/(1 + a*x)])/3 - a^3*PolyLog[2, 1 - 2/(1 - a*x)] + (5*a^3*PolyLog[2
, -1 + 2/(1 + a*x)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{x^4}\,dx &= \int \left(a^4\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{x^4} - \frac{2a^2\tanh^{-1}(ax)^2}{x^2}\right)dx \\
&= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\right) + a^4\int \tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx \\
&= -\frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2\tanh^{-1}(ax)^2}{x} + a^4x\tanh^{-1}(ax)^2 + \frac{1}{3}(2a)\int \frac{\tanh^{-1}(a}{x^3\,(1-a^2} \\
&= -a^3\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2\tanh^{-1}(ax)^2}{x} + a^4x\tanh^{-1}(ax)^2 + \frac{1}{3} \\
&= -\frac{a\tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2\tanh^{-1}(ax)^2}{x} + a^4 \\
&= -\frac{a^2}{3x} - \frac{a\tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2\tanh^{-1}(ax)}{x} \\
&= -\frac{a^2}{3x} + \frac{1}{3}a^3\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 153, normalized size = 0.92

$$\frac{1}{3}\left(-\frac{a^2}{x}+a^3\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{x^2}-8a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x^3}+\frac{6a^2\tanh^{-1}(ax)^2}{x}+3a^4x\tanh^{-1}(ax)^2-10a^3\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)-6a^3\tanh^{-1}(ax)\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+3a^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+5a^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] (-(a^2/x) + a^3*ArcTanh[a*x] - (a*ArcTanh[a*x])/x^2 - 8*a^3*ArcTanh[a*x]^2
- ArcTanh[a*x]^2/x^3 + (6*a^2*ArcTanh[a*x]^2)/x + 3*a^4*x*ArcTanh[a*x]^2 -
10*a^3*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 6*a^3*ArcTanh[a*x]*Log[1
 + E^(-2*ArcTanh[a*x])] + 3*a^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 5*a^3*Po
lyLog[2, E^(-2*ArcTanh[a*x])])/3
```

Maple [A]
time = 0.56, size = 208, normalized size = 1.25

method	result
derivativedivides	$a^3\Big(\operatorname{arctanh}(ax)^2\,ax-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}+\frac{2\operatorname{arctanh}(ax)^2}{ax}+\frac{8\operatorname{arctanh}(ax)\ln(ax+1)}{3}+\frac{8\operatorname{arctanh}(ax)\ln(ax-}{3}$
default	$a^3\Big(\operatorname{arctanh}(ax)^2\,ax-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}+\frac{2\operatorname{arctanh}(ax)^2}{ax}+\frac{8\operatorname{arctanh}(ax)\ln(ax+1)}{3}+\frac{8\operatorname{arctanh}(ax)\ln(ax-}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(arctanh(a*x)^2*a*x-1/3*arctanh(a*x)^2/a^3/x^3+2*arctanh(a*x)^2/a/x+8/3
*arctanh(a*x)*ln(a*x+1)+8/3*arctanh(a*x)*ln(a*x-1)-1/3*arctanh(a*x)/a^2/x^2
-10/3*arctanh(a*x)*ln(a*x)-1/6*ln(a*x-1)-1/3/a/x+1/6*ln(a*x+1)+5/3*dilog(a*
x+1)+5/3*ln(a*x)*ln(a*x+1)+5/3*dilog(a*x)-8/3*dilog(1/2*a*x+1/2)-4/3*ln(a*x
-1)*ln(1/2*a*x+1/2)+2/3*ln(a*x-1)^2-2/3*ln(a*x+1)^2+4/3*(ln(a*x+1)-ln(1/2*a
*x+1/2))*ln(-1/2*a*x+1/2))
```

Maxima [A]
time = 0.27, size = 203, normalized size = 1.22

$$-\frac{1}{6}\left(16\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-10\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)a+10\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)a-a\log(ax+1)+a\log(ax-1)+\frac{2\left(2ax\log(ax+1)^2-4ax\log(ax+1)\log(ax-1)-2ax\log(ax-1)^2+1\right)}{x}\right)a^2+\frac{1}{3}\left(8a^2\log(ax+1)+8a^2\log(ax-1)-10a^2\log(x)-\frac{1}{x^2}\right)a\,\mathrm{artanh}(ax)+\frac{1}{3}\left(3a^4x+\frac{6a^2x^2-1}{x^3}\right)\mathrm{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 10*(
log(a*x + 1)*log(x) + dilog(-a*x))*a + 10*(log(-a*x + 1)*log(x) + dilog(a*x
))*a - a*log(a*x + 1) + a*log(a*x - 1) + 2*(2*a*x*log(a*x + 1)^2 - 4*a*x*lo
g(a*x + 1)*log(a*x - 1) - 2*a*x*log(a*x - 1)^2 + 1)/x)*a^2 + 1/3*(8*a^2*log
```

```
(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a*arctanh(a*x) + 1/
3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**4,x)
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^4,x)
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^4, x)
```

3.212 $\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=214

$$-\frac{a^2}{12x^2}-\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{3a^3\tanh^{-1}(ax)}{2x}-\frac{3}{4}a^4\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{4x^4}+\frac{a^2\tanh^{-1}(ax)^2}{x^2}+2a^4\tanh^{-1}(ax)$$

```
[Out] -1/12*a^2/x^2-1/6*a*arctanh(a*x)/x^3+3/2*a^3*arctanh(a*x)/x-3/4*a^4*arctanh
(a*x)^2-1/4*arctanh(a*x)^2/x^4+a^2*arctanh(a*x)^2/x^2-2*a^4*arctanh(a*x)^2*
arctanh(-1+2/(-a*x+1))-4/3*a^4*ln(x)+2/3*a^4*ln(-a^2*x^2+1)-a^4*arctanh(a*x
)*polylog(2,1-2/(-a*x+1))+a^4*arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*a^4
*polylog(3,1-2/(-a*x+1))-1/2*a^4*polylog(3,-1+2/(-a*x+1))
```

Rubi [A]
time = 0.43, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6159, 6037, 6129, 272, 46, 36, 29, 31, 6095, 6033, 6199, 6205, 6745}

$$\frac{1}{2}a^4\text{Li}_3\left(1-\frac{2}{1-ax}\right)-\frac{1}{2}a^4\text{Li}_3\left(\frac{2}{1-ax}-1\right)-a^4\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)+a^4\text{Li}_2\left(\frac{2}{1-ax}-1\right)\tanh^{-1}(ax)-\frac{4}{3}a^4\log(x)-\frac{3}{4}a^4\tanh^{-1}(ax)^2+2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3a^3\tanh^{-1}(ax)}{2x}-\frac{a^2}{12x^2}+\frac{a^2\tanh^{-1}(ax)^2}{x^2}+\frac{2}{3}a^4\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)^2}{4x^4}-\frac{a\tanh^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] -1/12*a^2/x^2 - (a*ArcTanh[a*x])/(6*x^3) + (3*a^3*ArcTanh[a*x])/(2*x) - (3*
a^4*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(4*x^4) + (a^2*ArcTanh[a*x]^2)/x^2 +
 2*a^4*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (4*a^4*Log[x])/3 + (2*a^4*
Log[1 - a^2*x^2])/3 - a^4*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] + a^4*Ar
cTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + (a^4*PolyLog[3, 1 - 2/(1 - a*x)])
/2 - (a^4*PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2\tanh^{-1}(ax)^2}{x^5}\,dx = \int\left(\frac{\tanh^{-1}(ax)^2}{x^5}-\frac{2a^2\tanh^{-1}(ax)^2}{x^3}+\frac{a^4\tanh^{-1}(ax)^2}{x}\right)dx$$

$$= -\left((2a^2)\int\frac{\tanh^{-1}(ax)^2}{x^3}\,dx\right)+a^4\int\frac{\tanh^{-1}(ax)^2}{x}\,dx+\int\frac{\tanh^{-1}(ax)^2}{x^5}\,d$$

$$= -\frac{\tanh^{-1}(ax)^2}{4x^4}+\frac{a^2\tanh^{-1}(ax)^2}{x^2}+2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -\frac{\tanh^{-1}(ax)^2}{4x^4}+\frac{a^2\tanh^{-1}(ax)^2}{x^2}+2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{2a^3\tanh^{-1}(ax)}{x}-a^4\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{4x^4}+\frac{a^2}{}$$

$$= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{3a^3\tanh^{-1}(ax)}{2x}-\frac{3}{4}a^4\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{4x^4}+\frac{a}{}$$

$$= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{3a^3\tanh^{-1}(ax)}{2x}-\frac{3}{4}a^4\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{4x^4}+\frac{a}{}$$

$$= -\frac{a^2}{12x^2}-\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{3a^3\tanh^{-1}(ax)}{2x}-\frac{3}{4}a^4\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(a}{4x^4}$$

$$= -\frac{a^2}{12x^2}-\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{3a^3\tanh^{-1}(ax)}{2x}-\frac{3}{4}a^4\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(a}{4x^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 238, normalized size = 1.11

$$\frac{1}{24}\left(2a^4+ia^4\pi^3-\frac{2a^2}{x^2}-\frac{4a\tanh^{-1}(ax)}{x^3}+\frac{36a^3\tanh^{-1}(ax)}{x}-18a^4\tanh^{-1}(ax)^2-\frac{6\tanh^{-1}(ax)^2}{x^4}+\frac{24a^2\tanh^{-1}(ax)^2}{x^2}-16a^4\tanh^{-1}(ax)^3-24a^4\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+24a^4\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)-32a^4\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right)+24a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+24a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+12a^4\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)-12a^4\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] (2*a^4 + I*a^4*Pi^3 - (2*a^2)/x^2 - (4*a*ArcTanh[a*x])/x^3 + (36*a^3*ArcTan
h[a*x])/x - 18*a^4*ArcTanh[a*x]^2 - (6*ArcTanh[a*x]^2)/x^4 + (24*a^2*ArcTan
h[a*x]^2)/x^2 - 16*a^4*ArcTanh[a*x]^3 - 24*a^4*ArcTanh[a*x]^2*Log[1 + E^(-2
*ArcTanh[a*x])] + 24*a^4*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 32*a^
4*Log[(a*x)/Sqrt[1 - a^2*x^2]] + 24*a^4*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcT
anh[a*x])] + 24*a^4*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 12*a^4*Po
lyLog[3, -E^(-2*ArcTanh[a*x])] - 12*a^4*PolyLog[3, E^(2*ArcTanh[a*x])])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 22.70, size = 1124, normalized size = 5.25

method	result	size
derivativedivides	Expression too large to display	1124
default	Expression too large to display	1124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(arctanh(a*x)^2/a^2/x^2+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2
*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+
1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)
^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1
/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^2/a^4
/x^4+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*p
olylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*arctanh(a*x)^2+1/8*(-a*x*(-a^2*x^2
+1)^(1/2)+2*a^2*x^2+(-a^2*x^2+1)^(1/2)+a*x-1)*arctanh(a*x)/a^2/x^2-1/24*(-(
-a^2*x^2+1)^(1/2)*a^2*x^2+5*a^3*x^3+3*a*x*(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)
^(1/2)-3*a*x+2)*arctanh(a*x)/a^3/x^3-1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2-1/24*((-a^2*x^2+1)^(1/2)*a^2*x^2+5*a^3*x^3-3*a*x*(-a^2*x^2+1)^(1/
2)+2*(-a^2*x^2+1)^(1/2)-3*a*x+2)*arctanh(a*x)/a^3/x^3+1/8*(a*x*(-a^2*x^2+1)
^(1/2)+2*a^2*x^2-(-a^2*x^2+1)^(1/2)+a*x-1)*arctanh(a*x)/a^2/x^2-1/2*I*arcta
nh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2
+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^
2*x^2+1))-1/12/(a*x+1-(-a^2*x^2+1)^(1/2))*(-a^2*x^2+1)^(1/2)+1/12/((-a^2*x^
2+1)^(1/2)+a*x+1)*(-a^2*x^2+1)^(1/2)-1/24*(a*x-1)/((-a^2*x^2+1)^(1/2)-1)+1/
```

```
24*(a*x-1)/((-a^2*x^2+1)^(1/2)+1)+1/2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-4/
3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-4/3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2
*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^3+5/8*arctanh(a*x)*((-a^2*x^2+1)^(1/2)+a*x+1)/a/x+5/8*(a*x+1-(-a^2*
x^2+1)^(1/2))/a/x*arctanh(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*(4*a^2*x^2 - 1)*log(-a*x + 1)^2/x^4 - 1/4*integrate(-1/2*(2*(a^5*x^5 -
 a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (4*a^3*x^3 - a
*x + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1))*
log(-a*x + 1))/(a*x^6 - x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**5,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^5,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^5, x)
```

3.213 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^6} dx$

Optimal. Leaf size=157

$$-\frac{a^2}{30x^3}+\frac{11a^4}{30x}-\frac{11}{30}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}$$

```
[Out] -1/30*a^2/x^3+11/30*a^4/x-11/30*a^5*arctanh(a*x)-1/10*a*arctanh(a*x)/x^4+7/
15*a^3*arctanh(a*x)/x^2+8/15*a^5*arctanh(a*x)^2-1/5*arctanh(a*x)^2/x^5+2/3*
a^2*arctanh(a*x)^2/x^3-a^4*arctanh(a*x)^2/x+16/15*a^5*arctanh(a*x)*ln(2-2/(
a*x+1))-8/15*a^5*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.43, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6159, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$-\frac{8}{15}a^5\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{11}{30}a^5\tanh^{-1}(ax)+\frac{16}{15}a^5\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)+\frac{11a^4}{30x}-\frac{a^4\tanh^{-1}(ax)^2}{x}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}-\frac{a^2}{30x^3}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}-\frac{\tanh^{-1}(ax)^2}{5x^5}-\frac{a\tanh^{-1}(ax)}{10x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] -1/30*a^2/x^3 + (11*a^4)/(30*x) - (11*a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x
])/(10*x^4) + (7*a^3*ArcTanh[a*x])/(15*x^2) + (8*a^5*ArcTanh[a*x]^2)/15 - A
rcTanh[a*x]^2/(5*x^5) + (2*a^2*ArcTanh[a*x]^2)/(3*x^3) - (a^4*ArcTanh[a*x]^
2)/x + (16*a^5*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/15 - (8*a^5*PolyLog[2, -1
 + 2/(1 + a*x)])/15
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^6}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^6} - \frac{2a^2\tanh^{-1}(ax)^2}{x^4} + \frac{a^4\tanh^{-1}(ax)^2}{x^2}\right)dx$$

$$= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^6}\,d$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{2a^2\tanh^{-1}(ax)^2}{3x^3} - \frac{a^4\tanh^{-1}(ax)^2}{x} + \frac{1}{5}(2a)\int \frac{\tanh^{-1}(}{x^5(1-a}$$

$$= a^5\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{2a^2\tanh^{-1}(ax)^2}{3x^3} - \frac{a^4\tanh^{-1}(ax)^2}{x} + \frac{1}{5}($$

$$= -\frac{a\tanh^{-1}(ax)}{10x^4} + \frac{2a^3\tanh^{-1}(ax)}{3x^2} + \frac{1}{3}a^5\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{5x^5} + 2$$

$$= -\frac{a^2}{30x^3} + \frac{2a^4}{3x} - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{7a^3\tanh^{-1}(ax)}{15x^2} + \frac{8}{15}a^5\tanh^{-1}(ax)^2 - \text{t}$$

$$= -\frac{a^2}{30x^3} + \frac{11a^4}{30x} - \frac{2}{3}a^5\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{7a^3\tanh^{-1}(ax)}{15x^2} + \frac{8}{15}$$

$$= -\frac{a^2}{30x^3} + \frac{11a^4}{30x} - \frac{11}{30}a^5\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{7a^3\tanh^{-1}(ax)}{15x^2} + \frac{8}{1}$$

Mathematica [A]

time = 0.55, size = 118, normalized size = 0.75

$$\frac{a^2x^2(-1+11a^2x^2) + 2(-1+ax)^3(3+9ax+8a^2x^2)\tanh^{-1}(ax)^2 + ax\tanh^{-1}(ax)\left(-3+14a^2x^2-11a^4x^4+32a^4x^4\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right) - 16a^5x^5\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] (a^2*x^2*(-1 + 11*a^2*x^2) + 2*(-1 + a*x)^3*(3 + 9*a*x + 8*a^2*x^2)*ArcTanh
[a*x]^2 + a*x*ArcTanh[a*x]*(-3 + 14*a^2*x^2 - 11*a^4*x^4 + 32*a^4*x^4*Log[1
 - E^(-2*ArcTanh[a*x])]) - 16*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*
x^5)
```

Maple [A]

time = 0.57, size = 233, normalized size = 1.48

method	result
derivativedivides	$a^5\left(-\frac{\operatorname{arctanh}(ax)^2}{ax} - \frac{\operatorname{arctanh}(ax)^2}{5a^5x^5} + \frac{2\operatorname{arctanh}(ax)^2}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)}{10a^4x^4} + \frac{7\operatorname{arctanh}(ax)}{15a^2x^2} + \frac{16\operatorname{arctanh}(ax)\ln(a}{15}\right.$
default	$a^5\left(-\frac{\operatorname{arctanh}(ax)^2}{ax} - \frac{\operatorname{arctanh}(ax)^2}{5a^5x^5} + \frac{2\operatorname{arctanh}(ax)^2}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)}{10a^4x^4} + \frac{7\operatorname{arctanh}(ax)}{15a^2x^2} + \frac{16\operatorname{arctanh}(ax)\ln(a}{15}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-arctanh(a*x)^2/a/x-1/5*arctanh(a*x)^2/a^5/x^5+2/3*arctanh(a*x)^2/a^3/
x^3-1/10*arctanh(a*x)/a^4/x^4+7/15*arctanh(a*x)/a^2/x^2+16/15*arctanh(a*x)*
ln(a*x)-8/15*arctanh(a*x)*ln(a*x+1)-8/15*arctanh(a*x)*ln(a*x-1)-1/30/a^3/x^
3+11/30/a/x-11/60*ln(a*x+1)+11/60*ln(a*x-1)-8/15*dilog(a*x+1)-8/15*ln(a*x)*
ln(a*x+1)-8/15*dilog(a*x)-2/15*ln(a*x-1)^2+8/15*dilog(1/2*a*x+1/2)+4/15*ln(
a*x-1)*ln(1/2*a*x+1/2)+2/15*ln(a*x+1)^2-4/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln
(-1/2*a*x+1/2))
```

Maxima [A]
time = 0.27, size = 239, normalized size = 1.52

$$\frac{1}{60}\left(32\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^3-32\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a^3+32\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)a^3-11\,a^3\log\left(ax+1\right)+11\,a^3\log\left(ax-1\right)+\frac{2\left(4\,a^3x^3\log\left(ax+1\right)^2-8\,a^3x^3\log\left(ax+1\right)\log\left(ax-1\right)-4\,a^3x^3\log\left(ax-1\right)^2+11\,a^2x^2-1\right)}{x^3}\right)a^2-\frac{1}{30}\left(16\,a^4\log\left(a^2x^2-1\right)-16\,a^4\log\left(x^2\right)-\frac{14\,a^2x^2-3}{x^4}\right)a\,\mathrm{artanh}\left(ax\right)-\frac{\left(15\,a^4x^4-10\,a^2x^2+3\right)\mathrm{artanh}\left(ax\right)^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/60*(32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 32
*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 32*(log(-a*x + 1)*log(x) + dilog
(a*x))*a^3 - 11*a^3*log(a*x + 1) + 11*a^3*log(a*x - 1) + 2*(4*a^3*x^3*log(a
*x + 1)^2 - 8*a^3*x^3*log(a*x + 1)*log(a*x - 1) - 4*a^3*x^3*log(a*x - 1)^2
+ 11*a^2*x^2 - 1)/x^3)*a^2 - 1/30*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2
) - (14*a^2*x^2 - 3)/x^4)*a*arctanh(a*x) - 1/15*(15*a^4*x^4 - 10*a^2*x^2 +
3)*arctanh(a*x)^2/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**6,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^6,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^6, x)
```

3.214 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^7} \, dx$

Optimal. Leaf size=113

$$-\frac{a^2}{60x^4}+\frac{7a^4}{90x^2}-\frac{a\tanh^{-1}(ax)}{15x^5}+\frac{2a^3\tanh^{-1}(ax)}{9x^3}-\frac{a^5\tanh^{-1}(ax)}{3x}-\frac{(1-a^2x^2)^3\tanh^{-1}(ax)^2}{6x^6}+\frac{8}{45}a^6\log(x)-\frac{4}{45}a$$

```
[Out] -1/60*a^2/x^4+7/90*a^4/x^2-1/15*a*arctanh(a*x)/x^5+2/9*a^3*arctanh(a*x)/x^3
-1/3*a^5*arctanh(a*x)/x-1/6*(-a^2*x^2+1)^3*arctanh(a*x)^2/x^6+8/45*a^6*ln(x
)-4/45*a^6*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6155, 6159, 6037, 272, 46, 36, 29, 31}

$$\frac{8}{45}a^6\log(x)-\frac{a^5\tanh^{-1}(ax)}{3x}+\frac{7a^4}{90x^2}+\frac{2a^3\tanh^{-1}(ax)}{9x^3}-\frac{a^2}{60x^4}-\frac{(1-a^2x^2)^3\tanh^{-1}(ax)^2}{6x^6}-\frac{4}{45}a^6\log\left(1-a^2x^2\right)-\frac{a\tanh^{-1}(ax)}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

```
[Out] -1/60*a^2/x^4 + (7*a^4)/(90*x^2) - (a*ArcTanh[a*x])/(15*x^5) + (2*a^3*ArcTa
nh[a*x])/(9*x^3) - (a^5*ArcTanh[a*x])/(3*x) - ((1 - a^2*x^2)^3*ArcTanh[a*x]
^2)/(6*x^6) + (8*a^6*Log[x])/45 - (4*a^6*Log[1 - a^2*x^2])/45
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^7} dx = -\frac{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a \int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^6} dx$$
$$= -\frac{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a \int \left(\frac{\tanh^{-1}(ax)}{x^6} - \frac{2a^2 \tanh^{-1}(ax)}{x^4} + \frac{a^4 \text{ta}}{}\right.$$
$$= -\frac{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a \int \frac{\tanh^{-1}(ax)}{x^6} dx - \frac{1}{3}(2a^3) \int \frac{\tanh^{-1}(ax)}{x^4}$$
$$= -\frac{a \tanh^{-1}(ax)}{15x^5} + \frac{2a^3 \tanh^{-1}(ax)}{9x^3} - \frac{a^5 \tanh^{-1}(ax)}{3x} - \frac{(1-a^2x^2)^3 \tanh^{-1}(a}{6x^6}$$
$$= -\frac{a \tanh^{-1}(ax)}{15x^5} + \frac{2a^3 \tanh^{-1}(ax)}{9x^3} - \frac{a^5 \tanh^{-1}(ax)}{3x} - \frac{(1-a^2x^2)^3 \tanh^{-1}(a}{6x^6}$$
$$= -\frac{a \tanh^{-1}(ax)}{15x^5} + \frac{2a^3 \tanh^{-1}(ax)}{9x^3} - \frac{a^5 \tanh^{-1}(ax)}{3x} - \frac{(1-a^2x^2)^3 \tanh^{-1}(a}{6x^6}$$
$$= -\frac{a^2}{60x^4} + \frac{7a^4}{90x^2} - \frac{a \tanh^{-1}(ax)}{15x^5} + \frac{2a^3 \tanh^{-1}(ax)}{9x^3} - \frac{a^5 \tanh^{-1}(ax)}{3x} - \frac{(1-}{}$$

Mathematica [A]
time = 0.03, size = 99, normalized size = 0.88

$$\frac{-4ax(3-10a^2x^2+15a^4x^4)\tanh^{-1}(ax)+30(-1+a^2x^2)^3\tanh^{-1}(ax)^2+a^2x^2(-3+14a^2x^2+32a^4x^4\log(x)-16a^4x^4\log(1-a^2x^2))}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

```
[Out] (-4*a*x*(3 - 10*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x] + 30*(-1 + a^2*x^2)^3*Ar
cTanh[a*x]^2 + a^2*x^2*(-3 + 14*a^2*x^2 + 32*a^4*x^4*Log[x] - 16*a^4*x^4*Lo
g[1 - a^2*x^2]))/(180*x^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(97) = 194.
time = 0.54, size = 206, normalized size = 1.82

method	result
derivativedivides	$a^6\left(-\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)^2}{2a^4x^4}-\frac{\operatorname{arctanh}(ax)^2}{6a^6x^6}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{6}-\frac{\operatorname{arctanh}(ax)}{15a^5x^5}+\frac{2\operatorname{arctanh}(ax)}{9a^3x^3}\right.$
default	$a^6\left(-\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)^2}{2a^4x^4}-\frac{\operatorname{arctanh}(ax)^2}{6a^6x^6}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{6}-\frac{\operatorname{arctanh}(ax)}{15a^5x^5}+\frac{2\operatorname{arctanh}(ax)}{9a^3x^3}\right.$
risch	$\frac{(a^6x^6-3a^4x^4+3a^2x^2-1)\ln(ax+1)^2}{24x^6}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-20a^3x^3+45x^2\ln(-ax+1)a^2}{180x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] a^6*(-1/2*arctanh(a*x)^2/a^2/x^2+1/2*arctanh(a*x)^2/a^4/x^4-1/6*arctanh(a*x
)^2/a^6/x^6-1/6*arctanh(a*x)*ln(a*x-1)-1/15*arctanh(a*x)/a^5/x^5+2/9*arctan
h(a*x)/a^3/x^3-1/3*arctanh(a*x)/a/x+1/6*arctanh(a*x)*ln(a*x+1)-1/24*ln(a*x-
1)^2+1/12*ln(a*x-1)*ln(1/2*a*x+1/2)+1/12*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/
2*a*x+1/2)-1/24*ln(a*x+1)^2-4/45*ln(a*x-1)-1/60/a^4/x^4+7/90/a^2/x^2+8/45*l
n(a*x)-4/45*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 188, normalized size = 1.66

$$\frac{1}{360}\left(64\,a^4\log\left(x\right)-\frac{15\,a^4x^4\log\left(ax+1\right)^2+15\,a^4x^4\log\left(ax-1\right)^2+32\,a^4x^4\log\left(ax-1\right)-28\,a^2x^2-2\left(15\,a^4x^4\log\left(ax-1\right)-16\,a^4x^4\right)\log\left(ax+1\right)+6}{x^4}\right)a^2+\frac{1}{90}\left(15\,a^5\log\left(ax+1\right)-15\,a^5\log\left(ax-1\right)-\frac{2\left(15\,a^4x^4-10\,a^2x^2+3\right)}{x^5}\right)a\,\mathrm{artanh}\left(ax\right)-\frac{\left(3\,a^4x^4-3\,a^2x^2+1\right)\mathrm{artanh}\left(ax\right)^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/360*(64*a^4*log(x) - (15*a^4*x^4*log(a*x + 1)^2 + 15*a^4*x^4*log(a*x - 1)
^2 + 32*a^4*x^4*log(a*x - 1) - 28*a^2*x^2 - 2*(15*a^4*x^4*log(a*x - 1) - 16
*a^4*x^4)*log(a*x + 1) + 6)/x^4)*a^2 + 1/90*(15*a^5*log(a*x + 1) - 15*a^5*l
og(a*x - 1) - 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)/x^5)*a*arctanh(a*x) - 1/6*(3*
a^4*x^4 - 3*a^2*x^2 + 1)*arctanh(a*x)^2/x^6
```

Fricas [A]
time = 0.38, size = 132, normalized size = 1.17

$$-\frac{32\,a^6x^6\log\left(a^2x^2-1\right)-64\,a^6x^6\log\left(x\right)-28\,a^4x^4+6\,a^2x^2-15\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(15\,a^5x^5-10\,a^3x^3+3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{360\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/360*(32*a^6*x^6*log(a^2*x^2 - 1) - 64*a^6*x^6*log(x) - 28*a^4*x^4 + 6*a^
2*x^2 - 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))^
2 + 4*(15*a^5*x^5 - 10*a^3*x^3 + 3*a*x)*log(-(a*x + 1)/(a*x - 1)))/x^6
```

Sympy [A]
time = 0.68, size = 148, normalized size = 1.31

$$\begin{cases} \frac{8a^6\log\left(x\right)}{45}-\frac{8a^6\log\left(x-\frac{1}{a}\right)}{45}+\frac{a^6\,\mathrm{atanh}^2\left(ax\right)}{6}-\frac{8a^6\,\mathrm{atanh}\left(ax\right)}{45}-\frac{a^5\,\mathrm{atanh}\left(ax\right)}{3x}-\frac{a^4\,\mathrm{atanh}^2\left(ax\right)}{2x^2}+\frac{7a^4}{90x^2}+\frac{2a^3\,\mathrm{atanh}\left(ax\right)}{9x^3}+\frac{a^2\,\mathrm{atanh}^2\left(ax\right)}{2x^4}-\frac{a^2}{60x^4}-\frac{a\,\mathrm{atanh}\left(ax\right)}{15x^5}-\frac{\mathrm{atanh}^2\left(ax\right)}{6x^6} & \text{for}\: a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**7,x)
```

```
[Out] Piecewise((8*a**6*log(x)/45 - 8*a**6*log(x - 1/a)/45 + a**6*atanh(a*x)**2/6
 - 8*a**6*atanh(a*x)/45 - a**5*atanh(a*x)/(3*x) - a**4*atanh(a*x)**2/(2*x**
2) + 7*a**4/(90*x**2) + 2*a**3*atanh(a*x)/(9*x**3) + a**2*atanh(a*x)**2/(2*
```

```
x**4) - a**2/(60*x**4) - a*atanh(a*x)/(15*x**5) - atanh(a*x)**2/(6*x**6), N
e(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(96) = 192.
time = 0.41, size = 440, normalized size = 3.89

$$\frac{4}{45}\left(2a^5\log\left(-\frac{ax+1}{ax-1}-1\right)-2a^5\log\left(-\frac{ax+1}{ax-1}\right)+\frac{30(ax+1)^3a^5\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax-1)^3\left(\frac{(ax+1)^6}{(ax-1)^6}+\frac{6(ax+1)^5}{(ax-1)^5}+\frac{15(ax+1)^4}{(ax-1)^4}+\frac{20(ax+1)^3}{(ax-1)^3}+\frac{15(ax+1)^2}{(ax-1)^2}+\frac{6(ax+1)}{ax-1}+1\right)}+\frac{2\left(\frac{10(ax+1)^2a^5}{(ax-1)^2}+\frac{5(ax+1)a^5}{ax-1}+a^5\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^5}{(ax-1)^5}+\frac{5(ax+1)^4}{(ax-1)^4}+\frac{10(ax+1)^3}{(ax-1)^3}+\frac{10(ax+1)^2}{(ax-1)^2}+\frac{5(ax+1)}{ax-1}+1}-\frac{\frac{2(ax+1)^3a^5}{(ax-1)^3}+\frac{7(ax+1)^2a^5}{(ax-1)^2}+\frac{2(ax+1)a^5}{ax-1}}{\frac{(ax+1)^4}{(ax-1)^4}+\frac{4(ax+1)^3}{(ax-1)^3}+\frac{6(ax+1)^2}{(ax-1)^2}+\frac{4(ax+1)}{ax-1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="giac")
```

```
[Out] 4/45*(2*a^5*log(-(a*x + 1)/(a*x - 1) - 1) - 2*a^5*log(-(a*x + 1)/(a*x - 1))
 + 30*(a*x + 1)^3*a^5*log(-(a*x + 1)/(a*x - 1))^2/((a*x - 1)^3*((a*x + 1)^6
/(a*x - 1)^6 + 6*(a*x + 1)^5/(a*x - 1)^5 + 15*(a*x + 1)^4/(a*x - 1)^4 + 20*
(a*x + 1)^3/(a*x - 1)^3 + 15*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1
) + 1)) + 2*(10*(a*x + 1)^2*a^5/(a*x - 1)^2 + 5*(a*x + 1)*a^5/(a*x - 1) + a
^5)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^5/(a*x - 1)^5 + 5*(a*x + 1)^4/(a*x
 - 1)^4 + 10*(a*x + 1)^3/(a*x - 1)^3 + 10*(a*x + 1)^2/(a*x - 1)^2 + 5*(a*x
+ 1)/(a*x - 1) + 1) - (2*(a*x + 1)^3*a^5/(a*x - 1)^3 + 7*(a*x + 1)^2*a^5/(a
*x - 1)^2 + 2*(a*x + 1)*a^5/(a*x - 1))/((a*x + 1)^4/(a*x - 1)^4 + 4*(a*x +
1)^3/(a*x - 1)^3 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x + 1)/(a*x - 1) + 1))*
a
```

Mupad [B]
time = 1.58, size = 335, normalized size = 2.96

$$\frac{8a^6\ln(x)}{45}-\frac{\frac{3a^2}{4}-\frac{7a^4x^2}{2}}{45x^4}-\ln(1-ax)^2\left(\frac{\frac{a^4x^4}{2}-\frac{a^2x^2}{2}+\frac{1}{6}}{4x^6}-\frac{a^6}{24}\right)-\ln(ax+1)^2\left(\frac{\frac{a^4x^4}{8}-\frac{a^2x^2}{8}+\frac{1}{24}}{x^6}-\frac{a^6}{24}\right)-\ln(1-ax)\left(\frac{a\left(\frac{137a^5x^5}{2}-30a^4x^4+15a^3x^3-10a^2x^2+\frac{15ax}{2}-6\right)}{360x^5}-\ln(ax+1)\left(\frac{\frac{a^4x^4}{2}-\frac{a^2x^2}{2}+\frac{1}{6}}{2x^6}-\frac{a^6}{12}\right)-\frac{a(137a^5x^5+60a^4x^4+30a^3x^3+20a^2x^2+15ax+12)}{720x^5}+\frac{5a^8x^2-\frac{15a^9x^3}{2}}{60a^5x^5}+\frac{\frac{15a^9x^3}{2}+5a^8x^2}{60a^5x^5}\right)-\frac{4a^6\ln(a^2x^2-1)}{45}-\frac{a\ln(ax+1)\left(\frac{a^4x^4}{6}-\frac{a^2x^2}{9}+\frac{1}{30}\right)}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^7,x)
```

```
[Out] (8*a^6*log(x))/45 - ((3*a^2)/4 - (7*a^4*x^2)/2)/(45*x^4) - log(1 - a*x)^2*(
((a^4*x^4)/2 - (a^2*x^2)/2 + 1/6)/(4*x^6) - a^6/24) - log(a*x + 1)^2*(((a^4
*x^4)/8 - (a^2*x^2)/8 + 1/24)/x^6 - a^6/24) - log(1 - a*x)*((a*((15*a*x)/2
- 10*a^2*x^2 + 15*a^3*x^3 - 30*a^4*x^4 + (137*a^5*x^5)/2 - 6))/(360*x^5) -
log(a*x + 1)*(((a^4*x^4)/2 - (a^2*x^2)/2 + 1/6)/(2*x^6) - a^6/12) - (a*(15*
a*x + 20*a^2*x^2 + 30*a^3*x^3 + 60*a^4*x^4 + 137*a^5*x^5 + 12))/(720*x^5) +
 (5*a^8*x^2 - (15*a^9*x^3)/2)/(60*a^5*x^5) + (5*a^8*x^2 + (15*a^9*x^3)/2)/(
60*a^5*x^5)) - (4*a^6*log(a^2*x^2 - 1))/45 - (a*log(a*x + 1)*((a^4*x^4)/6 -
 (a^2*x^2)/9 + 1/30))/x^5
```

3.215 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^8} \, dx$

Optimal. Leaf size=183

$$-\frac{a^2}{105x^5}+\frac{17a^4}{630x^3}+\frac{a^6}{210x}-\frac{1}{210}a^7\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{21x^6}+\frac{9a^3\tanh^{-1}(ax)}{70x^4}-\frac{8a^5\tanh^{-1}(ax)}{105x^2}+\frac{8}{105}a^7\tanh$$

```
[Out] -1/105*a^2/x^5+17/630*a^4/x^3+1/210*a^6/x-1/210*a^7*arctanh(a*x)-1/21*a*arc
tanh(a*x)/x^6+9/70*a^3*arctanh(a*x)/x^4-8/105*a^5*arctanh(a*x)/x^2+8/105*a^
7*arctanh(a*x)^2-1/7*arctanh(a*x)^2/x^7+2/5*a^2*arctanh(a*x)^2/x^5-1/3*a^4*
arctanh(a*x)^2/x^3+16/105*a^7*arctanh(a*x)*ln(2-2/(a*x+1))-8/105*a^7*polylo
g(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.75, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6159, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$-\frac{8}{105}a^7\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{8}{105}a^7\tanh^{-1}(ax)^2-\frac{1}{210}a^7\tanh^{-1}(ax)+\frac{16}{105}a^7\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)+\frac{a^6}{210x}-\frac{8a^5\tanh^{-1}(ax)}{105x^2}+\frac{17a^4}{630x^3}-\frac{a^4\tanh^{-1}(ax)^2}{3x^3}+\frac{9a^3\tanh^{-1}(ax)}{70x^4}-\frac{a^2}{105x^5}+\frac{2a^2\tanh^{-1}(ax)^2}{5x^5}-\frac{\tanh^{-1}(ax)^2}{7x^7}-\frac{a\tanh^{-1}(ax)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

```
[Out] -1/105*a^2/x^5 + (17*a^4)/(630*x^3) + a^6/(210*x) - (a^7*ArcTanh[a*x])/210
- (a*ArcTanh[a*x])/(21*x^6) + (9*a^3*ArcTanh[a*x])/(70*x^4) - (8*a^5*ArcTan
h[a*x])/(105*x^2) + (8*a^7*ArcTanh[a*x]^2)/105 - ArcTanh[a*x]^2/(7*x^7) + (
2*a^2*ArcTanh[a*x]^2)/(5*x^5) - (a^4*ArcTanh[a*x]^2)/(3*x^3) + (16*a^7*ArcT
anh[a*x]*Log[2 - 2/(1 + a*x)])/105 - (8*a^7*PolyLog[2, -1 + 2/(1 + a*x)])/1
05
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^8}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^8} - \frac{2a^2\tanh^{-1}(ax)^2}{x^6} + \frac{a^4\tanh^{-1}(ax)^2}{x^4}\right)dx$$

$$= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^8}\, \ldots$$

$$= -\frac{\tanh^{-1}(ax)^2}{7x^7} + \frac{2a^2\tanh^{-1}(ax)^2}{5x^5} - \frac{a^4\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{7}(2a)\int \frac{\tanh^{-1}(\ldots}{x^7(1-a\ldots}$$

$$= -\frac{\tanh^{-1}(ax)^2}{7x^7} + \frac{2a^2\tanh^{-1}(ax)^2}{5x^5} - \frac{a^4\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{7}(2a)\int \frac{\tanh^{-1}(a\ldots}{x^7}$$

$$= -\frac{a\tanh^{-1}(ax)}{21x^6} + \frac{a^3\tanh^{-1}(ax)}{5x^4} - \frac{a^5\tanh^{-1}(ax)}{3x^2} + \frac{1}{3}a^7\tanh^{-1}(ax)^2 - \ldots$$

$$= -\frac{a^2}{105x^5} + \frac{a^4}{15x^3} - \frac{a^6}{3x} - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} + \frac{a^5\tanh^{-1}(ax\ldots}{15x^2}$$

$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{4a^6}{15x} + \frac{1}{3}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}\ldots}{70x^4}$$

$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{a^6}{210x} - \frac{4}{15}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh\ldots}{70x\ldots}$$

$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{a^6}{210x} - \frac{1}{210}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh\ldots}{70\ldots}$$

Mathematica [A]

time = 1.01, size = 140, normalized size = 0.77

$$\frac{a^2x^2(-6+17a^2x^2+3a^4x^4)+6(-15+42a^2x^2-35a^4x^4+8a^7x^7)\tanh^{-1}(ax)^2+3ax\tanh^{-1}(ax)\left(-10+27a^2x^2-16a^4x^4-a^6x^6+32a^6x^6\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-48a^7x^7\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)}{630x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

```
[Out] (a^2*x^2*(-6 + 17*a^2*x^2 + 3*a^4*x^4) + 6*(-15 + 42*a^2*x^2 - 35*a^4*x^4 +
 8*a^7*x^7)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[a*x]*(-10 + 27*a^2*x^2 - 16*a^4*
x^4 - a^6*x^6 + 32*a^6*x^6*Log[1 - E^(-2*ArcTanh[a*x])]) - 48*a^7*x^7*PolyL
og[2, E^(-2*ArcTanh[a*x])])/(630*x^7)
```

Maple [A]

time = 0.28, size = 253, normalized size = 1.38

method	result
derivativedivides	$a^7\left(-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3} - \frac{\operatorname{arctanh}(ax)^2}{7a^7x^7} + \frac{2\operatorname{arctanh}(ax)^2}{5a^5x^5} - \frac{\operatorname{arctanh}(ax)}{21a^6x^6} + \frac{9\operatorname{arctanh}(ax)}{70a^4x^4} - \frac{8\operatorname{arctanh}(ax)}{105a^2x^2} + 1\ldots\right.$

default	$a^7\left(-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)^2}{7a^7x^7}+\frac{2\operatorname{arctanh}(ax)^2}{5a^5x^5}-\frac{\operatorname{arctanh}(ax)}{21a^6x^6}+\frac{9\operatorname{arctanh}(ax)}{70a^4x^4}-\frac{8\operatorname{arctanh}(ax)}{105a^2x^2}+16\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a^7*(-1/3*arctanh(a*x)^2/a^3/x^3-1/7*arctanh(a*x)^2/a^7/x^7+2/5*arctanh(a*x
)^2/a^5/x^5-1/21*arctanh(a*x)/a^6/x^6+9/70*arctanh(a*x)/a^4/x^4-8/105*arcta
nh(a*x)/a^2/x^2+16/105*arctanh(a*x)*ln(a*x)-8/105*arctanh(a*x)*ln(a*x-1)-8/
105*arctanh(a*x)*ln(a*x+1)-8/105*dilog(a*x+1)-8/105*ln(a*x)*ln(a*x+1)-8/105
*dilog(a*x)-2/105*ln(a*x-1)^2+8/105*dilog(1/2*a*x+1/2)+4/105*ln(a*x-1)*ln(1
/2*a*x+1/2)+2/105*ln(a*x+1)^2-4/105*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x
+1/2)+1/210/a/x-1/105/a^5/x^5+17/630/a^3/x^3+1/420*ln(a*x-1)-1/420*ln(a*x+1
))
```

Maxima [A]
time = 0.26, size = 254, normalized size = 1.39

$$\frac{1}{1260}\left(96\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^5-96\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a^5+96\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)a^5-3\,a^5\log\left(ax+1\right)+3\,a^5\log\left(ax-1\right)+\frac{2\left(12\,a^5x^5\log\left(ax+1\right)^2-24\,a^5x^5\log\left(ax+1\right)\log\left(ax-1\right)-12\,a^5x^5\log\left(ax-1\right)^2+3\,a^4x^4+17\,a^2x^2-6\right)}{x^5}\right)a^2-\frac{1}{210}\left(16\,a^6\log\left(a^2x^2-1\right)-16\,a^6\log\left(x^2\right)+\frac{16\,a^4x^4-27\,a^2x^2+10}{x^6}\right)a\operatorname{artanh}\left(ax\right)-\frac{\left(35\,a^4x^4-42\,a^2x^2+15\right)\operatorname{artanh}\left(ax\right)^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/1260*(96*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^5 -
96*(log(a*x + 1)*log(x) + dilog(-a*x))*a^5 + 96*(log(-a*x + 1)*log(x) + dil
og(a*x))*a^5 - 3*a^5*log(a*x + 1) + 3*a^5*log(a*x - 1) + 2*(12*a^5*x^5*log(
a*x + 1)^2 - 24*a^5*x^5*log(a*x + 1)*log(a*x - 1) - 12*a^5*x^5*log(a*x - 1)
^2 + 3*a^4*x^4 + 17*a^2*x^2 - 6)/x^5)*a^2 - 1/210*(16*a^6*log(a^2*x^2 - 1)
- 16*a^6*log(x^2) + (16*a^4*x^4 - 27*a^2*x^2 + 10)/x^6)*a*arctanh(a*x) - 1/
105*(35*a^4*x^4 - 42*a^2*x^2 + 15)*arctanh(a*x)^2/x^7
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^8, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**8,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^8,x)
```

```
[Out] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^8, x)
```

3.216 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^9} dx$

Optimal. Leaf size=170

$$-\frac{a^2}{168x^6}+\frac{a^4}{84x^4}+\frac{5a^6}{504x^2}-\frac{a\tanh^{-1}(ax)}{28x^7}+\frac{a^3\tanh^{-1}(ax)}{12x^5}-\frac{a^5\tanh^{-1}(ax)}{36x^3}-\frac{a^7\tanh^{-1}(ax)}{12x}+\frac{1}{24}a^8\tanh^{-1}(ax)^2-$$

```
[Out] -1/168*a^2/x^6+1/84*a^4/x^4+5/504*a^6/x^2-1/28*a*arctanh(a*x)/x^7+1/12*a^3*
arctanh(a*x)/x^5-1/36*a^5*arctanh(a*x)/x^3-1/12*a^7*arctanh(a*x)/x+1/24*a^8
*arctanh(a*x)^2-1/8*arctanh(a*x)^2/x^8+1/3*a^2*arctanh(a*x)^2/x^6-1/4*a^4*a
rctanh(a*x)^2/x^4+4/63*a^8*ln(x)-2/63*a^8*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.61, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 56, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6159, 6037, 6129, 272, 46, 36, 29, 31, 6095}

$$\frac{4}{63}a^8\log(x)+\frac{1}{24}a^8\tanh^{-1}(ax)^2-\frac{a^7\tanh^{-1}(ax)}{12x}+\frac{5a^6}{504x^2}-\frac{a^5\tanh^{-1}(ax)}{36x^3}+\frac{a^4}{84x^4}-\frac{a^4\tanh^{-1}(ax)^2}{4x^4}+\frac{a^3\tanh^{-1}(ax)}{12x^5}-\frac{a^2}{168x^6}+\frac{a^2\tanh^{-1}(ax)^2}{3x^6}-\frac{2}{63}a^8\log\left(1-a^2x^2\right)-\frac{\tanh^{-1}(ax)^2}{8x^8}-\frac{a\tanh^{-1}(ax)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

```
[Out] -1/168*a^2/x^6 + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a*ArcTanh[a*x])/(28*x^
7) + (a^3*ArcTanh[a*x])/(12*x^5) - (a^5*ArcTanh[a*x])/(36*x^3) - (a^7*ArcTa
nh[a*x])/(12*x) + (a^8*ArcTanh[a*x]^2)/24 - ArcTanh[a*x]^2/(8*x^8) + (a^2*A
rcTanh[a*x]^2)/(3*x^6) - (a^4*ArcTanh[a*x]^2)/(4*x^4) + (4*a^8*Log[x])/63 -
 (2*a^8*Log[1 - a^2*x^2])/63
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6159

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^9}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^9} - \frac{2a^2\tanh^{-1}(ax)^2}{x^7} + \frac{a^4\tanh^{-1}(ax)^2}{x^5}\right)dx$$
$$= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x^7}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^5}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^9}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2\tanh^{-1}(ax)^2}{3x^6} - \frac{a^4\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{4}a\int \frac{\tanh^{-1}(ax)}{x^8(1-a^2x^2)}$$
$$= -\frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2\tanh^{-1}(ax)^2}{3x^6} - \frac{a^4\tanh^{-1}(ax)^2}{4x^4} + \frac{1}{4}a\int \frac{\tanh^{-1}(ax)}{x^8}\,dx$$
$$= -\frac{a\tanh^{-1}(ax)}{28x^7} + \frac{2a^3\tanh^{-1}(ax)}{15x^5} - \frac{a^5\tanh^{-1}(ax)}{6x^3} - \frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2 \tan}{}$$
$$= -\frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} + \frac{a^5\tanh^{-1}(ax)}{18x^3} - \frac{a^7\tanh^{-1}(ax)}{2x} + \frac{1}{4}a^8$$
$$= -\frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax)}{36x^3} + \frac{a^7\tanh^{-1}(ax)}{6x} - \frac{1}{12}a^8$$
$$= -\frac{a^2}{168x^6} + \frac{41a^4}{1680x^4} - \frac{29a^6}{840x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(}{36x^3}$$
$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{13a^6}{252x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax}{36x^3}$$
$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax}{36x^3}$$
$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a\tanh^{-1}(ax)}{28x^7} + \frac{a^3\tanh^{-1}(ax)}{12x^5} - \frac{a^5\tanh^{-1}(ax}{36x^3}$$

Mathematica [A]
time = 0.03, size = 124, normalized size = 0.73

$$\frac{-2ax(9-21a^2x^2+7a^4x^4+21a^6x^6)\tanh^{-1}(ax)+21(-1+a^2x^2)^3(3+a^2x^2)\tanh^{-1}(ax)^2+a^2x^2(-3+6a^2x^2+5a^4x^4+32a^6x^6\log(x)-16a^6x^6\log(1-a^2x^2))}{504x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

```
[Out] (-2*a*x*(9 - 21*a^2*x^2 + 7*a^4*x^4 + 21*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a
^2*x^2)^3*(3 + a^2*x^2)*ArcTanh[a*x]^2 + a^2*x^2*(-3 + 6*a^2*x^2 + 5*a^4*x^
4 + 32*a^6*x^6*Log[x] - 16*a^6*x^6*Log[1 - a^2*x^2]))/(504*x^8)
```

Maple [A]
time = 0.54, size = 226, normalized size = 1.33

method	result
derivativedivides	$a^8\left(\frac{\operatorname{arctanh}(ax)^2}{3a^6x^6}-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}-\frac{\operatorname{arctanh}(ax)^2}{8a^8x^8}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{24}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{24}-\frac{\operatorname{arctan}}{28a}\right.$ …
default	$a^8\left(\frac{\operatorname{arctanh}(ax)^2}{3a^6x^6}-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}-\frac{\operatorname{arctanh}(ax)^2}{8a^8x^8}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{24}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{24}-\frac{\operatorname{arctan}}{28a}\right.$ …
risch	$\frac{(a^8x^8-6a^4x^4+8a^2x^2-3)\ln(ax+1)^2}{96x^8}-\frac{(21a^8x^8\ln(-ax+1)+42a^7x^7+14a^5x^5-126x^4\ln(-ax+1)a^4-42a^3x^3+168x^2}{1008x^8}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] a^8*(1/3*arctanh(a*x)^2/a^6/x^6-1/4*arctanh(a*x)^2/a^4/x^4-1/8*arctanh(a*x)
^2/a^8/x^8+1/24*arctanh(a*x)*ln(a*x+1)-1/24*arctanh(a*x)*ln(a*x-1)-1/28*arc
tanh(a*x)/a^7/x^7+1/12*arctanh(a*x)/a^5/x^5-1/36*arctanh(a*x)/a^3/x^3-1/12*
arctanh(a*x)/a/x+1/48*ln(a*x-1)*ln(1/2*a*x+1/2)-1/96*ln(a*x-1)^2-1/96*ln(a*
x+1)^2+1/48*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-2/63*ln(a*x+1)-2/6
3*ln(a*x-1)-1/168/a^6/x^6+1/84/a^4/x^4+5/504/a^2/x^2+4/63*ln(a*x))
```

Maxima [A]
time = 0.26, size = 204, normalized size = 1.20

$$\frac{1}{2016}\left(128\,a^6\log(x)-\frac{21\,a^6x^6\log(ax+1)^2+21\,a^6x^6\log(ax-1)^2+64\,a^6x^6\log(ax-1)-20\,a^4x^4-24\,a^2x^2-2\,(21\,a^6x^6\log(ax-1)-32\,a^6x^6)\log(ax+1)+12}{x^6}\right)a^2+\frac{1}{504}\left(21\,a^7\log(ax+1)-21\,a^7\log(ax-1)-\frac{2\,(21\,a^6x^6+7\,a^4x^4-21\,a^2x^2+9)}{x^7}\right)a\operatorname{artanh}(ax)-\frac{(6\,a^4x^4-8\,a^2x^2+3)\operatorname{artanh}(ax)^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2016*(128*a^6*log(x) - (21*a^6*x^6*log(a*x + 1)^2 + 21*a^6*x^6*log(a*x -
1)^2 + 64*a^6*x^6*log(a*x - 1) - 20*a^4*x^4 - 24*a^2*x^2 - 2*(21*a^6*x^6*lo
g(a*x - 1) - 32*a^6*x^6)*log(a*x + 1) + 12)/x^6)*a^2 + 1/504*(21*a^7*log(a*
x + 1) - 21*a^7*log(a*x - 1) - 2*(21*a^6*x^6 + 7*a^4*x^4 - 21*a^2*x^2 + 9)/
x^7)*a*arctanh(a*x) - 1/24*(6*a^4*x^4 - 8*a^2*x^2 + 3)*arctanh(a*x)^2/x^8
```

Fricas [A]
time = 0.35, size = 148, normalized size = 0.87

$$-\frac{64\,a^8x^8\log(a^2x^2-1)-128\,a^8x^8\log(x)-20\,a^6x^6-24\,a^4x^4+12\,a^2x^2-21\,(a^8x^8-6\,a^4x^4+8\,a^2x^2-3)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(21\,a^7x^7+7\,a^5x^5-21\,a^3x^3+9\,ax)\log\left(-\frac{ax+1}{ax-1}\right)}{2016\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/2016*(64*a^8*x^8*log(a^2*x^2 - 1) - 128*a^8*x^8*log(x) - 20*a^6*x^6 - 24
*a^4*x^4 + 12*a^2*x^2 - 21*(a^8*x^8 - 6*a^4*x^4 + 8*a^2*x^2 - 3)*log(-(a*x
+ 1)/(a*x - 1))^2 + 4*(21*a^7*x^7 + 7*a^5*x^5 - 21*a^3*x^3 + 9*a*x)*log(-(a
*x + 1)/(a*x - 1)))/x^8
```

Sympy [A]
time = 0.93, size = 168, normalized size = 0.99

$$\begin{cases} \frac{4a^8\log(x)}{63} - \frac{4a^8\log\left(x-\frac{1}{a}\right)}{63} + \frac{a^8\operatorname{atanh}^2(ax)}{24} - \frac{4a^8\operatorname{atanh}(ax)}{63} - \frac{a^7\operatorname{atanh}(ax)}{12x} + \frac{5a^6}{504x^2} - \frac{a^5\operatorname{atanh}(ax)}{36x^3} - \frac{a^4\operatorname{atanh}^2(ax)}{4x^4} + \frac{a^4}{84x^4} + \frac{a^3\operatorname{atanh}(ax)}{12x^5} + \frac{a^2\operatorname{atanh}^2(ax)}{3x^6} - \frac{a^2}{168x^6} - \frac{a\operatorname{atanh}(ax)}{28x^7} - \frac{\operatorname{atanh}^2(ax)}{8x^8} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**9,x)
```

```
[Out] Piecewise((4*a**8*log(x)/63 - 4*a**8*log(x - 1/a)/63 + a**8*atanh(a*x)**2/2
4 - 4*a**8*atanh(a*x)/63 - a**7*atanh(a*x)/(12*x) + 5*a**6/(504*x**2) - a**
5*atanh(a*x)/(36*x**3) - a**4*atanh(a*x)**2/(4*x**4) + a**4/(84*x**4) + a**
3*atanh(a*x)/(12*x**5) + a**2*atanh(a*x)**2/(3*x**6) - a**2/(168*x**6) - a*
atanh(a*x)/(28*x**7) - atanh(a*x)**2/(8*x**8), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(144) = 288.
time = 0.41, size = 651, normalized size = 3.83

$$\frac{2}{63}\left(2a^7\log\left(-\frac{ax+1}{ax-1}-1\right) - 2a^7\log\left(-\frac{ax+1}{ax-1}\right) + \frac{84\left(\frac{(ax+1)^5a^7}{(ax-1)^5} - \frac{(ax+1)^4a^7}{(ax-1)^4} + \frac{(ax+1)^3a^7}{(ax-1)^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^8}{(ax-1)^8} + \frac{8(ax+1)^7}{(ax-1)^7} + \frac{28(ax+1)^6}{(ax-1)^6} + \frac{56(ax+1)^5}{(ax-1)^5} + \frac{70(ax+1)^4}{(ax-1)^4} + \frac{56(ax+1)^3}{(ax-1)^3} + \frac{28(ax+1)^2}{(ax-1)^2} + \frac{8(ax+1)}{ax-1} + 1} + \frac{2\left(\frac{28(ax+1)^4a^7}{(ax-1)^4} + \frac{7(ax+1)^3a^7}{(ax-1)^3} + \frac{21(ax+1)^2a^7}{(ax-1)^2} + \frac{7(ax+1)a^7}{ax-1} + a^7\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^7}{(ax-1)^7} + \frac{7(ax+1)^6}{(ax-1)^6} + \frac{21(ax+1)^5}{(ax-1)^5} + \frac{35(ax+1)^4}{(ax-1)^4} + \frac{35(ax+1)^3}{(ax-1)^3} + \frac{21(ax+1)^2}{(ax-1)^2} + \frac{7(ax+1)}{ax-1} + 1} - \frac{\frac{2(ax+1)^5a^7}{(ax-1)^5} + \frac{11(ax+1)^4a^7}{(ax-1)^4} + \frac{6(ax+1)^3a^7}{(ax-1)^3} + \frac{11(ax+1)^2a^7}{(ax-1)^2} + \frac{2(ax+1)a^7}{ax-1}}{\frac{(ax+1)^6}{(ax-1)^6} + \frac{6(ax+1)^5}{(ax-1)^5} + \frac{15(ax+1)^4}{(ax-1)^4} + \frac{20(ax+1)^3}{(ax-1)^3} + \frac{15(ax+1)^2}{(ax-1)^2} + \frac{6(ax+1)}{ax-1} + 1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="giac")
```

```
[Out] 2/63*(2*a^7*log(-(a*x + 1)/(a*x - 1) - 1) - 2*a^7*log(-(a*x + 1)/(a*x - 1))
 + 84*((a*x + 1)^5*a^7/(a*x - 1)^5 - (a*x + 1)^4*a^7/(a*x - 1)^4 + (a*x + 1
)^3*a^7/(a*x - 1)^3)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^8/(a*x - 1)^8 +
 8*(a*x + 1)^7/(a*x - 1)^7 + 28*(a*x + 1)^6/(a*x - 1)^6 + 56*(a*x + 1)^5/(a
*x - 1)^5 + 70*(a*x + 1)^4/(a*x - 1)^4 + 56*(a*x + 1)^3/(a*x - 1)^3 + 28*(a
*x + 1)^2/(a*x - 1)^2 + 8*(a*x + 1)/(a*x - 1) + 1) + 2*(28*(a*x + 1)^4*a^7/
(a*x - 1)^4 + 7*(a*x + 1)^3*a^7/(a*x - 1)^3 + 21*(a*x + 1)^2*a^7/(a*x - 1)^
2 + 7*(a*x + 1)*a^7/(a*x - 1) + a^7)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^7
/(a*x - 1)^7 + 7*(a*x + 1)^6/(a*x - 1)^6 + 21*(a*x + 1)^5/(a*x - 1)^5 + 35*
(a*x + 1)^4/(a*x - 1)^4 + 35*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)^2/(a*x
- 1)^2 + 7*(a*x + 1)/(a*x - 1) + 1) - (2*(a*x + 1)^5*a^7/(a*x - 1)^5 + 11*(
a*x + 1)^4*a^7/(a*x - 1)^4 + 6*(a*x + 1)^3*a^7/(a*x - 1)^3 + 11*(a*x + 1)^2
*a^7/(a*x - 1)^2 + 2*(a*x + 1)*a^7/(a*x - 1))/((a*x + 1)^6/(a*x - 1)^6 + 6*
(a*x + 1)^5/(a*x - 1)^5 + 15*(a*x + 1)^4/(a*x - 1)^4 + 20*(a*x + 1)^3/(a*x
- 1)^3 + 15*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1) + 1))*a
```

Mupad [B]
time = 2.72, size = 357, normalized size = 2.10

$$\frac{4a^8\ln(x)}{63} + \frac{a^8\ln(ax+1)^2}{96} + \frac{a^8\ln(1-ax)^2}{96} - \frac{\ln(ax+1)^2}{32x^8} - \frac{\ln(1-ax)^2}{32x^8} - \frac{2a^8\ln(a^2x^2-1)}{63} - \frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a^8\ln(ax+1)\ln(1-ax)}{48} + \frac{\ln(ax+1)\ln(1-ax)}{16x^8} + \frac{a^2\ln(ax+1)^2}{12x^6} - \frac{a^4\ln(ax+1)^2}{16x^4} + \frac{a^2\ln(1-ax)^2}{12x^6} - \frac{a^4\ln(1-ax)^2}{16x^4} - \frac{a\ln(ax+1)}{56x^7} + \frac{a\ln(1-ax)}{56x^7} + \frac{a^3\ln(ax+1)}{24x^5} - \frac{a^5\ln(ax+1)}{72x^3} - \frac{a^7\ln(ax+1)}{24x} - \frac{a^3\ln(1-ax)}{24x^5} + \frac{a^5\ln(1-ax)}{72x^3} + \frac{a^7\ln(1-ax)}{24x} - \frac{a^2\ln(ax+1)\ln(1-ax)}{6x^6} + \frac{a^4\ln(ax+1)\ln(1-ax)}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^9,x)
```

```
[Out] (4*a^8*log(x))/63 + (a^8*log(a*x + 1)^2)/96 + (a^8*log(1 - a*x)^2)/96 - log
(a*x + 1)^2/(32*x^8) - log(1 - a*x)^2/(32*x^8) - (2*a^8*log(a^2*x^2 - 1))/6
3 - a^2/(168*x^6) + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a^8*log(a*x + 1)*lo
g(1 - a*x))/48 + (log(a*x + 1)*log(1 - a*x))/(16*x^8) + (a^2*log(a*x + 1)^2
)/(12*x^6) - (a^4*log(a*x + 1)^2)/(16*x^4) + (a^2*log(1 - a*x)^2)/(12*x^6)
- (a^4*log(1 - a*x)^2)/(16*x^4) - (a*log(a*x + 1))/(56*x^7) + (a*log(1 - a*
x))/(56*x^7) + (a^3*log(a*x + 1))/(24*x^5) - (a^5*log(a*x + 1))/(72*x^3) -
(a^7*log(a*x + 1))/(24*x) - (a^3*log(1 - a*x))/(24*x^5) + (a^5*log(1 - a*x)
)/(72*x^3) + (a^7*log(1 - a*x))/(24*x) - (a^2*log(a*x + 1)*log(1 - a*x))/(6
*x^6) + (a^4*log(a*x + 1)*log(1 - a*x))/(8*x^4)
```

3.217 $\int (1-a^2x^2)^2 \tanh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=248

$$-\frac{1-a^2x^2}{20a}-x\tanh^{-1}(ax)-\frac{1}{10}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{2(1-a^2x^2)\tanh^{-1}(ax)^2}{5a}+\frac{3(1-a^2x^2)^2\tanh^{-1}(ax)^2}{20a}$$

```
[Out] 1/20*(a^2*x^2-1)/a-x*arctanh(a*x)-1/10*x*(-a^2*x^2+1)*arctanh(a*x)+2/5*(-a^
2*x^2+1)*arctanh(a*x)^2/a+3/20*(-a^2*x^2+1)^2*arctanh(a*x)^2/a+8/15*arctanh
(a*x)^3/a+8/15*x*arctanh(a*x)^3+4/15*x*(-a^2*x^2+1)*arctanh(a*x)^3+1/5*x*(-
a^2*x^2+1)^2*arctanh(a*x)^3-8/5*arctanh(a*x)^2*ln(2/(-a*x+1))/a-1/2*ln(-a^2
*x^2+1)/a-8/5*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+4/5*polylog(3,1-2/(-a*
x+1))/a
```

Rubi [A]
time = 0.19, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6091, 6021, 6131, 6055, 6095, 6205, 6745, 266, 6089}

$$-\frac{1-a^2x^2}{20a}-\frac{\log(1-a^2x^2)}{2a}+\frac{1}{5}x(1-a^2x^2)^2\tanh^{-1}(ax)^3+\frac{4}{15}x(1-a^2x^2)\tanh^{-1}(ax)^3+\frac{3(1-a^2x^2)^2\tanh^{-1}(ax)^2}{20a}+\frac{2(1-a^2x^2)\tanh^{-1}(ax)^2}{5a}-\frac{1}{10}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{4\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{5a}-\frac{8\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{5a}+\frac{8}{15}x\tanh^{-1}(ax)^3+\frac{8\tanh^{-1}(ax)^3}{15a}-x\tanh^{-1}(ax)-\frac{8\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

```
[Out] -1/20*(1 - a^2*x^2)/a - x*ArcTanh[a*x] - (x*(1 - a^2*x^2)*ArcTanh[a*x])/10
+ (2*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(5*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x]^
2)/(20*a) + (8*ArcTanh[a*x]^3)/(15*a) + (8*x*ArcTanh[a*x]^3)/15 + (4*x*(1 -
 a^2*x^2)*ArcTanh[a*x]^3)/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/5 - (8*Ar
cTanh[a*x]^2*Log[2/(1 - a*x)])/(5*a) - Log[1 - a^2*x^2]/(2*a) - (8*ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a) + (4*PolyLog[3, 1 - 2/(1 - a*x)])/(
5*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3\,dx &= \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{20a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3-\frac{3}{10}\int \left(1-a^2x^2\right. \\
&= -\frac{1-a^2x^2}{20a}-\frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a}+\frac{3(1}{} \\
&= -\frac{1-a^2x^2}{20a}-x\tanh^{-1}(ax)-\frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2\left(1-a^2x^2\right)\tan}{5a} \\
&= -\frac{1-a^2x^2}{20a}-x\tanh^{-1}(ax)-\frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2\left(1-a^2x^2\right)\tan}{5a} \\
&= -\frac{1-a^2x^2}{20a}-x\tanh^{-1}(ax)-\frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2\left(1-a^2x^2\right)\tan}{5a} \\
&= -\frac{1-a^2x^2}{20a}-x\tanh^{-1}(ax)-\frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2\left(1-a^2x^2\right)\tan}{5a} \\
&= -\frac{1-a^2x^2}{20a}-x\tanh^{-1}(ax)-\frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{2\left(1-a^2x^2\right)\tan}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.40, size = 183, normalized size = 0.74

$$\frac{-3+3a^2x^2-66ax\tanh^{-1}(ax)+6a^3x^3\tanh^{-1}(ax)+33\tanh^{-1}(ax)^2-42a^2x^2\tanh^{-1}(ax)^2+9a^4x^4\tanh^{-1}(ax)^2-32\tanh^{-1}(ax)^3+60ax\tanh^{-1}(ax)^3-40a^3x^3\tanh^{-1}(ax)^3+12a^5x^5\tanh^{-1}(ax)^3-96\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right)-30\log\left(1-a^2x^2\right)+96\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+48\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{60a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

```
[Out] (-3 + 3*a^2*x^2 - 66*a*x*ArcTanh[a*x] + 6*a^3*x^3*ArcTanh[a*x] + 33*ArcTanh
[a*x]^2 - 42*a^2*x^2*ArcTanh[a*x]^2 + 9*a^4*x^4*ArcTanh[a*x]^2 - 32*ArcTanh
[a*x]^3 + 60*a*x*ArcTanh[a*x]^3 - 40*a^3*x^3*ArcTanh[a*x]^3 + 12*a^5*x^5*Ar
cTanh[a*x]^3 - 96*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 30*Log[1 -
a^2*x^2] + 96*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 48*PolyLog[3,
 -E^(-2*ArcTanh[a*x])])/(60*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 30.23, size = 828, normalized size = 3.34

method	result	size

derivativedivides	Expression too large to display	828
default	Expression too large to display	828

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-2/3*arctanh(a*x)^3*a^3*x^3-4/5*I*Pi*arctanh(a*x)^2+arctanh(a*x)^3*a*x
+2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-1/2
0+1/20*a^2*x^2-11/10*a*x*arctanh(a*x)+1/10*a^3*x^3*arctanh(a*x)+3/20*a^4*x^
4*arctanh(a*x)^2-7/10*a^2*x^2*arctanh(a*x)^2+8/15*arctanh(a*x)^3+11/20*arct
anh(a*x)^2-arctanh(a*x)+4/5*arctanh(a*x)^2*ln(a*x+1)+4/5*arctanh(a*x)^2*ln(
a*x-1)-4/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))*arctanh(a*x)^2-2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2+2/5*I*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*arctanh(a*x)^2-2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^2+ln((a*x+1)^2/(-a^2*x^2+1)+1)+4
/5*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+4/5*I*Pi*csgn(I/((a*x+1)^2/(-
a^2*x^2+1)+1))^2*arctanh(a*x)^2-2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*ar
ctanh(a*x)^2-4/5*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+1
/5*arctanh(a*x)^3*a^5*x^5-8/5*arctanh(a*x)^2*ln(2)-8/5*arctanh(a*x)^2*ln((a
*x+1)/(-a^2*x^2+1)^(1/2))-8/5*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2400*(36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 + 480*a*x - 60
*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/
8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a -
 1/1440000*(288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x + 1
) - 6)*(a*x - 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*log
(-a*x + 1) - 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2
+ 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-a*x
+ 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 - 3*log
(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/432*(4*(9*log(-a*x + 1
)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x
```

```
 + 1)^3 - 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-
a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a - 1/8*in
tegrate(-1/150*(150*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*l
og(a*x + 1)^3 + (36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 - 450*
(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 + 480*
a*x - 60*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1))
/(a*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ax-1)^2 (ax+1)^2 \operatorname{atanh}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**3,x)
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)^3\,{\left(a^2\,x^2-1\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3*(a^2*x^2 - 1)^2,x)
[Out] int(atanh(a*x)^3*(a^2*x^2 - 1)^2, x)
```

3.218 $\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx = \int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Maple [A]
time = 54.24, size = 0, normalized size = 0.00

$$\int \frac{x(-a^2x^2+1)^2}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
[Out] int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
[Out] integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(ax-1)^2 (ax+1)^2}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2/atanh(a*x),x)
[Out] Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(a^2\,x^2-1\right)}^2}{\mathrm{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 - 1)^2)/atanh(a*x),x)
[Out] int((x*(a^2*x^2 - 1)^2)/atanh(a*x), x)
```

3.219 $\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/arctanh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx = \int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

```
[Out] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Maple [A]
time = 41.72, size = 0, normalized size = 0.00

$$\int \frac{(-a^2x^2+1)^2}{\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

<u>**Mupad**</u> [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a^2\,x^2-1\right)^2}{\operatorname{atanh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/atanh(a*x),x)
```

```
[Out] int((a^2*x^2 - 1)^2/atanh(a*x), x)
```

3.220 $\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
[Out] Defer[Int][(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx = \int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
[Out] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Maple [A]
time = 62.16, size = 0, normalized size = 0.00

$$\int \frac{(-a^2x^2+1)^2}{x\,\text{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2}{x \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/x/atanh(a*x),x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a^2 x^2-1\right)^2}{x \operatorname{atanh}(a x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/(x*atanh(a*x)),x)
[Out] int((a^2*x^2 - 1)^2/(x*atanh(a*x)), x)
```

3.221 $\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} \, dx = \int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} \, dx$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Maple [A]
time = 59.21, size = 0, normalized size = 0.00

$$\int \frac{x(-a^2x^2+1)^2}{\operatorname{arctanh}(ax)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)/(a*log(a*x + 1) - a*log(-a*x + 1))
+ integrate(-2*(7*a^6*x^6 - 15*a^4*x^4 + 9*a^2*x^2 - 1)/(a*log(a*x + 1) - a
*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(ax-1)^2\,(ax+1)^2}{\operatorname{atanh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(a^2x^2-1\right)^2}{\operatorname{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 - 1)^2)/atanh(a*x)^2,x)
[Out] int((x*(a^2*x^2 - 1)^2)/atanh(a*x)^2, x)
```

3.222 $\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2},x\right)$$

[Out] Unintegrable((-a^2*x^2+1)^2/arctanh(a*x)^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]

[Out] Defer[Int][(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2}\,dx = \int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]

[Out] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]

Maple [A]
time = 35.99, size = 0, normalized size = 0.00

$$\int \frac{(-a^2x^2+1)^2}{\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*log(a*x + 1) - a*log(-a*x + 1))
+ integrate(-12*(a^5*x^5 - 2*a^3*x^3 + a*x)/(log(a*x + 1) - log(-a*x + 1)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2(ax+1)^2}{\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a^2 x^2 - 1)^2}{\operatorname{atanh}(a x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/atanh(a*x)^2,x)
[Out] int((a^2*x^2 - 1)^2/atanh(a*x)^2, x)
```

3.223 $\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
[Out] Defer[Int][(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)^2}\,dx = \int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
[Out] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 68.78, size = 0, normalized size = 0.00

$$\int \frac{(-a^2x^2+1)^2}{x\,\mathrm{arctanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*x*log(a*x + 1) - a*x*log(-a*x +
1)) + integrate(-2*(5*a^6*x^6 - 9*a^4*x^4 + 3*a^2*x^2 + 1)/(a*x^2*log(a*x +
 1) - a*x^2*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)^2 (ax+1)^2}{x \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/x/atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a^2\,x^2-1\right)^2}{x\,\mathrm{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - 1)^2/(x*atanh(a*x)^2),x)
[Out] int((a^2*x^2 - 1)^2/(x*atanh(a*x)^2), x)
```

3.224 $\int (1-a^2x^2)^3 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=144

$$\frac{4(1-a^2x^2)}{35a}+\frac{3(1-a^2x^2)^2}{70a}+\frac{(1-a^2x^2)^3}{42a}+\frac{16}{35}x\tanh^{-1}(ax)+\frac{8}{35}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{6}{35}x(1-a^2x^2)^2\,\mathrm{ta}$$

```
[Out] 4/35*(-a^2*x^2+1)/a+3/70*(-a^2*x^2+1)^2/a+1/42*(-a^2*x^2+1)^3/a+16/35*x*arc
tanh(a*x)+8/35*x*(-a^2*x^2+1)*arctanh(a*x)+6/35*x*(-a^2*x^2+1)^2*arctanh(a*
x)+1/7*x*(-a^2*x^2+1)^3*arctanh(a*x)+8/35*ln(-a^2*x^2+1)/a
```

Rubi [A]
time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6089, 6021, 266}

$$\frac{(1-a^2x^2)^3}{42a}+\frac{3(1-a^2x^2)^2}{70a}+\frac{4(1-a^2x^2)}{35a}+\frac{8\log(1-a^2x^2)}{35a}+\frac{1}{7}x(1-a^2x^2)^3\tanh^{-1}(ax)+\frac{6}{35}x(1-a^2x^2)^2\tanh^{-1}(ax)+\frac{8}{35}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{16}{35}x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (4*(1 - a^2*x^2))/(35*a) + (3*(1 - a^2*x^2)^2)/(70*a) + (1 - a^2*x^2)^3/(42
*a) + (16*x*ArcTanh[a*x])/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x])/35 + (6*x*(
1 - a^2*x^2)^2*ArcTanh[a*x])/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x])/7 + (8*L
og[1 - a^2*x^2])/(35*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\int (1-a^2x^2)^3 \tanh^{-1}(ax)\,dx = \frac{(1-a^2x^2)^3}{42a} + \frac{1}{7}x(1-a^2x^2)^3 \tanh^{-1}(ax) + \frac{6}{7}\int (1-a^2x^2)^2 \tanh^{-1}(ax)\,dx$$
$$= \frac{3(1-a^2x^2)^2}{70a} + \frac{(1-a^2x^2)^3}{42a} + \frac{6}{35}x(1-a^2x^2)^2 \tanh^{-1}(ax) + \frac{1}{7}x(1-a^2x^2)^3$$
$$= \frac{4(1-a^2x^2)}{35a} + \frac{3(1-a^2x^2)^2}{70a} + \frac{(1-a^2x^2)^3}{42a} + \frac{8}{35}x(1-a^2x^2)\tanh^{-1}(ax) +$$
$$= \frac{4(1-a^2x^2)}{35a} + \frac{3(1-a^2x^2)^2}{70a} + \frac{(1-a^2x^2)^3}{42a} + \frac{16}{35}x\tanh^{-1}(ax) + \frac{8}{35}x(1-a^2$$
$$= \frac{4(1-a^2x^2)}{35a} + \frac{3(1-a^2x^2)^2}{70a} + \frac{(1-a^2x^2)^3}{42a} + \frac{16}{35}x\tanh^{-1}(ax) + \frac{8}{35}x(1-a^2$$

Mathematica [A]
time = 0.02, size = 79, normalized size = 0.55

$$\frac{-57a^2x^2 + 24a^4x^4 - 5a^6x^6 - 6ax(-35 + 35a^2x^2 - 21a^4x^4 + 5a^6x^6)\tanh^{-1}(ax) + 48\log(1-a^2x^2)}{210a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
[Out] (-57*a^2*x^2 + 24*a^4*x^4 - 5*a^6*x^6 - 6*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^
4 + 5*a^6*x^6)*ArcTanh[a*x] + 48*Log[1 - a^2*x^2])/(210*a)
```

Maple [A]
time = 0.26, size = 89, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{3\operatorname{arctanh}(ax)a^5x^5}{5}-a^3x^3\operatorname{arctanh}(ax)+ax\operatorname{arctanh}(ax)-\frac{a^6x^6}{42}+\frac{4a^4x^4}{35}-\frac{19a^2x^2}{70}+\frac{8\ln(ax-1)}{35}+\frac{8\ln(ax+1)}{35}}{a}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{3\operatorname{arctanh}(ax)a^5x^5}{5}-a^3x^3\operatorname{arctanh}(ax)+ax\operatorname{arctanh}(ax)-\frac{a^6x^6}{42}+\frac{4a^4x^4}{35}-\frac{19a^2x^2}{70}+\frac{8\ln(ax-1)}{35}+\frac{8\ln(ax+1)}{35}}{a}$
risch	$\left(-\frac{1}{14}a^6x^7+\frac{3}{10}a^4x^5-\frac{1}{2}a^2x^3+\frac{1}{2}x\right)\ln(ax+1)+\frac{a^6x^7\ln(-ax+1)}{14}-\frac{a^5x^6}{42}-\frac{3a^4x^5\ln(-ax+1)}{10}+$
meijerg	$-\frac{\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{\sqrt{a^2x^2}}-2\ln(-a^2x^2+1)}{4a}-\frac{\frac{a^2x^2(4a^4x^4+6a^2x^2+12)}{42}-\frac{2a^8x^8\left(\ln\left(1-\sqrt{a^2x^2}\right)\right.}{7\sqrt{a^2}}}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/7*arctanh(a*x)*a^7*x^7+3/5*arctanh(a*x)*a^5*x^5-a^3*x^3*arctanh(a*x
)+a*x*arctanh(a*x)-1/42*a^6*x^6+4/35*a^4*x^4-19/70*a^2*x^2+8/35*ln(a*x-1)+8
/35*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 82, normalized size = 0.57

$$-\frac{1}{210}\left(5a^4x^6-24a^2x^4+57x^2-\frac{48\log(ax+1)}{a^2}-\frac{48\log(ax-1)}{a^2}\right)a-\frac{1}{35}\left(5a^6x^7-21a^4x^5+35a^2x^3-35x\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*log(a*x
 - 1)/a^2)*a - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3 - 35*x)*arctanh(a*x
)
```

Fricas [A]
time = 0.38, size = 88, normalized size = 0.61

$$-\frac{5a^6x^6-24a^4x^4+57a^2x^2+3\left(5a^7x^7-21a^5x^5+35a^3x^3-35ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-48\log\left(a^2x^2-1\right)}{210a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*x^6 - 24*a^4*x^4 + 57*a^2*x^2 + 3*(5*a^7*x^7 - 21*a^5*x^5 + 3
5*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 48*log(a^2*x^2 - 1))/a
```

Sympy [A]
time = 0.81, size = 97, normalized size = 0.67

$$\begin{cases}-\frac{a^6x^7\operatorname{atanh}(ax)}{7}-\frac{a^5x^6}{42}+\frac{3a^4x^5\operatorname{atanh}(ax)}{5}+\frac{4a^3x^4}{35}-a^2x^3\operatorname{atanh}(ax)-\frac{19ax^2}{70}+x\operatorname{atanh}(ax)+\frac{16\log\left(x-\frac{1}{a}\right)}{35a}+\frac{16\operatorname{atanh}(ax)}{35a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x),x)
```

```
[Out] Piecewise((-a**6*x**7*atanh(a*x)/7 - a**5*x**6/42 + 3*a**4*x**5*atanh(a*x)/
5 + 4*a**3*x**4/35 - a**2*x**3*atanh(a*x) - 19*a*x**2/70 + x*atanh(a*x) + 1
6*log(x - 1/a)/(35*a) + 16*atanh(a*x)/(35*a), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(122) = 244.
time = 0.41, size = 303, normalized size = 2.10

$$\frac{8}{105}a\left(\frac{6\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2}-\frac{6\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2}-\frac{\frac{6(ax+1)^5}{(ax-1)^5}-\frac{33(ax+1)^4}{(ax-1)^4}+\frac{74(ax+1)^3}{(ax-1)^3}-\frac{33(ax+1)^2}{(ax-1)^2}+\frac{6(ax+1)}{ax-1}}{a^2\left(\frac{ax+1}{ax-1}-1\right)^6}-\frac{6\left(\frac{35(ax+1)^3}{(ax-1)^3}-\frac{21(ax+1)^2}{(ax-1)^2}+\frac{7(ax+1)}{ax-1}-1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 8/105*a*(6*log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - 6*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^2 - (6*(a*x + 1)^5/(a*x - 1)^5 - 33*(a*x + 1)^4/(a*x - 1)^4 +
74*(a*x + 1)^3/(a*x - 1)^3 - 33*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x
- 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^6) - 6*(35*(a*x + 1)^3/(a*x - 1)^3 - 2
1*(a*x + 1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(
a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)
/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^7))
```

Mupad [B]
time = 0.94, size = 80, normalized size = 0.56

$$x\,\mathrm{atanh}(a\,x)-\frac{19\,a\,x^2}{70}+\frac{8\,\ln\left(a^2\,x^2-1\right)}{35\,a}+\frac{4\,a^3\,x^4}{35}-\frac{a^5\,x^6}{42}-a^2\,x^3\,\mathrm{atanh}(a\,x)+\frac{3\,a^4\,x^5\,\mathrm{atanh}(a\,x)}{5}-\frac{a^6\,x^7\,\mathrm{atanh}(a\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)*(a^2*x^2 - 1)^3,x)
```

```
[Out] x*atanh(a*x) - (19*a*x^2)/70 + (8*log(a^2*x^2 - 1))/(35*a) + (4*a^3*x^4)/35
 - (a^5*x^6)/42 - a^2*x^3*atanh(a*x) + (3*a^4*x^5*atanh(a*x))/5 - (a^6*x^7*
atanh(a*x))/7
```

3.225 $\int (1-a^2x^2)^3 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=227

$$-\frac{38x}{105}+\frac{19a^2x^3}{315}-\frac{a^4x^5}{105}+\frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a}+\frac{3(1-a^2x^2)^2\tanh^{-1}(ax)}{35a}+\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{21a}+\frac{16\tan}{}$$

```
[Out] -38/105*x+19/315*a^2*x^3-1/105*a^4*x^5+8/35*(-a^2*x^2+1)*arctanh(a*x)/a+3/3
5*(-a^2*x^2+1)^2*arctanh(a*x)/a+1/21*(-a^2*x^2+1)^3*arctanh(a*x)/a+16/35*ar
ctanh(a*x)^2/a+16/35*x*arctanh(a*x)^2+8/35*x*(-a^2*x^2+1)*arctanh(a*x)^2+6/
35*x*(-a^2*x^2+1)^2*arctanh(a*x)^2+1/7*x*(-a^2*x^2+1)^3*arctanh(a*x)^2-32/3
5*arctanh(a*x)*ln(2/(-a*x+1))/a-16/35*polylog(2,1-2/(-a*x+1))/a
```

Rubi [A]

time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6091, 6021, 6131, 6055, 2449, 2352, 8, 200}

$$-\frac{1}{105}a^4x^5+\frac{19a^2x^3}{315}+\frac{1}{7}x(1-a^2x^2)^3\tanh^{-1}(ax)^2+\frac{6}{35}x(1-a^2x^2)^2\tanh^{-1}(ax)^2+\frac{8}{35}x(1-a^2x^2)\tanh^{-1}(ax)^2+\frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{21a}+\frac{3(1-a^2x^2)^2\tanh^{-1}(ax)}{35a}+\frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a}-\frac{16\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{35a}+\frac{16}{35}x\tanh^{-1}(ax)^2+\frac{16\tanh^{-1}(ax)^2}{35a}-\frac{32\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{35a}-\frac{38x}{105}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

```
[Out] (-38*x)/105 + (19*a^2*x^3)/315 - (a^4*x^5)/105 + (8*(1 - a^2*x^2)*ArcTanh[a
*x])/(35*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x])/(35*a) + ((1 - a^2*x^2)^3*Ar
cTanh[a*x])/(21*a) + (16*ArcTanh[a*x]^2)/(35*a) + (16*x*ArcTanh[a*x]^2)/35
+ (8*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]
^2)/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2)/7 - (32*ArcTanh[a*x]*Log[2/(1 -
 a*x)])/(35*a) - (16*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (1-a^2x^2)^3 \tanh^{-1}(ax)^2\,dx = \frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{21a} + \frac{1}{7}x(1-a^2x^2)^3\tanh^{-1}(ax)^2 - \frac{1}{21}\int (1-a^2x^2)$$

$$= \frac{3(1-a^2x^2)^2\tanh^{-1}(ax)}{35a} + \frac{(1-a^2x^2)^3\tanh^{-1}(ax)}{21a} + \frac{6}{35}x(1-a^2x^2)^2\tan$$

$$= -\frac{2x}{15} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a} + \frac{3(1-a^2x^2)^2\tanh^{-1}(}{35a}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a} + \frac{3(1-a^2x^2)^2\tanh^{-1}}{35a}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a} + \frac{3(1-a^2x^2)^2\tanh^{-1}}{35a}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a} + \frac{3(1-a^2x^2)^2\tanh^{-1}}{35a}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a} + \frac{3(1-a^2x^2)^2\tanh^{-1}}{35a}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8(1-a^2x^2)\tanh^{-1}(ax)}{35a} + \frac{3(1-a^2x^2)^2\tanh^{-1}}{35a}$$

Mathematica [A]
time = 0.84, size = 124, normalized size = 0.55

$$-\frac{114ax - 19a^3x^3 + 3a^5x^5 + 9(-1+ax)^4(16+29ax+20a^2x^2+5a^3x^3)\tanh^{-1}(ax)^2 + 3\tanh^{-1}(ax)\left(-38+57a^2x^2-24a^4x^4+5a^6x^6+96\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right) - 144\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

```
[Out] -1/315*(114*a*x - 19*a^3*x^3 + 3*a^5*x^5 + 9*(-1 + a*x)^4*(16 + 29*a*x + 20
*a^2*x^2 + 5*a^3*x^3)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-38 + 57*a^2*x^2 - 2
4*a^4*x^4 + 5*a^6*x^6 + 96*Log[1 + E^(-2*ArcTanh[a*x])]) - 144*PolyLog[2, -
E^(-2*ArcTanh[a*x])])/a
```

Maple [A]
time = 3.68, size = 222, normalized size = 0.98

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{3\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\operatorname{arctanh}(ax)^2 a^3 x^3+\operatorname{arctanh}(ax)^2 ax-\frac{\operatorname{arctanh}(ax)a^6x^6}{21}+\frac{8a^4x^4\operatorname{arctanh}(ax)}{35}-\frac{19a}{}$
default	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{3\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\operatorname{arctanh}(ax)^2 a^3 x^3+\operatorname{arctanh}(ax)^2 ax-\frac{\operatorname{arctanh}(ax)a^6x^6}{21}+\frac{8a^4x^4\operatorname{arctanh}(ax)}{35}-\frac{19a}{}$

risch	$-\frac{38x}{105}+\frac{a^6\ln(-ax+1)\ln(ax+1)x^7}{14}-\frac{1276\ln(ax-1)}{3675a}-\frac{3a^4\ln(-ax+1)\ln(ax+1)x^5}{10}+\frac{19a^2x^3}{315}+\frac{a^2\ln(-ax+1)\ln}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/7*arctanh(a*x)^2*a^7*x^7+3/5*arctanh(a*x)^2*a^5*x^5-arctanh(a*x)^2*
a^3*x^3+arctanh(a*x)^2*a*x-1/21*arctanh(a*x)*a^6*x^6+8/35*a^4*x^4*arctanh(a
*x)-19/35*a^2*x^2*arctanh(a*x)+16/35*arctanh(a*x)*ln(a*x-1)+16/35*arctanh(a
*x)*ln(a*x+1)+4/35*ln(a*x-1)^2-16/35*dilog(1/2*a*x+1/2)-8/35*ln(a*x-1)*ln(1
/2*a*x+1/2)-4/35*ln(a*x+1)^2+8/35*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1
/2)-1/105*a^5*x^5+19/315*a^3*x^3-38/105*a*x-19/105*ln(a*x-1)+19/105*ln(a*x+
1))
```

Maxima [A]
time = 0.27, size = 199, normalized size = 0.88

$$-\frac{1}{315}\,a^2\left(\frac{3\,a^5x^5-19\,a^3x^3+114\,ax+36\,\log\left(ax+1\right)^2-72\,\log\left(ax+1\right)\log\left(ax-1\right)-36\,\log\left(ax-1\right)^2+57\,\log\left(ax-1\right)}{a^3}+\frac{144\,\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a^3}-\frac{57\,\log\left(ax+1\right)}{a^3}\right)-\frac{1}{105}\left(5\,a^4x^6-24\,a^2x^4+57\,x^2-\frac{48\,\log\left(ax+1\right)}{a^2}-\frac{48\,\log\left(ax-1\right)}{a^2}\right)a\,\mathrm{artanh}\left(ax\right)-\frac{1}{35}\left(5\,a^6x^7-21\,a^4x^5+35\,a^2x^3-35\,x\right)\mathrm{artanh}\left(ax\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/315*a^2*((3*a^5*x^5 - 19*a^3*x^3 + 114*a*x + 36*log(a*x + 1)^2 - 72*log(
a*x + 1)*log(a*x - 1) - 36*log(a*x - 1)^2 + 57*log(a*x - 1))/a^3 + 144*(log
(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 - 57*log(a*x + 1)
/a^3) - 1/105*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*l
og(a*x - 1)/a^2)*a*arctanh(a*x) - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3
 - 35*x)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int 3a^2x^2\,\mathrm{atanh}^2\left(ax\right)dx-\int\left(-3a^4x^4\,\mathrm{atanh}^2\left(ax\right)\right)dx-\int a^6x^6\,\mathrm{atanh}^2\left(ax\right)dx-\int\left(-\mathrm{atanh}^2\left(ax\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x)**2,x)
```

```
[Out] -Integral(3*a**2*x**2*atanh(a*x)**2, x) - Integral(-3*a**4*x**4*atanh(a*x)*
*2, x) - Integral(a**6*x**6*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \operatorname{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2*(a^2*x^2 - 1)^3,x)
```

```
[Out] -int(atanh(a*x)^2*(a^2*x^2 - 1)^3, x)
```

3.226 $\int (1-a^2x^2)^3 \tanh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=338

$$-\frac{13(1-a^2x^2)}{210a}-\frac{(1-a^2x^2)^2}{140a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{13}{105}x(1-a^2x^2)\tanh^{-1}(ax)-\frac{1}{35}x(1-a^2x^2)^2\tanh^{-1}(ax)+$$

```
[Out] -13/210*(-a^2*x^2+1)/a-1/140*(-a^2*x^2+1)^2/a-14/15*x*arctanh(a*x)-13/105*x
*(-a^2*x^2+1)*arctanh(a*x)-1/35*x*(-a^2*x^2+1)^2*arctanh(a*x)+12/35*(-a^2*x
^2+1)*arctanh(a*x)^2/a+9/70*(-a^2*x^2+1)^2*arctanh(a*x)^2/a+1/14*(-a^2*x^2+
1)^3*arctanh(a*x)^2/a+16/35*arctanh(a*x)^3/a+16/35*x*arctanh(a*x)^3+8/35*x*
(-a^2*x^2+1)*arctanh(a*x)^3+6/35*x*(-a^2*x^2+1)^2*arctanh(a*x)^3+1/7*x*(-a^
2*x^2+1)^3*arctanh(a*x)^3-48/35*arctanh(a*x)^2*ln(2/(-a*x+1))/a-7/15*ln(-a^
2*x^2+1)/a-48/35*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+24/35*polylog(3,1-2
/(-a*x+1))/a
```

Rubi [A]
time = 0.25, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6091, 6021, 6131, 6055, 6095, 6205, 6745, 266, 6089}

$$-\frac{(1-a^2x^2)^2}{140a}-\frac{13(1-a^2x^2)}{210a}-\frac{7\log(1-a^2x^2)}{15a}+\frac{1}{7}x(1-a^2x^2)^3\tanh^{-1}(ax)^3+\frac{(1-a^2x^2)^3\tanh^{-1}(ax)^2}{14a}+\frac{6}{35}x(1-a^2x^2)^2\tanh^{-1}(ax)^3+\frac{9(1-a^2x^2)^2\tanh^{-1}(ax)^2}{70a}-\frac{1}{35}x(1-a^2x^2)^2\tanh^{-1}(ax)+\frac{8}{35}x(1-a^2x^2)\tanh^{-1}(ax)^3+\frac{12(1-a^2x^2)\tanh^{-1}(ax)^2}{35a}-\frac{13}{105}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{24\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{35a}-\frac{48\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{35a}+\frac{16}{35}x\tanh^{-1}(ax)^3+\frac{16\tanh^{-1}(ax)^3}{35a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{48\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{35a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

```
[Out] (-13*(1 - a^2*x^2))/(210*a) - (1 - a^2*x^2)^2/(140*a) - (14*x*ArcTanh[a*x])
/15 - (13*x*(1 - a^2*x^2)*ArcTanh[a*x])/105 - (x*(1 - a^2*x^2)^2*ArcTanh[a*
x])/35 + (12*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(35*a) + (9*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/(70*a) + ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(14*a) + (16*ArcTanh[
a*x]^3)/(35*a) + (16*x*ArcTanh[a*x]^3)/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x]
^3)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/35 + (x*(1 - a^2*x^2)^3*ArcTa
nh[a*x]^3)/7 - (48*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(35*a) - (7*Log[1 - a^2
*x^2])/(15*a) - (48*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a) + (24*
PolyLog[3, 1 - 2/(1 - a*x)])/(35*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
```

```
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned} \int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3\,dx &= \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{14a}+\frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3-\frac{1}{7}\int \left(1-a^2x^2\right)^2 \\ &= -\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{1}{35}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)+\frac{9\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{70a}+ \\ &= -\frac{13\left(1-a^2x^2\right)}{210a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax)-\frac{1}{35}x\left(1-a^2\right. \\ &= -\frac{13\left(1-a^2x^2\right)}{210a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1} \\ &= -\frac{13\left(1-a^2x^2\right)}{210a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1} \\ &= -\frac{13\left(1-a^2x^2\right)}{210a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1} \\ &= -\frac{13\left(1-a^2x^2\right)}{210a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1} \\ &= -\frac{13\left(1-a^2x^2\right)}{210a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{14}{15}x\tanh^{-1}(ax)-\frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1} \end{aligned}$$

Mathematica [A]
time = 0.72, size = 231, normalized size = 0.68

$$-\frac{29-32a^2x^2+3a^4x^4+456ax\tanh^{-1}(ax)-76a^3x^3\tanh^{-1}(ax)+12a^5x^5\tanh^{-1}(ax)-228\tanh^{-1}(ax)^2+342a^2x^2\tanh^{-1}(ax)^2-144a^4x^4\tanh^{-1}(ax)^2+30a^6x^6\tanh^{-1}(ax)^2+192\tanh^{-1}(ax)^3-420ax\tanh^{-1}(ax)^3+420a^3x^3\tanh^{-1}(ax)^3-252a^5x^5\tanh^{-1}(ax)^3+60a^7x^7\tanh^{-1}(ax)^3+576\tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+196\log\left(1-a^2x^2\right)-576\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-288\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{420a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

```
[Out] -1/420*(29 - 32*a^2*x^2 + 3*a^4*x^4 + 456*a*x*ArcTanh[a*x] - 76*a^3*x^3*Arc
Tanh[a*x] + 12*a^5*x^5*ArcTanh[a*x] - 228*ArcTanh[a*x]^2 + 342*a^2*x^2*ArcT
anh[a*x]^2 - 144*a^4*x^4*ArcTanh[a*x]^2 + 30*a^6*x^6*ArcTanh[a*x]^2 + 192*A
rcTanh[a*x]^3 - 420*a*x*ArcTanh[a*x]^3 + 420*a^3*x^3*ArcTanh[a*x]^3 - 252*a
^5*x^5*ArcTanh[a*x]^3 + 60*a^7*x^7*ArcTanh[a*x]^3 + 576*ArcTanh[a*x]^2*Log[
```

```
1 + E^(-2*ArcTanh[a*x])] + 196*Log[1 - a^2*x^2] - 576*ArcTanh[a*x]*PolyLog[
2, -E^(-2*ArcTanh[a*x])] - 288*PolyLog[3, -E^(-2*ArcTanh[a*x])])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 49.18, size = 978, normalized size = 2.89

method	result	size
derivativedivides	Expression too large to display	978
default	Expression too large to display	978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-arctanh(a*x)^3*a^3*x^3-1/14*arctanh(a*x)^2*a^6*x^6-24/35*I*arctanh(a*
x)^2*Pi+arctanh(a*x)^3*a*x+12/35*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))*arctanh(a*x)^2-13/105+13/105*a*x+12/35*a^4*x^4*arctanh(a*x)^2-57/
70*a^2*x^2*arctanh(a*x)^2+16/35*arctanh(a*x)^3+19/35*arctanh(a*x)^2+24/35*a
rctanh(a*x)^2*ln(a*x+1)+24/35*arctanh(a*x)^2*ln(a*x-1)+1/30*(a*x-1)^2+14/15
*ln((a*x+1)^2/(-a^2*x^2+1)+1)+24/35*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-12/3
5*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))^3-12/35*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+24/35*I*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-24/35*I*arctanh(a*x)^2*
Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/35*(a*x-1)^3-1/140*(a*x-1)^4+9/35
*(a*x-3)*(a*x+1)*arctanh(a*x)-11/105*(a^2*x^2-4*a*x+7)*(a*x+1)*arctanh(a*x)
-1/35*(a^4*x^4-6*a^3*x^3+16*a^2*x^2-26*a*x+31)*(a*x+1)*arctanh(a*x)-1/7*(a^
3*x^3-5*a^2*x^2+11*a*x-15)*(a*x+1)*arctanh(a*x)-1/7*arctanh(a*x)^3*a^7*x^7+
3/5*arctanh(a*x)^3*a^5*x^5-24/35*(a*x+1)*arctanh(a*x)-48/35*arctanh(a*x)^2*
ln(2)-48/35*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-48/35*arctanh(a*x
)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-12/35*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2-12/35*I*Pi*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*arctanh(a*x)^2+12/35*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-24/35*I*Pi
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh
(a*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/19600*(150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 840*a^4*x^4 + 1330*a^3*x
^3 - 1995*a^2*x^2 - 3360*a*x - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 - 3
5*a*x - 16)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(
-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a + 1/691488000*(36000*(343*lo
g(-a*x + 1)^3 - 147*log(-a*x + 1)^2 + 42*log(-a*x + 1) - 6)*(a*x - 1)^7 + 2
401000*(36*log(-a*x + 1)^3 - 18*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 1)*(a*x
 - 1)^6 + 2074464*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x +
 1) - 6)*(a*x - 1)^5 + 13505625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 +
12*log(-a*x + 1) - 3)*(a*x - 1)^4 + 48020000*(9*log(-a*x + 1)^3 - 9*log(-a*
x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 64827000*(4*log(-a*x + 1)^3 -
 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 86436000*(log(-a*x
+ 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a - 1/480000*(
288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x + 1) - 6)*(a*x
- 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*log(-a*x + 1) -
 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x
 + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^2 + 6*l
og(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2
 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/288*(4*(9*log(-a*x + 1)^3 - 9*log(
-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*
log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 -
 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/1
225*(1225*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x^
2 - a*x + 1)*log(a*x + 1)^3 + (150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 84
0*a^4*x^4 + 1330*a^3*x^3 - 1995*a^2*x^2 - 3675*(a^7*x^7 - a^6*x^6 - 3*a^5*x
^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 3360*a*x
 - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 - 35*a*x - 16)*log(a*x + 1))*lo
g(-a*x + 1))/(a*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int 3a^2x^2\operatorname{atanh}^3(ax)\,dx-\int\left(-3a^4x^4\operatorname{atanh}^3(ax)\right)\,dx-\int a^6x^6\operatorname{atanh}^3(ax)\,dx-\int\left(-\operatorname{atanh}^3(ax)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x)**3,x)
```

```
[Out] -Integral(3*a**2*x**2*atanh(a*x)**3, x) - Integral(-3*a**4*x**4*atanh(a*x)*
*3, x) - Integral(a**6*x**6*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \mathrm{atanh}(a\,x)^3\,{\left(a^2\,x^2-1\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3*(a^2*x^2 - 1)^3,x)
```

```
[Out] -int(atanh(a*x)^3*(a^2*x^2 - 1)^3, x)
```

3.227 $\int \frac{x^3 \tanh^{-1}(ax)}{1-a^2x^2} \, dx$

Optimal. Leaf size=87

$$-\frac{x}{2a^3}+\frac{\tanh^{-1}(ax)}{2a^4}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}$$

```
[Out] -1/2*x/a^3+1/2*arctanh(a*x)/a^4-1/2*x^2*arctanh(a*x)/a^2-1/2*arctanh(a*x)^2
/a^4+arctanh(a*x)*ln(2/(-a*x+1))/a^4+1/2*polylog(2,1-2/(-a*x+1))/a^4
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6127, 6037, 327, 212, 6131, 6055, 2449, 2352}

$$\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^4}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}-\frac{x}{2a^3}-\frac{x^2\tanh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -1/2*x/a^3 + ArcTanh[a*x]/(2*a^4) - (x^2*ArcTanh[a*x])/(2*a^2) - ArcTanh[a*
x]^2/(2*a^4) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 + PolyLog[2, 1 - 2/(1 -
a*x)]/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{1-a^2x^2}\,dx = -\frac{\int x\tanh^{-1}(ax)\,dx}{a^2} + \frac{\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^3} + \frac{\int \frac{x^2}{1-a^2x^2}\,dx}{2a}$$

$$= -\frac{x}{2a^3} - \frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{\int \frac{1}{1-a^2x^2}\,dx}{2a^3} - \frac{\int \frac{\mathrm{lo}}{}}{}$$

$$= -\frac{x}{2a^3} + \frac{\tanh^{-1}(ax)}{2a^4} - \frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{\mathrm{Su}}{}$$

$$= -\frac{x}{2a^3} + \frac{\tanh^{-1}(ax)}{2a^4} - \frac{x^2\tanh^{-1}(ax)}{2a^2} - \frac{\tanh^{-1}(ax)^2}{2a^4} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{\mathrm{Li}_2}{}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.69

$$\frac{-ax+\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(1-a^2x^2+2\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)-\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] (-(a*x) + ArcTanh[a*x]^2 + ArcTanh[a*x]*(1 - a^2*x^2 + 2*Log[1 + E^(-2*ArcT
anh[a*x])]) - PolyLog[2, -E^(-2*ArcTanh[a*x])])/(2*a^4)
```

Maple [A]
time = 0.35, size = 131, normalized size = 1.51

method	result
derivativedivides	$\frac{-\frac{a^2x^2\,\mathrm{arctanh}(ax)}{2}-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{2}-\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{2}-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}+\frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\right.}{4}}{a^4}$
default	$\frac{-\frac{a^2x^2\,\mathrm{arctanh}(ax)}{2}-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{2}-\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{2}-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}+\frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\right.}{4}}{a^4}$
risch	$\frac{\ln(-ax+1)x^2}{4a^2}-\frac{\ln(-ax+1)}{4a^4}-\frac{x}{2a^3}+\frac{\ln(-ax+1)^2}{8a^4}+\frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln(-ax+1)}{4a^4}-\frac{\mathrm{dilog}\left(-\frac{ax}{2}+\frac{1}{2}\right)}{4a^4}-\frac{\ln(ax+1)x^2}{4a^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/2*a^2*x^2*arctanh(a*x)-1/2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*x
)*ln(a*x+1)-1/2*a*x-1/4*ln(a*x-1)+1/4*ln(a*x+1)+1/2*dilog(1/2*a*x+1/2)+1/4*
```

```
ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^2-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*l
n(-1/2*a*x+1/2)+1/8*ln(a*x+1)^2)
```

Maxima [A]
time = 0.26, size = 120, normalized size = 1.38

$$-\frac{1}{8}a\left(\frac{4\,ax-\log\left(ax+1\right)^2+2\,\log\left(ax+1\right)\log\left(ax-1\right)+\log\left(ax-1\right)^2+2\,\log\left(ax-1\right)}{a^5}-\frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a^5}-\frac{2\,\log\left(ax+1\right)}{a^5}\right)-\frac{1}{2}\left(\frac{x^2}{a^2}+\frac{\log\left(a^2x^2-1\right)}{a^4}\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*a*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1
)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2
*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) - 1/2*(x^2/a^2 + log(a^2*x^2 - 1)/a^
4)*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3\,\mathrm{atanh}\,(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\mathrm{atanh}(a\,x)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x))/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^3*atanh(a*x))/(a^2*x^2 - 1), x)
```

3.228 $\int \frac{x^2 \tanh^{-1}(ax)}{1-a^2x^2} dx$

Optimal. Leaf size=42

$$-\frac{x \tanh^{-1}(ax)}{a^2} + \frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{\log(1-a^2x^2)}{2a^3}$$

```
[Out] -x*arctanh(a*x)/a^2+1/2*arctanh(a*x)^2/a^3-1/2*ln(-a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6127, 6021, 266, 6095}

$$\frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{x \tanh^{-1}(ax)}{a^2} - \frac{\log(1-a^2x^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

]

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \tanh^{-1}(ax)}{1-a^2x^2}\,dx &= -\frac{\int \tanh^{-1}(ax)\,dx}{a^2}+\frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x\tanh^{-1}(ax)}{a^2}+\frac{\tanh^{-1}(ax)^2}{2a^3}+\frac{\int \frac{x}{1-a^2x^2}\,dx}{a}\\
&= -\frac{x\tanh^{-1}(ax)}{a^2}+\frac{\tanh^{-1}(ax)^2}{2a^3}-\frac{\log(1-a^2x^2)}{2a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$
-\frac{x\tanh^{-1}(ax)}{a^2}+\frac{\tanh^{-1}(ax)^2}{2a^3}-\frac{\log(1-a^2x^2)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(38) = 76$.
time = 0.30, size = 114, normalized size = 2.71

method	result
risch	$\frac{\ln(ax+1)^2}{8a^3}-\frac{(2ax+\ln(-ax+1))\ln(ax+1)}{4a^3}+\frac{\ln(-ax+1)x}{2a^2}+\frac{\ln(-ax+1)^2}{8a^3}-\frac{\ln(a^2x^2-1)}{2a^3}$
derivativedivides	$\frac{-ax\,\mathrm{arctanh}(ax)-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{2}+\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(ax-1)^2}{8}-\frac{\ln(ax-1)}{2}-\frac{\ln(ax+1)}{2}+\frac{(\ln(\cdots}{\cdots}}{a^3}$
default	$\frac{-ax\,\mathrm{arctanh}(ax)-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{2}+\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(ax-1)^2}{8}-\frac{\ln(ax-1)}{2}-\frac{\ln(ax+1)}{2}+\frac{(\ln(\cdots}{\cdots}}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-a*x*arctanh(a*x)-1/2*arctanh(a*x)*ln(a*x-1)+1/2*arctanh(a*x)*ln(a*x
+1)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^2-1/2*ln(a*x-1)-1/2*ln(a*x+
1)+1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/8*ln(a*x+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(38) = 76$.
time = 0.26, size = 85, normalized size = 2.02

$$-\frac{1}{2}\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)+\frac{2(\log(ax-1)-2)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\log(ax-1)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x) + 1/8*(2*
(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log(a
*x - 1))/a^3
```

Fricas [A]
time = 0.36, size = 56, normalized size = 1.33

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,\log\left(a^2x^2-1\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a*x - 1))^2 + 4*log
(a^2*x^2 - 1))/a^3
```

Sympy [A]
time = 0.81, size = 41, normalized size = 0.98

$$\begin{cases} -\frac{x\operatorname{atanh}(ax)}{a^2}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3}+\frac{\operatorname{atanh}^2(ax)}{2a^3}-\frac{\operatorname{atanh}(ax)}{a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((-x*atanh(a*x)/a**2 - log(x - 1/a)/a**3 + atanh(a*x)**2/(2*a**3)
- atanh(a*x)/a**3, Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Mupad [B]
time = 0.96, size = 82, normalized size = 1.95

$$\frac{\ln(a\,x+1)^2}{8\,a^3}-\ln(1-a\,x)\,\left(\frac{\ln(a\,x+1)}{4\,a^3}-\frac{x}{2\,a^2}\right)+\frac{\ln(1-a\,x)^2}{8\,a^3}-\frac{\ln(a^2\,x^2-1)}{2\,a^3}-\frac{x\,\ln(a\,x+1)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x))/(a^2*x^2 - 1),x)
```

```
[Out] log(a*x + 1)^2/(8*a^3) - log(1 - a*x)*(log(a*x + 1)/(4*a^3) - x/(2*a^2)) +
log(1 - a*x)^2/(8*a^3) - log(a^2*x^2 - 1)/(2*a^3) - (x*log(a*x + 1))/(2*a^2
)
```

3.229 $\int \frac{x \tanh^{-1}(ax)}{1-a^2x^2} \, dx$

Optimal. Leaf size=54

$$-\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}$$

```
[Out] -1/2*arctanh(a*x)^2/a^2+arctanh(a*x)*ln(2/(-a*x+1))/a^2+1/2*polylog(2,1-2/(
-a*x+1))/a^2
```

Rubi [A]

time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6131, 6055, 2449, 2352}

$$\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -1/2*ArcTanh[a*x]^2/a^2 + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^2 + PolyLog[2,
1 - 2/(1 - a*x)]/(2*a^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x \tanh^{-1}(ax)}{1-a^2x^2}\,dx &= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a} \\
&= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a} \\
&= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-ax}\right)}{a^2} \\
&= -\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.81

$$
-\frac{-\tanh^{-1}(ax)\left(\tanh^{-1}(ax)+2\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -1/2*(-(ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 + E^(-2*ArcTanh[a*x])])) + Pol
yLog[2, -E^(-2*ArcTanh[a*x])])/a^2
```

Maple [A]
time = 0.27, size = 99, normalized size = 1.83

method	result
risch	$\frac{\ln(-ax+1)^2}{8a^2}+\frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln(-ax+1)}{4a^2}-\frac{\text{dilog}\left(-\frac{ax}{2}+\frac{1}{2}\right)}{4a^2}-\frac{\ln(ax+1)^2}{8a^2}-\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{4a^2}+\frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{4a^2}$
derivativedivides	$\frac{-\frac{\text{arctanh}(ax)\ln(ax-1)}{2}-\frac{\text{arctanh}(ax)\ln(ax+1)}{2}+\frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(ax-1)^2}{8}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln(-}{4}}{a^2}$
default	$\frac{-\frac{\text{arctanh}(ax)\ln(ax-1)}{2}-\frac{\text{arctanh}(ax)\ln(ax+1)}{2}+\frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(ax-1)^2}{8}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln(-}{4}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*x)*ln(a*x+1)+1/2*dilog(1/2
*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^2-1/4*(ln(a*x+1)-ln(1
/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/8*ln(a*x+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(47) = 94$.
time = 0.25, size = 125, normalized size = 2.31

$$-\frac{1}{8}a\left(\frac{\log(ax+1)^2+2\log(ax+1)\log(ax-1)-\log(ax-1)^2}{a^3}-\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^3}\right)+\frac{\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\log\left(a^2x^2-1\right)}{4a}-\frac{\operatorname{artanh}(ax)\log\left(a^2x^2-1\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a^3
 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3) + 1/4*(
log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a - 1/2*arctanh(a*x)*log(
a^2*x^2 - 1)/a^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\operatorname{atanh}(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x\,\mathrm{atanh}(a\,x)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x))/(a^2*x^2 - 1),x)
```

```
[Out] -int((x*atanh(a*x))/(a^2*x^2 - 1), x)
```

3.230 $\int \frac{\tanh^{-1}(ax)}{1-a^2x^2} dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

```
[Out] 1/2*arctanh(a*x)^2/a
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{6095\}$

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^2/(2*a)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx = \frac{\tanh^{-1}(ax)^2}{2a}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^2/(2*a)
```

Maple [A]
time = 0.22, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2}{2a}$	12
default	$\frac{\operatorname{arctanh}(ax)^2}{2a}$	12
risch	$\frac{\ln(ax+1)^2}{8a} - \frac{\ln(-ax+1)\ln(ax+1)}{4a} + \frac{\ln(-ax+1)^2}{8a}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(a*x)^2/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(11) = 22$.
time = 0.26, size = 65, normalized size = 5.00

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax) - \frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) + \log(ax-1)^2}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x) - 1/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/a
```

Fricas [A]
time = 0.38, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

Sympy [A]
time = 0.52, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
[Out] 1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

Mupad [B]
time = 0.87, size = 23, normalized size = 1.77

$$\frac{\left(\ln\left(a\,x+1\right)-\ln\left(1-a\,x\right)\right)^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(a^2*x^2 - 1),x)
[Out] (log(a*x + 1) - log(1 - a*x))^2/(8*a)
```

3.231 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{2}\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

```
[Out] 1/2*arctanh(a*x)^2+arctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6135, 6079, 2497}

$$-\frac{1}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1
 + a*x)]/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx &= \frac{1}{2}\tanh^{-1}(ax)^2 + \int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx \\
&= \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - a\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx \\
&= \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{2}\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\tanh^{-1}(ax)\left(\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

```
[Out] (ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2,
E^(-2*ArcTanh[a*x])])/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(41) = 82.
time = 0.28, size = 130, normalized size = 2.89

method	result
risch	$-\frac{\ln(ax+1)^2}{8}-\frac{\mathrm{dilog}(ax+1)}{2}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(-ax+1)^2}{8}+$ dilog(−
derivativedivides	$-\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{2}-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{2}+\mathrm{arctanh}\,(ax)\ln\,(ax)+\frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+$ ln(ax−1) ln(
default	$-\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{2}-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{2}+\mathrm{arctanh}\,(ax)\ln\,(ax)+\frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+$ ln(ax−1) ln(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(a*x)*ln(a*x+1)-1/2*arctanh(a*x)*ln(a*x-1)+arctanh(a*x)*ln(a*x)
+1/2*dilog(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^2-1/4*(
ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/8*ln(a*x+1)^2-1/2*dilog(a*x)-
1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(40) = 80.
time = 0.27, size = 132, normalized size = 2.93

$$\frac{1}{8}a\left(\frac{\log(ax+1)^2-2\log(ax+1)\log(ax-1)-\log(ax-1)^2}{a}+\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}-\frac{4\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)}{a}+\frac{4\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)}{a}\right)-\frac{1}{2}\left(\log\left(a^2x^2-1\right)-\log\left(x^2\right)\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*a*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a +
4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x
+ 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a) - 1
/2*(log(a^2*x^2 - 1) - log(x^2))*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(ax)}{a^2x^3 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)/(a**2*x**3 - x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)/(x*(a^2*x^2 - 1)), x)
```

3.232 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x^2\right)$$

[Out] -arctanh(a*x)/x+1/2*a*arctanh(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.350$, Rules used = {6129, 6037, 272, 36, 29, 31, 6095}

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]

[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\int \frac{1}{x(1-a^2x^2)}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(1-a^2x)}\,dx,x,x^2\right)\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{1-a^2x}\right.\\ &= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x^2\right)\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 41, normalized size = 1.00

$$-\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/
2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(37) = 74$.
time = 0.30, size = 120, normalized size = 2.93

method	result
risch	$\frac{a\ln(ax+1)^2}{8} - \frac{(ax\ln(-ax+1)+2)\ln(ax+1)}{4x} + \frac{a\ln(-ax+1)^2x+8a\ln(x)x-4a\ln(a^2x^2-1)x+4\ln(-ax+1)}{8x}$
derivativedivides	$a\left(-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} - \frac{\operatorname{arctanh}(ax)}{ax} + \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} + \frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4} - \frac{\ln(ax-1)^2}{8} - \ln\right.$
default	$a\left(-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} - \frac{\operatorname{arctanh}(ax)}{ax} + \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} + \frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4} - \frac{\ln(ax-1)^2}{8} - \ln\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)

[Out] a*(-1/2*arctanh(a*x)*ln(a*x-1)-arctanh(a*x)/a/x+1/2*arctanh(a*x)*ln(a*x+1)+
1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^2-1/2*ln(a*x-1)+ln(a*x)-1/2*ln(
a*x+1)+1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/8*ln(a*x+1)^2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.
time = 0.28, size = 82, normalized size = 2.00

$$\frac{1}{8}\left(2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2-\log\left(ax-1\right)^2-4\log\left(ax-1\right)+8\log\left(x\right)\right)a+\frac{1}{2}\left(a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2}{x}\right)\operatorname{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="maxima")

[Out] 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
4*log(a*x - 1) + 8*log(x))*a + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*
arctanh(a*x)

Fricas [A]
time = 0.38, size = 63, normalized size = 1.54

$$\frac{ax\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,ax\log\left(a^2x^2-1\right)+8\,ax\log\left(x\right)-4\,\log\left(-\frac{ax+1}{ax-1}\right)}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="fricas")

[Out] 1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^2 - 4*a*x*log(a^2*x^2 - 1) + 8*a*x*log(x
) - 4*log(-(a*x + 1)/(a*x - 1)))/x

Sympy [A]
time = 0.51, size = 37, normalized size = 0.90

$$\begin{cases} a\log(x) - a\log\left(x - \frac{1}{a}\right) + \frac{a\,\mathrm{atanh}^2(ax)}{2} - a\,\mathrm{atanh}(ax) - \frac{\mathrm{atanh}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((a*log(x) - a*log(x - 1/a) + a*atanh(a*x)**2/2 - a*atanh(a*x) - a
tanh(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^2), x)
```

Mupad [B]
time = 1.05, size = 80, normalized size = 1.95

$$\frac{a\ln(a\,x+1)^2}{8} + \frac{a\ln(1-a\,x)^2}{8} - \frac{\ln(a\,x+1)}{2\,x} + \frac{\ln(1-a\,x)}{2\,x} - \frac{a\,\ln(a^2\,x^2-1)}{2} + a\,\ln(x) - \frac{a\,\ln(a\,x+1)\,\ln(1-a\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] (a*log(a*x + 1)^2)/8 + (a*log(1 - a*x)^2)/8 - log(a*x + 1)/(2*x) + log(1 -
a*x)/(2*x) - (a*log(a^2*x^2 - 1))/2 + a*log(x) - (a*log(a*x + 1)*log(1 - a*
x))/4
```

3.233 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{a}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+a^2\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,\right.$$

```
[Out] -1/2*a/x+1/2*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1/2*a^2*arctanh(a*x)^2+a
^2*arctanh(a*x)*ln(2-2/(a*x+1))-1/2*a^2*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6129, 6037, 331, 212, 6135, 6079, 2497}

$$-\frac{1}{2}a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a^2\tanh^{-1}(ax)+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -1/2*a/x + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*ArcTanh[a*x]^
2)/2 + a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - (a^2*PolyLog[2, -1 + 2/(1 +
a*x)])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}\,dx+a^2\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\ &= -\frac{a}{2x}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+a^2\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)+\frac{1}{2}a^3\int \frac{}{1}\\ &= -\frac{a}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\tanh^{-1}(ax)^2+a^2\tanh^{-1}(ax)\log\left(2-\frac{}{1}\right.\end{aligned}$$

Mathematica [A]

time = 0.17, size = 60, normalized size = 0.71

$$-\frac{1}{2}a^2\left(\frac{1}{ax}-\tanh^{-1}(ax)\left(1-\frac{1}{a^2x^2}+\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -1/2*(a^2*(1/(a*x) - ArcTanh[a*x]*(1 - 1/(a^2*x^2) + ArcTanh[a*x] + 2*Log[1
 - E^(-2*ArcTanh[a*x])]) + PolyLog[2, E^(-2*ArcTanh[a*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(74) = 148$.
time = 0.38, size = 170, normalized size = 2.02

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\ln(ax-1)}{4}\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\ln(ax-1)}{4}\right.$
risch	$-\frac{a}{2x}-\frac{a^2\ln(ax)}{4}+\frac{a^2\ln(ax+1)}{4}-\frac{\ln(ax+1)}{4x^2}-\frac{a^2\ln(ax+1)^2}{8}-\frac{a^2\operatorname{dilog}(ax+1)}{2}-\frac{a^2\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arctanh(a*x)*ln(a*x+1)-1/2*arctanh(a*x)/a^2/x^2+arctanh(a*x)*ln(a
*x)-1/2*arctanh(a*x)*ln(a*x-1)-1/4*ln(a*x-1)+1/4*ln(a*x+1)-1/2/a/x+1/2*dilo
g(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^2-1/4*(ln(a*x+1)
-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/8*ln(a*x+1)^2-1/2*dilog(a*x)-1/2*dilog
(a*x+1)-1/2*ln(a*x)*ln(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(73) = 146$.
time = 0.27, size = 162, normalized size = 1.93

$$\frac{1}{8}\left(4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-4\left(\log(ax+1)\log(x)+\text{Li}_2(-ax)\right)a+4\left(\log(-ax+1)\log(x)+\text{Li}_2(ax)\right)a+2a\log(ax+1)-2a\log(ax-1)+\frac{ax\log(ax+1)^2-2ax\log(ax+1)\log(ax-1)-ax\log(ax-1)^2-4}{x}\right)a-\frac{1}{2}\left(a^2\log\left(a^2x^2-1\right)-a^2\log\left(x^2\right)+\frac{1}{x^2}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(log
(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*a
 + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*
x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a - 1/2*(a^2*log(a^2*x^2 -
 1) - a^2*log(x^2) + 1/x^2)*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(ax)}{a^2x^5 - x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)/(a**2*x**5 - x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\mathrm{atanh}(a\,x)}{x^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)/(x^3*(a^2*x^2 - 1)), x)
```

3.234 $\int \frac{x^3 \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=135

$$-\frac{x\tanh^{-1}(ax)}{a^3}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4}-\frac{\log(1-a^2x^2)}{2a^4}+\text{ta}$$

```
[Out] -x*arctanh(a*x)/a^3+1/2*arctanh(a*x)^2/a^4-1/2*x^2*arctanh(a*x)^2/a^2-1/3*a
rctanh(a*x)^3/a^4+arctanh(a*x)^2*ln(2/(-a*x+1))/a^4-1/2*ln(-a^2*x^2+1)/a^4+
arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^4-1/2*polylog(3,1-2/(-a*x+1))/a^4
```

Rubi [A]
time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6127, 6037, 6021, 266, 6095, 6131, 6055, 6205, 6745}

$$-\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^4}-\frac{x\tanh^{-1}(ax)}{a^3}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\log(1-a^2x^2)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^3) + ArcTanh[a*x]^2/(2*a^4) - (x^2*ArcTanh[a*x]^2)/(2*
a^2) - ArcTanh[a*x]^3/(3*a^4) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - Log
[1 - a^2*x^2]/(2*a^4) + (ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^4 - Po
lyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \tanh^{-1}(ax)^2}{1-a^2x^2}\,dx &= -\frac{\int x \tanh^{-1}(ax)^2\,dx}{a^2} + \frac{\int \frac{x \tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2} \\
&= -\frac{x^2 \tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^3} + \frac{\int \frac{x^2 \tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a} \\
&= -\frac{x^2 \tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\tanh^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\int \tanh^{-1}(ax)\,dx}{a^3} + \\
&= -\frac{x \tanh^{-1}(ax)}{a^3} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{x^2 \tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\tanh^{-1}(ax)^2 \mathrm{lo}}{a^4} \\
&= -\frac{x \tanh^{-1}(ax)}{a^3} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{x^2 \tanh^{-1}(ax)^2}{2a^2} - \frac{\tanh^{-1}(ax)^3}{3a^4} + \frac{\tanh^{-1}(ax)^2 \mathrm{lo}}{a^4}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 112, normalized size = 0.83

$$
-\frac{ax \tanh^{-1}(ax) - \frac{1}{2}(1-a^2x^2)\tanh^{-1}(ax)^2 - \frac{1}{3}\tanh^{-1}(ax)^3 - \tanh^{-1}(ax)^2 \log\left(1+e^{-2\tanh^{-1}(ax)}\right) - \log\left(\frac{1}{\sqrt{1-a^2x^2}}\right) + \tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((a*x*ArcTanh[a*x] - ((1 - a^2*x^2)*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^3/3 -
 ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - Log[1/Sqrt[1 - a^2*x^2]] + A
rcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*
x])]/2)/a^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 39.98, size = 728, normalized size = 5.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/2*a^2*x^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh
(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x
)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))
+1/12*arctanh(a*x)*(3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)*Pi+6*I
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh
(a*x)*Pi-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*Pi-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*arctanh(a*x)*Pi+3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)*Pi+3*I*csgn(I*(a*x+1)^2/(a^2*x^
```

```
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*Pi+3*I*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a
rctanh(a*x)*Pi+6*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*Pi-6*I
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*Pi+6*I*arctanh(a*x)*Pi-4
*arctanh(a*x)^2+12*arctanh(a*x)*ln(2)+6*arctanh(a*x)-12*a*x-12)+ln((a*x+1)^
2/(-a^2*x^2+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
[Out] -1/24*(3*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^4 +
1/4*integrate(-(a^3*x^3*log(a*x + 1)^2 - (a^3*x^3 + a^2*x^2 + (2*a^3*x^3 +
a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^5*x^2 - a^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{3}\operatorname{atanh}^{2}{\left(ax \right)}}{a^{2}x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1),x)
[Out] -Integral(x**3*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\mathrm{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

3.235 $\int \frac{x^2 \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=75

$$-\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}$$

```
[Out] -arctanh(a*x)^2/a^3-x*arctanh(a*x)^2/a^2+1/3*arctanh(a*x)^3/a^3+2*arctanh(a
*x)*ln(2/(-a*x+1))/a^3+polylog(2,1-2/(-a*x+1))/a^3
```

Rubi [A]
time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^3}{3a^3}-\frac{\tanh^{-1}(ax)^2}{a^3}+\frac{2\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -(ArcTanh[a*x]^2/a^3) - (x*ArcTanh[a*x]^2)/a^2 + ArcTanh[a*x]^3/(3*a^3) + (
2*ArcTanh[a*x]*Log[2/(1 - a*x)])/a^3 + PolyLog[2, 1 - 2/(1 - a*x)]/a^3
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx &= -\frac{\int \tanh^{-1}(ax)^2\,dx}{a^2}+\frac{\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^2}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}-\frac{2\int \text{l}}{}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{2\text{Sub}}{}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{\text{Li}_2(}{}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 59, normalized size = 0.79

$$-\frac{-\frac{1}{3}\tanh^{-1}(ax)\left(-3ax\tanh^{-1}(ax)+\tanh^{-1}(ax)\left(3+\tanh^{-1}(ax)\right)+6\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((-1/3*(ArcTanh[a*x]*(-3*a*x*ArcTanh[a*x] + ArcTanh[a*x]*(3 + ArcTanh[a*x]
) + 6*Log[1 + E^(-2*ArcTanh[a*x])])) + PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^
3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 41.03, size = 5330, normalized size = 71.07

method	result	size
derivativedivides	Expression too large to display	5330
default	Expression too large to display	5330

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(70) = 140.
time = 0.27, size = 200, normalized size = 2.67

$$-\frac{1}{2}\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^2-\frac{\frac{3(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\left(\log(ax-1)^2-4\log(ax-1)\right)\log(ax+1)+6\log(ax-1)^2}{a}-\frac{24\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}}{24a^2}+\frac{\left(2\left(\log(ax-1)-2\right)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\log(ax-1)\right)\operatorname{artanh}(ax)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x)^2 - 1/24*
((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3
*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24*
(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a)/a^2 + 1/4*(2*(
log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log(a*
x - 1))*arctanh(a*x)/a^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 \operatorname{atanh}^2(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2 \operatorname{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^2*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

3.236 $\int \frac{x \tanh^{-1}(ax)^2}{1-a^2x^2} \, dx$

Optimal. Leaf size=78

$$-\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2 \log \left(\frac{2}{1-ax}\right)}{a^2}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}$$

```
[Out] -1/3*arctanh(a*x)^3/a^2+arctanh(a*x)^2*ln(2/(-a*x+1))/a^2+arctanh(a*x)*poly
log(2,1-2/(-a*x+1))/a^2-1/2*polylog(3,1-2/(-a*x+1))/a^2
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6131, 6055, 6095, 6205, 6745}

$$-\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^2}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}-\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -1/3*ArcTanh[a*x]^3/a^2 + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^2 + (ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^2 - PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^2)
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^3}{3a^2} + \frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^3}{3a^2} + \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2} - \frac{2\int \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^3}{3a^2} + \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2} - \frac{\int \frac{\text{Li}_2\left(1-\frac{2}{1-}\right.}{1-a^2x^2}}{a}$$

$$= -\frac{\tanh^{-1}(ax)^3}{3a^2} + \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2} - \frac{\text{Li}_3\left(1-\frac{2}{1-}\right.}{2a^2}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.87

$$-\frac{-\frac{1}{3}\tanh^{-1}(ax)^3 - \tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + \tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((-1/3*ArcTanh[a*x]^3 - ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcT
anh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])
]/2)/a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 31.49, size = 638, normalized size = 8.18

method	result

derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^3}{3}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(ax+1)^2}{a^2x^2-1}\right)\right.}{}$
default	$-\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^3}{3}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(ax+1)^2}{a^2x^2-1}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a
*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3+1/4*(I*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^2-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^
2/(a^2*x^2-1))+2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+I*Pi*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-2*I
*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+2*I*Pi+4*ln(2))*arctanh(a*x)^2+arc
tanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*polylog(3,-(a*x+1)^2/(-a^2
*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(3*log(a*x + 1)*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^2 + 1/4*integrat
e(-(a*x*log(a*x + 1)^2 - (3*a*x + 1)*log(a*x + 1)*log(-a*x + 1))/(a^3*x^2 -
 a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \operatorname{atanh}^2(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1),x)
[Out] -Integral(x*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\,\operatorname{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^2)/(a^2*x^2 - 1),x)
[Out] -int((x*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

3.237 $\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

```
[Out] 1/3*arctanh(a*x)^3/a
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6095}

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^3/(3*a)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2} dx = \frac{\tanh^{-1}(ax)^3}{3a}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^3/(3*a)
```

Maple [A]
time = 0.23, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(ax)^3}{3a}$	12
default	$\frac{\operatorname{arctanh}(ax)^3}{3a}$	12
risch	$\frac{\ln(ax+1)^3}{24a}-\frac{\ln(-ax+1)\ln(ax+1)^2}{8a}+\frac{\ln(-ax+1)^2\ln(ax+1)}{8a}-\frac{\ln(-ax+1)^3}{24a}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctanh(a*x)^3/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(11) = 22$.
time = 0.26, size = 127, normalized size = 9.77

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2-\frac{\left(\log(ax+1)^2-2\log(ax+1)\log(ax-1)+\log(ax-1)^2\right)\operatorname{artanh}(ax)}{4a}+\frac{\log(ax+1)^3-3\log(ax+1)^2\log(ax-1)+3\log(ax+1)\log(ax-1)^2-\log(ax-1)^3}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^2 - 1/4*(log(a*x + 1)^2
- 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)*arctanh(a*x)/a + 1/24*(log(
a*x + 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2
- log(a*x - 1)^3)/a
```

Fricas [A]
time = 0.35, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

Sympy [A]
time = 1.00, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^{3}(ax)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.39, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

Mupad [B]
time = 0.95, size = 68, normalized size = 5.23

$$\frac{\ln\left(a\,x+1\right)^3}{24\,a}-\frac{\ln\left(1-a\,x\right)^3}{24\,a}+\frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^2}{8\,a}-\frac{\ln\left(a\,x+1\right)^2\,\ln\left(1-a\,x\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(a^2*x^2 - 1),x)
```

```
[Out] log(a*x + 1)^3/(24*a) - log(1 - a*x)^3/(24*a) + (log(a*x + 1)*log(1 - a*x)^
2)/(8*a) - (log(a*x + 1)^2*log(1 - a*x))/(8*a)
```

3.238 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=66

$$\frac{1}{3}\tanh^{-1}(ax)^3+\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,-1\right.$$

```
[Out] 1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2-2/(a*x+1))-arctanh(a*x)*polylog(2,-1
+2/(a*x+1))-1/2*polylog(3,-1+2/(a*x+1))
```

Rubi [A]
time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6135, 6079, 6095, 6203, 6745}

$$-\frac{1}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)-\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{3}\tanh^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyL
og[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{3}\tanh^{-1}(ax)^3 + \int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$
$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - (2a)\int \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,d$$
$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right) + a$$
$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{1}{2}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.91

$$-\frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right) + \tanh^{-1}(ax)\text{PolyLog}\left(2, e^{2\tanh^{-1}(ax)}\right) - \frac{1}{2}\text{PolyLog}\left(3, e^{2\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

```
[Out] -1/3*ArcTanh[a*x]^3 + ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + ArcTanh[
a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - PolyLog[3, E^(2*ArcTanh[a*x])]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 63.51, size = 1188, normalized size = 18.00

method	result	size
derivativedivides	Expression too large to display	1188
default	Expression too large to display	1188

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+ar
ctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(
-a^2*x^2+1)^(1/2))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*
x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3-1/2
*arctanh(a*x)^2*ln(a*x+1)-1/2*arctanh(a*x)^2*ln(a*x-1)+1/4*I*arctanh(a*x)^2
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/2*I*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)
^2/(-a^2*x^2+1)+1))^3+1/2*I*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/4*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*I*arctanh(a*x)^2*Pi*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*arctanh(a*x)^2*Pi-1/2*I*arctanh(a*x)^2*P
i*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^2-1/4*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*I*arcta
nh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^
2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I*arctanh(a*x)^2*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^2+1/2*I*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1
/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+arctanh(a*x)^2*ln(2)+arctanh(a*x)^2*ln
((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*log(a*x + 1)*log(-a*x + 1)^2 - 1/24*log(-a*x + 1)^3 + 1/4*integrate(((
a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1) - log(a*x + 1)^2)/(a^2*x^3 -
x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^3 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**2*x**3 - x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)^2}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^2/(x*(a^2*x^2 - 1)), x)
```

3.239 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)} dx$

Optimal. Leaf size=66

$$a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{1}{3}a\tanh^{-1}(ax)^3 + 2a\tanh^{-1}(ax)\log\left(2 - \frac{2}{1+ax}\right) - a\text{PolyLog}\left(2, -1 + \frac{2}{1+a$$

```
[Out] a*arctanh(a*x)^2-arctanh(a*x)^2/x+1/3*a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(
2-2/(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6129, 6037, 6135, 6079, 2497, 6095}

$$-a\text{Li}_2\left(\frac{2}{ax+1} - 1\right) + \frac{1}{3}a\tanh^{-1}(ax)^3 + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + 2a\log\left(2 - \frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a*ArcTanh[a*x]^3)/3 + 2*a*ArcTanh[a*
x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx\\
&= a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\
&= a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\\
&= a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 61, normalized size = 0.92

$$-a\left(-\frac{1}{3}\tanh^{-1}(ax)\left(-\frac{3\tanh^{-1}(ax)}{ax}+\tanh^{-1}(ax)\left(3+\tanh^{-1}(ax)\right)+6\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(a*(-1/3*(ArcTanh[a*x]*((-3*ArcTanh[a*x])/(a*x) + ArcTanh[a*x]*(3 + ArcTan
h[a*x]) + 6*Log[1 - E^(-2*ArcTanh[a*x])])) + PolyLog[2, E^(-2*ArcTanh[a*x])
]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 35.34, size = 4380, normalized size = 66.36

method	result	size
derivativedivides	Expression too large to display	4380
default	Expression too large to display	4380

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,(a*x+
1)/(-a^2*x^2+1)^(1/2))+polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*
x+1)/(-a^2*x^2+1)^(1/2))+1/3*arctanh(a*x)^3-arctanh(a*x)^2+1/4*I*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I
*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,-(a*x+1)/(-a^2
*x^2+1)^(1/2))-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*dilo
g((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))*arctanh(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/
2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a
rctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*ar
ctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*arctanh(
a*x)^2*ln(a*x+1)-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*I*Pi*csgn(I*(a*x+1)/(-a^2
*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)
^(1/2))+1/2*I*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*I*
arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/4*I*Pi*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x
^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*
x^2-1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*Pi*c
```

```
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))-1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*
x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x
^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/
2))-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*csgn(I*(a*x+1)/(-a^
2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-dilog((a*x+1
)/(-a^2*x^2+1)^(1/2))+dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*Pi*polylog(
2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*Pi*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2
))-arctanh(a*x)^2/a/x-1/2*I*Pi*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*d
ilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(
(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*(a*x+
1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*
x^2+1)^(1/2))+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2))+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^
2-1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x
^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))-1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*arctanh(a*x)^2+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a
^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)*ln(1
-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^
2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/4*I*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(63) = 126$.
time = 0.27, size = 237, normalized size = 3.59

$$-\frac{1}{24}a^2\left(\frac{3\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)^2-\log\left(ax+1\right)^3+\log\left(ax-1\right)^3-3\left(\log\left(ax-1\right)^2-4\log\left(ax-1\right)\right)\log\left(ax+1\right)+6\log\left(ax-1\right)^2}{a}-\frac{24\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}+\frac{24\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)}{a}-\frac{24\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(ax\right)\right)}{a}\right)+\frac{1}{4}\left(2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2-\log\left(ax-1\right)^2-4\log\left(ax-1\right)+8\log\left(x\right)\right)a\,\mathrm{artanh}\left(ax\right)+\frac{1}{2}\left(a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2}{x}\right)\mathrm{artanh}\left(ax\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*a^2*((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x
- 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^
2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*
(log(a*x + 1)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*
x))/a) + 1/4*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x
- 1)^2 - 4*log(a*x - 1) + 8*log(x))*a*arctanh(a*x) + 1/2*(a*log(a*x + 1) -
a*log(a*x - 1) - 2/x)*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^4 - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1),x)
[Out] -Integral(atanh(a*x)**2/(a**2*x**4 - x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x^2*(a^2*x^2 - 1)),x)
[Out] -int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)), x)
```

3.240 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=138

$$-\frac{a\tanh^{-1}(ax)}{x}+\frac{1}{2}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}+\frac{1}{3}a^2\tanh^{-1}(ax)^3+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\tanh^{-1}$$

```
[Out] -a*arctanh(a*x)/x+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+1/3*a^2*arc
tanh(a*x)^3+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+a^2*arctanh(a*x)^2*ln(2-2/(a*x
+1))-a^2*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*a^2*polylog(3,-1+2/(a*x+1
))
```

Rubi [A]
time = 0.24, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6129, 6037, 272, 36, 29, 31, 6095, 6135, 6079, 6203, 6745}

$$-\frac{1}{2}a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)-a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{3}a^2\tanh^{-1}(ax)^3+\frac{1}{2}a^2\tanh^{-1}(ax)^2+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}-\frac{a\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) + (
a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2 + a^2*ArcTanh
[a*x]^2*Log[2 - 2/(1 + a*x)] - a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)
] - (a^2*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

<u>Rule 6745</u>

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

<u>Rubi steps</u>

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx \\
&= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx + a^2\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx \\
&= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) + a\int \frac{\tanh^{-1}(}{x^2} \\
&= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax \\
&= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax \\
&= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax \\
&= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\log(x) - \frac{1}{2}
\end{aligned}$$

<u>Mathematica</u> [**C**] Result contains complex when optimal does not.
time = 0.24, size = 133, normalized size = 0.96

$$-a^2\left(-\frac{i\pi^3}{24}+\frac{\tanh^{-1}(ax)}{ax}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a^2x^2}+\frac{1}{3}\tanh^{-1}(ax)^3-\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)-\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -(a^2*((-1/24*I)*Pi^3 + ArcTanh[a*x]/(a*x) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)
/(2*a^2*x^2) + ArcTanh[a*x]^3/3 - ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])
] - Log[(a*x)/Sqrt[1 - a^2*x^2]] - ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x
])] + PolyLog[3, E^(2*ArcTanh[a*x])]/2))
```

<u>Maple</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 85.71, size = 1277, normalized size = 9.25

method	result	size

derivativedivides	Expression too large to display	1277
default	Expression too large to display	1277

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x+1)-1/2*arctanh
(a*x)^2/a^2/x^2+arctanh(a*x)^2*ln(a*x)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+
1)^(1/2))-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(
a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*p
olylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/12*arctanh(a*x)*(-6*I*csgn(I/((a*x+
1)^2/(-a^2*x^2+1)+1))^2*Pi*arctanh(a*x)*a*x+6*I*csgn(I*((a*x+1)^2/(-a^2*x^2
+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*arctanh(a*x)*a*x+3*I*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2*Pi*arctanh(a*x)*a*x-6*I*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*Pi*arctanh(a*x)*a*x-6*
I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^2*Pi*arctanh(a*x)*a*x+6*I*Pi*arctanh(a*x)*a*x+3*I*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*arctanh(a*x)*a*x+6*I*csgn(I*(a*x+1)/(-a^2
*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi*arctanh(a*x)*a*x+3*I*csgn
(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*arctanh(a
*x)*a*x-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^2*Pi*arctanh(a*x)*a*x+6*I*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))^3*Pi*arctanh(a*x)*a*x+6*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2
/(-a^2*x^2+1)-1))*Pi*arctanh(a*x)*a*x+3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^3*Pi*arctanh(a*x)*a*x-3*I*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))*Pi*arctanh(a*x)*a*x+12*ln(2)*arctanh(a*x)*a*x-4*arcta
nh(a*x)^2*a*x+6*a*x*arctanh(a*x)-12*a*x-12)/a/x+ln((a*x+1)/(-a^2*x^2+1)^(1/
2)-1)+ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(a^2*x^2*log(-a*x + 1)^3 + 3*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)
^2)/x^2 + 1/4*integrate(-(log(a*x + 1)^2 - (a^2*x^2 + a*x + (a^4*x^4 + a^3*
x^3 + 2)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^5 - x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^5 - x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1),x)
[Out] -Integral(atanh(a*x)**2/(a**2*x**5 - x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,\left(a^2\,x^2-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x^3*(a^2*x^2 - 1)),x)
[Out] -int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)), x)
```

3.241 $\int \frac{x^3 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx$

Optimal. Leaf size=205

$$-\frac{3\tanh^{-1}(ax)^2}{2a^4}-\frac{3x\tanh^{-1}(ax)^2}{2a^3}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^4}+\frac{3\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}$$

```
[Out] -3/2*arctanh(a*x)^2/a^4-3/2*x*arctanh(a*x)^2/a^3+1/2*arctanh(a*x)^3/a^4-1/2
*x^2*arctanh(a*x)^3/a^2-1/4*arctanh(a*x)^4/a^4+3*arctanh(a*x)*ln(2/(-a*x+1)
)/a^4+arctanh(a*x)^3*ln(2/(-a*x+1))/a^4+3/2*polylog(2,1-2/(-a*x+1))/a^4+3/2
*arctanh(a*x)^2*polylog(2,1-2/(-a*x+1))/a^4-3/2*arctanh(a*x)*polylog(3,1-2/
(-a*x+1))/a^4+3/4*polylog(4,1-2/(-a*x+1))/a^4
```

Rubi [A]
time = 0.33, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6127, 6037, 6021, 6131, 6055, 2449, 2352, 6095, 6205, 6209, 6745}

$$\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{4a^4}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{2a^4}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{2a^4}-\frac{\tanh^{-1}(ax)^4}{4a^4}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{3\tanh^{-1}(ax)^2}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^4}+\frac{3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}-\frac{3x\tanh^{-1}(ax)^2}{2a^3}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] (-3*ArcTanh[a*x]^2)/(2*a^4) - (3*x*ArcTanh[a*x]^2)/(2*a^3) + ArcTanh[a*x]^3
/(2*a^4) - (x^2*ArcTanh[a*x]^3)/(2*a^2) - ArcTanh[a*x]^4/(4*a^4) + (3*ArcTa
nh[a*x]*Log[2/(1 - a*x)])/a^4 + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 + (3*
PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(2*a^4) - (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^4) +
 (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx &= -\frac{\int x \tanh^{-1}(ax)^3\,dx}{a^2}+\frac{\int \frac{x \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^4}+\frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a^3}+\frac{3\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a}\\
&= -\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^4}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^4}-\frac{3\int \tanh^{-1}(ax)^2\,dx}{2a^3}\\
&= -\frac{3x\tanh^{-1}(ax)^2}{2a^3}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^4}+\frac{\tanh^{-1}(ax)^3}{a^4}\\
&= -\frac{3\tanh^{-1}(ax)^2}{2a^4}-\frac{3x\tanh^{-1}(ax)^2}{2a^3}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax}{4a^4}\\
&= -\frac{3\tanh^{-1}(ax)^2}{2a^4}-\frac{3x\tanh^{-1}(ax)^2}{2a^3}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax}{4a^4}\\
&= -\frac{3\tanh^{-1}(ax)^2}{2a^4}-\frac{3x\tanh^{-1}(ax)^2}{2a^3}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax}{4a^4}\\
&= -\frac{3\tanh^{-1}(ax)^2}{2a^4}-\frac{3x\tanh^{-1}(ax)^2}{2a^3}+\frac{\tanh^{-1}(ax)^3}{2a^4}-\frac{x^2\tanh^{-1}(ax)^3}{2a^2}-\frac{\tanh^{-1}(ax}{4a^4}
\end{aligned}
$$

Mathematica [A]

time = 0.19, size = 142, normalized size = 0.69

$$
-\frac{-6\tanh^{-1}(ax)^2+6ax\tanh^{-1}(ax)^2-2(1-a^2x^2)\tanh^{-1}(ax)^3-\tanh^{-1}(ax)^4-12\tanh^{-1}(ax)\log\left(1+e^{-2\tanh^{-1}(ax)}\right)-4\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+6\left(1+\tanh^{-1}(ax)^2\right)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)}{4a^4}
$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]

[Out] -1/4*(-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 - 2*(1 - a^2*x^2)*ArcTanh[a*x]^3 - ArcTanh[a*x]^4 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(1 + ArcTanh[a*x]^2)*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/a^4

Maple [A]
time = 52.64, size = 217, normalized size = 1.06

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^4}{4}-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3)(ax-1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)+\frac{3\operatorname{arctanh}(ax)^2\operatorname{polylog}}{2}$
default	$-\frac{\operatorname{arctanh}(ax)^4}{4}-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3)(ax-1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)+\frac{3\operatorname{arctanh}(ax)^2\operatorname{polylog}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)

[Out] 1/a^4*(-1/4*arctanh(a*x)^4-1/2*arctanh(a*x)^2*(a*x*arctanh(a*x)+arctanh(a*x)+3)*(a*x-1)+arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3*arctanh(a*x)^2+3*arctanh(a*x)*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")

[Out] 1/64*(4*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^4 - 1/8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)^3 - 6*a^3*x^3*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^3 + a^2*x^2 + (2*a^3*x^3 + a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^5*x^2 - a^3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 \operatorname{atanh}^3(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^3 \operatorname{atanh}(a\,x)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^3)/(a^2*x^2 - 1),x)
```

```
[Out] -int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

3.242 $\int \frac{x^2 \tanh^{-1}(ax)^3}{1-a^2x^2} dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}$$

```
[Out] -arctanh(a*x)^3/a^3-x*arctanh(a*x)^3/a^2+1/4*arctanh(a*x)^4/a^3+3*arctanh(a
*x)^2*ln(2/(-a*x+1))/a^3+3*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^3-3/2*pol
ylog(3,1-2/(-a*x+1))/a^3
```

Rubi [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6127, 6021, 6131, 6055, 6095, 6205, 6745}

$$-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^3}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}+\frac{\tanh^{-1}(ax)^4}{4a^3}-\frac{\tanh^{-1}(ax)^3}{a^3}+\frac{3\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -(ArcTanh[a*x]^3/a^3) - (x*ArcTanh[a*x]^3)/a^2 + ArcTanh[a*x]^4/(4*a^3) + (
3*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^3 + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(
1 - a*x)])/a^3 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx &= -\frac{\int \tanh^{-1}(ax)^3\,dx}{a^2}+\frac{\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}\\ &= -\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a}\\ &= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}-\frac{6\int\\ &= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\,\mathrm{ta}\\ &= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\,\mathrm{ta}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 78, normalized size = 0.76

$$\frac{\tanh^{-1}(ax)^2\left((4-4ax)\tanh^{-1}(ax)+\tanh^{-1}(ax)^2+12\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right)-12\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*((4 - 4*a*x)*ArcTanh[a*x] + ArcTanh[a*x]^2 + 12*Log[1 + E^(
-2*ArcTanh[a*x])]) - 12*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6*P
olyLog[3, -E^(-2*ArcTanh[a*x])])/(4*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 59.02, size = 736, normalized size = 7.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-arctanh(a*x)^3*a*x-1/2*arctanh(a*x)^3*ln(a*x-1)+1/2*arctanh(a*x)^3*
ln(a*x+1)-arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*arctanh(a*x)^4-
1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/4*I*Pi*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3-1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3-1/4*I*Pi*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3+3
*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3*arctanh(a*x)*polylog(2,-(a*x
+1)^2/(-a^2*x^2+1))-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))^3*arctanh(a*x)^3+1/2*I*Pi*arctanh(a*x)^3+1/4*I*Pi*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3+1/2*I*Pi*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))^3*arctanh(a*x)^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/4
*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-arctanh(a*x)^3-3/2*polylog(3,-(a*x+
1)^2/(-a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(4*(2*a*x - log(a*x + 1) - 2)*log(-a*x + 1)^3 + log(-a*x + 1)^4 - 6*(4
*(a*x + 1)*log(a*x + 1) - log(a*x + 1)^2)*log(-a*x + 1)^2)/a^3 + 1/8*integr
ate(-1/2*(2*a^2*x^2*log(a*x + 1)^3 - 3*((2*a^2*x^2 - a*x - 1)*log(a*x + 1)^
```

```
2 + 4*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^4*x^2 - a^2), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}\operatorname{atanh}^{3}(ax)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1),x)
[Out] -Integral(x**2*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,{\mathrm{atanh}\left(a\,x\right)}^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^3)/(a^2*x^2 - 1),x)
[Out] -int((x^2*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

3.243 $\int \frac{x \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}(ax)\text{PolyLog}(3,1-$$

```
[Out] -1/4*arctanh(a*x)^4/a^2+arctanh(a*x)^3*ln(2/(-a*x+1))/a^2+3/2*arctanh(a*x)^
2*polylog(2,1-2/(-a*x+1))/a^2-3/2*arctanh(a*x)*polylog(3,1-2/(-a*x+1))/a^2+
3/4*polylog(4,1-2/(-a*x+1))/a^2
```

Rubi [A]
time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6131, 6055, 6095, 6205, 6209, 6745}

$$\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{4a^2}+\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{2a^2}-\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -1/4*ArcTanh[a*x]^4/a^2 + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^2 + (3*ArcTan
h[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2) - (3*ArcTanh[a*x]*PolyLog[3,
1 - 2/(1 - a*x)])/(2*a^2) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^2)
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\int \frac{\tanh^{-}}{}}{}$$
$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}}{}$$
$$= -\frac{\tanh^{-1}(ax)^4}{4a^2}+\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}}{}$$

Mathematica [A]

time = 0.05, size = 87, normalized size = 0.81

$$-\frac{-\tanh^{-1}(ax)^4-4\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]

[Out] -1/4*(-ArcTanh[a*x]^4 - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/a^2

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 46.66, size = 670, normalized size = 6.20

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(ax+1)^2}{a^2x^2-1}\right)\ldots\right.}{\ldots}}{\ldots}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(ax+1)^2}{a^2x^2-1}\right)\ldots\right.}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)

[Out] 1/a^2*(-1/2*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4+1/4*(I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3-2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+2*I*Pi+4*ln(2))*arctanh(a*x)^3+3/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")

[Out] 1/64*(4*log(a*x + 1)*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^2 - 1/8*integrate(1/2*(2*a*x*log(a*x + 1)^3 - 6*a*x*log(a*x + 1)^2*log(-a*x + 1) + 3*(3*a*x + 1)*log(a*x + 1)*log(-a*x + 1)^2)/(a^3*x^2 - a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \operatorname{atanh}^{3}(ax)}{a^{2}x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1),x)
[Out] -Integral(x*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x \operatorname{atanh}(a\,x)^{3}}{a^{2}\,x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^3)/(a^2*x^2 - 1),x)
[Out] -int((x*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

3.244 $\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2} dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

```
[Out] 1/4*arctanh(a*x)^4/a
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6095}

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^4/(4*a)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2} dx = \frac{\tanh^{-1}(ax)^4}{4a}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^4/(4*a)
```

Maple [A]
time = 0.33, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(ax)^4}{4a}$	12
default	$\frac{\operatorname{arctanh}(ax)^4}{4a}$	12
risch	$\frac{\ln(ax+1)^4}{64a} - \frac{\ln(-ax+1)\ln(ax+1)^3}{16a} + \frac{3\ln(-ax+1)^2\ln(ax+1)^2}{32a} - \frac{\ln(-ax+1)^3\ln(ax+1)}{16a} + \frac{\ln(-ax+1)^4}{64a}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctanh(a*x)^4/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(11) = 22$.
time = 0.26, size = 209, normalized size = 16.08

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3 + \frac{1}{64}a\left(\frac{8\left(\log(ax+1)^3 - 3\log(ax+1)^2\log(ax-1) + 3\log(ax+1)\log(ax-1)^2 - \log(ax-1)^3\right)\operatorname{artanh}(ax)}{a^2} - \frac{\log(ax+1)^4 - 4\log(ax+1)^3\log(ax-1) + 6\log(ax+1)^2\log(ax-1)^2 - 4\log(ax+1)\log(ax-1)^3 + \log(ax-1)^4}{a^2}\right) - \frac{3\left(\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) + \log(ax-1)^2\right)\operatorname{artanh}(ax)^2}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^3 + 1/64*a*(8*(log(a*x +
 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2 - log
(a*x - 1)^3)*arctanh(a*x)/a^2 - (log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(a*x
- 1) + 6*log(a*x + 1)^2*log(a*x - 1)^2 - 4*log(a*x + 1)*log(a*x - 1)^3 + lo
g(a*x - 1)^4)/a^2) - 3/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + lo
g(a*x - 1)^2)*arctanh(a*x)^2/a
```

Fricas [A]
time = 0.35, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

Sympy [A]
time = 0.68, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^{4}(ax)}{4a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.39, size = 22, normalized size = 1.69

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

Mupad [B]
time = 0.90, size = 90, normalized size = 6.92

$$\frac{\ln\left(a\,x+1\right)^4}{64\,a}+\frac{\ln\left(1-a\,x\right)^4}{64\,a}-\frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^3}{16\,a}-\frac{\ln\left(a\,x+1\right)^3\,\ln\left(1-a\,x\right)}{16\,a}+\frac{3\ln\left(a\,x+1\right)^2\ln\left(1-a\,x\right)^2}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(a^2*x^2 - 1),x)
```

```
[Out] log(a*x + 1)^4/(64*a) + log(1 - a*x)^4/(64*a) - (log(a*x + 1)*log(1 - a*x)^
3)/(16*a) - (log(a*x + 1)^3*log(1 - a*x))/(16*a) + (3*log(a*x + 1)^2*log(1
- a*x)^2)/(32*a)
```

3.245 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=91

$$\frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)-\frac{3}{2}\tanh^{-1}(ax)\text{P}$$

```
[Out] 1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*polylo
g(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylog(4,-1
+2/(a*x+1))
```

Rubi [A]
time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6135, 6079, 6095, 6203, 6207, 6745}

$$-\frac{3}{4}\text{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-\frac{3}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}\tanh^{-1}(ax)^4+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*
PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*
x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])/4
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx &= \frac{1}{4}\tanh^{-1}(ax)^4+\int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx\\ &= \frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)-(3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\\ &= \frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)+\\ &= \frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)-\\ &= \frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)-\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.91

$$-\frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{4}\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

```
[Out] -1/4*ArcTanh[a*x]^4 + ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] + (3*ArcTa
nh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*PolyLog[3, E^
(2*ArcTanh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 81.98, size = 1245, normalized size = 13.68

method	result	size
derivativedivides	Expression too large to display	1245
default	Expression too large to display	1245

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*arctanh(a*x)^3*Pi+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2
+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))+6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*po
lylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)
^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arcta
nh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*
x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4-1/2*arctanh(a*x)^3*ln(a*x-1)-1/
2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+ar
ctanh(a*x)^3*ln(2)+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(
a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*arctanh(a*x)^3*Pi*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2-1/4*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I*arctanh(a*x)^3*Pi*csgn(
I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/
(-a^2*x^2+1)+1))^2-1/2*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*I*ar
ctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*
x^2-1))+1/2*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(
a*x+1)^2/(a^2*x^2-1))^2+1/4*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(
-a^2*x^2+1)+1))^2+1/2*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-
a^2*x^2+1)+1))^3+1/4*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+ar
ctanh(a*x)^3*ln(a*x)-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/16*log(a*x + 1)*log(-a*x + 1)^3 + 1/64*log(-a*x + 1)^4 - 1/8*integrate(1/
2*(3*(a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1)^2 + 2*log(a*x + 1)^3 -
6*log(a*x + 1)^2*log(-a*x + 1))/(a^2*x^3 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{3}{\left(ax \right)}}{a^{2}x^{3} - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**3 - x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{{\mathrm{atanh}\left(a\,x\right)}^3}{x\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^3/(x*(a^2*x^2 - 1)), x)
```

3.246 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)}\,dx$

Optimal. Leaf size=90

$$a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{1}{4}a\tanh^{-1}(ax)^4+3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-3a\tanh^{-1}(ax)\text{PolyLog}($$

```
[Out] a*arctanh(a*x)^3-arctanh(a*x)^3/x+1/4*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*l
n(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+
2/(a*x+1))
```

Rubi [A]

time = 0.20, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6129, 6037, 6135, 6079, 6095, 6203, 6745}

$$-\frac{3}{2}a\text{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+3a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/x + (a*ArcTanh[a*x]^4)/4 + 3*a*ArcTanh[a*
x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] -
 (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx \\
&= -\frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx \\
&= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx \\
&= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + 3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) \\
&= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + 3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) \\
&= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + 3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 93, normalized size = 1.03

$$-a\left(-\frac{i\pi^3}{8}+\tanh^{-1}(ax)^3+\frac{\tanh^{-1}(ax)^3}{ax}-\frac{1}{4}\tanh^{-1}(ax)^4-3\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(a*((-1/8*I)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - ArcTanh[a*x]^4
/4 - 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[
2, E^(2*ArcTanh[a*x])] + (3*PolyLog[3, E^(2*ArcTanh[a*x])])/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 67.87, size = 810, normalized size = 9.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*arctanh(a*x)^3*ln(a*x+1)-arctanh(a*x)^3/x/a-1/2*arctanh(a*x)^3*ln(a*
x-1)+1/2*I*arctanh(a*x)^3*Pi+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))*arctanh(a*x)^3-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/4*I*P
i*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctan
h(a*x)^3-6*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(3,(a*x+1)/(-a^2
*x^2+1)^(1/2))+1/4*arctanh(a*x)^4-arctanh(a*x)^3-1/2*I*Pi*csgn(I*(a*x+1)/(-
a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3+1/4*I*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
```

```
x^2+1)+1))^2*arctanh(a*x)^3+1/2*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^3-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3-1/4*I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)
^3-1/2*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-arctanh(a*x
)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))+3*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*
x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,-(a*x+1)/(-a
^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(a*x*log(-a*x + 1)^4 - 4*(a*x*log(a*x + 1) + 2*a*x - 2)*log(-a*x + 1)^
3 + 6*(a*x*log(a*x + 1)^2 - 4*(a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/x -
1/8*integrate(1/2*(2*log(a*x + 1)^3 + 3*((a^3*x^3 + a^2*x^2 - 2)*log(a*x +
1)^2 - 4*(a^3*x^3 + 2*a^2*x^2 + a*x)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^4
- x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{3}(ax)}{a^{2}x^{4}-x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**4 - x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\mathrm{atanh}(a\,x)^3}{x^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)), x)
```

3.247 $\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=200

$$\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{2x^2}+\frac{1}{4}a^2\tanh^{-1}(ax)^4+3a^2\tanh^{-1}(ax)\log\Big(2$$

```
[Out] 3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x+1/2*a^2*arctanh(a*x)^3-1/2*ar
ctanh(a*x)^3/x^2+1/4*a^2*arctanh(a*x)^4+3*a^2*arctanh(a*x)*ln(2-2/(a*x+1))+
a^2*arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/(a*x+1))-3/2*a^2*
arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))-3/2*a^2*arctanh(a*x)*polylog(3,-1+2/
(a*x+1))-3/4*a^2*polylog(4,-1+2/(a*x+1))
```

Rubi [A]
time = 0.34, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6129, 6037, 6135, 6079, 2497, 6095, 6203, 6207, 6745}

$$-\frac{3}{2}a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{3}{4}a^2\text{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-\frac{3}{2}a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}a^2\tanh^{-1}(ax)^4+\frac{1}{2}a^2\tanh^{-1}(ax)^3+\frac{3}{2}a^2\tanh^{-1}(ax)^2+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3+3a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^3}{2x^2}-\frac{3a\tanh^{-1}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3
)/2 - ArcTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^4)/4 + 3*a^2*ArcTanh[a*x]*
Log[2 - 2/(1 + a*x)] + a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*a^2*Pol
yLog[2, -1 + 2/(1 + a*x)])/2 - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 +
 a*x)])/2 - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*a^2*Po
lyLog[4, -1 + 2/(1 + a*x)])/4
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx \\
&= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx + a^2\int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,d \\
&= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) + \frac{1}{2}(3a)\int \\
&= -\frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tanh^{-1}(\\
&= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(\\
&= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(\\
&= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 165, normalized size = 0.82

$$-\frac{1}{64}a^2\left(-\pi^4-96\tanh^{-1}(ax)^2+\frac{96\tanh^{-1}(ax)^2}{ax}+\frac{32(1-a^2x^2)\tanh^{-1}(ax)^3}{a^2x^2}+16\tanh^{-1}(ax)^4-192\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)-64\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)+96\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+96\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-48\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -1/64*(a^2*(-Pi^4 - 96*ArcTanh[a*x]^2 + (96*ArcTanh[a*x]^2)/(a*x) + (32*(1
- a^2*x^2)*ArcTanh[a*x]^3)/(a^2*x^2) + 16*ArcTanh[a*x]^4 - 192*ArcTanh[a*x]
*Log[1 - E^(-2*ArcTanh[a*x])] - 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x]
)] + 96*PolyLog[2, E^(-2*ArcTanh[a*x])] - 96*ArcTanh[a*x]^2*PolyLog[2, E^(2
*ArcTanh[a*x])] + 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] - 48*PolyL
og[4, E^(2*ArcTanh[a*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(182) = 364.
time = 85.05, size = 369, normalized size = 1.84

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3ax)(ax-1)}{2a^2x^2}+\operatorname{arctanh}(ax)^3\ln\left(1+\right.\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3ax)(ax-1)}{2a^2x^2}+\operatorname{arctanh}(ax)^3\ln\left(1+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4*arctanh(a*x)^4+1/2*arctanh(a*x)^2*(a*x*arctanh(a*x)+arctanh(a*x)+
3*a*x)*(a*x-1)/a^2/x^2+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*ar
ctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(
3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arc
tanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*
x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2
))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2+3*arctanh(a*x)*
ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3
*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*polylog(2,(a*x+1)/(-a^2*x^
2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(a^2*x^2*log(-a*x + 1)^4 + 4*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)^
3)/x^2 - 1/8*integrate(1/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x +
1) + 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 + 2)*log(a*x + 1))*log(-a*x + 1)
^2)/(a^2*x^5 - x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^3(ax)}{a^2x^5 - x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1),x)
[Out] -Integral(atanh(a*x)**3/(a**2*x**5 - x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x^3*(a^2*x^2 - 1)),x)
[Out] -int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)), x)
```

3.248 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{1-a^2x^2}\,dx$

Optimal. Leaf size=15

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

```
[Out] 2/3*arctanh(a*x)^(3/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{6095\}$

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{1-a^2x^2}\,dx = \frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

Maple [A]
time = 0.72, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}(ax)^{\frac{3}{2}}}{3a}$	12
default	$\frac{2\,\mathrm{arctanh}(ax)^{\frac{3}{2}}}{3a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/3*arctanh(a*x)^(3/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.33, size = 25, normalized size = 1.67

$$\frac{\sqrt{2}\,\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

Sympy [A]
time = 0.70, size = 14, normalized size = 0.93

$$\begin{cases} \frac{2\,\mathrm{atanh}^{\frac{3}{2}}(ax)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((2*atanh(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.39, size = 25, normalized size = 1.67

$$\frac{\sqrt{2}\,\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

Mupad [B]
time = 0.87, size = 11, normalized size = 0.73

$$\frac{2\,\mathrm{atanh}(a\,x)^{3/2}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1),x)
```

```
[Out] (2*atanh(a*x)^(3/2))/(3*a)
```

3.249 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
[Out] Defer[Int][x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx = \int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Maple [A]
time = 4.14, size = 0, normalized size = 0.00

$$\int \frac{x}{(-a^2x^2+1)\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x),x)
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate(x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{2}x^{2}\operatorname{atanh}{\left(ax \right)} - \operatorname{atanh}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x),x)
[Out] -Integral(x/(a**2*x**2*atanh(a*x) - atanh(a*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)*(a^2*x^2 - 1)),x)
[Out] -int(x/(atanh(a*x)*(a^2*x^2 - 1)), x)
```

3.250 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] ln(arctanh(a*x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6093}

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Log[ArcTanh[a*x]]/a
```

Rule 6093

```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx=\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 1.00

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Log[ArcTanh[a*x]]/a
```

Maple [A]
time = 0.31, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln(\operatorname{arctanh}(ax))}{a}$	10
default	$\frac{\ln(\operatorname{arctanh}(ax))}{a}$	10
risch	$\frac{\ln(\ln(ax+1)-\ln(-ax+1))}{a}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] ln(arctanh(a*x))/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.26, size = 21, normalized size = 2.33

$$\frac{\log\left(-\log\left(ax+1\right)+\log\left(-ax+1\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] log(-log(a*x + 1) + log(-a*x + 1))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.
time = 0.35, size = 20, normalized size = 2.22

$$\frac{\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] log(log(-(a*x + 1)/(a*x - 1)))/a
```

Sympy [A]
time = 0.77, size = 7, normalized size = 0.78

$$\frac{\log\left(\operatorname{atanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] log(atanh(a*x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.39, size = 21, normalized size = 2.33

$$\frac{\log\left(\left|\log\left(-\frac{ax+1}{ax-1}\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] log(abs(log(-(a*x + 1)/(a*x - 1))))/a
```

Mupad [B]
time = 0.82, size = 9, normalized size = 1.00

$$\frac{\ln\left(\operatorname{atanh}(a\,x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)*(a^2*x^2 - 1)),x)
```

```
[Out] log(atanh(a*x))/a
```

3.251 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\,(1-a^2x^2)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\,(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\,(1-a^2x^2)\tanh^{-1}(ax)}\,dx = \int \frac{1}{x\,(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Maple [A]
time = 4.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(-a^2x^2+1)\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
[Out] -integrate(1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^3\operatorname{atanh}(ax) - x\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x),x)
[Out] -Integral(1/(a**2*x**3*atanh(a*x) - x*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{1}{x\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)*(a^2*x^2 - 1)),x)
[Out] -int(1/(x*atanh(a*x)*(a^2*x^2 - 1)), x)
```

3.252 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$-\frac{x}{a\tanh^{-1}(ax)}+\frac{\mathrm{Int}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a}$$

```
[Out] -x/a/arctanh(a*x)+Unintegrable(1/arctanh(a*x),x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*ArcTanh[a*x])) + Defer[Int][ArcTanh[a*x]^(-1), x]/a
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx=-\frac{x}{a\tanh^{-1}(ax)}+\frac{\int \frac{1}{\tanh^{-1}(ax)}\,dx}{a}$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 4.13, size = 0, normalized size = 0.00

$$\int \frac{x}{(-a^2x^2+1)\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x/(a*log(a*x + 1) - a*log(-a*x + 1)) - 2*integrate(-1/(a*log(a*x + 1) -
a*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^2*(a^2*x^2 - 1)),x)
[Out] -int(x/(atanh(a*x)^2*(a^2*x^2 - 1)), x)
```

3.253 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{a\tanh^{-1}(ax)}$$

[Out] -1/a/arctanh(a*x)

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6095}

$$-\frac{1}{a\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]

[Out] -(1/(a*ArcTanh[a*x]))

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\tanh^{-1}(ax)}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{a\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]

[Out] -(1/(a*ArcTanh[a*x]))

Maple [A]
time = 0.28, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\frac{1}{a \operatorname{arctanh}(ax)}$	12
default	$-\frac{1}{a \operatorname{arctanh}(ax)}$	12
risch	$\frac{2}{a(-\ln(ax+1)+\ln(-ax+1))}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/arctanh(a*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.26, size = 23, normalized size = 2.09

$$-\frac{2}{a\log(ax+1) - a\log(-ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/(a*log(a*x + 1) - a*log(-a*x + 1))
```

Fricas [A]
time = 0.36, size = 22, normalized size = 2.00

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [A]
time = 0.47, size = 8, normalized size = 0.73

$$-\frac{1}{a\operatorname{atanh}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -1/(a*atanh(a*x))
```

Giac [A]
time = 0.39, size = 22, normalized size = 2.00

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1)))
```

Mupad [B]
time = 0.78, size = 23, normalized size = 2.09

$$-\frac{2}{a\,\left(\ln\left(a\,x+1\right)-\ln\left(1-a\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^2*(a^2*x^2 - 1)),x)
[Out] -2/(a*(log(a*x + 1) - log(1 - a*x)))
```

3.254 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{ax\tanh^{-1}(ax)}-\frac{\text{Int}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}$$

```
[Out] -1/a/x/arctanh(a*x)-Unintegrable(1/x^2/arctanh(a*x),x)/a
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
[Out] -(1/(a*x*ArcTanh[a*x])) - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}\,dx=-\frac{1}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)}\,dx}{a}$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 4.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)\operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) + 2*integrate(-1/(a*x^2*log(a*x +
 1) - a*x^2*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}x^{3}\operatorname{atanh}^{2}\left(ax\right) - x\operatorname{atanh}^{2}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^2*(a^2*x^2 - 1)),x)
[Out] -int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)), x)
```

3.255 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=31

$$-\frac{x}{2a\tanh^{-1}(ax)^2}+\frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)^2},x\right)}{2a}$$

```
[Out] -1/2*x/a/arctanh(a*x)^2+1/2*Unintegrable(1/arctanh(a*x)^2,x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] -1/2*x/(a*ArcTanh[a*x]^2) + Defer[Int][ArcTanh[a*x]^(-2), x]/(2*a)
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx=-\frac{x}{2a\tanh^{-1}(ax)^2}+\frac{\int \frac{1}{\tanh^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 4.28, size = 0, normalized size = 0.00

$$\int \frac{x}{(-a^2x^2+1)\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*lo
g(a*x + 1)^2 - 2*a^2*log(a*x + 1)*log(-a*x + 1) + a^2*log(-a*x + 1)^2) + 2*
integrate(-x/(log(a*x + 1) - log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{2}x^{2}\operatorname{atanh}^{3}(ax) - \operatorname{atanh}^{3}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^3*(a^2*x^2 - 1)),x)
[Out] -int(x/(atanh(a*x)^3*(a^2*x^2 - 1)), x)
```

3.256 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

```
[Out] -1/2/a/arctanh(a*x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6095}

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] -1/2*1/(a*ArcTanh[a*x]^2)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\tanh^{-1}(ax)^2}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] -1/2*1/(a*ArcTanh[a*x]^2)
```

Maple [A]
time = 0.32, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{2a\,\mathrm{arctanh}(ax)^{2}}$	12
default	$-\frac{1}{2a\,\mathrm{arctanh}(ax)^{2}}$	12
risch	$-\frac{2}{a(-\ln(ax+1)+\ln(-ax+1))^{2}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/arctanh(a*x)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(11) = 22$.
time = 0.26, size = 42, normalized size = 3.23

$$-\frac{2}{a\log\left(ax+1\right)^{2}-2\,a\log\left(ax+1\right)\log\left(-ax+1\right)+a\log\left(-ax+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2/(a*log(a*x + 1)^2 - 2*a*log(a*x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2)
```

Fricas [A]
time = 0.37, size = 22, normalized size = 1.69

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [A]
time = 0.90, size = 12, normalized size = 0.92

$$-\frac{1}{2a\,\mathrm{atanh}^{2}\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -1/(2*a*atanh(a*x)**2)
```

Giac [A]
time = 0.39, size = 22, normalized size = 1.69

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

Mupad [B]
time = 0.80, size = 23, normalized size = 1.77

$$-\frac{2}{a\left(\ln\left(a\,x+1\right)-\ln\left(1-a\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^3*(a^2*x^2 - 1)),x)
[Out] -2/(a*(log(a*x + 1) - log(1 - a*x))^2)
```

3.257 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2ax\tanh^{-1}(ax)^2}-\frac{\text{Int}\left(\frac{1}{x^2\tanh^{-1}(ax)^2},x\right)}{2a}$$

```
[Out] -1/2/a/x/arctanh(a*x)^2-1/2*Unintegrable(1/x^2/arctanh(a*x)^2,x)/a
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] -1/2*1/(a*x*ArcTanh[a*x]^2) - Defer[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx=-\frac{1}{2ax\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 4.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-a^2x^2+1)\,\mathrm{arctanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*x^
2*log(a*x + 1)^2 - 2*a^2*x^2*log(a*x + 1)*log(-a*x + 1) + a^2*x^2*log(-a*x
+ 1)^2) - 2*integrate(-1/(a^2*x^3*log(a*x + 1) - a^2*x^3*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}x^{3}\operatorname{atanh}^{3}\left(ax\right) - x\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^3), x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(x*atanh(a*x)^3*(a^2*x^2 - 1)),x)

[Out] -int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)), x)

3.258 $\int \frac{\tanh^{-1}(ax)^p}{1-a^2x^2} dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}(ax)^{1+p}}{a(1+p)}$$

[Out] arctanh(a*x)^(1+p)/a/(1+p)

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6095}

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]^p/(1 - a^2*x^2),x]

[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^p}{1-a^2x^2} dx = \frac{\tanh^{-1}(ax)^{1+p}}{a(1+p)}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)^{1+p}}{a(1+p)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a*x]^p/(1 - a^2*x^2),x]

[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))

Maple [A]
time = 0.28, size = 18, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(ax)^{p+1}}{a(p+1)}$	18
default	$\frac{\operatorname{arctanh}(ax)^{p+1}}{a(p+1)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^p/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(a*x)^(p+1)/a/(p+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(arctanh(a*x)^p/(a^2*x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(17) = 34$.
time = 0.36, size = 84, normalized size = 4.94

$$\frac{\cosh\left(p\log\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+\log\left(-\frac{ax+1}{ax-1}\right)\sinh\left(p\log\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(p*log(1/2*log(-(a*x + 1)/(a*x - 1))))*log(-(a*x + 1)/(a*x - 1)) +
 log(-(a*x + 1)/(a*x - 1))*sinh(p*log(1/2*log(-(a*x + 1)/(a*x - 1)))))/(a*p
 + a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.87, size = 26, normalized size = 1.53

$$\begin{cases}\frac{\begin{cases}\frac{\operatorname{atanh}^{p+1}(ax)}{p+1} & \text{for}\ p\neq -1\\ \log\left(\operatorname{atanh}(ax)\right) & \text{otherwise}\end{cases}}{a} & \text{for}\ a\neq 0\\ 0^{p}x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**p/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((Piecewise((atanh(a*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(atanh(a
*x)), True))/a, Ne(a, 0)), (0**p*x, True))
```

Giac [A]
time = 0.40, size = 30, normalized size = 1.76

$$\frac{\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)^{p+1}}{a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] (1/2*log(-(a*x + 1)/(a*x - 1)))^(p + 1)/(a*(p + 1))
```

Mupad [B]
time = 1.05, size = 33, normalized size = 1.94

$$\begin{cases} \frac{\ln(\mathrm{atanh}(a\,x))}{a} & \text{if } p=-1 \\ \frac{\mathrm{atanh}(a\,x)^{p+1}}{a\,(p+1)} & \text{if } p\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^p/(a^2*x^2 - 1),x)
```

```
[Out] piecewise(p == -1, log(atanh(a*x))/a, p ~= -1, atanh(a*x)^(p + 1)/(a*(p + 1
)))
```

3.259 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^2} \, dx$

Optimal. Leaf size=109

$$-\frac{x}{4a^3(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{4a^4}+\frac{\tanh^{-1}(ax)}{2a^4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}$$

```
[Out] -1/4*x/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)/a^4+1/2*arctanh(a*x)/a^4/(-a^2*x^2
+1)+1/2*arctanh(a*x)^2/a^4-arctanh(a*x)*ln(2/(-a*x+1))/a^4-1/2*polylog(2,1-
2/(-a*x+1))/a^4
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6175, 6131, 6055, 2449, 2352, 6141, 205, 212}

$$-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{\tanh^{-1}(ax)}{4a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{2a^4(1-a^2x^2)}-\frac{x}{4a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/4*x/(a^3*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^4) + ArcTanh[a*x]/(2*a^4*(1
- a^2*x^2)) + ArcTanh[a*x]^2/(2*a^4) - (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4
- PolyLog[2, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx = \frac{\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{a^2} - \frac{\int \frac{x \tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\int \frac{1}{(1-a^2x^2)^2}\,dx}{2a^3} - \frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^3}$$

$$= -\frac{x}{4a^3(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\int \frac{1}{1-a^2x}}{4a^3}$$

$$= -\frac{x}{4a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{}{1}\right.}{a^4}$$

$$= -\frac{x}{4a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{}{1}\right.}{a^4}$$

Mathematica [A]

time = 0.10, size = 64, normalized size = 0.59

$$-\frac{4\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\left(\cosh\left(2\tanh^{-1}(ax)\right) - 4\log\left(1+e^{-2\tanh^{-1}(ax)}\right)\right) - 4\text{PolyLog}\left(2, -e^{-2\tanh^{-1}(ax)}\right) + \sinh\left(2\tanh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/8*(4*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 4*Log[1 + E
^(-2*ArcTanh[a*x])]) - 4*PolyLog[2, -E^(-2*ArcTanh[a*x])] + Sinh[2*ArcTanh[
a*x]])/a^4
```

Maple [A]

time = 0.75, size = 159, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)}{4ax+4} + \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} + \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} - \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4} + \frac{\ln(ax-1)^2}{8} + \frac{(\ln}{}}{a^4}$
default	$\frac{\frac{\operatorname{arctanh}(ax)}{4ax+4} + \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} + \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} - \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4} + \frac{\ln(ax-1)^2}{8} + \frac{(\ln}{}}{a^4}$
risch	$\frac{\ln(ax+1)^2}{8a^4} + \frac{\ln(ax-1)}{16a^4} - \frac{\ln(ax+1)x}{16a^3(ax-1)} - \frac{\ln(ax+1)}{16a^4(ax-1)} + \frac{\ln(ax+1)}{8a^4(ax+1)} + \frac{1}{8a^4(ax+1)} + \frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{4a^4} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*arctanh(a*x)/(a*x+1)+1/2*arctanh(a*x)*ln(a*x+1)-1/4*arctanh(a*x)
/(a*x-1)+1/2*arctanh(a*x)*ln(a*x-1)-1/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x-1)*ln
(1/2*a*x+1/2)+1/8*ln(a*x-1)^2+1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1
/2)-1/8*ln(a*x+1)^2+1/8/(a*x+1)-1/8*ln(a*x+1)+1/8/(a*x-1)+1/8*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 177, normalized size = 1.62

$$-\frac{1}{8}a\left(\frac{(a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)-(a^2x^2-1)\log(ax-1)^2-2ax-(a^2x^2-1)\log(ax-1)}{a^7x^2-a^5}+\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^5}+\frac{\log(ax+1)}{a^5}\right)-\frac{1}{2}\left(\frac{1}{a^6x^2-a^4}-\frac{\log(a^2x^2-1)}{a^4}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2*a*x - (a^2*x^2 - 1)*log(a*x - 1))
/(a^7*x^2 - a^5) + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/
2))/a^5 + log(a*x + 1)/a^5) - 1/2*(1/(a^6*x^2 - a^4) - log(a^2*x^2 - 1)/a^4
)*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atanh}(a\,x)}{{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^3*atanh(a*x))/(a^2*x^2 - 1)^2, x)
```

3.260 $\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^2} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{4a^3(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^2}{4a^3}$$

```
[Out] -1/4/a^3/(-a^2*x^2+1)+1/2*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-1/4*arctanh(a*x)^
2/a^3
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6145, 6095}

$$-\frac{\tanh^{-1}(ax)^2}{4a^3}+\frac{x\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{1}{4a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/4*1/(a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*a^2*(1 - a^2*x^2)) - ArcTa
nh[a*x]^2/(4*a^3)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6145

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]
 - Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x])
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q,
-5/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx &= -\frac{1}{4a^3(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{2a^2}\\ &= -\frac{1}{4a^3(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^2}{4a^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.79

$$\frac{1 - 2ax\tanh^{-1}(ax) + (1 - a^2x^2)\tanh^{-1}(ax)^2}{4a^3(-1 + a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (1 - a^2*x^2)*ArcTanh[a*x]^2)/(4*a^3*(-1 + a^2*x^
2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(51) = 102.
time = 0.73, size = 134, normalized size = 2.35

method	result
risch	$-\frac{\ln(ax+1)^2}{16a^3} + \frac{(x^2\ln(-ax+1)a^2 - 2ax - \ln(-ax+1))\ln(ax+1)}{8a^3(a^2x^2-1)} - \frac{a^2x^2\ln(-ax+1)^2 - 4ax\ln(-ax+1) - \ln(-ax+1)^2 - 4}{16a^3(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax+1)} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} + \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4} + \frac{\ln(ax+1)^2}{16} - \frac{\left(\ln(ax+1) - \ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{8} - }{a^3}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax+1)} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} + \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4} + \frac{\ln(ax+1)^2}{16} - \frac{\left(\ln(ax+1) - \ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{8} - }{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/4*arctanh(a*x)/(a*x+1)-1/4*arctanh(a*x)*ln(a*x+1)-1/4*arctanh(a*x
)/(a*x-1)+1/4*arctanh(a*x)*ln(a*x-1)+1/16*ln(a*x+1)^2-1/8*(ln(a*x+1)-ln(1/2
*a*x+1/2))*ln(-1/2*a*x+1/2)-1/8*ln(a*x-1)*ln(1/2*a*x+1/2)+1/16*ln(a*x-1)^2+
1/8/(a*x-1)-1/8/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(49) = 98.
time = 0.25, size = 126, normalized size = 2.21

$$-\frac{1}{4}\left(\frac{2x}{a^4x^2 - a^2} + \frac{\log(ax+1)}{a^3} - \frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax) + \frac{((a^2x^2-1)\log(ax+1)^2 - 2(a^2x^2-1)\log(ax+1)\log(ax-1) + (a^2x^2-1)\log(ax-1)^2 + 4)a}{16(a^6x^2 - a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x) + 1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(
a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a/(a^6*x^2 - a^4)
```

Fricas [A]
time = 0.36, size = 65, normalized size = 1.14

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\,(a^5x^2-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^5*x^2 - a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,\mathrm{atanh}\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

Mupad [B]
time = 0.96, size = 110, normalized size = 1.93

$$\ln\left(1-a\,x\right)\,\left(\frac{\ln\left(a\,x+1\right)}{8\,a^3}+\frac{x}{2\,a^2\,\left(2\,a^2\,x^2-2\right)}\right)-\frac{{\ln\left(a\,x+1\right)}^2}{16\,a^3}-\frac{{\ln\left(1-a\,x\right)}^2}{16\,a^3}-\frac{1}{2\,a^2\,\left(2\,a-2\,a^3\,x^2\right)}-\frac{x\,\ln\left(a\,x+1\right)}{4\,a^3\,\left(a\,x^2-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(1 - a*x)*(log(a*x + 1)/(8*a^3) + x/(2*a^2*(2*a^2*x^2 - 2))) - log(a*x +
 1)^2/(16*a^3) - log(1 - a*x)^2/(16*a^3) - 1/(2*a^2*(2*a - 2*a^3*x^2)) - (x
*log(a*x + 1))/(4*a^3*(a*x^2 - 1/a))
```

3.261 $\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=55

$$-\frac{x}{4a(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{4a^2}+\frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}$$

```
[Out] -1/4*x/a/(-a^2*x^2+1)-1/4*arctanh(a*x)/a^2+1/2*arctanh(a*x)/a^2/(-a^2*x^2+1
)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6141, 205, 212}

$$-\frac{x}{4a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/4*x/(a*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^2) + ArcTanh[a*x]/(2*a^2*(1 -
a^2*x^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx &= \frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)} - \frac{\int \frac{1}{(1-a^2x^2)^2}\,dx}{2a} \\
&= -\frac{x}{4a(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)} - \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a} \\
&= -\frac{x}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{4a^2} + \frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.20

$$\frac{2ax - 4\tanh^{-1}(ax) + (-1+a^2x^2)\log(1-ax) + \log(1+ax) - a^2x^2\log(1+ax)}{8a^2(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] (2*a*x - 4*ArcTanh[a*x] + (-1 + a^2*x^2)*Log[1 - a*x] + Log[1 + a*x] - a^2*
x^2*Log[1 + a*x])/(8*a^2*(-1 + a^2*x^2))
```

Maple [A]
time = 0.31, size = 57, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{2(a^2x^2-1)}+\frac{1}{8ax+8}-\frac{\ln(ax+1)}{8}+\frac{1}{8ax-8}+\frac{\ln(ax-1)}{8}}{a^2}$	57
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{2(a^2x^2-1)}+\frac{1}{8ax+8}-\frac{\ln(ax+1)}{8}+\frac{1}{8ax-8}+\frac{\ln(ax-1)}{8}}{a^2}$	57
risch	$-\frac{\ln(ax+1)}{4a^2(a^2x^2-1)}-\frac{a^2x^2\ln(ax+1)-x^2\ln(-ax+1)a^2-2ax-\ln(ax+1)-\ln(-ax+1)}{8a^2(ax-1)(ax+1)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2/(a^2*x^2-1)*arctanh(a*x)+1/8/(a*x+1)-1/8*ln(a*x+1)+1/8/(a*x-1)+
1/8*ln(a*x-1))
```

Maxima [A]
time = 0.25, size = 62, normalized size = 1.13

$$\frac{\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}}{8a} - \frac{\operatorname{artanh}(ax)}{2(a^2x^2-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)/a - 1/2*arctanh(a
*x)/((a^2*x^2 - 1)*a^2)
```

Fricas [**A**]
time = 0.43, size = 48, normalized size = 0.87

$$\frac{2\,ax - (a^2x^2 + 1)\log\left(-\frac{ax+1}{ax-1}\right)}{8\,(a^4x^2 - a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [**A**]
time = 1.08, size = 61, normalized size = 1.11

$$\begin{cases} -\frac{a^2x^2\operatorname{atanh}(ax)}{4a^4x^2-4a^2} + \frac{ax}{4a^4x^2-4a^2} - \frac{\operatorname{atanh}(ax)}{4a^4x^2-4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Piecewise((-a**2*x**2*atanh(a*x)/(4*a**4*x**2 - 4*a**2) + a*x/(4*a**4*x**2
- 4*a**2) - atanh(a*x)/(4*a**4*x**2 - 4*a**2), Ne(a, 0)), (0, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(47) = 94.
time = 0.40, size = 154, normalized size = 2.80

$$-\frac{1}{16}\left(\left(\frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right) - \frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/16*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*((a*
x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x -
 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) - (a*x + 1)/((a*x - 1)*a^3) + (a
*x - 1)/((a*x + 1)*a^3))*a
```

Mupad [B]
time = 0.91, size = 37, normalized size = 0.67

$$-\frac{\operatorname{atanh}(a\,x)}{4\,a^2}-\frac{\frac{\operatorname{atanh}(a\,x)}{2}-\frac{a\,x}{4}}{a^2\,\left(a^2\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x))/(a^2*x^2 - 1)^2,x)
[Out] - atanh(a*x)/(4*a^2) - (atanh(a*x)/2 - (a*x)/4)/(a^2*(a^2*x^2 - 1))
```

3.262 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4a}$$

```
[Out] -1/4/a/(-a^2*x^2+1)+1/2*x*arctanh(a*x)/(-a^2*x^2+1)+1/4*arctanh(a*x)^2/a
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6103, 267}

$$-\frac{1}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x
]^2/(4*a)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx &= \frac{x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4a}-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}\,dx\\ &= -\frac{1}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.81

$$\frac{1-2ax\tanh^{-1}(ax)+(-1+a^2x^2)\tanh^{-1}(ax)^2}{4a\,(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (-1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a*(-1 + a^2*x^2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(48) = 96.
time = 0.73, size = 134, normalized size = 2.48

method	result
risch	$\frac{\ln(ax+1)^2}{16a}-\frac{(x^2\ln(-ax+1)a^2+2ax-\ln(-ax+1))\ln(ax+1)}{8(a^2x^2-1)a}+\frac{a^2x^2\ln(-ax+1)^2+4ax\ln(-ax+1)-\ln(-ax+1)^2+4}{16a(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\mathrm{arctanh}(ax)}{4(ax+1)}+\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{4}-\frac{\mathrm{arctanh}(ax)}{4(ax-1)}-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{4}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{8}-\frac{\ln(ax-1)^2}{16}+\frac{(\ln(ax+1)-\ln(}{}}{a}$
default	$\frac{-\frac{\mathrm{arctanh}(ax)}{4(ax+1)}+\frac{\mathrm{arctanh}(ax)\ln(ax+1)}{4}-\frac{\mathrm{arctanh}(ax)}{4(ax-1)}-\frac{\mathrm{arctanh}(ax)\ln(ax-1)}{4}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{8}-\frac{\ln(ax-1)^2}{16}+\frac{(\ln(ax+1)-\ln(}{}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4*arctanh(a*x)/(a*x+1)+1/4*arctanh(a*x)*ln(a*x+1)-1/4*arctanh(a*x)/
(a*x-1)-1/4*arctanh(a*x)*ln(a*x-1)+1/8*ln(a*x-1)*ln(1/2*a*x+1/2)-1/16*ln(a*
x-1)^2+1/8*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/16*ln(a*x+1)^2+1/
8/(a*x-1)-1/8/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(46) = 92.
time = 0.26, size = 122, normalized size = 2.26

$$-\frac{1}{4}\left(\frac{2\,x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)-\frac{\left((a^2x^2-1)\log(ax+1)^2-2\,(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2-4\right)a}{16\,(a^4x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x) - 1
/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x -
1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a/(a^4*x^2 - a^2)
```

Fricas [A]
time = 0.34, size = 64, normalized size = 1.19

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(a^{2}x^{2}-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-4}{16\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(ax\right)}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(46) = 92.
time = 1.26, size = 255, normalized size = 4.72

$$\frac{1}{8}\,a^{2}\left(\frac{(ax-1)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}-1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}+1}\right)}{(ax+1)a^{4}}+\frac{ax-1}{(ax+1)a^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*((a*x - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(
a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)
/((a*x + 1)*a/(a*x - 1) - a) - 1)) - 1)/(a*((a*x + 1)/(a*x - 1) + 1)/(a - a
*(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1
)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + 1))/((a*x + 1)*a^4) +
(a*x - 1)/((a*x + 1)*a^4))
```

Mupad [B]
time = 0.97, size = 106, normalized size = 1.96

$$\frac{\ln(a\,x+1)^2}{16\,a}-\ln(1-a\,x)\,\left(\frac{\ln(a\,x+1)}{8\,a}-\frac{x}{2\,\left(2\,a^2\,x^2-2\right)}\right)+\frac{\ln(1-a\,x)^2}{16\,a}+\frac{1}{2\,a\,\left(2\,a^2\,x^2-2\right)}-\frac{x\,\ln(a\,x+1)}{4\,a\,\left(a\,x^2-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(a*x + 1)^2/(16*a) - log(1 - a*x)*(log(a*x + 1)/(8*a) - x/(2*(2*a^2*x^2
- 2))) + log(1 - a*x)^2/(16*a) + 1/(2*a*(2*a^2*x^2 - 2)) - (x*log(a*x + 1))
/(4*a*(a*x^2 - 1/a))
```

3.263 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{ax}{4(1-a^2x^2)}-\frac{1}{4}\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{2}\text{PolyLog}\left(2,-\right.$$

```
[Out] -1/4*a*x/(-a^2*x^2+1)-1/4*arctanh(a*x)+1/2*arctanh(a*x)/(-a^2*x^2+1)+1/2*ar
ctanh(a*x)^2+arctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6177, 6135, 6079, 2497, 6141, 205, 212}

$$-\frac{ax}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}-\frac{1}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{4}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] -1/4*(a*x)/(1 - a^2*x^2) - ArcTanh[a*x]/4 + ArcTanh[a*x]/(2*(1 - a^2*x^2))
+ ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/
(1 + a*x)]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&= \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\
&= -\frac{ax}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{4}\\
&= -\frac{ax}{4\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\log\Bigg(2
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.69

$$\frac{1}{8}\left(4\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)\left(\cosh\left(2\tanh^{-1}(ax)\right)+4\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-\sinh\left(2\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (4*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] + 4*Log[1 - E^(-2*
ArcTanh[a*x])]) - 4*PolyLog[2, E^(-2*ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]])
/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(81) = 162.
time = 0.43, size = 190, normalized size = 2.09

method	result
derivativedivides	$\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-$ di
default	$\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-$ di
risch	$-\frac{\ln(ax+1)^2}{8}-\frac{\operatorname{dilog}(ax+1)}{2}+\frac{\ln(ax-1)}{16}-\frac{\ln(ax+1)(ax+1)}{16(ax-1)}+\frac{\ln(ax+1)}{8ax+8}+\frac{1}{8ax+8}-\frac{(\ln(ax+1)-\ln(\frac{ax}{2}+\frac{1}{2}))\ \mathrm{l}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(a*x)*ln(a*x)-1/4*arctanh(a*x)/(a*x-1)-1/2*arctanh(a*x)*ln(a*x-1)+1/
4*arctanh(a*x)/(a*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*dilog(a*x)-1/2*dilog(
a*x+1)-1/2*ln(a*x)*ln(a*x+1)+1/2*dilog(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*
x+1/2)-1/8*ln(a*x-1)^2-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/8
*ln(a*x+1)^2+1/8/(a*x+1)-1/8*ln(a*x+1)+1/8/(a*x-1)+1/8*ln(a*x-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(78) = 156.
time = 0.27, size = 206, normalized size = 2.26

$$\frac{1}{8}a\left(\frac{(a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)-(a^2x^2-1)\log(ax-1)^2+2ax}{a^3x^2-a}+\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}-\frac{4\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)}{a}+\frac{4\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)}{a}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)-\frac{1}{2}\left(\frac{1}{a^2x^2-1}+\log\left(a^2x^2-1\right)-\log\left(x^2\right)\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
 - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 2*a*x)/(a^3*x^2 - a) + 4*(log(a*x -
1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x + 1)*log(x) +
 dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a - log(a*x + 1)/a
```

```
+ log(a*x - 1)/a) - 1/2*(1/(a^2*x^2 - 1) + log(a^2*x^2 - 1) - log(x^2))*arc
tanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
[Out] integral(arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**2,x)
[Out] Integral(atanh(a*x)/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="giac")
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x\,(a^2\,x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x*(a^2*x^2 - 1)^2),x)
[Out] int(atanh(a*x)/(x*(a^2*x^2 - 1)^2), x)
```

3.264 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a}{4(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{3}{4}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x^2\right)$$

```
[Out] -1/4*a/(-a^2*x^2+1)-arctanh(a*x)/x+1/2*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+3/4*
a*arctanh(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6177, 6129, 6037, 272, 36, 29, 31, 6095, 6103, 267}

$$-\frac{a}{4(1-a^2x^2)}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)}+a\log(x)+\frac{3}{4}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] -1/4*a/(1 - a^2*x^2) - ArcTanh[a*x]/x + (a^2*x*ArcTanh[a*x])/(2*(1 - a^2*x^
2)) + (3*a*ArcTanh[a*x]^2)/4 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx$$

$$= \frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{4}a\tanh^{-1}(ax)^2 + a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx - \frac{1}{2}a^3\int \frac{x}{(1-a^2x^2)^2}\,dx +$$

$$= -\frac{a}{4(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{3}{4}a\tanh^{-1}(ax)^2 + a\int \frac{1}{x(1-a^2x^2}$$

$$= -\frac{a}{4(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{3}{4}a\tanh^{-1}(ax)^2 + \frac{1}{2}a\text{Subst}\Bigg(\int \frac{}{x(}$$

$$= -\frac{a}{4(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{3}{4}a\tanh^{-1}(ax)^2 + \frac{1}{2}a\text{Subst}\Bigg(\int \frac{1}{x}$$

$$= -\frac{a}{4(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{3}{4}a\tanh^{-1}(ax)^2 + a\log(x) - \frac{1}{2}a\,\text{lo}$$

Mathematica [A]
time = 0.07, size = 77, normalized size = 0.94

$$\frac{1}{4}\left(-\frac{2(-2+3a^2x^2)\tanh^{-1}(ax)}{x(-1+a^2x^2)} + 3a\tanh^{-1}(ax)^2 + a\left(\frac{1}{-1+a^2x^2} + 4\log(ax) - 2\log\left(1-a^2x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] ((-2*(-2 + 3*a^2*x^2)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)) + 3*a*ArcTanh[a*x]^2
 + a*((-1 + a^2*x^2)^(-1) + 4*Log[a*x] - 2*Log[1 - a^2*x^2]))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(74) = 148.
time = 0.37, size = 164, normalized size = 2.00

method	result
derivativedivides	$a\Big(-\frac{\text{arctanh}(ax)}{4(ax-1)} - \frac{3\,\text{arctanh}(ax)\ln(ax-1)}{4} - \frac{\text{arctanh}(ax)}{ax} - \frac{\text{arctanh}(ax)}{4(ax+1)} + \frac{3\,\text{arctanh}(ax)\ln(ax+1)}{4} + \frac{3\ln(ax}{}$
default	$a\Big(-\frac{\text{arctanh}(ax)}{4(ax-1)} - \frac{3\,\text{arctanh}(ax)\ln(ax-1)}{4} - \frac{\text{arctanh}(ax)}{ax} - \frac{\text{arctanh}(ax)}{4(ax+1)} + \frac{3\,\text{arctanh}(ax)\ln(ax+1)}{4} + \frac{3\ln(ax}{}$
risch	$\frac{3a\ln(ax+1)^2}{16} - \frac{(3a^3x^3\ln(-ax+1)+6a^2x^2-3ax\ln(-ax+1)-4)\ln(ax+1)}{8x(a^2x^2-1)} + \frac{3a^3x^3\ln(-ax+1)^2+16\ln(x)a^3x^3-8\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/4*arctanh(a*x)/(a*x-1)-3/4*arctanh(a*x)*ln(a*x-1)-arctanh(a*x)/a/x-1/
4*arctanh(a*x)/(a*x+1)+3/4*arctanh(a*x)*ln(a*x+1)+3/8*ln(a*x-1)*ln(1/2*a*x+
1/2)-3/16*ln(a*x-1)^2+3/8*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-3/16
*ln(a*x+1)^2-1/2*ln(a*x-1)+1/8/(a*x-1)+ln(a*x)-1/2*ln(a*x+1)-1/8/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(72) = 144$.
time = 0.26, size = 150, normalized size = 1.83

$$-\frac{1}{16}\,a\left(\frac{3\,(a^2x^2-1)\log(ax+1)^2-6\,(a^2x^2-1)\log(ax+1)\log(ax-1)+3\,(a^2x^2-1)\log(ax-1)^2-4}{a^2x^2-1}+8\,\log(ax+1)+8\,\log(ax-1)-16\,\log(x)\right)+\frac{1}{4}\left(3\,a\log(ax+1)-3\,a\log(ax-1)-\frac{2\,(3\,a^2x^2-2)}{a^2x^3-x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/16*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*x + 1)*log
(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8*log(a*x +
 1) + 8*log(a*x - 1) - 16*log(x)) + 1/4*(3*a*log(a*x + 1) - 3*a*log(a*x - 1
) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 118, normalized size = 1.44

$$\frac{3\,(a^3x^3-ax)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,ax-8\,(a^3x^3-ax)\log(a^2x^2-1)+16\,(a^3x^3-ax)\log(x)-4\,(3\,a^2x^2-2)\log\left(-\frac{ax+1}{ax-1}\right)}{16\,(a^2x^3-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3*x^3 - a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x - 8*(a^3*x^3 -
a*x)*log(a^2*x^2 - 1) + 16*(a^3*x^3 - a*x)*log(x) - 4*(3*a^2*x^2 - 2)*log(-
(a*x + 1)/(a*x - 1)))/(a^2*x^3 - x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.
time = 1.97, size = 253, normalized size = 3.09

$$\begin{cases}\frac{4a^3x^3\log(x)}{4a^2x^3-4x}-\frac{4a^3x^3\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x}+\frac{3a^3x^3\operatorname{atanh}^2(ax)}{4a^2x^3-4x}-\frac{4a^3x^3\operatorname{atanh}(ax)}{4a^2x^3-4x}-\frac{6a^2x^2\operatorname{atanh}(ax)}{4a^2x^3-4x}-\frac{4ax\log(x)}{4a^2x^3-4x}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x}-\frac{3ax\operatorname{atanh}^2(ax)}{4a^2x^3-4x}+\frac{4ax\operatorname{atanh}(ax)}{4a^2x^3-4x}+\frac{ax}{4a^2x^3-4x}+\frac{4\operatorname{atanh}(ax)}{4a^2x^3-4x} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Piecewise((4*a**3*x**3*log(x)/(4*a**2*x**3 - 4*x) - 4*a**3*x**3*log(x - 1/a
)/(4*a**2*x**3 - 4*x) + 3*a**3*x**3*atanh(a*x)**2/(4*a**2*x**3 - 4*x) - 4*a
**3*x**3*atanh(a*x)/(4*a**2*x**3 - 4*x) - 6*a**2*x**2*atanh(a*x)/(4*a**2*x*
*3 - 4*x) - 4*a*x*log(x)/(4*a**2*x**3 - 4*x) + 4*a*x*log(x - 1/a)/(4*a**2*x
**3 - 4*x) - 3*a*x*atanh(a*x)**2/(4*a**2*x**3 - 4*x) + 4*a*x*atanh(a*x)/(4*
```

```
a**2*x**3 - 4*x) + a*x/(4*a**2*x**3 - 4*x) + 4*atanh(a*x)/(4*a**2*x**3 - 4*
x), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^2), x)
```

Mupad [B]
time = 1.15, size = 132, normalized size = 1.61

$$\frac{3\,a\ln\left(a\,x+1\right)^2}{16}+\frac{3\,a\ln\left(1-a\,x\right)^2}{16}+\frac{a}{2\,\left(2\,a^2\,x^2-2\right)}-\frac{a\,\ln\left(a^2\,x^2-1\right)}{2}+a\,\ln\left(x\right)-\ln\left(1-a\,x\right)\,\left(\frac{\frac{3\,a^2\,x^2}{2}-1}{2\,x-2\,a^2\,x^3}+\frac{3\,a\,\ln\left(a\,x+1\right)}{8}\right)+\frac{\ln\left(a\,x+1\right)\,\left(\frac{3\,a\,x^2}{4}-\frac{1}{2\,a}\right)}{\frac{x}{a}-a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] (3*a*log(a*x + 1)^2)/16 + (3*a*log(1 - a*x)^2)/16 + a/(2*(2*a^2*x^2 - 2)) -
 (a*log(a^2*x^2 - 1))/2 + a*log(x) - log(1 - a*x)*(((3*a^2*x^2)/2 - 1)/(2*x
 - 2*a^2*x^3) + (3*a*log(a*x + 1))/8) + (log(a*x + 1)*((3*a*x^2)/4 - 1/(2*a
)))/(x/a - a*x^3)
```

3.265 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)^2} \, dx$

Optimal. Leaf size=123

$$-\frac{a}{2x}-\frac{a^3x}{4(1-a^2x^2)}+\frac{1}{4}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{a^2\tanh^{-1}(ax)}{2(1-a^2x^2)}+a^2\tanh^{-1}(ax)^2+2a^2\tanh^{-1}(ax)\log\left(2\right.$$

```
[Out] -1/2*a/x-1/4*a^3*x/(-a^2*x^2+1)+1/4*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1
/2*a^2*arctanh(a*x)/(-a^2*x^2+1)+a^2*arctanh(a*x)^2+2*a^2*arctanh(a*x)*ln(2
-2/(a*x+1))-a^2*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.27, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6177, 6129, 6037, 331, 212, 6135, 6079, 2497, 6141, 205}

$$-a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{a^2\tanh^{-1}(ax)}{2(1-a^2x^2)}+a^2\tanh^{-1}(ax)^2+\frac{1}{4}a^2\tanh^{-1}(ax)+2a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{a^3x}{4(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] -1/2*a/x - (a^3*x)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x])/4 - ArcTanh[a*x]/
(2*x^2) + (a^2*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + a^2*ArcTanh[a*x]^2 + 2*a^2
*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a^2*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx\right) + a^4\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}\,dx + 2\left(\frac{1}{2}a^2\tanh^{-1}(ax)^2 + a^2\right.$$

$$= -\frac{a}{2x} - \frac{a^3x}{4(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2(1-a^2x^2)} - \frac{1}{4}a^3\int \frac{1}{1-a^2x^2}\,dx + \frac{1}{2}a^3\int$$

$$= -\frac{a}{2x} - \frac{a^3x}{4(1-a^2x^2)} + \frac{1}{4}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2(1-a^2x^2)} + 2\left(\frac{1}{2}a^2\tan\right.$$

Mathematica [A]

time = 0.24, size = 83, normalized size = 0.67

$$\frac{1}{8}a^2\left(-\frac{4}{ax}+8\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)\left(2-\frac{2}{a^2x^2}+\cosh\left(2\tanh^{-1}(ax)\right)+8\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-8\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-\sinh\left(2\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*(-4/(a*x) + 8*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(2 - 2/(a^2*x^2) + Cosh[
2*ArcTanh[a*x]] + 8*Log[1 - E^(-2*ArcTanh[a*x])]) - 8*PolyLog[2, E^(-2*ArcT
anh[a*x])] - Sinh[2*ArcTanh[a*x]]))/8
```

Maple [A]

time = 0.44, size = 213, normalized size = 1.73

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+2\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\operatorname{arctanh}(ax)\ln(ax+1)-\frac{\operatorname{arctan}}{4(ax}\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+2\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\operatorname{arctanh}(ax)\ln(ax+1)-\frac{\operatorname{arctan}}{4(ax}\right.$

risch	$-\frac{a}{2x}+\frac{a^2\ln(ax-1)}{16}-\frac{a^2\ln(-ax-1)}{16}-\frac{a^2}{8(-ax+1)}-\frac{a^2\ln(-ax+1)}{8(-ax+1)}+\frac{a^2\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln(-ax+1)}{2}+\frac{a^2\ln(-ax+}{-16ax-16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arctanh(a*x)/a^2/x^2+2*arctanh(a*x)*ln(a*x)+1/4*arctanh(a*x)/(a*x
+1)-arctanh(a*x)*ln(a*x+1)-1/4*arctanh(a*x)/(a*x-1)-arctanh(a*x)*ln(a*x-1)-
dilog(a*x)-dilog(a*x+1)-ln(a*x)*ln(a*x+1)-1/4*ln(a*x-1)^2+dilog(1/2*a*x+1/2
)+1/2*ln(a*x-1)*ln(1/2*a*x+1/2)-1/2*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x
+1/2)+1/4*ln(a*x+1)^2-1/2/a/x+1/8/(a*x+1)+1/8*ln(a*x+1)+1/8/(a*x-1)-1/8*ln(
a*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(110) = 220$.
time = 0.27, size = 233, normalized size = 1.89

$$\frac{1}{8}\left(8\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-8\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a+8\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2(ax)\right)a+a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2\left(a^2x^2-\left(a^3x^3-ax\right)\log\left(ax+1\right)^2+2\left(a^3x^3-ax\right)\log\left(ax+1\right)\log\left(ax-1\right)+\left(a^3x^3-ax\right)\log\left(ax-1\right)^2-2\right)}{a^2x^3-x}\right)a-\frac{1}{2}\left(2a^2\log\left(a^2x^2-1\right)-2a^2\log\left(x^2\right)+\frac{2a^2x^2-1}{a^2x^4-x^2}\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 8*(log
(a*x + 1)*log(x) + dilog(-a*x))*a + 8*(log(-a*x + 1)*log(x) + dilog(a*x))*a
 + a*log(a*x + 1) - a*log(a*x - 1) - 2*(a^2*x^2 - (a^3*x^3 - a*x)*log(a*x +
 1)^2 + 2*(a^3*x^3 - a*x)*log(a*x + 1)*log(a*x - 1) + (a^3*x^3 - a*x)*log(a
*x - 1)^2 - 2)/(a^2*x^3 - x))*a - 1/2*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x
^2) + (2*a^2*x^2 - 1)/(a^2*x^4 - x^2))*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(ax\right)}{x^3\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**2,x)
[Out] Integral(atanh(a*x)/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)}{x^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^3*(a^2*x^2 - 1)^2),x)
[Out] int(atanh(a*x)/(x^3*(a^2*x^2 - 1)^2), x)
```

3.266 $\int \frac{x^3 \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} \, dx$

Optimal. Leaf size=161

$$\frac{1}{4a^4(1-a^2x^2)} - \frac{x\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4} - \tanh$$

```
[Out] 1/4/a^4/(-a^2*x^2+1)-1/2*x*arctanh(a*x)/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)^2
/a^4+1/2*arctanh(a*x)^2/a^4/(-a^2*x^2+1)+1/3*arctanh(a*x)^3/a^4-arctanh(a*x
)^2*ln(2/(-a*x+1))/a^4-arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^4+1/2*polylog
(3,1-2/(-a*x+1))/a^4
```

Rubi [A]
time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6175, 6131, 6055, 6095, 6205, 6745, 6141, 6103, 267}

$$\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^4} - \frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2}{4a^4} - \frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^4} + \frac{1}{4a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^4(1-a^2x^2)} - \frac{x\tanh^{-1}(ax)}{2a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] 1/(4*a^4*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a^3*(1 - a^2*x^2)) - ArcTanh[
a*x]^2/(4*a^4) + ArcTanh[a*x]^2/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(3*a
^4) - (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - (ArcTanh[a*x]*PolyLog[2, 1 -
2/(1 - a*x)])/a^4 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx = \frac{\int \frac{x \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{a^2} - \frac{\int \frac{x \tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a^3} - \frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^3}$$

$$= -\frac{x \tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2 \log}{a^4}$$

$$= \frac{1}{4a^4\left(1-a^2x^2\right)} - \frac{x \tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} -$$

$$= \frac{1}{4a^4\left(1-a^2x^2\right)} - \frac{x \tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} -$$

Mathematica [A]
time = 0.10, size = 103, normalized size = 0.64

$$\frac{-\frac{1}{3}\tanh^{-1}(ax)^3 + \frac{1}{8}\left(1+2\tanh^{-1}(ax)^2\right)\cosh\left(2\tanh^{-1}(ax)\right) - \tanh^{-1}(ax)^2\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + \tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) - \frac{1}{4}\tanh^{-1}(ax)\sinh\left(2\tanh^{-1}(ax)\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-1/3*ArcTanh[a*x]^3 + ((1 + 2*ArcTanh[a*x]^2)*Cosh[2*ArcTanh[a*x]])/8 - Ar
cTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]*PolyLog[2, -E^(-2*
ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])]/2 - (ArcTanh[a*x]*Sinh[2*
ArcTanh[a*x]])/4)/a^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 64.10, size = 735, normalized size = 4.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/4*arctanh(a*x)^2/(a*x-1)+1/2*arctanh(a*x)^2*ln(a*x-1)+1/4*arctanh
(a*x)^2/(a*x+1)+1/2*arctanh(a*x)^2*ln(a*x+1)-arctanh(a*x)^2*ln((a*x+1)/(-a^
2*x^2+1)^(1/2))+1/3*arctanh(a*x)^3-1/8*arctanh(a*x)*(a*x-1)/(a*x+1)-1/16*(a
*x-1)/(a*x+1)+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)-1/16*(a*x+1)/(a*x-1)-arctanh
(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a*x+1)^2/(-a^2*x^2
+1))-1/4*(2*I*Pi+I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
```

```
)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))+2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+2*I*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-2*I*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1+4
*ln(2))*arctanh(a*x)^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -3/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^
3), x) - 1/4*a^2*integrate(x^2*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*
x^2 + a^3), x) - 1/32*(a*(2/(a^7*x - a^6) - log(a*x + 1)/a^6 + log(a*x - 1)
/a^6) + 4*log(-a*x + 1)/(a^7*x^2 - a^5))*a + 1/4*a*integrate(x*log(a*x + 1)
*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x) + 1/24*((a^2*x^2 - 1)*log(-a
*x + 1)^3 + 3*((a^2*x^2 - 1)*log(a*x + 1) - 1)*log(-a*x + 1)^2)/(a^6*x^2 -
a^4) + 1/4*integrate(a^3*x^3*log(a*x + 1)^2/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
 + 1/4*integrate(log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
 + 1/4*integrate(log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^2(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atanh}(a\,x)^2}{{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^2, x)
```

3.267 $\int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=94

$$\frac{x}{4a^2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{4a^3}-\frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^3}{6a^3}$$

```
[Out] 1/4*x/a^2/(-a^2*x^2+1)+1/4*arctanh(a*x)/a^3-1/2*arctanh(a*x)/a^3/(-a^2*x^2+
1)+1/2*x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)-1/6*arctanh(a*x)^3/a^3
```

Rubi [A]

time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6147, 6141, 205, 212}

$$-\frac{\tanh^{-1}(ax)^3}{6a^3}+\frac{\tanh^{-1}(ax)}{4a^3}+\frac{x}{4a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] x/(4*a^2*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a^3) - ArcTanh[a*x]/(2*a^3*(1 - a
^2*x^2)) + (x*ArcTanh[a*x]^2)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(6*a^3
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6147

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> Simp[-(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[b*(p/(2*c)), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2
), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx &= \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3} - \frac{\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{a}\\ &= -\frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{(1-a^2x^2)^2}\,dx}{2a^2}\\ &= \frac{x}{4a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a^2}\\ &= \frac{x}{4a^2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{4a^3} - \frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.99

$$\frac{12\tanh^{-1}(ax) - 12ax\tanh^{-1}(ax)^2 + (4-4a^2x^2)\tanh^{-1}(ax)^3 - 3(2ax + (-1+a^2x^2)\log(1-ax) + (1-a^2x^2)\log(1+ax))}{24a^3(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + (4 - 4*a^2*x^2)*ArcTanh[a*x]^3 -
 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(24*
a^3*(-1 + a^2*x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 51.87, size = 1340, normalized size = 14.26

method	result
risch	$-\frac{\ln(ax+1)^3}{48a^3} + \frac{(x^2\ln(-ax+1)a^2-2ax-\ln(-ax+1))\ln(ax+1)^2}{16a^3(a^2x^2-1)} - \frac{(a^2x^2\ln(-ax+1)^2-4ax\ln(-ax+1)-\ln(-ax+1)^2}{16a^3(ax-1)(ax+1)}$
derivativedivides	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/4*arctanh(a*x)^2/(a*x-1)+1/4*arctanh(a*x)^2*ln(a*x-1)-1/4*arctanh
(a*x)^2/(a*x+1)-1/4*arctanh(a*x)^2*ln(a*x+1)+1/2*arctanh(a*x)^2*ln((a*x+1)/
(-a^2*x^2+1)^(1/2))+1/24*(-4*arctanh(a*x)^3*a^2*x^2-6*I*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2*Pi-3*I*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))*arctanh(a*x)^2*Pi+3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi-6*I*Pi*
arctanh(a*x)^2*a^2*x^2-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))^2*arctanh(a*x)^2*Pi-6*a*x+4*arctanh(a*x)^3+6*a^2*x^2*arcta
nh(a*x)+6*arctanh(a*x)-3*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)*arctanh(a*x)^2*a^2*x^2+3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*arctanh(a*x)^2*Pi-6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)
^2*a^2*x^2+3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^2*x^2+3*
I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x
)^2*a^2*x^2+6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^2*
x^2+6*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-3*I*csgn(I*(
a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-6*I*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*arctanh(a*x)^2*Pi+6*I*arctanh(a*x)^2*Pi-3*I*Pi*csgn(I*(a*x+1)
^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*
arctanh(a*x)^2*a^2*x^2+3*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(
a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2+6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^
2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*a^2*x^2+3*I*Pi*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^2*x^2)/(a*x-1)/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(81) = 162$.
time = 0.26, size = 273, normalized size = 2.90

$$-\frac{1}{4}\left(\frac{2x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^2-\frac{\left((a^2x^2-1)\log(ax+1)^3-3(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3+12ax-3\left(2a^2x^2-(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)+6(a^2x^2-1)\log(ax-1)\right)a^2}{48(a^7x^2-a^5)}+\frac{\left((a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2+4\right)a\operatorname{artanh}(ax)}{8(a^6x^2-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x)^2 - 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*
log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*(2*a^2*x^2 - (a^2*
```

```
x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 - 1)*log(a*x - 1))*a
^2/(a^7*x^2 - a^5) + 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*lo
g(a*x + 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)/
(a^6*x^2 - a^4)
```

Fricas [A]
time = 0.39, size = 96, normalized size = 1.02

$$-\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^2 -
a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\operatorname{atanh}^2\left(ax\right)}{\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

Mupad [B]
time = 1.76, size = 231, normalized size = 2.46

$$\frac{\ln\left(1-a\,x\right)}{4\,a^3-4\,a^5\,x^2}-\frac{{\ln\left(a\,x+1\right)}^3}{48\,a^3}+\frac{{\ln\left(1-a\,x\right)}^3}{48\,a^3}+\frac{x}{4\,a^2-4\,a^4\,x^2}-\frac{\ln\left(a\,x+1\right)}{4\,\left(a^3-a^5\,x^2\right)}+\frac{x\,{\ln\left(1-a\,x\right)}^2}{8\,a^2-8\,a^4\,x^2}-\frac{\ln\left(a\,x+1\right)\,{\ln\left(1-a\,x\right)}^2}{16\,a^3}+\frac{{\ln\left(a\,x+1\right)}^2\,\ln\left(1-a\,x\right)}{16\,a^3}+\frac{x\,{\ln\left(a\,x+1\right)}^2}{8\,\left(a^2-a^4\,x^2\right)}-\frac{x\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)}{4\,a^2-4\,a^4\,x^2}-\frac{\mathrm{atan}\left(a\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(1 - a*x)/(4*a^3 - 4*a^5*x^2) - log(a*x + 1)^3/(48*a^3) + log(1 - a*x)^3
/(48*a^3) + x/(4*a^2 - 4*a^4*x^2) - (atan(a*x*1i)*1i)/(4*a^3) - log(a*x + 1
)/(4*(a^3 - a^5*x^2)) + (x*log(1 - a*x)^2)/(8*a^2 - 8*a^4*x^2) - (log(a*x +
 1)*log(1 - a*x)^2)/(16*a^3) + (log(a*x + 1)^2*log(1 - a*x))/(16*a^3) + (x*
log(a*x + 1)^2)/(8*(a^2 - a^4*x^2)) - (x*log(a*x + 1)*log(1 - a*x))/(4*a^2
- 4*a^4*x^2)
```

3.268 $\int \frac{x \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=82

$$\frac{1}{4a^2(1-a^2x^2)} - \frac{x \tanh^{-1}(ax)}{2a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^2}{4a^2} + \frac{\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)}$$

```
[Out] 1/4/a^2/(-a^2*x^2+1)-1/2*x*arctanh(a*x)/a/(-a^2*x^2+1)-1/4*arctanh(a*x)^2/a
^2+1/2*arctanh(a*x)^2/a^2/(-a^2*x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6141, 6103, 267}

$$\frac{1}{4a^2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{x \tanh^{-1}(ax)}{2a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] 1/(4*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^2/(4*a^2) + ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx &= \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}-\frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a}\\
&= -\frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^2}+\frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}+\frac{1}{2}\int\frac{x}{\left(1-a^2x^2\right)^2}\,dx\\
&= \frac{1}{4a^2\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^2}+\frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.52

$$
\frac{1-2ax\tanh^{-1}(ax)+\left(1+a^2x^2\right)\tanh^{-1}(ax)^2}{4a^2-4a^4x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a^2 - 4*a^4*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(74) = 148$.
time = 0.38, size = 153, normalized size = 1.87

method	result
risch	$-\frac{\left(a^2x^2+1\right)\ln(ax+1)^2}{16a^2(ax-1)(ax+1)}+\frac{\left(x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1)\right)\ln(ax+1)}{8a^2(ax-1)(ax+1)}-\frac{a^2x^2\ln(-ax+1)^2+4ax\ln(-ax+1)+\ln\cdots}{16a^2(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2}{2\left(a^2x^2-1\right)}+\frac{\operatorname{arctanh}(ax)}{4ax-4}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\cdots\right.}{8}\cdots}{a^2}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2}{2\left(a^2x^2-1\right)}+\frac{\operatorname{arctanh}(ax)}{4ax-4}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\cdots\right.}{8}\cdots}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2/(a^2*x^2-1)*arctanh(a*x)^2+1/4*arctanh(a*x)/(a*x-1)+1/4*arctanh
(a*x)*ln(a*x-1)+1/4*arctanh(a*x)/(a*x+1)-1/4*arctanh(a*x)*ln(a*x+1)-1/8*(ln
(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/16*ln(a*x+1)^2-1/8*ln(a*x-1)*ln
(1/2*a*x+1/2)+1/16*ln(a*x-1)^2+1/8/(a*x+1)-1/8/(a*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(71) = 142$.
time = 0.26, size = 146, normalized size = 1.78

$$\frac{\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)}{4a} + \frac{(a^2x^2-1)\log(ax+1)^2 - 2(a^2x^2-1)\log(ax+1)\log(ax-1) + (a^2x^2-1)\log(ax-1)^2 - 4}{16(a^4x^2-a^2)} - \frac{\operatorname{artanh}(ax)^2}{2(a^2x^2-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)/a +
1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x -
 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^4*x^2 - a^2) - 1/2*arctanh(a*x)^
2/((a^2*x^2 - 1)*a^2)
```

Fricas [A]
time = 0.35, size = 66, normalized size = 0.80

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right) - (a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 4}{16\,(a^4x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1))^2 - 4)/(a^4*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [A]
time = 0.40, size = 140, normalized size = 1.71

$$-\frac{1}{32}\left(\left(\frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 2\left(\frac{ax+1}{(ax-1)a^3} - \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{2(ax+1)}{(ax-1)a^3} + \frac{2(ax-1)}{(ax+1)a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/32*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*x +
1)/(a*x - 1))^2 - 2*((a*x + 1)/((a*x - 1)*a^3) - (a*x - 1)/((a*x + 1)*a^3))
*log(-(a*x + 1)/(a*x - 1)) + 2*(a*x + 1)/((a*x - 1)*a^3) + 2*(a*x - 1)/((a*
x + 1)*a^3))*a
```

Mupad [B]
time = 1.16, size = 198, normalized size = 2.41

$$\ln(1-a\,x)\,\left(\frac{\frac{x}{2}-\frac{1}{2\,a}}{4\,a-4\,a^3\,x^2}+\frac{\frac{x}{2}+\frac{1}{2\,a}}{4\,a-4\,a^3\,x^2}+\ln(a\,x+1)\,\left(\frac{1}{8\,a^2}+\frac{1}{2\,a^2\,\left(2\,a^2\,x^2-2\right)}\right)\right)-\ln(1-a\,x)^2\,\left(\frac{1}{16\,a^2}+\frac{1}{2\,a^2\,\left(4\,a^2\,x^2-4\right)}\right)-\frac{1}{2\,a^2\,\left(2\,a^2\,x^2-2\right)}-\ln(a\,x+1)^2\,\left(\frac{1}{8\,a^3\,\left(a\,x^2-\frac{1}{a}\right)}+\frac{1}{16\,a^2}\right)+\frac{x\,\ln(a\,x+1)}{4\,a^2\,\left(a\,x^2-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(1 - a*x)*((x/2 - 1/(2*a))/(4*a - 4*a^3*x^2) + (x/2 + 1/(2*a))/(4*a - 4*
a^3*x^2) + log(a*x + 1)*(1/(8*a^2) + 1/(2*a^2*(2*a^2*x^2 - 2)))) - log(1 -
a*x)^2*(1/(16*a^2) + 1/(2*a^2*(4*a^2*x^2 - 4))) - 1/(2*a^2*(2*a^2*x^2 - 2))
 - log(a*x + 1)^2*(1/(8*a^3*(a*x^2 - 1/a)) + 1/(16*a^2)) + (x*log(a*x + 1))
/(4*a^2*(a*x^2 - 1/a))
```

3.269 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=88

$$\frac{x}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{4a}-\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}$$

```
[Out] 1/4*x/(-a^2*x^2+1)+1/4*arctanh(a*x)/a-1/2*arctanh(a*x)/a/(-a^2*x^2+1)+1/2*x
*arctanh(a*x)^2/(-a^2*x^2+1)+1/6*arctanh(a*x)^3/a
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6103, 6141, 205, 212}

$$\frac{x}{4(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{\tanh^{-1}(ax)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

```
[Out] x/(4*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a) - ArcTanh[a*x]/(2*a*(1 - a^2*x^2))
 + (x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx &= \frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}-a\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{1}{2}\int \frac{1}{(1-a^2x^2)^2}\,dx\\ &= \frac{x}{4(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{1}{4}\int \frac{1}{1-a^2x^2}\,dx\\ &= \frac{x}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{4a}-\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 93, normalized size = 1.06

$$\frac{12\tanh^{-1}(ax)-12ax\tanh^{-1}(ax)^2+4(-1+a^2x^2)\tanh^{-1}(ax)^3-3(2ax+(-1+a^2x^2)\log(1-ax)+(1-a^2x^2)\log(1+ax))}{24a(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

```
[Out] (12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^3
- 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(24
*a*(-1 + a^2*x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 71.18, size = 1340, normalized size = 15.23

method	result
risch	$\frac{\ln(ax+1)^3}{48a}-\frac{(x^2\ln(-ax+1)a^2+2ax-\ln(-ax+1))\ln(ax+1)^2}{16(a^2x^2-1)a}+\frac{(a^2x^2\ln(-ax+1)^2+4ax\ln(-ax+1)-\ln(-ax+1)^2\cdots}{16a(ax-1)(ax+1)}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4*arctanh(a*x)^2/(a*x-1)-1/4*arctanh(a*x)^2*ln(a*x-1)-1/4*arctanh(a
*x)^2/(a*x+1)+1/4*arctanh(a*x)^2*ln(a*x+1)-1/2*arctanh(a*x)^2*ln((a*x+1)/(-
a^2*x^2+1)^(1/2))-1/24*(-4*arctanh(a*x)^3*a^2*x^2-6*I*csgn(I*(a*x+1)^2/(a^2
*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2*Pi-3*I*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))*arctanh(a*x)^2*Pi+3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi-6*I*Pi*ar
ctanh(a*x)^2*a^2*x^2-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2
*x^2+1)^(1/2))^2*arctanh(a*x)^2*Pi+6*a*x+4*arctanh(a*x)^3-6*a^2*x^2*arctanh
(a*x)-6*arctanh(a*x)-3*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*
arctanh(a*x)^2*a^2*x^2+3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a
rctanh(a*x)^2*Pi-6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2
*a^2*x^2+3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^2*x^2+3*I*
Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^
2*a^2*x^2+6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^2*x^
2+6*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-3*I*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-6*I*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))^2*arctanh(a*x)^2*Pi+6*I*arctanh(a*x)^2*Pi-3*I*Pi*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*ar
ctanh(a*x)^2*a^2*x^2+3*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2+6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+
1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*a^2*x^2+3*I*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*arctanh(a*x)^2*a^2*x^2)/(a*x-1)/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(75) = 150$.
time = 0.27, size = 268, normalized size = 3.05

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2+\frac{\left((a^2x^2-1)\log(ax+1)^3-3(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3-12ax+3\left(2a^2x^2+(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)-6(a^2x^2-1)\log(ax-1)\right)a^2}{48(a^5x^2-a^3)}-\frac{\left((a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2-4\right)a\operatorname{artanh}(ax)}{8(a^4x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2 +
 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2 -
```

```
1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^
5*x^2 - a^3) - 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x
+ 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/(a^4*x
^2 - a^2)
```

Fricas [A]
time = 0.37, size = 95, normalized size = 1.08

$$-\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^3*x^2 -
a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [A]
time = 1.42, size = 88, normalized size = 1.00

$$\frac{1}{16}\,a^2\left(\frac{(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax+1)a^4}+\frac{2\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)}{(ax+1)a^4}+\frac{2\,(ax-1)}{(ax+1)a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*a^2*((a*x - 1)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)*a^4) + 2*(a*x -
1)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)*a^4) + 2*(a*x - 1)/((a*x + 1)*a^4))
```

Mupad [B]
time = 1.50, size = 213, normalized size = 2.42

$$\frac{\ln(a\,x+1)^3}{48\,a}-\frac{\ln(a\,x+1)}{4\,(a-a^3\,x^2)}-\frac{\ln(1-a\,x)^3}{48\,a}-\frac{x}{4\,a^2\,x^2-4}+\frac{\ln(1-a\,x)}{4\,a-4\,a^3\,x^2}+\frac{\ln(a\,x+1)\,\ln(1-a\,x)^2}{16\,a}-\frac{\ln(a\,x+1)^2\,\ln(1-a\,x)}{16\,a}-\frac{x\ln(a\,x+1)^2}{8\,(a^2\,x^2-1)}-\frac{x\ln(1-a\,x)^2}{2\,(4\,a^2\,x^2-4)}+\frac{x\,\ln(a\,x+1)\,\ln(1-a\,x)}{4\,a^2\,x^2-4}-\frac{\operatorname{atan}(a\,x\,1\mathrm{i})\,1\mathrm{i}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(a^2*x^2 - 1)^2,x)
```

```
[Out] log(a*x + 1)^3/(48*a) - log(a*x + 1)/(4*(a - a^3*x^2)) - log(1 - a*x)^3/(48
*a) - x/(4*a^2*x^2 - 4) - (atan(a*x*1i)*1i)/(4*a) + log(1 - a*x)/(4*a - 4*a
^3*x^2) + (log(a*x + 1)*log(1 - a*x)^2)/(16*a) - (log(a*x + 1)^2*log(1 - a*
x))/(16*a) - (x*log(a*x + 1)^2)/(8*(a^2*x^2 - 1)) - (x*log(1 - a*x)^2)/(2*(
4*a^2*x^2 - 4)) + (x*log(a*x + 1)*log(1 - a*x))/(4*a^2*x^2 - 4)
```

3.270 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=136

$$\frac{1}{4(1-a^2x^2)}-\frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)}-\frac{1}{4}\tanh^{-1}(ax)^2+\frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{1}{3}\tanh^{-1}(ax)^3+\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right.$$

```
[Out] 1/4/(-a^2*x^2+1)-1/2*a*x*arctanh(a*x)/(-a^2*x^2+1)-1/4*arctanh(a*x)^2+1/2*a
rctanh(a*x)^2/(-a^2*x^2+1)+1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2-2/(a*x+1)
)-arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*polylog(3,-1+2/(a*x+1))
```

Rubi [A]
time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6177, 6135, 6079, 6095, 6203, 6745, 6141, 6103, 267}

$$\frac{1}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)}-\frac{1}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)-\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{3}\tanh^{-1}(ax)^3-\frac{1}{4}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] 1/(4*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) - ArcTanh[a*x]^2
/4 + ArcTanh[a*x]^2/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*L
og[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3
, -1 + 2/(1 + a*x)]/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^2}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx$$
$$= \frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)} + \frac{1}{3}\tanh^{-1}(ax)^3 - a\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$
$$= -\frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)} - \frac{1}{4}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)} + \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2 \mathrm{lo}$$
$$= \frac{1}{4(1-a^2x^2)} - \frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)} - \frac{1}{4}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)} + \frac{1}{3}\tanh^{-1}(ax)^3 + \mathrm{t}$$
$$= \frac{1}{4(1-a^2x^2)} - \frac{ax\tanh^{-1}(ax)}{2(1-a^2x^2)} - \frac{1}{4}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{2(1-a^2x^2)} + \frac{1}{3}\tanh^{-1}(ax)^3 + \mathrm{t}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 106, normalized size = 0.78

$$\frac{1}{24}\left(i\pi^3 - 8\tanh^{-1}(ax)^3 + 3\cosh\left(2\tanh^{-1}(ax)\right) + 6\tanh^{-1}(ax)^2\cosh\left(2\tanh^{-1}(ax)\right) + 24\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 12\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) - 6\tanh^{-1}(ax)\sinh\left(2\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (I*Pi^3 - 8*ArcTanh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*Cosh
[2*ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 24*ArcTa
nh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - 12*PolyLog[3, E^(2*ArcTanh[a*x])]
- 6*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 116.12, size = 1290, normalized size = 9.49

method	result	size
derivativedivides	Expression too large to display	1290
default	Expression too large to display	1290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*arctanh(a*x)*(a*x-1)/(a*x+1)+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)-1/4*I*Pi
*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-2*polylog(3,
-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*
Pi*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+
```

```
1/2*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2
/(a^2*x^2-1))^2+1/4*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I*Pi*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2-1/2*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*
x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/4*I*Pi*arctanh(a*x)^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arc
tanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^2*csg
n(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/4*I*Pi*arcta
nh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/4*I*Pi*arctanh(a*x)^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*I*Pi*arctanh(a*x)^2*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)
^2/(-a^2*x^2+1)+1))^3-1/3*arctanh(a*x)^3-1/4*arctanh(a*x)^2+1/4*I*Pi*arctan
h(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*arctanh(a*x)^2*ln(a*x+1)-1/2*arctanh(a*x)^2
*ln(a*x-1)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1+
(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
+arctanh(a*x)^2*ln(a*x)+1/2*I*Pi*arctanh(a*x)^2-1/4*arctanh(a*x)^2/(a*x-1)+
1/4*arctanh(a*x)^2/(a*x+1)-1/16*(a*x-1)/(a*x+1)-1/16*(a*x+1)/(a*x-1)+arctan
h(a*x)^2*ln(2)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*integrate(x^4*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x),
 x) + 1/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3
 + x), x) - 1/32*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)
 + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^2 - 1/4*a^2*integrate(x^2*log(a*x + 1
)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/4*a*integrate(x*log(a*x +
 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) + 1/4*a*integrate(x*log(-a*
x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/24*((a^2*x^2 - 1)*log(-a*x + 1)^3
+ 3*((a^2*x^2 - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^2*x^2 - 1) + 1/4*i
ntegrate(log(a*x + 1)^2/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/2*integrate(log(a
*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^2), x)
```

3.271 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=142

$$\frac{a^2x}{4(1-a^2x^2)}+\frac{1}{4}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{2(1-a^2x^2)}+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{a^2x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{1}{2}a\tanh^{-1}(ax$$

```
[Out] 1/4*a^2*x/(-a^2*x^2+1)+1/4*a*arctanh(a*x)-1/2*a*arctanh(a*x)/(-a^2*x^2+1)+a
*arctanh(a*x)^2-arctanh(a*x)^2/x+1/2*a^2*x*arctanh(a*x)^2/(-a^2*x^2+1)+1/2*
a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(2-2/(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A]

time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6177, 6129, 6037, 6135, 6079, 2497, 6095, 6103, 6141, 205, 212}

$$\frac{a^2x}{4(1-a^2x^2)}+\frac{a^2x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{a\tanh^{-1}(ax)}{2(1-a^2x^2)}-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{4}a\tanh^{-1}(ax)+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*x)/(4*(1 - a^2*x^2)) + (a*ArcTanh[a*x])/4 - (a*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2)/(2*
(1 - a^2*x^2)) + (a*ArcTanh[a*x]^3)/2 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x
)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx \\
&= \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{6}a\tanh^{-1}(ax)^3+a^2\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx-a^3\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx \\
&= -\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{x}+\frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{2}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh}{x\,(1-} \\
&= \frac{a^2x}{4\left(1-a^2x^2\right)}-\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{2} \\
&= \frac{a^2x}{4\left(1-a^2x^2\right)}+\frac{1}{4}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{a^2x}{2} \\
&= \frac{a^2x}{4\left(1-a^2x^2\right)}+\frac{1}{4}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{a^2x}{2}
\end{aligned}
$$

Mathematica [A]

time = 0.18, size = 97, normalized size = 0.68

$$
\frac{4ax\tanh^{-1}(ax)^3-2ax\tanh^{-1}(ax)\left(\cosh\left(2\tanh^{-1}(ax)\right)-8\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-8ax\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)+ax\sinh\left(2\tanh^{-1}(ax)\right)+2\tanh^{-1}(ax)^2\left(-4+4ax+ax\sinh\left(2\tanh^{-1}(ax)\right)\right)}{8x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (4*a*x*ArcTanh[a*x]^3 - 2*a*x*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 8*Log[1
- E^(-2*ArcTanh[a*x])]) - 8*a*x*PolyLog[2, E^(-2*ArcTanh[a*x])] + a*x*Sinh[
```

```
2*ArcTanh[a*x]] + 2*ArcTanh[a*x]^2*(-4 + 4*a*x + a*x*Sinh[2*ArcTanh[a*x]]))
/(8*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 79.48, size = 4474, normalized size = 31.51

method	result	size
derivativedivides	Expression too large to display	4474
default	Expression too large to display	4474

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*arctanh(a*x)*(a*x-1)/(a*x+1)+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)+3/4*I*
Pi*arctanh(a*x)^2-3/8*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*d
ilog((a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+pol
ylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)
^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))+3/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-3/8*I*Pi*csgn
(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(
1/2))-3/8*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,-(a
*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*arctanh(a*x)^3-arctanh(a*x)^
2-3/8*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)*ln(1
-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*arctanh(a*x)^2*ln(a*x+1)-3/4*arctanh(a*x)^
2*ln(a*x-1)+3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*polylog(2,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^3*arctanh(a*x)^2+3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(a*x+1)/(-a^2*x
^2+1)^(1/2))-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^
```

```
2-1))^3*arctanh(a*x)^2+3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog((a*x+
1)/(-a^2*x^2+1)^(1/2))-3/4*I*Pi*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2
))+3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2-3
/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)
^(1/2))-3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*
x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arct
anh(a*x)^2-3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog((a*x+1)/(-a^
2*x^2+1)^(1/2))+3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)
/(-a^2*x^2+1)^(1/2))-3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+
(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*p
olylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))
^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+dilog
(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*Pi*p
olylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*Pi*polylog(2,-(a*x+1)/(-a^2*x^2+
1)^(1/2))-3/4*I*Pi*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*Pi*dilog(1+(a*x+
1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2/a/x+3/8*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2)
)-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*Pi*cs
gn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))-3/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+
1)/(-a^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1
/2))+3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^2*arctanh(a*x)^2-3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))-3/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(
a^2*x^2-1))^2*arctanh(a*x)^2+3/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,(a*x+1)...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(128) = 256$.
time = 0.28, size = 406, normalized size = 2.86

$$\frac{1}{16}a^2\left(\frac{(a^2x^2-1)\log(ax+1)^3-(a^2x^2-1)\log(ax-1)^3+(4a^2x^2-3(a^2x^2-1)\log(ax-1)-4)\log(ax+1)^2-4(a^2x^2-1)\log(ax-1)^2-4ax+(3(a^2x^2-1)\log(ax-1)^2-8(a^2x^2-1)\log(ax-1))\log(ax+1)}{a^3x^2-a}+\frac{16\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}-\frac{16(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax))}{a}+\frac{16(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax))}{a}+\frac{2\log(ax+1)}{a}-\frac{2\log(ax-1)}{a}\right)-\frac{1}{8}a\left(\frac{3(a^2x^2-1)\log(ax+1)^2-6(a^2x^2-1)\log(ax+1)\log(ax-1)+3(a^2x^2-1)\log(ax-1)^2-4}{a^2x^2-1}+8\log(ax+1)+8\log(ax-1)-16\log(x)\right)\mathrm{artanh}(ax)+\frac{1}{4}\left(3a\log(ax+1)-3a\log(ax-1)-\frac{2(3a^2x^2-2)}{a^2x^3-x}\right)\mathrm{artanh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/16*a^2*(((a^2*x^2 - 1)*log(a*x + 1)^3 - (a^2*x^2 - 1)*log(a*x - 1)^3 + (4
*a^2*x^2 - 3*(a^2*x^2 - 1)*log(a*x - 1) - 4)*log(a*x + 1)^2 - 4*(a^2*x^2 -
1)*log(a*x - 1)^2 - 4*a*x + (3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 8*(a^2*x^2 -
1)*log(a*x - 1))*log(a*x + 1))/(a^3*x^2 - a) + 16*(log(a*x - 1)*log(1/2*a*x
 + 1/2) + dilog(-1/2*a*x + 1/2))/a - 16*(log(a*x + 1)*log(x) + dilog(-a*x))
/a + 16*(log(-a*x + 1)*log(x) + dilog(a*x))/a + 2*log(a*x + 1)/a - 2*log(a*
x - 1)/a) - 1/8*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*
x + 1)*log(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8
*log(a*x + 1) + 8*log(a*x - 1) - 16*log(x))*arctanh(a*x) + 1/4*(3*a*log(a*x
 + 1) - 3*a*log(a*x - 1) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
[Out] integral(arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}(ax)}{x^{2}(ax-1)^{2}(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**2,x)
[Out] Integral(atanh(a*x)**2/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,(a^2\,x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^2),x)
[Out] int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^2), x)
```

3.272 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=205

$$\frac{a^2}{4(1-a^2x^2)}-\frac{a\tanh^{-1}(ax)}{x}-\frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{4}a^2\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{2x^2}+\frac{a^2\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+\frac{2}{3}a^2\tanh$$

```
[Out] 1/4*a^2/(-a^2*x^2+1)-a*arctanh(a*x)/x-1/2*a^3*x*arctanh(a*x)/(-a^2*x^2+1)+1
/4*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+1/2*a^2*arctanh(a*x)^2/(-a^2*x
^2+1)+2/3*a^2*arctanh(a*x)^3+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+2*a^2*arctanh
(a*x)^2*ln(2-2/(a*x+1))-2*a^2*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-a^2*poly
log(3,-1+2/(a*x+1))
```

Rubi [A]
time = 0.49, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6177, 6129, 6037, 272, 36, 29, 31, 6095, 6135, 6079, 6203, 6745, 6141, 6103, 267}

$$-a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)-2a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{a^2}{4(1-a^2x^2)}-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+\frac{a^2\tanh^{-1}(ax)^2}{2(1-a^2x^2)}+a^2\log(x)+\frac{2}{3}a^2\tanh^{-1}(ax)^3+\frac{1}{4}a^2\tanh^{-1}(ax)^2+2a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2-\frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^2}{2x^2}-\frac{a\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] a^2/(4*(1 - a^2*x^2)) - (a*ArcTanh[a*x])/x - (a^3*x*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + (a^2*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(2*x^2) + (a^2*ArcTanh[a
*x]^2)/(2*(1 - a^2*x^2)) + (2*a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log
[1 - a^2*x^2])/2 + 2*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a^2*ArcTan
h[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - a^2*PolyLog[3, -1 + 2/(1 + a*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3 (1-a^2x^2)^2}\,dx = a^2 \int \frac{\tanh^{-1}(ax)^2}{x (1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3 (1-a^2x^2)}\,dx$$
$$= 2\left(a^2 \int \frac{\tanh^{-1}(ax)^2}{x (1-a^2x^2)}\,dx\right) + a^4 \int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(ax)^2}{2(1-a^2x^2)} + a\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx + 2\left(\frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\right.$$
$$= -\frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)} - \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(ax)^2}{2(1-a^2x^2)} + a\int \frac{\tanh^{-}}{x^{}}$$
$$= \frac{a^2}{4(1-a^2x^2)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} +$$
$$= \frac{a^2}{4(1-a^2x^2)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} +$$
$$= \frac{a^2}{4(1-a^2x^2)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} +$$
$$= \frac{a^2}{4(1-a^2x^2)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 146, normalized size = 0.71

$$a^2\left(2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{1}{24}\left(2i\pi^3-16\tanh^{-1}(ax)^3+3\cosh\left(2\tanh^{-1}(ax)\right)+6\tanh^{-1}(ax)^2\left(2-\frac{2}{a^2x^2}+\cosh\left(2\tanh^{-1}(ax)\right)+8\log\left(1-e^{2\tanh^{-1}(ax)}\right)\right)+24\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right)-24\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-\frac{6\tanh^{-1}(ax)\left(4+ax\sinh\left(2\tanh^{-1}(ax)\right)\right)}{ax}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] a^2*(2*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((2*I)*Pi^3 - 16*ArcTa
nh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*(2 - 2/(a^2*x^2) + Co
sh[2*ArcTanh[a*x]] + 8*Log[1 - E^(2*ArcTanh[a*x])]) + 24*Log[(a*x)/Sqrt[1 -
 a^2*x^2]] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] - (6*ArcTanh[a*x]*(4 + a*x*S
inh[2*ArcTanh[a*x]]))/(a*x))/24)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 402.64, size = 2444, normalized size = 11.92

method	result	size
derivativedivides	Expression too large to display	2444
default	Expression too large to display	2444

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arctanh(a*x)^2/a^2/x^2+2*arctanh(a*x)^2*ln(a*x)+1/4*arctanh(a*x)^
2/(a*x+1)-arctanh(a*x)^2*ln(a*x+1)-1/4*arctanh(a*x)^2/(a*x-1)-arctanh(a*x)^
2*ln(a*x-1)+2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)^
2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(
1/2))+4*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-4*polylog(3,-(a
*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
+4*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-4*polylog(3,(a*x+1)/(
-a^2*x^2+1)^(1/2))-1/48*(32*arctanh(a*x)^3*a^3*x^3-32*arctanh(a*x)^3*a*x+12
*arctanh(a*x)^2*a*x-48*I*arctanh(a*x)^2*Pi*a^3*x^3+24*I*arctanh(a*x)^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*a^3*x^3-48*I*arctanh(a*x)^2*c
sgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*Pi*a^3*x^3-24*I*arc
tanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*a*x+48*I*arct
anh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*Pi*a*x+
9*a*x-48*a*x*arctanh(a*x)+48*a^3*x^3*arctanh(a*x)+24*a^2*x^2*arctanh(a*x)-4
8*arctanh(a*x)+3*a^3*x^3-12*arctanh(a*x)^2*a^3*x^3-24*I*arctanh(a*x)^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*a^3*x^3+96*ln(2)*arctanh(a*x)^2*a*x-96*ln(2)
*arctanh(a*x)^2*a^3*x^3+48*I*arctanh(a*x)^2*Pi*a*x-48*I*arctanh(a*x)^2*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*a^3*x^3-48*I
*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*a^3*x^3+48*I*arctan
h(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*a^3*x^3+24*I*arctanh(a*x)^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*a*x+24*I*arctanh(a*x)^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*a*x+48*I*arctanh(a*x)^2*csg
n(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*a*x+48*I*ar
ctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*a*x-48*I*arctanh(a*x)^
2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*a*x-24*I*arctanh(a*x)^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*a^3*x^3+48*I*arctanh(a
*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi
*a*x-24*I*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*a*x+24*I*arctanh(a*x)^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*a*x+24*I*arcta
nh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I
/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*a*x-48*I*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))*Pi*a*x-48*I*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*Pi*a*x-48*I*arctanh
(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*
```

```
Pi*a^3*x^3+24*I*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*a^3*x^3-24*I*arctanh(a*x)^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*a^3*
x^3-24*I*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*a^3*x^3+48*I*arctanh(a*x)^2*c
sgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))*Pi*a^3*x^3+48*I*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a
^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1
))*Pi*a^3*x^3)/(a*x+1)/a/x/(a*x-1)+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+ln(1+(a
*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^6*integrate(x^6*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3
), x) + 1/2*a^5*integrate(x^5*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x
^5 + x^3), x) - 1/16*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/
a^3) + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^4 - 1/2*a^4*integrate(x^4*log(a*x
 + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/2*a^3*integrate(x^3
*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) + 1/2*a^3*integ
rate(x^3*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a^2*integrate(
x^2*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a*integrate(x*log(-
a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/24*(2*(a^4*x^4 - a^2*x^2)*log(
-a*x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1) - 1)*log(-a
*x + 1)^2)/(a^2*x^4 - x^2) + 1/4*integrate(log(a*x + 1)^2/(a^4*x^7 - 2*a^2*
x^5 + x^3), x) - 1/2*integrate(log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*
x^5 + x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^3(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)^2), x)
```

3.273 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=227

$$-\frac{3x}{8a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)}{8a^4}+\frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^3}{4a^4}+\frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(a\ldots}{4a^4}$$

```
[Out] -3/8*x/a^3/(-a^2*x^2+1)-3/8*arctanh(a*x)/a^4+3/4*arctanh(a*x)/a^4/(-a^2*x^2
+1)-3/4*x*arctanh(a*x)^2/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)^3/a^4+1/2*arctan
h(a*x)^3/a^4/(-a^2*x^2+1)+1/4*arctanh(a*x)^4/a^4-arctanh(a*x)^3*ln(2/(-a*x+
1))/a^4-3/2*arctanh(a*x)^2*polylog(2,1-2/(-a*x+1))/a^4+3/2*arctanh(a*x)*pol
ylog(3,1-2/(-a*x+1))/a^4-3/4*polylog(4,1-2/(-a*x+1))/a^4
```

Rubi [A]
time = 0.29, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6175, 6131, 6055, 6095, 6205, 6209, 6745, 6141, 6103, 205, 212}

$$-\frac{3\text{Li}_4\left(1-\frac{2}{1-ax}\right)}{4a^4}-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{2a^4}+\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{2a^4}+\frac{\tanh^{-1}(ax)^4}{4a^4}-\frac{\tanh^{-1}(ax)^3}{4a^4}-\frac{3\tanh^{-1}(ax)}{8a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^3}{a^4}+\frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)}-\frac{3x}{8a^3\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^4) + (3*ArcTanh[a*x])/
(4*a^4*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) - ArcTan
h[a*x]^3/(4*a^4) + ArcTanh[a*x]^3/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(4
*a^4) - (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 - (3*ArcTanh[a*x]^2*PolyLog[2
, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/
(2*a^4) - (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx &= \frac{\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2} \\
&= \frac{\tanh^{-1}(ax)^3}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a^3} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx}{2a^3} \\
&= -\frac{3x\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\tanh^{-1}(ax)^3 \,\mathrm{lo}}{a^4} \\
&= \frac{3\tanh^{-1}(ax)}{4a^4(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \\
&= -\frac{3x}{8a^3(1-a^2x^2)} + \frac{3\tanh^{-1}(ax)}{4a^4(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4(1-a^2x} \\
&= -\frac{3x}{8a^3(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)}{8a^4} + \frac{3\tanh^{-1}(ax)}{4a^4(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^4}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 139, normalized size = 0.61

$$\frac{-4\tanh^{-1}(ax)^4 + 6\tanh^{-1}(ax)\cosh\left(2\tanh^{-1}(ax)\right) + 4\tanh^{-1}(ax)^3\cosh\left(2\tanh^{-1}(ax)\right) - 16\tanh^{-1}(ax)^3\log\left(1+e^{-2\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 12\mathrm{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right) - 3\sinh\left(2\tanh^{-1}(ax)\right) - 6\tanh^{-1}(ax)^2\sinh\left(2\tanh^{-1}(ax)\right)}{16a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-4*ArcTanh[a*x]^4 + 6*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3
*Cosh[2*ArcTanh[a*x]] - 16*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 24
*ArcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 24*ArcTanh[a*x]*PolyLog[
3, -E^(-2*ArcTanh[a*x])] + 12*PolyLog[4, -E^(-2*ArcTanh[a*x])] - 3*Sinh[2*A
rcTanh[a*x]] - 6*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/(16*a^4)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 107.56, size = 806, normalized size = 3.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*arctanh(a*x)^3/(a*x+1)+1/2*arctanh(a*x)^3*ln(a*x+1)-1/4*arctanh(
a*x)^3/(a*x-1)+1/2*arctanh(a*x)^3*ln(a*x-1)-arctanh(a*x)^3*ln((a*x+1)/(-a^2
*x^2+1)^(1/2))+1/4*arctanh(a*x)^4-3/16*arctanh(a*x)^2*(a*x-1)/(a*x+1)-3/16*
arctanh(a*x)*(a*x-1)/(a*x+1)-3/32*(a*x-1)/(a*x+1)+3/16*(a*x+1)*arctanh(a*x)
^2/(a*x-1)-3/16*arctanh(a*x)*(a*x+1)/(a*x-1)+3/32*(a*x+1)/(a*x-1)-3/2*arcta
nh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2*arctanh(a*x)*polylog(3,-(a
*x+1)^2/(-a^2*x^2+1))-3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-1/4*(2*I*Pi+I*
Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2-I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+2
*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+
1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*
x^2-1))-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^3+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1+4*ln(2))*arctanh(a*x
)^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) - 1)*l
og(-a*x + 1)^3)/(a^6*x^2 - a^4) + 1/8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)
^3 - 6*a^3*x^3*log(a*x + 1)^2*log(-a*x + 1) - 3*(a*x - (3*a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^7*x^4 - 2*a^5*x^2 + a^3),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atanh}^3(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \operatorname{atanh}(a\,x)^3}{\left(a^2\,x^2-1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^2, x)
```

3.274 $\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=121

$$-\frac{3}{8a^3(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{8a^3}-\frac{3\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^4}{8a^3}$$

```
[Out] -3/8/a^3/(-a^2*x^2+1)+3/4*x*arctanh(a*x)/a^2/(-a^2*x^2+1)+3/8*arctanh(a*x)^
2/a^3-3/4*arctanh(a*x)^2/a^3/(-a^2*x^2+1)+1/2*x*arctanh(a*x)^3/a^2/(-a^2*x^
2+1)-1/8*arctanh(a*x)^4/a^3
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6147, 6141, 6103, 267}

$$-\frac{\tanh^{-1}(ax)^4}{8a^3}+\frac{3\tanh^{-1}(ax)^2}{8a^3}+\frac{x\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)}-\frac{3}{8a^3(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] -3/(8*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*a^2*(1 - a^2*x^2)) + (3*Ar
cTanh[a*x]^2)/(8*a^3) - (3*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) + (x*ArcTa
nh[a*x]^3)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(8*a^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6147

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> Simp[-(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[b*(p/(2*c)), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2
), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{x\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^4}{8a^3} - \frac{3\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx}{2a}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^4}{8a^3} + \frac{3\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{2a^2}$$

$$= \frac{3x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} + \frac{3\tanh^{-1}(ax)^2}{8a^3} - \frac{3\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^4}{8a^3} -$$

$$= -\frac{3}{8a^3(1-a^2x^2)} + \frac{3x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} + \frac{3\tanh^{-1}(ax)^2}{8a^3} - \frac{3\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax}{2a^2(1-a^2x}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.60

$$\frac{3-6ax\tanh^{-1}(ax)+3(1+a^2x^2)\tanh^{-1}(ax)^2-4ax\tanh^{-1}(ax)^3+(1-a^2x^2)\tanh^{-1}(ax)^4}{8a^3(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a*
x]^3 + (1 - a^2*x^2)*ArcTanh[a*x]^4)/(8*a^3*(-1 + a^2*x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 144.89, size = 1357, normalized size = 11.21

method	result
risch	$-\frac{\ln(ax+1)^4}{128a^3} + \frac{(x^2\ln(-ax+1)a^2-2ax-\ln(-ax+1))\ln(ax+1)^3}{32a^3(a^2x^2-1)} - \frac{3\left(a^2x^2\ln(-ax+1)^2-2a^2x^2-4ax\ln(-ax+1)-\ln(\right.}{64a^3(ax-1)(ax+1)}$
derivativedivides	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/4*arctanh(a*x)^3/(a*x+1)-1/4*arctanh(a*x)^3*ln(a*x+1)-1/4*arctanh
(a*x)^3/(a*x-1)+1/4*arctanh(a*x)^3*ln(a*x-1)+1/2*arctanh(a*x)^3*ln((a*x+1)/
(-a^2*x^2+1)^(1/2))+1/16*(3+2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*arctanh(a*x)^3*Pi+2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*arctanh(a*x)^3
*a^2*x^2-4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*arctanh(a*x)^3*a^2*x^2
-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*arctanh(a*x)^3*a^2*x
^2+3*a^2*x^2-12*a*x*arctanh(a*x)+6*a^2*x^2*arctanh(a*x)^2+2*arctanh(a*x)^4+
6*arctanh(a*x)^2+2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^3*Pi*arctanh(a*x)^3*a^2*x^2+4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*
arctanh(a*x)^3*a^2*x^2-2*arctanh(a*x)^4*a^2*x^2+2*I*csgn(I*(a*x+1)/(-a^2*x^
2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*arctanh(a*x)^3*a^2*x^2+4*I*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi*arctan
h(a*x)^3*a^2*x^2-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*arctanh(a*x)^3*a^2*x^2+2*I*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2*Pi*arctanh(a*x)^3*a^2*x^2+4*I*arctanh(a*x)^3*Pi-2*I*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^3*arctanh(a*x)^3*Pi+4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*arctanh(a*x)^3*Pi-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^3*arctanh(a*x)^3*Pi-4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(
a*x)^3*Pi-4*I*Pi*arctanh(a*x)^3*a^2*x^2-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^3*Pi-4*I*csgn(I*(a*x+1)^2/
(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3*Pi+2*I*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*arctanh(a*x)^3*Pi-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-
a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi)/(a*x
-1)/(a*x+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(105) = 210$.
time = 0.30, size = 465, normalized size = 3.84

$$-\frac{1}{4}\left(\frac{2x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^3+\frac{3\left((a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2+4\right)a\operatorname{artanh}(ax)^2}{16(a^6x^2-a^4)}+\frac{1}{128}\left(\frac{\left((a^2x^2-1)\log(ax+1)^4-4(a^2x^2-1)\log(ax+1)^3\log(ax-1)+(a^2x^2-1)\log(ax-1)^4-6\left(2a^2x^2-(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)^2-12(a^2x^2-1)\log(ax-1)^2-4\left((a^2x^2-1)\log(ax-1)^3-6(a^2x^2-1)\log(ax-1)\right)\log(ax+1)+48\right)a^2}{a^8x^2-a^6}-\frac{8\left((a^2x^2-1)\log(ax+1)^3-3(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3+12ax-3\left(2a^2x^2-(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)+6(a^2x^2-1)\log(ax-1)\right)a\operatorname{artanh}(ax)}{a^7x^2-a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x)^3 + 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*lo
```

```
g(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)^2/(a^6*x^2 -
a^4) + 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^
3*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 - 6*(2*a^2*x^2 - (a^2*x^2 - 1
)*log(a*x - 1)^2 - 2)*log(a*x + 1)^2 - 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*
((a^2*x^2 - 1)*log(a*x - 1)^3 - 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1)
+ 48)*a^2/(a^8*x^2 - a^6) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 -
1)*log(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*
(2*a^2*x^2 - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 -
1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^2 - a^5))*a
```

Fricas [A]
time = 0.36, size = 114, normalized size = 0.94

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+48\,ax\log\left(-\frac{ax+1}{ax-1}\right)-12\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-48}{128\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*x
+ 1)/(a*x - 1))^2 - 48)/(a^5*x^2 - a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,\mathrm{atanh}^3\,(ax)}{\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

Mupad [B]
time = 1.71, size = 410, normalized size = 3.39

$$\frac{3\ln\left(a\,x+1\right)^2}{32\,a^3}-\frac{3\ln\left(a\,x+1\right)^2}{16\,\left(a^3-a^5\,x^2\right)}+\frac{3\ln\left(1-a\,x\right)^2}{32\,a^3}-\frac{\ln\left(a\,x+1\right)^4}{128\,a^3}-\frac{\ln\left(1-a\,x\right)^4}{128\,a^3}-\frac{3\ln\left(1-a\,x\right)^2}{16\,a^3-16\,a^5\,x^2}-\frac{3}{2\,\left(4\,a^3-4\,a^5\,x^2\right)}-\frac{x\ln\left(1-a\,x\right)^3}{2\,\left(8\,a^2-8\,a^4\,x^2\right)}-\frac{3\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)}{16\,a^3}+\frac{3\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)}{8\,a^3-8\,a^5\,x^2}+\frac{3\,x\,\ln\left(a\,x+1\right)}{8\,\left(a^2-a^4\,x^2\right)}+\frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^3}{32\,a^3}+\frac{\ln\left(a\,x+1\right)^3\,\ln\left(1-a\,x\right)}{32\,a^3}-\frac{6\,x\,\ln\left(1-a\,x\right)}{16\,a^2-16\,a^4\,x^2}+\frac{x\ln\left(a\,x+1\right)^3}{16\,\left(a^2-a^4\,x^2\right)}-\frac{3\ln\left(a\,x+1\right)^2\ln\left(1-a\,x\right)^2}{64\,a^3}+\frac{6\,x\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^2}{32\,a^2-32\,a^4\,x^2}-\frac{6\,x\ln\left(a\,x+1\right)^2\,\ln\left(1-a\,x\right)}{32\,a^2-32\,a^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (3*log(a*x + 1)^2)/(32*a^3) - (3*log(a*x + 1)^2)/(16*(a^3 - a^5*x^2)) + (3*
log(1 - a*x)^2)/(32*a^3) - log(a*x + 1)^4/(128*a^3) - log(1 - a*x)^4/(128*a
^3) - (3*log(1 - a*x)^2)/(16*a^3 - 16*a^5*x^2) - 3/(2*(4*a^3 - 4*a^5*x^2))
- (x*log(1 - a*x)^3)/(2*(8*a^2 - 8*a^4*x^2)) - (3*log(a*x + 1)*log(1 - a*x)
)/(16*a^3) + (3*log(a*x + 1)*log(1 - a*x))/(8*a^3 - 8*a^5*x^2) + (3*x*log(a
*x + 1))/(8*(a^2 - a^4*x^2)) + (log(a*x + 1)*log(1 - a*x)^3)/(32*a^3) + (lo
g(a*x + 1)^3*log(1 - a*x))/(32*a^3) - (6*x*log(1 - a*x))/(16*a^2 - 16*a^4*x
^2) + (x*log(a*x + 1)^3)/(16*(a^2 - a^4*x^2)) - (3*log(a*x + 1)^2*log(1 - a
*x)^2)/(64*a^3) + (6*x*log(a*x + 1)*log(1 - a*x)^2)/(32*a^2 - 32*a^4*x^2) -
 (6*x*log(a*x + 1)^2*log(1 - a*x))/(32*a^2 - 32*a^4*x^2)
```

3.275 $\int \frac{x \tanh^{-1}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=119

$$-\frac{3x}{8a(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)}{8a^2}+\frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)}-\frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^3}{4a^2}+\frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)}$$

```
[Out] -3/8*x/a/(-a^2*x^2+1)-3/8*arctanh(a*x)/a^2+3/4*arctanh(a*x)/a^2/(-a^2*x^2+1
)-3/4*x*arctanh(a*x)^2/a/(-a^2*x^2+1)-1/4*arctanh(a*x)^3/a^2+1/2*arctanh(a*
x)^3/a^2/(-a^2*x^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6141, 6103, 205, 212}

$$-\frac{3x}{8a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^3}{4a^2}-\frac{3\tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^2) + (3*ArcTanh[a*x])/(4
*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^3/(4*a^2) + ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx}{2a}$$

$$= -\frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} + \frac{3}{2}\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx$$

$$= \frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\int \frac{1}{(1-a^2x^2)^2}\,d}{4a}$$

$$= -\frac{3x}{8a(1-a^2x^2)} + \frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2}$$

$$= -\frac{3x}{8a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)}{8a^2} + \frac{3\tanh^{-1}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\tanh^{-1}(ax)^2}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{4a^2}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.76

$$\frac{6ax-12\tanh^{-1}(ax)+12ax\tanh^{-1}(ax)^2-4(1+a^2x^2)\tanh^{-1}(ax)^3+3(-1+a^2x^2)\log(1-ax)-3(-1+a^2x^2)\log(1+ax)}{16a^2(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (6*a*x - 12*ArcTanh[a*x] + 12*a*x*ArcTanh[a*x]^2 - 4*(1 + a^2*x^2)*ArcTanh[
a*x]^3 + 3*(-1 + a^2*x^2)*Log[1 - a*x] - 3*(-1 + a^2*x^2)*Log[1 + a*x])/(16
*a^2*(-1 + a^2*x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 166.97, size = 1359, normalized size = 11.42

method	result

risch	$-\frac{(a^2x^2+1)\ln(ax+1)^3}{32a^2(ax-1)(ax+1)}+\frac{3(x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1))\ln(ax+1)^2}{32a^2(ax-1)(ax+1)}-\frac{3\left(a^2x^2\ln(-ax+1)^2+4ax\ln(-ax+1)+\right.}{32a^2(ax-1)(ax}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2/(a^2*x^2-1)*arctanh(a*x)^3+3/8*arctanh(a*x)^2/(a*x+1)-3/8*arcta
nh(a*x)^2*ln(a*x+1)+3/8*arctanh(a*x)^2/(a*x-1)+3/8*arctanh(a*x)^2*ln(a*x-1)
+3/4*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*(-4*arctanh(a*x)^3*
a^2*x^2-3*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn
(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*Pi+3*I*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(
a*x)^2*Pi-3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x
)^2*a^2*x^2+6*a*x+4*arctanh(a*x)^3-6*a^2*x^2*arctanh(a*x)+3*I*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*Pi+3*I*Pi*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2-6*I*Pi*cs
gn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2+3*I*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^2*x^2+6*I*Pi*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*arctanh(a*x)^2*a^2*x^2-6*arctanh(a*x)-3*I*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^2*Pi-6*I*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^
2*Pi-6*I*Pi*arctanh(a*x)^2*a^2*x^2+6*I*arctanh(a*x)^2*Pi-3*I*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi+6*I*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-3*I*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^3*arctanh(a*x)^2*Pi-6*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(
a*x)^2*Pi-3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2+3*I*Pi*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2
+6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*
arctanh(a*x)^2*a^2*x^2+3*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*a^2*x^2)/(a
*x-1)/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(103) = 206$.
time = 0.27, size = 298, normalized size = 2.50

$$\frac{3\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2}{8a}-\frac{\frac{\left((a^2x^2-1)\log(ax+1)^3-3(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3-12ax+3\left(2a^2x^2+(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)-6(a^2x^2-1)\log(ax-1)\right)a^2}{a^5x^2-a^3}-\frac{6\left((a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2-4\right)a\operatorname{artanh}(ax)}{a^4x^2-a^2}}{32a}-\frac{\operatorname{artanh}(ax)^3}{2(a^2x^2-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2/a
- 1/32*(((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(
a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2
- 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2/(
a^5*x^2 - a^3) - 6*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x
+ 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/(a^4*x
^2 - a^2))/a - 1/2*arctanh(a*x)^3/((a^2*x^2 - 1)*a^2)
```

Fricas [**A**]
time = 0.45, size = 97, normalized size = 0.82

$$\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{2} - \left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3} + 12\,ax - 6\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{32\left(a^{4}x^{2}-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a
^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\operatorname{atanh}^{3}\left(ax\right)}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [**A**]
time = 0.41, size = 192, normalized size = 1.61

$$-\frac{1}{64}\left(\left(\frac{ax+1}{(ax-1)a^{3}} + \frac{ax-1}{(ax+1)a^{3}}\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3} - 3\left(\frac{ax+1}{(ax-1)a^{3}} - \frac{ax-1}{(ax+1)a^{3}}\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2} + 6\left(\frac{ax+1}{(ax-1)a^{3}} + \frac{ax-1}{(ax+1)a^{3}}\right)\log\left(-\frac{ax+1}{ax-1}\right) - \frac{6\,(ax+1)}{(ax-1)a^{3}} + \frac{6\,(ax-1)}{(ax+1)a^{3}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/64*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*x +
1)/(a*x - 1))^3 - 3*((a*x + 1)/((a*x - 1)*a^3) - (a*x - 1)/((a*x + 1)*a^3))
*log(-(a*x + 1)/(a*x - 1))^2 + 6*((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a
*x + 1)*a^3))*log(-(a*x + 1)/(a*x - 1)) - 6*(a*x + 1)/((a*x - 1)*a^3) + 6*(
a*x - 1)/((a*x + 1)*a^3))*a
```

Mupad [B]
time = 1.76, size = 239, normalized size = 2.01

$$-\frac{6\ln\left(1-a\,x\right)-6\ln\left(a\,x+1\right)+12\,a\,x-\ln\left(a\,x+1\right)^3+\ln\left(1-a\,x\right)^3-3\ln\left(a\,x+1\right)\ln\left(1-a\,x\right)^2+3\ln\left(a\,x+1\right)^2\ln\left(1-a\,x\right)-a^2\,x^2\,\left(6\ln\left(a\,x+1\right)-6\ln\left(1-a\,x\right)\right)-a^2\,x^2\ln\left(a\,x+1\right)^3+a^2\,x^2\ln\left(1-a\,x\right)^3+6\,a\,x\ln\left(a\,x+1\right)^2+6\,a\,x\ln\left(1-a\,x\right)^2-12\,a\,x\ln\left(a\,x+1\right)\ln\left(1-a\,x\right)-3\,a^2\,x^2\ln\left(a\,x+1\right)\ln\left(1-a\,x\right)^2+3\,a^2\,x^2\ln\left(a\,x+1\right)^2\ln\left(1-a\,x\right)}{32\,a^2-32\,a^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

```
[Out] -(6*log(1 - a*x) - 6*log(a*x + 1) + 12*a*x - log(a*x + 1)^3 + log(1 - a*x)^
3 - 3*log(a*x + 1)*log(1 - a*x)^2 + 3*log(a*x + 1)^2*log(1 - a*x) - a^2*x^2
*(6*log(a*x + 1) - 6*log(1 - a*x)) - a^2*x^2*log(a*x + 1)^3 + a^2*x^2*log(1
 - a*x)^3 + 6*a*x*log(a*x + 1)^2 + 6*a*x*log(1 - a*x)^2 - 12*a*x*log(a*x +
1)*log(1 - a*x) - 3*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 + 3*a^2*x^2*log(a*x
 + 1)^2*log(1 - a*x))/(32*a^2 - 32*a^4*x^2)
```

3.276 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=115

$$-\frac{3}{8a(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{8a}-\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}$$

```
[Out] -3/8/a/(-a^2*x^2+1)+3/4*x*arctanh(a*x)/(-a^2*x^2+1)+3/8*arctanh(a*x)^2/a-3/
4*arctanh(a*x)^2/a/(-a^2*x^2+1)+1/2*x*arctanh(a*x)^3/(-a^2*x^2+1)+1/8*arcta
nh(a*x)^4/a
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6103, 6141, 267}

$$-\frac{3}{8a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}+\frac{3\tanh^{-1}(ax)^2}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

```
[Out] -3/(8*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*ArcTanh[
a*x]^2)/(8*a) - (3*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)
/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

GtQ[p, 0] && NeQ[q, -1]

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx &= \frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}-\frac{1}{2}(3a)\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx\\ &= -\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}+\frac{3}{2}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx\\ &= \frac{3x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{8a}-\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}-\frac{1}{4}\\ &= -\frac{3}{8a(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{8a}-\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.62

$$\frac{3-6ax\tanh^{-1}(ax)+3(1+a^2x^2)\tanh^{-1}(ax)^2-4ax\tanh^{-1}(ax)^3+(-1+a^2x^2)\tanh^{-1}(ax)^4}{8a(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

```
[Out] (3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a*
x]^3 + (-1 + a^2*x^2)*ArcTanh[a*x]^4)/(8*a*(-1 + a^2*x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 123.62, size = 1357, normalized size = 11.80

method	result
risch	$\frac{\ln(ax+1)^4}{128a}-\frac{(x^2\ln(-ax+1)a^2+2ax-\ln(-ax+1))\ln(ax+1)^3}{32(a^2x^2-1)a}+\frac{3\left(a^2x^2\ln(-ax+1)^2+2a^2x^2+4ax\ln(-ax+1)-\ln(-a\right.}{64a(ax-1)(ax+1)}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4*arctanh(a*x)^3/(a*x+1)+1/4*arctanh(a*x)^3*ln(a*x+1)-1/4*arctanh(a
*x)^3/(a*x-1)-1/4*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln((a*x+1)/(-
a^2*x^2+1)^(1/2))-1/16*(-3+2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
```

```
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*arctanh(a*x)^3*Pi+2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*arctanh(a*x)^3*
a^2*x^2-4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*arctanh(a*x)^3*a^2*x^2-
2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*arctanh(a*x)^3*a^2*x^
2-3*a^2*x^2+12*a*x*arctanh(a*x)-6*a^2*x^2*arctanh(a*x)^2+2*arctanh(a*x)^4-6
*arctanh(a*x)^2+2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3*Pi*arctanh(a*x)^3*a^2*x^2+4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*a
rctanh(a*x)^3*a^2*x^2-2*arctanh(a*x)^4*a^2*x^2+2*I*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*arctanh(a*x)^3*a^2*x^2+4*I*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi*arctanh
(a*x)^3*a^2*x^2-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*arctanh(a*x)^3*a^2*x^2+2*I*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*Pi*arctanh(a*x)^3*a^2*x^2+4*I*arctanh(a*x)^3*Pi-2*I*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^3*arctanh(a*x)^3*Pi+4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3
*arctanh(a*x)^3*Pi-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^3*arctanh(a*x)^3*Pi-4*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a
*x)^3*Pi-4*I*Pi*arctanh(a*x)^3*a^2*x^2-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^3*Pi-4*I*csgn(I*(a*x+1)^2/(
a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3*Pi+2*I*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*arctanh(a*x)^3*Pi-2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi)/(a*x-
1)/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(99) = 198$.
time = 0.28, size = 459, normalized size = 3.99

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3-\frac{3\left((a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2-4\right)a\operatorname{artanh}(ax)^2}{16(a^4x^2-a^2)}-\frac{1}{128}\left(\frac{\left((a^2x^2-1)\log(ax+1)^4-4(a^2x^2-1)\log(ax+1)^3\log(ax-1)+(a^2x^2-1)\log(ax-1)^4+6\left(2a^2x^2+(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)^2+12(a^2x^2-1)\log(ax-1)^2-4\left((a^2x^2-1)\log(ax-1)^3+6(a^2x^2-1)\log(ax-1)\right)\log(ax+1)-48\right)a^2}{a^6x^2-a^4}-\frac{8\left((a^2x^2-1)\log(ax+1)^3-3(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3-12ax+3\left(2a^2x^2+(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+1)-6(a^2x^2-1)\log(ax-1)\right)a\operatorname{artanh}(ax)}{a^5x^2-a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^3 -
 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
- 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)^2/(a^4*x^2 - a^2) -
 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^3*log(
a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 + 6*(2*a^2*x^2 + (a^2*x^2 - 1)*log(
a*x - 1)^2 - 2)*log(a*x + 1)^2 + 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*((a^2*
x^2 - 1)*log(a*x - 1)^3 + 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1) - 48)*
a^2/(a^6*x^2 - a^4) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log
(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2
*x^2 + (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log
(a*x - 1))*a*arctanh(a*x)/(a^5*x^2 - a^3))*a
```

Fricas [A]
time = 0.36, size = 113, normalized size = 0.98

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{3}-\left(a^{2}x^{2}-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+48\,ax\log\left(-\frac{ax+1}{ax-1}\right)-12\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-48}{128\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*x
+ 1)/(a*x - 1))^2 - 48)/(a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^{3}\left(ax\right)}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [A]
time = 1.52, size = 122, normalized size = 1.06

$$\frac{1}{32}\,a^{2}\left(\frac{(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^{3}}{(ax+1)a^{4}}+\frac{3\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^{2}}{(ax+1)a^{4}}+\frac{6\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)}{(ax+1)a^{4}}+\frac{6\,(ax-1)}{(ax+1)a^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/32*a^2*((a*x - 1)*log(-(a*x + 1)/(a*x - 1))^3/((a*x + 1)*a^4) + 3*(a*x -
1)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)*a^4) + 6*(a*x - 1)*log(-(a*x + 1)
/(a*x - 1))/((a*x + 1)*a^4) + 6*(a*x - 1)/((a*x + 1)*a^4))
```

Mupad [B]
time = 1.72, size = 378, normalized size = 3.29

$$\frac{3\ln\left(a\,x+1\right)^2}{32\,a}-\frac{3}{2\,\left(4\,a-4\,a^3\,x^2\right)}-\frac{3\ln\left(1-a\,x\right)^2}{16\,a-16\,a^3\,x^2}+\frac{3\ln\left(1-a\,x\right)^2}{32\,a}+\frac{\ln\left(a\,x+1\right)^4}{128\,a}+\frac{\ln\left(1-a\,x\right)^4}{128\,a}-\frac{3\ln\left(a\,x+1\right)^2}{16\,\left(a-a^3\,x^2\right)}-\frac{3\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)}{16\,a}-\frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^3}{32\,a}-\frac{\ln\left(a\,x+1\right)^3\,\ln\left(1-a\,x\right)}{32\,a}-\frac{3\,x\,\ln\left(a\,x+1\right)}{8\,\left(a^2\,x^2-1\right)}+\frac{6\,x\,\ln\left(1-a\,x\right)}{16\,a^2\,x^2-16}+\frac{3\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)}{8\,a-8\,a^3\,x^2}+\frac{3\ln\left(a\,x+1\right)^2\ln\left(1-a\,x\right)^2}{64\,a}-\frac{x\ln\left(a\,x+1\right)^3}{16\,\left(a^2\,x^2-1\right)}+\frac{x\ln\left(1-a\,x\right)^3}{2\,\left(8\,a^2\,x^2-8\right)}-\frac{6\,x\,\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^2}{32\,a^2\,x^2-32}+\frac{6\,x\ln\left(a\,x+1\right)^2\,\ln\left(1-a\,x\right)}{32\,a^2\,x^2-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(a^2*x^2 - 1)^2,x)
```

```
[Out] (3*log(a*x + 1)^2)/(32*a) - 3/(2*(4*a - 4*a^3*x^2)) - (3*log(1 - a*x)^2)/(1
6*a - 16*a^3*x^2) + (3*log(1 - a*x)^2)/(32*a) + log(a*x + 1)^4/(128*a) + lo
g(1 - a*x)^4/(128*a) - (3*log(a*x + 1)^2)/(16*(a - a^3*x^2)) - (3*log(a*x +
 1)*log(1 - a*x))/(16*a) - (log(a*x + 1)*log(1 - a*x)^3)/(32*a) - (log(a*x
+ 1)^3*log(1 - a*x))/(32*a) - (3*x*log(a*x + 1))/(8*(a^2*x^2 - 1)) + (6*x*l
og(1 - a*x))/(16*a^2*x^2 - 16) + (3*log(a*x + 1)*log(1 - a*x))/(8*a - 8*a^3
*x^2) + (3*log(a*x + 1)^2*log(1 - a*x)^2)/(64*a) - (x*log(a*x + 1)^3)/(16*(
a^2*x^2 - 1)) + (x*log(1 - a*x)^3)/(2*(8*a^2*x^2 - 8)) - (6*x*log(a*x + 1)*
log(1 - a*x)^2)/(32*a^2*x^2 - 32) + (6*x*log(a*x + 1)^2*log(1 - a*x))/(32*a
^2*x^2 - 32)
```

3.277 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=193

$$-\frac{3ax}{8(1-a^2x^2)}-\frac{3}{8}\tanh^{-1}(ax)+\frac{3\tanh^{-1}(ax)}{4(1-a^2x^2)}-\frac{3ax\tanh^{-1}(ax)^2}{4(1-a^2x^2)}-\frac{1}{4}\tanh^{-1}(ax)^3+\frac{\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\frac{1}{4}\tanh^{-1}(a$$

```
[Out] -3/8*a*x/(-a^2*x^2+1)-3/8*arctanh(a*x)+3/4*arctanh(a*x)/(-a^2*x^2+1)-3/4*a*
x*arctanh(a*x)^2/(-a^2*x^2+1)-1/4*arctanh(a*x)^3+1/2*arctanh(a*x)^3/(-a^2*x
^2+1)+1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*
polylog(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylo
g(4,-1+2/(a*x+1))
```

Rubi [A]
time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6177, 6135, 6079, 6095, 6203, 6207, 6745, 6141, 6103, 205, 212}

$$-\frac{3ax}{8(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{2(1-a^2x^2)}-\frac{3ax\tanh^{-1}(ax)^2}{4(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)}{4(1-a^2x^2)}-\frac{3}{4}\mathrm{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-\frac{3}{2}\mathrm{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}\tanh^{-1}(ax)^4-\frac{1}{4}\tanh^{-1}(ax)^3-\frac{3}{8}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a*x)/(8*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/8 + (3*ArcTanh[a*x])/(4*(1 -
a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]^3/4 + A
rcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 -
 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcT
anh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])
/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
```

```
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx \\
&= \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{4}\tanh^{-1}(ax)^4-\frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx \\
&= -\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^3+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{4}\tanh^{-1}(ax)^4+\tanh^{-1}(ax)^3\,\mathrm{lo} \\
&= \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^3+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{4}\tanh^{-1}(ax)^4+ \\
&= -\frac{3ax}{8\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^3+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+ \\
&= -\frac{3ax}{8\left(1-a^2x^2\right)}-\frac{3}{8}\tanh^{-1}(ax)+\frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}-\frac{1}{4}\tanh^{-1}(ax)^3+
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 135, normalized size = 0.70

$$
\frac{1}{64}\left(\pi^4-16\tanh^{-1}(ax)^4+24\tanh^{-1}(ax)\cosh\left(2\tanh^{-1}(ax)\right)+16\tanh^{-1}(ax)^3\cosh\left(2\tanh^{-1}(ax)\right)+64\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)+96\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)+48\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)-12\sinh\left(2\tanh^{-1}(ax)\right)-24\tanh^{-1}(ax)^2\sinh\left(2\tanh^{-1}(ax)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (Pi^4 - 16*ArcTanh[a*x]^4 + 24*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 16*ArcTa
nh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x
])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*Po
lyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])] - 12*Sinh[
2*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/64
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 276.65, size = 1387, normalized size = 7.19

method	result	size
derivativedivides	Expression too large to display	1387
default	Expression too large to display	1387

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*arctanh(a*x)*(a*x-1)/(a*x+1)-3/16*arctanh(a*x)*(a*x+1)/(a*x-1)-3/16*a
rctanh(a*x)^2*(a*x-1)/(a*x+1)+1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a
^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^
2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^3+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^3+1/2*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))^3-1/2*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2
*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^3+6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^
2*x^2+1)^(1/2))-1/4*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))+3/16*(a*x+1)*arctanh(a*x)^2/(a*x-1)+1/2*I*Pi*arctanh(a*x)^3+3*a
rctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^
2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctan
h(a*x)^4-1/4*arctanh(a*x)^3+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*arctanh(a*x)^3*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I*Pi*a
rctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^
2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1
)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/4*I*Pi*arctanh(a*x)
^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2-1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I*Pi*arctan
h(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*arctanh(a*x)^3/(a*x+1)-1/2*arctanh(a*x
)^3*ln(a*x+1)-1/4*arctanh(a*x)^3/(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x-1)+arcta
nh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/32*(a*x-1)/(a*x+1)+3/32*(a*x+1)/
```

```
(a*x-1)+arctanh(a*x)^3*ln(a*x)-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+
ln(2)*arctanh(a*x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) + 1)*lo
g(-a*x + 1)^3)/(a^2*x^2 - 1) - 1/8*integrate(-1/2*(2*log(a*x + 1)^3 - 6*log
(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 - a^2*x^2
 - a*x - 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{x(ax-1)^{2}(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^2),x)
[Out] int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^2), x)
```

3.278 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)^2} dx$

Optimal. Leaf size=191

$$-\frac{3a}{8(1-a^2x^2)}+\frac{3a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{3}{8}a\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{4(1-a^2x^2)}+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{a^2x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}$$

```
[Out] -3/8*a/(-a^2*x^2+1)+3/4*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+3/8*a*arctanh(a*x)^
2-3/4*a*arctanh(a*x)^2/(-a^2*x^2+1)+a*arctanh(a*x)^3-arctanh(a*x)^3/x+1/2*a
^2*x*arctanh(a*x)^3/(-a^2*x^2+1)+3/8*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*ln
(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+2
/(a*x+1))
```

Rubi [A]
time = 0.31, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6177, 6129, 6037, 6135, 6079, 6095, 6203, 6745, 6103, 6141, 267}

$$-\frac{3a}{8(1-a^2x^2)}+\frac{a^2x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}-\frac{3a\tanh^{-1}(ax)^2}{4(1-a^2x^2)}+\frac{3a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)}-\frac{3}{2}a\text{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{3}{8}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{3}{8}a\tanh^{-1}(ax)^2+3a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a)/(8*(1 - a^2*x^2)) + (3*a^2*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*a*
ArcTanh[a*x]^2)/8 - (3*a*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + a*ArcTanh[a*x]
^3 - ArcTanh[a*x]^3/x + (a^2*x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (3*a*Arc
Tanh[a*x]^4)/8 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
```

```
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)}\,dx$$

$$= \frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{8}a\tanh^{-1}(ax)^4 + a^2\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx - \frac{1}{2}\left(3a^3\right)\int \frac{x\tanh^{-1}(ax}{\left(1-a^2x^2\right)}$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{x} + \frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tan}{x\,(}$$

$$= \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)}{x}$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(a$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(a$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(a$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 144, normalized size = 0.75

$$\frac{1}{16}a\left(2i\pi^3 - 16\tanh^{-1}(ax)^3 - \frac{16\tanh^{-1}(ax)^3}{ax} + 6\tanh^{-1}(ax)^4 - 3\cosh\left(2\tanh^{-1}(ax)\right) - 6\tanh^{-1}(ax)^2\cosh\left(2\tanh^{-1}(ax)\right) + 48\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right) + 48\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 24\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 6\tanh^{-1}(ax)\sinh\left(2\tanh^{-1}(ax)\right) + 4\tanh^{-1}(ax)^3\sinh\left(2\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (a*((2*I)*Pi^3 - 16*ArcTanh[a*x]^3 - (16*ArcTanh[a*x]^3)/(a*x) + 6*ArcTanh[
a*x]^4 - 3*Cosh[2*ArcTanh[a*x]] - 6*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]] + 4
8*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 48*ArcTanh[a*x]*PolyLog[2, E
^(2*ArcTanh[a*x])] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*Sin
h[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3*Sinh[2*ArcTanh[a*x]]))/16
```

Maple [A]
time = 138.15, size = 271, normalized size = 1.42

method	result
derivativedivides	$a\left(\frac{3\,\mathrm{arctanh}(ax)^4}{8}-\frac{(ax+1)\left(4\,\mathrm{arctanh}(ax)^3-6\,\mathrm{arctanh}(ax)^2+6\,\mathrm{arctanh}(ax)-3\right)}{32(ax-1)}+\frac{\left(4\,\mathrm{arctanh}(ax)^3+6\,\mathrm{arctanh}(a\right.}{32a}\right.$
default	$a\left(\frac{3\,\mathrm{arctanh}(ax)^4}{8}-\frac{(ax+1)\left(4\,\mathrm{arctanh}(ax)^3-6\,\mathrm{arctanh}(ax)^2+6\,\mathrm{arctanh}(ax)-3\right)}{32(ax-1)}+\frac{\left(4\,\mathrm{arctanh}(ax)^3+6\,\mathrm{arctanh}(a\right.}{32a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(3/8*arctanh(a*x)^4-1/32*(a*x+1)*(4*arctanh(a*x)^3-6*arctanh(a*x)^2+6*arc
tanh(a*x)-3)/(a*x-1)+1/32*(4*arctanh(a*x)^3+6*arctanh(a*x)^2+6*arctanh(a*x)
+3)*(a*x-1)/(a*x+1)+arctanh(a*x)^3/a/x*(a*x-1)-2*arctanh(a*x)^3+3*arctanh(a
*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,-(a*x+1)/(-
a^2*x^2+1)^(1/2))-6*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2
*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^
2+1)^(1/2))-6*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^2(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,(a^2\,x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^2), x)
```

3.279 $\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=302

$$-\frac{3a^3x}{8(1-a^2x^2)}-\frac{3}{8}a^2\tanh^{-1}(ax)+\frac{3a^2\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{2x}-\frac{3a^3x\tanh^{-1}(ax)^2}{4(1-a^2x^2)}+$$

```
[Out] -3/8*a^3*x/(-a^2*x^2+1)-3/8*a^2*arctanh(a*x)+3/4*a^2*arctanh(a*x)/(-a^2*x^2
+1)+3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x-3/4*a^3*x*arctanh(a*x)^2/
(-a^2*x^2+1)+1/4*a^2*arctanh(a*x)^3-1/2*arctanh(a*x)^3/x^2+1/2*a^2*arctanh(
a*x)^3/(-a^2*x^2+1)+1/2*a^2*arctanh(a*x)^4+3*a^2*arctanh(a*x)*ln(2-2/(a*x+1
))+2*a^2*arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/(a*x+1))-3*a
^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))-3*a^2*arctanh(a*x)*polylog(3,-1+2
/(a*x+1))-3/2*a^2*polylog(4,-1+2/(a*x+1))
```

Rubi [A]
time = 0.66, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6177, 6129, 6037, 6135, 6079, 2497, 6095, 6203, 6207, 6745, 6141, 6103, 205, 212}

$$-\frac{3}{2}a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\text{Li}_4\left(\frac{2}{ax+1}-1\right)-3a^2\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-3a^2\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{a^2\tanh^{-1}(ax)^3}{2(1-a^2x^2)}+\frac{3a^2\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{1}{2}a^2\tanh^{-1}(ax)^4+\frac{1}{4}a^2\tanh^{-1}(ax)^3+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3}{8}a^2\tanh^{-1}(ax)+2a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3+3a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{3a^3x}{8(1-a^2x^2)}-\frac{3a^3x\tanh^{-1}(ax)^2}{4(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^3}{2x^2}-\frac{3a\tanh^{-1}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a^3*x)/(8*(1 - a^2*x^2)) - (3*a^2*ArcTanh[a*x])/8 + (3*a^2*ArcTanh[a*x]
)/(4*(1 - a^2*x^2)) + (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x)
 - (3*a^3*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x]^3)/4 - Ar
cTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (a^2*ArcTan
h[a*x]^4)/2 + 3*a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + 2*a^2*ArcTanh[a*x]^
3*Log[2 - 2/(1 + a*x)] - (3*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - 3*a^2*Arc
Tanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)] - 3*a^2*ArcTanh[a*x]*PolyLog[3, -1
 + 2/(1 + a*x)] - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)}\,dx\\
&= 2\left(a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx\right)+a^4\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx\\
&= -\frac{\tanh^{-1}(ax)^3}{2x^2}+\frac{a^2\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx+2\left(\frac{1}{4}a^2\tanh^{-1}(ax)^4\right.\\
&= -\frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}-\frac{1}{4}a^2\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{2x^2}+\frac{a^2\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{2}(3a)\int\\
&= \frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(ax)^2}{2x}-\frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}+\frac{1}{4}a^2\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(}{2x^2}\\
&= -\frac{3a^3x}{8\left(1-a^2x^2\right)}+\frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{2x}-\frac{3a^3x\tanh^{-}}{4\left(1-a^2\right.}\\
&= -\frac{3a^3x}{8\left(1-a^2x^2\right)}-\frac{3}{8}a^2\tanh^{-1}(ax)+\frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(}{2x}\\
&= -\frac{3a^3x}{8\left(1-a^2x^2\right)}-\frac{3}{8}a^2\tanh^{-1}(ax)+\frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)}+\frac{3}{2}a^2\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(}{2x}
\end{aligned}
$$

Mathematica [A]
time = 0.43, size = 215, normalized size = 0.71

$$\frac{1}{32}a^2\left(\pi^4+48\tanh^{-1}(ax)^2-\frac{48\tanh^{-1}(ax)^2}{ax}-\frac{16(1-a^2x^2)\tanh^{-1}(ax)^3}{a^2x^2}-16\tanh^{-1}(ax)^4+12\tanh^{-1}(ax)\cosh\left(2\tanh^{-1}(ax)\right)+8\tanh^{-1}(ax)^3\cosh\left(2\tanh^{-1}(ax)\right)+96\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)+64\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right)-48\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)+96\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)+48\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)-6\sinh\left(2\tanh^{-1}(ax)\right)-12\tanh^{-1}(ax)^2\sinh\left(2\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*(Pi^4 + 48*ArcTanh[a*x]^2 - (48*ArcTanh[a*x]^2)/(a*x) - (16*(1 - a^2*x
^2)*ArcTanh[a*x]^3)/(a^2*x^2) - 16*ArcTanh[a*x]^4 + 12*ArcTanh[a*x]*Cosh[2*
ArcTanh[a*x]] + 8*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 96*ArcTanh[a*x]*Log
[1 - E^(-2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] -
 48*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*Arc
Tanh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4
, E^(2*ArcTanh[a*x])] - 6*Sinh[2*ArcTanh[a*x]] - 12*ArcTanh[a*x]^2*Sinh[2*A
rcTanh[a*x]]))/32
```

Maple [A]
time = 306.89, size = 447, normalized size = 1.48

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{2}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax)^3-6\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-3\right)}{32(ax-1)}-\frac{\left(4\operatorname{arctanh}(ax)^3+6\operatorname{arctanh}\right.}{32}\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{2}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax)^3-6\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-3\right)}{32(ax-1)}-\frac{\left(4\operatorname{arctanh}(ax)^3+6\operatorname{arctanh}\right.}{32}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arctanh(a*x)^4-1/32*(a*x+1)*(4*arctanh(a*x)^3-6*arctanh(a*x)^2+6*
arctanh(a*x)-3)/(a*x-1)-1/32*(4*arctanh(a*x)^3+6*arctanh(a*x)^2+6*arctanh(a
*x)+3)*(a*x-1)/(a*x+1)+1/2*arctanh(a*x)^2*(a*x*arctanh(a*x)+arctanh(a*x)+3*
a*x)*(a*x-1)/a^2/x^2-3*arctanh(a*x)^2+3*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))+3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)*ln(1-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(
a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)^2*polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2))-12*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+12*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)^3*ln(1-(a*x+1)/(-
a^2*x^2+1)^(1/2))+6*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12
*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+12*polylog(4,(a*x+1)/(-
a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/32*((a^4*x^4 - a^2*x^2)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4 - a^2
*x^2)*log(a*x + 1) - 1)*log(-a*x + 1)^3)/(a^2*x^4 - x^2) - 1/8*integrate(-1
/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x + 1) - 3*(2*a^4*x^4 + 2*a^
3*x^3 - a^2*x^2 - a*x + 2*(a^6*x^6 + a^5*x^5 - a^4*x^4 - a^3*x^3 - 1)*log(a
*x + 1))*log(-a*x + 1)^2)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**2,x)
[Out] Integral(atanh(a*x)**3/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)^2),x)
[Out] int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)^2), x)
```

3.280 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=103

$$\frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}$$

```
[Out] 1/3*arctanh(a*x)^(3/2)/a+1/32*erf(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)/a-1/32*erfi(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+1/2*x*arctan
h(a*x)^(1/2)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6103, 6181, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

```
[Out] (x*Sqrt[ArcTanh[a*x]])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^(3/2)/(3*a) + (Sqrt
[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*S
qrt[ArcTanh[a*x]]])/(16*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^2}\,dx = \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} - \frac{1}{4}a\int \frac{x}{(1-a^2x^2)^2\sqrt{\tanh^{-1}(ax)}}\,dx$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} - \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \tanh^{-1}(ax)\right)}{4a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \tanh^{-1}(ax)\right)}{4a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \tanh^{-1}(ax)\right)}{8a}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \tanh^{-1}(ax)\right)}{16a} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} + \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\tanh^{-1}(ax)}\right)}{8a} - \frac{\text{Subs}}{}$$

$$= \frac{x\sqrt{\tanh^{-1}(ax)}}{2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^{3/2}}{3a} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\surd\right.}{}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.84

$$\sqrt{\tanh^{-1}(ax)}\left(-\frac{x}{2(-1+a^2x^2)} + \frac{\tanh^{-1}(ax)}{3a}\right) - \frac{\sqrt{\frac{\pi}{2}}\left(-\text{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right) + \text{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

```
[Out] Sqrt[ArcTanh[a*x]]*(-1/2*x/(-1 + a^2*x^2) + ArcTanh[a*x]/(3*a)) - (Sqrt[Pi/
2]*(-Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]]))/(
16*a)
```

Maple [F]
time = 6.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(-a^2x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}(ax)}}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(sqrt(atanh(a*x))/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{atanh}\left(a\,x\right)}}{{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^2,x)
[Out] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^2, x)
```

3.281 $\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=40

$$\frac{\operatorname{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}-\frac{3\log\left(\tanh^{-1}(ax)\right)}{2a^5}+\frac{\operatorname{Int}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a^4}$$

[Out] 1/2*Chi(2*arctanh(a*x))/a^5-3/2*ln(arctanh(a*x))/a^5+Unintegrable(1/arctanh(a*x),x)/a^4

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Int[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] Defer[Int][x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]

Rubi steps

$$\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx=\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 3.74, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]

Maple [A]
time = 34.47, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-a^2x^2+1\right)^2 \operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**4/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
[Out] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

3.282 $\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=43

$$\frac{\text{Shi}(2\tanh^{-1}(ax))}{2a^4} - \frac{\text{Int}\left(\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}, x\right)}{a^2}$$

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^4-Unintegrable(x/(-a^2*x^2+1)/arctanh(a*x),x)/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx = \int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Mathematica [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Maple [A]
time = 37.94, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-a^2x^2+1)^2 \operatorname{arctanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
[Out] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

3.283 $\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

```
[Out] 1/2*Chi(2*arctanh(a*x))/a^3-1/2*ln(arctanh(a*x))/a^3
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6181, 3393, 3382}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^3}$$
$$= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Mathematica [A]
time = 0.05, size = 27, normalized size = 1.00

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

Maple [A]
time = 5.58, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\ln(\mathrm{arctanh}(ax))}{2}+\frac{\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{2}}{a^3}$	22
default	$\frac{-\frac{\ln(\mathrm{arctanh}(ax))}{2}+\frac{\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{2}}{a^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2*ln(arctanh(a*x))+1/2*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(23) = 46$.
time = 0.48, size = 58, normalized size = 2.15

$$-\frac{2 \log \left(\log \left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a*x + 1)/(a*x - 1))
 - log_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax-1)^2\,(ax+1)^2\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

3.284 $\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=14

$$\frac{\text{Shi}(2\tanh^{-1}(ax))}{2a^2}$$

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6181, 5556, 12, 3379}

$$\frac{\text{Shi}(2\tanh^{-1}(ax))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^2}$$
$$= \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A]
time = 0.04, size = 14, normalized size = 1.00

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Maple [A]
time = 5.72, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{2a^2}$	13
default	$\frac{\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{2a^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")

[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.
time = 0.35, size = 38, normalized size = 2.71

$$\frac{\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")

[Out] 1/4*(log_integral(-(a*x + 1)/(a*x - 1)) - log_integral(-(a*x - 1)/(a*x + 1)
))/a^2

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^2\,(ax+1)^2\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x),x)

[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")

[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(atanh(a*x)*(a^2*x^2 - 1)^2),x)

[Out] int(x/(atanh(a*x)*(a^2*x^2 - 1)^2), x)

3.285 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}(2\tanh^{-1}(ax))}{2a} + \frac{\log(\tanh^{-1}(ax))}{2a}$$

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/2*ln(arctanh(a*x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6115, 3393, 3382}

$$\frac{\text{Chi}(2\tanh^{-1}(ax))}{2a} + \frac{\log(\tanh^{-1}(ax))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\log\left(\tanh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a}$$
$$= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a} + \frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 1.00

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a} + \frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

Maple [A]
time = 2.25, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\frac{\ln(\text{arctanh}(ax))}{2}+\frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{2}}{a}$	22
default	$\frac{\frac{\ln(\text{arctanh}(ax))}{2}+\frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{2}}{a}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*ln(arctanh(a*x))+1/2*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.
time = 0.34, size = 54, normalized size = 2.00

$$\frac{2 \log \left(\log \left(-\frac{ax+1}{ax-1}\right)\right) + \log\_integral\left(-\frac{ax+1}{ax-1}\right) + \log\_integral\left(-\frac{ax-1}{ax+1}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a*x + 1)/(a*x - 1))
+ log_integral(-(a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

3.286 $\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)+\mathrm{Int}\left(\frac{1}{x\,(1-a^2x^2)\tanh^{-1}(ax)},x\right)$$

[Out] 1/2*Shi(2*arctanh(a*x))+Unintegrable(1/x/(-a^2*x^2+1)/arctanh(a*x),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\,(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] Defer[Int][1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]

Rubi steps

$$\int \frac{1}{x\,(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx=\int \frac{1}{x\,(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]

[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]

Maple [A]
time = 15.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(-a^2x^2+1)^2\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

<u>Mupad</u> [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x \operatorname{atanh}(a x) \left(a^2 x^2 - 1\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

3.287 $\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=61

$$\frac{x}{a^3\tanh^{-1}(ax)} - \frac{x}{a^3(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^4} - \frac{\text{Int}\left(\frac{1}{\tanh^{-1}(ax)}, x\right)}{a^3}$$

```
[Out] x/a^3/arctanh(a*x)-x/a^3/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))/a^4-
Unintegrable(1/arctanh(a*x),x)/a^3
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] x/(a^3*ArcTanh[a*x]) - x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*
ArcTanh[a*x]]/a^4 - Defer[Int][ArcTanh[a*x]^(-1), x]/a^3
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx = \frac{\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^2} \, dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2} \, dx}{a^2}$$

$$= \frac{x}{a^3\tanh^{-1}(ax)} - \frac{x}{a^3(1-a^2x^2)\tanh^{-1}(ax)} - \frac{\int \frac{1}{\tanh^{-1}(ax)} \, dx}{a^3} + \frac{\int \frac{1}{(1-a^2x^2)^2 \text{ta}}}{a^3}$$

$$= \frac{x}{a^3\tanh^{-1}(ax)} - \frac{x}{a^3(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x} \, dx, x, \tanh^{-1}\right.}{a^4}$$

$$= \frac{x}{a^3\tanh^{-1}(ax)} - \frac{x}{a^3(1-a^2x^2)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right) dx,\right.}{a^4}$$

$$= \frac{x}{a^3\tanh^{-1}(ax)} - \frac{x}{a^3(1-a^2x^2)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} \, dx, x, \tanh^{-}\right.}{2a^4}$$

$$= \frac{x}{a^3\tanh^{-1}(ax)} - \frac{x}{a^3(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^4} - \frac{\int \frac{1}{\tanh^{-}}}{a}$$

Mathematica [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 32.05, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-a^2x^2+1\right)^2 \operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
[Out] 2*x^3/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) + integrat
e(-2*(a^2*x^4 - 3*x^2)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 -
 2*a^3*x^2 + a)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax-1\right)^2 \left(ax+1\right)^2 \operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

3.288 $\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=38

$$-\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}$$

```
[Out] -x^2/a/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a^3
```

Rubi [A]
time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6153, 6181, 5556, 12, 3379}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx &= -\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{2\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx}{a}\\
&= -\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^3}\\
&= -\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx,x,\tanh^{-1}(ax)\right)}{a^3}\\
&= -\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^3}\\
&= -\frac{x^2}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 0.95

$$\frac{x^2}{a(-1+a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
[Out] x^2/(a*(-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Maple [A]
time = 6.84, size = 36, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{a^3}$	36
default	$\frac{\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{a^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh
(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x^2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - 4*integr
ate(-x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(36) = 72$.
time = 0.35, size = 111, normalized size = 2.92

$$\frac{4\,a^2x^2 + \left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^5x^2-a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2 + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x
^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^
5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax-1\right)^2\left(ax+1\right)^2\mathrm{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
[Out] int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

3.289 $\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=36

$$-\frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

```
[Out] -x/a/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))/a^2
```

Rubi [A]
time = 0.16, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6179, 6181, 3393, 3382, 6115}

$$\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2} - \frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/a^2
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
```

```
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx &= -\frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\int \frac{1}{(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx}{a} + a\int \frac{x^2}{(1-a^2x^2)^2\tanh
\\
&= -\frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{\text{Subst}\left(\right.}{}
\\
&= -\frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^2} +
\\
&= -\frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^2}
\\
&= -\frac{x}{a(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.89

$$\frac{\frac{ax}{(-1+a^2x^2)\tanh^{-1}(ax)} + \text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
[Out] ((a*x)/((-1 + a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]])/a^2
```

Maple [A]
time = 6.88, size = 28, normalized size = 0.78

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{a^2}$	28
default	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{a^2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*sinh(2*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - integrate(
-2*(a^2*x^2 + 1)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3
*x^2 + a)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(34) = 68$.
time = 0.36, size = 106, normalized size = 2.94

$$\frac{4\,ax + \left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + \left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^4x^2-a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 -
 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^
2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
[Out] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

3.290 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=35

$$-\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a}$$

```
[Out] -1/a/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6113, 6181, 5556, 12, 3379}

$$\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a}-\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
```

```
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx &= -\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}+(2a)\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx\\ &= -\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.86

$$\frac{\frac{1}{(-1+a^2x^2)\tanh^{-1}(ax)}+\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] (1/((-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]])/a
```

Maple [A]
time = 6.61, size = 36, normalized size = 1.03

method	result	size

derivativedivides	$\frac{-\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{a}$	36
default	$\frac{-\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{a}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh(
a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -4*a*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^
2*x^2 + 1)*log(-a*x + 1)), x) + 2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 -
a)*log(-a*x + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(33) = 66.
time = 0.39, size = 102, normalized size = 2.91

$$\frac{\left(\left(a^{2} x^{2}-1\right) \operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{2} x^{2}-1\right) \operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right) \log\left(-\frac{ax+1}{ax-1}\right)+4}{2\left(a^{3} x^{2}-a\right) \log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) + 4)/((a^3*x^2 -
a)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}^{2}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
[Out] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

3.291 $\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\mathrm{Chi}\big(2\tanh^{-1}(ax)\big)-\frac{\mathrm{Int}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}$$

```
[Out] -1/a/x/arctanh(a*x)-a*x/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))-Unint
egrable(1/x^2/arctanh(a*x),x)/a
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral
[2*ArcTanh[a*x]] - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx &= a^2\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx+\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)}\,dx}{a}+a\int \frac{}{(1}\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)}\,dx}{a}+\mathrm{Subst}\Big(\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)}\,dx}{a}-\mathrm{Subst}\Big(\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+2\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx\right.\right.\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\mathrm{Chi}\big(2\tanh^{-1}(ax)\big)-\frac{\int \frac{1}{x^2}}{}\end{aligned}$$

Mathematica [A]
time = 3.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 12.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^2 \operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2/((a^3*x^3 - a*x)*log(a*x + 1) - (a^3*x^3 - a*x)*log(-a*x + 1)) - integrat
e(-2*(3*a^2*x^2 - 1)/((a^5*x^6 - 2*a^3*x^4 + a*x^2)*log(a*x + 1) - (a^5*x^6
 - 2*a^3*x^4 + a*x^2)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax-1\right)^2\left(ax+1\right)^2 \operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

3.292 $\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=102

$$\frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3 (1-a^2x^2) \tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4 (1-a^2x^2) \tanh^{-1}(ax)} + \frac{\mathrm{Shi}(2\tanh^{-1}(ax))}{a^4} - \frac{\mathrm{Int}\left(\frac{1}{\tanh^{-1}(a$$

```
[Out] 1/2*x/a^3/arctanh(a*x)^2-1/2*x/a^3/(-a^2*x^2+1)/arctanh(a*x)^2+1/2*(-a^2*x^
2-1)/a^4/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a^4-1/2*Unintegrable
(1/arctanh(a*x)^2,x)/a^3
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] x/(2*a^3*ArcTanh[a*x]^2) - x/(2*a^3*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^
2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^
4 - Defer[Int][ArcTanh[a*x]^(-2), x]/(2*a^3)
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx = \frac{\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2) \tanh^{-1}(ax)^3} \, dx}{a^2}$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3 (1-a^2x^2) \tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4 (1-a^2x^2) \tanh^{-1}(ax$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3 (1-a^2x^2) \tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4 (1-a^2x^2) \tanh^{-1}(ax$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3 (1-a^2x^2) \tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4 (1-a^2x^2) \tanh^{-1}(ax$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3 (1-a^2x^2) \tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4 (1-a^2x^2) \tanh^{-1}(ax$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3 (1-a^2x^2) \tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4 (1-a^2x^2) \tanh^{-1}(ax$$

Mathematica [A]
time = 7.05, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-a^2x^2)^2\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 29.26, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-a^2x^2+1)^2\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x^3 - (a^2*x^4 - 3*x^2)*log(a*x + 1) + (a^2*x^4 - 3*x^2)*log(-a*x + 1)
)/((a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x
 + 1) + (a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-2*(a^4*x^5 - 2*a^2*x^
3 + 3*x)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1) - (a^6*x^4 - 2*a^4*x^2 +
 a^2)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

3.293 $\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\text{Chi}(2\tanh^{-1}(ax))}{a^3}$$

```
[Out] -1/2*x^2/a/(-a^2*x^2+1)/arctanh(a*x)^2-x/a^2/(-a^2*x^2+1)/arctanh(a*x)+Chi(
2*arctanh(a*x))/a^3
```

Rubi [A]
time = 0.19, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6153, 6179, 6181, 3393, 3382, 6115}

$$\frac{\text{Chi}(2\tanh^{-1}(ax))}{a^3}-\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(a^2*(1 - a^2*x^2)*ArcTanh[a*
x]) + CoshIntegral[2*ArcTanh[a*x]]/a^3
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx = -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} + \frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx}{a}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}}}{a^2}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\right.}{}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \right.\right.}{}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2}{x}\right.}{}$$

$$= -\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(a\right.}{a^3}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 0.73

$$\frac{\frac{ax(ax+2\tanh^{-1}(ax))}{(-1+a^2x^2)\tanh^{-1}(ax)^2}+2\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] ((a*x*(a*x + 2*ArcTanh[a*x]))/((-1 + a^2*x^2)*ArcTanh[a*x]^2) + 2*CoshInteg
ral[2*ArcTanh[a*x]])/(2*a^3)
```

Maple [A]
time = 6.86, size = 51, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{1}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{a^3}$	51
default	$\frac{\frac{1}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{a^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2*sinh(
2*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(a*x^2 + x*log(a*x + 1) - x*log(-a*x + 1))/((a^4*x^2 - a^2)*log(a*x + 1)^
2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^4*x^2 - a^2)*log(-a*x
 + 1)^2) - integrate(-2*(a^2*x^2 + 1)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x
+ 1) - (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.
time = 0.44, size = 131, normalized size = 2.05

$$\frac{4\,a^2x^2+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{2\,(a^5x^2-a^3)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2 + 4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integ
ral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)
))*log(-(a*x + 1)/(a*x - 1))^2)/((a^5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1))^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^3{(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

3.294 $\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=72

$$-\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

```
[Out] -1/2*x/a/(-a^2*x^2+1)/arctanh(a*x)^2+1/2*(-a^2*x^2-1)/a^2/(-a^2*x^2+1)/arct
anh(a*x)+Shi(2*arctanh(a*x))/a^2
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6143, 6181, 5556, 12, 3379}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^2
)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
```

```
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2(1-a^2x^2)\tanh^{-1}(ax)} + 2\int \frac{}{(1-a^2x^2)}$$

$$= -\frac{x}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{2\text{Subst}\left(\int \frac{\cosh(}{}\right.}{}$$

$$= -\frac{x}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{2\text{Subst}\left(\int \frac{\sinh(}{2x}\right.}{}$$

$$= -\frac{x}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x}{x}\right.}{}$$

$$= -\frac{x}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Shi}\left(2\tanh^{-1}(\right.}{a^2}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.92

$$\frac{ax + (1+a^2x^2)\tanh^{-1}(ax) + 2(-1+a^2x^2)\tanh^{-1}(ax)^2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2(-1+a^2x^2)\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] (a*x + (1 + a^2*x^2)*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*SinhInt
egral[2*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

Maple [A]
time = 6.96, size = 43, normalized size = 0.60

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{a^2}$	43
default	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{a^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/2/arctanh(a*x)*cosh(2*arc
tanh(a*x))+Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (a^2*x^2 + 1)*log(a*x + 1) - (a^2*x^2 + 1)*log(-a*x + 1))/((a^4*x^
2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a
^4*x^2 - a^2)*log(-a*x + 1)^2) - 4*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.
time = 0.40, size = 135, normalized size = 1.88

$$\frac{\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,ax+2\,(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,(a^4x^2-a^2)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x + 2*(a^
2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^2 - a^2)*log(-(a*x + 1)/(a*x
- 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

3.295 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=58

$$-\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)/arctanh(a*x)^2-x/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctan
h(a*x))/a
```

Rubi [A]
time = 0.17, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6113, 6179, 6181, 3393, 3382, 6115}

$$-\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/((1 - a^2*x^2)*ArcTanh[a*x]) +
CoshIntegral[2*ArcTanh[a*x]]/a
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-a^2x^2)^2\tanh^{-1}(ax)^3}\,dx &= -\frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}+a\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx \\
&= -\frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}+a^2\int \frac{x^2}{(1-a^2x^2)^2 \ldots} \\
&= -\frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\mathrm{Subst}\left(\int \frac{\cosh^2(x)}{x}\,d\ldots\right.}{a\ldots} \\
&= -\frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\mathrm{Subst}\left(\int\left(\frac{1}{2x}-\ldots\right.\right.}{\ldots} \\
&= -\frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}+2\frac{\mathrm{Subst}\left(\int \frac{\cosh(2x)}{x}\ldots\right.}{2\ldots} \\
&= -\frac{1}{2a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}+\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.04, size = 58, normalized size = 1.00

$$\frac{1+2ax\tanh^{-1}(ax)+2(-1+a^2x^2)\tanh^{-1}(ax)^2\mathrm{Chi}\big(2\tanh^{-1}(ax)\big)}{2a\,(-1+a^2x^2)\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] (1 + 2*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*CoshIntegral[2*Ar
cTanh[a*x]])/(2*a*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

Maple [A]
time = 6.82, size = 51, normalized size = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{a}$	51
default	$\frac{-\frac{1}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{a}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2*sinh(2
*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(a*x*log(a*x + 1) - a*x*log(-a*x + 1) + 1)/((a^3*x^2 - a)*log(a*x + 1)^2
- 2*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1) + (a^3*x^2 - a)*log(-a*x + 1)^
2) - integrate(-2*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (
a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(53) = 106$.
time = 0.38, size = 122, normalized size = 2.10

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4}{2\,(a^3x^2-a)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integral(-(a*x +
1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x
 + 1)/(a*x - 1))^2 + 4)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^3{(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

3.296 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=99

$$-\frac{1}{2ax\tanh^{-1}(ax)^2}-\frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(ax)}+\text{Shi}\left(2\tanh^{-1}(ax)\right)-\frac{\text{Int}\left(\frac{1}{x^2\tanh^{-1}(\right.}{2a}$$

```
[Out] -1/2/a/x/arctanh(a*x)^2-1/2*a*x/(-a^2*x^2+1)/arctanh(a*x)^2+1/2*(-a^2*x^2-1
)/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))-1/2*Unintegrable(1/x^2/arct
anh(a*x)^2,x)/a
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 +
 a^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]] - D
efer[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx = a^2 \int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} \, dx + \int \frac{1}{x(1-a^2x^2) \tanh^{-1}(ax)^3} \, dx$$
$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(a}$$
$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(a}$$
$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(a}$$
$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(a}$$
$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(a}$$

Mathematica [A]
time = 2.57, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 20.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^2\operatorname{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (3*a^2*x^2 - 1)*log(a*x + 1) - (3*a^2*x^2 - 1)*log(-a*x + 1))/((a^
4*x^4 - a^2*x^2)*log(a*x + 1)^2 - 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1)*log(-a
*x + 1) + (a^4*x^4 - a^2*x^2)*log(-a*x + 1)^2) - integrate(-2*(3*a^4*x^4 -
2*a^2*x^2 + 1)/((a^6*x^7 - 2*a^4*x^5 + a^2*x^3)*log(a*x + 1) - (a^6*x^7 - 2
*a^4*x^5 + a^2*x^3)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
[Out] int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

3.297 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=97

$$-\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}$$

```
[Out] -1/3/a/(-a^2*x^2+1)/arctanh(a*x)^3-1/3*x/(-a^2*x^2+1)/arctanh(a*x)^2+1/3*(-
a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/3*Shi(2*arctanh(a*x))/a
```

Rubi [A]
time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6113, 6143, 6181, 5556, 12, 3379}

$$-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}+\frac{2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

```
[Out] -1/3*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(3*(1 - a^2*x^2)*ArcTanh[a*x]^2
) - (1 + a^2*x^2)/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*SinhIntegral[2*ArcT
anh[a*x]])/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
```

```
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^2\tanh^{-1}(ax)^4}\,dx = -\frac{1}{3a(1-a^2x^2)\tanh^{-1}(ax)^3}+\frac{1}{3}(2a)\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{1}{3a(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1+a^{2}}{3a(1-a^2x^2)}$$

$$= -\frac{1}{3a(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1+a^{2}}{3a(1-a^2x^2)}$$

$$= -\frac{1}{3a(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1+a^{2}}{3a(1-a^2x^2)}$$

$$= -\frac{1}{3a(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1+a^{2}}{3a(1-a^2x^2)}$$

$$= -\frac{1}{3a(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1+a^{2}}{3a(1-a^2x^2)}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.75

$$\frac{1+ax\tanh^{-1}(ax)+(1+a^2x^2)\tanh^{-1}(ax)^2+2(-1+a^2x^2)\tanh^{-1}(ax)^3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{3a(-1+a^2x^2)\tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

```
[Out] (1 + a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*Arc
Tanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]])/(3*a*(-1 + a^2*x^2)*ArcTanh[a*x]^
3)
```

Maple [A]
time = 6.91, size = 68, normalized size = 0.70

method	result	size
derivativedivides	$\frac{-\frac{1}{6\,\mathrm{arctanh}(ax)^{3}}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)^{3}}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)^{2}}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{3\,\mathrm{arctanh}(ax)}+\frac{2\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{3}}{a}$	68
default	$\frac{-\frac{1}{6\,\mathrm{arctanh}(ax)^{3}}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)^{3}}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)^{2}}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{3\,\mathrm{arctanh}(ax)}+\frac{2\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{3}}{a}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/6/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6*sinh(2
*arctanh(a*x))/arctanh(a*x)^2-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*Shi
(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -8*a*integrate(-1/3*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/3*(2*a*x*log(a*x + 1) + (a^2*x^2 + 1)
*log(a*x + 1)^2 + (a^2*x^2 + 1)*log(-a*x + 1)^2 - 2*(a*x + (a^2*x^2 + 1)*lo
g(a*x + 1))*log(-a*x + 1) + 4)/((a^3*x^2 - a)*log(a*x + 1)^3 - 3*(a^3*x^2 -
 a)*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x +
1)^2 - (a^3*x^2 - a)*log(-a*x + 1)^3)
```

Fricas [A]
time = 0.41, size = 151, normalized size = 1.56

$$\frac{\left(\left(a^{2}x^{2}-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{2}x^{2}-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+2\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}+8}{3\left(a^{3}x^{2}-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^3 + 4*a*x*log(-(a
*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 8)/((a^3
*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**4,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^4\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^2), x)
```

3.298 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^5} dx$

Optimal. Leaf size=120

$$-\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4}-\frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{3(1-a^2x^2)\tanh^{-1}(a$$

```
[Out] -1/4/a/(-a^2*x^2+1)/arctanh(a*x)^4-1/6*x/(-a^2*x^2+1)/arctanh(a*x)^3+1/12*(
-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^2-1/3*x/(-a^2*x^2+1)/arctanh(a*x)+1
/3*Chi(2*arctanh(a*x))/a
```

Rubi [A]
time = 0.21, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6113, 6143, 6179, 6181, 3393, 3382, 6115}

$$-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{a^2x^2+1}{12a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4}+\frac{\text{Chi}(2\tanh^{-1}(ax))}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

```
[Out] -1/4*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(6*(1 - a^2*x^2)*ArcTanh[a*x]^3
) - (1 + a^2*x^2)/(12*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(3*(1 - a^2*x^2)*
ArcTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^5}\,dx = -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} + \frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^4}\,dx$$
$$= -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3} - \frac{1+a^2}{12a(1-a^2x^2)\,\mathrm{t}}$$
$$= -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3} - \frac{1+a^2}{12a(1-a^2x^2)\,\mathrm{t}}$$
$$= -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3} - \frac{1+a^2}{12a(1-a^2x^2)\,\mathrm{t}}$$
$$= -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3} - \frac{1+a^2}{12a(1-a^2x^2)\,\mathrm{t}}$$
$$= -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3} - \frac{1+a^2}{12a(1-a^2x^2)\,\mathrm{t}}$$
$$= -\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3} - \frac{1+a^2}{12a(1-a^2x^2)\,\mathrm{t}}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.70

$$\frac{3+2ax\tanh^{-1}(ax)+(1+a^2x^2)\tanh^{-1}(ax)^2+4ax\tanh^{-1}(ax)^3+4(-1+a^2x^2)\tanh^{-1}(ax)^4\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{12a\left(-1+a^2x^2\right)\tanh^{-1}(ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
[Out] (3 + 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[a*x]
^3 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]])/(12*a*(-
1 + a^2*x^2)*ArcTanh[a*x]^4)
```

Maple [A]
time = 6.87, size = 83, normalized size = 0.69

method	result
derivativedivides	$\frac{-\frac{1}{8\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^4}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^3}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(2}{3}}{a}$
default	$\frac{-\frac{1}{8\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^4}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^3}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(2}{3}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/8/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12*sinh(
2*arctanh(a*x))/arctanh(a*x)^3-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/6
*sinh(2*arctanh(a*x))/arctanh(a*x)+1/3*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*x*log(a*x + 1)^3 - 2*a*x*log(-a*x + 1)^3 + 4*a*x*log(a*x + 1) + (a
^2*x^2 + 1)*log(a*x + 1)^2 + (a^2*x^2 + 6*a*x*log(a*x + 1) + 1)*log(-a*x +
1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*a*x + (a^2*x^2 + 1)*log(a*x + 1))*log(-a
*x + 1) + 12)/((a^3*x^2 - a)*log(a*x + 1)^4 - 4*(a^3*x^2 - a)*log(a*x + 1)^
3*log(-a*x + 1) + 6*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*(a^3*x
^2 - a)*log(a*x + 1)*log(-a*x + 1)^3 + (a^3*x^2 - a)*log(-a*x + 1)^4) - int
egrate(-2/3*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^
4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.36, size = 171, normalized size = 1.42

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{3} + \left(\left(a^{2}x^{2}-1\right)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + \left(a^{2}x^{2}-1\right)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4} + 8\,ax\log\left(-\frac{ax+1}{ax-1}\right) + 2\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2} + 24}{6\left(a^{3}x^{2}-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(4*a*x*log(-(a*x + 1)/(a*x - 1))^3 + ((a^2*x^2 - 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a
*x + 1)/(a*x - 1))^4 + 8*a*x*log(-(a*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*lo
g(-(a*x + 1)/(a*x - 1))^2 + 24)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^{2}\left(ax+1\right)^{2}\operatorname{atanh}^{5}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**5,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^5\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^2),x)
[Out] int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^2), x)
```

3.299 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^6}\, dx$

Optimal. Leaf size=154

$$-\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}}$$

```
[Out] -1/5/a/(-a^2*x^2+1)/arctanh(a*x)^5-1/10*x/(-a^2*x^2+1)/arctanh(a*x)^4+1/30*
(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^3-1/15*x/(-a^2*x^2+1)/arctanh(a*x)
^2+1/15*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/15*Shi(2*arctanh(a*x))/a
```

Rubi [A]
time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6113, 6143, 6181, 5556, 12, 3379}

$$-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{a^2x^2+1}{15a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{a^2x^2+1}{30a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}+\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

```
[Out] -1/5*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(10*(1 - a^2*x^2)*ArcTanh[a*x]^
4) - (1 + a^2*x^2)/(30*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(15*(1 - a^2*x^2
)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(15*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Si
nhIntegral[2*ArcTanh[a*x]])/(15*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^6}\,dx = -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} + \frac{1}{5}(2a)\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^5}\,dx$$

$$= -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{x}{10(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{1+a}{30a(1-a^2x^2)}$$

$$= -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{x}{10(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{1+a}{30a(1-a^2x^2)}$$

$$= -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{x}{10(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{1+a}{30a(1-a^2x^2)}$$

$$= -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{x}{10(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{1+a}{30a(1-a^2x^2)}$$

$$= -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{x}{10(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{1+a}{30a(1-a^2x^2)}$$

$$= -\frac{1}{5a(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{x}{10(1-a^2x^2)\tanh^{-1}(ax)^4} - \frac{1+a}{30a(1-a^2x^2)}$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 0.66

$$\frac{6+3ax\tanh^{-1}(ax)+(1+a^2x^2)\tanh^{-1}(ax)^2+2ax\tanh^{-1}(ax)^3+2(1+a^2x^2)\tanh^{-1}(ax)^4+4(-1+a^2x^2)\tanh^{-1}(ax)^5\text{Shi}(2\tanh^{-1}(ax))}{30a(-1+a^2x^2)\tanh^{-1}(ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

```
[Out] (6 + 3*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*a*x*ArcTanh[a*x]
^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^5*SinhI
ntegral[2*ArcTanh[a*x]])/(30*a*(-1 + a^2*x^2)*ArcTanh[a*x]^5)
```

Maple [A]
time = 6.88, size = 98, normalized size = 0.64

method	result
derivativedivides	$\frac{-\frac{1}{10\,\text{arctanh}(ax)^5}-\frac{\cosh(2\,\text{arctanh}(ax))}{10\,\text{arctanh}(ax)^5}-\frac{\sinh(2\,\text{arctanh}(ax))}{20\,\text{arctanh}(ax)^4}-\frac{\cosh(2\,\text{arctanh}(ax))}{30\,\text{arctanh}(ax)^3}-\frac{\sinh(2\,\text{arctanh}(ax))}{30\,\text{arctanh}(ax)^2}-\frac{\cosh(2\,\text{arctanh}(ax))}{15\,\text{arctanh}(ax)}+2}{a}$
default	$\frac{-\frac{1}{10\,\text{arctanh}(ax)^5}-\frac{\cosh(2\,\text{arctanh}(ax))}{10\,\text{arctanh}(ax)^5}-\frac{\sinh(2\,\text{arctanh}(ax))}{20\,\text{arctanh}(ax)^4}-\frac{\cosh(2\,\text{arctanh}(ax))}{30\,\text{arctanh}(ax)^3}-\frac{\sinh(2\,\text{arctanh}(ax))}{30\,\text{arctanh}(ax)^2}-\frac{\cosh(2\,\text{arctanh}(ax))}{15\,\text{arctanh}(ax)}+2}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/10/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/arc
tanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1
/30*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/15/arctanh(a*x)*cosh(2*arctanh(a*
x))+2/15*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="maxima")
```

```
[Out] -8*a*integrate(-1/15*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/15*(2*a*x*log(a*x + 1)^3 + (a^2*x^2
+ 1)*log(a*x + 1)^4 + (a^2*x^2 + 1)*log(-a*x + 1)^4 - 2*(a*x + 2*(a^2*x^2 +
 1)*log(a*x + 1))*log(-a*x + 1)^3 + 12*a*x*log(a*x + 1) + 2*(a^2*x^2 + 1)*l
og(a*x + 1)^2 + 2*(a^2*x^2 + 3*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(a*x +
 1)^2 + 1)*log(-a*x + 1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*(a^2*x^2 + 1)*log(
a*x + 1)^3 + 6*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 48)/((a^
3*x^2 - a)*log(a*x + 1)^5 - 5*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1) +
```

```
10*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^3*x^2 - a)*log(a*x
+ 1)^2*log(-a*x + 1)^3 + 5*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^4 - (a^
3*x^2 - a)*log(-a*x + 1)^5)
```

Fricas [A]
time = 0.34, size = 200, normalized size = 1.30

$$\frac{\left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^5+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+2\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+24\,ax\log\left(-\frac{ax+1}{ax-1}\right)+4\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+96}{15\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log
_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^5 + 4*a*x*log(-(
a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 24*a*
x*log(-(a*x + 1)/(a*x - 1)) + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 +
 96)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^6\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**6,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^6\,\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^2), x)
```

3.300 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^7} dx$

Optimal. Leaf size=177

$$-\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6}-\frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a(1-a^2x^2)\tanh^{-1}(ax)^4}-\frac{x}{45(1-a^2x^2)\tanh$$

```
[Out] -1/6/a/(-a^2*x^2+1)/arctanh(a*x)^6-1/15*x/(-a^2*x^2+1)/arctanh(a*x)^5+1/60*
(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^4-1/45*x/(-a^2*x^2+1)/arctanh(a*x)
^3+1/90*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^2-2/45*x/(-a^2*x^2+1)/arct
anh(a*x)+2/45*Chi(2*arctanh(a*x))/a
```

Rubi [A]
time = 0.28, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6113, 6143, 6179, 6181, 3393, 3382, 6115}

$$-\frac{2x}{45(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{45(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5}-\frac{a^2x^2+1}{90a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{60a(1-a^2x^2)\tanh^{-1}(ax)^4}-\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6}+\frac{2\text{Chi}(2\tanh^{-1}(ax))}{45a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

```
[Out] -1/6*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^6) - x/(15*(1 - a^2*x^2)*ArcTanh[a*x]^
5) - (1 + a^2*x^2)/(60*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(45*(1 - a^2*x^2
)*ArcTanh[a*x]^3) - (1 + a^2*x^2)/(90*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*
x)/(45*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*CoshIntegral[2*ArcTanh[a*x]])/(45*a
)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
```

```
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^7}\,dx = -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} + \frac{1}{3}a\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^6}\,dx$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

$$= -\frac{1}{6a(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5} - \frac{1+}{60a(1-a^2x}$$

Mathematica [A]
time = 0.05, size = 112, normalized size = 0.63

$$\frac{30+12ax\tanh^{-1}(ax)+3(1+a^2x^2)\tanh^{-1}(ax)^2+4ax\tanh^{-1}(ax)^3+2(1+a^2x^2)\tanh^{-1}(ax)^4+8ax\tanh^{-1}(ax)^5+8(-1+a^2x^2)\tanh^{-1}(ax)^6\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{180a(-1+a^2x^2)\tanh^{-1}(ax)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

```
[Out] (30 + 12*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[
a*x]^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 8*a*x*ArcTanh[a*x]^5 + 8*(-1 + a^
2*x^2)*ArcTanh[a*x]^6*CoshIntegral[2*ArcTanh[a*x]])/(180*a*(-1 + a^2*x^2)*A
rcTanh[a*x]^6)
```

Maple [A]
time = 6.90, size = 113, normalized size = 0.64

method	result
derivativedivides	$\frac{-\frac{1}{12\,\mathrm{arctanh}(ax)^6}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^6}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^5}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{60\,\mathrm{arctanh}(ax)^4}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{90\,\mathrm{arctanh}(ax)^3}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{90\,\mathrm{arctanh}(ax)^2}-\mathrm{s}}{a}$

default	$\frac{-\frac{1}{12\,\mathrm{arctanh}(ax)^6}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^6}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^5}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{60\,\mathrm{arctanh}(ax)^4}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{90\,\mathrm{arctanh}(ax)^3}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{90\,\mathrm{arctanh}(ax)^2}-\mathrm{sin}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/12/arctanh(a*x)^6-1/12/arctanh(a*x)^6*cosh(2*arctanh(a*x))-1/30/arc
tanh(a*x)^5*sinh(2*arctanh(a*x))-1/60/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1
/90*sinh(2*arctanh(a*x))/arctanh(a*x)^3-1/90/arctanh(a*x)^2*cosh(2*arctanh(
a*x))-1/45*sinh(2*arctanh(a*x))/arctanh(a*x)+2/45*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="maxima")
```

```
[Out] 2/45*(2*a*x*log(a*x + 1)^5 - 2*a*x*log(-a*x + 1)^5 + 4*a*x*log(a*x + 1)^3 +
 (a^2*x^2 + 1)*log(a*x + 1)^4 + (a^2*x^2 + 10*a*x*log(a*x + 1) + 1)*log(-a*
x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + a*x + (a^2*x^2 + 1)*log(a*x + 1))*log(
-a*x + 1)^3 + 48*a*x*log(a*x + 1) + 6*(a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(10*
a*x*log(a*x + 1)^3 + 3*a^2*x^2 + 6*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(a
*x + 1)^2 + 3)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x + 1)^4 + 6*a*x*log(a*x +
1)^2 + 2*(a^2*x^2 + 1)*log(a*x + 1)^3 + 24*a*x + 6*(a^2*x^2 + 1)*log(a*x +
1))*log(-a*x + 1) + 240)/((a^3*x^2 - a)*log(a*x + 1)^6 - 6*(a^3*x^2 - a)*lo
g(a*x + 1)^5*log(-a*x + 1) + 15*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^
2 - 20*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^3 + 15*(a^3*x^2 - a)*log(
a*x + 1)^2*log(-a*x + 1)^4 - 6*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^5 +
 (a^3*x^2 - a)*log(-a*x + 1)^6) - integrate(-4/45*(a^2*x^2 + 1)/((a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.36, size = 220, normalized size = 1.24

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^5+\left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^6+8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+2\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+96\,ax\log\left(-\frac{ax+1}{ax-1}\right)+12\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+480}{45\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="fricas")
```

```
[Out] 1/45*(4*a*x*log(-(a*x + 1)/(a*x - 1))^5 + ((a^2*x^2 - 1)*log_integral(-(a*x
 + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(
a*x + 1)/(a*x - 1))^6 + 8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)
*log(-(a*x + 1)/(a*x - 1))^4 + 96*a*x*log(-(a*x + 1)/(a*x - 1)) + 12*(a^2*x
```

```
^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 480)/((a^3*x^2 - a)*log(-(a*x + 1)/(a
*x - 1))^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^7(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**7,x)
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^7\,\left(a^2\,x^2-1\right)^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^7*(a^2*x^2 - 1)^2),x)
[Out] int(1/(atanh(a*x)^7*(a^2*x^2 - 1)^2), x)
```

3.301 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^8} \, dx$

Optimal. Leaf size=211

$$-\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7}-\frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a(1-a^2x^2)\tanh^{-1}(ax)^5}-\frac{x}{105(1-a^2x^2)\tanh$$

```
[Out] -1/7/a/(-a^2*x^2+1)/arctanh(a*x)^7-1/21*x/(-a^2*x^2+1)/arctanh(a*x)^6+1/105
*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^5-1/105*x/(-a^2*x^2+1)/arctanh(a*
x)^4+1/315*(-a^2*x^2-1)/a/(-a^2*x^2+1)/arctanh(a*x)^3-2/315*x/(-a^2*x^2+1)/
arctanh(a*x)^2-2/315*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)+4/315*Shi(2*ar
ctanh(a*x))/a
```

Rubi [A]
time = 0.28, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6113, 6143, 6181, 5556, 12, 3379}

$$-\frac{2x}{315(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{105(1-a^2x^2)\tanh^{-1}(ax)^4}-\frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6}-\frac{2(a^2x^2+1)}{315a(1-a^2x^2)\tanh^{-1}(ax)}-\frac{a^2x^2+1}{315a(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{a^2x^2+1}{105a(1-a^2x^2)\tanh^{-1}(ax)^5}-\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7}+\frac{4\text{Shi}(2\tanh^{-1}(ax))}{315a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

```
[Out] -1/7*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^7) - x/(21*(1 - a^2*x^2)*ArcTanh[a*x]^
6) - (1 + a^2*x^2)/(105*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(105*(1 - a^2*x
^2)*ArcTanh[a*x]^4) - (1 + a^2*x^2)/(315*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) -
(2*x)/(315*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*(1 + a^2*x^2))/(315*a*(1 - a^
2*x^2)*ArcTanh[a*x]) + (4*SinhIntegral[2*ArcTanh[a*x]])/(315*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^8}\,dx = -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} + \frac{1}{7}(2a)\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^7}\,dx$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

$$= -\frac{1}{7a(1-a^2x^2)\tanh^{-1}(ax)^7} - \frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6} - \frac{1+ \cdots}{105a(1-a^2x^2 \cdots}$$

Mathematica [A]

time = 0.06, size = 128, normalized size = 0.61

$$\frac{45+15ax\tanh^{-1}(ax)+3(1+a^2x^2)\tanh^{-1}(ax)^2+3ax\tanh^{-1}(ax)^3+(1+a^2x^2)\tanh^{-1}(ax)^4+2ax\tanh^{-1}(ax)^5+2(1+a^2x^2)\tanh^{-1}(ax)^6+4(-1+a^2x^2)\tanh^{-1}(ax)^7\text{Shi}(2\tanh^{-1}(ax))}{315a(-1+a^2x^2)\tanh^{-1}(ax)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

```
[Out] (45 + 15*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[
a*x]^3 + (1 + a^2*x^2)*ArcTanh[a*x]^4 + 2*a*x*ArcTanh[a*x]^5 + 2*(1 + a^2*x
^2)*ArcTanh[a*x]^6 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^7*SinhIntegral[2*ArcTanh
[a*x]])/(315*a*(-1 + a^2*x^2)*ArcTanh[a*x]^7)
```

Maple [A]

time = 5.89, size = 128, normalized size = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{14\,\text{arctanh}(ax)^7}-\frac{\cosh(2\,\text{arctanh}(ax))}{14\,\text{arctanh}(ax)^7}-\frac{\sinh(2\,\text{arctanh}(ax))}{42\,\text{arctanh}(ax)^6}-\frac{\cosh(2\,\text{arctanh}(ax))}{105\,\text{arctanh}(ax)^5}-\frac{\sinh(2\,\text{arctanh}(ax))}{210\,\text{arctanh}(ax)^4}-\frac{\cosh(2\,\text{arctanh}(ax))}{315\,\text{arctanh}(ax)^3}-\frac{\sin\cdots}{31\cdots}}{a}$

default	$\frac{-\frac{1}{14\,\mathrm{arctanh}(ax)^7}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{14\,\mathrm{arctanh}(ax)^7}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{42\,\mathrm{arctanh}(ax)^6}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{105\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{210\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{315\,\mathrm{arctanh}(ax)^3}-\ \mathrm{s}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/14/arctanh(a*x)^7-1/14/arctanh(a*x)^7*cosh(2*arctanh(a*x))-1/42/arc
tanh(a*x)^6*sinh(2*arctanh(a*x))-1/105/arctanh(a*x)^5*cosh(2*arctanh(a*x))-
1/210/arctanh(a*x)^4*sinh(2*arctanh(a*x))-1/315/arctanh(a*x)^3*cosh(2*arcta
nh(a*x))-1/315*sinh(2*arctanh(a*x))/arctanh(a*x)^2-2/315/arctanh(a*x)*cosh(
2*arctanh(a*x))+4/315*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="maxima")
```

```
[Out] -16*a*integrate(-1/315*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4
 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 4/315*(2*a*x*log(a*x + 1)^5 + (a^2*x
^2 + 1)*log(a*x + 1)^6 + (a^2*x^2 + 1)*log(-a*x + 1)^6 - 2*(a*x + 3*(a^2*x^
2 + 1)*log(a*x + 1))*log(-a*x + 1)^5 + 12*a*x*log(a*x + 1)^3 + 2*(a^2*x^2 +
 1)*log(a*x + 1)^4 + (2*a^2*x^2 + 10*a*x*log(a*x + 1) + 15*(a^2*x^2 + 1)*lo
g(a*x + 1)^2 + 2)*log(-a*x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + 5*(a^2*x^2 +
1)*log(a*x + 1)^3 + 3*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)^3 +
 240*a*x*log(a*x + 1) + 24*(a^2*x^2 + 1)*log(a*x + 1)^2 + (20*a*x*log(a*x +
 1)^3 + 15*(a^2*x^2 + 1)*log(a*x + 1)^4 + 24*a^2*x^2 + 36*a*x*log(a*x + 1)
+ 12*(a^2*x^2 + 1)*log(a*x + 1)^2 + 24)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x
+ 1)^4 + 3*(a^2*x^2 + 1)*log(a*x + 1)^5 + 18*a*x*log(a*x + 1)^2 + 4*(a^2*x^
2 + 1)*log(a*x + 1)^3 + 120*a*x + 24*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x +
 1) + 1440)/((a^3*x^2 - a)*log(a*x + 1)^7 - 7*(a^3*x^2 - a)*log(a*x + 1)^6*
log(-a*x + 1) + 21*(a^3*x^2 - a)*log(a*x + 1)^5*log(-a*x + 1)^2 - 35*(a^3*x
^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^3 + 35*(a^3*x^2 - a)*log(a*x + 1)^3*lo
g(-a*x + 1)^4 - 21*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^5 + 7*(a^3*x^
2 - a)*log(a*x + 1)*log(-a*x + 1)^6 - (a^3*x^2 - a)*log(-a*x + 1)^7)
```

Fricas [A]
time = 0.37, size = 249, normalized size = 1.18

$$\frac{2\left(\left(\left(a^{2}x^{2}-1\right)\log\_\mathrm{integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{2}x^{2}-1\right)\log\_\mathrm{integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{7}+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{5}+2\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{6}+24\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{3}+4\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+480\,ax\log\left(-\frac{ax+1}{ax-1}\right)+48\left(a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}+2880\right)}{315\left(a^{3}x^{2}-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="fricas")
```

```
[Out] 2/315*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*lo
g_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^7 + 4*a*x*log(-
(a*x + 1)/(a*x - 1))^5 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^6 + 24*a
*x*log(-(a*x + 1)/(a*x - 1))^3 + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^
4 + 480*a*x*log(-(a*x + 1)/(a*x - 1)) + 48*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*
x - 1))^2 + 2880)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^2(ax+1)^2\operatorname{atanh}^8(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**8,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\operatorname{atanh}(a\,x)^8\,(a^2\,x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^8*(a^2*x^2 - 1)^2),x)
```

```
[Out] int(1/(atanh(a*x)^8*(a^2*x^2 - 1)^2), x)
```

3.302 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=77

$$-\frac{x^3}{16a\left(1-a^2x^2\right)^2}+\frac{3x}{32a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)}{32a^4}+\frac{x^4\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}$$

```
[Out] -1/16*x^3/a/(-a^2*x^2+1)^2+3/32*x/a^3/(-a^2*x^2+1)-3/32*arctanh(a*x)/a^4+1/
4*x^4*arctanh(a*x)/(-a^2*x^2+1)^2
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6155, 294, 212}

$$-\frac{3\tanh^{-1}(ax)}{32a^4}+\frac{x^4\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{x^3}{16a\left(1-a^2x^2\right)^2}+\frac{3x}{32a^3\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*x^3/(a*(1 - a^2*x^2)^2) + (3*x)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a
*x])/(32*a^4) + (x^4*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
```

```
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx &= \frac{x^4\tanh^{-1}(ax)}{4(1-a^2x^2)^2}-\frac{1}{4}a\int \frac{x^4}{(1-a^2x^2)^3}\,dx\\ &= -\frac{x^3}{16a(1-a^2x^2)^2}+\frac{x^4\tanh^{-1}(ax)}{4(1-a^2x^2)^2}+\frac{3\int \frac{x^2}{(1-a^2x^2)^2}\,dx}{16a}\\ &= -\frac{x^3}{16a(1-a^2x^2)^2}+\frac{3x}{32a^3(1-a^2x^2)}+\frac{x^4\tanh^{-1}(ax)}{4(1-a^2x^2)^2}-\frac{3\int \frac{1}{1-a^2x^2}\,dx}{32a^3}\\ &= -\frac{x^3}{16a(1-a^2x^2)^2}+\frac{3x}{32a^3(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)}{32a^4}+\frac{x^4\tanh^{-1}(ax)}{4(1-a^2x^2)^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.27

$$-\frac{x}{16a^3(-1+a^2x^2)^2}-\frac{5x}{32a^3(-1+a^2x^2)}+\frac{(-1+2a^2x^2)\tanh^{-1}(ax)}{4a^4(-1+a^2x^2)^2}-\frac{5\log(1-ax)}{64a^4}+\frac{5\log(1+ax)}{64a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*x/(a^3*(-1 + a^2*x^2)^2) - (5*x)/(32*a^3*(-1 + a^2*x^2)) + ((-1 + 2*a
^2*x^2)*ArcTanh[a*x])/(4*a^4*(-1 + a^2*x^2)^2) - (5*Log[1 - a*x])/(64*a^4)
+ (5*Log[1 + a*x])/(64*a^4)
```

Maple [A]
time = 0.61, size = 110, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}+\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}-\frac{1}{64(ax-1)^2}-\frac{5}{64(ax-1)}-\frac{5\ln(ax-1)}{64}+\frac{1}{64(ax+1)^2}-\frac{5}{64(ax+1)}+\frac{5\ln(ax-}{64}}{a^4}$
default	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}+\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}-\frac{1}{64(ax-1)^2}-\frac{5}{64(ax-1)}-\frac{5\ln(ax-1)}{64}+\frac{1}{64(ax+1)^2}-\frac{5}{64(ax+1)}+\frac{5\ln(ax-}{64}}{a^4}$
risch	$\frac{(2a^2x^2-1)\ln(ax+1)}{8a^4(a^2x^2-1)^2}+\frac{5\ln(-ax-1)a^4x^4-5\ln(ax-1)a^4x^4-10a^3x^3-10\ln(-ax-1)a^2x^2+10\ln(ax-1)a^2x^2-16x^2\ln(-}{64a^4(ax+1)(ax-1)(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/16*arctanh(a*x)/(a*x-1)^2+3/16*arctanh(a*x)/(a*x-1)+1/16*arctanh(a
*x)/(a*x+1)^2-3/16*arctanh(a*x)/(a*x+1)-1/64/(a*x-1)^2-5/64/(a*x-1)-5/64*ln
(a*x-1)+1/64/(a*x+1)^2-5/64/(a*x+1)+5/64*ln(a*x+1))
```

Maxima [**A**]
time = 0.26, size = 99, normalized size = 1.29

$$-\frac{1}{64}a\left(\frac{2\left(5a^2x^3-3x\right)}{a^8x^4-2a^6x^2+a^4}-\frac{5\log\left(ax+1\right)}{a^5}+\frac{5\log\left(ax-1\right)}{a^5}\right)+\frac{\left(2a^2x^2-1\right)\operatorname{artanh}\left(ax\right)}{4\left(a^8x^4-2a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/(a^8*x^4 - 2*a^
6*x^2 + a^4)
```

Fricas [**A**]
time = 0.44, size = 71, normalized size = 0.92

$$-\frac{10a^3x^3-6ax-\left(5a^4x^4+6a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\left(a^8x^4-2a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*a^3*x^3 - 6*a*x - (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x
 - 1)))/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(65) = 130.
time = 0.84, size = 158, normalized size = 2.05

$$\begin{cases}\frac{5a^4x^4\operatorname{atanh}\left(ax\right)}{32a^8x^4-64a^6x^2+32a^4}-\frac{5a^3x^3}{32a^8x^4-64a^6x^2+32a^4}+\frac{6a^2x^2\operatorname{atanh}\left(ax\right)}{32a^8x^4-64a^6x^2+32a^4}+\frac{3ax}{32a^8x^4-64a^6x^2+32a^4}-\frac{3\operatorname{atanh}\left(ax\right)}{32a^8x^4-64a^6x^2+32a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((5*a**4*x**4*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) -
 5*a**3*x**3/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 6*a**2*x**2*atanh(a*
x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 3*a*x/(32*a**8*x**4 - 64*a**6*
x**2 + 32*a**4) - 3*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4), Ne(
a, 0)), (0, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(66) = 132.

time = 0.41, size = 239, normalized size = 3.10

$$\frac{1}{256}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{\frac{(ax+1)^2a^5}{(ax-1)^2}+\frac{4(ax+1)a^5}{ax-1}}{a^{10}}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)+\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}-\frac{\frac{(ax+1)^2a^5}{(ax-1)^2}+\frac{8(ax+1)a^5}{ax-1}}{a^{10}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/256*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + ((a*x
 + 1)^2*a^5/(a*x - 1)^2 + 4*(a*x + 1)*a^5/(a*x - 1))/a^10)*log(-(a*((a*x +
1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1)
+ 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + (a*x - 1)^2*(8*(a*x + 1)/(a*x - 1)
 + 1)/((a*x + 1)^2*a^5) - ((a*x + 1)^2*a^5/(a*x - 1)^2 + 8*(a*x + 1)*a^5/(a
*x - 1))/a^10)*a
```

Mupad [**B**]

time = 1.39, size = 83, normalized size = 1.08

$$\frac{5\,\mathrm{atanh}(a\,x)}{32\,a^4}+\frac{\frac{\ln(1-a\,x)}{8}-\frac{\ln(a\,x+1)}{8}+\frac{3\,a\,x}{32}+x^2\left(\frac{a^2\,\ln(a\,x+1)}{4}-\frac{a^2\,\ln(1-a\,x)}{4}\right)-\frac{5\,a^3\,x^3}{32}}{a^4\,\left(a^2\,x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

```
[Out] (5*atanh(a*x))/(32*a^4) + (log(1 - a*x)/8 - log(a*x + 1)/8 + (3*a*x)/32 + x
^2*((a^2*log(a*x + 1))/4 - (a^2*log(1 - a*x))/4) - (5*a^3*x^3)/32)/(a^4*(a^
2*x^2 - 1)^2)
```

3.303 $\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=100

$$-\frac{1}{16a^3(1-a^2x^2)^2}+\frac{1}{16a^3(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}-\frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)^2}{16a^3}$$

```
[Out] -1/16/a^3/(-a^2*x^2+1)^2+1/16/a^3/(-a^2*x^2+1)+1/4*x*arctanh(a*x)/a^2/(-a^2
*x^2+1)^2-1/8*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-1/16*arctanh(a*x)^2/a^3
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6145, 6103, 267}

$$-\frac{\tanh^{-1}(ax)^2}{16a^3}-\frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}+\frac{1}{16a^3(1-a^2x^2)}-\frac{1}{16a^3(1-a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*1/(a^3*(1 - a^2*x^2)^2) + 1/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])
/(4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x])/(8*a^2*(1 - a^2*x^2)) - ArcTanh
[a*x]^2/(16*a^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6145

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]
 - Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x])
```

```
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q,
-5/2]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{1}{16a^3(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{4a^2}$$

$$= -\frac{1}{16a^3(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^2}{16a^3} + \frac{\int \frac{x}{(1-a^2x^2)^2}}{8a}$$

$$= -\frac{1}{16a^3(1-a^2x^2)^2} + \frac{1}{16a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)} - \frac{\tanh^{-1}(}{16a^3}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.61

$$-\frac{a^2x^2 - 2(ax + a^3x^3)\tanh^{-1}(ax) + (-1+a^2x^2)^2\tanh^{-1}(ax)^2}{16a^3(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*(a^2*x^2 - 2*(a*x + a^3*x^3)*ArcTanh[a*x] + (-1 + a^2*x^2)^2*ArcTanh[
a*x]^2)/(a^3*(-1 + a^2*x^2)^2)
```

Maple [A]
time = 0.78, size = 178, normalized size = 1.78

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax+16}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{16}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax-16}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)}{32}}{a^3}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax+16}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{16}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax-16}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)}{32}}{a^3}$
risch	$-\frac{\ln(ax+1)^2}{64a^3}+\frac{\left(x^4\ln(-ax+1)a^4+2a^3x^3-2x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1)\right)\ln(ax+1)}{32a^3(a^2x^2-1)^2}-a^4x^4\ln(-ax+1)^2+4a^3x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/16*arctanh(a*x)/(a*x+1)^2+1/16*arctanh(a*x)/(a*x+1)-1/16*arctanh(
a*x)*ln(a*x+1)+1/16*arctanh(a*x)/(a*x-1)^2+1/16*arctanh(a*x)/(a*x-1)+1/16*a
```

```
rctanh(a*x)*ln(a*x-1)-1/32*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/6
4*ln(a*x+1)^2-1/32*ln(a*x-1)*ln(1/2*a*x+1/2)+1/64*ln(a*x-1)^2-1/64/(a*x+1)^
2+1/64/(a*x+1)-1/64/(a*x-1)^2-1/64/(a*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(86) = 172$.
time = 0.27, size = 179, normalized size = 1.79

$$\frac{1}{16}\left(\frac{2\left(a^{2}x^{3}+x\right)}{a^{6}x^{4}-2\,a^{4}x^{2}+a^{2}}-\frac{\log\left(ax+1\right)}{a^{3}}+\frac{\log\left(ax-1\right)}{a^{3}}\right)\operatorname{artanh}\left(ax\right)-\frac{\left(4\,a^{2}x^{2}-\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(ax+1\right)^{2}+2\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(ax+1\right)\log\left(ax-1\right)-\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(ax-1\right)^{2}\right)a}{64\left(a^{8}x^{4}-2\,a^{6}x^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x) - 1/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*lo
g(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.95

$$-\frac{4\,a^{2}x^{2}+\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-4\left(a^{3}x^{3}+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\left(a^{7}x^{4}-2\,a^{5}x^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*a^2*x^2 + (a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 -
4*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{2}\operatorname{atanh}\left(ax\right)}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

Mupad [B]
time = 1.19, size = 150, normalized size = 1.50

$$\ln(1-a\,x)\left(\frac{\ln(a\,x+1)}{32\,a^3}-\frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{2\,a^4\,x^4-4\,a^2\,x^2+2}\right)-\frac{\ln(a\,x+1)^2}{64\,a^3}-\frac{\ln(1-a\,x)^2}{64\,a^3}-\frac{x^2}{2\,(8\,a^5\,x^4-16\,a^3\,x^2+8\,a)}+\frac{\ln(a\,x+1)\,\left(\frac{x}{16\,a^3}+\frac{x^3}{16\,a}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(1 - a*x)*(log(a*x + 1)/(32*a^3) - (x/(8*a^2) + x^3/8)/(2*a^4*x^4 - 4*a^
2*x^2 + 2)) - log(a*x + 1)^2/(64*a^3) - log(1 - a*x)^2/(64*a^3) - x^2/(2*(8
*a - 16*a^3*x^2 + 8*a^5*x^4)) + (log(a*x + 1)*(x/(16*a^3) + x^3/(16*a)))/(1
/a - 2*a*x^2 + a^3*x^4)
```

3.304 $\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=75

$$-\frac{x}{16a\left(1-a^2x^2\right)^2}-\frac{3x}{32a\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)}{32a^2}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}$$

```
[Out] -1/16*x/a/(-a^2*x^2+1)^2-3/32*x/a/(-a^2*x^2+1)-3/32*arctanh(a*x)/a^2+1/4*ar
ctanh(a*x)/a^2/(-a^2*x^2+1)^2
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6141, 205, 212}

$$-\frac{3x}{32a\left(1-a^2x^2\right)}-\frac{x}{16a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*x/(a*(1 - a^2*x^2)^2) - (3*x)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])
/(32*a^2) + ArcTanh[a*x]/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx &= \frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx}{4a} \\
&= -\frac{x}{16a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx}{16a} \\
&= -\frac{x}{16a\left(1-a^2x^2\right)^2}-\frac{3x}{32a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\int \frac{1}{1-a^2x^2}\,dx}{32a} \\
&= -\frac{x}{16a\left(1-a^2x^2\right)^2}-\frac{3x}{32a\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)}{32a^2}+\frac{\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.17

$$
-\frac{x}{16a\left(-1+a^2x^2\right)^2}+\frac{3x}{32a\left(-1+a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4a^2\left(-1+a^2x^2\right)^2}+\frac{3\log(1-ax)}{64a^2}-\frac{3\log(1+ax)}{64a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*x/(a*(-1 + a^2*x^2)^2) + (3*x)/(32*a*(-1 + a^2*x^2)) + ArcTanh[a*x]/(
4*a^2*(-1 + a^2*x^2)^2) + (3*Log[1 - a*x])/(64*a^2) - (3*Log[1 + a*x])/(64*
a^2)
```

Maple [A]
time = 0.58, size = 75, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)}{4\left(a^2x^2-1\right)^2}-\frac{1}{64(ax-1)^2}+\frac{3}{64(ax-1)}+\frac{3\ln(ax-1)}{64}+\frac{1}{64(ax+1)^2}+\frac{3}{64(ax+1)}-\frac{3\ln(ax+1)}{64}}{a^2}$
default	$\frac{\frac{\operatorname{arctanh}(ax)}{4\left(a^2x^2-1\right)^2}-\frac{1}{64(ax-1)^2}+\frac{3}{64(ax-1)}+\frac{3\ln(ax-1)}{64}+\frac{1}{64(ax+1)^2}+\frac{3}{64(ax+1)}-\frac{3\ln(ax+1)}{64}}{a^2}$
risch	$\frac{\ln(ax+1)}{8a^2(a^2x^2-1)^2}+\frac{3x^4\ln(-ax+1)a^4-3\ln(ax+1)a^4x^4+6a^3x^3-6x^2\ln(-ax+1)a^2+6a^2x^2\ln(ax+1)-10ax-5\ln(-ax+1)}{64a^2(ax+1)(ax-1)(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4/(a^2*x^2-1)^2*arctanh(a*x)-1/64/(a*x-1)^2+3/64/(a*x-1)+3/64*ln(a
*x-1)+1/64/(a*x+1)^2+3/64/(a*x+1)-3/64*ln(a*x+1))
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.09

$$\frac{\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}}{64\,a}+\frac{\operatorname{artanh}(ax)}{4\,(a^2x^2-1)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)/a + 1/4*arctanh(a*x)/((a^2*x^2 - 1)^2*a^2)
```

Fricas [A]
time = 0.34, size = 71, normalized size = 0.95

$$\frac{6\,a^3x^3-10\,ax-(3\,a^4x^4-6\,a^2x^2-5)\log\left(-\frac{ax+1}{ax-1}\right)}{64\,(a^6x^4-2\,a^4x^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(6*a^3*x^3 - 10*a*x - (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x
- 1)))/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(61) = 122.
time = 0.85, size = 158, normalized size = 2.11

$$\begin{cases} -\frac{3a^4x^4\operatorname{atanh}(ax)}{32a^6x^4-64a^4x^2+32a^2}+\frac{3a^3x^3}{32a^6x^4-64a^4x^2+32a^2}+\frac{6a^2x^2\operatorname{atanh}(ax)}{32a^6x^4-64a^4x^2+32a^2}-\frac{5ax}{32a^6x^4-64a^4x^2+32a^2}+\frac{5\operatorname{atanh}(ax)}{32a^6x^4-64a^4x^2+32a^2} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((-3*a**4*x**4*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2)
+ 3*a**3*x**3/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) + 6*a**2*x**2*atanh(a
*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) - 5*a*x/(32*a**6*x**4 - 64*a**4
*x**2 + 32*a**2) + 5*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2), Ne
(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(64) = 128.
time = 0.40, size = 239, normalized size = 3.19

$$-\frac{1}{256}\left(2\left(\frac{(ax-1)^2\left(\frac{4\,(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{\frac{(ax+1)^2a^3}{(ax-1)^2}-\frac{4\,(ax+1)a^3}{ax-1}}{a^6}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)+\frac{(ax-1)^2\left(\frac{8\,(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}+\frac{\frac{(ax+1)^2a^3}{(ax-1)^2}-\frac{8\,(ax+1)a^3}{ax-1}}{a^6}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")

[Out] -1/256*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - ((a*x + 1)^2*a^3/(a*x - 1)^2 - 4*(a*x + 1)*a^3/(a*x - 1))/a^6)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + (a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) + ((a*x + 1)^2*a^3/(a*x - 1)^2 - 8*(a*x + 1)*a^3/(a*x - 1))/a^6)*a

Mupad [B]
time = 1.08, size = 105, normalized size = 1.40

$$\frac{\frac{3\,\ln(a\,x-1)}{64}-\frac{3\,\ln(a\,x+1)}{64}}{a^2}+\frac{\frac{\mathrm{atanh}(a\,x)}{4}-x^2\,\left(a^2\,\left(\frac{3\,\ln(a\,x-1)}{32}-\frac{3\,\ln(a\,x+1)}{32}\right)-2\,a^2\,\left(\frac{3\,\ln(a\,x-1)}{64}-\frac{3\,\ln(a\,x+1)}{64}\right)\right)-\frac{5\,a\,x}{32}+\frac{3\,a^3\,x^3}{32}}{a^2\,{\left(a^2\,x^2-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x*atanh(a*x))/(a^2*x^2 - 1)^3,x)

[Out] ((3*log(a*x - 1))/64 - (3*log(a*x + 1))/64)/a^2 + (atanh(a*x)/4 - x^2*(a^2*((3*log(a*x - 1))/32 - (3*log(a*x + 1))/32) - 2*a^2*((3*log(a*x - 1))/64 - (3*log(a*x + 1))/64)) - (5*a*x)/32 + (3*a^3*x^3)/32)/(a^2*(a^2*x^2 - 1)^2)

3.305 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=94

$$-\frac{1}{16a(1-a^2x^2)^2}-\frac{3}{16a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{8(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

```
[Out] -1/16/a/(-a^2*x^2+1)^2-3/16/a/(-a^2*x^2+1)+1/4*x*arctanh(a*x)/(-a^2*x^2+1)^
2+3/8*x*arctanh(a*x)/(-a^2*x^2+1)+3/16*arctanh(a*x)^2/a
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6107, 6103, 267}

$$-\frac{3}{16a(1-a^2x^2)}-\frac{1}{16a(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{8(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4(1-a^2x^2)^2}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/16*1/(a*(1 - a^2*x^2)^2) - 3/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(4*
(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^2
)/(16*a)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
```

```
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{1}{16a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{4\,(1-a^2x^2)^2}+\frac{3}{4}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx$$
$$= -\frac{1}{16a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{4\,(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{8\,(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{16a}-\frac{1}{8}(3a)\int \frac{x}{(1-a$$
$$= -\frac{1}{16a\,(1-a^2x^2)^2}-\frac{3}{16a\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{4\,(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{8\,(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.69

$$\frac{-4+3a^2x^2+(10ax-6a^3x^3)\tanh^{-1}(ax)+3(-1+a^2x^2)^2\tanh^{-1}(ax)^2}{16a\,(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

```
[Out] (-4 + 3*a^2*x^2 + (10*a*x - 6*a^3*x^3)*ArcTanh[a*x] + 3*(-1 + a^2*x^2)^2*Ar
cTanh[a*x]^2)/(16*a*(-1 + a^2*x^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(84) = 168.
time = 0.77, size = 178, normalized size = 1.89

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{16}+\frac{3\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right.}{32}}{a}$
default	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{16}+\frac{3\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right.}{32}}{a}$
risch	$\frac{3\ln(ax+1)^2}{64a}-\frac{\left(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax+3\ln(-ax+1)\right)\ln(ax+1)}{32(a^2x^2-1)^2a}+\frac{3a^4x^4\ln(-ax+1)^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/16*arctanh(a*x)/(a*x-1)^2-3/16*arctanh(a*x)/(a*x-1)-3/16*arctanh(a*x
)*ln(a*x-1)-1/16*arctanh(a*x)/(a*x+1)^2-3/16*arctanh(a*x)/(a*x+1)+3/16*arct
```

```
anh(a*x)*ln(a*x+1)+3/32*ln(a*x-1)*ln(1/2*a*x+1/2)-3/64*ln(a*x-1)^2+3/32*(ln
(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-3/64*ln(a*x+1)^2-1/64/(a*x-1)^2+7
/64/(a*x-1)-1/64/(a*x+1)^2-7/64/(a*x+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(80) = 160$.
time = 0.26, size = 182, normalized size = 1.94

$$-\frac{1}{16}\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)+\frac{\left(12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^2+6\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)\log\left(ax-1\right)-3\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^2-16\right)a}{64\,(a^6x^4-2\,a^4x^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x) + 1/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 +
 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1)
- 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a/(a^6*x^4 - 2*a^4*x^2 +
 a^2)
```

Fricas [**A**]
time = 0.34, size = 97, normalized size = 1.03

$$\frac{12\,a^2x^2+3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-16}{64\,(a^5x^4-2\,a^3x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
- 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^5*x^4 - 2*a^3*x^
2 + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{atanh}\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

Mupad [B]
time = 1.37, size = 154, normalized size = 1.64

$$\frac{\frac{3\,a\,x^2}{2}-\frac{2}{a}}{8\,a^4\,x^4-16\,a^2\,x^2+8}-\ln\left(1-a\,x\right)\,\left(\frac{3\,\ln\left(a\,x+1\right)}{32\,a}+\frac{\frac{5\,x}{8}-\frac{3\,a^2\,x^3}{8}}{2\,a^4\,x^4-4\,a^2\,x^2+2}\right)+\frac{3\ln\left(a\,x+1\right)^2}{64\,a}+\frac{3\ln\left(1-a\,x\right)^2}{64\,a}+\frac{\ln\left(a\,x+1\right)\,\left(\frac{5\,x}{16\,a}-\frac{3\,a\,x^3}{16}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(a^2*x^2 - 1)^3,x)
```

```
[Out] ((3*a*x^2)/2 - 2/a)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 - a*x)*((3*log(a*x
 + 1))/(32*a) + ((5*x)/8 - (3*a^2*x^3)/8)/(2*a^4*x^4 - 4*a^2*x^2 + 2)) + (3
*log(a*x + 1)^2)/(64*a) + (3*log(1 - a*x)^2)/(64*a) + (log(a*x + 1)*((5*x)/
(16*a) - (3*a*x^3)/16))/(1/a - 2*a*x^2 + a^3*x^4)
```

3.306 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{ax}{16(1-a^2x^2)^2}-\frac{11ax}{32(1-a^2x^2)}-\frac{11}{32}\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{4(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)$$

```
[Out] -1/16*a*x/(-a^2*x^2+1)^2-11/32*a*x/(-a^2*x^2+1)-11/32*arctanh(a*x)+1/4*arct
anh(a*x)/(-a^2*x^2+1)^2+1/2*arctanh(a*x)/(-a^2*x^2+1)+1/2*arctanh(a*x)^2+ar
ctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6177, 6135, 6079, 2497, 6141, 205, 212}

$$-\frac{11ax}{32(1-a^2x^2)}-\frac{ax}{16(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{4(1-a^2x^2)^2}-\frac{1}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{11}{32}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] -1/16*(a*x)/(1 - a^2*x^2)^2 - (11*a*x)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x
])/32 + ArcTanh[a*x]/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(2*(1 - a^2*x^2)) +
 ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(
1 + a*x)]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^3}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx + \int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^2}\,dx \\
&= \frac{\tanh^{-1}(ax)}{4(1-a^2x^2)^2} - \frac{1}{4}a\int \frac{1}{(1-a^2x^2)^3}\,dx + a^2\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx \\
&= -\frac{ax}{16(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{2}\tanh^{-1}(ax)^2 - \frac{1}{16}(3a)\int \frac{1}{(1-a \cdots} \\
&= -\frac{ax}{16(1-a^2x^2)^2} - \frac{11ax}{32(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{2(1-a^2x^2)} + \frac{1}{2}\tanh^{-1}(ax)^2 + \cdots \\
&= -\frac{ax}{16(1-a^2x^2)^2} - \frac{11ax}{32(1-a^2x^2)} - \frac{11}{32}\tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)}{4(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{2(1-a^2x^2)} + \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.63

$$\frac{1}{128}\left(64\tanh^{-1}(ax)^2 + 4\tanh^{-1}(ax)\left(12\cosh\left(2\tanh^{-1}(ax)\right) + \cosh\left(4\tanh^{-1}(ax)\right) + 32\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right) - 64\text{PolyLog}\left(2, e^{-2\tanh^{-1}(ax)}\right) - 24\sinh\left(2\tanh^{-1}(ax)\right) - \sinh\left(4\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (64*ArcTanh[a*x]^2 + 4*ArcTanh[a*x]*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTa
nh[a*x]] + 32*Log[1 - E^(-2*ArcTanh[a*x])]) - 64*PolyLog[2, E^(-2*ArcTanh[a
*x])] - 24*Sinh[2*ArcTanh[a*x]] - Sinh[4*ArcTanh[a*x]])/128
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(115) = 230$.
time = 0.92, size = 234, normalized size = 1.81

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2} - \frac{5\operatorname{arctanh}(ax)}{16(ax-1)} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} + \operatorname{arctanh}(ax)\ln(ax) + \frac{\operatorname{arctanh}(ax)}{16(ax+1)^2} + \frac{5\operatorname{arctan}\cdots}{16(ax\cdots}$
default	$\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2} - \frac{5\operatorname{arctanh}(ax)}{16(ax-1)} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} + \operatorname{arctanh}(ax)\ln(ax) + \frac{\operatorname{arctanh}(ax)}{16(ax+1)^2} + \frac{5\operatorname{arctan}\cdots}{16(ax\cdots}$
risch	$\frac{11\ln(ax-1)}{128} - \frac{5\ln(ax+1)(ax+1)}{64(ax-1)} + \frac{1}{64ax-64} - \frac{\ln(ax+1)(ax+1)(ax-3)}{128(ax-1)^2} - \frac{\ln(ax+1)^2}{8} + \frac{\ln(ax+1)}{32(ax+1)^2} + \frac{1}{64(a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*arctanh(a*x)/(a*x-1)^2-5/16*arctanh(a*x)/(a*x-1)-1/2*arctanh(a*x)*ln(a
*x-1)+arctanh(a*x)*ln(a*x)+1/16*arctanh(a*x)/(a*x+1)^2+5/16*arctanh(a*x)/(a
```

```
*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x
)*ln(a*x+1)+1/2*dilog(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x
-1)^2+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/64
/(a*x-1)^2+11/64/(a*x-1)+11/64*ln(a*x-1)+1/64/(a*x+1)^2+11/64/(a*x+1)-11/64
*ln(a*x+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(110) = 220$.
time = 0.27, size = 268, normalized size = 2.08

$$\frac{1}{64}a\left(\frac{2\left(11a^3x^3+4\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)^2-8\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)\log\left(ax-1\right)-4\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)^2-13ax\right)}{a^5x^4-2a^3x^2+a}+\frac{32\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}-\frac{32\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)}{a}+\frac{32\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(ax\right)\right)}{a}-\frac{11\log\left(ax+1\right)}{a}+\frac{11\log\left(ax-1\right)}{a}\right)-\frac{1}{4}\left(\frac{2a^2x^2-3}{a^4x^4-2a^2x^2+1}+2\log\left(a^2x^2-1\right)-2\log\left(x^2\right)\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a*(2*(11*a^3*x^3 + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 - 8*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 4*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x - 1)^2 - 13*a*x)/(a^5*x^4 - 2*a^3*x^2 + a) + 32*(log(a*x - 1)*lo
g(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 32*(log(a*x + 1)*log(x) + dil
og(-a*x))/a + 32*(log(-a*x + 1)*log(x) + dilog(a*x))/a - 11*log(a*x + 1)/a
+ 11*log(a*x - 1)/a) - 1/4*((2*a^2*x^2 - 3)/(a^4*x^4 - 2*a^2*x^2 + 1) + 2*l
og(a^2*x^2 - 1) - 2*log(x^2))*arctanh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{atanh}\left(ax\right)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\mathrm{atanh}(a\,x)}{x\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)/(x*(a^2*x^2 - 1)^3), x)
```

3.307 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{16(1-a^2x^2)^2}-\frac{7a}{16(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2}+\frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)}+\frac{15}{16}a\tanh^{-1}(ax)^2+a\log($$

```
[Out] -1/16*a/(-a^2*x^2+1)^2-7/16*a/(-a^2*x^2+1)-arctanh(a*x)/x+1/4*a^2*x*arctanh
(a*x)/(-a^2*x^2+1)^2+7/8*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+15/16*a*arctanh(a*
x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6177, 6129, 6037, 272, 36, 29, 31, 6095, 6103, 267, 6107}

$$-\frac{7a}{16(1-a^2x^2)}-\frac{a}{16(1-a^2x^2)^2}-\frac{1}{2}a\log(1-a^2x^2)+\frac{7a^2x\tanh^{-1}(ax)}{8(1-a^2x^2)}+\frac{a^2x\tanh^{-1}(ax)}{4(1-a^2x^2)^2}+a\log(x)+\frac{15}{16}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -1/16*a/(1 - a^2*x^2)^2 - (7*a)/(16*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*
x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x])/(8*(1 - a^2*x^
2)) + (15*a*ArcTanh[a*x]^2)/16 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
```

```
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx+a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,d$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{7}{16}a\tanh^{-1}(ax)^2+a^2\int \frac{\text{ta}}{}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax}{8\left(1-a^2x^2\right)}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax}{8\left(1-a^2x^2\right)}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax}{8\left(1-a^2x^2\right)}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax}{8\left(1-a^2x^2\right)}$$

Mathematica [A]
time = 0.08, size = 94, normalized size = 0.76

$$\frac{1}{16}\left(-\frac{2(8-25a^2x^2+15a^4x^4)\tanh^{-1}(ax)}{x\left(-1+a^2x^2\right)^2}+15a\tanh^{-1}(ax)^2+a\left(\frac{-8+7a^2x^2}{\left(-1+a^2x^2\right)^2}+16\log(x)-8\log\left(1-a^2x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] ((-2*(8 - 25*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)^2) + 15*
a*ArcTanh[a*x]^2 + a*((-8 + 7*a^2*x^2)/(-1 + a^2*x^2)^2 + 16*Log[x] - 8*Log
[1 - a^2*x^2]))/16
```

Maple [A]
time = 0.81, size = 208, normalized size = 1.69

method	result
derivativedivides	$a\left(\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{7\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{15\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{7\operatorname{arctanh}(ax)}{16(ax+1)}+\right.$

default	$a\left(\frac{\operatorname{arctanh}(ax)}{16(ax-1)^{2}}-\frac{7\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{15\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^{2}}-\frac{7\operatorname{arctanh}(ax)}{16(ax+1)}+\right.$
risch	$\frac{15a\ln(ax+1)^{2}}{64}-\frac{(15x^{5}\ln(-ax+1)a^{5}+30a^{4}x^{4}-30a^{3}x^{3}\ln(-ax+1)-50a^{2}x^{2}+15ax\ln(-ax+1)+16)\ln(ax+1)}{32x(a^{2}x^{2}-1)^{2}}+\frac{15a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/16*arctanh(a*x)/(a*x-1)^2-7/16*arctanh(a*x)/(a*x-1)-15/16*arctanh(a*x)
*ln(a*x-1)-arctanh(a*x)/a/x-1/16*arctanh(a*x)/(a*x+1)^2-7/16*arctanh(a*x)/(
a*x+1)+15/16*arctanh(a*x)*ln(a*x+1)+15/32*ln(a*x-1)*ln(1/2*a*x+1/2)-15/64*l
n(a*x-1)^2+15/32*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-15/64*ln(a*x+
1)^2-1/64/(a*x-1)^2+15/64/(a*x-1)-1/2*ln(a*x-1)+ln(a*x)-1/64/(a*x+1)^2-15/6
4/(a*x+1)-1/2*ln(a*x+1))
```

Maxima [A]
time = 0.29, size = 204, normalized size = 1.66

$$\frac{1}{64}a\left(\frac{28a^{2}x^{2}-15(a^{4}x^{4}-2a^{2}x^{2}+1)\log(ax+1)^{2}+30(a^{4}x^{4}-2a^{2}x^{2}+1)\log(ax+1)\log(ax-1)-15(a^{4}x^{4}-2a^{2}x^{2}+1)\log(ax-1)^{2}-32}{a^{4}x^{4}-2a^{2}x^{2}+1}-32\log(ax+1)-32\log(ax-1)+64\log(x)\right)+\frac{1}{16}\left(15a\log(ax+1)-15a\log(ax-1)-\frac{2(15a^{4}x^{4}-25a^{2}x^{2}+8)}{a^{4}x^{5}-2a^{2}x^{3}+x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a*((28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2 +
 1)*log(a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 32*l
og(a*x - 1) + 64*log(x)) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1) - 2*
(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)
```

Fricas [A]
time = 0.34, size = 161, normalized size = 1.31

$$\frac{28a^{3}x^{3}+15(a^{5}x^{5}-2a^{3}x^{3}+ax)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-32ax-32(a^{5}x^{5}-2a^{3}x^{3}+ax)\log(a^{2}x^{2}-1)+64(a^{5}x^{5}-2a^{3}x^{3}+ax)\log(x)-4(15a^{4}x^{4}-25a^{2}x^{2}+8)\log\left(-\frac{ax+1}{ax-1}\right)}{64(a^{4}x^{5}-2a^{2}x^{3}+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(28*a^3*x^3 + 15*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))
^2 - 32*a*x - 32*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a^2*x^2 - 1) + 64*(a^5*x^5
 - 2*a^3*x^3 + a*x)*log(x) - 4*(15*a^4*x^4 - 25*a^2*x^2 + 8)*log(-(a*x + 1)
/(a*x - 1)))/(a^4*x^5 - 2*a^2*x^3 + x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(107) = 214$.
time = 1.67, size = 549, normalized size = 4.46

$$\begin{cases}\frac{16a^{5}x^{5}\log(x)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{16a^{5}x^{5}\log\left(x-\frac{1}{a}\right)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{15a^{5}x^{5}\operatorname{atanh}^{2}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{16a^{5}x^{5}\operatorname{atanh}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{30a^{4}x^{4}\operatorname{atanh}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{32a^{3}x^{3}\log(x)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{32a^{3}x^{3}\log\left(x-\frac{1}{a}\right)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{30a^{3}x^{3}\operatorname{atanh}^{2}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{32a^{3}x^{3}\operatorname{atanh}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{7a^{3}x^{3}}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{50a^{2}x^{2}\operatorname{atanh}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{16ax\log(x)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{16ax\log\left(x-\frac{1}{a}\right)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}+\frac{15ax\operatorname{atanh}^{2}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{16ax\operatorname{atanh}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{8ax}{16a^{4}x^{5}-32a^{2}x^{3}+16x}-\frac{16\operatorname{atanh}(ax)}{16a^{4}x^{5}-32a^{2}x^{3}+16x} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((16*a**5*x**5*log(x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a*
*5*x**5*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 15*a**5*x**5*at
anh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a**5*x**5*atanh(a*x)/
(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 30*a**4*x**4*atanh(a*x)/(16*a**4*x**
5 - 32*a**2*x**3 + 16*x) - 32*a**3*x**3*log(x)/(16*a**4*x**5 - 32*a**2*x**3
 + 16*x) + 32*a**3*x**3*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) -
 30*a**3*x**3*atanh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 32*a**3*
x**3*atanh(a*x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 7*a**3*x**3/(16*a**4
*x**5 - 32*a**2*x**3 + 16*x) + 50*a**2*x**2*atanh(a*x)/(16*a**4*x**5 - 32*a
**2*x**3 + 16*x) + 16*a*x*log(x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*
a*x*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 15*a*x*atanh(a*x)**
2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a*x*atanh(a*x)/(16*a**4*x**5 -
32*a**2*x**3 + 16*x) - 8*a*x/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*atan
h(a*x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x^2), x)
```

Mupad [B]
time = 1.39, size = 183, normalized size = 1.49

$$\frac{15\,a\ln\left(a\,x+1\right)^2}{64}-\frac{4\,a-\frac{7\,a^3\,x^2}{2}}{8\,a^4\,x^4-16\,a^2\,x^2+8}+\frac{15\,a\ln\left(1-a\,x\right)^2}{64}-\frac{a\,\ln\left(a^2\,x^2-1\right)}{2}+a\,\ln\left(x\right)+\ln\left(1-a\,x\right)\,\left(\frac{\frac{15\,a^4\,x^4}{8}-\frac{25\,a^2\,x^2}{8}+1}{2\,a^4\,x^5-4\,a^2\,x^3+2\,x}-\frac{15\,a\,\ln\left(a\,x+1\right)}{32}\right)-\frac{\ln\left(a\,x+1\right)\,\left(\frac{1}{2\,a}-\frac{25\,a\,x^2}{16}+\frac{15\,a^3\,x^4}{16}\right)}{\frac{x}{a}-2\,a\,x^3+a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)/(x^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] (15*a*log(a*x + 1)^2)/64 - (4*a - (7*a^3*x^2)/2)/(8*a^4*x^4 - 16*a^2*x^2 +
8) + (15*a*log(1 - a*x)^2)/64 - (a*log(a^2*x^2 - 1))/2 + a*log(x) + log(1 -
 a*x)*(((15*a^4*x^4)/8 - (25*a^2*x^2)/8 + 1)/(2*x - 4*a^2*x^3 + 2*a^4*x^5)
- (15*a*log(a*x + 1))/32) - (log(a*x + 1)*(1/(2*a) - (25*a*x^2)/16 + (15*a^
3*x^4)/16))/(x/a - 2*a*x^3 + a^3*x^5)
```

3.308 $\int \frac{x^3 \tanh^{-1}(ax)^2}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=127

$$\frac{x^4}{32(1-a^2x^2)^2}-\frac{3}{32a^4(1-a^2x^2)}-\frac{x^3\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{16a^3(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{32a^4}+\frac{x^4\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}$$

```
[Out] 1/32*x^4/(-a^2*x^2+1)^2-3/32/a^4/(-a^2*x^2+1)-1/8*x^3*arctanh(a*x)/a/(-a^2*
x^2+1)^2+3/16*x*arctanh(a*x)/a^3/(-a^2*x^2+1)-3/32*arctanh(a*x)^2/a^4+1/4*x
^4*arctanh(a*x)^2/(-a^2*x^2+1)^2
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6155, 6149, 6145, 6095}

$$-\frac{3\tanh^{-1}(ax)^2}{32a^4}+\frac{x^4}{32(1-a^2x^2)^2}+\frac{x^4\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}-\frac{x^3\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3}{32a^4(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{16a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] x^4/(32*(1 - a^2*x^2)^2) - 3/(32*a^4*(1 - a^2*x^2)) - (x^3*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(16*a^3*(1 - a^2*x^2)) - (3*ArcTan
h[a*x]^2)/(32*a^4) + (x^4*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6145

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]
 - Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x])
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q,
-5/2]
```

Rule 6149

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(q_), x_Symbol] :> Simp[(-b)*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)),
x] + (-Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*
```

```
(a + b*ArcTanh[c*x]), x], x] + Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a
 + b*ArcTanh[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c
^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{x^4 \tanh^{-1}(ax)^2}{4(1-a^2x^2)^2} - \frac{1}{2}a \int \frac{x^4 \tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx$$

$$= \frac{x^4}{32(1-a^2x^2)^2} - \frac{x^3 \tanh^{-1}(ax)}{8a(1-a^2x^2)^2} + \frac{x^4 \tanh^{-1}(ax)^2}{4(1-a^2x^2)^2} + \frac{3\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{8a}$$

$$= \frac{x^4}{32(1-a^2x^2)^2} - \frac{3}{32a^4(1-a^2x^2)} - \frac{x^3 \tanh^{-1}(ax)}{8a(1-a^2x^2)^2} + \frac{3x \tanh^{-1}(ax)}{16a^3(1-a^2x^2)} + \frac{x^4 \tanh^{-1}(a}{4(1-a^2x}$$

$$= \frac{x^4}{32(1-a^2x^2)^2} - \frac{3}{32a^4(1-a^2x^2)} - \frac{x^3 \tanh^{-1}(ax)}{8a(1-a^2x^2)^2} + \frac{3x \tanh^{-1}(ax)}{16a^3(1-a^2x^2)} - \frac{3\tanh^{-1}(a}{32a^4}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.56

$$\frac{-4+5a^2x^2+(6ax-10a^3x^3)\tanh^{-1}(ax)+(-3+6a^2x^2+5a^4x^4)\tanh^{-1}(ax)^2}{32a^4(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-4 + 5*a^2*x^2 + (6*a*x - 10*a^3*x^3)*ArcTanh[a*x] + (-3 + 6*a^2*x^2 + 5*a
^4*x^4)*ArcTanh[a*x]^2)/(32*a^4*(-1 + a^2*x^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(115) = 230.
time = 1.58, size = 238, normalized size = 1.87

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)^2}{16(ax+1)}+\frac{\operatorname{arctanh}(ax)^2}{16(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)^2}{16(ax-1)}+\frac{\operatorname{arctanh}(ax)}{32(ax+1)^2}-\frac{5\operatorname{arctanh}(ax)}{32(ax+1)}+\frac{5\operatorname{arctanh}(ax)\ln(ax+1)}{32}-\frac{\operatorname{arctanh}(}{32(ax-1}}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)^2}{16(ax+1)}+\frac{\operatorname{arctanh}(ax)^2}{16(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)^2}{16(ax-1)}+\frac{\operatorname{arctanh}(ax)}{32(ax+1)^2}-\frac{5\operatorname{arctanh}(ax)}{32(ax+1)}+\frac{5\operatorname{arctanh}(ax)\ln(ax+1)}{32}-\frac{\operatorname{arctanh}(}{32(ax-1}}{}$
risch	$\frac{(5a^4x^4+6a^2x^2-3)\ln(ax+1)^2}{128a^4(a^2x^2-1)^2}-\frac{(5x^4\ln(-ax+1)a^4+10a^3x^3+6x^2\ln(-ax+1)a^2-6ax-3\ln(-ax+1))\ln(ax+1)}{64a^4(ax+1)(ax-1)(a^2x^2-1)}+5$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/16*arctanh(a*x)^2/(a*x+1)^2-3/16*arctanh(a*x)^2/(a*x+1)+1/16*arcta
nh(a*x)^2/(a*x-1)^2+3/16*arctanh(a*x)^2/(a*x-1)+1/32*arctanh(a*x)/(a*x+1)^2
-5/32*arctanh(a*x)/(a*x+1)+5/32*arctanh(a*x)*ln(a*x+1)-1/32*arctanh(a*x)/(a
*x-1)^2-5/32*arctanh(a*x)/(a*x-1)-5/32*arctanh(a*x)*ln(a*x-1)-5/128*ln(a*x-
1)^2+5/64*ln(a*x-1)*ln(1/2*a*x+1/2)+5/64*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/
2*a*x+1/2)-5/128*ln(a*x+1)^2+1/128/(a*x+1)^2-9/128/(a*x+1)+1/128/(a*x-1)^2+
9/128/(a*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(110) = 220$.
time = 0.27, size = 226, normalized size = 1.78

$$-\frac{1}{32}\,a\left(\frac{2\,(5\,a^2x^3-3\,x)}{a^8x^4-2\,a^6x^2+a^4}-\frac{5\,\log(ax+1)}{a^5}+\frac{5\,\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)+\frac{\left(20\,a^2x^2-5\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2+10\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1)-5\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^2-16\right)a^2}{128\,(a^{10}x^4-2\,a^8x^2+a^6)}+\frac{(2\,a^2x^2-1)\operatorname{artanh}(ax)^2}{4\,(a^8x^4-2\,a^6x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/32*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x) + 1/128*(20*a^2*x^2 - 5*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x + 1)^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log
(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a^2/(a^10*x^4
- 2*a^8*x^2 + a^6) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/(a^8*x^4 - 2*a^6*x^
2 + a^4)
```

Fricas [A]
time = 0.36, size = 99, normalized size = 0.78

$$\frac{20\,a^2x^2+(5\,a^4x^4+6\,a^2x^2-3)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,(5\,a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-16}{128\,(a^8x^4-2\,a^6x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*a^2*x^2 + (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1))^2
 - 4*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^8*x^4 - 2*a^6*x
^2 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 \operatorname{atanh}^2(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**3*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(110) = 220.
time = 0.40, size = 250, normalized size = 1.97

$$\frac{1}{512}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{4(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+2\left(\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}-\frac{(ax+1)^2}{(ax-1)^2a^5}-\frac{8(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{(ax-1)^2\left(\frac{16(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{16(ax+1)}{(ax-1)a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/512*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (a*x
+ 1)^2/((a*x - 1)^2*a^5) + 4*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/(a*x
 - 1))^2 + 2*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) - (
a*x + 1)^2/((a*x - 1)^2*a^5) - 8*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/
(a*x - 1)) + (a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (
a*x + 1)^2/((a*x - 1)^2*a^5) + 16*(a*x + 1)/((a*x - 1)*a^5))*a
```

Mupad [B]
time = 1.53, size = 372, normalized size = 2.93

$$\ln(a\,x+1)^2\left(\frac{5}{128\,a^4}-\frac{\frac{1}{16\,a^5}-\frac{x^2}{8\,a^3}}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}\right)-\ln(1-a\,x)\left(\frac{\frac{3\,x}{8}+a\,x^2-\frac{3}{4\,a}-\frac{5\,a^2\,x^3}{8}}{8\,a^7\,x^4-16\,a^5\,x^2+8\,a^3}+\frac{\frac{3\,x}{8}-a\,x^2+\frac{3}{4\,a}-\frac{5\,a^2\,x^3}{8}}{8\,a^7\,x^4-16\,a^5\,x^2+8\,a^3}-\ln(a\,x+1)\left(\frac{\frac{1}{4\,a^4}-\frac{x^2}{2\,a^2}}{2\,a^4\,x^4-4\,a^2\,x^2+2}-\frac{5\,(a^4\,x^4-2\,a^2\,x^2+1)}{32\,a^4\,(2\,a^4\,x^4-4\,a^2\,x^2+2)}\right)\right)-\ln(1-a\,x)^2\left(\frac{\frac{1}{4\,a^4}-\frac{x^2}{2\,a^2}}{4\,a^4\,x^4-8\,a^2\,x^2+4}-\frac{5}{128\,a^4}\right)-\frac{\frac{2}{a^2}-\frac{5\,x^2}{2}}{16\,a^6\,x^4-32\,a^4\,x^2+16\,a^2}+\frac{\ln(a\,x+1)\left(\frac{3\,x}{32\,a^4}-\frac{5\,x^3}{32\,a^2}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(a*x + 1)^2*(5/(128*a^4) - (1/(16*a^5) - x^2/(8*a^3))/(1/a - 2*a*x^2 + a
^3*x^4)) - log(1 - a*x)*(((3*x)/8 + a*x^2 - 3/(4*a) - (5*a^2*x^3)/8)/(8*a^3
 - 16*a^5*x^2 + 8*a^7*x^4) + ((3*x)/8 - a*x^2 + 3/(4*a) - (5*a^2*x^3)/8)/(8
*a^3 - 16*a^5*x^2 + 8*a^7*x^4) - log(a*x + 1)*((1/(4*a^4) - x^2/(2*a^2))/(2
*a^4*x^4 - 4*a^2*x^2 + 2) - (5*(a^4*x^4 - 2*a^2*x^2 + 1))/(32*a^4*(2*a^4*x^
4 - 4*a^2*x^2 + 2)))) - log(1 - a*x)^2*((1/(4*a^4) - x^2/(2*a^2))/(4*a^4*x^
4 - 8*a^2*x^2 + 4) - 5/(128*a^4)) - (2/a^2 - (5*x^2)/2)/(16*a^2 - 32*a^4*x^
2 + 16*a^6*x^4) + (log(a*x + 1)*((3*x)/(32*a^4) - (5*x^3)/(32*a^2)))/(1/a -
 2*a*x^2 + a^3*x^4)
```

3.309 $\int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=163

$$\frac{x}{32a^2\,(1-a^2x^2)^2}-\frac{x}{64a^2\,(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{64a^3}-\frac{\tanh^{-1}(ax)}{8a^3\,(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{8a^3\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4a^2\,(1-a^2x^2)^2}-\frac{x\,\mathrm{tan}}{8a^2\,(1}$$

```
[Out] 1/32*x/a^2/(-a^2*x^2+1)^2-1/64*x/a^2/(-a^2*x^2+1)-1/64*arctanh(a*x)/a^3-1/8
*arctanh(a*x)/a^3/(-a^2*x^2+1)^2+1/8*arctanh(a*x)/a^3/(-a^2*x^2+1)+1/4*x*ar
ctanh(a*x)^2/a^2/(-a^2*x^2+1)^2-1/8*x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)-1/24*
arctanh(a*x)^3/a^3
```

Rubi [A]

time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6175, 6103, 6141, 205, 212, 6111}

$$-\frac{\tanh^{-1}(ax)^3}{24a^3}-\frac{\tanh^{-1}(ax)}{64a^3}-\frac{x}{64a^2\,(1-a^2x^2)}+\frac{x}{32a^2\,(1-a^2x^2)^2}-\frac{x\tanh^{-1}(ax)^2}{8a^2\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4a^2\,(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{8a^3\,(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a^3\,(1-a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] x/(32*a^2*(1 - a^2*x^2)^2) - x/(64*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]/(64*a^
3) - ArcTanh[a*x]/(8*a^3*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(8*a^3*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x]^2)/(8*a^
2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(24*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
```

```
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{(1-a^2x^2)^3}}{8a^2}$$

$$= \frac{x}{32a^2(1-a^2x^2)^2} - \frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}}{8a^2(1-}$$

$$= \frac{x}{32a^2(1-a^2x^2)^2} - \frac{13x}{64a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)} + \frac{x\tanh^{-}}{4a^2(1-}$$

$$= \frac{x}{32a^2(1-a^2x^2)^2} - \frac{x}{64a^2(1-a^2x^2)} - \frac{13\tanh^{-1}(ax)}{64a^3} - \frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2} + \frac{\tanh^{-1}}{8a^3(1-}$$

$$= \frac{x}{32a^2(1-a^2x^2)^2} - \frac{x}{64a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{64a^3} - \frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2} + \frac{\tanh^{-1}(a}{8a^3(1-a^2}$$

Mathematica [A]
time = 0.05, size = 121, normalized size = 0.74

$$\frac{6ax(1+a^2x^2) - 48a^2x^2\tanh^{-1}(ax) + 48(ax+a^3x^3)\tanh^{-1}(ax)^2 - 16(-1+a^2x^2)^2\tanh^{-1}(ax)^3 + 3(-1+a^2x^2)^2\log(1-ax) - 3(-1+a^2x^2)^2\log(1+ax)}{384a^3(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (6*a*x*(1 + a^2*x^2) - 48*a^2*x^2*ArcTanh[a*x] + 48*(a*x + a^3*x^3)*ArcTanh
[a*x]^2 - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3 + 3*(-1 + a^2*x^2)^2*Log[1 - a
*x] - 3*(-1 + a^2*x^2)^2*Log[1 + a*x])/(384*a^3*(-1 + a^2*x^2)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 237.51, size = 2028, normalized size = 12.44

method	result
risch	$-\frac{\ln(ax+1)^3}{192a^3} + \frac{(x^4\ln(-ax+1)a^4+2a^3x^3-2x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1))\ln(ax+1)^2}{64a^3(a^2x^2-1)^2} - \frac{(a^4x^4\ln(-ax+1)^2+}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/16*arctanh(a*x)^2/(a*x+1)^2+1/16*arctanh(a*x)^2/(a*x+1)-1/16*arct
anh(a*x)^2*ln(a*x+1)+1/16*arctanh(a*x)^2/(a*x-1)^2+1/16*arctanh(a*x)^2/(a*x
-1)+1/16*arctanh(a*x)^2*ln(a*x-1)+1/8*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1
)^(1/2))+1/192*(16*arctanh(a*x)^3*a^2*x^2-24*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))^2*arctanh(a*x)^2*a^2*x^2+6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1))^3-12*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1
2*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+6*I*Pi*arctanh(a
*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+3*a*x+12*I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*a^2*x^2+6
*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*
x)^2*a^4*x^4-6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(
a*x)^2*a^4*x^4-3*a^4*x^4*arctanh(a*x)-8*arctanh(a*x)^3-18*a^2*x^2*arctanh(a
*x)-3*arctanh(a*x)+24*I*Pi*arctanh(a*x)^2*a^2*x^2-6*I*Pi*arctanh(a*x)^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^2+6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1))+12*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+6*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+
1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))^2+3*a^3*x^3-8*arctanh(a*x)^3*a^4*x^4-12*I*Pi*arctanh(a*x)^2*a^4*x^4-12*
I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arc
tanh(a*x)^2*a^2*x^2-24*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+
1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*a^2*x^2+6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+
1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^4*x^4+12*I*Pi*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*
x)^2*a^4*x^4+12*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2-12*I*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))*arctanh(a*x)^2*a^2*x^2-12*I*Pi*arctanh(a*x)^2-6*I*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*arctanh(a*x)^2*a^4*x^4+6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*a^4*x^4+6
*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^4*x^4-12*I*Pi*csgn(I
/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^4*x^4+12*I*Pi*csgn(I/((a*x+
1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^4*x^4-12*I*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2-12*I*Pi*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^2*x^2+24*I*Pi*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2-6*I*Pi*arctanh(a*x)^2*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)))/(a*x+1)^2/(a*x-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(141) = 282$.

time = 0.29, size = 388, normalized size = 2.38

$$\frac{1}{16}\left(\frac{2\,(a^2x^3+x)}{a^6x^4-2\,a^4x^2+a^2}-\frac{\log\left(ax+1\right)}{a^3}+\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)^2+\frac{\left(6\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^3+6\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^2\log\left(ax-1\right)+2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^3+6\,ax-3\,(a^4x^4-2\,a^2x^2+2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^2+1)\log\left(ax+1\right)+3\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)\right)a^2}{384\,(a^9x^4-2\,a^7x^2+a^5)}-\frac{\left(4\,a^2x^2-(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^2+2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)\log\left(ax-1\right)-(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^2\right)a\operatorname{artanh}\left(ax\right)}{32\,(a^8x^4-2\,a^6x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x)^2 + 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1)
 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*
x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^9*x^4 - 2*a^7*x^2 + a^5) - 1/32*
(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)
^2)*a*arctanh(a*x)/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Fricas [A]
time = 0.42, size = 136, normalized size = 0.83

$$\frac{6\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,(a^3x^3+ax)\log\left(-\frac{ax+1}{ax-1}\right)^2+6\,ax-3\,(a^4x^4+6\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{384\,(a^7x^4-2\,a^5x^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^3
+ 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 6*a*x - 3*(a^4*x^4 + 6*a
^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2\operatorname{atanh}^2\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

Mupad [B]
time = 1.97, size = 350, normalized size = 2.15

$$\ln\left(1-a\,x\right)\,\left(\frac{\frac{3\,a\,x^3}{2}-\frac{x}{2\,a}+x^2}{32\,a^5\,x^4-64\,a^3\,x^2+32\,a}+\frac{\frac{x}{2\,a}-\frac{3\,a\,x^3}{2}+x^2}{32\,a^5\,x^4-64\,a^3\,x^2+32\,a}+\frac{{\ln\left(a\,x+1\right)}^2}{64\,a^3}-\frac{\ln\left(a\,x+1\right)\,\left(2\,a^2\,x^3+2\,x\right)}{32\,a^6\,x^4-64\,a^4\,x^2+32\,a^2}\right)+\frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{8\,a^4\,x^4-16\,a^2\,x^2+8}-{\ln\left(1-a\,x\right)}^2\,\left(\frac{\ln\left(a\,x+1\right)}{64\,a^3}-\frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{4\,a^4\,x^4-8\,a^2\,x^2+4}\right)-\frac{{\ln\left(a\,x+1\right)}^3}{192\,a^3}+\frac{{\ln\left(1-a\,x\right)}^3}{192\,a^3}+\frac{{\ln\left(a\,x+1\right)}^2\,\left(\frac{x}{32\,a^3}+\frac{x^3}{32\,a}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}-\frac{x^2\,\ln\left(a\,x+1\right)}{16\,a^2\,\left(\frac{1}{a}-2\,a\,x^2+a^3\,x^4\right)}+\frac{\mathrm{atan}\left(a\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{64\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(1 - a*x)*(((3*a*x^3)/2 - x/(2*a) + x^2)/(32*a - 64*a^3*x^2 + 32*a^5*x^4
) + (x/(2*a) - (3*a*x^3)/2 + x^2)/(32*a - 64*a^3*x^2 + 32*a^5*x^4) + log(a*
x + 1)^2/(64*a^3) - (log(a*x + 1)*(2*x + 2*a^2*x^3))/(32*a^2 - 64*a^4*x^2 +
 32*a^6*x^4)) + (x/(8*a^2) + x^3/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 -
a*x)^2*(log(a*x + 1)/(64*a^3) - (x/(8*a^2) + x^3/8)/(4*a^4*x^4 - 8*a^2*x^2
+ 4)) - log(a*x + 1)^3/(192*a^3) + log(1 - a*x)^3/(192*a^3) + (atan(a*x*1i)
*1i)/(64*a^3) + (log(a*x + 1)^2*(x/(32*a^3) + x^3/(32*a)))/(1/a - 2*a*x^2 +
 a^3*x^4) - (x^2*log(a*x + 1))/(16*a^2*(1/a - 2*a*x^2 + a^3*x^4))
```

3.310 $\int \frac{x \tanh^{-1}(ax)^2}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=125

$$\frac{1}{32a^2(1-a^2x^2)^2}+\frac{3}{32a^2(1-a^2x^2)}-\frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3x\tanh^{-1}(ax)}{16a(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{32a^2}+\frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2}$$

```
[Out] 1/32/a^2/(-a^2*x^2+1)^2+3/32/a^2/(-a^2*x^2+1)-1/8*x*arctanh(a*x)/a/(-a^2*x^
2+1)^2-3/16*x*arctanh(a*x)/a/(-a^2*x^2+1)-3/32*arctanh(a*x)^2/a^2+1/4*arcta
nh(a*x)^2/a^2/(-a^2*x^2+1)^2
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6141, 6107, 6103, 267}

$$\frac{3}{32a^2(1-a^2x^2)}+\frac{1}{32a^2(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{3x\tanh^{-1}(ax)}{16a(1-a^2x^2)}-\frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)^2}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] 1/(32*a^2*(1 - a^2*x^2)^2) + 3/(32*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(16*a*(1 - a^2*x^2)) - (3*ArcTanh[
a*x]^2)/(32*a^2) + ArcTanh[a*x]^2/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
```

```
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} - \frac{\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx}{2a}$$
$$= \frac{1}{32a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx}{8a}$$
$$= \frac{1}{32a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)}{16a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)^2}{32a^2} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x}$$
$$= \frac{1}{32a^2(1-a^2x^2)^2} + \frac{3}{32a^2(1-a^2x^2)} - \frac{x\tanh^{-1}(ax)}{8a(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)}{16a(1-a^2x^2)} - \frac{3\tanh^{-1}(a}{32a^2}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.57

$$\frac{4-3a^2x^2+2ax(-5+3a^2x^2)\tanh^{-1}(ax)+(5+6a^2x^2-3a^4x^4)\tanh^{-1}(ax)^2}{32a^2(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (4 - 3*a^2*x^2 + 2*a*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x] + (5 + 6*a^2*x^2 - 3*a
^4*x^4)*ArcTanh[a*x]^2)/(32*a^2*(-1 + a^2*x^2)^2)
```

Maple [A]
time = 0.78, size = 197, normalized size = 1.58

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2}{4\left(a^2x^2-1\right)^2}+\frac{\operatorname{arctanh}(ax)}{32(ax+1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax+1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{32}-\frac{\operatorname{arctanh}(ax)}{32(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax-1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{32}-\frac{3\ln(a}{}}{}$

default	$\frac{\frac{\operatorname{arctanh}(ax)^2}{4\left(a^2x^2-1\right)^2}+\frac{\operatorname{arctanh}(ax)}{32(ax+1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax+1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{32}-\frac{\operatorname{arctanh}(ax)}{32(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax-1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{32}-\frac{3\ln}{}}{}$
risch	$-\frac{(3a^4x^4-6a^2x^2-5)\ln(ax+1)^2}{128a^2(ax+1)(ax-1)(a^2x^2-1)}+\frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax-5\ln(-ax+1))\ln(ax+1)}{64a^2(ax+1)(ax-1)(a^2x^2-1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4/(a^2*x^2-1)^2*arctanh(a*x)^2+1/32*arctanh(a*x)/(a*x+1)^2+3/32*ar
ctanh(a*x)/(a*x+1)-3/32*arctanh(a*x)*ln(a*x+1)-1/32*arctanh(a*x)/(a*x-1)^2+
3/32*arctanh(a*x)/(a*x-1)+3/32*arctanh(a*x)*ln(a*x-1)-3/64*ln(a*x-1)*ln(1/2
*a*x+1/2)+3/128*ln(a*x-1)^2-3/64*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/
2)+3/128*ln(a*x+1)^2+1/128/(a*x-1)^2-7/128/(a*x-1)+1/128/(a*x+1)^2+7/128/(a
*x+1))
```

Maxima [A]
time = 0.26, size = 206, normalized size = 1.65

$$\frac{\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}\right)\operatorname{artanh}(ax)}{32\,a}-\frac{12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2+6\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1)-3\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^2-16}{128\,(a^6x^4-2\,a^4x^2+a^2)}+\frac{\operatorname{artanh}(ax)^2}{4\,(a^2x^2-1)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)*arctanh(a*x)/a - 1/128*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1
) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)/(a^6*x^4 - 2*a^4*x^2 +
 a^2) + 1/4*arctanh(a*x)^2/((a^2*x^2 - 1)^2*a^2)
```

Fricas [A]
time = 0.36, size = 99, normalized size = 0.79

$$-\frac{12\,a^2x^2+(3\,a^4x^4-6\,a^2x^2-5)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-16}{128\,(a^6x^4-2\,a^4x^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*a^2*x^2 + (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))^
2 - 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^6*x^4 - 2*a^4*
x^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(108) = 216.

time = 0.41, size = 251, normalized size = 2.01

$$-\frac{1}{512}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{4(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+2\left(\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}+\frac{(ax+1)^2}{(ax-1)^2a^3}-\frac{8(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{(ax-1)^2\left(\frac{16(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{16(ax+1)}{(ax-1)a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/512*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - (a*x
 + 1)^2/((a*x - 1)^2*a^3) + 4*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)/(a*
x - 1))^2 + 2*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) +
(a*x + 1)^2/((a*x - 1)^2*a^3) - 8*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)
/(a*x - 1)) + (a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) -
(a*x + 1)^2/((a*x - 1)^2*a^3) + 16*(a*x + 1)/((a*x - 1)*a^3))*a
```

Mupad [B]

time = 1.48, size = 319, normalized size = 2.55

$$\ln(ax+1)^2\left(\frac{1}{16a^3\left(\frac{1}{a}-2ax^2+a^3x^4\right)}-\frac{3}{128a^2}\right)-\ln(1-ax)^2\left(\frac{3}{128a^2}-\frac{1}{4a^2\left(4a^4x^4-8a^2x^2+4\right)}\right)-\ln(1-ax)\left(\frac{\frac{1}{4a}-\frac{5x}{8}+\frac{3a^2x^3}{8}}{8a^5x^4-16a^3x^2+8a}-\frac{\frac{5x}{8}+\frac{1}{4a}-\frac{3a^2x^3}{8}}{8a^5x^4-16a^3x^2+8a}+\ln(ax+1)\left(\frac{1}{4a^2\left(2a^4x^4-4a^2x^2+2\right)}-\frac{3\left(a^4x^4-2a^2x^2+1\right)}{32a^2\left(2a^4x^4-4a^2x^2+2\right)}\right)\right)+\frac{\frac{2}{a^2}-\frac{3x^2}{2}}{16a^4x^4-32a^2x^2+16}-\frac{\ln(ax+1)\left(\frac{5x}{32a^2}-\frac{3x^3}{32}\right)}{\frac{1}{a}-2ax^2+a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

```
[Out] log(a*x + 1)^2*(1/(16*a^3*(1/a - 2*a*x^2 + a^3*x^4)) - 3/(128*a^2)) - log(1
 - a*x)^2*(3/(128*a^2) - 1/(4*a^2*(4*a^4*x^4 - 8*a^2*x^2 + 4))) - log(1 - a
*x)*((1/(4*a) - (5*x)/8 + (3*a^2*x^3)/8)/(8*a - 16*a^3*x^2 + 8*a^5*x^4) - (
(5*x)/8 + 1/(4*a) - (3*a^2*x^3)/8)/(8*a - 16*a^3*x^2 + 8*a^5*x^4) + log(a*x
 + 1)*(1/(4*a^2*(2*a^4*x^4 - 4*a^2*x^2 + 2)) - (3*(a^4*x^4 - 2*a^2*x^2 + 1)
)/(32*a^2*(2*a^4*x^4 - 4*a^2*x^2 + 2)))) + (2/a^2 - (3*x^2)/2)/(16*a^4*x^4
- 32*a^2*x^2 + 16) - (log(a*x + 1)*((5*x)/(32*a^2) - (3*x^3)/32))/(1/a - 2*
a*x^2 + a^3*x^4)
```

3.311 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=151

$$\frac{x}{32(1-a^2x^2)^2}+\frac{15x}{64(1-a^2x^2)}+\frac{15\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}+\frac{3x\tanh^{-1}}{8(1-a^2}$$

```
[Out] 1/32*x/(-a^2*x^2+1)^2+15/64*x/(-a^2*x^2+1)+15/64*arctanh(a*x)/a-1/8*arctanh
(a*x)/a/(-a^2*x^2+1)^2-3/8*arctanh(a*x)/a/(-a^2*x^2+1)+1/4*x*arctanh(a*x)^2
/(-a^2*x^2+1)^2+3/8*x*arctanh(a*x)^2/(-a^2*x^2+1)+1/8*arctanh(a*x)^3/a
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6111, 6103, 6141, 205, 212}

$$\frac{15x}{64(1-a^2x^2)}+\frac{x}{32(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^2}{8(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^3}{8a}+\frac{15\tanh^{-1}(ax)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

```
[Out] x/(32*(1 - a^2*x^2)^2) + (15*x)/(64*(1 - a^2*x^2)) + (15*ArcTanh[a*x])/(64*
a) - ArcTanh[a*x]/(8*a*(1 - a^2*x^2)^2) - (3*ArcTanh[a*x])/(8*a*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x]^2)/(8*(1 -
 a^2*x^2)) + ArcTanh[a*x]^3/(8*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
```

```
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx &= -\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}+\frac{1}{8}\int\frac{1}{(1-a^2x^2)^3}\,dx+\frac{3}{4}\int\frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx\\ &= \frac{x}{32(1-a^2x^2)^2}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^2}{8(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{8a}+\\ &= \frac{x}{32(1-a^2x^2)^2}+\frac{3x}{64(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}+\\ &= \frac{x}{32(1-a^2x^2)^2}+\frac{15x}{64(1-a^2x^2)}+\frac{3\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}+\\ &= \frac{x}{32(1-a^2x^2)^2}+\frac{15x}{64(1-a^2x^2)}+\frac{15\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}+\end{aligned}$$

Mathematica [A]
time = 0.05, size = 127, normalized size = 0.84

$$\frac{1}{128}\left(\frac{4x}{(-1+a^2x^2)^2}-\frac{30x}{-1+a^2x^2}+\frac{16(-4+3a^2x^2)\tanh^{-1}(ax)}{a(-1+a^2x^2)^2}-\frac{16x(-5+3a^2x^2)\tanh^{-1}(ax)^2}{(-1+a^2x^2)^2}+\frac{16\tanh^{-1}(ax)^3}{a}-\frac{15\log(1-ax)}{a}+\frac{15\log(1+ax)}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

```
[Out] ((4*x)/(-1 + a^2*x^2)^2 - (30*x)/(-1 + a^2*x^2) + (16*(-4 + 3*a^2*x^2)*ArcT
anh[a*x])/(a*(-1 + a^2*x^2)^2) - (16*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x]^2)/(-1
 + a^2*x^2)^2 + (16*ArcTanh[a*x]^3)/a - (15*Log[1 - a*x])/a + (15*Log[1 + a
*x])/a)/128
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 257.52, size = 2028, normalized size = 13.43

method	result
risch	$\frac{\ln(ax+1)^3}{64a} - \frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax+3\ln(-ax+1))\ln(ax+1)^2}{64(a^2x^2-1)^2 a} + \frac{\left(3a^4x^4\ln(-ax+1)\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/16*arctanh(a*x)^2/(a*x-1)^2-3/16*arctanh(a*x)^2/(a*x-1)-3/16*arctanh
(a*x)^2*ln(a*x-1)-1/16*arctanh(a*x)^2/(a*x+1)^2-3/16*arctanh(a*x)^2/(a*x+1)
+3/16*arctanh(a*x)^2*ln(a*x+1)-3/8*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(
1/2))-1/64*(16*arctanh(a*x)^3*a^2*x^2-24*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*arctanh(a*x)^2*a^2*x^2+6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*
x^2-1))^3-12*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+12*I*
Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+6*I*Pi*arctanh(a*x)^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-17*a*x+12*I*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*a^2*x^2+6*I*
Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^
2*a^4*x^4-6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x
)^2*a^4*x^4-15*a^4*x^4*arctanh(a*x)-8*arctanh(a*x)^3+6*a^2*x^2*arctanh(a*x)
+17*arctanh(a*x)+24*I*Pi*arctanh(a*x)^2*a^2*x^2-6*I*Pi*arctanh(a*x)^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2+6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*
(a*x+1)^2/(a^2*x^2-1))+12*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(
1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+6*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)
^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^2+15*a^3*x^3-8*arctanh(a*x)^3*a^4*x^4-12*I*Pi*arctanh(a*x)^2*a^4*x^4-12*I
*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arct
anh(a*x)^2*a^2*x^2-24*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*a^2*x^2+6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1
)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^4*x^4+12*I*Pi*csg
n(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x
```

```
)^2*a^4*x^4+12*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2-12*I*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*arctanh(a*x)^2*a^2*x^2-12*I*Pi*arctanh(a*x)^2-6*I*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*arctanh(a*x)^2*a^4*x^4+6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*a^4*x^4+6*
I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^4*x^4-12*I*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^4*x^4+12*I*Pi*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^4*x^4-12*I*Pi*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2-12*I*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^2*x^2+24*I*Pi*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2-6*I*Pi*arctanh(a*x)^2*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)))/(a*x+1)^2/(a*x-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(129) = 258$.
time = 0.28, size = 392, normalized size = 2.60

$$-\frac{1}{16}\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(30\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^3+6\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^2\log\left(ax-1\right)+2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^3-34\,ax-3\,(5\,a^4x^4-10\,a^2x^2+2\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^2+5)\log\left(ax+1\right)+15\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)\right)a^2}{128\,(a^7x^4-2\,a^5x^2+a^3)}+\frac{\left(12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^2+6\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)\log\left(ax-1\right)-3\,(a^4x^4-2\,a^2x^2+1)\log\left(ax-1\right)^2-16\right)a\,\operatorname{artanh}\left(ax\right)}{32\,(a^6x^4-2\,a^4x^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^
2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x
- 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 -
 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)
+ 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2 + a^3
) + 1/32*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

Fricas [A]
time = 0.43, size = 137, normalized size = 0.91

$$-\frac{30\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^3+4\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-34\,ax-(15\,a^4x^4-6\,a^2x^2-17)\log\left(-\frac{ax+1}{ax-1}\right)}{128\,(a^5x^4-2\,a^3x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^
3 + 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 34*a*x - (15*a^4*x^
4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

Mupad [B]
time = 2.06, size = 358, normalized size = 2.37

$$\frac{\frac{17x}{8}-\frac{15a^2x^3}{8}}{8a^4x^4-16a^2x^2+8}-\ln(1-ax)\left(\frac{3\ln(ax+1)^2}{64a}-\frac{\frac{7x}{2}-3ax^2+\frac{4}{a}-\frac{5a^2x^3}{2}}{32a^4x^4-64a^2x^2+32}+\frac{\frac{7x}{2}+3ax^2-\frac{4}{a}-\frac{5a^2x^3}{2}}{32a^4x^4-64a^2x^2+32}+\frac{\ln(ax+1)\,(10x-6a^2x^3)}{32a^4x^4-64a^2x^2+32}\right)+\ln(1-ax)^2\left(\frac{3\ln(ax+1)}{64a}+\frac{\frac{5x}{8}-\frac{3a^2x^3}{8}}{4a^4x^4-8a^2x^2+4}\right)+\frac{\ln(ax+1)^3}{64a}-\frac{\ln(1-ax)^3}{64a}-\frac{\ln(ax+1)\left(\frac{1}{4a^2}-\frac{3x^2}{16}\right)}{\frac{1}{a}-2ax^2+a^3x^4}+\frac{\ln(ax+1)^2\left(\frac{5x}{32a}-\frac{3ax^3}{32}\right)}{\frac{1}{a}-2ax^2+a^3x^4}-\frac{\operatorname{atan}(ax\,1\mathrm{i})\,15\mathrm{i}}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(a^2*x^2 - 1)^3,x)
```

```
[Out] ((17*x)/8 - (15*a^2*x^3)/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 - a*x)*((3
*log(a*x + 1)^2)/(64*a) - ((7*x)/2 - 3*a*x^2 + 4/a - (5*a^2*x^3)/2)/(32*a^4
*x^4 - 64*a^2*x^2 + 32) + ((7*x)/2 + 3*a*x^2 - 4/a - (5*a^2*x^3)/2)/(32*a^4
*x^4 - 64*a^2*x^2 + 32) + (log(a*x + 1)*(10*x - 6*a^2*x^3))/(32*a^4*x^4 - 6
4*a^2*x^2 + 32)) + log(1 - a*x)^2*((3*log(a*x + 1))/(64*a) + ((5*x)/8 - (3*
a^2*x^3)/8)/(4*a^4*x^4 - 8*a^2*x^2 + 4)) + log(a*x + 1)^3/(64*a) - log(1 -
a*x)^3/(64*a) - (atan(a*x*1i)*15i)/(64*a) - (log(a*x + 1)*(1/(4*a^2) - (3*x
^2)/16))/(1/a - 2*a*x^2 + a^3*x^4) + (log(a*x + 1)^2*((5*x)/(32*a) - (3*a*x
^3)/32))/(1/a - 2*a*x^2 + a^3*x^4)
```

3.312 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=196

$$\frac{1}{32\,(1-a^2x^2)^2}+\frac{11}{32\,(1-a^2x^2)}-\frac{ax\tanh^{-1}(ax)}{8\,(1-a^2x^2)^2}-\frac{11ax\tanh^{-1}(ax)}{16\,(1-a^2x^2)}-\frac{11}{32}\tanh^{-1}(ax)^2+\frac{\tanh^{-1}(ax)^2}{4\,(1-a^2x^2)^2}+\frac{\tanh^{-1}(}{2\,(1-a}$$

```
[Out] 1/32/(-a^2*x^2+1)^2+11/32/(-a^2*x^2+1)-1/8*a*x*arctanh(a*x)/(-a^2*x^2+1)^2-
11/16*a*x*arctanh(a*x)/(-a^2*x^2+1)-11/32*arctanh(a*x)^2+1/4*arctanh(a*x)^2
/(-a^2*x^2+1)^2+1/2*arctanh(a*x)^2/(-a^2*x^2+1)+1/3*arctanh(a*x)^3+arctanh(
a*x)^2*ln(2-2/(a*x+1))-arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*polylog(3,-
1+2/(a*x+1))
```

Rubi [A]
time = 0.34, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {6177, 6135, 6079, 6095, 6203, 6745, 6141, 6103, 267, 6107}

$$\frac{11}{32\,(1-a^2x^2)}+\frac{1}{32\,(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^2}{2\,(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4\,(1-a^2x^2)^2}-\frac{11ax\tanh^{-1}(ax)}{16\,(1-a^2x^2)}-\frac{ax\tanh^{-1}(ax)}{8\,(1-a^2x^2)^2}-\frac{1}{2}\mathrm{Li}_3\left(\frac{2}{ax+1}-1\right)-\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{3}\tanh^{-1}(ax)^3-\frac{11}{32}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] 1/(32*(1 - a^2*x^2)^2) + 11/(32*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(8*(1 -
 a^2*x^2)^2) - (11*a*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^
2)/32 + ArcTanh[a*x]^2/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^2/(2*(1 - a^2*x^2
)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*
PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
```

```
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^3}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx\\
&= \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{1}{2}a\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx+a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx\\
&= \frac{1}{32\left(1-a^2x^2\right)^2}-\frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\tanh^{-1}(ax)^3-\frac{1}{8}\\
&= \frac{1}{32\left(1-a^2x^2\right)^2}-\frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)}-\frac{11}{32}\tanh^{-1}(ax)^2+\frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)}\\
&= \frac{1}{32\left(1-a^2x^2\right)^2}+\frac{11}{32\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)}-\frac{11}{32}\tanh^{-1}(ax\\
&= \frac{1}{32\left(1-a^2x^2\right)^2}+\frac{11}{32\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)}-\frac{11}{32}\tanh^{-1}(ax
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 129, normalized size = 0.66

$$\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{1}{768}\left(32i\pi^3-256\tanh^{-1}(ax)^3+144\cosh\left(2\tanh^{-1}(ax)\right)+3\cosh\left(4\tanh^{-1}(ax)\right)+24\tanh^{-1}(ax)^2\left(12\cosh\left(2\tanh^{-1}(ax)\right)+\cosh\left(4\tanh^{-1}(ax)\right)+32\log\left(1-e^{2\tanh^{-1}(ax)}\right)\right)-384\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-12\tanh^{-1}(ax)\left(24\sinh\left(2\tanh^{-1}(ax)\right)+\sinh\left(4\tanh^{-1}(ax)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((32*I)*Pi^3 - 256*ArcTanh[a*
x]^3 + 144*Cosh[2*ArcTanh[a*x]] + 3*Cosh[4*ArcTanh[a*x]] + 24*ArcTanh[a*x]^
2*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTanh[a*x]] + 32*Log[1 - E^(2*ArcTanh
[a*x])]) - 384*PolyLog[3, E^(2*ArcTanh[a*x])] - 12*ArcTanh[a*x]*(24*Sinh[2*
ArcTanh[a*x]] + Sinh[4*ArcTanh[a*x]]))/768
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 424.51, size = 1392, normalized size = 7.10

method	result	size
derivativedivides	Expression too large to display	1392
default	Expression too large to display	1392

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*arctanh(a*x)*(a*x-1)/(a*x+1)+3/16*arctanh(a*x)*(a*x+1)/(a*x-1)+1/128*
arctanh(a*x)*(a*x-1)^2/(a*x+1)^2-1/4*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*pol
ylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^2*csgn(I*((a*x+1)^
2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a
^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+2*arctanh(a*x)*polylog(2,-(a*x+1)/
(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/
3*arctanh(a*x)^3-11/32*arctanh(a*x)^2+1/2*I*Pi*arctanh(a*x)^2-1/128*arctanh
(a*x)*(a*x+1)^2/(a*x-1)^2-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*l
n(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*ln((a
*x+1)^2/(-a^2*x^2+1)-1)+1/4*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+1/2*I*Pi*arctanh(a*x)^2*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/4*I*Pi*arctanh(a*
x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2-1/4*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I*Pi*arct
anh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-5/16*arctanh(a*x)^2/(a*x-1)+5/16*arctanh
(a*x)^2/(a*x+1)+1/16*arctanh(a*x)^2/(a*x-1)^2+1/16*arctanh(a*x)^2/(a*x+1)^2
+1/2*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*I*Pi*arct
anh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*Pi*arctanh(a*x)^2*csg
n(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/4*I*Pi*arcta
nh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/4*I*
Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/512*(a*x+1)^2/(a*x-1)^2
+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1-(a*x+1
)/(-a^2*x^2+1)^(1/2))-1/2*I*Pi*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-3/32*
(a*x-1)/(a*x+1)-3/32*(a*x+1)/(a*x-1)+ln(2)*arctanh(a*x)^2+1/512*(a*x-1)^2/(
a*x+1)^2+arctanh(a*x)^2*ln(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^6*integrate(1/2*x^6*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 +
 3*a^2*x^3 - x), x) + 1/2*a^5*integrate(1/2*x^5*log(a*x + 1)*log(-a*x + 1)/
(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/256*(a*(2*(5*a^2*x^2 + 3*a*x
- 6)/(a^8*x^3 - a^7*x^2 - a^6*x + a^5) - 5*log(a*x + 1)/a^5 + 5*log(a*x - 1
)/a^5) + 16*(2*a^2*x^2 - 1)*log(-a*x + 1)/(a^8*x^4 - 2*a^6*x^2 + a^4))*a^4
- a^4*integrate(1/2*x^4*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3
*a^2*x^3 - x), x) - a^3*integrate(1/2*x^3*log(a*x + 1)*log(-a*x + 1)/(a^6*x
^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + 1/2*a^3*integrate(1/2*x^3*log(-a*x +
1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 3/512*(a*(2*(3*a^2*x^2 - 3*a
*x - 2)/(a^6*x^3 - a^5*x^2 - a^4*x + a^3) - 3*log(a*x + 1)/a^3 + 3*log(a*x
- 1)/a^3) - 16*log(-a*x + 1)/(a^6*x^4 - 2*a^4*x^2 + a^2))*a^2 + 1/2*a^2*int
egrate(1/2*x^2*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3
- x), x) + 1/2*a*integrate(1/2*x*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^
4*x^5 + 3*a^2*x^3 - x), x) - 3/4*a*integrate(1/2*x*log(-a*x + 1)/(a^6*x^7 -
 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/48*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*
x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log
(-a*x + 1)^2)/(a^4*x^4 - 2*a^2*x^2 + 1) - 1/2*integrate(1/2*log(a*x + 1)^2/
(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + integrate(1/2*log(a*x + 1)*log(
-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^6x^7 - 3a^4x^5 + 3a^2x^3 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\mathrm{atanh}(a\,x)^2}{x\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^3), x)
```

3.313 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=209

$$\frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{31}{64}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] 1/32*a^2*x/(-a^2*x^2+1)^2+31/64*a^2*x/(-a^2*x^2+1)+31/64*a*arctanh(a*x)-1/8
*a*arctanh(a*x)/(-a^2*x^2+1)^2-7/8*a*arctanh(a*x)/(-a^2*x^2+1)+a*arctanh(a*
x)^2-arctanh(a*x)^2/x+1/4*a^2*x*arctanh(a*x)^2/(-a^2*x^2+1)^2+7/8*a^2*x*arc
tanh(a*x)^2/(-a^2*x^2+1)+5/8*a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(2-2/(a*x+
1))-a*polylog(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.36, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {6177, 6129, 6037, 6135, 6079, 2497, 6095, 6103, 6141, 205, 212, 6111}

$$\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{5}{8}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{31}{64}a\tanh^{-1}(ax)+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] (a^2*x)/(32*(1 - a^2*x^2)^2) + (31*a^2*x)/(64*(1 - a^2*x^2)) + (31*a*ArcTan
h[a*x])/64 - (a*ArcTanh[a*x])/(8*(1 - a^2*x^2)^2) - (7*a*ArcTanh[a*x])/(8*(
1 - a^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2
)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^2)/(8*(1 - a^2*x^2)) + (5*a*A
rcTanh[a*x]^3)/8 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1
+ 2/(1 + a*x)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^3}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx \\
&= -\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{1}{8}a^2\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx+\frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax}{(1-a^2x^2} \\
&= \frac{a^2x}{32\left(1-a^2x^2\right)^2}-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{7}{24}a\tanh \\
&= \frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{3a^2x}{64\left(1-a^2x^2\right)}-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{x}+ \\
&= \frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{3}{64}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} \\
&= \frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{31}{64}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} \\
&= \frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{31}{64}a\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 127, normalized size = 0.61

$$-a\left(-\frac{5}{8}\tanh^{-1}(ax)^3+\frac{1}{64}\tanh^{-1}(ax)\left(32\cosh\left(2\tanh^{-1}(ax)\right)+\cosh\left(4\tanh^{-1}(ax)\right)-128\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{4}\sinh\left(2\tanh^{-1}(ax)\right)+\tanh^{-1}(ax)^2\left(-1+\frac{1}{ax}+\frac{ax}{-1+a^2x^2}-\frac{1}{32}\sinh\left(4\tanh^{-1}(ax)\right)\right)-\frac{1}{256}\sinh\left(4\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*((-5*ArcTanh[a*x]^3)/8 + (ArcTanh[a*x]*(32*Cosh[2*ArcTanh[a*x]] + Cosh[
4*ArcTanh[a*x]] - 128*Log[1 - E^(-2*ArcTanh[a*x])]))/64 + PolyLog[2, E^(-2*
ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]]/4 + ArcTanh[a*x]^2*(-1 + 1/(a*x) + (a
*x)/(-1 + a^2*x^2) - Sinh[4*ArcTanh[a*x]]/32) - Sinh[4*ArcTanh[a*x]]/256))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 563.27, size = 4576, normalized size = 21.89

method	result	size
derivativedivides	Expression too large to display	4576
default	Expression too large to display	4576

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/4*arctanh(a*x)*(a*x-1)/(a*x+1)+1/4*arctanh(a*x)*(a*x+1)/(a*x-1)-1/128*
arctanh(a*x)*(a*x-1)^2/(a*x+1)^2+15/32*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))*arctanh(a*x)^2+15/16*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I
*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,-(a*x+1)/(-a^2*
x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*ln(1+(a*
x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/3
2*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*Pi*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1
+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*
Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog
(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*a
rctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*Pi*csgn(I*(a*x+1)/(-a^
2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2+15/16*I*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/3
2*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a
*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1
5/32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))^2*arctanh(a*x)^2+15/32*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*Pi*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I
*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arct
anh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arc
tanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)*ln(1-(a*x+1)/(-a
^2*x^2+1)^(1/2))-15/32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x
^2+1)^(1/2))-15/32*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2
```

```
/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*dilo
g((a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+5/8*arctanh(a*x)^3-arct
anh(a*x)^2+15/32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2
))-15/32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*di
log(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+15/32*I*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1/
128*arctanh(a*x)*(a*x+1)^2/(a*x-1)^2-15/16*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2
+1)+1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+15/16*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+
1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*Pi*arcta
nh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*Pi*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^3*arctanh(a*x)^2+15/16*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+
1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*Pi*csgn(I/((a*x+1)^2/(
-a^2*x^2+1)+1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*Pi*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/(-a^2*...
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(186) = 372$.
time = 0.29, size = 534, normalized size = 2.56

$$-\frac{1}{128}a^2\left(\frac{2\left(31a^3x^3-5\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)^3+5\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)^3-\left(16a^4x^4-32a^2x^2-15\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)+16\right)\log\left(ax+1\right)^2+16\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)^2-33ax-\left(15\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)^2-32\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)\right)\log\left(ax+1\right)\right)}{a^5x^4-2a^3x^2+a}-\frac{128\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}+\frac{128\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)}{a}-\frac{128\left(\log\left(-ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(ax\right)\right)}{a}-\frac{31\log\left(ax+1\right)}{a}+\frac{31\log\left(ax-1\right)}{a}\right)+\frac{1}{32}a\left(\frac{28a^2x^2-15\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)^2+30\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)\log\left(ax-1\right)-15\left(a^4x^4-2a^2x^2+1\right)\log\left(ax-1\right)^2-32}{a^4x^4-2a^2x^2+1}-32\log\left(ax+1\right)-32\log\left(ax-1\right)+64\log\left(x\right)\right)\operatorname{artanh}\left(ax\right)+\frac{1}{16}\left(15a\log\left(ax+1\right)-15a\log\left(ax-1\right)-\frac{2\left(15a^4x^4-25a^2x^2+8\right)}{a^4x^5-2a^2x^3+x}\right)\operatorname{artanh}\left(ax\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/128*a^2*(2*(31*a^3*x^3 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 5*
(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - (16*a^4*x^4 - 32*a^2*x^2 - 15*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1) + 16)*log(a*x + 1)^2 + 16*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x - 1)^2 - 33*a*x - (15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a
*x - 1)^2 - 32*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))/(a^5*x
^4 - 2*a^3*x^2 + a) - 128*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x
 + 1/2))/a + 128*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 128*(log(-a*x + 1)
*log(x) + dilog(a*x))/a - 31*log(a*x + 1)/a + 31*log(a*x - 1)/a) + 1/32*a*(
(28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(
a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 32*log(a*x -
 1) + 64*log(x))*arctanh(a*x) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1)
 - 2*(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)^
2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{2}(ax)}{a^{6}x^{8} - 3a^{4}x^{6} + 3a^{2}x^{4} - x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\operatorname{atanh}(a\,x)}^2}{x^2\,{\left(a^2\,x^2-1\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^3), x)
```

3.314 $\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=192

$$-\frac{3x^3}{128a\,(1-a^2x^2)^2}+\frac{45x}{256a^3\,(1-a^2x^2)}+\frac{27\tanh^{-1}(ax)}{256a^4}+\frac{3x^4\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}-\frac{9\tanh^{-1}(ax)}{32a^4\,(1-a^2x^2)}-\frac{3x^3\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2}$$

```
[Out] -3/128*x^3/a/(-a^2*x^2+1)^2+45/256*x/a^3/(-a^2*x^2+1)+27/256*arctanh(a*x)/a
^4+3/32*x^4*arctanh(a*x)/(-a^2*x^2+1)^2-9/32*arctanh(a*x)/a^4/(-a^2*x^2+1)-
3/16*x^3*arctanh(a*x)^2/a/(-a^2*x^2+1)^2+9/32*x*arctanh(a*x)^2/a^3/(-a^2*x^
2+1)-3/32*arctanh(a*x)^3/a^4+1/4*x^4*arctanh(a*x)^3/(-a^2*x^2+1)^2
```

Rubi [A]
time = 0.19, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6155, 6151, 6147, 6141, 205, 212, 294}

$$-\frac{3\tanh^{-1}(ax)^3}{32a^4}+\frac{27\tanh^{-1}(ax)}{256a^4}+\frac{x^4\tanh^{-1}(ax)^3}{4\,(1-a^2x^2)^2}+\frac{3x^4\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}-\frac{3x^3}{128a\,(1-a^2x^2)^2}-\frac{3x^3\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2}-\frac{9\tanh^{-1}(ax)}{32a^4\,(1-a^2x^2)}+\frac{45x}{256a^3\,(1-a^2x^2)}+\frac{9x\tanh^{-1}(ax)^2}{32a^3\,(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*x^3)/(128*a*(1 - a^2*x^2)^2) + (45*x)/(256*a^3*(1 - a^2*x^2)) + (27*Arc
Tanh[a*x])/(256*a^4) + (3*x^4*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) - (9*ArcTa
nh[a*x])/(32*a^4*(1 - a^2*x^2)) - (3*x^3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2
)^2) + (9*x*ArcTanh[a*x]^2)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32
*a^4) + (x^4*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6147

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> Simp[-(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[b*(p/(2*c)), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2
), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6151

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Simp[(-b)*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a +
b*ArcTanh[c*x])^(p - 1)/(c*d*m^2)), x] + (-Dist[f^2*((m - 1)/(c^2*d*m)), In
t[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b
^2*p*((p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2),
 x], x] + Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/
(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
EqQ[m + 2*q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^3} \, dx = \frac{x^4 \tanh^{-1}(ax)^3}{4\,(1-a^2x^2)^2} - \frac{1}{4}(3a) \int \frac{x^4 \tanh^{-1}(ax)^2}{(1-a^2x^2)^3} \, dx$$

$$= \frac{3x^4 \tanh^{-1}(ax)}{32\,(1-a^2x^2)^2} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2} + \frac{x^4 \tanh^{-1}(ax)^3}{4\,(1-a^2x^2)^2} + \frac{9 \int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} \, dx}{16a} - \frac{1}{32}(3$$

$$= -\frac{3x^3}{128a\,(1-a^2x^2)^2} + \frac{3x^4 \tanh^{-1}(ax)}{32\,(1-a^2x^2)^2} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2} + \frac{9x \tanh^{-1}(ax)^2}{32a^3\,(1-a^2x^2)} - \frac{3\,\mathrm{ta}}{}$$

$$= -\frac{3x^3}{128a\,(1-a^2x^2)^2} + \frac{9x}{256a^3\,(1-a^2x^2)} + \frac{3x^4 \tanh^{-1}(ax)}{32\,(1-a^2x^2)^2} - \frac{9 \tanh^{-1}(ax)}{32a^4\,(1-a^2x^2)} - \frac{3x^3}{16a}$$

$$= -\frac{3x^3}{128a\,(1-a^2x^2)^2} + \frac{45x}{256a^3\,(1-a^2x^2)} - \frac{9 \tanh^{-1}(ax)}{256a^4} + \frac{3x^4 \tanh^{-1}(ax)}{32\,(1-a^2x^2)^2} - \frac{9\,\mathrm{tan}}{32a^4\,(}$$

$$= -\frac{3x^3}{128a\,(1-a^2x^2)^2} + \frac{45x}{256a^3\,(1-a^2x^2)} + \frac{27 \tanh^{-1}(ax)}{256a^4} + \frac{3x^4 \tanh^{-1}(ax)}{32\,(1-a^2x^2)^2} - \frac{9\,\mathrm{ta}}{32a^4}$$

Mathematica [A]
time = 0.06, size = 135, normalized size = 0.70

$$\frac{48(-4+5a^2x^2)\tanh^{-1}(ax) - 48ax(-3+5a^2x^2)\tanh^{-1}(ax)^2 + 16(-3+6a^2x^2+5a^4x^4)\tanh^{-1}(ax)^3 + 3\left(30ax - 34a^3x^3 - 17(-1+a^2x^2)^2\log(1-ax) + 17(-1+a^2x^2)^2\log(1+ax)\right)}{512a^4\,(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (48*(-4 + 5*a^2*x^2)*ArcTanh[a*x] - 48*a*x*(-3 + 5*a^2*x^2)*ArcTanh[a*x]^2
+ 16*(-3 + 6*a^2*x^2 + 5*a^4*x^4)*ArcTanh[a*x]^3 + 3*(30*a*x - 34*a^3*x^3 -
 17*(-1 + a^2*x^2)^2*Log[1 - a*x] + 17*(-1 + a^2*x^2)^2*Log[1 + a*x]))/(512
*a^4*(-1 + a^2*x^2)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 370.82, size = 2088, normalized size = 10.88

method	result
risch	$\frac{(5a^4x^4+6a^2x^2-3)\ln(ax+1)^3}{256a^4(a^2x^2-1)^2} - \frac{3(5x^4\ln(-ax+1)a^4+10a^3x^3+6x^2\ln(-ax+1)a^2-6ax-3\ln(-ax+1))\ln(ax+1)^2}{256a^4(ax+1)(ax-1)(a^2x^2-1)} +$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/16*arctanh(a*x)^3/(a*x+1)^2-3/16*arctanh(a*x)^3/(a*x+1)+1/16*arcta
nh(a*x)^3/(a*x-1)^2+3/16*arctanh(a*x)^3/(a*x-1)+3/64*arctanh(a*x)^2/(a*x+1)
^2-15/64*arctanh(a*x)^2/(a*x+1)+15/64*arctanh(a*x)^2*ln(a*x+1)-3/64*arctanh
(a*x)^2/(a*x-1)^2-15/64*arctanh(a*x)^2/(a*x-1)-15/64*arctanh(a*x)^2*ln(a*x-
1)-15/32*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/256*(-80*arctanh(a
*x)^3*a^2*x^2+30*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a
*x)^2*Pi*a^4*x^4-60*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctan
h(a*x)^2*Pi*a^2*x^2+45*a*x+51*a^4*x^4*arctanh(a*x)+40*arctanh(a*x)^3+18*a^2
*x^2*arctanh(a*x)-45*arctanh(a*x)+30*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))+60*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arct
anh(a*x)^2*Pi*a^4*x^4-30*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^3*arctanh(a*x)^2*Pi*a^4*x^4-30*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*a
rctanh(a*x)^2*Pi*a^4*x^4-60*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(
a*x)^2*Pi*a^4*x^4-120*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2
*Pi*a^2*x^2+60*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3
*arctanh(a*x)^2*Pi*a^2*x^2+60*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x
)^2*Pi*a^2*x^2+120*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*Pi
*a^2*x^2-51*a^3*x^3+40*arctanh(a*x)^3*a^4*x^4+60*I*Pi*arctanh(a*x)^2*csgn(I
/((a*x+1)^2/(-a^2*x^2+1)+1))^3-30*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-30*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^3-60*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
^2+60*I*Pi*arctanh(a*x)^2+60*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*Pi*a^2*x^2
-60*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi*a^2*x^2+60*I*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi*a^2*x^2+120*I*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(
a*x)^2*Pi*a^2*x^2-30*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*Pi*a^4*x^4+30*I*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1))*arctanh(a*x)^2*Pi*a^4*x^4-30*I*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2
))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi*a^4*x^4-60*I*csgn(I*(a
*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*Pi
*a^4*x^4+60*I*arctanh(a*x)^2*Pi*a^4*x^4-120*I*arctanh(a*x)^2*Pi*a^2*x^2-30*
I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+30*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2
-30*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1))-60*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))^2)/(a*x+1)^2/(a*x-1)^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 437 vs.

$2(167) = 334.$
time = 0.29, size = 437, normalized size = 2.28

$$-\frac{3}{64}a\left(\frac{2\,(5\,a^2x^3-3\,x)}{a^8x^4-2\,a^6x^2+a^4}-\frac{5\,\log(ax+1)}{a^5}+\frac{5\,\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)^2+\frac{(2\,a^2x^2-1)\operatorname{artanh}(ax)^3}{4\,(a^8x^4-2\,a^6x^2+a^4)}-\frac{1}{512}\left(\frac{(102\,a^3x^3-10\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^3+30\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2\log(ax-1)+10\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^3-90\,ax-3\,(17\,a^4x^4-34\,a^2x^2+10\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^2+17)\log(ax+1)+51\,(a^4x^4-2\,a^2x^2+1)\log(ax-1))a^2}{a^{11}x^4-2\,a^9x^2+a^7}-\frac{12\,(20\,a^2x^2-5\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2+10\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1)-5\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^2-16)a\operatorname{artanh}(ax)}{a^{10}x^4-2\,a^8x^2+a^6}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -3/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x)^2 + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^
3/(a^8*x^4 - 2*a^6*x^2 + a^4) - 1/512*((102*a^3*x^3 - 10*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x + 1)^3 + 30*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*
x - 1) + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 90*a*x - 3*(17*a^4*x
^4 - 34*a^2*x^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 17)*log(a*x
 + 1) + 51*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^11*x^4 - 2*a^9*x^
2 + a^7) - 12*(20*a^2*x^2 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 10
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^10*x^4 - 2*a^8*x^2 + a^6))*a
```

Fricas [A]
time = 0.41, size = 140, normalized size = 0.73

$$-\frac{102\,a^3x^3-2\,(5\,a^4x^4+6\,a^2x^2-3)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,(5\,a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-90\,ax-3\,(17\,a^4x^4+6\,a^2x^2-15)\log\left(-\frac{ax+1}{ax-1}\right)}{512\,(a^8x^4-2\,a^6x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/512*(102*a^3*x^3 - 2*(5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1
))^3 + 12*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 90*a*x - 3*(17*
a^4*x^4 + 6*a^2*x^2 - 15)*log(-(a*x + 1)/(a*x - 1)))/(a^8*x^4 - 2*a^6*x^2 +
 a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3\operatorname{atanh}^3(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**3*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(167) = 334.

time = 0.43, size = 341, normalized size = 1.78

$$\frac{1}{2048}\left(4\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{4(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+6\left(\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}-\frac{(ax+1)^2}{(ax-1)^2a^5}-\frac{8(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+6\left(\frac{(ax-1)^2\left(\frac{16(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{16(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{3(ax-1)^2\left(\frac{32(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}-\frac{3(ax+1)^2}{(ax-1)^2a^5}-\frac{96(ax+1)}{(ax-1)a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(4*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (a*x
 + 1)^2/((a*x - 1)^2*a^5) + 4*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/(a*
x - 1))^3 + 6*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) -
(a*x + 1)^2/((a*x - 1)^2*a^5) - 8*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)
/(a*x - 1))^2 + 6*((a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^
5) + (a*x + 1)^2/((a*x - 1)^2*a^5) + 16*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*
x + 1)/(a*x - 1)) + 3*(a*x - 1)^2*(32*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2
*a^5) - 3*(a*x + 1)^2/((a*x - 1)^2*a^5) - 96*(a*x + 1)/((a*x - 1)*a^5))*a
```

Mupad [B]

time = 3.09, size = 414, normalized size = 2.16

$$\frac{48\ln(1-ax)-48\ln(ax+1)+51\,\mathrm{atanh}(ax)+45ax-3\ln(ax+1)^3+3\ln(1-ax)^3-9\ln(ax+1)\ln(1-ax)^2+9\ln(ax+1)^2\ln(1-ax)-51a^3x^3+6a^2x^2\ln(ax+1)^3-6a^2x^2\ln(1-ax)^3-30a^3x^3\ln(ax+1)^2-30a^3x^3\ln(1-ax)^2+5a^4x^4\ln(ax+1)^3-5a^4x^4\ln(1-ax)^3-102a^2x^2\,\mathrm{atanh}(ax)+51a^4x^4\,\mathrm{atanh}(ax)+18ax\ln(ax+1)^2+18ax\ln(1-ax)^2+60a^2x^2\ln(ax+1)-60a^2x^2\ln(1-ax)-36ax\ln(ax+1)\ln(1-ax)+18a^2x^2\ln(ax+1)\ln(1-ax)^2-18a^2x^2\ln(ax+1)^2\ln(1-ax)+15a^4x^4\ln(ax+1)\ln(1-ax)^2-15a^4x^4\ln(ax+1)^2\ln(1-ax)+60a^3x^3\ln(ax+1)\ln(1-ax)}{256a^4\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

```
[Out] (48*log(1 - a*x) - 48*log(a*x + 1) + 51*atanh(a*x) + 45*a*x - 3*log(a*x + 1
)^3 + 3*log(1 - a*x)^3 - 9*log(a*x + 1)*log(1 - a*x)^2 + 9*log(a*x + 1)^2*l
og(1 - a*x) - 51*a^3*x^3 + 6*a^2*x^2*log(a*x + 1)^3 - 6*a^2*x^2*log(1 - a*x
)^3 - 30*a^3*x^3*log(a*x + 1)^2 - 30*a^3*x^3*log(1 - a*x)^2 + 5*a^4*x^4*log
(a*x + 1)^3 - 5*a^4*x^4*log(1 - a*x)^3 - 102*a^2*x^2*atanh(a*x) + 51*a^4*x^
4*atanh(a*x) + 18*a*x*log(a*x + 1)^2 + 18*a*x*log(1 - a*x)^2 + 60*a^2*x^2*l
og(a*x + 1) - 60*a^2*x^2*log(1 - a*x) - 36*a*x*log(a*x + 1)*log(1 - a*x) +
18*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 - 18*a^2*x^2*log(a*x + 1)^2*log(1 -
a*x) + 15*a^4*x^4*log(a*x + 1)*log(1 - a*x)^2 - 15*a^4*x^4*log(a*x + 1)^2*l
og(1 - a*x) + 60*a^3*x^3*log(a*x + 1)*log(1 - a*x))/(256*a^4*(a^2*x^2 - 1)^
2)
```

3.315 $\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=215

$$-\frac{3}{128a^3 (1-a^2x^2)^2}+\frac{3}{128a^3 (1-a^2x^2)}+\frac{3x \tanh^{-1}(ax)}{32a^2 (1-a^2x^2)^2}-\frac{3x \tanh^{-1}(ax)}{64a^2 (1-a^2x^2)}-\frac{3 \tanh^{-1}(ax)^2}{128a^3}-\frac{3 \tanh^{-1}(ax)}{16a^3 (1-a^2x}$$

```
[Out] -3/128/a^3/(-a^2*x^2+1)^2+3/128/a^3/(-a^2*x^2+1)+3/32*x*arctanh(a*x)/a^2/(-
a^2*x^2+1)^2-3/64*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-3/128*arctanh(a*x)^2/a^3-
3/16*arctanh(a*x)^2/a^3/(-a^2*x^2+1)^2+3/16*arctanh(a*x)^2/a^3/(-a^2*x^2+1)
+1/4*x*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^2-1/8*x*arctanh(a*x)^3/a^2/(-a^2*x^2
+1)-1/32*arctanh(a*x)^4/a^3
```

Rubi [A]

time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6175, 6103, 6141, 267, 6111, 6107}

$$-\frac{\tanh^{-1}(ax)^4}{32a^3}-\frac{3 \tanh^{-1}(ax)^2}{128a^3}-\frac{x \tanh^{-1}(ax)^3}{8a^2 (1-a^2x^2)}+\frac{x \tanh^{-1}(ax)^3}{4a^2 (1-a^2x^2)^2}-\frac{3x \tanh^{-1}(ax)}{64a^2 (1-a^2x^2)}+\frac{3x \tanh^{-1}(ax)}{32a^2 (1-a^2x^2)^2}+\frac{3}{128a^3 (1-a^2x^2)}-\frac{3}{128a^3 (1-a^2x^2)^2}+\frac{3 \tanh^{-1}(ax)^2}{16a^3 (1-a^2x^2)}-\frac{3 \tanh^{-1}(ax)^2}{16a^3 (1-a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] -3/(128*a^3*(1 - a^2*x^2)^2) + 3/(128*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x
])/(32*a^2*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(64*a^2*(1 - a^2*x^2)) - (
3*ArcTanh[a*x]^2)/(128*a^3) - (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)^2) +
 (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*a^2*(1 -
 a^2*x^2)^2) - (x*ArcTanh[a*x]^3)/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(3
2*a^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^3} dx = \frac{\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^3} dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^2} dx}{a^2}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{16a^3(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)^3}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^4}{8a^3} + \frac{3\int \frac{\tanh^{-1}}{(1-a^2}}{8a^2}$$

$$= -\frac{3}{128a^3(1-a^2x^2)^2} + \frac{3x\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} - \frac{3\tanh^{-1}(ax)^2}{16a^3(1-a^2x^2)^2} + \frac{3\tanh^{-1}(ax)^2}{4a^3(1-a^2x^2)} + \frac{x\,\mathrm{t}}{4a^}$$

$$= -\frac{3}{128a^3(1-a^2x^2)^2} + \frac{3x\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} - \frac{39x\tanh^{-1}(ax)}{64a^2(1-a^2x^2)} - \frac{39\tanh^{-1}(ax)^2}{128a^3} - \frac{3}{16}$$

$$= -\frac{3}{128a^3(1-a^2x^2)^2} + \frac{39}{128a^3(1-a^2x^2)} + \frac{3x\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)}{64a^2(1-a^2x^2)} - \frac{3}{}$$

$$= -\frac{3}{128a^3(1-a^2x^2)^2} + \frac{3}{128a^3(1-a^2x^2)} + \frac{3x\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)}{64a^2(1-a^2x^2)} - \frac{3}{}$$

Mathematica [A]
time = 0.05, size = 107, normalized size = 0.50

$$\frac{-3a^2x^2 + 6(ax + a^3x^3)\tanh^{-1}(ax) - 3(1 + 6a^2x^2 + a^4x^4)\tanh^{-1}(ax)^2 + 16(ax + a^3x^3)\tanh^{-1}(ax)^3 - 4(-1 + a^2x^2)^2\tanh^{-1}(ax)^4}{128a^3(-1 + a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*a^2*x^2 + 6*(a*x + a^3*x^3)*ArcTanh[a*x] - 3*(1 + 6*a^2*x^2 + a^4*x^4)*
ArcTanh[a*x]^2 + 16*(a*x + a^3*x^3)*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*Arc
Tanh[a*x]^4)/(128*a^3*(-1 + a^2*x^2)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 387.63, size = 2059, normalized size = 9.58

method	result
risch	$-\frac{\ln(ax+1)^4}{512a^3} + \frac{(x^4\ln(-ax+1)a^4+2a^3x^3-2x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1))\ln(ax+1)^3}{128a^3(a^2x^2-1)^2} - \frac{3\left(2a^4x^4\ln(-ax+1)^2\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/16*arctanh(a*x)^3/(a*x+1)^2+1/16*arctanh(a*x)^3/(a*x+1)-1/16*arct
anh(a*x)^3*ln(a*x+1)+1/16*arctanh(a*x)^3/(a*x-1)^2+1/16*arctanh(a*x)^3/(a*x
-1)+1/16*arctanh(a*x)^3*ln(a*x-1)+1/8*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1
)^(1/2))-1/1024*(3-32*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+32*I*Pi*arctan
h(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^2-32*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(
1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+32*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*arctanh(a*x)^3*a^4*x^4+64*I*Pi*arctanh(a*x)^3*a^4*x^4-128
*I*Pi*arctanh(a*x)^3*a^2*x^2+18*a^2*x^2-48*a*x*arctanh(a*x)-64*I*Pi*arctanh
(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+
3*a^4*x^4-64*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*
x)^3*a^2*x^2-48*a^3*x^3*arctanh(a*x)+24*a^4*x^4*arctanh(a*x)^2+144*a^2*x^2*
arctanh(a*x)^2+32*arctanh(a*x)^4+24*arctanh(a*x)^2+32*arctanh(a*x)^4*a^4*x^
4-64*arctanh(a*x)^4*a^2*x^2-32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3*a^4*x^4+64*I*Pi*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))^3*arctanh(a*x)^3*a^4*x^4-32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3
*arctanh(a*x)^3*a^4*x^4-64*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctan
h(a*x)^3*a^4*x^4-128*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)
^3*a^2*x^2+64*I*Pi*arctanh(a*x)^3+64*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))^3-32*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^3-32*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-
1))^3-64*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+64*I*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3*a^
2*x^2+64*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3*a^2*x^2+128*I*
Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3*a^2*x^2+32*I*Pi*arct
anh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+64*I*Pi*csgn(I*(a
*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*a^
2*x^2+128*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2
-1))^2*arctanh(a*x)^3*a^2*x^2-32*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3*a^4
*x^4+32*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*a^4*x^4-32*I*Pi*csgn(I*(a*x+1)/(-
a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*a^4*x^4-64
*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*ar
ctanh(a*x)^3*a^4*x^4+64*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3*a^2*x^2-64*I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*arctanh(a*x)^3*a^2*x^2)/(a*x+1)^2/(a*x-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(187) = 374$.

time = 0.28, size = 657, normalized size = 3.06

$$\frac{1}{16}\left(\frac{2\left(a^{2}x^{3}+x\right)}{a^{6}x^{4}-2a^{4}x^{2}+a^{2}}-\frac{\log\left(ax+1\right)}{a^{3}}+\frac{\log\left(ax-1\right)}{a^{3}}\right)\operatorname{artanh}\left(ax\right)^{3}-\frac{3\left(4a^{2}x^{2}-\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax+1\right)^{2}+2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax+1\right)\log\left(ax-1\right)-\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{2}\right)a\operatorname{artanh}\left(ax\right)^{2}}{64\left(a^{8}x^{4}-2a^{6}x^{2}+a^{4}\right)}+\frac{1}{512}\left(\frac{\left(\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax+1\right)^{4}-4\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax+1\right)^{3}\log\left(ax-1\right)+\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{4}-12a^{2}x^{2}+3\left(a^{4}x^{4}-2a^{2}x^{2}+2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{2}+1\right)\log\left(ax+1\right)^{2}+3\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{2}-2\left(2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{3}+3\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)\right)\log\left(ax+1\right)\right)a^{2}}{a^{10}x^{4}-2a^{8}x^{2}+a^{6}}+\frac{4\left(6a^{3}x^{3}-2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax+1\right)^{3}+6\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax+1\right)^{2}\log\left(ax-1\right)+2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{3}+6ax-3\left(a^{4}x^{4}-2a^{2}x^{2}+2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)^{2}+1\right)\log\left(ax+1\right)+3\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(ax-1\right)\right)a\operatorname{artanh}\left(ax\right)}{a^{9}x^{4}-2a^{7}x^{2}+a^{5}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x)^3 - 3/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a*arctanh(a*x)^2/(a^8*x^4 - 2*a^6*x
^2 + a^4) + 1/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 - 4*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log
(a*x - 1)^4 - 12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2
+ 1)*log(a*x - 1)^2 + 1)*log(a*x + 1)^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a
*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 3*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))*a^2/(a^10*x^4 - 2*a^8*x^2 + a^6) +
 4*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log
(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)
)*a*arctanh(a*x)/(a^9*x^4 - 2*a^7*x^2 + a^5))*a
```

Fricas [A]
time = 0.42, size = 161, normalized size = 0.75

$$-\frac{\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+12\,a^{2}x^{2}-8\left(a^{3}x^{3}+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}+3\left(a^{4}x^{4}+6\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-12\left(a^{3}x^{3}+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{512\left(a^{7}x^{4}-2\,a^{5}x^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/512*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 12*a^2*x^2
- 8*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(a^4*x^4 + 6*a^2*x^2 +
1)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1
)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{2}\operatorname{atanh}^{3}\left(ax\right)}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

[Out] -Integral(x**2*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")

[Out] integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)

Mupad [B]
time = 3.19, size = 831, normalized size = 3.87

$$\frac{3\ln(ax+1)\ln(1-ax)}{4(64a^7x^4-128a^5x^2+64a^3)}-\frac{3\ln(1-ax)^2}{512a^3}-\frac{\ln(ax+1)^4}{512a^3}-\frac{\ln(1-ax)^4}{512a^3}-\frac{3x^2}{2(64a^5x^4-128a^3x^2+64a)}-\frac{x\ln(1-ax)^3}{8(8a^6x^4-16a^4x^2+8a^2)}-\frac{6x^2\ln(1-ax)^2}{128a^5x^4-256a^3x^2+128a}-\frac{3\ln(ax+1)^2}{512a^3}+\frac{x^3\ln(ax+1)^3}{64(a^4x^4-2a^2x^2+1)}-\frac{x^3\ln(1-ax)^3}{8(8a^4x^4-16a^2x^2+8)}+\frac{3x\ln(ax+1)}{128(a^6x^4-2a^4x^2+a^2)}+\frac{\ln(ax+1)\ln(1-ax)^3}{128a^3}+\frac{\ln(ax+1)^3\ln(1-ax)}{128a^3}-\frac{3x\ln(1-ax)}{128a^6x^4-256a^4x^2+128a^2}-\frac{3x^2\ln(ax+1)^2}{64(a^5x^4-2a^3x^2+a)}+\frac{x\ln(ax+1)^3}{64(a^6x^4-2a^4x^2+a^2)}-\frac{3\ln(ax+1)^2\ln(1-ax)^2}{256a^3}+\frac{3x^3\ln(ax+1)}{128(a^4x^4-2a^2x^2+1)}-\frac{3ax^3\ln(1-ax)}{128a^5x^4-256a^3x^2+128a}+\frac{6x\ln(ax+1)\ln(1-ax)^2}{128a^6x^4-256a^4x^2+128a^2}-\frac{6x\ln(ax+1)^2\ln(1-ax)}{128a^6x^4-256a^4x^2+128a^2}+\frac{6x^2\ln(ax+1)\ln(1-ax)}{64a^5x^4-128a^3x^2+64a}+\frac{6a^2x^3\ln(ax+1)\ln(1-ax)^2}{128a^6x^4-256a^4x^2+128a^2}-\frac{6a^2x^3\ln(ax+1)^2\ln(1-ax)}{128a^6x^4-256a^4x^2+128a^2}-\frac{3a^2x^2\ln(ax+1)\ln(1-ax)}{2(64a^7x^4-128a^5x^2+64a^3)}+\frac{3a^4x^4\ln(ax+1)\ln(1-ax)}{4(64a^7x^4-128a^5x^2+64a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)

[Out] (3*log(a*x + 1)*log(1 - a*x))/(4*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4)) - (3*
log(1 - a*x)^2)/(512*a^3) - log(a*x + 1)^4/(512*a^3) - log(1 - a*x)^4/(512*
a^3) - (3*x^2)/(2*(64*a - 128*a^3*x^2 + 64*a^5*x^4)) - (x*log(1 - a*x)^3)/(
8*(8*a^2 - 16*a^4*x^2 + 8*a^6*x^4)) - (6*x^2*log(1 - a*x)^2)/(128*a - 256*a
^3*x^2 + 128*a^5*x^4) - (3*log(a*x + 1)^2)/(512*a^3) + (x^3*log(a*x + 1)^3)
/(64*(a^4*x^4 - 2*a^2*x^2 + 1)) - (x^3*log(1 - a*x)^3)/(8*(8*a^4*x^4 - 16*a
^2*x^2 + 8)) + (3*x*log(a*x + 1))/(128*(a^2 - 2*a^4*x^2 + a^6*x^4)) + (log(
a*x + 1)*log(1 - a*x)^3)/(128*a^3) + (log(a*x + 1)^3*log(1 - a*x))/(128*a^3
) - (3*x*log(1 - a*x))/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (3*x^2*log(a
x + 1)^2)/(64(a - 2*a^3*x^2 + a^5*x^4)) + (x*log(a*x + 1)^3)/(64*(a^2 - 2
*a^4*x^2 + a^6*x^4)) - (3*log(a*x + 1)^2*log(1 - a*x)^2)/(256*a^3) + (3*x^3
*log(a*x + 1))/(128*(a^4*x^4 - 2*a^2*x^2 + 1)) - (3*a*x^3*log(1 - a*x))/(12
8*a - 256*a^3*x^2 + 128*a^5*x^4) + (6*x*log(a*x + 1)*log(1 - a*x)^2)/(128*a
^2 - 256*a^4*x^2 + 128*a^6*x^4) - (6*x*log(a*x + 1)^2*log(1 - a*x))/(128*a^
2 - 256*a^4*x^2 + 128*a^6*x^4) + (6*x^2*log(a*x + 1)*log(1 - a*x))/(64*a -
128*a^3*x^2 + 64*a^5*x^4) + (6*a^2*x^3*log(a*x + 1)*log(1 - a*x)^2)/(128*a^
2 - 256*a^4*x^2 + 128*a^6*x^4) - (6*a^2*x^3*log(a*x + 1)^2*log(1 - a*x))/(1
28*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (3*a^2*x^2*log(a*x + 1)*log(1 - a*x))
/(2*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4)) + (3*a^4*x^4*log(a*x + 1)*log(1 -
a*x))/(4*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4))

3.316 $\int \frac{x \tanh^{-1}(ax)^3}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=188

$$-\frac{3x}{128a\left(1-a^2x^2\right)^2}-\frac{45x}{256a\left(1-a^2x^2\right)}-\frac{45\tanh^{-1}(ax)}{256a^2}+\frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2}+\frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}$$

```
[Out] -3/128*x/a/(-a^2*x^2+1)^2-45/256*x/a/(-a^2*x^2+1)-45/256*arctanh(a*x)/a^2+3
/32*arctanh(a*x)/a^2/(-a^2*x^2+1)^2+9/32*arctanh(a*x)/a^2/(-a^2*x^2+1)-3/16
*x*arctanh(a*x)^2/a/(-a^2*x^2+1)^2-9/32*x*arctanh(a*x)^2/a/(-a^2*x^2+1)-3/3
2*arctanh(a*x)^3/a^2+1/4*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^2
```

Rubi [A]

time = 0.12, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6141, 6111, 6103, 205, 212}

$$-\frac{45x}{256a\left(1-a^2x^2\right)}-\frac{3x}{128a\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{9x\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^3}{32a^2}-\frac{45\tanh^{-1}(ax)}{256a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*x)/(128*a*(1 - a^2*x^2)^2) - (45*x)/(256*a*(1 - a^2*x^2)) - (45*ArcTanh
[a*x])/(256*a^2) + (3*ArcTanh[a*x])/(32*a^2*(1 - a^2*x^2)^2) + (9*ArcTanh[a
*x])/(32*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) -
 (9*x*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32*a^2) +
ArcTanh[a*x]^3/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
```

```
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^3}\,dx = \frac{\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx}{4a}$$
$$= \frac{3\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)^2}{16a(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{1}{(1-a^2x^2)^3}\,dx}{32a} - \frac{9\int \frac{\tanh^{-1}(}{(1-a^2}}{16a}$$
$$= -\frac{3x}{128a(1-a^2x^2)^2} + \frac{3\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} - \frac{3x\tanh^{-1}(ax)^2}{16a(1-a^2x^2)^2} - \frac{9x\tanh^{-1}(ax)^2}{32a(1-a^2x^2)} - \frac{3\tanh}{3}$$
$$= -\frac{3x}{128a(1-a^2x^2)^2} - \frac{9x}{256a(1-a^2x^2)} + \frac{3\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} + \frac{9\tanh^{-1}(ax)}{32a^2(1-a^2x^2)} - \frac{3x\tan}{16a(1}$$
$$= -\frac{3x}{128a(1-a^2x^2)^2} - \frac{45x}{256a(1-a^2x^2)} - \frac{9\tanh^{-1}(ax)}{256a^2} + \frac{3\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} + \frac{9\tanh^{}}{32a^2(1-}$$
$$= -\frac{3x}{128a(1-a^2x^2)^2} - \frac{45x}{256a(1-a^2x^2)} - \frac{45\tanh^{-1}(ax)}{256a^2} + \frac{3\tanh^{-1}(ax)}{32a^2(1-a^2x^2)^2} + \frac{9\tanh}{32a^2(1}$$

Mathematica [A]

time = 0.04, size = 148, normalized size = 0.79

$$\frac{-102ax+90a^3x^3-48(-4+3a^2x^2)\tanh^{-1}(ax)+48ax(-5+3a^2x^2)\tanh^{-1}(ax)^2+(80+96a^2x^2-48a^4x^4)\tanh^{-1}(ax)^3+45(-1+a^2x^2)^2\log(1-ax)-45\log(1+ax)+90a^2x^2\log(1+ax)-45a^4x^4\log(1+ax)}{512a^2(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-102*a*x + 90*a^3*x^3 - 48*(-4 + 3*a^2*x^2)*ArcTanh[a*x] + 48*a*x*(-5 + 3*
a^2*x^2)*ArcTanh[a*x]^2 + (80 + 96*a^2*x^2 - 48*a^4*x^4)*ArcTanh[a*x]^3 + 4
5*(-1 + a^2*x^2)^2*Log[1 - a*x] - 45*Log[1 + a*x] + 90*a^2*x^2*Log[1 + a*x]
 - 45*a^4*x^4*Log[1 + a*x])/(512*a^2*(-1 + a^2*x^2)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 444.55, size = 2047, normalized size = 10.89

method	result
risch	$-\frac{(3a^4x^4-6a^2x^2-5)\ln(ax+1)^3}{256a^2(ax+1)(ax-1)(a^2x^2-1)}+\frac{3(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax-5\ln(-ax+1))\ln(ax+1)^2}{256a^2(ax+1)(ax-1)(a^2x^2-1)}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4/(a^2*x^2-1)^2*arctanh(a*x)^3+3/64*arctanh(a*x)^2/(a*x+1)^2+9/64*
arctanh(a*x)^2/(a*x+1)-9/64*arctanh(a*x)^2*ln(a*x+1)-3/64*arctanh(a*x)^2/(a
*x-1)^2+9/64*arctanh(a*x)^2/(a*x-1)+9/64*arctanh(a*x)^2*ln(a*x-1)+9/32*arct
anh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/256*(-16*arctanh(a*x)^3*a^2*x^2
+6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^4*x
^4-12*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^
2*x^2-6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-12*I*Pi*arctanh
(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+17*a*x+12*I*Pi*arctanh(a*x)^2*
a^4*x^4-6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^3+15*a^4*x^4*arctanh(a*x)+8*arctanh(a*x)^3+12*I*Pi*arctanh(a*x)
^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3-6*a^2*x^2*arctanh(a*x)-17*arctanh(a
*x)+6*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+24
*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^2*x^2+12*I*Pi*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^4*x^4-6*I*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^4*x^4-6*
I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^4*x^4-12*I*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^4*x^4-24*I*Pi*csgn(I/((a*x+1
```

```
)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2+12*I*Pi*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*a^2*x^2+12*I*Pi*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*a^2*x^2-15*a^3*x^3+8*arctanh(a*x)^
3*a^4*x^4+12*I*Pi*arctanh(a*x)^2+12*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^2*x^2+24*I*Pi*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*a^2*
x^2-6*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^4*x^4+6*I*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a
rctanh(a*x)^2*a^4*x^4-6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*a^4*x^4-12*I*Pi*csgn(I*(a*x+1)/(-a^2*x^
2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2*a^4*x^4+12*I*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/
(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*a^2*x^2-12*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x
)^2*a^2*x^2-24*I*Pi*arctanh(a*x)^2*a^2*x^2-6*I*Pi*arctanh(a*x)^2*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2+6*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-6*I*Pi*arctanh(a*x)^2*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-12*I*Pi*arctanh(
a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2)/
(a*x+1)^2/(a*x-1)^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(163) = 326$.

time = 0.28, size = 422, normalized size = 2.24

$$\frac{3\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}\right)\operatorname{artanh}\left(ax\right)^2}{64\,a}+\frac{3\left(\frac{\left(30\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^3+6\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2\log(ax-1)+2\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^3-34\,ax-3\left(5\,a^4x^4-10\,a^2x^2+2\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^2+5\right)\log(ax+1)+15\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)\right)a^2}{a^7x^4-2\,a^5x^2+a^3}-\frac{4\left(12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2+6\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1)-3\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^2-16\right)a\operatorname{artanh}(ax)}{a^6x^4-2\,a^4x^2+a^2}\right)}{512\,a}+\frac{\operatorname{artanh}\left(ax\right)^3}{4\left(a^2x^2-1\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)*arctanh(a*x)^2/a + 3/512*((30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*
x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4
 - 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1
) + 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2 + a
^3) - 4*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)
*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2))/a + 1/4*a
rctanh(a*x)^3/((a^2*x^2 - 1)^2*a^2)
```

Fricas [**A**]

time = 0.40, size = 140, normalized size = 0.74

$$\frac{90\,a^3x^3-2\,(3\,a^4x^4-6\,a^2x^2-5)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-102\,ax-3\,(15\,a^4x^4-6\,a^2x^2-17)\log\left(-\frac{ax+1}{ax-1}\right)}{512\,(a^6x^4-2\,a^4x^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(90*a^3*x^3 - 2*(3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))
^3 + 12*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 102*a*x - 3*(15*a
^4*x^4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^6*x^4 - 2*a^4*x^2 +
a^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(163) = 326.
time = 0.43, size = 342, normalized size = 1.82

$$-\frac{1}{2048}\left(4\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{4(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+6\left(\frac{(ax-1)^2\left(\frac{8(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}+\frac{(ax+1)^2}{(ax-1)^2a^3}-\frac{8(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+6\left(\frac{(ax-1)^2\left(\frac{16(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{16(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{3(ax-1)^2\left(\frac{32(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}+\frac{3(ax+1)^2}{(ax-1)^2a^3}-\frac{96(ax+1)}{(ax-1)a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(4*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - (a*
x + 1)^2/((a*x - 1)^2*a^3) + 4*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)/(a
*x - 1))^3 + 6*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) +
 (a*x + 1)^2/((a*x - 1)^2*a^3) - 8*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1
)/(a*x - 1))^2 + 6*((a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a
^3) - (a*x + 1)^2/((a*x - 1)^2*a^3) + 16*(a*x + 1)/((a*x - 1)*a^3))*log(-(a
*x + 1)/(a*x - 1)) + 3*(a*x - 1)^2*(32*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^
2*a^3) + 3*(a*x + 1)^2/((a*x - 1)^2*a^3) - 96*(a*x + 1)/((a*x - 1)*a^3))*a
```

Mupad [**B**]
time = 2.69, size = 414, normalized size = 2.20

$$-\frac{48\ln(1-ax)-48\ln(ax+1)+45\operatorname{atanh}(ax)+51ax-5\ln(ax+1)^3+5\ln(1-ax)^3-15\ln(ax+1)\ln(1-ax)^2+15\ln(ax+1)^2\ln(1-ax)-45a^3x^3-6a^2x^2\ln(ax+1)^3+6a^2x^2\ln(1-ax)^3-18a^3x^3\ln(ax+1)^2-18a^3x^3\ln(1-ax)^2+3a^4x^4\ln(ax+1)^3-3a^4x^4\ln(1-ax)^3-90a^2x^2\operatorname{atanh}(ax)+45a^4x^4\operatorname{atanh}(ax)+30ax\ln(ax+1)^2+30ax\ln(1-ax)^2+36a^2x^2\ln(ax+1)-36a^2x^2\ln(1-ax)-60ax\ln(ax+1)\ln(1-ax)-18a^2x^2\ln(ax+1)\ln(1-ax)^2+18a^2x^2\ln(ax+1)^2\ln(1-ax)+9a^4x^4\ln(ax+1)\ln(1-ax)^2-9a^4x^4\ln(ax+1)^2\ln(1-ax)+36a^3x^3\ln(ax+1)\ln(1-ax)}{256a^2(a^2x^2-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

```
[Out] -(48*log(1 - a*x) - 48*log(a*x + 1) + 45*atanh(a*x) + 51*a*x - 5*log(a*x +
1)^3 + 5*log(1 - a*x)^3 - 15*log(a*x + 1)*log(1 - a*x)^2 + 15*log(a*x + 1)^
```

```
2*log(1 - a*x) - 45*a^3*x^3 - 6*a^2*x^2*log(a*x + 1)^3 + 6*a^2*x^2*log(1 -
a*x)^3 - 18*a^3*x^3*log(a*x + 1)^2 - 18*a^3*x^3*log(1 - a*x)^2 + 3*a^4*x^4*
log(a*x + 1)^3 - 3*a^4*x^4*log(1 - a*x)^3 - 90*a^2*x^2*atanh(a*x) + 45*a^4*
x^4*atanh(a*x) + 30*a*x*log(a*x + 1)^2 + 30*a*x*log(1 - a*x)^2 + 36*a^2*x^2
*log(a*x + 1) - 36*a^2*x^2*log(1 - a*x) - 60*a*x*log(a*x + 1)*log(1 - a*x)
- 18*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 + 18*a^2*x^2*log(a*x + 1)^2*log(1
- a*x) + 9*a^4*x^4*log(a*x + 1)*log(1 - a*x)^2 - 9*a^4*x^4*log(a*x + 1)^2*l
og(1 - a*x) + 36*a^3*x^3*log(a*x + 1)*log(1 - a*x))/(256*a^2*(a^2*x^2 - 1)^
2)
```

3.317 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=203

$$-\frac{3}{128a\,(1-a^2x^2)^2}-\frac{45}{128a\,(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}+\frac{45x\tanh^{-1}(ax)}{64\,(1-a^2x^2)}+\frac{45\tanh^{-1}(ax)^2}{128a}-\frac{3\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2}$$

```
[Out] -3/128/a/(-a^2*x^2+1)^2-45/128/a/(-a^2*x^2+1)+3/32*x*arctanh(a*x)/(-a^2*x^2
+1)^2+45/64*x*arctanh(a*x)/(-a^2*x^2+1)+45/128*arctanh(a*x)^2/a-3/16*arctan
h(a*x)^2/a/(-a^2*x^2+1)^2-9/16*arctanh(a*x)^2/a/(-a^2*x^2+1)+1/4*x*arctanh(
a*x)^3/(-a^2*x^2+1)^2+3/8*x*arctanh(a*x)^3/(-a^2*x^2+1)+3/32*arctanh(a*x)^4
/a
```

Rubi [A]
time = 0.13, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6111, 6103, 6141, 267, 6107}

$$-\frac{45}{128a\,(1-a^2x^2)}-\frac{3}{128a\,(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^3}{8\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{4\,(1-a^2x^2)^2}-\frac{9\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2}+\frac{45x\tanh^{-1}(ax)}{64\,(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}+\frac{3\tanh^{-1}(ax)^4}{32a}+\frac{45\tanh^{-1}(ax)^2}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

```
[Out] -3/(128*a*(1 - a^2*x^2)^2) - 45/(128*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/
(32*(1 - a^2*x^2)^2) + (45*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) + (45*ArcTanh
[a*x]^2)/(128*a) - (3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) - (9*ArcTanh[a
*x]^2)/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (3*x
*ArcTanh[a*x]^3)/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^4)/(32*a)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^3}\,dx &= -\frac{3\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)^3}{4\,(1-a^2x^2)^2}+\frac{3}{8}\int\frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx+\frac{3}{4}\int\frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^2}\,dx\\
&= -\frac{3}{128a\,(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)^2}{16a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)^3}{4\,(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(\ldots}{8\,(1-a^2\ldots}\\
&= -\frac{3}{128a\,(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}+\frac{9x\tanh^{-1}(ax)}{64\,(1-a^2x^2)}+\frac{9\tanh^{-1}(ax)^2}{128a}-\frac{3\tanh^{-1}(a\ldots}{16a\,(1-a^2\ldots}\\
&= -\frac{3}{128a\,(1-a^2x^2)^2}-\frac{9}{128a\,(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}+\frac{45x\tanh^{-1}(ax)}{64\,(1-a^2x^2)}+\frac{45\tanh^{-1}\ldots}{128a\ldots}\\
&= -\frac{3}{128a\,(1-a^2x^2)^2}-\frac{45}{128a\,(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{32\,(1-a^2x^2)^2}+\frac{45x\tanh^{-1}(ax)}{64\,(1-a^2x^2)}+\frac{45\tanh^{-1}\ldots}{128a\ldots}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 111, normalized size = 0.55

$$\frac{-48+45a^2x^2+(102ax-90a^3x^3)\tanh^{-1}(ax)+3(-17-6a^2x^2+15a^4x^4)\tanh^{-1}(ax)^2+(80ax-48a^3x^3)\tanh^{-1}(ax)^3+12(-1+a^2x^2)^2\tanh^{-1}(ax)^4}{128a\,(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

```
[Out] (-48 + 45*a^2*x^2 + (102*a*x - 90*a^3*x^3)*ArcTanh[a*x] + 3*(-17 - 6*a^2*x^
2 + 15*a^4*x^4)*ArcTanh[a*x]^2 + (80*a*x - 48*a^3*x^3)*ArcTanh[a*x]^3 + 12*
(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)/(128*a*(-1 + a^2*x^2)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 466.62, size = 2059, normalized size = 10.14

method	result
risch	$\frac{3\ln(ax+1)^4}{512a} - \frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax+3\ln(-ax+1))\ln(ax+1)^3}{128(a^2x^2-1)^2a} + \frac{3\left(6a^4x^4\ln(-ax+\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/16*arctanh(a*x)^3/(a*x+1)^2-3/16*arctanh(a*x)^3/(a*x+1)+3/16*arctan
h(a*x)^3*ln(a*x+1)+1/16*arctanh(a*x)^3/(a*x-1)^2-3/16*arctanh(a*x)^3/(a*x-1
)-3/16*arctanh(a*x)^3*ln(a*x-1)-3/8*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^
(1/2))-3/1024*(65-32*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))+32*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*arctanh(a*x)^3*Pi*a^4*x^4+32*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*
x)^3*Pi*a^4*x^4-64*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3*Pi
*a^4*x^4+64*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3*Pi*a^4*x^
4-64*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a
*x)^3*Pi*a^2*x^2-64*I*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3*Pi*a^2
*x^2+128*I*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3*Pi*a^2*x^2-3
2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi*a^4*x^
4+64*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi*a^2
*x^2+6*a^2*x^2-272*a*x*arctanh(a*x)-63*a^4*x^4+240*a^3*x^3*arctanh(a*x)-120
*a^4*x^4*arctanh(a*x)^2+48*a^2*x^2*arctanh(a*x)^2-32*arctanh(a*x)^4+136*arc
tanh(a*x)^2-32*arctanh(a*x)^4*a^4*x^4+64*arctanh(a*x)^4*a^2*x^2-128*I*csgn(
I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3*Pi*a^2*x^2+32*I*csgn(I*(a*x+
1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi*a^
4*x^4+64*I*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))
^2*arctanh(a*x)^3*Pi*a^4*x^4-32*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi*a^4*x^4+3
2*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x
```

```
+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi*a^4*x^4-64*I*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi*a^2*x^2-128
*I*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arcta
nh(a*x)^3*Pi*a^2*x^2+64*I*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi*a^2*x^2-64*I*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-
a^2*x^2+1)+1))*arctanh(a*x)^3*Pi*a^2*x^2+32*I*Pi*arctanh(a*x)^3*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2+128*I*arctanh(a*x)^3*Pi*a^2*x^2-64*I*arctanh(a*x)^3*Pi*a^4*x^4-64*I
*Pi*arctanh(a*x)^3+32*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+64*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-32*I*Pi*arctanh(a*x)^3*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^2+32*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-64*I*Pi*a
rctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+64*I*Pi*arctanh(a*x)^3*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+32*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3)/(a*x+1)^2/(a*x-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(175) = 350$.
time = 0.32, size = 663, normalized size = 3.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^3 + 3/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1
) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)^2/(a^6*
x^4 - 2*a^4*x^2 + a^2) - 3/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 -
 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2
*x^2 + 1)*log(a*x - 1)^4 - 60*a^2*x^2 + 3*(5*a^4*x^4 - 10*a^2*x^2 + 2*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)^2 + 15*(a^4*x^4 - 2*a
^2*x^2 + 1)*log(a*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3
+ 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1) + 64)*a^2/(a^8*x^
4 - 2*a^6*x^2 + a^4) + 4*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x
+ 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 - 10*a^2*x^2 + 2
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1) + 15*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^4 - 2*a^5*x^2 + a^3))*a
```

Fricas [A]
time = 0.35, size = 166, normalized size = 0.82

$$\frac{3\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+180\,a^{2}x^{2}-8\left(3\,a^{3}x^{3}-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}+3\left(15\,a^{4}x^{4}-6\,a^{2}x^{2}-17\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-12\left(15\,a^{3}x^{3}-17\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-192}{512\left(a^{5}x^{4}-2\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")

[Out] 1/512*(3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 180*a^2*x^2 - 8*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(15*a^4*x^4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^3*x^3 - 17*a*x)*log(-(a*x + 1)/(a*x - 1)) - 192)/(a^5*x^4 - 2*a^3*x^2 + a)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{3}(ax)}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**3,x)

[Out] -Integral(atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")

[Out] integrate(-arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)

Mupad [B]
time = 2.29, size = 736, normalized size = 3.63

$$\frac{\frac{45ax^2}{2}-\frac{24}{a}}{64a^4x^4-128a^2x^2+64}-\ln(ax+1)^2\left(\frac{\frac{3}{16a^2}-\frac{9x^2}{64}}{\frac{1}{a}-2ax^2+a^3x^4}-\frac{45}{512a}\right)-\ln(1-ax)^3\left(\frac{3\ln(ax+1)}{128a}+\frac{\frac{5x}{8}-\frac{3a^2x^3}{8}}{8a^4x^4-16a^2x^2+8}\right)-\ln(1-ax)\left(\frac{3\ln(ax+1)^3}{128a}+\ln(ax+1)\left(\frac{\frac{21x}{2}+9ax^2-\frac{12}{a}-\frac{15a^2x^3}{2}}{64a^4x^4-128a^2x^2+64}-\frac{\frac{21x}{2}-9ax^2+\frac{12}{a}-\frac{15a^2x^3}{2}}{64a^4x^4-128a^2x^2+64}+\frac{45(a^4x^4-2a^2x^2+1)}{4a(64a^4x^4-128a^2x^2+64)}\right)+\frac{\frac{9x}{2}+\frac{3ax^2}{2}-\frac{3}{2a}-\frac{9a^2x^3}{2}}{128a^4x^4-256a^2x^2+128}+\frac{21x+\frac{33ax^2}{2}-\frac{39}{2a}-18a^2x^3}{128a^4x^4-256a^2x^2+128}+\frac{\frac{51x}{2}-18ax^2+\frac{21}{a}-\frac{45a^2x^3}{2}}{128a^4x^4-256a^2x^2+128}+\frac{\ln(ax+1)^2(30x-18a^2x^3)}{128a^4x^4-256a^2x^2+128}\right)+\frac{3\ln(ax+1)^4}{512a}+\frac{3\ln(1-ax)^4}{512a}+\ln(1-ax)^2\left(\frac{9\ln(ax+1)^2}{256a}+\frac{45}{512a}-\frac{\frac{21x}{2}-9ax^2+\frac{12}{a}-\frac{15a^2x^3}{2}}{128a^4x^4-256a^2x^2+128}+\frac{\frac{21x}{2}+9ax^2-\frac{12}{a}-\frac{15a^2x^3}{2}}{128a^4x^4-256a^2x^2+128}+\frac{\ln(ax+1)(30x-18a^2x^3)}{128a^4x^4-256a^2x^2+128}\right)+\frac{\ln(ax+1)\left(\frac{51x}{128a}-\frac{45ax^3}{128}\right)}{\frac{1}{a}-2ax^2+a^3x^4}+\frac{\ln(ax+1)^3\left(\frac{5x}{64a}-\frac{3ax^3}{64}\right)}{\frac{1}{a}-2ax^2+a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-atanh(a*x)^3/(a^2*x^2 - 1)^3,x)

[Out] ((45*a*x^2)/2 - 24/a)/(64*a^4*x^4 - 128*a^2*x^2 + 64) - log(a*x + 1)^2*((3/(16*a^2) - (9*x^2)/64)/(1/a - 2*a*x^2 + a^3*x^4) - 45/(512*a)) - log(1 - a*x)^3*((3*log(a*x + 1))/(128*a) + ((5*x)/8 - (3*a^2*x^3)/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8)) - log(1 - a*x)*((3*log(a*x + 1)^3)/(128*a) + log(a*x + 1)*(((21*x)/2 + 9*a*x^2 - 12/a - (15*a^2*x^3)/2)/(64*a^4*x^4 - 128*a^2*x^2 + 64) - ((21*x)/2 - 9*a*x^2 + 12/a - (15*a^2*x^3)/2)/(64*a^4*x^4 - 128*a^2*x^2 + 64) + (45*(a^4*x^4 - 2*a^2*x^2 + 1))/(4*a*(64*a^4*x^4 - 128*a^2*x^2 + 64))) + ((9*x)/2 + (3*a*x^2)/2 - 3/(2*a) - (9*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2*x^2 + 128) + (21*x + (33*a*x^2)/2 - 39/(2*a) - 18*a^2*x^3)/(128*a^4*x^4 -

```
256*a^2*x^2 + 128) + ((51*x)/2 - 18*a*x^2 + 21/a - (45*a^2*x^3)/2)/(128*a^4
*x^4 - 256*a^2*x^2 + 128) + (log(a*x + 1)^2*(30*x - 18*a^2*x^3))/(128*a^4*x
^4 - 256*a^2*x^2 + 128)) + (3*log(a*x + 1)^4)/(512*a) + (3*log(1 - a*x)^4)/
(512*a) + log(1 - a*x)^2*((9*log(a*x + 1)^2)/(256*a) + 45/(512*a) - ((21*x)
/2 - 9*a*x^2 + 12/a - (15*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2*x^2 + 128) + (
(21*x)/2 + 9*a*x^2 - 12/a - (15*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2*x^2 + 12
8) + (log(a*x + 1)*(30*x - 18*a^2*x^3))/(128*a^4*x^4 - 256*a^2*x^2 + 128))
+ (log(a*x + 1)*((51*x)/(128*a) - (45*a*x^3)/128))/(1/a - 2*a*x^2 + a^3*x^4
) + (log(a*x + 1)^3*((5*x)/(64*a) - (3*a*x^3)/64))/(1/a - 2*a*x^2 + a^3*x^4
)
```

3.318 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=277

$$-\frac{3ax}{128\left(1-a^2x^2\right)^2}-\frac{141ax}{256\left(1-a^2x^2\right)}-\frac{141}{256}\tanh^{-1}(ax)+\frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}-3$$

```
[Out] -3/128*a*x/(-a^2*x^2+1)^2-141/256*a*x/(-a^2*x^2+1)-141/256*arctanh(a*x)+3/3
2*arctanh(a*x)/(-a^2*x^2+1)^2+33/32*arctanh(a*x)/(-a^2*x^2+1)-3/16*a*x*arct
anh(a*x)^2/(-a^2*x^2+1)^2-33/32*a*x*arctanh(a*x)^2/(-a^2*x^2+1)-11/32*arcta
nh(a*x)^3+1/4*arctanh(a*x)^3/(-a^2*x^2+1)^2+1/2*arctanh(a*x)^3/(-a^2*x^2+1)
+1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*polyl
og(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylog(4,-
1+2/(a*x+1))
```

Rubi [A]

time = 0.45, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {6177, 6135, 6079, 6095, 6203, 6207, 6745, 6141, 6103, 205, 212, 6111}

$$-\frac{141ax}{256\left(1-a^2x^2\right)}-\frac{3ax}{128\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{33ax\tanh^{-1}(ax)^2}{32\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3}{4}\text{Li}_4\left(\frac{2}{ax+1}-1\right)-\frac{3}{2}\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)^2-\frac{3}{2}\text{Li}_3\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{1}{4}\tanh^{-1}(ax)^4-\frac{11}{32}\tanh^{-1}(ax)^3-\frac{141}{256}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (-3*a*x)/(128*(1 - a^2*x^2)^2) - (141*a*x)/(256*(1 - a^2*x^2)) - (141*ArcTa
nh[a*x])/256 + (3*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) + (33*ArcTanh[a*x])/(3
2*(1 - a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (33*a*x*Ar
cTanh[a*x]^2)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^3)/32 + ArcTanh[a*x]^3/
(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 +
 ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/
(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLo
g[4, -1 + 2/(1 + a*x)])/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
```

```
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6207

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx + a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}$$

$$= \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 -$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} - \frac{33ax\tanh^{-1}(ax)^2}{32\left(1-a^2x^2\right)} - \frac{11}{32}\tan$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} - \frac{9ax}{256\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(}{16\left(1-a^2x\right.}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} - \frac{141ax}{256\left(1-a^2x^2\right)} - \frac{9}{256}\tanh^{-1}(ax) + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{33\tanh^{-1}(}{32\left(1-a^2x\right.}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2} - \frac{141ax}{256\left(1-a^2x^2\right)} - \frac{141}{256}\tanh^{-1}(ax) + \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{33\tanh^{-1}(}{32\left(1-a^2x\right.}$$

Mathematica [A]
time = 0.15, size = 189, normalized size = 0.68

$$\frac{16\pi^4 - 256\tanh^{-1}(ax)^4 + 576\tanh^{-1}(ax)\cosh\left(2\tanh^{-1}(ax)\right) + 384\tanh^{-1}(ax)^3\cosh\left(2\tanh^{-1}(ax)\right) + 12\tanh^{-1}(ax)\cosh\left(4\tanh^{-1}(ax)\right) + 32\tanh^{-1}(ax)^3\cosh\left(4\tanh^{-1}(ax)\right) + 1024\tanh^{-1}(ax)^3\log\left(1-e^{2\tanh^{-1}(ax)}\right) + 1536\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 1536\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 768\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right) - 288\sinh\left(2\tanh^{-1}(ax)\right) - 576\tanh^{-1}(ax)^2\sinh\left(2\tanh^{-1}(ax)\right) - 3\sinh\left(4\tanh^{-1}(ax)\right) - 24\tanh^{-1}(ax)^2\sinh\left(4\tanh^{-1}(ax)\right)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (16*Pi^4 - 256*ArcTanh[a*x]^4 + 576*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 384
*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 12*ArcTanh[a*x]*Cosh[4*ArcTanh[a*x]]
 + 32*ArcTanh[a*x]^3*Cosh[4*ArcTanh[a*x]] + 1024*ArcTanh[a*x]^3*Log[1 - E^(
2*ArcTanh[a*x])] + 1536*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 153
6*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 768*PolyLog[4, E^(2*ArcTanh
[a*x])] - 288*Sinh[2*ArcTanh[a*x]] - 576*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]
] - 3*Sinh[4*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[4*ArcTanh[a*x]])/1024
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.24, size = 1533, normalized size = 5.53

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x)
```

```
[Out] -9/32*arctanh(a*x)*(a*x-1)/(a*x+1)-9/32*arctanh(a*x)*(a*x+1)/(a*x-1)+3/512*
arctanh(a*x)*(a*x-1)^2/(a*x+1)^2-1/4*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))+1/2*I*Pi*arctanh(a*x)^3+6*polylog(4,-(a*x+1)/(
-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6
*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4-11/3
2*arctanh(a*x)^3+1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))+3/512*arctanh(a*x)*(a*x+1)^2/(a*x-1)^2+ln(2)*arctanh
(a*x)^3-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/16*arctanh(a*x)^3/(a*
x+1)^2+1/16*arctanh(a*x)^3/(a*x-1)^2+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/4*I*Pi*arctanh(a*x)^3*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^3-1/2*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))^2+1/2*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/
2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^3+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*l
n(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/256*arctanh(a*x)^2*(a*x+1)^2/(a*x-1)^2-9/
32*arctanh(a*x)^2*(a*x-1)/(a*x+1)+3/256*arctanh(a*x)^2*(a*x-1)^2/(a*x+1)^2+
9/32*(a*x+1)*arctanh(a*x)^2/(a*x-1)-3/2048*(a*x+1)^2/(a*x-1)^2+5/16*arctanh
(a*x)^3/(a*x+1)-1/2*arctanh(a*x)^3*ln(a*x+1)-5/16*arctanh(a*x)^3/(a*x-1)-1/
2*arctanh(a*x)^3*ln(a*x-1)+arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-9/
64*(a*x-1)/(a*x+1)+9/64*(a*x+1)/(a*x-1)+3/2048*(a*x-1)^2/(a*x+1)^2+arctanh(
a*x)^3*ln(a*x)+1/2*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))^2-1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I*
Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^
2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I*Pi*arctanh(a*x)^3*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^2+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))+1/4*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-
a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4
 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log(-a*x + 1)^3)/(a^4*x^4 - 2*a^2*x^2 +
 1) - 1/8*integrate(1/4*(4*log(a*x + 1)^3 - 12*log(a*x + 1)^2*log(-a*x + 1)
```

```
 + 3*(2*a^4*x^4 + 2*a^3*x^3 - 3*a^2*x^2 - 3*a*x + 2*(a^6*x^6 + a^5*x^5 - 2*
a^4*x^4 - 2*a^3*x^3 + a^2*x^2 + a*x + 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^
6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
[Out] integral(-arctanh(a*x)^3/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}^{3}(ax)}{a^{6}x^{7}-3a^{4}x^{5}+3a^{2}x^{3}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**3,x)
[Out] -Integral(atanh(a*x)**3/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="giac")
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\operatorname{atanh}(a\,x)}^3}{x\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x*(a^2*x^2 - 1)^3),x)
[Out] -int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^3), x)
```

3.319 $\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=281

$$-\frac{3a}{128\left(1-a^2x^2\right)^2}-\frac{93a}{128\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{93}{128}a\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(a$$

```
[Out] -3/128*a/(-a^2*x^2+1)^2-93/128*a/(-a^2*x^2+1)+3/32*a^2*x*arctanh(a*x)/(-a^2
*x^2+1)^2+93/64*a^2*x*arctanh(a*x)/(-a^2*x^2+1)+93/128*a*arctanh(a*x)^2-3/1
6*a*arctanh(a*x)^2/(-a^2*x^2+1)^2-21/16*a*arctanh(a*x)^2/(-a^2*x^2+1)+a*arc
tanh(a*x)^3-arctanh(a*x)^3/x+1/4*a^2*x*arctanh(a*x)^3/(-a^2*x^2+1)^2+7/8*a^
2*x*arctanh(a*x)^3/(-a^2*x^2+1)+15/32*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*l
n(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+
2/(a*x+1))
```

Rubi [A]
time = 0.50, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6177, 6129, 6037, 6135, 6079, 6095, 6203, 6745, 6103, 6141, 267, 6111, 6107}

$$-\frac{93a}{128\left(1-a^2x^2\right)}-\frac{3a}{128\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^3}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{21a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3}{2}a\text{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\tanh^{-1}(ax)+\frac{15}{32}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)^3}{x}+\frac{93}{128}a\tanh^{-1}(ax)^2+3a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] (-3*a)/(128*(1 - a^2*x^2)^2) - (93*a)/(128*(1 - a^2*x^2)) + (3*a^2*x*ArcTan
h[a*x])/(32*(1 - a^2*x^2)^2) + (93*a^2*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) +
 (93*a*ArcTanh[a*x]^2)/128 - (3*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (2
1*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/
x + (a^2*x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^3)/(
8*(1 - a^2*x^2)) + (15*a*ArcTanh[a*x]^4)/32 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/
(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3
, -1 + 2/(1 + a*x)])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

```
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^3}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}\,dx \\
&= -\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{1}{8}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx + \frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}\cdots}{(1-\cdots} \\
&= -\frac{3a}{128\left(1-a^2x^2\right)^2} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\,\mathrm{t}\cdots}{8\,(1\cdots} \\
&= -\frac{3a}{128\left(1-a^2x^2\right)^2} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{9a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{9}{128}a\tanh^{-1}(ax)^2 - \frac{3a\,\mathrm{t}\cdots}{16\,(\cdots} \\
&= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{9a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\cdots \\
&= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{93a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\cdots \\
&= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{93a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\cdots \\
&= -\frac{3a}{128\left(1-a^2x^2\right)^2} - \frac{93a}{128\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} + \frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)} + \frac{93}{128}a\cdots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.43, size = 218, normalized size = 0.78

$$-a\left(-\frac{i\pi^3}{8}+\tanh^{-1}(ax)^3+\frac{\tanh^{-1}(ax)^3}{ax}-\frac{ax\tanh^{-1}(ax)^3}{1-a^2x^2}-\frac{15}{32}\tanh^{-1}(ax)^4+\frac{3}{8}\cosh\left(2\tanh^{-1}(ax)\right)+\frac{3}{4}\tanh^{-1}(ax)^2\cosh\left(2\tanh^{-1}(ax)\right)+\frac{3\cosh\left(4\tanh^{-1}(ax)\right)}{1024}+\frac{3}{128}\tanh^{-1}(ax)^2\cosh\left(4\tanh^{-1}(ax)\right)-3\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-\frac{3}{4}\tanh^{-1}(ax)\sinh\left(2\tanh^{-1}(ax)\right)-\frac{3}{256}\tanh^{-1}(ax)\sinh\left(4\tanh^{-1}(ax)\right)-\frac{1}{32}\tanh^{-1}(ax)^3\sinh\left(4\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*((-1/8*I)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - (a*x*ArcTanh[a
*x]^3)/(1 - a^2*x^2) - (15*ArcTanh[a*x]^4)/32 + (3*Cosh[2*ArcTanh[a*x]])/8
+ (3*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]])/4 + (3*Cosh[4*ArcTanh[a*x]])/1024
 + (3*ArcTanh[a*x]^2*Cosh[4*ArcTanh[a*x]])/128 - 3*ArcTanh[a*x]^2*Log[1 - E
^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + (3*Pol
yLog[3, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/4 -
(3*ArcTanh[a*x]*Sinh[4*ArcTanh[a*x]])/256 - (ArcTanh[a*x]^3*Sinh[4*ArcTanh[
a*x]])/32))
```

Maple [A]
time = 17.69, size = 351, normalized size = 1.25

method	result

derivativedivides	$a\left(\frac{15\,\mathrm{arctanh}(ax)^4}{32}+\frac{\left(32\,\mathrm{arctanh}(ax)^3-24\,\mathrm{arctanh}(ax)^2+12\,\mathrm{arctanh}(ax)-3\right)(ax+1)^2}{2048(ax-1)^2}-\frac{(ax+1)\left(4\,\mathrm{arctanh}(ax)^3\right.}{}\right.$
default	$a\left(\frac{15\,\mathrm{arctanh}(ax)^4}{32}+\frac{\left(32\,\mathrm{arctanh}(ax)^3-24\,\mathrm{arctanh}(ax)^2+12\,\mathrm{arctanh}(ax)-3\right)(ax+1)^2}{2048(ax-1)^2}-\frac{(ax+1)\left(4\,\mathrm{arctanh}(ax)^3\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(15/32*arctanh(a*x)^4+1/2048*(32*arctanh(a*x)^3-24*arctanh(a*x)^2+12*arct
anh(a*x)-3)*(a*x+1)^2/(a*x-1)^2-1/16*(a*x+1)*(4*arctanh(a*x)^3-6*arctanh(a*
x)^2+6*arctanh(a*x)-3)/(a*x-1)+1/16*(4*arctanh(a*x)^3+6*arctanh(a*x)^2+6*ar
ctanh(a*x)+3)*(a*x-1)/(a*x+1)-1/2048*(32*arctanh(a*x)^3+24*arctanh(a*x)^2+1
2*arctanh(a*x)+3)*(a*x-1)^2/(a*x+1)^2+arctanh(a*x)^3/a/x*(a*x-1)-2*arctanh(
a*x)^3+3*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*pol
ylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+
3*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,
-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\mathrm{atanh}^3\,(ax)}{a^6x^8-3a^4x^6+3a^2x^4-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\mathrm{atanh}(a\,x)^3}{x^2\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^3), x)
```

3.320 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}$$

```
[Out] 1/4*arctanh(a*x)^(3/2)/a+1/32*erf(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)/a-1/32*erfi(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+1/256*erf(2*
arctanh(a*x)^(1/2))*Pi^(1/2)/a-1/256*erfi(2*arctanh(a*x)^(1/2))*Pi^(1/2)/a+
1/4*sinh(2*arctanh(a*x))*arctanh(a*x)^(1/2)/a+1/32*sinh(4*arctanh(a*x))*arc
tanh(a*x)^(1/2)/a
```

Rubi [A]
time = 0.15, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6115, 3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

```
[Out] ArcTanh[a*x]^(3/2)/(4*a) + (Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(256*a) + (
Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi]*Erfi[2*Sqrt[
ArcTanh[a*x]]])/(256*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16
*a) + (Sqrt[ArcTanh[a*x]]*Sinh[2*ArcTanh[a*x]])/(4*a) + (Sqrt[ArcTanh[a*x]]
*Sinh[4*ArcTanh[a*x]])/(32*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^3}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\,\cosh^4(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8} + \frac{1}{2}\sqrt{x}\,\cosh(2x) + \frac{1}{8}\sqrt{x}\,\cosh(4x)\right)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\text{Subst}\left(\int \sqrt{x}\,\cosh(4x)\,dx, x, \tanh^{-1}(ax)\right)}{8a} + \frac{\text{Subst}\left(\int \sqrt{x}\,\cosh(2x\right.}{2\ldots}$$
$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\tanh\ldots\right.}{32a}$$
$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\tanh\ldots\right.}{32a}$$
$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{4a} + \frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\tanh\ldots\right.}{32a}$$
$$= \frac{\tanh^{-1}(ax)^{3/2}}{4a} + \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a} - \ldots$$

Mathematica [A]
time = 0.30, size = 152, normalized size = 0.90

$$\frac{\frac{32\sqrt{\tanh^{-1}(ax)}\left(5ax-3a^3x^3+2(-1+a^2x^2)^2\tanh^{-1}(ax)\right)}{(-1+a^2x^2)^2} + \frac{\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\tanh^{-1}(ax)\right)}{\sqrt{-\tanh^{-1}(ax)}} + \frac{8\sqrt{2}\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\tanh^{-1}(ax)\right)}{\sqrt{-\tanh^{-1}(ax)}} - 8\sqrt{2}\,\Gamma\left(\frac{1}{2},2\tanh^{-1}(ax)\right) - \Gamma\left(\frac{1}{2},4\tanh^{-1}(ax)\right)}{256a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

```
[Out] ((32*Sqrt[ArcTanh[a*x]]*(5*a*x - 3*a^3*x^3 + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x
]))/(-1 + a^2*x^2)^2 + (Sqrt[ArcTanh[a*x]]*Gamma[1/2, -4*ArcTanh[a*x]])/Sqr
t[-ArcTanh[a*x]] + (8*Sqrt[2]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -2*ArcTanh[a*x]
])/Sqrt[-ArcTanh[a*x]] - 8*Sqrt[2]*Gamma[1/2, 2*ArcTanh[a*x]] - Gamma[1/2,
4*ArcTanh[a*x]])/(256*a)
```

Maple [F]
time = 9.58, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(-a^2x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\operatorname{atanh}\left(ax\right)}}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(sqrt(atanh(a*x))/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{\operatorname{atanh}\left(a\,x\right)}}{{\left(a^2\,x^2-1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1)^3,x)
[Out] int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1)^3, x)
```

3.321 $\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Defer[Int][x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Mathematica [A]
time = 6.95, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Maple [A]
time = 5.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-a^2x^2+1)^3 \operatorname{arctanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-x^6/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{a^6x^6\operatorname{atanh}(ax) - 3a^4x^4\operatorname{atanh}(ax) + 3a^2x^2\operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**6/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^6}{\operatorname{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.322 $\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Defer[Int][x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Mathematica [A]
time = 14.59, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Maple [A]
time = 5.28, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(-a^2x^2+1)^3 \operatorname{arctanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^5}{a^6x^6\operatorname{atanh}(ax) - 3a^4x^4\operatorname{atanh}(ax) + 3a^2x^2\operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**5/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^5}{\operatorname{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
[Out] -int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.323 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=41

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

```
[Out] -1/2*Chi(2*arctanh(a*x))/a^5+1/8*Chi(4*arctanh(a*x))/a^5+3/8*ln(arctanh(a*x
))/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 3393, 3382}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -1/2*CoshIntegral[2*ArcTanh[a*x]]/a^5 + CoshIntegral[4*ArcTanh[a*x]]/(8*a^5
) + (3*Log[ArcTanh[a*x]])/(8*a^5)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} - \frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x} dx, x, \tanh^{-1}(ax)\right)}{8a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(\cdots}{x}\right.}{}$$
$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5} + \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.76

$$\frac{-4\text{Chi}\left(2\tanh^{-1}(ax)\right) + \text{Chi}\left(4\tanh^{-1}(ax)\right) + 3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (-4*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*ArcTanh[a*x]] + 3*Log[Arc
Tanh[a*x]])/(8*a^5)
```

Maple [A]
time = 1.64, size = 31, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\frac{3\ln(\text{arctanh}(ax))}{8} - \frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{2} + \frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{8}}{a^5}$	31
default	$\frac{\frac{3\ln(\text{arctanh}(ax))}{8} - \frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{2} + \frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{8}}{a^5}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(3/8*ln(arctanh(a*x))-1/2*Chi(2*arctanh(a*x))+1/8*Chi(4*arctanh(a*x))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(35) = 70.

time = 0.38, size = 118, normalized size = 2.88

$$\frac{6 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + \mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - 4\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - 4\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a
*x + 1)) - 4*log_integral(-(a*x + 1)/(a*x - 1)) - 4*log_integral(-(a*x - 1)
/(a*x + 1)))/a^5
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{a^6x^6\,\mathrm{atanh}\,(ax) - 3a^4x^4\,\mathrm{atanh}\,(ax) + 3a^2x^2\,\mathrm{atanh}\,(ax) - \mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^4}{\mathrm{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.324 $\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=29

$$-\frac{\text{Shi}(2\tanh^{-1}(ax))}{4a^4}+\frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^4}$$

```
[Out] -1/4*Shi(2*arctanh(a*x))/a^4+1/8*Shi(4*arctanh(a*x))/a^4
```

Rubi [A]
time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 5556, 3379}

$$\frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^4}-\frac{\text{Shi}(2\tanh^{-1}(ax))}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -1/4*SinhIntegral[2*ArcTanh[a*x]]/a^4 + SinhIntegral[4*ArcTanh[a*x]]/(8*a^4
)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4}$$

Mathematica [A]
time = 0.08, size = 24, normalized size = 0.83

$$\frac{-2\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] (-2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^4)
```

Maple [A]
time = 1.96, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8} - \frac{\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{4}}{a^4}$	24
default	$\frac{\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8} - \frac{\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{4}}{a^4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^4*(1/8*Shi(4*arctanh(a*x))-1/4*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(25) = 50$.

time = 0.41, size = 102, normalized size = 3.52

$$\frac{\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-2\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+2\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*log_integral(-(a*x + 1
)/(a*x - 1)) + 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^4
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{a^6x^6\,\text{atanh}\,(ax)-3a^4x^4\,\text{atanh}\,(ax)+3a^2x^2\,\text{atanh}\,(ax)-\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x^3}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.325 $\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

```
[Out] 1/8*Chi(4*arctanh(a*x))/a^3-1/8*ln(arctanh(a*x))/a^3
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 5556, 3382}

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[4*ArcTanh[a*x]]/(8*a^3) - Log[ArcTanh[a*x]]/(8*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8x} + \frac{\cosh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x} dx, x, \tanh^{-1}(ax)\right)}{8a^3}$$
$$= \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 0.81

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right) - \log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] (CoshIntegral[4*ArcTanh[a*x]] - Log[ArcTanh[a*x]])/(8*a^3)
```

Maple [A]
time = 1.44, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\ln(\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{8}}{a^3}$	22
default	$\frac{-\frac{\ln(\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{8}}{a^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(-1/8*ln(arctanh(a*x))+1/8*Chi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(23) = 46$.

time = 0.47, size = 88, normalized size = 3.26

$$-\frac{2\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/16*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral((a^2*x^2 + 2*a*x + 1
)/(a^2*x^2 - 2*a*x + 1)) - log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*
a*x + 1)))/a^3
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{a^6x^6\,\mathrm{atanh}\,(ax)-3a^4x^4\,\mathrm{atanh}\,(ax)+3a^2x^2\,\mathrm{atanh}\,(ax)-\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$-\int\frac{x^2}{\mathrm{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.326 $\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}(2\tanh^{-1}(ax))}{4a^2} + \frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^2}$$

```
[Out] 1/4*Shi(2*arctanh(a*x))/a^2+1/8*Shi(4*arctanh(a*x))/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6181, 5556, 3379}

$$\frac{\text{Shi}(2\tanh^{-1}(ax))}{4a^2} + \frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(4*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a^2}$$
$$= \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Mathematica [A]
time = 0.07, size = 24, normalized size = 0.83

$$\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)+\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
[Out] (2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^2)
```

Maple [A]
time = 1.57, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{4}}{a^2}$	24
default	$\frac{\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{4}}{a^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/8*Shi(4*arctanh(a*x))+1/4*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(25) = 50$.
time = 0.42, size = 102, normalized size = 3.52

$$\frac{\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+2\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-2\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 2*log_integral(-(a*x + 1
)/(a*x - 1)) - 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{a^6x^6\,\text{atanh}\,(ax)-3a^4x^4\,\text{atanh}\,(ax)+3a^2x^2\,\text{atanh}\,(ax)-\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*at
anh(a*x) - atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int\frac{x}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.327 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=41

$$\frac{\text{Chi}(2\tanh^{-1}(ax))}{2a}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/8*Chi(4*arctanh(a*x))/a+3/8*ln(arctanh(a*x))/a
```

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6115, 3393, 3382}

$$\frac{\text{Chi}(2\tanh^{-1}(ax))}{2a}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + CoshIntegral[4*ArcTanh[a*x]]/(8*a) + (
3*Log[ArcTanh[a*x]])/(8*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} + \frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x} dx, x, \tanh^{-1}(ax)\right)}{8a} + \frac{\text{Subst}\left(\int \frac{\cosh(2}{x}\right.}{}$$
$$= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a} + \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.80

$$-\frac{-4\text{Chi}\left(2\tanh^{-1}(ax)\right) - \text{Chi}\left(4\tanh^{-1}(ax)\right) - 3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -1/8*(-4*CoshIntegral[2*ArcTanh[a*x]] - CoshIntegral[4*ArcTanh[a*x]] - 3*Lo
g[ArcTanh[a*x]])/a
```

Maple [A]
time = 1.59, size = 31, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\frac{3\ln(\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{2}+\frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{8}}{a}$	31
default	$\frac{\frac{3\ln(\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{2}+\frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{8}}{a}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(3/8*ln(arctanh(a*x))+1/2*Chi(2*arctanh(a*x))+1/8*Chi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")

[Out] -integrate(1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(35) = 70$.
time = 0.41, size = 118, normalized size = 2.88

$$\frac{6 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + \mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) + 4\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + 4\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")

[Out] 1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 4*log_integral(-(a*x + 1)/(a*x - 1)) + 4*log_integral(-(a*x - 1)/(a*x + 1)))/a

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x),x)

[Out] -Integral(1/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*atanh(a*x) - atanh(a*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")

[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(atanh(a*x)*(a^2*x^2 - 1)^3),x)

[Out] -int(1/(atanh(a*x)*(a^2*x^2 - 1)^3), x)

3.328 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=49

$$\frac{3}{4}\text{Shi}\left(2\tanh^{-1}(ax)\right)+\frac{1}{8}\text{Shi}\left(4\tanh^{-1}(ax)\right)-\text{Int}\left(\frac{1}{x\left(-1+a^2x^2\right)\tanh^{-1}(ax)},x\right)$$

```
[Out] 3/4*Shi(2*arctanh(a*x))+1/8*Shi(4*arctanh(a*x))-Unintegrable(1/x/(a^2*x^2-1
)/arctanh(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx = \int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Maple [A]
time = 7.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3 \operatorname{arctanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{6}x^{7}\operatorname{atanh}(ax) - 3a^{4}x^{5}\operatorname{atanh}(ax) + 3a^{2}x^{3}\operatorname{atanh}(ax) - x\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x) - 3*a**4*x**5*atanh(a*x) + 3*a**2*x**3*at
anh(a*x) - x*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

3.329 $\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=102

$$-\frac{x}{a^5 \tanh^{-1}(ax)}-\frac{x}{a^5 (1-a^2x^2)^2 \tanh^{-1}(ax)}+\frac{2x}{a^5 (1-a^2x^2) \tanh^{-1}(ax)}-\frac{3\text{Chi}(2\tanh^{-1}(ax))}{2a^6}+\frac{\text{Chi}(4\tanh^{-1}$$

[Out] -x/a^5/arctanh(a*x)-x/a^5/(-a^2*x^2+1)^2/arctanh(a*x)+2*x/a^5/(-a^2*x^2+1)/arctanh(a*x)-3/2*Chi(2*arctanh(a*x))/a^6+1/2*Chi(4*arctanh(a*x))/a^6+Unintegrable(1/arctanh(a*x),x)/a^5

Rubi [A]
time = 0.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$$

Verification is not applicable to the result.

[In] Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]

[Out] -(x/(a^5*ArcTanh[a*x])) - x/(a^5*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (2*x)/(a^5*(1 - a^2*x^2)*ArcTanh[a*x]) - (3*CoshIntegral[2*ArcTanh[a*x]])/(2*a^6) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^6) + Defer[Int][ArcTanh[a*x]^(-1), x]/a^5

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{x^3}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx}{a^2} - \frac{\int \frac{x^3}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$= \frac{\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx}{a^4} - 2\frac{\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx}{a^4} + \frac{\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}}{a^4}$$

$$= -\frac{x}{a^5\tanh^{-1}(ax)} - \frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\int \frac{1}{\tanh^{-1}(ax)}\,dx}{a^5} + \frac{\int \frac{}{(1-a^2x}}{}$$

$$= -\frac{x}{a^5\tanh^{-1}(ax)} - \frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \tan\right.}{a^6}$$

$$= -\frac{x}{a^5\tanh^{-1}(ax)} - \frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} - 2\left(-\frac{x}{a^5\left(1-a^2x^2\right)\tanh^{-}}\right.$$

$$= -\frac{x}{a^5\tanh^{-1}(ax)} - \frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tan\right.}{8a^6}$$

$$= -\frac{x}{a^5\tanh^{-1}(ax)} - \frac{x}{a^5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^6} - 2\left(\right.$$

Mathematica [A]
time = 11.33, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
[Out] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-a^2x^2+1\right)^3\operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^5/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) - integrate(-2*(a^2*x^6 - 5*x^4)/((a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{5}}{a^{6}x^{6}\operatorname{atanh}^{2}(ax) - 3a^{4}x^{4}\operatorname{atanh}^{2}(ax) + 3a^{2}x^{2}\operatorname{atanh}^{2}(ax) - \operatorname{atanh}^{2}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**5/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^5}{\operatorname{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
[Out] -int(x^5/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.330 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=53

$$-\frac{x^4}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

```
[Out] -x^4/a/(-a^2*x^2+1)^2/arctanh(a*x)-Shi(2*arctanh(a*x))/a^5+1/2*Shi(4*arctan
h(a*x))/a^5
```

Rubi [A]
time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6153, 6181, 5556, 3379}

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}-\frac{x^4}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) - SinhIntegral[2*ArcTanh[a*x]]/a^5
+ SinhIntegral[4*ArcTanh[a*x]]/(2*a^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx = -\frac{x^4}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{4\int \frac{x^3}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx}{a}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{4\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{4\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)\,dx, x, \tanh^{-1}\right.}{a^5}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^5} - \text{Subs}$$

$$= -\frac{x^4}{a(1-a^2x^2)^2\tanh^{-1}(ax)} - \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Mathematica [A]
time = 0.10, size = 49, normalized size = 0.92

$$\frac{-\frac{2a^4x^4}{(-1+a^2x^2)^2\tanh^{-1}(ax)} - 2\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] ((-2*a^4*x^4)/((-1 + a^2*x^2)^2*ArcTanh[a*x]) - 2*SinhIntegral[2*ArcTanh[a*
x]] + SinhIntegral[4*ArcTanh[a*x]])/(2*a^5)
```

Maple [A]
time = 3.66, size = 62, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{3}{8\operatorname{arctanh}(ax)} + \frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)} - \text{hyperbolicSineIntegral}(2\operatorname{arctanh}(ax)) - \frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} + \frac{\text{hyperbolicSineIntegral}(4}{2}}{a^5}$

default	$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}-\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicSineIntegral}(4\,\mathrm{ar}}{2}}{a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-3/8/arctanh(a*x)+1/2/arctanh(a*x)*cosh(2*arctanh(a*x))-Shi(2*arctan
h(a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^4/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + 8*integrate(-x^3/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*lo
g(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(50) = 100.
time = 0.34, size = 232, normalized size = 4.38

$$-\frac{8\,a^4x^4-\left(\left(a^4x^4-2\,a^2x^2+1\right)\log\_\mathrm{integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\left(a^4x^4-2\,a^2x^2+1\right)\log\_\mathrm{integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-2\left(a^4x^4-2\,a^2x^2+1\right)\log\_\mathrm{integral}\left(-\frac{ax+1}{ax-1}\right)+2\left(a^4x^4-2\,a^2x^2+1\right)\log\_\mathrm{integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^9x^4-2\,a^7x^2+a^5\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^4*x^4 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_in
tegral(-(a*x + 1)/(a*x - 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^9*x^4 - 2*a^7*x^2 + a^5)
*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{a^6x^6\,\mathrm{atanh}^2\,(ax)-3a^4x^4\,\mathrm{atanh}^2\,(ax)+3a^2x^2\,\mathrm{atanh}^2\,(ax)-\mathrm{atanh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^4}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^4/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.331 $\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=55

$$-\frac{x^3}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4}$$

```
[Out] -x^3/a/(-a^2*x^2+1)^2/arctanh(a*x)-1/2*Chi(2*arctanh(a*x))/a^4+1/2*Chi(4*ar
ctanh(a*x))/a^4
```

Rubi [A]
time = 0.36, antiderivative size = 76, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6175, 6179, 6181, 3393, 3382, 6115, 5556}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4}+\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x])) + x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]
) - CoshIntegral[2*ArcTanh[a*x]]/(2*a^4) + CoshIntegral[4*ArcTanh[a*x]]/(2*
a^4)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx = \frac{\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx}{a^2}$$

$$= -\frac{x}{a^3 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{x}{a^3 (1-a^2x^2) \tanh^{-1}(ax)} + \frac{\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}}}{a^3}$$

$$= -\frac{x}{a^3 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{x}{a^3 (1-a^2x^2) \tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\right.}{}$$

$$= -\frac{x}{a^3 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{x}{a^3 (1-a^2x^2) \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} -\right.\right.}{}$$

$$= -\frac{x}{a^3 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{x}{a^3 (1-a^2x^2) \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\right.}{}$$

$$= -\frac{x}{a^3 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{x}{a^3 (1-a^2x^2) \tanh^{-1}(ax)} - \frac{\text{Chi}\left(2\tanh^{-1}(a\right.}{2a^4}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 1.45

$$\frac{-2a^3x^3 - (-1+a^2x^2)^2 \tanh^{-1}(ax)\text{Chi}\left(2\tanh^{-1}(ax)\right) + (-1+a^2x^2)^2 \tanh^{-1}(ax)\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4 (-1+a^2x^2)^2 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a^3*x^3 - (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] +
(-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^4*(-1 + a^
2*x^2)^2*ArcTanh[a*x])
```

Maple [A]
time = 3.66, size = 54, normalized size = 0.98

method	result	s
derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{2}+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}-\frac{\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{2}}{a^4}$	5
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{2}+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}-\frac{\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{2}}{a^4}$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/8/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))+1/4*s
inh(2*arctanh(a*x))/arctanh(a*x)-1/2*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + integrate(-2*(a^2*x^4 + 3*x^2)/((a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(50) = 100.
time = 0.37, size = 231, normalized size = 4.20

$$-\frac{8\,a^{3}x^{3}-\left(\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\_integral\left(\frac{a^{2}x^{2}+2\,ax+1}{a^{2}x^{2}-2\,ax+1}\right)+\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\_integral\left(\frac{a^{2}x^{2}-2\,ax+1}{a^{2}x^{2}+2\,ax+1}\right)-\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\_integral\left(-\frac{ax+1}{ax-1}\right)-\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\_integral\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^{8}x^{4}-2\,a^{6}x^{2}+a^{4}\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*x^3 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_inte
gral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x -
 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log
(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{3}}{a^{6}x^{6}\operatorname{atanh}^{2}\left(ax\right)-3a^{4}x^{4}\operatorname{atanh}^{2}\left(ax\right)+3a^{2}x^{2}\operatorname{atanh}^{2}\left(ax\right)-\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^3}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
[Out] -int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.332 $\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=41

$$-\frac{x^2}{a(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\mathrm{Shi}(4\tanh^{-1}(ax))}{2a^3}$$

```
[Out] -x^2/a/(-a^2*x^2+1)^2/arctanh(a*x)+1/2*Shi(4*arctanh(a*x))/a^3
```

Rubi [A]
time = 0.21, antiderivative size = 60, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6175, 6113, 6181, 5556, 12, 3379}

$$\frac{\mathrm{Shi}(4\tanh^{-1}(ax))}{2a^3} + \frac{1}{a^3(1-a^2x^2)\tanh^{-1}(ax)} - \frac{1}{a^3(1-a^2x^2)^2 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x])) + 1/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]
) + SinhIntegral[4*ArcTanh[a*x]]/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
```

```
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx}{a^2} - \frac{\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$= -\frac{1}{a^3(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{1}{a^3(1-a^2x^2)\tanh^{-1}(ax)} - \frac{2\int \frac{x}{(1-a^2x^2)^2 \tanh}}{a}$$

$$= -\frac{1}{a^3(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{1}{a^3(1-a^2x^2)\tanh^{-1}(ax)} - \frac{2\text{Subst}\left(\int \frac{\cosh(x}{}\right.}{}$$

$$= -\frac{1}{a^3(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{1}{a^3(1-a^2x^2)\tanh^{-1}(ax)} - \frac{2\text{Subst}\left(\int \frac{\sinh(2x}{2x}\right.}{}$$

$$= -\frac{1}{a^3(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{1}{a^3(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\right.}{}$$

$$= -\frac{1}{a^3(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{1}{a^3(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Shi}\left(4\tanh^{-1}(a\right.}{2a^3}$$

Mathematica [A]
time = 0.13, size = 56, normalized size = 1.37

$$\frac{-2a^2x^2 + (-1+a^2x^2)^2 \tanh^{-1}(ax)\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3(-1+a^2x^2)^2 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a^2*x^2 + (-1 + a^2*x^2)^2*ArcTanh[a*x]*SinhIntegral[4*ArcTanh[a*x]])/(
2*a^3*(-1 + a^2*x^2)^2*ArcTanh[a*x])
```

Maple [A]
time = 3.66, size = 38, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\frac{1}{8\,\mathrm{arctanh}(ax)}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))}{2}}{a^3}$	38
default	$\frac{\frac{1}{8\,\mathrm{arctanh}(ax)}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))}{2}}{a^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/8/arctanh(a*x)-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arc
tanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + integrate(-4*(a^2*x^3 + x)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(38) = 76.
time = 0.35, size = 164, normalized size = 4.00

$$-\frac{8\,a^2x^2-\left(\left(a^4x^4-2\,a^2x^2+1\right)\log\_\mathrm{integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\left(a^4x^4-2\,a^2x^2+1\right)\log\_\mathrm{integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^7x^4-2\,a^5x^2+a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
```

```
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^
4 - 2*a^5*x^2 + a^3)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.333 $\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=53

$$-\frac{x}{a(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{\text{Chi}(2\tanh^{-1}(ax))}{2a^2}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{2a^2}$$

```
[Out] -x/a/(-a^2*x^2+1)^2/arctanh(a*x)+1/2*Chi(2*arctanh(a*x))/a^2+1/2*Chi(4*arct
anh(a*x))/a^2
```

Rubi [A]
time = 0.17, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6179, 6181, 5556, 3382, 6115, 3393}

$$\frac{\text{Chi}(2\tanh^{-1}(ax))}{2a^2}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{2a^2}-\frac{x}{a(1-a^2x^2)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/(2*a^2
) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
```

```
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx &= -\frac{x}{a\,(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\int \frac{1}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx}{a} + (3a)\int \frac{x}{(1-a^2x^2)^3 \ldots} \\
&= -\frac{x}{a\,(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{3\text{Subs}\ldots}{\ldots} \\
&= -\frac{x}{a\,(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh\ldots\right)}{a^2} \\
&= -\frac{x}{a\,(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2} + \frac{3\text{Subs}\ldots}{\ldots} \\
&= -\frac{x}{a\,(1-a^2x^2)^2\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 1.42

$$
\frac{-2ax+(-1+a^2x^2)^2\tanh^{-1}(ax)\text{Chi}\left(2\tanh^{-1}(ax)\right)+(-1+a^2x^2)^2\tanh^{-1}(ax)\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2\,(-1+a^2x^2)^2\tanh^{-1}(ax)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a*x + (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] + (-1
+ a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^
2)^2*ArcTanh[a*x])
```

Maple [A]
time = 3.66, size = 54, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{2}}{a^2}$
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))}{2}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/8/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-1/4*s
inh(2*arctanh(a*x))/arctanh(a*x)+1/2*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*lo
g(-a*x + 1)) + integrate(-2*(3*a^2*x^2 + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(48) = 96$.
time = 0.38, size = 225, normalized size = 4.25

$$-\frac{8\,ax-\left(\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^6x^4-2\,a^4x^2+a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*x - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 -
```

```
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral
(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a
*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{6}x^{6}\operatorname{atanh}^{2}(ax) - 3a^{4}x^{4}\operatorname{atanh}^{2}(ax) + 3a^{2}x^{2}\operatorname{atanh}^{2}(ax) - \operatorname{atanh}^{2}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
[Out] -Integral(x/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*x
**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
[Out] -int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.334 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{a(1-a^2x^2)^2 \tanh^{-1}(ax)}+\frac{\mathrm{Shi}(2\tanh^{-1}(ax))}{a}+\frac{\mathrm{Shi}(4\tanh^{-1}(ax))}{2a}$$

```
[Out] -1/a/(-a^2*x^2+1)^2/arctanh(a*x)+Shi(2*arctanh(a*x))/a+1/2*Shi(4*arctanh(a*
x))/a
```

Rubi [A]
time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6113, 6181, 5556, 3379}

$$-\frac{1}{a(1-a^2x^2)^2 \tanh^{-1}(ax)}+\frac{\mathrm{Shi}(2\tanh^{-1}(ax))}{a}+\frac{\mathrm{Shi}(4\tanh^{-1}(ax))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a + Si
nhIntegral[4*ArcTanh[a*x]]/(2*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx &= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)}+(4a)\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx\\ &= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{4\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{4\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)\,dx,x,\tanh^{-1}(\right.}{a}\\ &= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Subst}(}{}\\ &= -\frac{1}{a(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 43, normalized size = 0.88

$$\frac{-\frac{2}{(-1+a^2x^2)^2\tanh^{-1}(ax)}+2\text{Shi}\left(2\tanh^{-1}(ax)\right)+\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2/((-1 + a^2*x^2)^2*ArcTanh[a*x]) + 2*SinhIntegral[2*ArcTanh[a*x]] + Sinh
Integral[4*ArcTanh[a*x]])/(2*a)
```

Maple [A]
time = 3.70, size = 60, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{3}{8\,\text{arctanh}(ax)}-\frac{\cosh(2\,\text{arctanh}(ax))}{2\,\text{arctanh}(ax)}+\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))-\frac{\cosh(4\,\text{arctanh}(ax))}{8\,\text{arctanh}(ax)}+\frac{\text{hyperbolicSineIntegral}(4\,\text{ar}}{2}}{a}$

default	$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicSineIntegral}(4}{2}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-3/8/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh(
a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 8*a*integrate(-x/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x) - 2/((a^5*x^4 - 2*a^3*
x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(46) = 92.
time = 0.37, size = 222, normalized size = 4.53

$$\frac{\left((a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+2\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-2\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)-8}{4\,(a^5x^4-2\,a^3x^2+a)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2
 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x +
1)/(a^2*x^2 + 2*a*x + 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1)) - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x
+ 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6\,\mathrm{atanh}^2\,(ax)-3a^4x^4\,\mathrm{atanh}^2\,(ax)+3a^2x^2\,\mathrm{atanh}^2\,(ax)-\mathrm{atanh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*x
**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.335 $\int \frac{1}{x(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=99

$$-\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\frac{3}{2}\text{Chi}\left(2\tanh^{-1}(ax)\right)+\frac{1}{2}\text{Chi}\left(4\tanh^{-1}(\right.$$

```
[Out] -1/a/x/arctanh(a*x)-a*x/(-a^2*x^2+1)^2/arctanh(a*x)-a*x/(-a^2*x^2+1)/arctan
h(a*x)+3/2*Chi(2*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-Unintegrable(1/x^2/a
rctanh(a*x),x)/a
```

Rubi [A]
time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)^2*ArcTanh[a*x]) - (a*x)/((1
- a^2*x^2)*ArcTanh[a*x]) + (3*CoshIntegral[2*ArcTanh[a*x]])/2 + CoshIntegra
l[4*ArcTanh[a*x]]/2 - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx = a^2\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx + \int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{ax}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + a\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx + a^2\int \frac{}{(1-}$$

$$= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 3$$

$$= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 3$$

$$= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{1}{8}$$

$$= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{1}{2}$$

$$= -\frac{1}{ax\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} - \frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{3}{2}$$

Mathematica [A]
time = 3.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 5.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3\operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/((a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) - (a^5*x^5 - 2*a^3*x^3 + a*x)*
log(-a*x + 1)) + integrate(-2*(5*a^2*x^2 - 1)/((a^7*x^8 - 3*a^5*x^6 + 3*a^3
*x^4 - a*x^2)*log(a*x + 1) - (a^7*x^8 - 3*a^5*x^6 + 3*a^3*x^4 - a*x^2)*log(
-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^7\operatorname{atanh}^2(ax) - 3a^4x^5\operatorname{atanh}^2(ax) + 3a^2x^3\operatorname{atanh}^2(ax) - x\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x)**2 - 3*a**4*x**5*atanh(a*x)**2 + 3*a**2*x
**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x\,\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

3.336 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=100

$$-\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2}-\frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{2x}{a^4(1-a^2x^2)\tanh^{-1}(ax)}-\frac{\text{Chi}(2\tanh^{-1}(ax))}{a^5}+\text{C}$$

```
[Out] -1/2*x^4/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2*x/a^4/(-a^2*x^2+1)^2/arctanh(a*x
)+2*x/a^4/(-a^2*x^2+1)/arctanh(a*x)-Chi(2*arctanh(a*x))/a^5+Chi(4*arctanh(a
*x))/a^5
```

Rubi [A]
time = 0.43, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6153, 6175, 6179, 6181, 3393, 3382, 6115, 5556}

$$-\frac{\text{Chi}(2\tanh^{-1}(ax))}{a^5}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{a^5}-\frac{x^4}{2a(1-a^2x^2)^2\tanh^{-1}(ax)^2}+\frac{2x}{a^4(1-a^2x^2)\tanh^{-1}(ax)}-\frac{2x}{a^4(1-a^2x^2)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/(a^4*(1 - a^2*x^2)^2*Ar
cTanh[a*x]) + (2*x)/(a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - CoshIntegral[2*ArcTa
nh[a*x]]/a^5 + CoshIntegral[4*ArcTanh[a*x]]/a^5
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx = -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{2\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx}{a}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{2\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx}{a^3} - \frac{2\int \frac{x}{(1-a^2x^2)^2 \tanh}}{a^3}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)}$$

$$= -\frac{x^4}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4(1-a^2x^2)}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.60

$$-\frac{\frac{a^3x^3(ax+4\tanh^{-1}(ax))}{(-1+a^2x^2)^2\tanh^{-1}(ax)^2} + 2\text{Chi}\left(2\tanh^{-1}(ax)\right) - 2\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*((a^3*x^3*(a*x + 4*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTanh[a*x]^2) +
2*CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]])/a^5
```

Maple [A]
time = 2.73, size = 90, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{3}{16\,\mathrm{arctanh}(ax)^2}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}-\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-}{a^5}$
default	$\frac{-\frac{3}{16\,\mathrm{arctanh}(ax)^2}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}-\mathrm{hyperbolicCosineIntegral}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-}{a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-3/16/arctanh(a*x)^2+1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))+1/2*sin
h(2*arctanh(a*x))/arctanh(a*x)-Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh
(4*arctanh(a*x))-1/4/arctanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a*x^4 + 2*x^3*log(a*x + 1) - 2*x^3*log(-a*x + 1))/((a^6*x^4 - 2*a^4*x^2
 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)*log(-a*
x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + integrate(-4*(a^2*x
^4 + 3*x^2)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1) - (a^8*x^
6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(95) = 190.
time = 0.35, size = 256, normalized size = 2.56

$$-\frac{4\,a^{4}x^{4}+8\,a^{3}x^{3}\log\left(-\frac{ax+1}{ax-1}\right)-\left(\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(\frac{a^{2}x^{2}+2\,ax+1}{a^{2}x^{2}-2\,ax+1}\right)+\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(\frac{a^{2}x^{2}-2\,ax+1}{a^{2}x^{2}+2\,ax+1}\right)-\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}}{2\left(a^{9}x^{4}-2\,a^{7}x^{2}+a^{5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*x^4 + 8*a^3*x^3*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x
^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^
4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)
) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4
 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x -
 1))^2)/((a^9*x^4 - 2*a^7*x^2 + a^5)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{4}}{a^{6}x^{6}\operatorname{atanh}^{3}\left(ax\right)-3a^{4}x^{4}\operatorname{atanh}^{3}\left(ax\right)+3a^{2}x^{2}\operatorname{atanh}^{3}\left(ax\right)-\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4}{\operatorname{atanh}(a\,x)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
[Out] -int(x^4/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

3.337 $\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=107

$$-\frac{x^3}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{3x^2}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x^4}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}-$$

```
[Out] -1/2*x^3/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-3/2*x^2/a^2/(-a^2*x^2+1)^2/arctanh
(a*x)-1/2*x^4/(-a^2*x^2+1)^2/arctanh(a*x)-1/2*Shi(2*arctanh(a*x))/a^4+Shi(4
*arctanh(a*x))/a^4
```

Rubi [A]

time = 0.45, antiderivative size = 160, normalized size of antiderivative = 1.50, number of steps used = 25, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6175, 6143, 6181, 5556, 12, 3379, 6179, 6113}

$$-\frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\mathrm{Shi}\left(4\tanh^{-1}(ax)\right)}{a^4}+\frac{a^2x^2+1}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{3}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2}{a^4\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(2*a^3*(1 - a^2*x^2)*ArcTan
h[a*x]^2) - 2/(a^4*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 3/(2*a^4*(1 - a^2*x^2)*A
rcTanh[a*x]) + (1 + a^2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - SinhInteg
ral[2*ArcTanh[a*x]]/(2*a^4) + SinhIntegral[4*ArcTanh[a*x]]/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx = \frac{\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3}\,dx}{a^2}$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} + \frac{1}{2a^4(1-a$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{2a^4(1-a}$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^4(1-a^2}$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^4(1-a^2}$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^4(1-a^2}$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^4(1-a^2}$$

$$= -\frac{x}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{x}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^4(1-a^2}$$

Mathematica [A]
time = 0.17, size = 66, normalized size = 0.62

$$-\frac{\frac{a^2x^2(ax+(3+a^2x^2)\tanh^{-1}(ax))}{(-1+a^2x^2)^2\tanh^{-1}(ax)^2} + \mathrm{Shi}(2\tanh^{-1}(ax)) - 2\mathrm{Shi}(4\tanh^{-1}(ax))}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*((a^2*x^2*(a*x + (3 + a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTan
h[a*x]^2) + SinhIntegral[2*ArcTanh[a*x]] - 2*SinhIntegral[4*ArcTanh[a*x]])/
a^4
```

Maple [A]
time = 2.85, size = 82, normalized size = 0.77

method	result

derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^2}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}-}{a^4}$
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^2}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}-}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))-1/4/arctanh(a*x)*cosh(4*ar
ctanh(a*x))+Shi(4*arctanh(a*x))+1/8*sinh(2*arctanh(a*x))/arctanh(a*x)^2+1/4
/arctanh(a*x)*cosh(2*arctanh(a*x))-1/2*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x^3 + (a^2*x^4 + 3*x^2)*log(a*x + 1) - (a^2*x^4 + 3*x^2)*log(-a*x + 1
))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a
^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^
2) + integrate(-2*(5*a^2*x^3 + 3*x)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2
)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(96) = 192$.
time = 0.42, size = 267, normalized size = 2.50

$$-\frac{8\,a^3x^3-\left(2\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-2\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(a^4x^4+3\,a^2x^2)\log\left(-\frac{ax+1}{ax-1}\right)}{4\,(a^8x^4-2\,a^6x^2+a^4)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*x^3 - (2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*
x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a
^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_
integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^4*x^4 + 3*a^2*x^2)*l
og(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log(-(a*x + 1)/(a*x
- 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3}{a^6x^6\,\mathrm{atanh}^3\,(ax)-3a^4x^4\,\mathrm{atanh}^3\,(ax)+3a^2x^2\,\mathrm{atanh}^3\,(ax)-\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3}{\mathrm{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

3.338 $\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=86

$$-\frac{x^2}{2a\left(-1+a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{2x}{a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}+\frac{x}{a^2\left(1-a^2x^2\right) \tanh^{-1}(ax)}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}$$

```
[Out] -1/2*x^2/a/(a^2*x^2-1)^2/arctanh(a*x)^2-2*x/a^2/(-a^2*x^2+1)^2/arctanh(a*x)
+x/a^2/(-a^2*x^2+1)/arctanh(a*x)+Chi(4*arctanh(a*x))/a^3
```

Rubi [A]
time = 0.61, antiderivative size = 109, normalized size of antiderivative = 1.27, number of steps used = 22, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6175, 6113, 6179, 6181, 3393, 3382, 6115, 5556}

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}+\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{1}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a^3*(1 - a^2*x^2)*ArcTan
h[a*x]^2) - (2*x)/(a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/(a^2*(1 - a^2*x^2)
*ArcTanh[a*x]) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx = \frac{\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx}{a^2} - \frac{\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3}\,dx}{a^2}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{\int \frac{x}{(1-a^2x^2)^2 \tanh}}{a}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^2(1-a^2x}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^2(1-a^2x}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^2(1-a^2x}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^2(1-a^2x}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{}{a^2(1-a^2x}$$

Mathematica [A]
time = 0.15, size = 56, normalized size = 0.65

$$-\frac{x\left(ax+2(1+a^2x^2)\tanh^{-1}(ax)\right)}{2a^2(-1+a^2x^2)^2\tanh^{-1}(ax)^2} + \frac{\operatorname{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*(x*(a*x + 2*(1 + a^2*x^2)*ArcTanh[a*x]))/(a^2*(-1 + a^2*x^2)^2*ArcTanh
[a*x]^2) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

Maple [A]
time = 2.80, size = 51, normalized size = 0.59

method	result	size
derivativedivides	$\frac{\frac{1}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{a^3}$	51
default	$\frac{\frac{1}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{a^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/16/arctanh(a*x)^2-1/16/arctanh(a*x)^2*cosh(4*arctanh(a*x))-1/4/arc
tanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a*x^2 + (a^2*x^3 + x)*log(a*x + 1) - (a^2*x^3 + x)*log(-a*x + 1))/((a^6
*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(
a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + int
egrate(-2*(a^4*x^4 + 6*a^2*x^2 + 1)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2
)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(83) = 166.
time = 0.34, size = 193, normalized size = 2.24

$$-\frac{4\,a^2x^2-\left(\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(a^3x^3+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^7x^4-2\,a^5x^2+a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^
3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^4 - 2*a^5*x^2 + a^3)*log(-(
a*x + 1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{a^6x^6\operatorname{atanh}^3(ax)-3a^4x^4\operatorname{atanh}^3(ax)+3a^2x^2\operatorname{atanh}^3(ax)-\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
[Out] -int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

3.339 $\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=100

$$-\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{3}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}-$$

```
[Out] -1/2*x/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2/a^2/(-a^2*x^2+1)^2/arctanh(a*x)+3/
2/a^2/(-a^2*x^2+1)/arctanh(a*x)+1/2*Shi(2*arctanh(a*x))/a^2+Shi(4*arctanh(a
*x))/a^2
```

Rubi [A]
time = 0.33, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6179, 6175, 6113, 6181, 5556, 12, 3379}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{3}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - 2/(a^2*(1 - a^2*x^2)^2*ArcTanh[
a*x]) + 3/(2*a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]
/(2*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx = -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} \, dx}{2a} + \frac{1}{2}(3a) \int \frac{}{(1-a}$$

$$= -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{1}{2a^2(1-a^2x^2)^2 \tanh^{-1}(ax)} + 2 \int \frac{}{(1-a}$$

$$= -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{}{2a^2(1-a^2x}$$

$$= -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{}{2a^2(1-a^2x}$$

$$= -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{}{2a^2(1-a^2x}$$

$$= -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{}{2a^2(1-a^2x}$$

$$= -\frac{x}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{}{2a^2(1-a^2x}$$

Mathematica [A]
time = 0.13, size = 96, normalized size = 0.96

$$-\frac{ax + \tanh^{-1}(ax) + 3a^2x^2 \tanh^{-1}(ax) - (-1+a^2x^2)^2 \tanh^{-1}(ax)^2 \text{Shi}(2\tanh^{-1}(ax)) - 2(-1+a^2x^2)^2 \tanh^{-1}(ax)^2 \text{Shi}(4\tanh^{-1}(ax))}{2a^2(-1+a^2x^2)^2 \tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*(a*x + ArcTanh[a*x] + 3*a^2*x^2*ArcTanh[a*x] - (-1 + a^2*x^2)^2*ArcTan
h[a*x]^2*SinhIntegral[2*ArcTanh[a*x]] - 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*S
inhIntegral[4*ArcTanh[a*x]])/(a^2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

Maple [A]
time = 2.95, size = 82, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\text{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax)}{4\,\mathrm{arctanh}(ax)}}{a^2}$
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\text{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax)}{4\,\mathrm{arctanh}(ax)}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))-1/4/arctanh(a*x)*cosh(4*ar
ctanh(a*x))+Shi(4*arctanh(a*x))-1/8*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/4
/arctanh(a*x)*cosh(2*arctanh(a*x))+1/2*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (3*a^2*x^2 + 1)*log(a*x + 1) - (3*a^2*x^2 + 1)*log(-a*x + 1))/((a
^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*lo
g(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + i
ntegrate(-2*(3*a^2*x^3 + 5*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*
x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(91) = 182$.
time = 0.38, size = 256, normalized size = 2.56

$$\frac{\left(2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\operatorname{log\_integral}\left(\frac{a^{2}x^{2}+2ax+1}{a^{2}x^{2}-2ax+1}\right)-2\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\operatorname{log\_integral}\left(\frac{a^{2}x^{2}-2ax+1}{a^{2}x^{2}+2ax+1}\right)+\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-8ax-4\left(3a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^{6}x^{4}-2a^{4}x^{2}+a^{2}\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^2 - 8*a*x - 4*(3*a^2*x^2 + 1)*log(-(a*x +
1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a^{6}x^{6}\operatorname{atanh}^{3}\left(ax\right)-3a^{4}x^{4}\operatorname{atanh}^{3}\left(ax\right)+3a^{2}x^{2}\operatorname{atanh}^{3}\left(ax\right)-\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*x
**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\mathrm{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

3.340 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=69

$$-\frac{1}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{2x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2*x/(-a^2*x^2+1)^2/arctanh(a*x)+Chi(2*
arctanh(a*x))/a+Chi(4*arctanh(a*x))/a
```

Rubi [A]
time = 0.20, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6113, 6179, 6181, 5556, 3382, 6115, 3393}

$$-\frac{2x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/((1 - a^2*x^2)^2*ArcTanh[
a*x]) + CoshIntegral[2*ArcTanh[a*x]]/a + CoshIntegral[4*ArcTanh[a*x]]/a
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
```

```
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx = -\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + (2a) \int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$$

$$= -\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} + 2\int \frac{1}{(1-a^2x^2)^3}$$

$$= -\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2\text{Subst}\left(\int \frac{\cosh^4(x}{x}\right.}{}$$

$$= -\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{2\text{Subst}\left(\int \left(\frac{3}{8x} +\right.\right.}{}$$

$$= -\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\right.}{}$$

$$= -\frac{1}{2a(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{2x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax\right.}{a}$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 1.25

$$\frac{-1-4ax\tanh^{-1}(ax)+2(-1+a^2x^2)^2\tanh^{-1}(ax)^2\text{Chi}(2\tanh^{-1}(ax))+2(-1+a^2x^2)^2\tanh^{-1}(ax)^2\text{Chi}(4\tanh^{-1}(ax))}{2a(-1+a^2x^2)^2\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] (-1 - 4*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[2
*ArcTanh[a*x]] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[4*ArcTanh[a
*x]])/(2*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

Maple [A]
time = 2.92, size = 88, normalized size = 1.28

method	result
derivativedivides	$\frac{-\frac{3}{16\,\text{arctanh}(ax)^2}-\frac{\cosh(2\,\text{arctanh}(ax))}{4\,\text{arctanh}(ax)^2}-\frac{\sinh(2\,\text{arctanh}(ax))}{2\,\text{arctanh}(ax)}+\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))-\frac{\cosh(4\,\text{arctanh}(ax))}{16\,\text{arctanh}(ax)^2}-}{a}$
default	$\frac{-\frac{3}{16\,\text{arctanh}(ax)^2}-\frac{\cosh(2\,\text{arctanh}(ax))}{4\,\text{arctanh}(ax)^2}-\frac{\sinh(2\,\text{arctanh}(ax))}{2\,\text{arctanh}(ax)}+\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))-\frac{\cosh(4\,\text{arctanh}(ax))}{16\,\text{arctanh}(ax)^2}-}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-3/16/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2*sinh(
2*arctanh(a*x))/arctanh(a*x)+Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh(4
*arctanh(a*x))-1/4/arctanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a*x*log(a*x + 1) - 2*a*x*log(-a*x + 1) + 1)/((a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^2 - 2*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1) +
 (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^2) + integrate(-4*(3*a^2*x^2 + 1)/
((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6*x^6 - 3*a^4*x^4
+ 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(65) = 130$.
time = 0.39, size = 241, normalized size = 3.49

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4}{2\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*x*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_inte
gral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 +
1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2
*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1
)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4)/((a^
5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{6}x^{6}\operatorname{atanh}^{3}{\left(ax \right)} - 3a^{4}x^{4}\operatorname{atanh}^{3}{\left(ax \right)} + 3a^{2}x^{2}\operatorname{atanh}^{3}{\left(ax \right)} - \operatorname{atanh}^{3}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*x
**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
[Out] -int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

3.341 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=176

$$-\frac{1}{2ax\tanh^{-1}(ax)^2}-\frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{2}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{}{2\,(1-}$$

```
[Out] -1/2/a/x/arctanh(a*x)^2-1/2*a*x/(-a^2*x^2+1)^2/arctanh(a*x)^2-1/2*a*x/(-a^2
*x^2+1)/arctanh(a*x)^2-2/(-a^2*x^2+1)^2/arctanh(a*x)+3/2/(-a^2*x^2+1)/arcta
nh(a*x)+1/2*(-a^2*x^2-1)/(-a^2*x^2+1)/arctanh(a*x)+3/2*Shi(2*arctanh(a*x))+
Shi(4*arctanh(a*x))-1/2*Unintegrable(1/x^2/arctanh(a*x)^2,x)/a
```

Rubi [A]
time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (a
*x)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - 2/((1 - a^2*x^2)^2*ArcTanh[a*x]) + 3
/(2*(1 - a^2*x^2)*ArcTanh[a*x]) - (1 + a^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*
x]) + (3*SinhIntegral[2*ArcTanh[a*x]])/2 + SinhIntegral[4*ArcTanh[a*x]] - D
efer[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx = a^2\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx + \int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2}a\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2}\,dx + a^2\int$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax}$$

Mathematica [A]
time = 3.65, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 5.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-a^2x^2+1)^3 \operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (5*a^2*x^2 - 1)*log(a*x + 1) - (5*a^2*x^2 - 1)*log(-a*x + 1))/((a
^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(a*x + 1)^2 - 2*(a^6*x^6 - 2*a^4*x^4 + a^2
*x^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(-a*x
 + 1)^2) + integrate(-2*(10*a^4*x^4 - 3*a^2*x^2 + 1)/((a^8*x^9 - 3*a^6*x^7
+ 3*a^4*x^5 - a^2*x^3)*log(a*x + 1) - (a^8*x^9 - 3*a^6*x^7 + 3*a^4*x^5 - a^
2*x^3)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{6}x^{7}\operatorname{atanh}^{3}(ax) - 3a^{4}x^{5}\operatorname{atanh}^{3}(ax) + 3a^{2}x^{3}\operatorname{atanh}^{3}(ax) - x\operatorname{atanh}^{3}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x)**3 - 3*a**4*x**5*atanh(a*x)**3 + 3*a**2*x
**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
[Out] -int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

3.342 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{8}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{2}{a\left(1-a^2x^2\right)\tanh^{-1}}$$

```
[Out] -1/3/a/(-a^2*x^2+1)^2/arctanh(a*x)^3-2/3*x/(-a^2*x^2+1)^2/arctanh(a*x)^2-8/
3/a/(-a^2*x^2+1)^2/arctanh(a*x)+2/a/(-a^2*x^2+1)/arctanh(a*x)+2/3*Shi(2*arc
tanh(a*x))/a+4/3*Shi(4*arctanh(a*x))/a
```

Rubi [A]
time = 0.35, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6113, 6179, 6175, 6181, 5556, 12, 3379}

$$-\frac{2x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}+\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}+\frac{4\text{Shi}\left(4\tanh^{-1}(ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

```
[Out] -1/3*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) - (2*x)/(3*(1 - a^2*x^2)^2*ArcTan
h[a*x]^2) - 8/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 2/(a*(1 - a^2*x^2)*ArcTa
nh[a*x]) + (2*SinhIntegral[2*ArcTanh[a*x]])/(3*a) + (4*SinhIntegral[4*ArcTa
nh[a*x]])/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^4}\,dx = -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} + \frac{1}{3}(4a)\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{2}{3}\int \frac{}{(1-a^2}$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{}{3a\,(1-a^2x^2}$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{}{3a\,(1-a^2x^2}$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{}{3a\,(1-a^2x^2}$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{}{3a\,(1-a^2x^2}$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{}{3a\,(1-a^2x^2}$$
$$= -\frac{1}{3a\,(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2x}{3\,(1-a^2x^2)^2 \tanh^{-1}(ax)^2} - \frac{}{3a\,(1-a^2x^2}$$

Mathematica [A]
time = 0.15, size = 108, normalized size = 0.86

$$-\frac{1+2ax\tanh^{-1}(ax)+2\tanh^{-1}(ax)^2+6a^2x^2\tanh^{-1}(ax)^2-2(-1+a^2x^2)^2\tanh^{-1}(ax)^3\mathrm{Shi}(2\tanh^{-1}(ax))-4(-1+a^2x^2)^2\tanh^{-1}(ax)^3\mathrm{Shi}(4\tanh^{-1}(ax))}{3a\,(-1+a^2x^2)^2\tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

```
[Out] -1/3*(1 + 2*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2
- 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]] - 4*(-1 +
a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2
*ArcTanh[a*x]^3)
```

Maple [A]
time = 3.63, size = 122, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{8\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^3}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{3\operatorname{arctanh}(ax)}+\frac{2\,\mathrm{hyperbolicSineIntegral}(2\operatorname{arctanh}(ax))}{3}-\frac{\cosh}{24}}{a}$

default	$\frac{-\frac{1}{8\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^3}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{3\operatorname{arctanh}(ax)}+\frac{2\,\text{hyperbolicSineIntegral}(2\operatorname{arctanh}(ax))}{3}-\frac{\cosh(4}{24\operatorname{ar}}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x,method=_RETURNVERBOSE)

[Out] 1/a*(-1/8/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*Shi(2*arctanh(a*x))-1/24/arctanh(a*x)^3*cosh(4*arctanh(a*x))-1/12/arctanh(a*x)^2*sinh(4*arctanh(a*x))-1/3/arctanh(a*x)*cosh(4*arctanh(a*x))+4/3*Shi(4*arctanh(a*x)))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="maxima")

[Out] -4/3*(2*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 1)*log(-a*x + 1)^2 - 2*(a*x + (3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 2)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^3 - 3*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1)^2 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^3) + integrate(-8/3*(3*a^3*x^3 + 5*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(111) = 222$.
time = 0.34, size = 272, normalized size = 2.18

$$\frac{\left(2\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-2\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^3-8ax\log\left(-\frac{ax+1}{ax-1}\right)-4\left(3a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-8}{3\left(a^5x^4-2a^3x^2+a\right)\log\left(-\frac{ax+1}{ax-1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="fricas")

[Out] 1/3*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^3 - 8*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6 \operatorname{atanh}^4(ax) - 3a^4x^4 \operatorname{atanh}^4(ax) + 3a^2x^2 \operatorname{atanh}^4(ax) - \operatorname{atanh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**4,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**4 - 3*a**4*x**4*atanh(a*x)**4 + 3*a**2*x
**2*atanh(a*x)**4 - atanh(a*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^4\,{\left(a^2\,x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^4*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^3), x)
```

3.343 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^5}\,dx$

Optimal. Leaf size=170

$$-\frac{1}{4a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}-\frac{x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}-\frac{2}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-}}$$

```
[Out] -1/4/a/(-a^2*x^2+1)^2/arctanh(a*x)^4-1/3*x/(-a^2*x^2+1)^2/arctanh(a*x)^3-2/
3/a/(-a^2*x^2+1)^2/arctanh(a*x)^2+1/2/a/(-a^2*x^2+1)/arctanh(a*x)^2-8/3*x/(
-a^2*x^2+1)^2/arctanh(a*x)+x/(-a^2*x^2+1)/arctanh(a*x)+1/3*Chi(2*arctanh(a*
x))/a+4/3*Chi(4*arctanh(a*x))/a
```

Rubi [A]
time = 0.66, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6113, 6179, 6175, 6181, 3393, 3382, 6115, 5556}

$$\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}+\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{2}{3a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{1}{4a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}+\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{3a}+\frac{4\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

```
[Out] -1/4*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^4) - x/(3*(1 - a^2*x^2)^2*ArcTanh[a*
x]^3) - 2/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a*(1 - a^2*x^2)*ArcTa
nh[a*x]^2) - (8*x)/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/((1 - a^2*x^2)*ArcT
anh[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a) + (4*CoshIntegral[4*ArcTanh[
a*x]])/(3*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx &= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} + a\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^4}\,dx \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} + \frac{1}{3}\int \frac{}{(1-a^2x} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{1}{6a\left(1-a^2x^2\right)} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)} \\
&= -\frac{1}{4a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a\left(1-a^2x^2\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 132, normalized size = 0.78

$$
-\frac{3+4ax\tanh^{-1}(ax)+2\tanh^{-1}(ax)^2+6a^2x^2\tanh^{-1}(ax)^2+20ax\tanh^{-1}(ax)^3+12a^3x^3\tanh^{-1}(ax)^3-4\left(-1+a^2x^2\right)^2\tanh^{-1}(ax)^4\operatorname{Chi}\left(2\tanh^{-1}(ax)\right)-16\left(-1+a^2x^2\right)^2\tanh^{-1}(ax)^4\operatorname{Chi}\left(4\tanh^{-1}(ax)\right)}{12a\left(-1+a^2x^2\right)^2\tanh^{-1}(ax)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

```
[Out] -1/12*(3 + 4*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2
 + 20*a*x*ArcTanh[a*x]^3 + 12*a^3*x^3*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*A
rcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*
x]^4*CoshIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)
```

Maple [A]
time = 2.74, size = 152, normalized size = 0.89

method	result

derivativedivides	$\frac{-\frac{3}{32\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^4}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^3}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegr}}{3}}{}$
default	$\frac{-\frac{3}{32\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^4}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^3}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{12\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{6\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegr}}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-3/32/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12*sinh
(2*arctanh(a*x))/arctanh(a*x)^3-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/
6*sinh(2*arctanh(a*x))/arctanh(a*x)+1/3*Chi(2*arctanh(a*x))-1/32/arctanh(a*
x)^4*cosh(4*arctanh(a*x))-1/24/arctanh(a*x)^3*sinh(4*arctanh(a*x))-1/12/arc
tanh(a*x)^2*cosh(4*arctanh(a*x))-1/3/arctanh(a*x)*sinh(4*arctanh(a*x))+4/3*
Chi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="maxima")
```

```
[Out] -2/3*((3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - (3*a^3*x^3 + 5*a*x)*log(-a*x + 1
)^3 + 4*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 3*
(3*a^3*x^3 + 5*a*x)*log(a*x + 1) + 1)*log(-a*x + 1)^2 - (3*(3*a^3*x^3 + 5*a
*x)*log(a*x + 1)^2 + 4*a*x + 2*(3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)
+ 6)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4 - 4*(a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^3*log(-a*x + 1) + 6*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^2
*log(-a*x + 1)^2 - 4*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1)^3
 + (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^4) + integrate(-2/3*(3*a^4*x^4 +
 24*a^2*x^2 + 5)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(151) = 302$.
time = 0.49, size = 303, normalized size = 1.78

$$\frac{\left(4\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+4\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^4-4\left(3\,a^3x^3+5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3-16\,ax\log\left(-\frac{ax+1}{ax-1}\right)-4\left(3\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-24}{6\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-\frac{ax+1}{ax-1}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="fricas")
```

```
[Out] 1/6*((4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
```

```
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(
a*x - 1))^3 - 16*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3*a^2*x^2 + 1)*log(-(a*
x + 1)/(a*x - 1))^2 - 24)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x -
1))^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^6x^6 \operatorname{atanh}^5(ax) - 3a^4x^4 \operatorname{atanh}^5(ax) + 3a^2x^2 \operatorname{atanh}^5(ax) - \operatorname{atanh}^5(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**5,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**5 - 3*a**4*x**4*atanh(a*x)**5 + 3*a**2*x
**2*atanh(a*x)**5 - atanh(a*x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\operatorname{atanh}(a\,x)^5\,(a^2\,x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^5*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^3), x)
```

3.344 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^6} \, dx$

Optimal. Leaf size=257

$$-\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5}-\frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4}-\frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}+\frac{1}{5a\left(1-a^2x^2\right) \tan}$$

```
[Out] -1/5/a/(-a^2*x^2+1)^2/arctanh(a*x)^5-1/5*x/(-a^2*x^2+1)^2/arctanh(a*x)^4-4/
15/a/(-a^2*x^2+1)^2/arctanh(a*x)^3+1/5/a/(-a^2*x^2+1)/arctanh(a*x)^3-8/15*x
/(-a^2*x^2+1)^2/arctanh(a*x)^2+1/5*x/(-a^2*x^2+1)/arctanh(a*x)^2-32/15/a/(-
a^2*x^2+1)^2/arctanh(a*x)+8/5/a/(-a^2*x^2+1)/arctanh(a*x)+1/5*(a^2*x^2+1)/a
/(-a^2*x^2+1)/arctanh(a*x)+2/15*Shi(2*arctanh(a*x))/a+16/15*Shi(4*arctanh(a
*x))/a
```

Rubi [A]
time = 0.91, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6113, 6179, 6175, 6143, 6181, 5556, 12, 3379}

$$\frac{x}{5\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{8x}{15\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{x}{5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}+\frac{a^2x^2+1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{8}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{32}{15a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{4}{15a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}-\frac{1}{5a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^5}+\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{15a}+\frac{16\text{Shi}\left(4\tanh^{-1}(ax)\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

```
[Out] -1/5*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^5) - x/(5*(1 - a^2*x^2)^2*ArcTanh[a*
x]^4) - 4/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + 1/(5*a*(1 - a^2*x^2)*ArcT
anh[a*x]^3) - (8*x)/(15*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(5*(1 - a^2*x^2
)*ArcTanh[a*x]^2) - 32/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 8/(5*a*(1 - a^
2*x^2)*ArcTanh[a*x]) + (1 + a^2*x^2)/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*
SinhIntegral[2*ArcTanh[a*x]])/(15*a) + (16*SinhIntegral[4*ArcTanh[a*x]])/(1
5*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6143

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTanh[c*x])^(p +
 2)/(d + e*x^2)^2), x], x] + Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p +
2)/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^6}\,dx = -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} + \frac{1}{5}(4a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} + \frac{1}{5}\int \frac{}{(1-a^2}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{}{15a\left(1-a^2x\right.}$$

Mathematica [A]

time = 0.17, size = 166, normalized size = 0.65

$$-\frac{3+3ax\tanh^{-1}(ax)+\tanh^{-1}(ax)^2+3a^2x^2\tanh^{-1}(ax)^2+5ax\tanh^{-1}(ax)^3+3a^3x^3\tanh^{-1}(ax)^3+5\tanh^{-1}(ax)^4+24a^2x^2\tanh^{-1}(ax)^4+3a^4x^4\tanh^{-1}(ax)^4-2(-1+a^2x^2)^2\tanh^{-1}(ax)^5\operatorname{Shi}\left(2\tanh^{-1}(ax)\right)-16(-1+a^2x^2)^2\tanh^{-1}(ax)^5\operatorname{Shi}\left(4\tanh^{-1}(ax)\right)}{15a\left(-1+a^2x^2\right)^2\tanh^{-1}(ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

```
[Out] -1/15*(3 + 3*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 +
 5*a*x*ArcTanh[a*x]^3 + 3*a^3*x^3*ArcTanh[a*x]^3 + 5*ArcTanh[a*x]^4 + 24*a^
```

```
2*x^2*ArcTanh[a*x]^4 + 3*a^4*x^4*ArcTanh[a*x]^4 - 2*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^5*SinhIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5*
SinhIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5)
```

Maple [A]
time = 3.71, size = 182, normalized size = 0.71

method	result
derivativedivides	$-\frac{3}{40\,\mathrm{arctanh}(ax)^5}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{10\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{20\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^3}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{15\,\mathrm{arctanh}(ax)}+\frac{2\,\mathrm{h}}{}$
default	$-\frac{3}{40\,\mathrm{arctanh}(ax)^5}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{10\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{20\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^3}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{15\,\mathrm{arctanh}(ax)}+\frac{2\,\mathrm{h}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-3/40/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/arc
tanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1
/30*sinh(2*arctanh(a*x))/arctanh(a*x)^2-1/15/arctanh(a*x)*cosh(2*arctanh(a*
x))+2/15*Shi(2*arctanh(a*x))-1/40/arctanh(a*x)^5*cosh(4*arctanh(a*x))-1/40/
arctanh(a*x)^4*sinh(4*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(4*arctanh(a*x)
)-1/15/arctanh(a*x)^2*sinh(4*arctanh(a*x))-4/15/arctanh(a*x)*cosh(4*arctanh
(a*x))+16/15*Shi(4*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="maxima")
```

```
[Out] -2/15*((3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^4 + (3*a^4*x^4 + 24*a^2*x^
2 + 5)*log(-a*x + 1)^4 + 2*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - 2*(3*a^3*x^
3 + 5*a*x + 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1))*log(-a*x + 1)^3 +
24*a*x*log(a*x + 1) + 4*(3*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(6*a^2*x^2 + 3*(
3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^2 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x
+ 1) + 2)*log(-a*x + 1)^2 - 2*(2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^
3 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^2 + 12*a*x + 4*(3*a^2*x^2 + 1)*log(a
*x + 1))*log(-a*x + 1) + 48)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^5 - 5*
(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4*log(-a*x + 1) + 10*(a^5*x^4 - 2*a^
3*x^2 + a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^5*x^4 - 2*a^3*x^2 + a)*lo
g(a*x + 1)^2*log(-a*x + 1)^3 + 5*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log
(-a*x + 1)^4 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^5) + integrate(-8/15
*(15*a^3*x^3 + 17*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)
- (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [A]
time = 0.38, size = 341, normalized size = 1.33

$$\frac{\left(8\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(\frac{a^{2}x^{2}+2ax+1}{a^{2}x^{2}-2ax+1}\right)-8\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(\frac{a^{2}x^{2}-2ax+1}{a^{2}x^{2}+2ax+1}\right)+\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{5}-2\left(3a^{4}x^{4}+24a^{2}x^{2}+5\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}-4\left(3a^{3}x^{3}+5ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}-48ax\log\left(-\frac{ax+1}{ax-1}\right)-8\left(3a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-96}{15\left(a^{5}x^{4}-2a^{3}x^{2}+a\right)\log\left(-\frac{ax+1}{ax-1}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) - 8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a
*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
 + 1)))*log(-(a*x + 1)/(a*x - 1))^5 - 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(-(
a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 -
 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 8*(3*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1))^2 - 96)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{a^{6}x^{6}\operatorname{atanh}^{6}(ax)-3a^{4}x^{4}\operatorname{atanh}^{6}(ax)+3a^{2}x^{2}\operatorname{atanh}^{6}(ax)-\operatorname{atanh}^{6}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**6,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**6 - 3*a**4*x**4*atanh(a*x)**6 + 3*a**2*x
**2*atanh(a*x)**6 - atanh(a*x)**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int\frac{1}{\operatorname{atanh}(a\,x)^{6}\,{\left(a^{2}\,x^{2}-1\right)}^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(atanh(a*x)^6*(a^2*x^2 - 1)^3),x)
```

```
[Out] -int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^3), x)
```

3.345 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^4}\,dx$

Optimal. Leaf size=134

$$-\frac{1}{36a\,(1-a^2x^2)^3}-\frac{5}{96a\,(1-a^2x^2)^2}-\frac{5}{32a\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{6\,(1-a^2x^2)^3}+\frac{5x\tanh^{-1}(ax)}{24\,(1-a^2x^2)^2}+\frac{5x\tanh^{-1}(ax)}{16\,(1-a^2x^2)}+\frac{5\tanh}{3}$$

[Out] -1/36/a/(-a^2*x^2+1)^3-5/96/a/(-a^2*x^2+1)^2-5/32/a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)/(-a^2*x^2+1)^3+5/24*x*arctanh(a*x)/(-a^2*x^2+1)^2+5/16*x*arctanh(a*x)/(-a^2*x^2+1)+5/32*arctanh(a*x)^2/a

Rubi [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6107, 6103, 267}

$$-\frac{5}{32a\,(1-a^2x^2)}-\frac{5}{96a\,(1-a^2x^2)^2}-\frac{1}{36a\,(1-a^2x^2)^3}+\frac{5x\tanh^{-1}(ax)}{16\,(1-a^2x^2)}+\frac{5x\tanh^{-1}(ax)}{24\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{6\,(1-a^2x^2)^3}+\frac{5\tanh^{-1}(ax)^2}{32a}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]

[Out] -1/36*1/(a*(1 - a^2*x^2)^3) - 5/(96*a*(1 - a^2*x^2)^2) - 5/(32*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x])/(24*(1 - a^2*x^2)^2) + (5*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^2)/(32*a)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
```

```
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^4}\,dx = -\frac{1}{36a\,(1-a^2x^2)^3} + \frac{x\tanh^{-1}(ax)}{6\,(1-a^2x^2)^3} + \frac{5}{6}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= -\frac{1}{36a\,(1-a^2x^2)^3} - \frac{5}{96a\,(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)}{6\,(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)}{24\,(1-a^2x^2)^2} + \frac{5}{8}\int \frac{\tanh^{-1}(}{(1-a^2}$$
$$= -\frac{1}{36a\,(1-a^2x^2)^3} - \frac{5}{96a\,(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)}{6\,(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)}{24\,(1-a^2x^2)^2} + \frac{5x\tanh^{-1}(a}{16\,(1-a^2x^2}$$
$$= -\frac{1}{36a\,(1-a^2x^2)^3} - \frac{5}{96a\,(1-a^2x^2)^2} - \frac{5}{32a\,(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)}{6\,(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(a}{24\,(1-a^2x^2}$$

Mathematica [A]
time = 0.07, size = 81, normalized size = 0.60

$$\frac{68-105a^2x^2+45a^4x^4-6ax(33-40a^2x^2+15a^4x^4)\tanh^{-1}(ax)+45(-1+a^2x^2)^3\tanh^{-1}(ax)^2}{288a\,(-1+a^2x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
[Out] (68 - 105*a^2*x^2 + 45*a^4*x^4 - 6*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTa
nh[a*x] + 45*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2)/(288*a*(-1 + a^2*x^2)^3)
```

Maple [A]
time = 0.72, size = 222, normalized size = 1.66

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{48(ax-1)^3}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5\operatorname{arctanh}(ax)}{32(ax-1)}-\frac{5\operatorname{arctanh}(ax)\ln(ax-1)}{32}-\frac{\operatorname{arctanh}(ax)}{48(ax+1)^3}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{5\operatorname{arctanh}(ax)}{32(ax+1)}+\frac{5\operatorname{arctanh}(ax)}{32}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{48(ax-1)^3}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5\operatorname{arctanh}(ax)}{32(ax-1)}-\frac{5\operatorname{arctanh}(ax)\ln(ax-1)}{32}-\frac{\operatorname{arctanh}(ax)}{48(ax+1)^3}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{5\operatorname{arctanh}(ax)}{32(ax+1)}+\frac{5\operatorname{arctanh}(ax)}{32}}{}$
risch	$\frac{5\ln(ax+1)^2}{128a}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-80a^3x^3+45x^2\ln(-ax+1)a^2+66ax-15\ln(-ax+1)}{192(a^2x^2-1)^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/48*arctanh(a*x)/(a*x-1)^3+1/16*arctanh(a*x)/(a*x-1)^2-5/32*arctanh(
a*x)/(a*x-1)-5/32*arctanh(a*x)*ln(a*x-1)-1/48*arctanh(a*x)/(a*x+1)^3-1/16*a
rctanh(a*x)/(a*x+1)^2-5/32*arctanh(a*x)/(a*x+1)+5/32*arctanh(a*x)*ln(a*x+1)
+5/64*ln(a*x-1)*ln(1/2*a*x+1/2)-5/128*ln(a*x-1)^2+5/64*(ln(a*x+1)-ln(1/2*a*
x+1/2))*ln(-1/2*a*x+1/2)-5/128*ln(a*x+1)^2+1/288/(a*x-1)^3-7/384/(a*x-1)^2+
37/384/(a*x-1)-1/288/(a*x+1)^3-7/384/(a*x+1)^2-37/384/(a*x+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(114) = 228$.
time = 0.27, size = 240, normalized size = 1.79

$$-\frac{1}{96}\left(\frac{2\,(15\,a^4x^5 - 40\,a^2x^3 + 33\,x)}{a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1} - \frac{15\,\log(ax+1)}{a} + \frac{15\,\log(ax-1)}{a}\right)\operatorname{artanh}(ax) + \frac{(180\,a^4x^4 - 420\,a^2x^2 - 45\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log(ax+1)^2 + 90\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log(ax+1)\log(ax-1) - 45\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log(ax-1)^2 + 272)a}{1152\,(a^8x^6 - 3\,a^6x^4 + 3\,a^4x^2 - a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x) + 1/1152*(180*a^
4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)
^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1) - 4
5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a/(a^8*x^6 -
3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

Fricas [A]
time = 0.36, size = 131, normalized size = 0.98

$$\frac{180\,a^4x^4 - 420\,a^2x^2 + 45\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 12\,(15\,a^5x^5 - 40\,a^3x^3 + 33\,ax)\log\left(-\frac{ax+1}{ax-1}\right) + 272}{1152\,(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(180*a^4*x^4 - 420*a^2*x^2 + 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1
)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*log(-
(a*x + 1)/(a*x - 1)) + 272)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(ax-1)^4\,(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a^2*x^2 - 1)^4, x)
```

Mupad [B]
time = 1.30, size = 206, normalized size = 1.54

$$\frac{\frac{34}{3\,a}-\frac{35\,a\,x^2}{2}+\frac{15\,a^3\,x^4}{2}}{48\,a^6\,x^6-144\,a^4\,x^4+144\,a^2\,x^2-48}-\ln\left(1-a\,x\right)\,\left(\frac{5\,\ln\left(a\,x+1\right)}{64\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{2\,a^6\,x^6-6\,a^4\,x^4+6\,a^2\,x^2-2}\right)+\frac{5\,{\ln\left(a\,x+1\right)}^2}{128\,a}+\frac{5\,{\ln\left(1-a\,x\right)}^2}{128\,a}-\frac{\ln\left(a\,x+1\right)\,\left(\frac{11\,x}{32\,a}-\frac{5\,a\,x^3}{12}+\frac{5\,a^3\,x^5}{32}\right)}{3\,a\,x^2-\frac{1}{a}-3\,a^3\,x^4+a^5\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(a^2*x^2 - 1)^4,x)
```

```
[Out] (34/(3*a) - (35*a*x^2)/2 + (15*a^3*x^4)/2)/(144*a^2*x^2 - 144*a^4*x^4 + 48*
a^6*x^6 - 48) - log(1 - a*x)*((5*log(a*x + 1))/(64*a) - ((11*x)/16 - (5*a^2
*x^3)/6 + (5*a^4*x^5)/16)/(6*a^2*x^2 - 6*a^4*x^4 + 2*a^6*x^6 - 2)) + (5*log
(a*x + 1)^2)/(128*a) + (5*log(1 - a*x)^2)/(128*a) - (log(a*x + 1)*((11*x)/(
32*a) - (5*a*x^3)/12 + (5*a^3*x^5)/32))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^
6)
```

3.346 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^4}\,dx$

Optimal. Leaf size=214

$$\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{245\tanh^{-1}(ax)}{1152a}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}-$$

```
[Out] 1/108*x/(-a^2*x^2+1)^3+65/1728*x/(-a^2*x^2+1)^2+245/1152*x/(-a^2*x^2+1)+245
/1152*arctanh(a*x)/a-1/18*arctanh(a*x)/a/(-a^2*x^2+1)^3-5/48*arctanh(a*x)/a
/(-a^2*x^2+1)^2-5/16*arctanh(a*x)/a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)^2/(-a^2
*x^2+1)^3+5/24*x*arctanh(a*x)^2/(-a^2*x^2+1)^2+5/16*x*arctanh(a*x)^2/(-a^2*
x^2+1)+5/48*arctanh(a*x)^3/a
```

Rubi [A]
time = 0.12, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6111, 6103, 6141, 205, 212}

$$\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{x}{108\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^2}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}+\frac{5\tanh^{-1}(ax)^3}{48a}+\frac{245\tanh^{-1}(ax)}{1152a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

```
[Out] x/(108*(1 - a^2*x^2)^3) + (65*x)/(1728*(1 - a^2*x^2)^2) + (245*x)/(1152*(1
- a^2*x^2)) + (245*ArcTanh[a*x])/(1152*a) - ArcTanh[a*x]/(18*a*(1 - a^2*x^2
)^3) - (5*ArcTanh[a*x])/(48*a*(1 - a^2*x^2)^2) - (5*ArcTanh[a*x])/(16*a*(1
- a^2*x^2)) + (x*ArcTanh[a*x]^2)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^2)
/(24*(1 - a^2*x^2)^2) + (5*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + (5*ArcTan
h[a*x]^3)/(48*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^4}\,dx &= -\frac{\tanh^{-1}(ax)}{18a\,(1-a^2x^2)^3}+\frac{x\tanh^{-1}(ax)^2}{6\,(1-a^2x^2)^3}+\frac{1}{18}\int \frac{1}{(1-a^2x^2)^4}\,dx+\frac{5}{6}\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx\\
&= \frac{x}{108\,(1-a^2x^2)^3}-\frac{\tanh^{-1}(ax)}{18a\,(1-a^2x^2)^3}-\frac{5\tanh^{-1}(ax)}{48a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)^2}{6\,(1-a^2x^2)^3}+\cdots\\
&= \frac{x}{108\,(1-a^2x^2)^3}+\frac{65x}{1728\,(1-a^2x^2)^2}-\frac{\tanh^{-1}(ax)}{18a\,(1-a^2x^2)^3}-\frac{5\tanh^{-1}(ax)}{48a\,(1-a^2x^2)^2}+\cdots\\
&= \frac{x}{108\,(1-a^2x^2)^3}+\frac{65x}{1728\,(1-a^2x^2)^2}+\frac{65x}{1152\,(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{18a\,(1-a^2x^2)^3}-\cdots\\
&= \frac{x}{108\,(1-a^2x^2)^3}+\frac{65x}{1728\,(1-a^2x^2)^2}+\frac{245x}{1152\,(1-a^2x^2)}+\frac{65\tanh^{-1}(ax)}{1152a}-\cdots\\
&= \frac{x}{108\,(1-a^2x^2)^3}+\frac{65x}{1728\,(1-a^2x^2)^2}+\frac{245x}{1152\,(1-a^2x^2)}+\frac{245\tanh^{-1}(ax)}{1152a}-\cdots
\end{aligned}
$$

Mathematica [A]

time = 0.08, size = 157, normalized size = 0.73

$$\frac{-\frac{64x}{(-1+a^2x^2)^3}+\frac{260x}{(-1+a^2x^2)^2}-\frac{1470x}{-1+a^2x^2}+\frac{48(68-105a^2x^2+45a^4x^4)\tanh^{-1}(ax)}{a(-1+a^2x^2)^3}-\frac{144x(33-40a^2x^2+15a^4x^4)\tanh^{-1}(ax)^2}{(-1+a^2x^2)^3}+\frac{720\tanh^{-1}(ax)^3}{a}-\frac{735\log(1-ax)}{a}+\frac{735\log(1+ax)}{a}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

```
[Out] ((-64*x)/(-1 + a^2*x^2)^3 + (260*x)/(-1 + a^2*x^2)^2 - (1470*x)/(-1 + a^2*x
^2) + (48*(68 - 105*a^2*x^2 + 45*a^4*x^4)*ArcTanh[a*x])/(a*(-1 + a^2*x^2)^3
) - (144*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^2)/(-1 + a^2*x^2)^3
+ (720*ArcTanh[a*x]^3)/a - (735*Log[1 - a*x])/a + (735*Log[1 + a*x])/a)/691
2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 10.11, size = 2716, normalized size = 12.69

method	result
risch	$\frac{5\ln(ax+1)^3}{384a}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-80a^3x^3+45x^2\ln(-ax+1)a^2+66ax-15\ln(-ax+1))\ \mathrm{l}}{384(a^2x^2-1)^3a}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/48*arctanh(a*x)^2/(a*x-1)^3+1/16*arctanh(a*x)^2/(a*x-1)^2-5/32*arct
anh(a*x)^2/(a*x-1)-5/32*arctanh(a*x)^2*ln(a*x-1)-1/48*arctanh(a*x)^2/(a*x+1
)^3-1/16*arctanh(a*x)^2/(a*x+1)^2-5/32*arctanh(a*x)^2/(a*x+1)+5/32*arctanh(
a*x)^2*ln(a*x+1)-5/16*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/3456*
(1080*arctanh(a*x)^3*a^2*x^2-897*a*x-735*a^5*x^5-1125*a^4*x^4*arctanh(a*x)-
360*arctanh(a*x)^3-315*a^2*x^2*arctanh(a*x)+897*arctanh(a*x)+1600*a^3*x^3-1
080*arctanh(a*x)^3*a^4*x^4+810*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*a^4*x^4-1620*I*Pi*arctanh(a*x)^2*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))^3*a^4*x^4-810*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^4*x^4+810*I*Pi*arctanh(a*x)^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2
+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^2*x^2+270*I*Pi*arctanh(a*x)^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^6*x^6-540*I*Pi*arctanh(a*x)^2*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a^6*x^6-2
70*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a^6*x^6+270*I*Pi*arctanh(a*x)^2*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/
```

```
(a^2*x^2-1))*a^6*x^6+810*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1
/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^4*x^4+1620*I*Pi*arctanh(a*x)^2*csgn(
I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a^4*x^4+810*I
*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a^4*x^4-810*I*Pi*arctanh(a*x)^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*a^4*x^4-810*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^2*x^2-1620*I*Pi*arctanh(a*x)^2*csgn(I*(a
*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a^2*x^2-810*I*Pi*
arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a^2*x^2+810*I*Pi*arctanh(a*x)^2*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*a^2*x^2-270*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*a^6*x^6+540*I*Pi*arctanh(a*x)^2*a^6*x^6-1620*I
*Pi*arctanh(a*x)^2*a^4*x^4+1620*I*Pi*arctanh(a*x)^2*a^2*x^2+270*I*Pi*arctan
h(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))
+540*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^2+270*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-270*I*Pi*arctan
h(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^2-270*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^3*a^6*x^6+540*I*Pi*arctanh(a*x)^2*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))^3*a^6*x^6-270*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1))^3*a^6*x^6-540*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*a^6*x^6-270*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))+810*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*a^4*x^4
+1620*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*a^4*x^4-810*
I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3*a^2*x^2+1620*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*a
^2*x^2-810*I*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*a^2*x^2-1620
*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*a^2*x^2+360*arcta
nh(a*x)^3*a^6*x^6+735*arctanh(a*x)*a^6*x^6-540*I*Pi*arctanh(a*x)^2+270*I*Pi
*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-
540*I*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+270*I*Pi*arcta
nh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+540*I*Pi*arctanh(a*x)^2*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))^2)/(a*x-1)^3/(a*x+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(183) = 366$.
time = 0.29, size = 516, normalized size = 2.41

$$-\frac{1}{96}\left(\frac{2\left(15\,a^4x^5-40\,a^2x^3+33\,x\right)}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}-\frac{15\,\log\left(ax+1\right)}{a}+\frac{15\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(1470\,a^5x^5-3200\,a^3x^3-90\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax+1\right)^3+270\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax+1\right)^2\log\left(ax-1\right)+90\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax-1\right)^3+1794\,ax-15\left(49\,a^6x^6-147\,a^4x^4+147\,a^2x^2+18\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax-1\right)^2-49\right)\log\left(ax+1\right)+735\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax-1\right)\right)a^2}{6912\left(a^9x^6-3\,a^7x^4+3\,a^5x^2-a^3\right)}+\frac{\left(180\,a^4x^4-420\,a^2x^2-45\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax+1\right)^2+90\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax+1\right)\log\left(ax-1\right)-45\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(ax-1\right)^2+272\right)a\,\operatorname{artanh}\left(ax\right)}{576\left(a^8x^6-3\,a^6x^4+3\,a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/6912*(1470
*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
+ 1)^3 + 270*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x -
 1) + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x -
15*(49*a^6*x^6 - 147*a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^
2*x^2 - 1)*log(a*x - 1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3
*a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 - a^3) + 1
/576*(180*a^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*
log(a*x + 1)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(
a*x - 1) - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a
*arctanh(a*x)/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

Fricas [A]
time = 0.39, size = 179, normalized size = 0.84

$$-\frac{1470\,a^5x^5 - 3200\,a^3x^3 - 90\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 36\,(15\,a^5x^5 - 40\,a^3x^3 + 33\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 1794\,ax - 3\,(245\,a^6x^6 - 375\,a^4x^4 - 105\,a^2x^2 + 299)\log\left(-\frac{ax+1}{ax-1}\right)}{6912\,(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] -1/6912*(1470*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)*log(-(a*x + 1)/(a*x - 1))^3 + 36*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*lo
g(-(a*x + 1)/(a*x - 1))^2 + 1794*a*x - 3*(245*a^6*x^6 - 375*a^4*x^4 - 105*a
^2*x^2 + 299)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{(ax-1)^4\,(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)**2/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="giac")

[Out] integrate(arctanh(a*x)^2/(a^2*x^2 - 1)^4, x)

Mupad [B]
time = 2.49, size = 493, normalized size = 2.30

$$\ln(1-a\,x)^2\left(\frac{5\,\ln(a\,x+1)}{128\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{4\,a^6\,x^6-12\,a^4\,x^4+12\,a^2\,x^2-4}\right)-\frac{\frac{245\,a^4\,x^5}{8}-\frac{200\,a^2\,x^3}{3}+\frac{299\,x}{8}}{144\,a^6\,x^6-432\,a^4\,x^4+432\,a^2\,x^2-144}-\ln(1-a\,x)\left(\frac{5\ln(a\,x+1)^2}{128\,a}+\frac{\frac{37\,x}{2}-35\,a\,x^2+\frac{68}{3\,a}-\frac{82\,a^2\,x^3}{3}+15\,a^3\,x^4+\frac{23\,a^4\,x^5}{2}}{192\,a^6\,x^6-576\,a^4\,x^4+576\,a^2\,x^2-192}-\frac{\frac{37\,x}{2}+35\,a\,x^2-\frac{68}{3\,a}-\frac{82\,a^2\,x^3}{3}-15\,a^3\,x^4+\frac{23\,a^4\,x^5}{2}}{192\,a^6\,x^6-576\,a^4\,x^4+576\,a^2\,x^2-192}-\frac{\ln(a\,x+1)\,\left(10\,a^4\,x^5-\frac{80\,a^2\,x^3}{3}+22\,x\right)}{64\,a^6\,x^6-192\,a^4\,x^4+192\,a^2\,x^2-64}\right)+\frac{5\ln(a\,x+1)^3}{384\,a}-\frac{5\ln(1-a\,x)^3}{384\,a}+\frac{\ln(a\,x+1)\,\left(\frac{17}{72\,a^2}-\frac{35\,x^2}{96}+\frac{5\,a^2\,x^4}{32}\right)}{3\,a\,x^2-\frac{1}{a}-3\,a^3\,x^4+a^5\,x^6}-\frac{\ln(a\,x+1)^2\,\left(\frac{11\,x}{64\,a}-\frac{5\,a\,x^3}{24}+\frac{5\,a^3\,x^5}{64}\right)}{3\,a\,x^2-\frac{1}{a}-3\,a^3\,x^4+a^5\,x^6}-\frac{\mathrm{atan}(a\,x\,1\mathrm{i})\,245\mathrm{i}}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a*x)^2/(a^2*x^2 - 1)^4,x)

[Out] log(1 - a*x)^2*((5*log(a*x + 1))/(128*a) - ((11*x)/16 - (5*a^2*x^3)/6 + (5*a^4*x^5)/16)/(12*a^2*x^2 - 12*a^4*x^4 + 4*a^6*x^6 - 4)) - ((299*x)/8 - (200*a^2*x^3)/3 + (245*a^4*x^5)/8)/(432*a^2*x^2 - 432*a^4*x^4 + 144*a^6*x^6 - 144) - log(1 - a*x)*((5*log(a*x + 1)^2)/(128*a) + ((37*x)/2 - 35*a*x^2 + 68/(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(576*a^2*x^2 - 576*a^4*x^4 + 192*a^6*x^6 - 192) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a^2*x^3)/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(576*a^2*x^2 - 576*a^4*x^4 + 192*a^6*x^6 - 192) - (log(a*x + 1)*(22*x - (80*a^2*x^3)/3 + 10*a^4*x^5))/(192*a^2*x^2 - 192*a^4*x^4 + 64*a^6*x^6 - 64)) + (5*log(a*x + 1)^3)/(384*a) - (5*log(1 - a*x)^3)/(384*a) - (atan(a*x*1i)*245i)/(1152*a) + (log(a*x + 1)*(17/(72*a^2) - (35*x^2)/96 + (5*a^2*x^4)/32))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6) - (log(a*x + 1)^2*((11*x)/(64*a) - (5*a*x^3)/24 + (5*a^3*x^5)/64))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6)

3.347 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^4} \, dx$

Optimal. Leaf size=291

$$-\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{245}{768a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}+\frac{245x\tanh^{-1}(ax}{384\left(1-a^2x^2\right)}$$

```
[Out] -1/216/a/(-a^2*x^2+1)^3-65/2304/a/(-a^2*x^2+1)^2-245/768/a/(-a^2*x^2+1)+1/3
6*x*arctanh(a*x)/(-a^2*x^2+1)^3+65/576*x*arctanh(a*x)/(-a^2*x^2+1)^2+245/38
4*x*arctanh(a*x)/(-a^2*x^2+1)+245/768*arctanh(a*x)^2/a-1/12*arctanh(a*x)^2/
a/(-a^2*x^2+1)^3-5/32*arctanh(a*x)^2/a/(-a^2*x^2+1)^2-15/32*arctanh(a*x)^2/
a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)^3/(-a^2*x^2+1)^3+5/24*x*arctanh(a*x)^3/(-
a^2*x^2+1)^2+5/16*x*arctanh(a*x)^3/(-a^2*x^2+1)+5/64*arctanh(a*x)^4/a
```

Rubi [A]
time = 0.22, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6111, 6103, 6141, 267, 6107}

$$-\frac{245}{768a\left(1-a^2x^2\right)}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^3}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^3}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}-\frac{15\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)}-\frac{5\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right)^3}+\frac{245x\tanh^{-1}(ax)}{384\left(1-a^2x^2\right)}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{5\tanh^{-1}(ax)^4}{64a}+\frac{245\tanh^{-1}(ax)^2}{768a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

```
[Out] -1/216*1/(a*(1 - a^2*x^2)^3) - 65/(2304*a*(1 - a^2*x^2)^2) - 245/(768*a*(1
- a^2*x^2)) + (x*ArcTanh[a*x])/(36*(1 - a^2*x^2)^3) + (65*x*ArcTanh[a*x])/(
576*(1 - a^2*x^2)^2) + (245*x*ArcTanh[a*x])/(384*(1 - a^2*x^2)) + (245*ArcT
anh[a*x]^2)/(768*a) - ArcTanh[a*x]^2/(12*a*(1 - a^2*x^2)^3) - (5*ArcTanh[a*
x]^2)/(32*a*(1 - a^2*x^2)^2) - (15*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) + (
x*ArcTanh[a*x]^3)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^3)/(24*(1 - a^2*x
^2)^2) + (5*x*ArcTanh[a*x]^3)/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^4)/(64*a
)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^4}\,dx &= -\frac{\tanh^{-1}(ax)^2}{12a\,(1-a^2x^2)^3}+\frac{x\tanh^{-1}(ax)^3}{6\,(1-a^2x^2)^3}+\frac{1}{6}\int\frac{\tanh^{-1}(ax)}{(1-a^2x^2)^4}\,dx+\frac{5}{6}\int\frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^3}\,dx\\
&= -\frac{1}{216a\,(1-a^2x^2)^3}+\frac{x\tanh^{-1}(ax)}{36\,(1-a^2x^2)^3}-\frac{\tanh^{-1}(ax)^2}{12a\,(1-a^2x^2)^3}-\frac{5\tanh^{-1}(ax)^2}{32a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}}{6\,(1-a}\\
&= -\frac{1}{216a\,(1-a^2x^2)^3}-\frac{65}{2304a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{36\,(1-a^2x^2)^3}+\frac{65x\tanh^{-1}(ax)}{576\,(1-a^2x^2)^2}-\frac{\tanh}{12a\,(1}\\
&= -\frac{1}{216a\,(1-a^2x^2)^3}-\frac{65}{2304a\,(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{36\,(1-a^2x^2)^3}+\frac{65x\tanh^{-1}(ax)}{576\,(1-a^2x^2)^2}+\frac{65x\,\mathrm{ta}}{384\,(}\\
&= -\frac{1}{216a\,(1-a^2x^2)^3}-\frac{65}{2304a\,(1-a^2x^2)^2}-\frac{65}{768a\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{36\,(1-a^2x^2)^3}+\frac{65x\,\mathrm{ta}}{576\,(1}\\
&= -\frac{1}{216a\,(1-a^2x^2)^3}-\frac{65}{2304a\,(1-a^2x^2)^2}-\frac{245}{768a\,(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{36\,(1-a^2x^2)^3}+\frac{65x\,\mathrm{ta}}{576\,(1}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 143, normalized size = 0.49

$$\frac{2432-4605a^2x^2+2205a^4x^4-6ax(897-1600a^2x^2+735a^4x^4)\tanh^{-1}(ax)+9(299-105a^2x^2-375a^4x^4+245a^6x^6)\tanh^{-1}(ax)^2-144ax(33-40a^2x^2+15a^4x^4)\tanh^{-1}(ax)^3+540(-1+a^2x^2)^3\tanh^{-1}(ax)^4}{6912a\,(-1+a^2x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

```
[Out] (2432 - 4605*a^2*x^2 + 2205*a^4*x^4 - 6*a*x*(897 - 1600*a^2*x^2 + 735*a^4*x
^4)*ArcTanh[a*x] + 9*(299 - 105*a^2*x^2 - 375*a^4*x^4 + 245*a^6*x^6)*ArcTan
h[a*x]^2 - 144*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^3 + 540*(-1
+ a^2*x^2)^3*ArcTanh[a*x]^4)/(6912*a*(-1 + a^2*x^2)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.23, size = 2761, normalized size = 9.49

method	result
risch	$\frac{5\ln(ax+1)^4}{1024a}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-80a^3x^3+45x^2\ln(-ax+1)a^2+66ax-15\ln(-ax+1))\ \mathrm{l}}{768(a^2x^2-1)^3a}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/48*arctanh(a*x)^3/(a*x-1)^3+1/16*arctanh(a*x)^3/(a*x-1)^2-5/32*arct
anh(a*x)^3/(a*x-1)-5/32*arctanh(a*x)^3*ln(a*x-1)-1/48*arctanh(a*x)^3/(a*x+1
)^3-1/16*arctanh(a*x)^3/(a*x+1)^2-5/32*arctanh(a*x)^3/(a*x+1)+5/32*arctanh(
a*x)^3*ln(a*x+1)-5/16*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/55296
*(9971+17640*arctanh(a*x)^2*a^6*x^6-35280*arctanh(a*x)*a^5*x^5-8385*a^2*x^2
-43056*a*x*arctanh(a*x)+9485*a^6*x^6-10815*a^4*x^4+76800*a^3*x^3*arctanh(a*
x)-27000*a^4*x^4*arctanh(a*x)^2-7560*a^2*x^2*arctanh(a*x)^2-4320*arctanh(a*
x)^4+21528*arctanh(a*x)^2-12960*arctanh(a*x)^4*a^4*x^4+12960*arctanh(a*x)^4
*a^2*x^2+4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3-8640*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))^3+4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+8640*I*Pi*arc
tanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-4320*I*Pi*arctanh(a*x)^3*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))^2-25920*I*Pi*arctanh(a*x)^3*a^4*x^4+25920*I*Pi*arctanh(a*x)^3*a^
2*x^2+4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*
(a*x+1)^2/(a^2*x^2-1))+8640*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+4320*I*Pi*arctanh(a*x)^3*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2-4320*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
```

```
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))-8640*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*a^6*x^6
+12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^3*a^4*x^4-25920*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))^3*a^4*x^4+12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*a^
4*x^4+25920*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*a^4*x^
4-12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^3*a^2*x^2+25920*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^3*a^2*x^2-12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*a
^2*x^2-25920*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*a^2*x
^2-4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^3*a^6*x^6+12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(
1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^4*x^4+25920*I*Pi*arctanh(a*x)^3*csg
n(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a^4*x^4+129
60*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a^4*x^4-12960*I*Pi*arctanh(a*x)^3
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*a^4*x^4-12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1
)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^2*x^2-25920*I*Pi*arctanh(a*x)^3*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a^2*x^2-
12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a^2*x^2+12960*I*Pi*arctanh(a*x
)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*a^2*x^2-4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*a^6*x^6-8640*I*Pi*arctanh(a*x)^3
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*a^6*x^6
-4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*a^6*x^6+4320*I*Pi*arctanh(a*x)
^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*a^6*x^6-8640*I*Pi*arctanh(a*x)^3+4320*arctanh(a*x)^4*a^6*x
^6+4320*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*
a^6*x^6-12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1))*a^4*x^4+12960*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1))*a^2*x^2+8640*I*Pi*arctanh(a*x)^3*a^6*x^6+8640*I*Pi*arctanh(a*x)
^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*a^6*x^6-4320*I*Pi*arctanh(a*x)^3*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^3*a^6*x^6)/(a*x-1)^3/(a*x+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(251) = 502$.
time = 0.29, size = 871, normalized size = 2.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^3 + 1/384*(180*a
^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1
)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1) -
45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a*arctanh(a*
x)^2/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2) + 1/27648*((8820*a^4*x^4 - 135
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^4 + 540*(a^6*x^6 - 3*a^
4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3*log(a*x - 1) - 135*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^4 - 18420*a^2*x^2 - 45*(49*a^6*x^6 - 147*a
^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1
)^2 - 49)*log(a*x + 1)^2 - 2205*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a
*x - 1)^2 + 90*(6*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 49
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1) + 9728)*a
^2/(a^10*x^6 - 3*a^8*x^4 + 3*a^6*x^2 - a^4) - 12*(1470*a^5*x^5 - 3200*a^3*x
^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3 + 270*(a^6*x^6
 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x - 1) + 90*(a^6*x^6 - 3
*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x - 15*(49*a^6*x^6 - 147*
a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x -
1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
 - 1))*a*arctanh(a*x)/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 - a^3))*a
```

Fricas [A]
time = 0.48, size = 216, normalized size = 0.74

$$\frac{8820\,a^4x^4+135\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^4-18420\,a^2x^2-72\,(15\,a^5x^5-40\,a^3x^3+33\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3+9\,(245\,a^6x^6-375\,a^4x^4-105\,a^2x^2+299)\log\left(-\frac{ax+1}{ax-1}\right)^2-12\,(735\,a^5x^5-1600\,a^3x^3+897\,ax)\log\left(-\frac{ax+1}{ax-1}\right)+9728}{27648\,(a^7x^6-3\,a^5x^4+3\,a^3x^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/27648*(8820*a^4*x^4 + 135*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x
 + 1)/(a*x - 1))^4 - 18420*a^2*x^2 - 72*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*
log(-(a*x + 1)/(a*x - 1))^3 + 9*(245*a^6*x^6 - 375*a^4*x^4 - 105*a^2*x^2 +
299)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(735*a^5*x^5 - 1600*a^3*x^3 + 897*a*x
)*log(-(a*x + 1)/(a*x - 1)) + 9728)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}{\left(ax \right)}}{\left(ax - 1\right)^{4} \left(ax + 1\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**4,x)
[Out] Integral(atanh(a*x)**3/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/(a^2*x^2 - 1)^4, x)
```

Mupad [B]
time = 2.97, size = 1041, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(a^2*x^2 - 1)^4,x)
[Out] (1216/(3*a) - (1535*a*x^2)/2 + (735*a^3*x^4)/2)/(3456*a^2*x^2 - 3456*a^4*x^
4 + 1152*a^6*x^6 - 1152) - log(1 - a*x)^3*((5*log(a*x + 1))/(256*a) - ((11*
x)/16 - (5*a^2*x^3)/6 + (5*a^4*x^5)/16)/(24*a^2*x^2 - 24*a^4*x^4 + 8*a^6*x^
6 - 8)) + (5*log(a*x + 1)^4)/(1024*a) + (5*log(1 - a*x)^4)/(1024*a) + log(1
 - a*x)^2*((15*log(a*x + 1)^2)/(512*a) + 245/(3072*a) + ((37*x)/2 - 35*a*x^
2 + 68/(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(768*a^2*x^2 -
 768*a^4*x^4 + 256*a^6*x^6 - 256) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a
^2*x^3)/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(768*a^2*x^2 - 768*a^4*x^4 + 256*a
^6*x^6 - 256) - (log(a*x + 1)*(66*x - 80*a^2*x^3 + 30*a^4*x^5))/(768*a^2*x^
2 - 768*a^4*x^4 + 256*a^6*x^6 - 256)) + log(a*x + 1)^2*((17/(96*a^2) - (35*
x^2)/128 + (15*a^2*x^4)/128)/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6) + 245/(3
072*a)) + log(1 - a*x)*((36*x + 22*a*x^2 - 23/(2*a) - 67*a^2*x^3 - (21*a^3*
x^4)/2 + 31*a^4*x^5)/(2304*a^2*x^2 - 2304*a^4*x^4 + 768*a^6*x^6 - 768) - (5
*log(a*x + 1)^3)/(256*a) - log(a*x + 1)*(((37*x)/2 - 35*a*x^2 + 68/(3*a) -
(82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(384*a^2*x^2 - 384*a^4*x^4 +
128*a^6*x^6 - 128) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a^2*x^3)/3 - 15*
a^3*x^4 + (23*a^4*x^5)/2)/(384*a^2*x^2 - 384*a^4*x^4 + 128*a^6*x^6 - 128) +
 (245*(3*a^2*x^2 - 3*a^4*x^4 + a^6*x^6 - 1))/(12*a*(384*a^2*x^2 - 384*a^4*x
^4 + 128*a^6*x^6 - 128))) + ((227*x)/2 + 173*a*x^2 - 593/(6*a) - (599*a^2*x
^3)/3 - (159*a^3*x^4)/2 + (183*a^4*x^5)/2)/(2304*a^2*x^2 - 2304*a^4*x^4 + 7
68*a^6*x^6 - 768) + ((299*x)/2 - 195*a*x^2 + 331/(3*a) - (800*a^2*x^3)/3 +
90*a^3*x^4 + (245*a^4*x^5)/2)/(2304*a^2*x^2 - 2304*a^4*x^4 + 768*a^6*x^6 -
768) + (log(a*x + 1)^2*(66*x - 80*a^2*x^3 + 30*a^4*x^5))/(768*a^2*x^2 - 768
*a^4*x^4 + 256*a^6*x^6 - 256)) - (log(a*x + 1)*((299*x)/(768*a) - (25*a*x^3
)/36 + (245*a^3*x^5)/768))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6) - (log(a*x
 + 1)^3*((11*x)/(128*a) - (5*a*x^3)/48 + (5*a^3*x^5)/128))/(3*a*x^2 - 1/a -
 3*a^3*x^4 + a^5*x^6)
```

3.348 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^4}\,dx$

Optimal. Leaf size=252

$$\frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\,\mathrm{Erf}\left(\sqrt{6}\,\sqrt{\tanh^{-1}\right.}}{768a}$$

```
[Out] 5/24*arctanh(a*x)^(3/2)/a+1/4608*erf(6^(1/2)*arctanh(a*x)^(1/2))*6^(1/2)*Pi
^(1/2)/a-1/4608*erfi(6^(1/2)*arctanh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a+15/512*
erf(2^(1/2)*arctanh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a-15/512*erfi(2^(1/2)*arct
anh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+3/512*erf(2*arctanh(a*x)^(1/2))*Pi^(1/2)
/a-3/512*erfi(2*arctanh(a*x)^(1/2))*Pi^(1/2)/a+15/64*sinh(2*arctanh(a*x))*a
rctanh(a*x)^(1/2)/a+3/64*sinh(4*arctanh(a*x))*arctanh(a*x)^(1/2)/a+1/192*si
nh(6*arctanh(a*x))*arctanh(a*x)^(1/2)/a
```

Rubi [A]
time = 0.21, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6115, 3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\,\mathrm{Erf}\left(\sqrt{6}\,\sqrt{\tanh^{-1}(ax)}\right)}{768a}-\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}-\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\tanh^{-1}(ax)}\right)}{256a}-\frac{\sqrt{\frac{\pi}{6}}\,\mathrm{Erfi}\left(\sqrt{6}\,\sqrt{\tanh^{-1}(ax)}\right)}{768a}+\frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\tanh^{-1}(ax)\right)}{64a}+\frac{\sqrt{\tanh^{-1}(ax)}\,\sinh\left(6\tanh^{-1}(ax)\right)}{192a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

```
[Out] (5*ArcTanh[a*x]^(3/2))/(24*a) + (3*Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(512
*a) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a) + (Sqrt[Pi/6]
*Erf[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) - (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcTanh
[a*x]]])/(512*a) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a)
 - (Sqrt[Pi/6]*Erfi[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) + (15*Sqrt[ArcTanh
[a*x]]*Sinh[2*ArcTanh[a*x]])/(64*a) + (3*Sqrt[ArcTanh[a*x]]*Sinh[4*ArcTanh[
a*x]])/(64*a) + (Sqrt[ArcTanh[a*x]]*Sinh[6*ArcTanh[a*x]])/(192*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^4}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\,\cosh^6(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\sqrt{x}}{16} + \frac{15}{32}\sqrt{x}\,\cosh(2x) + \frac{3}{16}\sqrt{x}\,\cosh(4x) + \frac{1}{32}\sqrt{x}\,\cosh(6x)\right) dx, x, \tan\right.}{a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a} + \frac{\text{Subst}\left(\int \sqrt{x}\,\cosh(6x)\,dx, x, \tanh^{-1}(ax)\right)}{32a} + \frac{3\text{Subst}\left(\int \sqrt{x}\,\cosh(\right.}{1}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a} + \frac{15\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{64a} + \frac{3\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\,\mathrm{t}\right.}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a} + \frac{15\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{64a} + \frac{3\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\,\mathrm{t}\right.}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a} + \frac{15\sqrt{\tanh^{-1}(ax)}\,\sinh\left(2\tanh^{-1}(ax)\right)}{64a} + \frac{3\sqrt{\tanh^{-1}(ax)}\,\sinh\left(4\,\mathrm{t}\right.}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a} + \frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}$$

Mathematica [A]
time = 0.43, size = 257, normalized size = 1.02

$$\frac{-\frac{3168x\sqrt{\tanh^{-1}(ax)}}{(-1+a^2x^2)^3} + \frac{3840a^2x^3\sqrt{\tanh^{-1}(ax)}}{(-1+a^2x^2)^3} - \frac{1440a^4x^5\sqrt{\tanh^{-1}(ax)}}{(-1+a^2x^2)^3} + \frac{960\tanh^{-1}(ax)^{3/2}}{a} + \frac{\sqrt{6}\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-6\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}} + \frac{27\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}} + \frac{135\sqrt{2}\sqrt{\tanh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}} - \frac{135\sqrt{2}\,\Gamma\left(\frac{1}{2},2\tanh^{-1}(ax)\right)}{a} - \frac{27\Gamma\left(\frac{1}{2},4\tanh^{-1}(ax)\right)}{a} - \frac{\sqrt{6}\,\Gamma\left(\frac{1}{2},6\tanh^{-1}(ax)\right)}{a}}{4608}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

```
[Out] ((-3168*x*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)^3 + (3840*a^2*x^3*Sqrt[ArcTanh
[a*x]])/(-1 + a^2*x^2)^3 - (1440*a^4*x^5*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)
^3 + (960*ArcTanh[a*x]^(3/2))/a + (Sqrt[6]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -6
*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (27*Sqrt[ArcTanh[a*x]]*Gamma[1/2,
 -4*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (135*Sqrt[2]*Sqrt[ArcTanh[a*x]
]*Gamma[1/2, -2*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) - (135*Sqrt[2]*Gamma
[1/2, 2*ArcTanh[a*x]])/a - (27*Gamma[1/2, 4*ArcTanh[a*x]])/a - (Sqrt[6]*Gam
ma[1/2, 6*ArcTanh[a*x]])/a)/4608
```

Maple [F]
time = 11.58, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\mathrm{arctanh}\left(ax\right)}}{\left(-a^2x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}(ax)}}{(ax-1)^4 (ax+1)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(sqrt(atanh(a*x))/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\operatorname{atanh}(a\,x)}}{{\left(a^2\,x^2-1\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^4,x)
[Out] int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^4, x)
```

3.349 $\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Defer[Int][x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx = \int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Mathematica [A]
time = 7.72, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A]
time = 7.45, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(-a^2x^2+1)^4 \operatorname{arctanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^8/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*x
)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**8/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^8}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^8/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.350 $\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Defer[Int][x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx = \int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Mathematica [A]
time = 65.91, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A]
time = 7.20, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-a^2x^2+1)^4 \operatorname{arctanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^7/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*x
)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**7/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^7}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^7/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.351 $\int \frac{x^6}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}$$

```
[Out] 15/32*Chi(2*arctanh(a*x))/a^7-3/16*Chi(4*arctanh(a*x))/a^7+1/32*Chi(6*arcta
nh(a*x))/a^7-5/16*ln(arctanh(a*x))/a^7
```

Rubi [A]
time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 3393, 3382}

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a^7) - (3*CoshIntegral[4*ArcTanh[a*x]
])/(16*a^7) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^7) - (5*Log[ArcTanh[a*x]])
/(16*a^7)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^6}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^6(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^7}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{5}{16x} - \frac{15\cosh(2x)}{32x} + \frac{3\cosh(4x)}{16x} - \frac{\cosh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^7}$$

$$= -\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^7} - \frac{3\text{Subst}\left(\int \frac{\cosh}{}\right.}{}$$

$$= \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7} - \frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7} - \frac{5\log}{}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.73

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right) - 6\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) - 10\log\left(\tanh^{-1}(ax)\right)}{32a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]] - 6*CoshIntegral[4*ArcTanh[a*x]] + CoshInt
egral[6*ArcTanh[a*x]] - 10*Log[ArcTanh[a*x]])/(32*a^7)
```

Maple [A]
time = 1.40, size = 40, normalized size = 0.73

method	result
derivativedivides	$\frac{-\frac{5\ln(\text{arctanh}(ax))}{16}+\frac{15\,\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32}-\frac{3\,\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16}+\frac{\text{hyperbolicCosineIntegra}}{32}}{a^7}$
default	$\frac{-\frac{5\ln(\text{arctanh}(ax))}{16}+\frac{15\,\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32}-\frac{3\,\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16}+\frac{\text{hyperbolicCosineIntegra}}{32}}{a^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^7*(-5/16*ln(arctanh(a*x))+15/32*Chi(2*arctanh(a*x))-3/16*Chi(4*arctanh(
a*x))+1/32*Chi(6*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(47) = 94$.
time = 0.38, size = 220, normalized size = 4.00

$$-\frac{20 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right) + 6\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + 6\,\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - 15\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - 15\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x
^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_integr
al((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^2 -
 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 15*log_integral(-(a*x + 1)/(a*x - 1))
- 15*log_integral(-(a*x - 1)/(a*x + 1)))/a^7
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(ax-1)^4\,(ax+1)^4\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**6/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.352 $\int \frac{x^5}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}(2\tanh^{-1}(ax))}{32a^6} - \frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^6} + \frac{\text{Shi}(6\tanh^{-1}(ax))}{32a^6}$$

```
[Out] 5/32*Shi(2*arctanh(a*x))/a^6-1/8*Shi(4*arctanh(a*x))/a^6+1/32*Shi(6*arctanh
(a*x))/a^6
```

Rubi [A]
time = 0.10, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 5556, 3379}

$$\frac{5\text{Shi}(2\tanh^{-1}(ax))}{32a^6} - \frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^6} + \frac{\text{Shi}(6\tanh^{-1}(ax))}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^6) - SinhIntegral[4*ArcTanh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^6)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} - \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^6} +$$
$$= \frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Mathematica [A]
time = 0.10, size = 33, normalized size = 0.77

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right) - 4\text{Shi}\left(4\tanh^{-1}(ax)\right) + \text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]] - 4*SinhIntegral[4*ArcTanh[a*x]] + SinhInte
gral[6*ArcTanh[a*x]])/(32*a^6)
```

Maple [A]
time = 1.62, size = 33, normalized size = 0.77

method	result	size
derivativedivides	$\frac{-\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicSineIntegral}(6\,\text{arctanh}(ax))}{32}+\frac{5\,\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{32}}{a^6}$	33
default	$\frac{-\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicSineIntegral}(6\,\text{arctanh}(ax))}{32}+\frac{5\,\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{32}}{a^6}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(-1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(a*x))+5/32*Shi(2*arctanh
(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(37) = 74$.
time = 0.37, size = 200, normalized size = 4.65

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)-4\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+4\,\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+5\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-5\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) - 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2
*a*x + 1)) + 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) +
5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x + 1))
)/a^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(ax-1)^4\,(ax+1)^4\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**5/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.353 $\int \frac{x^4}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5}-\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5}+\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5}$$

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^5-1/16*Chi(4*arctanh(a*x))/a^5+1/32*Chi(6*arcta
nh(a*x))/a^5+1/16*ln(arctanh(a*x))/a^5
```

Rubi [A]
time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 5556, 3382}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5}-\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5}+\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -1/32*CoshIntegral[2*ArcTanh[a*x]]/a^5 - CoshIntegral[4*ArcTanh[a*x]]/(16*a
^5) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^5) + Log[ArcTanh[a*x]]/(16*a^5)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16x} - \frac{\cosh(2x)}{32x} - \frac{\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$

$$= \frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\right.}{3}$$

$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5} - \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5} + \frac{\log\left(\tan\right.}{1}$$

Mathematica [A]
time = 0.05, size = 40, normalized size = 0.73

$$\frac{-\text{Chi}\left(2\tanh^{-1}(ax)\right) - 2\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) + 2\log\left(\tanh^{-1}(ax)\right)}{32a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] (-CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]] + CoshInteg
ral[6*ArcTanh[a*x]] + 2*Log[ArcTanh[a*x]])/(32*a^5)
```

Maple [A]
time = 1.41, size = 40, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{\ln(\text{arctanh}(ax))}{16} - \frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32} - \frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16} + \frac{\text{hyperbolicCosineIntegral}(6\,\text{arct}}{32}}{a^5}$
default	$\frac{\frac{\ln(\text{arctanh}(ax))}{16} - \frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32} - \frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16} + \frac{\text{hyperbolicCosineIntegral}(6\,\text{arct}}{32}}{a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^5*(1/16*ln(arctanh(a*x))-1/32*Chi(2*arctanh(a*x))-1/16*Chi(4*arctanh(a*
x))+1/32*Chi(6*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(47) = 94$.
time = 0.38, size = 216, normalized size = 3.93

$$\frac{4 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) + \text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right) - 2\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - 2\,\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - \text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^2
 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3 -
 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integral
((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2 - 2
*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - log_integral(-(a*x + 1)/(a*x - 1)) - log
_integral(-(a*x - 1)/(a*x + 1)))/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(ax-1)^4\,(ax+1)^4\,\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**4/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.354 $\int \frac{x^3}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=29

$$-\frac{3\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}+\frac{\mathrm{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}$$

```
[Out] -3/32*Shi(2*arctanh(a*x))/a^4+1/32*Shi(6*arctanh(a*x))/a^4
```

Rubi [A]
time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 5556, 3379}

$$\frac{\mathrm{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}-\frac{3\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]])/(32*a^4) + SinhIntegral[6*ArcTanh[a*x]]/(
32*a^4)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(2x)}{32x} + \frac{\sinh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^4} - \frac{3\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^4}$$
$$= -\frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}$$

Mathematica [A]
time = 0.08, size = 24, normalized size = 0.83

$$\frac{-3\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[6*ArcTanh[a*x]])/(32*a^4)
```

Maple [A]
time = 1.39, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\text{hyperbolicSineIntegral}(6\,\text{arctanh}(ax))}{32} - \frac{3\,\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{32}}{a^4}$	24
default	$\frac{\frac{\text{hyperbolicSineIntegral}(6\,\text{arctanh}(ax))}{32} - \frac{3\,\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{32}}{a^4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^4*(1/32*Shi(6*arctanh(a*x))-3/32*Shi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(25) = 50$.
time = 0.42, size = 136, normalized size = 4.69

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)-3\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+3\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) - 3*log_integral(-(a*x + 1)/(a*x - 1)) + 3*log_inte
gral(-(a*x - 1)/(a*x + 1)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax-1)^4\,(ax+1)^4\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**4/atanh(a*x),x)
[Out] Integral(x**3/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\text{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.355 $\int \frac{x^2}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=55

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^3+1/16*Chi(4*arctanh(a*x))/a^3+1/32*Chi(6*arcta
nh(a*x))/a^3-1/16*ln(arctanh(a*x))/a^3
```

Rubi [A]
time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6181, 5556, 3382}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -1/32*CoshIntegral[2*ArcTanh[a*x]]/a^3 + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16x} - \frac{\cosh(2x)}{32x} + \frac{\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(}{x}\right.}{}$$
$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3} - \frac{\log\left(\text{t}\right.}{}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 1.00

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] -1/32*CoshIntegral[2*ArcTanh[a*x]]/a^3 + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Maple [A]
time = 1.40, size = 40, normalized size = 0.73

method	result
derivativedivides	$\frac{-\frac{\ln(\text{arctanh}(ax))}{16} - \frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32} + \frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16} + \frac{\text{hyperbolicCosineIntegral}(6}{32}}{a^3}$
default	$\frac{-\frac{\ln(\text{arctanh}(ax))}{16} - \frac{\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32} + \frac{\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16} + \frac{\text{hyperbolicCosineIntegral}(6}{32}}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(-1/16*ln(arctanh(a*x))-1/32*Chi(2*arctanh(a*x))+1/16*Chi(4*arctanh(a
*x))+1/32*Chi(6*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(47) = 94$.
time = 0.39, size = 216, normalized size = 3.93

$$-\frac{4\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)-2\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-2\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^3
- 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integra
l((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2 -
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + log_integral(-(a*x + 1)/(a*x - 1)) + lo
g_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax-1)^4\,(ax+1)^4\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**2/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\mathrm{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.356 $\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}(2\tanh^{-1}(ax))}{32a^2}+\frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^2}+\frac{\text{Shi}(6\tanh^{-1}(ax))}{32a^2}$$

```
[Out] 5/32*Shi(2*arctanh(a*x))/a^2+1/8*Shi(4*arctanh(a*x))/a^2+1/32*Shi(6*arctanh
(a*x))/a^2
```

Rubi [A]
time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6181, 5556, 3379}

$$\frac{5\text{Shi}(2\tanh^{-1}(ax))}{32a^2}+\frac{\text{Shi}(4\tanh^{-1}(ax))}{8a^2}+\frac{\text{Shi}(6\tanh^{-1}(ax))}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^2} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2}$$
$$= \frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Mathematica [A]
time = 0.11, size = 43, normalized size = 1.00

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Maple [A]
time = 1.56, size = 33, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicSineIntegral}(6\,\text{arctanh}(ax))}{32}+\frac{5\,\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{32}}{a^2}$	33
default	$\frac{\frac{\text{hyperbolicSineIntegral}(4\,\text{arctanh}(ax))}{8}+\frac{\text{hyperbolicSineIntegral}(6\,\text{arctanh}(ax))}{32}+\frac{5\,\text{hyperbolicSineIntegral}(2\,\text{arctanh}(ax))}{32}}{a^2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(a*x))+5/32*Shi(2*arctanh(
a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(37) = 74$.

time = 0.38, size = 200, normalized size = 4.65

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right)+4\,\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-4\,\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+5\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-5\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) + 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2
*a*x + 1)) - 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) +
5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x + 1))
)/a^2
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^4\,(ax+1)^4\,\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.357 $\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=55

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] 15/32*Chi(2*arctanh(a*x))/a+3/16*Chi(4*arctanh(a*x))/a+1/32*Chi(6*arctanh(a
*x))/a+5/16*ln(arctanh(a*x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6115, 3393, 3382}

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a) + (3*CoshIntegral[4*ArcTanh[a*x]])
/(16*a) + CoshIntegral[6*ArcTanh[a*x]]/(32*a) + (5*Log[ArcTanh[a*x]])/(16*a
)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5}{16x} + \frac{15\cosh(2x)}{32x} + \frac{3\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{5\log\left(\tanh^{-1}(ax)\right)}{16a} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a} + \frac{3\text{Subst}\left(\int \frac{\cosh(4}{x}\right.}{}$$
$$= \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a} + \frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a} + \frac{5\log\left(\right.}{}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.73

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right) + 6\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) + 10\log\left(\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]] + 6*CoshIntegral[4*ArcTanh[a*x]] + CoshInt
egral[6*ArcTanh[a*x]] + 10*Log[ArcTanh[a*x]])/(32*a)
```

Maple [A]
time = 1.51, size = 40, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{5\ln(\text{arctanh}(ax))}{16} + \frac{15\,\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32} + \frac{3\,\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16} + \frac{\text{hyperbolicCosineIntegral}(}{32}}{a}$
default	$\frac{\frac{5\ln(\text{arctanh}(ax))}{16} + \frac{15\,\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{32} + \frac{3\,\text{hyperbolicCosineIntegral}(4\,\text{arctanh}(ax))}{16} + \frac{\text{hyperbolicCosineIntegral}(}{32}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(5/16*ln(arctanh(a*x))+15/32*Chi(2*arctanh(a*x))+3/16*Chi(4*arctanh(a*x
))+1/32*Chi(6*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(47) = 94$.
time = 0.35, size = 216, normalized size = 3.93

$$\frac{20\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+6\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+6\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+15\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+15\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3
- 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_integra
l((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^2 -
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 15*log_integral(-(a*x + 1)/(a*x - 1)) +
 15*log_integral(-(a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^4\,(ax+1)^4\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(1/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.358 $\int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)} dx = \int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A]
time = 7.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-a^2x^2+1)^4 \operatorname{arctanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^8*x^9 - 4*a^6*x^7 + 6*a^4*x^5 - 4*a^2*x^3 + x)*arctanh(a*x))
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax-1\right)^{4}\left(ax+1\right)^{4}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(1/(x*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \operatorname{atanh}(a x) (a^2 x^2 - 1)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.359 $\int \frac{1}{x^2(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2 (1-a^2x^2)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \tanh^{-1}(ax)} dx = \int \frac{1}{x^2 (1-a^2x^2)^4 \tanh^{-1}(ax)} dx$$

Mathematica [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \tanh^{-1}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A]
time = 8.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (-a^2x^2+1)^4 \operatorname{arctanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^8*x^10 - 4*a^6*x^8 + 6*a^4*x^6 - 4*a^2*x^4 + x^2)*arctanh(a*
x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(ax - 1\right)^{4} \left(ax + 1\right)^{4} \operatorname{atanh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(1/(x**2*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,\mathrm{atanh}\left(a\,x\right)\,{\left(a^2\,x^2-1\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(a*x)*(a^2*x^2 - 1)^4),x)
[Out] int(1/(x^2*atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

3.360 $\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=67

$$-\frac{x}{a(1-a^2x^2)^3 \tanh^{-1}(ax)}+\frac{5\text{Chi}(2\tanh^{-1}(ax))}{16a^2}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{2a^2}+\frac{3\text{Chi}(6\tanh^{-1}(ax))}{16a^2}$$

```
[Out] -x/a/(-a^2*x^2+1)^3/arctanh(a*x)+5/16*Chi(2*arctanh(a*x))/a^2+1/2*Chi(4*arc
tanh(a*x))/a^2+3/16*Chi(6*arctanh(a*x))/a^2
```

Rubi [A]
time = 0.20, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6179, 6181, 5556, 3382, 6115, 3393}

$$\frac{5\text{Chi}(2\tanh^{-1}(ax))}{16a^2}+\frac{\text{Chi}(4\tanh^{-1}(ax))}{2a^2}+\frac{3\text{Chi}(6\tanh^{-1}(ax))}{16a^2}-\frac{x}{a(1-a^2x^2)^3 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (5*CoshIntegral[2*ArcTanh[a*x]])/(1
6*a^2) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2) + (3*CoshIntegral[6*ArcTanh[a
*x]])/(16*a^2)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1-a^2x^2)^4\tanh^{-1}(ax)^2}\,dx &= -\frac{x}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{\int\frac{1}{(1-a^2x^2)^4\tanh^{-1}(ax)}\,dx}{a}+(5a)\int\frac{}{(1-a^2x^2)}\\ &= -\frac{x}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int\frac{\cosh^6(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^2}+\frac{5\text{Sub}}{}\\ &= -\frac{x}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int\left(\frac{5}{16x}+\frac{15\cosh(2x)}{32x}+\frac{3\cosh(4x)}{16x}+\frac{\cosh(}{32}\right.\right.}{a^2}\\ &= -\frac{x}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int\frac{\cosh(6x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{32a^2}-\frac{5\text{Sub}}{}\\ &= -\frac{x}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}\end{aligned}$$

Mathematica [A]

time = 0.13, size = 56, normalized size = 0.84

$$\frac{\frac{16ax}{(-1+a^2x^2)^3\tanh^{-1}(ax)}+5\text{Chi}\left(2\tanh^{-1}(ax)\right)+8\text{Chi}\left(4\tanh^{-1}(ax)\right)+3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] ((16*a*x)/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + 5*CoshIntegral[2*ArcTanh[a*x]]
+ 8*CoshIntegral[4*ArcTanh[a*x]] + 3*CoshIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

Maple [A]
time = 3.87, size = 78, normalized size = 1.16

method	result
derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{2}-\frac{\sinh(6\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}+\frac{3\,\mathrm{hyperbolicCosineIntegral}(6\,\mathrm{arctanh}(ax))}{16}-5}{a^2}$
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{hyperbolicCosineIntegral}(4\,\mathrm{arctanh}(ax))}{2}-\frac{\sinh(6\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}+\frac{3\,\mathrm{hyperbolicCosineIntegral}(6\,\mathrm{arctanh}(ax))}{16}-5}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/8/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-1/32/
arctanh(a*x)*sinh(6*arctanh(a*x))+3/16*Chi(6*arctanh(a*x))-5/32*sinh(2*arct
anh(a*x))/arctanh(a*x)+5/16*Chi(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*
x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)) - integrate(-2*(5*a^2*x^2 + 1)/((a^9*x^
8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(a*x + 1) - (a^9*x^8 - 4*a^7*
x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(59) = 118.
time = 0.38, size = 418, normalized size = 6.24

$$\frac{64\,ax+\left(3\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+3\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+8\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+8\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+5\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+5\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{32\,(a^8x^6-3\,a^6x^4+3\,a^4x^2-a^2)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(64*a*x + (3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*
x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 3*(a^6*x^
6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 8*(a^6*x^
6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2
+ 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*
x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4
*x^2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}(a\,x)^2\,{\left(a^2\,x^2-1\right)}^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^4), x)
```

3.361 $\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=66

$$-\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -1/a/(-a^2*x^2+1)^3/arctanh(a*x)+15/16*Shi(2*arctanh(a*x))/a+3/4*Shi(4*arct
anh(a*x))/a+3/16*Shi(6*arctanh(a*x))/a
```

Rubi [A]
time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6113, 6181, 5556, 3379}

$$-\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (15*SinhIntegral[2*ArcTanh[a*x]])/(
16*a) + (3*SinhIntegral[4*ArcTanh[a*x]])/(4*a) + (3*SinhIntegral[6*ArcTanh[
a*x]])/(16*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2}\,dx = -\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)} + (6a)\int \frac{x}{(1-a^2x^2)^4\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)} + \frac{6\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)} + \frac{6\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right)\,dx\right.}{a}$$

$$= -\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)} + \frac{3\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{3\text{Su}}{}$$

$$= -\frac{1}{a(1-a^2x^2)^3\tanh^{-1}(ax)} + \frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a} + \frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}$$

Mathematica [A]
time = 0.14, size = 56, normalized size = 0.85

$$\frac{\frac{1}{(-1+a^2x^2)^3\tanh^{-1}(ax)} + \frac{15}{16}\text{Shi}\left(2\tanh^{-1}(ax)\right) + \frac{3}{4}\text{Shi}\left(4\tanh^{-1}(ax)\right) + \frac{3}{16}\text{Shi}\left(6\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] (1/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + (15*SinhIntegral[2*ArcTanh[a*x]])/16 +
 (3*SinhIntegral[4*ArcTanh[a*x]])/4 + (3*SinhIntegral[6*ArcTanh[a*x]])/16)/
a
```

Maple [A]
time = 3.64, size = 86, normalized size = 1.30

method	result

derivativedivides	$\frac{-\frac{5}{16\,\mathrm{arctanh}(ax)}-\frac{15\cosh(2\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}+\frac{15\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{16}-\frac{3\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)}+\frac{3\,\mathrm{hyperbolicSineIntegral}}{4}}{a}$
default	$\frac{-\frac{5}{16\,\mathrm{arctanh}(ax)}-\frac{15\cosh(2\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}+\frac{15\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arctanh}(ax))}{16}-\frac{3\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)}+\frac{3\,\mathrm{hyperbolicSineIntegral}}{4}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-5/16/arctanh(a*x)-15/32/arctanh(a*x)*cosh(2*arctanh(a*x))+15/16*Shi(2
*arctanh(a*x))-3/16/arctanh(a*x)*cosh(4*arctanh(a*x))+3/4*Shi(4*arctanh(a*x
))-1/32/arctanh(a*x)*cosh(6*arctanh(a*x))+3/16*Shi(6*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -12*a*integrate(-x/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a
*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)),
 x) + 2/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*
a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(58) = 116$.
time = 0.41, size = 413, normalized size = 6.26

$$\frac{3\left(\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+4\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-4\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+5\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-5\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+64}{32\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(3*((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - (a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^
3 + 3*a^2*x^2 + 3*a*x + 1)) + 4*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_i
ntegral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1
)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x -
 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x
 + 1)))*log(-(a*x + 1)/(a*x - 1)) + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^4),x)
[Out] int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^4), x)
```

3.362 $\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=114

$$-\frac{x}{2a(1-a^2x^2)^3\tanh^{-1}(ax)^2}-\frac{3}{a^2(1-a^2x^2)^3\tanh^{-1}(ax)}+\frac{5}{2a^2(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{5\text{Shi}(2\tanh^{-1}(ax))}{16a^2}$$

```
[Out] -1/2*x/a/(-a^2*x^2+1)^3/arctanh(a*x)^2-3/a^2/(-a^2*x^2+1)^3/arctanh(a*x)+5/
2/a^2/(-a^2*x^2+1)^2/arctanh(a*x)+5/16*Shi(2*arctanh(a*x))/a^2+Shi(4*arctan
h(a*x))/a^2+9/16*Shi(6*arctanh(a*x))/a^2
```

Rubi [A]
time = 0.35, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6179, 6175, 6113, 6181, 5556, 3379}

$$\frac{5\text{Shi}(2\tanh^{-1}(ax))}{16a^2}+\frac{\text{Shi}(4\tanh^{-1}(ax))}{a^2}+\frac{9\text{Shi}(6\tanh^{-1}(ax))}{16a^2}-\frac{x}{2a(1-a^2x^2)^3\tanh^{-1}(ax)^2}+\frac{5}{2a^2(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{3}{a^2(1-a^2x^2)^3\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - 3/(a^2*(1 - a^2*x^2)^3*ArcTanh[
a*x]) + 5/(2*a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (5*SinhIntegral[2*ArcTanh[
a*x]])/(16*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2 + (9*SinhIntegral[6*ArcT
anh[a*x]])/(16*a^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
```

```
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} + \frac{\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2}\,dx}{2a} + \frac{1}{2}(5a)\int \frac{}{(1-a^2}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{1}{2a^2\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + 3\int \frac{}{(1-a^2x}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right.}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right.}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right.}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right.}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3}{a^2\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{5}{2a^2\left(1-a^2x^2\right.}$$

Mathematica [A]
time = 0.40, size = 73, normalized size = 0.64

$$\frac{\frac{8\left(ax+\left(1+5a^2x^2\right)\tanh^{-1}(ax)\right)}{\left(-1+a^2x^2\right)^3 \tanh^{-1}(ax)^2} + 5\mathrm{Shi}\left(2\tanh^{-1}(ax)\right) + 16\mathrm{Shi}\left(4\tanh^{-1}(ax)\right) + 9\mathrm{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] ((8*(a*x + (1 + 5*a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^3*ArcTanh[a*x]^2)
 + 5*SinhIntegral[2*ArcTanh[a*x]] + 16*SinhIntegral[4*ArcTanh[a*x]] + 9*Sin
hIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

Maple [A]
time = 3.71, size = 121, normalized size = 1.06

method	result
derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))-\frac{\sinh(6\,\mathrm{arctanh}(ax))}{64\,\mathrm{arctanh}(ax)^2}-\frac{3\cosh(6\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}}{a^2}$
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{hyperbolicSineIntegral}(4\,\mathrm{arctanh}(ax))-\frac{\sinh(6\,\mathrm{arctanh}(ax))}{64\,\mathrm{arctanh}(ax)^2}-\frac{3\cosh(6\,\mathrm{arctanh}(ax))}{32\,\mathrm{arctanh}(ax)}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))-1/4/arctanh(a*x)*cosh(4*ar
ctanh(a*x))+Shi(4*arctanh(a*x))-1/64/arctanh(a*x)^2*sinh(6*arctanh(a*x))-3/
32/arctanh(a*x)*cosh(6*arctanh(a*x))+9/16*Shi(6*arctanh(a*x))-5/64*sinh(2*a
rctanh(a*x))/arctanh(a*x)^2-5/32/arctanh(a*x)*cosh(2*arctanh(a*x))+5/16*Shi
(2*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (5*a^2*x^2 + 1)*log(a*x + 1) - (5*a^2*x^2 + 1)*log(-a*x + 1))/((a^
8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^8*x^6 - 3*a^6*x^
4 + 3*a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^8*x^6 - 3*a^6*x^4 + 3*
a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-4*(5*a^2*x^3 + 4*x)/((a^8*x^8
- 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^
6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(103) = 206$.
time = 0.36, size = 447, normalized size = 3.92

$$\frac{\left(9\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-9\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+16\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-16\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+5\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-5\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+64\,ax+32\,(5\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{32\,(a^8x^6-3\,a^6x^4+3\,a^4x^2-a^2)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((9*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - 9*(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*
x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 16*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*lo
g_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 16*(a^6*x^6 - 3*a
^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x
 + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a
*x - 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 64*a*x + 32*(5*a^2*x^2 + 1)*log(-
(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-(a*x +
1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax-1)^4\,(ax+1)^4\,\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{atanh}\,(a\,x)^3\,(a^2\,x^2-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^4), x)
```

3.363 $\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=89

$$-\frac{1}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{3x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a}+\frac{9\text{Chi}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^3/arctanh(a*x)^2-3*x/(-a^2*x^2+1)^3/arctanh(a*x)+15/16*
Chi(2*arctanh(a*x))/a+3/2*Chi(4*arctanh(a*x))/a+9/16*Chi(6*arctanh(a*x))/a
```

Rubi [A]
time = 0.22, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6113, 6179, 6181, 5556, 3382, 6115, 3393}

$$-\frac{3x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}+\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a}+\frac{9\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - (3*x)/((1 - a^2*x^2)^3*ArcTanh[
a*x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/(16*a) + (3*CoshIntegral[4*ArcTan
h[a*x]])/(2*a) + (9*CoshIntegral[6*ArcTanh[a*x]])/(16*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\,(1-a^2x^2)^3 \tanh^{-1}(ax)^2} + (3a)\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\,(1-a^2x^2)^3 \tanh^{-1}(ax)^2} - \frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)} + 3\int \frac{}{(1-a^2x^2)}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^3 \tanh^{-1}(ax)^2} - \frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)} + \frac{3\text{Subst}\left(\int \frac{\cosh^6}{x}\right.}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^3 \tanh^{-1}(ax)^2} - \frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)} + \frac{3\text{Subst}\left(\int \left(\frac{5}{16x}\right.\right.}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^3 \tanh^{-1}(ax)^2} - \frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)} + \frac{3\text{Subst}\left(\int \frac{\cosh(}{x}\right.}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^3 \tanh^{-1}(ax)^2} - \frac{3x}{(1-a^2x^2)^3 \tanh^{-1}(ax)} + \frac{15\text{Chi}\left(2\tanh^{-}\right.}{16a}$$

Mathematica [A]
time = 0.10, size = 83, normalized size = 0.93

$$\frac{1}{16}\left(\frac{8}{a\,(-1+a^2x^2)^3 \tanh^{-1}(ax)^2} + \frac{48x}{(-1+a^2x^2)^3 \tanh^{-1}(ax)} + \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a} + \frac{24\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a} + \frac{9\text{Chi}\left(6\tanh^{-1}(ax)\right)}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] (8/(a*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2) + (48*x)/((-1 + a^2*x^2)^3*ArcTanh[a
*x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/a + (24*CoshIntegral[4*ArcTanh[a*x
]])/a + (9*CoshIntegral[6*ArcTanh[a*x]])/a)/16
```

Maple [A]
time = 2.72, size = 131, normalized size = 1.47

method	result
derivativedivides	$\frac{-\frac{5}{32\,\text{arctanh}(ax)^2}-\frac{15\cosh(2\,\text{arctanh}(ax))}{64\,\text{arctanh}(ax)^2}-\frac{15\sinh(2\,\text{arctanh}(ax))}{32\,\text{arctanh}(ax)}+\frac{15\,\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{16}-\frac{3\cosh(4\,\text{arctanh}(}{32\,\text{arctanh}(ax}}{}$
default	$\frac{-\frac{5}{32\,\text{arctanh}(ax)^2}-\frac{15\cosh(2\,\text{arctanh}(ax))}{64\,\text{arctanh}(ax)^2}-\frac{15\sinh(2\,\text{arctanh}(ax))}{32\,\text{arctanh}(ax)}+\frac{15\,\text{hyperbolicCosineIntegral}(2\,\text{arctanh}(ax))}{16}-\frac{3\cosh(4\,\text{arctanh}(}{32\,\text{arctanh}(ax}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-5/32/arctanh(a*x)^2-15/64/arctanh(a*x)^2*cosh(2*arctanh(a*x))-15/32*s
inh(2*arctanh(a*x))/arctanh(a*x)+15/16*Chi(2*arctanh(a*x))-3/32/arctanh(a*x
)^2*cosh(4*arctanh(a*x))-3/8/arctanh(a*x)*sinh(4*arctanh(a*x))+3/2*Chi(4*ar
ctanh(a*x))-1/64/arctanh(a*x)^2*cosh(6*arctanh(a*x))-3/32/arctanh(a*x)*sinh
(6*arctanh(a*x))+9/16*Chi(6*arctanh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(3*a*x*log(a*x + 1) - 3*a*x*log(-a*x + 1) + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*
a^3*x^2 - a)*log(a*x + 1)^2 - 2*(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a
*x + 1)*log(-a*x + 1) + (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)
^2) - integrate(-6*(5*a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^
2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)
*log(-a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(79) = 158$.
time = 0.36, size = 435, normalized size = 4.89

$$\frac{192\,ax\log\left(-\frac{ax+1}{ax-1}\right)+3\left(3\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+3\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+8\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+8\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+5\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+5\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+64}{32\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(192*a*x*log(-(a*x + 1)/(a*x - 1)) + 3*(3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)) + 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(
a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a
^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a
^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2
 + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-1\right)^4\left(ax+1\right)^4\operatorname{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^3\,{\left(a^2\,x^2-1\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^4),x)
```

```
[Out] int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^4), x)
```

3.364 $\int \frac{x^5 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=139

$$-\frac{5x\sqrt{1-a^2x^2}}{24a^5}-\frac{x^3\sqrt{1-a^2x^2}}{20a^3}+\frac{89\text{ArcSin}(ax)}{120a^6}-\frac{8\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^6}-\frac{4x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}-x$$

```
[Out] 89/120*arcsin(a*x)/a^6-5/24*x*(-a^2*x^2+1)^(1/2)/a^5-1/20*x^3*(-a^2*x^2+1)^
(1/2)/a^3-8/15*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^6-4/15*x^2*arctanh(a*x)*(-
a^2*x^2+1)^(1/2)/a^4-1/5*x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6163, 327, 222, 6141}

$$\frac{89\text{ArcSin}(ax)}{120a^6}-\frac{x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5a^2}-\frac{8\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^6}-\frac{5x\sqrt{1-a^2x^2}}{24a^5}-\frac{4x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}-\frac{x^3\sqrt{1-a^2x^2}}{20a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-5*x*Sqrt[1 - a^2*x^2])/(24*a^5) - (x^3*Sqrt[1 - a^2*x^2])/(20*a^3) + (89*
ArcSin[a*x])/(120*a^6) - (8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^6) - (4*x
^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^4*Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(5*a^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int \frac{x^5 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5a^2} + \frac{4\int \frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{5a^2} + \frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}\,dx}{5a}$$

$$= -\frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{4x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4} - \frac{x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5a^2} + \frac{8\int \cdots}{\cdots}$$

$$= -\frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{8\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^6} - \frac{4x^2\sqrt{1-a^2x^2}\,\tan\cdots}{15a^4}$$

$$= -\frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^3\sqrt{1-a^2x^2}}{20a^3} + \frac{89\sin^{-1}(ax)}{120a^6} - \frac{8\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^6} - \frac{4x^2\cdots}{\cdots}$$

Mathematica [A]
time = 0.07, size = 79, normalized size = 0.57

$$-\frac{ax\sqrt{1-a^2x^2}\,(25+6a^2x^2) - 89\text{ArcSin}(ax) + 8\sqrt{1-a^2x^2}\,(8+4a^2x^2+3a^4x^4)\tanh^{-1}(ax)}{120a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/120*(a*x*Sqrt[1 - a^2*x^2]*(25 + 6*a^2*x^2) - 89*ArcSin[a*x] + 8*Sqrt[1
- a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/a^6
```

Maple [C] Result contains complex when optimal does not.
time = 2.71, size = 120, normalized size = 0.86

method	result

default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(24a^4x^4\operatorname{arctanh}(ax)+6a^3x^3+32a^2x^2\operatorname{arctanh}(ax)+25ax+64\operatorname{arctanh}(ax))}{120a^6}+\frac{89i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2}}\right.}{120a^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/a^6*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*a^3*x^3+32*a
^2*x^2*arctanh(a*x)+25*a*x+64*arctanh(a*x))+89/120*I*ln((a*x+1)/(-a^2*x^2+1
)^(1/2)+I)/a^6-89/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^6
```

Maxima [A]
time = 0.47, size = 163, normalized size = 1.17

$$-\frac{1}{120}a\left(\frac{3\left(\frac{2\sqrt{-a^2x^2+1}\,x^3}{a^2}+\frac{3\sqrt{-a^2x^2+1}\,x}{a^4}-\frac{3\arcsin(ax)}{a^5}\right)}{a^2}+\frac{16\left(\frac{\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{\arcsin(ax)}{a^3}\right)}{a^4}-\frac{64\arcsin(ax)}{a^7}\right)-\frac{1}{15}\left(\frac{3\sqrt{-a^2x^2+1}\,x^4}{a^2}+\frac{4\sqrt{-a^2x^2+1}\,x^2}{a^4}+\frac{8\sqrt{-a^2x^2+1}}{a^6}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*a*(3*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2 + 1)*x/a^4 - 3*
arcsin(a*x)/a^5)/a^2 + 16*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)/a^4
- 64*arcsin(a*x)/a^7) - 1/15*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 0.65

$$-\frac{\left(6\,a^3x^3+25\,ax+4\left(3\,a^4x^4+4\,a^2x^2+8\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+178\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*((6*a^3*x^3 + 25*a*x + 4*(3*a^4*x^4 + 4*a^2*x^2 + 8)*log(-(a*x + 1)/
(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 178*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))
)/a^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**5*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\mathrm{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^5*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.365 $\int \frac{x^4 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=197

$$-\frac{5\sqrt{1-a^2x^2}}{8a^5}+\frac{(1-a^2x^2)^{3/2}}{12a^5}-\frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2}-\frac{3\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{4a^5}$$

```
[Out] 1/12*(-a^2*x^2+1)^(3/2)/a^5-3/4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctan
h(a*x)/a^5-3/8*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+3/8*I*polyl
og(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5-5/8*(-a^2*x^2+1)^(1/2)/a^5-3/8*x*a
rctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/4*x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/
a^2
```

Rubi [A]
time = 0.14, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6163, 272, 45, 267, 6097}

$$-\frac{3\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a^5}-\frac{3i\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{3i\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2}+\frac{(1-a^2x^2)^{3/2}}{12a^5}-\frac{5\sqrt{1-a^2x^2}}{8a^5}-\frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-5*Sqrt[1 - a^2*x^2])/(8*a^5) + (1 - a^2*x^2)^(3/2)/(12*a^5) - (3*x*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/(8*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4
*a^2) - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^5) - (((3
*I)/8)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (((3*I)/8)*Pol
yLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= -\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} + \frac{3\int \frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{4a^2} + \frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}\,dx}{4a} \\
&= -\frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} + \frac{3\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^4} + \frac{3}{\cdots} \\
&= -\frac{3\sqrt{1-a^2x^2}}{8a^5} - \frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4a^2} - \frac{3\tan^{-1}\cdots}{\cdots} \\
&= -\frac{5\sqrt{1-a^2x^2}}{8a^5} + \frac{(1-a^2x^2)^{3/2}}{12a^5} - \frac{3x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\,\tanh\cdots}{4a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.33, size = 160, normalized size = 0.81

$$
\frac{\sqrt{1-a^2x^2}\left(-13-2a^2x^2-15ax\tanh^{-1}(ax)-6ax(-1+a^2x^2)\tanh^{-1}(ax)-\frac{9i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}-\frac{9i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{24a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-13 - 2*a^2*x^2 - 15*a*x*ArcTanh[a*x] - 6*a*x*(-1 + a^2
*x^2)*ArcTanh[a*x] - ((9*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1
 + I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((9*I)*(PolyLog[2, (-I)/E^ArcTan
h[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^5)
```

Maple [A]
time = 1.78, size = 175, normalized size = 0.89

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(6a^3x^3\,\mathrm{arctanh}(ax)+2a^2x^2+9ax\,\mathrm{arctanh}(ax)+13)}{24a^5}-\frac{3i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\mathrm{arctanh}(ax)}{8a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2+9*a*x*
arctanh(a*x)+13)-3/8*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+
3/8*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5-3/8*I*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.366 $\int \frac{x^3 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=87

$$-\frac{x\sqrt{1-a^2x^2}}{6a^3}+\frac{5\text{ArcSin}(ax)}{6a^4}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2}$$

```
[Out] 5/6*arcsin(a*x)/a^4-1/6*x*(-a^2*x^2+1)^(1/2)/a^3-2/3*arctanh(a*x)*(-a^2*x^2
+1)^(1/2)/a^4-1/3*x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6163, 327, 222, 6141}

$$\frac{5\text{ArcSin}(ax)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^4}-\frac{x\sqrt{1-a^2x^2}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/6*(x*Sqrt[1 - a^2*x^2])/a^3 + (5*ArcSin[a*x])/(6*a^4) - (2*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int \frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^2} + \frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{3a}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{6a^3} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2} + \frac{\int \frac{1}{\sqrt{1-}}}{6}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{6a^3} + \frac{5\sin^{-1}(ax)}{6a^4} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(}{3a^2}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.69

$$-\frac{ax\sqrt{1-a^2x^2} - 5\text{ArcSin}(ax) + 2\sqrt{1-a^2x^2}\,(2+a^2x^2)\tanh^{-1}(ax)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
[Out] -1/6*(a*x*Sqrt[1 - a^2*x^2] - 5*ArcSin[a*x] + 2*Sqrt[1 - a^2*x^2]*(2 + a^2*
x^2)*ArcTanh[a*x])/a^4
```

Maple [C] Result contains complex when optimal does not.
time = 1.56, size = 99, normalized size = 1.14

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\operatorname{arctanh}(ax)+ax+4\operatorname{arctanh}(ax))}{6a^4} + \frac{5i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{6a^4} - \frac{5i\ln\left(\frac{ax+}{\sqrt{-a^2x}}\right.}{6a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x+4*arctanh(a*x
))+5/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-5/6*I*ln((a*x+1)/(-a^2*x^2+1)
^(1/2)-I)/a^4
```

Maxima [A]
time = 0.47, size = 88, normalized size = 1.01

$$-\frac{1}{6}\,a\left(\frac{\frac{\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{\arcsin(ax)}{a^3}}{a^2}-\frac{4\,\arcsin\left(ax\right)}{a^5}\right)-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*((sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)/a^2 - 4*arcsin(a*x)/a^
5) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arctanh(a*
x)
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.83

$$-\frac{\sqrt{-a^2x^2+1}\,\left(ax+\left(a^2x^2+2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)+10\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1))) +
10*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\,\mathrm{atanh}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.367 $\int \frac{x^2 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{1-a^2x^2}}{2a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2a^2}-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3}-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a^3}+$$

```
[Out] -arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/2*I*polylog(2,-I*(
-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a^3-1/2*(-a^2*x^2+1)^(1/2)/a^3-1/2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/
a^2
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$,
Rules used = {6163, 267, 6097}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/2*Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) - (
ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - ((I/2)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}}\,dx}{2a}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2} - \frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{i\mathrm{Li}_2}{}$$

Mathematica [A]
time = 0.16, size = 125, normalized size = 0.86

$$-\frac{\sqrt{1-a^2x^2}+ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)+i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+i\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-i\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/2*(Sqrt[1 - a^2*x^2] + a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + I*ArcTanh[a*
x]*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + I
*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x]])/a^3
```

Maple [A]
time = 1.56, size = 154, normalized size = 1.05

method	result
default	$-\frac{(ax\,\mathrm{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^3} - \frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\mathrm{arctanh}(ax)}{2a^3} + \frac{i\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right.}{2a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3-1/2*I*ln(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/
```

```
2))*arctanh(a*x)/a^3-1/2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/2*I*
dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.368 $\int \frac{x \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=32

$$\frac{\text{ArcSin}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}$$

```
[Out] arcsin(a*x)/a^2-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6141, 222}

$$\frac{\text{ArcSin}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{a} \\ &= \frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.91

$$\frac{\operatorname{ArcSin}(ax)-\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (ArcSin[a*x] - Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Maple [C] Result contains complex when optimal does not.
time = 1.32, size = 81, normalized size = 2.53

method	result	size
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{arctanh}(ax)}{a^2}+\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{a^2}-\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-i\right)}{a^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)+I*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)+I)/a^2-I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

Maxima [A]
time = 0.47, size = 30, normalized size = 0.94

$$-\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{a^2}+\frac{\arcsin(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*arctanh(a*x)/a^2 + arcsin(a*x)/a^2
```

Fricas [A]
time = 0.41, size = 58, normalized size = 1.81

$$-\frac{\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)+4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)) + 4*arctan((sqrt(-a^2*x^
2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A]
time = 0.44, size = 47, normalized size = 1.47

$$\frac{\arcsin(ax)\operatorname{sgn}(a)}{a|a|} - \frac{\sqrt{-a^2x^2+1}\,\log\left(-\frac{ax+1}{ax-1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/2*sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1))/a^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\mathrm{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.369 $\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] -2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-I*polylog(2,-I*(-a*x
+1)^(1/2)/(a*x+1)^(1/2))/a+I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6097}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx=-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 0.05, size = 76, normalized size = 0.80

$$-\frac{i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
[Out] ((-I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]])
 + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a
```

Maple [A]
time = 2.87, size = 113, normalized size = 1.19

method	result
default	$\frac{i\left(\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)-\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)+\operatorname{dilog}\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)-\operatorname{dilog}\left(1+\frac{i(ax+1)}{\sqrt{-}}\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] I/a*(arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+I*(a
*x+1)/(-a^2*x^2+1)^(1/2))+dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(1/2), x)
```

3.370 $\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=75

$$-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

```
[Out] -2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(
1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6165}

$$\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1
- a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx=-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.76

$$\tanh^{-1}(ax)\left(\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\log\left(1+e^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ArcTanh[a*x]*(Log[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])]) + Po
lyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A]
time = 1.97, size = 99, normalized size = 1.32

method	result
default	$\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{arctanh}(ax)\ln\left(1+\frac{}{\sqrt{-}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1
)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-polylog(2,-(a*x+1)/(
-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.371 $\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6155, 272, 65, 214}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
```

```
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\ &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 1.14

$$-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+a\log(x)-a\log\left(1+\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + a*Log[x] - a*Log[1 + Sqrt[1 - a^2*x
^2]]
```

Maple [A]
time = 1.65, size = 72, normalized size = 1.71

method	result	size
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{arctanh}(ax)}{x}-a\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+a\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)/x-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Maxima [A]
time = 0.47, size = 51, normalized size = 1.21

$$-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}\ \operatorname{artanh}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*arctanh
(a*x)/x
```

Fricas [A]
time = 0.39, size = 58, normalized size = 1.38

$$\frac{2\,ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\ \log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*log(-(a*x +
 1)/(a*x - 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(38) = 76.
time = 0.45, size = 111, normalized size = 2.64

$$-\frac{1}{2}a\left(\log\left(\sqrt{-a^2x^2+1}+1\right)-\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{x|a|}\right)\log\left(-\frac{ax+1}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*(log(sqrt(-a^2*x^2 + 1) + 1) - log(-sqrt(-a^2*x^2 + 1) + 1)) + 1/4*(
a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(x*abs(a)))*log(-(a*x + 1)/(a*x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

3.372 $\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.$$

```
[Out] -a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/2*a^2*polylog(2,-
(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2
))-1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6173, 270, 6165}

$$\frac{1}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+a^2(-\tanh^{-1}(ax))\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 - (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
```

```
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\frac{1}{2}$$

Mathematica [A]
time = 0.48, size = 126, normalized size = 0.92

$$\frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-4\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-4\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 + 4*Arc
Tanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] + 4*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*PolyLog[2, E^(-ArcTanh[a*x])]
- ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A]
time = 1.77, size = 141, normalized size = 1.03

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(ax+\operatorname{arctanh}(ax))}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a^2\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2+1/2*a^2*arctanh(a*x)*l
n(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))-1/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,
-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

3.373 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3}-\frac{10\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^4}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^4}-$$

```
[Out] -10/3*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4-5/3*I*polylog(2
,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+5/3*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x
+1)^(1/2))/a^4-1/3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x*arctanh(a*x)*(-a^2*x^2+1)^(
1/2)/a^3-2/3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arctanh(a*x)^2*(
-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.27, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6163, 267, 6097, 6141}

$$-\frac{10\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{3a^4}-\frac{5i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}+\frac{5i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^2}-\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^4}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/3*Sqrt[1 - a^2*x^2]/a^4 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^3) - (
10*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^4) - (2*Sqrt[1 -
a^2*x^2]*ArcTanh[a*x]^2)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(
3*a^2) - (((5*I)/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (
((5*I)/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx &= -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^2}+\frac{2\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{3a^2}+\frac{2\int \frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a}\\ &= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(a}{3a^2}\\ &= -\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3}-\frac{10\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^4}-\end{aligned}$$

Mathematica [A]
time = 0.30, size = 160, normalized size = 0.78

$$\frac{\sqrt{1-a^2x^2}\left(-1-ax\tanh^{-1}(ax)-3\tanh^{-1}(ax)^2+(1-a^2x^2)\tanh^{-1}(ax)^2-\frac{5i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}-\frac{5i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1 - a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + (1 - a^2*x^2
)*ArcTanh[a*x]^2 - ((5*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
 I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((5*I)*(PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(3*a^4)
```

Maple [A]
time = 0.69, size = 175, normalized size = 0.85

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\left(a^2x^2\operatorname{arctanh}(ax)^2+ax\operatorname{arctanh}(ax)+2\operatorname{arctanh}(ax)^2+1\right)}{3a^4}-\frac{5i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{arcta}}{3a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)+
2*arctanh(a*x)^2+1)-5/3*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a
^4+5/3*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4-5/3*I*dilog(1+
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

[Out] Integral(x**3*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.374 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=161

$$\frac{\text{ArcSin}(ax)}{a^3}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2}+\frac{\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3}-\frac{i\tanh$$

```
[Out] arcsin(a*x)/a^3+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^3-I*arc
tanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+I*arctanh(a*x)*polyl
og(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^
(1/2))/a^3-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-arctanh(a*x)*(-a^2
*x^2+1)^(1/2)/a^3-1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.25, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6163, 6141, 222, 6099, 4265, 2611, 2320, 6724}

$$\frac{\text{ArcSin}(ax)}{a^3}+\frac{\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^3 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^3 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^2) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^3 -
(I*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (I*ArcTanh[a*x]*Poly
Log[2, I*E^ArcTanh[a*x]])/a^3 + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - (
I*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \text{t}\right.}{2a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(}\right.}{}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(}\right.}{}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(}\right.}{}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(}\right.}{}$$

Mathematica [**A**]
time = 0.53, size = 188, normalized size = 1.17

$$\frac{\sqrt{1-a^2x^2}\left(-2\tanh^{-1}(ax) - ax\tanh^{-1}(ax)^2 - \frac{i\left(4i\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right) + \tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - 2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right) - 2\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2*ArcTanh[a*x] - a*x*ArcTanh[a*x]^2 - (I*((4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a^
3)
```

Maple [**F**]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arctanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.375 $\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{4\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^2}-\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^2}+\frac{2i\text{PolyLog}\left(\right.}{}$$

```
[Out] -4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^2-2*I*polylog(2,-I*(
-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2+2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a^2-arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6141, 6097}

$$-\frac{4\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^2}-\frac{2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}+\frac{2i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^2 - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x]^2)/a^2 - ((2*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a^2 + ((2*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^2} + \frac{2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^2} - \frac{2i\text{Li}_2\left(-\frac{i\sqrt{1-a}}{\sqrt{1+a}}\right.}{a^2}$$

Mathematica [A]
time = 0.16, size = 104, normalized size = 0.87

$$-\frac{\tanh^{-1}(ax)\left(\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+2i\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)+2i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + (2*I)*(Log[1 - I/E^ArcTan
h[a*x]] - Log[1 + I/E^ArcTanh[a*x]])) + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]])/a^2)
```

Maple [A]
time = 0.65, size = 151, normalized size = 1.26

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{arctanh}(ax)^2}{a^2} - \frac{2i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{arctanh}(ax)}{a^2} + \frac{2i\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\,\text{a}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2-2*I*ln(1+I*(a*x+1)/(-a^2*x^2
+1)^(1/2))*arctanh(a*x)/a^2+2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(
a*x)/a^2-2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+2*I*dilog(1-I*(a*x+1
)/(-a^2*x^2+1)^(1/2))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \operatorname{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.376 $\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a}-\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}$$

```
[Out] 2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-2*I*arctanh(a*x)*poly
log(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+2*I*arctanh(a*x)*polylog(2,I*(a*x+1)
/(-a^2*x^2+1)^(1/2))/a+2*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-2*I*p
olylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6099, 4265, 2611, 2320, 6724}

$$\frac{2\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a}-\frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{2i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - ((2*I)*ArcTanh[a*x]*PolyLog[2
, (-I)*E^ArcTanh[a*x]])/a + ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]
])/a + ((2*I)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((2*I)*PolyLog[3, I*E^Ar
cTanh[a*x]])/a
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{(2i)\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \tanh^{-1}(ax)\right)}{a} + \frac{(}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{I}}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{I}}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{I}}{}$$

Mathematica [A]
time = 0.07, size = 119, normalized size = 1.16

$$\frac{i\left(-\tanh^{-1}(ax)^2\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)-2\tanh^{-1}(ax)\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)-2\left(\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (I*(-(ArcTanh[a*x]^2*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]
)) - 2*ArcTanh[a*x]*(PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTa
nh[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcTanh[a*x]] - PolyLog[3, I/E^ArcTanh[a*x
]])))/a
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(a x\right)^{2}}{\sqrt{-a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}\left(a x\right)}{\sqrt{-\left(a x-1\right)\left(a x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(1/2), x)
```

3.377 $\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=68

$$-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right.$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a
^2*x^2+1)^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6167, 4267, 2611, 2320, 6724}

$$-2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*PolyLog[2, -E^Ar
cTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 2*PolyLog[3, -E^A
rcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx &= \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)\\
&= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \tanh^{-1}(ax)\right) +\\
&= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{I}\\
&= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{I}\\
&= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{I}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 100, normalized size = 1.47

$$\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 2\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Log[1 + E^(-ArcT
anh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 2*ArcTanh[a*x]
*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyL
og[3, E^(-ArcTanh[a*x])]
```

Maple [A]
time = 0.65, size = 158, normalized size = 2.32

method	result
default	$\operatorname{arctanh}(ax)^2 \ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+2\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-2\operatorname{polylog}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a
*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a
*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-
a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.378 $\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-2a\text{PolyLog}$$

```
[Out] -4*a*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a*polylog(2,-(-a*
x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-arcta
nh(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6155, 6165}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}+2a\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-4a\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a
*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x} + (2a)\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x} - 4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + 2a\mathrm{Li}_2\left(-\frac{\sqrt{1-a}}{\sqrt{1+a}}\right.$$

Mathematica [A]
time = 0.35, size = 89, normalized size = 0.85

$$-\frac{\tanh^{-1}(ax)\left(\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+2ax\left(-\log\left(1-e^{-\tanh^{-1}(ax)}\right)+\log\left(1+e^{-\tanh^{-1}(ax)}\right)\right)\right)}{x}+2a\mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-2a\mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
[Out] -((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2*a*x*(-Log[1 - E^(-ArcTa
nh[a*x])] + Log[1 + E^(-ArcTanh[a*x])])))/x) + 2*a*PolyLog[2, -E^(-ArcTanh[
a*x])] - 2*a*PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A]
time = 0.66, size = 131, normalized size = 1.25

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{arctanh}(ax)^2}{x}+2a\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+2a\operatorname{polylog}\left(2,-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/x+2*a*arctanh(a*x)*ln(1-(a*x+1)/(-
a^2*x^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*arctanh(a*x
)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}(ax)}{x^{2}\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\operatorname{atanh}(a\,x)}^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

3.379 $\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=152

$$-\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-a^2\tanh^{-1}\left(\sqrt{1-\right.$$

```
[Out] -a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-a^2*x
^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ar
ctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)*(
-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6173, 6155, 272, 65, 214, 6167, 4267, 2611, 2320, 6724}

$$-a^2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+a^2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+a^2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-a^2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}+a^2\left(-\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] - a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] + a^2*PolyLog[3, -E^ArcTanh[a*x]] - a^2*Poly
Log[3, E^ArcTanh[a*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
```

```
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}+a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}+\frac{1}{2}a^2\text{Subst}\left(\int x^2\text{csch}(x)\,dx,\right.\\
&= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh\\
&= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh\\
&= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh\\
&= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh\end{aligned}$$

Mathematica [A]
time = 0.82, size = 188, normalized size = 1.24

$$\frac{1}{8}a^2\left(-4\tanh^{-1}(ax)\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\tanh^{-1}(ax)^2\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right)-4\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right)+8\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+8\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-8\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+8\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-8\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^2\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4\tanh^{-1}(ax)\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2 + 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]^2*Lo
g[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 8*ArcTanh[a*x]*Pol
yLog[2, -E^(-ArcTanh[a*x])] - 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])]
```

```
+ 8*PolyLog[3, -E^(-ArcTanh[a*x])] - 8*PolyLog[3, E^(-ArcTanh[a*x])] - ArcT
anh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A]
time = 0.69, size = 231, normalized size = 1.52

method	result
default	$-\frac{\sqrt{-\left(a x -1\right) \left(a x +1\right)}\, \operatorname{arctanh}\left(a x \right) \left(2 a x +\operatorname{arctanh}\left(a x \right)\right)}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}\left(a x \right)^{2} \ln \left(1-\frac{a x +1}{\sqrt{-a^{2} x^{2}+1}}\right)}{2}+a^{2} \operatorname{arctan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2+1/2*a^2
*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*
a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}{\left(a x \right)}}{x^{3} \sqrt{- \left(a x - 1\right) \left(a x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

3.380 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=219

$$\frac{\text{ArcSin}(ax)}{a^4}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3}+\frac{5\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4}-2\sqrt{}$$

```
[Out] arcsin(a*x)/a^4+5*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^4-5*I
*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5*I*arctanh(a*x)
*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5*I*polylog(3,-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))/a^4-5*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-arctanh(
a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^3-2/3
*arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arctanh(a*x)^3*(-a^2*x^2+1)^
(1/2)/a^2
```

Rubi [A]
time = 0.36, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6163, 6141, 222, 6099, 4265, 2611, 2320, 6724}

$$\frac{\text{ArcSin}(ax)}{a^4}+\frac{5\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{5i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{5i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^2}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^4}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^4 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^3) + (5*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^4
- (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^3)/(3*a^2) - ((5*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])
/a^4 + ((5*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 + ((5*I)*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a^4 - ((5*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{3a^2} + \frac{\int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(\cdots}{3a^2}$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\cdots}{3a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh\cdots}\right.}{\cdots}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh\cdots}\right.}{\cdots}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh\cdots}\right.}{\cdots}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh\cdots}\right.}{\cdots}$$

Mathematica [A]
time = 0.62, size = 215, normalized size = 0.98

$$\frac{\sqrt{1-a^2x^2}\left(-3ax\tanh^{-1}(ax)^2+2(1-a^2x^2)\tanh^{-1}(ax)^3-6\tanh^{-1}(ax)\left(1+\tanh^{-1}(ax)^2\right)-\frac{3i\left(4i\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+5\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-5\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+10\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-10\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+10\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-10\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-3*a*x*ArcTanh[a*x]^2 + 2*(1 - a^2*x^2)*ArcTanh[a*x]^3
- 6*ArcTanh[a*x]*(1 + ArcTanh[a*x]^2) - ((3*I)*((4*I)*ArcTan[Tanh[ArcTanh[a
*x]/2]] + 5*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 5*ArcTanh[a*x]^2*Log
[1 + I/E^ArcTanh[a*x]] + 10*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] -
10*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 10*PolyLog[3, (-I)/E^ArcTanh
[a*x]] - 10*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(6*a^4)
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \operatorname{atanh}^{3}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.381 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=305

$$-\frac{6\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3}-\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}+\frac{\text{ArcTan}\left(e^{\tanh^{-1}}\right.}{}$$

```
[Out] -6*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3+arctan((a*x+1)/(-a
^2*x^2+1)^(1/2))*arctanh(a*x)^3/a^3-3/2*I*arctanh(a*x)^2*polylog(2,-I*(a*x+
1)/(-a^2*x^2+1)^(1/2))/a^3+3/2*I*arctanh(a*x)^2*polylog(2,I*(a*x+1)/(-a^2*x
^2+1)^(1/2))/a^3-3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+3*I*pol
ylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+3*I*arctanh(a*x)*polylog(3,-I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^
2+1)^(1/2))/a^3-3*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+3*I*polylo
g(4,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
/a^3-1/2*x*arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]

time = 0.24, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6163, 6141, 6097, 6099, 4265, 2611, 6744, 2320, 6724}

$$\frac{\tanh^{-1}(ax)^3\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh^{-1}(ax)\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i\text{Li}_4\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{3i\text{Li}_4\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2}-\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - (3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^2)/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(2*a^2
) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 - (((3*I)/2)*ArcTanh[a*x]^2
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2
, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 +
a*x]])/a^3 + ((3*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((3*
I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]*
PolyLog[3, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])
/a^3 + ((3*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
```

```
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{3\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{2a}$$

$$= -\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2a^2} + \frac{\mathrm{Subst}\left(\int x^3\mathrm{sech}(x)\,dx, x\right.}{2a^3}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tan}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tan}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tan}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tan}{2a^2}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\,\tan}{2a^2}$$

Mathematica [A]
time = 2.77, size = 570, normalized size = 1.87

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (1
92*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 - (64*I)*
a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + 384*ArcTanh[a*x]
*Log[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] - 384*Arc
Tanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh
[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTan
h[a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(Pi^2 - (4*I)*Pi*ArcTan
h[a*x] - 4*(2 + ArcTanh[a*x]^2))*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 384*Poly
Log[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x
]] - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog
[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*Ar
cTanh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-
I)*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[
4, (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a^3
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.382 $\int \frac{x \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=128

$$\frac{6\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i}{}$$

```
[Out] 6*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^2-6*I*arctanh(a*x)*po
lylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+6*I*arctanh(a*x)*polylog(2,I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^2+6*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
^2-6*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2-arctanh(a*x)^3*(-a^2*x^2
+1)^(1/2)/a^2
```

Rubi [A]
time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6141, 6099, 4265, 2611, 2320, 6724}

$$\frac{6\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^2}-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}-\frac{6i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (6*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/a^2 - ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^2 + ((
6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^2 + ((6*I)*PolyLog[3, (-I
)*E^ArcTanh[a*x]])/a^2 - ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2} + \frac{3\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2} + \frac{3\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$

$$= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2} - \frac{(6i)\text{Subst}\left(\int x\log\left(\right.\right.}{}$$

$$= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2} - \frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-\right.}{a^2}$$

$$= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2} - \frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-\right.}{a^2}$$

$$= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^2} - \frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-\right.}{a^2}$$

Mathematica [**A**]
time = 0.13, size = 157, normalized size = 1.23

$$-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3 + 3i\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - 3i\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right) + 6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - 6i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right) + 6i\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right) - 6i\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 + (3*I)*ArcTanh[a*x]^2*Log[1 - I/E^ArcT
anh[a*x]] - (3*I)*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*ArcTanh[
a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*PolyLog[2, I/E^Ar
cTanh[a*x]] + (6*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (6*I)*PolyLog[3, I/E^
ArcTanh[a*x]])/a^2)
```

Maple [**F**]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x\,\text{arctanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}^{3}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.383 $\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=153

$$\frac{2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,i\right.}{a}$$

```
[Out] 2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a-3*I*arctanh(a*x)^2*po
lylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+3*I*arctanh(a*x)^2*polylog(2,I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a+6*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a^2*x^2+
1)^(1/2))/a-6*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-6*I*
polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+6*I*polylog(4,I*(a*x+1)/(-a^2*x^
2+1)^(1/2))/a
```

Rubi [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6099, 4265, 2611, 6744, 2320, 6724}

$$\frac{2\tanh^{-1}(ax)^3\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a}-\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{6i\tanh^{-1}(ax)\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{6i\tanh^{-1}(ax)\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{6i\text{Li}_4\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{6i\text{Li}_4\left(ie^{\tanh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - ((3*I)*ArcTanh[a*x]^2*PolyLog
[2, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[
a*x]])/a + ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((6*I)*
ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a - ((6*I)*PolyLog[4, (-I)*E^Arc
Tanh[a*x]])/a + ((6*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{(3i)\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 451 vs. $2(153) = 306$.
time = 0.27, size = 451, normalized size = 2.95

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-1/64*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (32
*I)*Pi*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[
a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - (96*I)*Pi*ArcTanh[
a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*
x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] + (96*I)*Pi*ArcTanh[a*
x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + 64*
ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*A
rcTanh[a*x])/4]] - 48*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^ArcTanh
[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - 48*Pi^2*PolyL
og[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*
x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcTanh[a*x]*PolyLog
[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]]
 - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)/E^ArcTanh[
a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

Maple [**F**]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(1/2), x)
```

3.384 $\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=102

$$-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3-3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tan\right.$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-3*arctanh(a*x)^2*poly
log(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arct
anh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {6167, 4267, 2611, 6744, 2320, 6724}

$$-3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-6\text{Li}_4\left(-e^{\tanh^{-1}(ax)}\right)+6\text{Li}_4\left(e^{\tanh^{-1}(ax)}\right)-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^
ArcTanh[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x
]*PolyLog[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] -
 6*PolyLog[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$

$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \tanh^{-1}(ax)\right)$$

$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(a$$

$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(a$$

$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(a$$

$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(a$$

Mathematica [A]
time = 0.10, size = 146, normalized size = 1.43

$$\frac{1}{8}\left(\pi^4 - 2\tanh^{-1}(ax)^4 - 8\tanh^{-1}(ax)^3\log\left(1+e^{-\tanh^{-1}(ax)}\right) + 8\tanh^{-1}(ax)^3\log\left(1-e^{\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right) + 48\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 48\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{\tanh^{-1}(ax)}\right) + 48\text{PolyLog}\left(4,-e^{-\tanh^{-1}(ax)}\right) + 48\text{PolyLog}\left(4,e^{\tanh^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*
ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^(-
ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[
a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[
a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*x]])/
8
```

Maple [A]
time = 0.68, size = 215, normalized size = 2.11

method	result
default	$\operatorname{arctanh}(ax)^3 \ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-6\operatorname{arctanh}($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,
(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(
1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3*ln(1+(a*x+1)/(
-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/
(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.385 $\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=98

$$-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x}-6a\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6$$

```
[Out] -6*a*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-6*a*arctanh(a*x)*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(
a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6155, 6167, 4267, 2611, 2320, 6724}

$$-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x}-6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+6a\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-6a\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x]*P
olyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLog[3
, E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} + (3a)\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$

$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - (6a)\text{Subst}\left(\int x\right.$$

$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{L}$$

$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{L}$$

$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{L}$$

Mathematica [A]
time = 0.26, size = 131, normalized size = 1.34

$$a\left(-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{ax}+3\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+6\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(a*x)) + 3*ArcTanh[a*x]^2*Log[1 - E
^(-ArcTanh[a*x])] - 3*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 6*ArcTanh
[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*PolyLog[2, E^(-ArcTan
h[a*x])] + 6*PolyLog[3, -E^(-ArcTanh[a*x])] - 6*PolyLog[3, E^(-ArcTanh[a*x]
)])
```

Maple [A]
time = 0.68, size = 190, normalized size = 1.94

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{arctanh}(ax)^3}{x}+3a\operatorname{arctanh}(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+6a\operatorname{arctanh}(ax$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^3/x+3*a*arctanh(a*x)^2*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-
6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(
-a^2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{x^{2}\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^{3}}{x^{2}\,\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

3.386 $\int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=267

$$-\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3-6a^2\tanh^{-1}(ax$$

```
[Out] -a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-6*a^2*arctanh(a*x)*
arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+3*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3*a^2*polylog(2,(
-a*x+1)^(1/2)/(a*x+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^
2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*
polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)
^(1/2))-3/2*a*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^3*(-a^2*
x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.30, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6173, 6155, 6165, 6167, 4267, 2611, 6744, 2320, 6724}

$$3a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+3a^2\tanh^{-1}(ax)\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-3a^2\tanh^{-1}(ax)\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-3a^2\text{Li}_4\left(-e^{\tanh^{-1}(ax)}\right)+3a^2\text{Li}_4\left(e^{\tanh^{-1}(ax)}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3-6a^2\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a^2*ArcTan
h[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*ArcTanh[a*x]^2*PolyLog
[2, -E^ArcTanh[a*x]])/2 + (3*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]])
/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*PolyLog[2, Sq
rt[1 - a*x]/Sqrt[1 + a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]]
 - 3*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 3*a^2*PolyLog[4, -E^ArcT
anh[a*x]] + 3*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
```

```
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int\frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int\frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}+\frac{1}{2}a^2\mathrm{Subst}\left(\int x^3\mathrm{csch}(x)\,d\right.$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan$$

Mathematica [**A**]
time = 6.81, size = 301, normalized size = 1.13

$$\frac{1}{16}a\left(a\pi^4-2a\tanh^{-1}(ax)^4-12a\tanh^{-1}(ax)^2\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-2a\tanh^{-1}(ax)^3\mathrm{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+48a\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-48a\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)-8a\tanh^{-1}(ax)^3\log\left(1+e^{-\tanh^{-1}(ax)}\right)+8a\tanh^{-1}(ax)^3\log\left(1-e^{\tanh^{-1}(ax)}\right)+24a\left(2+\tanh^{-1}(ax)^2\right)\mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-48a\mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+24a\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+48a\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-48a\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,e^{\tanh^{-1}(ax)}\right)+48a\mathrm{PolyLog}\left(4,-e^{-\tanh^{-1}(ax)}\right)+48a\mathrm{PolyLog}\left(4,e^{\tanh^{-1}(ax)}\right)+12a\tanh^{-1}(ax)^2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*(a*Pi^4 - 2*a*ArcTanh[a*x]^4 - 12*a*ArcTanh[a*x]^2*Coth[ArcTanh[a*x]/2]
- 2*a*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2 + 48*a*ArcTanh[a*x]*Log[1 - E^(
-ArcTanh[a*x])] - 48*a*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] - 8*a*ArcTan
h[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*a*ArcTanh[a*x]^3*Log[1 - E^ArcTanh[
a*x]] + 24*a*(2 + ArcTanh[a*x]^2)*PolyLog[2, -E^(-ArcTanh[a*x])] - 48*a*Pol
yLog[2, E^(-ArcTanh[a*x])] + 24*a*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]]
 + 48*a*ArcTanh[a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*a*ArcTanh[a*x]*Pol
yLog[3, E^ArcTanh[a*x]] + 48*a*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*a*PolyLo
g[4, E^ArcTanh[a*x]] + 12*a*ArcTanh[a*x]^2*Tanh[ArcTanh[a*x]/2] - (4*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x]^3*Tanh[ArcTanh[a*x]/2])/x))/16
```

Maple [A]
time = 0.73, size = 386, normalized size = 1.45

method	result
default	$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\ \mathrm{arctanh}(ax)^{2}(3ax+\mathrm{arctanh}(ax))}{2x^{2}}+\frac{a^{2}\,\mathrm{arctanh}(ax)^{3}\ln\left(1-\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}+\frac{3a^{2}\,\mathrm{arctan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2*(3*a*x+arctanh(a*x))/x^2+1/2*a
^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*p
olylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(
-a^2*x^2+1)^(1/2))+3*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*arct
anh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))-3*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*
x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2))-3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-(
a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(atanh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

3.387 $\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx = \int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx$$

Mathematica [A]
time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Maple [A]
time = 4.18, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atanh}(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**m*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atanh}(a x)}{(1 - a^2 x^2)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^m*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

3.388 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=74

$$-\frac{x}{a^3\sqrt{1-a^2x^2}}-\frac{\text{ArcSin}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4}$$

```
[Out] -arcsin(a*x)/a^4-x/a^3/(-a^2*x^2+1)^(1/2)+arctanh(a*x)/a^4/(-a^2*x^2+1)^(1/
2)+arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A]
time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6175, 6141, 222, 197}

$$-\frac{\text{ArcSin}(ax)}{a^4}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{x}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(x/(a^3*Sqrt[1 - a^2*x^2])) - ArcSin[a*x]/a^4 + ArcTanh[a*x]/(a^4*Sqrt[1 -
 a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
```

```
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx &= \frac{\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= \frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3}\\ &= -\frac{x}{a^3\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{a^4} + \frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 1.03

$$\frac{ax\sqrt{1-a^2x^2} + (1-a^2x^2)\,\text{ArcSin}(ax) + \sqrt{1-a^2x^2}\,(-2+a^2x^2)\tanh^{-1}(ax)}{a^4\,(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] + (1 - a^2*x^2)*ArcSin[a*x] + Sqrt[1 - a^2*x^2]*(-2
+ a^2*x^2)*ArcTanh[a*x])/(a^4*(-1 + a^2*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 1.59, size = 144, normalized size = 1.95

method	result
default	$-\frac{(\text{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2a^4(ax-1)} + \frac{(\text{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^4(ax+1)} + \frac{\sqrt{-(ax-1)(a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x-1)+1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x+1)+(-(a*x-1)*(a*x+1))^(1/2)*arctanh(
a*x)/a^4+I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4-I*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)+I)/a^4
```

Maxima [A]
time = 0.46, size = 96, normalized size = 1.30

$$a\left(\frac{\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin(ax)}{a^3}}{a^2}-\frac{2\,x}{\sqrt{-a^2x^2+1}\,a^4}\right)-\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*((x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3)/a^2 - 2*x/(sqrt(-a^2*x^2
+ 1)*a^4)) - (x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4))*ar
ctanh(a*x)
```

Fricas [A]
time = 0.39, size = 94, normalized size = 1.27

$$\frac{4\left(a^2x^2-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\left(2\,ax+\left(a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^2*x^2 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(2*a*x + (a^2*x^2 - 2)*log(-(a*x + 1)/(a*x - 1))))/(a^6*x^2 - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\operatorname{atanh}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atanh}(a x)}{(1 - a^2 x^2)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^3*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

3.389 $\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3}+\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3}-\frac{i\text{PolyLo}}{}$$

```
[Out] 2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3+I*polylog(2,-I*(-a*
x+1)^(1/2)/(a*x+1)^(1/2))/a^3-I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
^3-1/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6145, 6097}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}+\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{1}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) + (
2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 + (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 - (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a^3
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6145

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]
 - Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x])
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q,
```

-5/2]

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{1}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{1}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} + \frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} + \frac{i\text{Li}_2\left(-\frac{i\sqrt{}}{\sqrt{1}}\right.}{a^3}$$

Mathematica [A]
time = 0.20, size = 121, normalized size = 0.88

$$\frac{i\left(\frac{i}{\sqrt{1-a^2x^2}} - \frac{iax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right) + \text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - \text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (I*(I/Sqrt[1 - a^2*x^2] - (I*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[
a*x]*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + P
olyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a^3
```

Maple [A]
time = 1.11, size = 190, normalized size = 1.39

method	result
default	$-\frac{(\text{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2a^3(ax-1)} - \frac{(\text{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^3(ax+1)} + \frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+}}\right.}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x-1)-1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x+1)+I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*arctanh(a*x)/a^3-I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+
I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-I*dilog(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atanh}(a\,x)}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

3.390 $\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=43

$$-\frac{x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] -x/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6141, 197}

$$\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx &= \frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{a}\\ &= -\frac{x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.63

$$\frac{-ax+\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-(a*x) + ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 1.07, size = 66, normalized size = 1.53

method	result	size
default	$-\frac{(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)a^2}+\frac{(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2(ax+1)a^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)/a^2+1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)/a^2
```

Maxima [A]
time = 0.25, size = 39, normalized size = 0.91

$$-\frac{x}{\sqrt{-a^2x^2+1}\,a}+\frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a^2)
```

Fricas [A]
time = 0.38, size = 51, normalized size = 1.19

$$\frac{\sqrt{-a^2x^2+1}\left(2\,ax-\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(2*a*x - log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [**A**]
time = 0.43, size = 61, normalized size = 1.42

$$\frac{\sqrt{-a^2x^2+1}\,x}{(a^2x^2-1)a}+\frac{\log\left(-\frac{ax+1}{ax-1}\right)}{2\sqrt{-a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*a) + 1/2*log(-(a*x + 1)/(a*x - 1))/(sqr
t(-a^2*x^2 + 1)*a^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \operatorname{atanh}(a\,x)}{{(1-a^2\,x^2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

3.391 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -1/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6105}

$$\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
[Out] -(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx=-\frac{1}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.68

$$\frac{-1+ax\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
[Out] (-1 + a*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 1.10, size = 38, normalized size = 0.95

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, (a x \,\mathrm{arctanh}(a x)-1)}{a (a^{2} x^{2}-1)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(a*x*arctanh(a*x)-1)/(a^2*x^2-1)
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.90

$$\frac{x \,\mathrm{artanh}\left(a x\right)}{\sqrt{-a^{2} x^{2} + 1}} - \frac{1}{\sqrt{-a^{2} x^{2} + 1}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] x*arctanh(a*x)/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.18

$$-\frac{\sqrt{-a^{2} x^{2} + 1} \left(a x \log\left(-\frac{a x + 1}{a x - 1}\right) - 2\right)}{2 \left(a^{3} x^{2} - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atanh}\left(a x\right)}{\left(-\left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.43, size = 59, normalized size = 1.48

$$-\frac{\sqrt{-a^2x^2+1}\,x\log\left(-\frac{ax+1}{ax-1}\right)}{2\,(a^2x^2-1)}-\frac{1}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1) - 1/(sqrt
(-a^2*x^2 + 1)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(3/2), x)
```

3.392 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=112

$$-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{PolyLog}\left(2,\right.$$

```
[Out] -2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(
1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-a*x/(-a^2*x^2+1
)^(1/2)+arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6177, 6165, 6141, 197}

$$-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] - 2*ArcTanh[a*x
]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
+ a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
```

```
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^{3/2}}\,dx = a^2\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \mathrm{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \mathrm{Li}_2\left(\frac{\sqrt{1-}}{\sqrt{1-}}\right.$$
$$= -\frac{ax}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \mathrm{Li}_2\left(-\frac{\sqrt{1-a}}{\sqrt{1+a}}\right.$$

Mathematica [A]
time = 0.15, size = 97, normalized size = 0.87

$$-\frac{ax}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right) + \mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - \mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]*
Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + Poly
Log[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A]
time = 1.72, size = 157, normalized size = 1.40

method	result
default	$-\frac{(\mathrm{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)} + \frac{(\mathrm{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2ax+2} + \mathrm{arctanh}(ax)\ln\left(1\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)+1)
*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1
/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))-polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x*(1 - a^2*x^2)^(3/2)),x)
[Out] int(atanh(a*x)/(x*(1 - a^2*x^2)^(3/2)), x)
```

3.393 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-a/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x)/(-a^
2*x^2+1)^(1/2)-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 6155, 272, 65, 214, 6105}

$$-\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -(a/Sqrt[1 - a^2*x^2]) + (a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/x - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+\frac{1}{2}a\text{Subst}\Bigg(\int \frac{1}{x\sqrt{1-\cdots}}$$

$$= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\text{Subst}\Big(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x\cdots}{a}$$

$$= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-a\tanh^{-1}\Big(\sqrt{1-a^2x^2}\cdots$$

Mathematica [A]
time = 0.08, size = 89, normalized size = 1.09

$$\frac{\left(-1+2a^2x^2\right)\tanh^{-1}(ax)+ax\left(-1+\sqrt{1-a^2x^2}\,\log(x)-\sqrt{1-a^2x^2}\,\log\left(1+\sqrt{1-a^2x^2}\right)\right)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] ((-1 + 2*a^2*x^2)*ArcTanh[a*x] + a*x*(-1 + Sqrt[1 - a^2*x^2]*Log[x] - Sqrt[
1 - a^2*x^2]*Log[1 + Sqrt[1 - a^2*x^2]]))/(x*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 1.34, size = 132, normalized size = 1.61

method	result
default	$-\frac{a(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}-\frac{(\operatorname{arctanh}(ax)+1)a\sqrt{-(ax-1)(ax+1)}}{2(ax+1)}-\frac{\sqrt{-(ax-1)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)-1/2*(arctanh(a*x)+
1)*a*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)
/x+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.25, size = 84, normalized size = 1.02

$$-a\left(\frac{1}{\sqrt{-a^2x^2+1}}+\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)\right)+\left(\frac{2\,a^2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(1/sqrt(-a^2*x^2 + 1) + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))) + (
2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 107, normalized size = 1.30

$$-\frac{2\,a^3x^3-2\,ax-2\,(a^3x^3-ax)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\,\left(2\,ax-(2\,a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\,(a^2x^3-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*x^3 - 2*a*x - 2*(a^3*x^3 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x)
 - sqrt(-a^2*x^2 + 1)*(2*a*x - (2*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/
(a^2*x^3 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x^2\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(74) = 148.
time = 0.43, size = 155, normalized size = 1.89

$$-\frac{1}{2}\,a\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{1}{2}\,a\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{2\sqrt{-a^2x^2+1}\,a^2x}{a^2x^2-1}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{x|a|}\right)\log\left(-\frac{ax+1}{ax-1}\right)-\frac{a}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(sqrt(-a^2*x^2 + 1) + 1) + 1/2*a*log(-sqrt(-a^2*x^2 + 1) + 1) + 1
/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 2*sqrt(-a^2*x^2 + 1)*a
^2*x/(a^2*x^2 - 1) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a)))*log(-(a*x
+ 1)/(a*x - 1)) - a/sqrt(-a^2*x^2 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

3.394 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-3a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.$$

```
[Out] -3*a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/2*a^2*polylog(2
,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1
/2))-a^3*x/(-a^2*x^2+1)^(1/2)+a^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-1/2*a*(-a
^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6177, 6173, 270, 6165, 6141, 197}

$$\frac{3}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-3a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3x}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a^3*x)/Sqrt[1 - a^2*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x) + (a^2*ArcTanh[a
*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - 3*a^2*A
rcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (3*a^2*PolyLog[2, -(Sqrt
[1 - a*x]/Sqrt[1 + a*x])])/2 - (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx+a^2\\ &= -\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-3a^2\tanh^{-1}(ax)\tanh^{-}\\ &= -\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}-3a^2\tan\end{aligned}$$

Mathematica [A]

time = 1.10, size = 182, normalized size = 1.02

$$\frac{1}{8}a^2\left(-\frac{8ax}{\sqrt{1-a^2x^2}}+\frac{8\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{ax\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}-\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+12\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-12\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+12\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-12\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*((-8*a*x)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*
x*Csch[ArcTanh[a*x]/2]^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]*Csch[ArcTanh[a*x
]/2]^2 + 12*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]*Log[1
 + E^(-ArcTanh[a*x])] + 12*PolyLog[2, -E^(-ArcTanh[a*x])] - 12*PolyLog[2, E
^(-ArcTanh[a*x])] - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*
x]/2]))/8
```

Maple [A]
time = 1.38, size = 205, normalized size = 1.15

method	result
default	$-\frac{a^2(\text{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{(\text{arctanh}(ax)+1)a^2\sqrt{-(ax-1)(ax+1)}}{2ax+2}-\sqrt{-(ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x
)+1)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x
+arctanh(a*x))/x^2+3/2*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/
2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)*ln(1+(a*x+
1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{x^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

3.395 $\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx = \int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^m \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Maple [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
[Out] int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{atanh}^{2}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x**m*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atanh}(a\,x)^2}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^m*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

3.396 $\int \frac{x^3 \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}}+\frac{4\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax}{a^4}$$

```
[Out] 4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4+2*I*polylog(2,-I*(-
a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4-2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a^4+2/a^4/(-a^2*x^2+1)^(1/2)-2*x*arctanh(a*x)/a^3/(-a^2*x^2+1)^(1/2)+arc
tanh(a*x)^2/a^4/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A]
time = 0.22, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6175, 6141, 6097, 6105}

$$\frac{4\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^4}+\frac{2i\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}-\frac{2i\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}+\frac{2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^4}+\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 2/(a^4*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) + (4
*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^4 + ArcTanh[a*x]^2/(a^
4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^4 + ((2*I)*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 - ((2*I)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx &= \frac{\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{a^2}-\frac{\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= \frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{a^4}-\frac{2\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx}{a^3}-\frac{2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^3}\\ &= \frac{2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}}+\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax}{a^4\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 165, normalized size = 0.89

$$\frac{2i\,\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+\frac{2+(2-a^2x^2)\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)\left(ax-i\sqrt{1-a^2x^2}\,\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+i\sqrt{1-a^2x^2}\,\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)-2i\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (2 + (2 - a^2*x^2)*ArcTanh[a*x]^2
- 2*ArcTanh[a*x]*(a*x - I*Sqrt[1 - a^2*x^2]*Log[1 - I/E^ArcTanh[a*x]] + I*S
qrt[1 - a^2*x^2]*Log[1 + I/E^ArcTanh[a*x]]) - (2*I)*Sqrt[1 - a^2*x^2]*PolyL
og[2, I/E^ArcTanh[a*x]])/Sqrt[1 - a^2*x^2])/a^4
```

Maple [A]
time = 0.69, size = 230, normalized size = 1.24

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^{2}-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2a^{4}(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^{2}+2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+}}{2a^{4}(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x+1)
+(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/a^4+2*I*ln(1+I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^4-2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*
x)/a^4+2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-2*I*dilog(1-I*(a*x+1)/
(-a^2*x^2+1)^(1/2))/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\operatorname{atanh}^{2}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atanh}(a\,x)^2}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

3.397 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{2x}{a^2\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3}+\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(\right.}{a^3}$$

```
[Out] -2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^3+2*I*arctanh(a*x)*p
olylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-2*I*arctanh(a*x)*polylog(2,I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a^3-2*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^3+2*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+2*x/a^2/(-a^2*x^2+1)^(1
/2)-2*arctanh(a*x)/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)
^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6099, 4265, 2611, 2320, 6724, 6109, 197}

$$-\frac{2\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{2i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{2x}{a^2\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*x)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) +
(x*ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcT
anh[a*x]^2)/a^3 + ((2*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3
- ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^3 - ((2*I)*PolyLog[3,
 (-I)*E^ArcTanh[a*x]])/a^3 + ((2*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^3} + \frac{2\int \frac{}{(1-}}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^3}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^3}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^3}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^3}$$

Mathematica [A]
time = 0.22, size = 193, normalized size = 1.13

$$\frac{i\left(-\frac{2iax}{\sqrt{1-a^2x^2}} + \frac{2i\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{iax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - 2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right) - 2\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (I*(((-2*I)*a*x)/Sqrt[1 - a^2*x^2] + ((2*I)*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
 - (I*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - I/E^Ar
cTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*Pol
yLog[2, (-I)/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]]
+ 2*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/a^3
```

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x**2*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\mathrm{atanh}\left(a\,x\right)}^2}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

3.398 $\int \frac{x \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=68

$$\frac{2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] 2/a^2/(-a^2*x^2+1)^(1/2)-2*x*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)
^2/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6141, 6105}

$$\frac{2}{a^2\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 2/(a^2*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + ArcT
anh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx &= \frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx}{a}\\ &= \frac{2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.50

$$\frac{2-2ax\tanh^{-1}(ax)+\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2 - 2*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 0.64, size = 82, normalized size = 1.21

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)a^2}+\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)/a^2
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)/a^2
```

Maxima [A]
time = 0.26, size = 62, normalized size = 0.91

$$-\frac{2\,x\,\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a}+\frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,a^2}+\frac{2}{\sqrt{-a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x*arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^2/(sqrt(-a^2*x^2 +
1)*a^2) + 2/(sqrt(-a^2*x^2 + 1)*a^2)
```

Fricas [A]
time = 0.35, size = 69, normalized size = 1.01

$$\frac{\sqrt{-a^2x^2+1}\left(4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\log\left(-\frac{ax+1}{ax-1}\right)^2-8\right)}{4\,(a^4x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a
*x - 1))^2 - 8)/(a^4*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \operatorname{atanh}(a\,x)^2}{{(1-a^2\,x^2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

3.399 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2x}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}$$

```
[Out] 2*x/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^2
/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6109, 197}

$$\frac{2x}{\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTa
nh[a*x]^2)/Sqrt[1 - a^2*x^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx &= -\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+2\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx\\ &= \frac{2x}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.60

$$\frac{2ax - 2\tanh^{-1}(ax) + ax\tanh^{-1}(ax)^2}{a\sqrt{1 - a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
[Out] (2*a*x - 2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 0.64, size = 49, normalized size = 0.78

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(\operatorname{arctanh}(ax)^2ax+2ax-2\operatorname{arctanh}(ax)\right)}{a(a^2x^2-1)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^2*a*x+2*a*x-2*arctanh(a*x))/(a^2*x^2-
1)
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.90

$$\frac{x\,\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}} + \frac{2\,x}{\sqrt{-a^2x^2+1}} - \frac{2\,\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1) + 2*x/sqrt(-a^2*x^2 + 1) - 2*arctanh(a*
x)/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A]
time = 0.38, size = 69, normalized size = 1.10

$$-\frac{\sqrt{-a^2x^2+1}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^2 + 8\,ax - 4\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{4\,(a^3x^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1))^2 + 8*a*x - 4*log(-(
a*x + 1)/(a*x - 1)))/(a^3*x^2 - a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\operatorname{atanh}(a\,x)}^2}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(3/2), x)
```

3.400 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)\text{PolyLog}\Big(2,-$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a
^2*x^2+1)^(1/2))+2/(-a^2*x^2+1)^(1/2)-2*a*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
+arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6177, 6167, 4267, 2611, 2320, 6724, 6141, 6105}

$$\frac{2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh
[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x
]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-(2a)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx+\text{Subst}\left(\int x^2\text{csch}(x)\,dx,x,\tanh^{-1}(ax)\right)\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2\\
&= \frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2\end{aligned}$$

Mathematica [A]
time = 0.17, size = 159, normalized size = 1.25

$$\frac{2}{\sqrt{1-a^2x^2}}-\frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[
a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[
a*x])] - 2*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-A
rcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a*x])]
```

Maple [A]
time = 0.68, size = 232, normalized size = 1.83

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+}}{2ax+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2
*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)+arctanh
```

```
(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(
-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2
+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}\left(ax\right)}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)**2/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(3/2)),x)
[Out] int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(3/2)), x)
```

3.401 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{2a^2x}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-a}}{\sqrt{1+a}}\right.$$

```
[Out] -4*a*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a*polylog(2,-(-a*
x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a^2
*x/(-a^2*x^2+1)^(1/2)-2*a*arctanh(a*x)/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x
)^2/(-a^2*x^2+1)^(1/2)-arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]

time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6177, 6155, 6165, 6109, 197}

$$\frac{2a^2x}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}+2a\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*x)/Sqrt[1 - a^2*x^2] - (2*a*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (a^2*x
*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x -
 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(S
qrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
```

```
1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}+(2a)\int \frac{\tanh^{-1}(}{x\sqrt{1-a}\\ &= \frac{2a^2x}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-4a\,t\end{aligned}$$

Mathematica [A]
time = 0.75, size = 215, normalized size = 1.26

$$\frac{a\left(4ax-4\tanh^{-1}(ax)+2ax\tanh^{-1}(ax)^2-\frac{1}{2}ax\tanh^{-1}(ax)^2\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+4\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-4\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\frac{2\left(-1+a^2x^2\right)\tanh^{-1}(ax)^2\sinh^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{ax}\right)}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a*(4*a*x - 4*ArcTanh[a*x] + 2*a*x*ArcTanh[a*x]^2 - (a*x*ArcTanh[a*x]^2*Csc
h[ArcTanh[a*x]/2]^2)/2 + 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-ArcTa
nh[a*x])] - 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 4
*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*Sqrt[1 - a^2*x^2]*Pol
```

```
yLog[2, E^(-ArcTanh[a*x])] - (2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*Sinh[ArcTanh[
a*x]/2]^2)/(a*x)))/(2*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 0.70, size = 207, normalized size = 1.21

method	result
default	$-\frac{a\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}-\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)a\sqrt{-(ax-1)(}}{2(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*a*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)-1
/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(
a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/x+2*a*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*
x^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*arctanh(a*x)*ln
(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

3.402 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{2a^2}{\sqrt{1-a^2x^2}}-\frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+\frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-3a^2$$

```
[Out] -3*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-a^2
*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*
a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(3,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2/
(-a^2*x^2+1)^(1/2)-2*a^3*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)+a^2*arctanh(a*x)
^2/(-a^2*x^2+1)^(1/2)-a*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^
2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.55, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6177, 6173, 6155, 272, 65, 214, 6167, 4267, 2611, 2320, 6724, 6141, 6105}

$$-3a^2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+3a^2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+3a^2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-3a^2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)+\frac{2a^2}{\sqrt{1-a^2x^2}}+\frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2}-3a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-\frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (2*a^2)/Sqrt[1 - a^2*x^2] - (2*a^3*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*S
qrt[1 - a^2*x^2]*ArcTanh[a*x])/x + (a^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] -
 (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]
*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1 - a^2*x^2]] - 3*a^2*ArcTanh[a*x]*PolyL
og[2, -E^ArcTanh[a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 3*
a^2*PolyLog[3, -E^ArcTanh[a*x]] - 3*a^2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx + a^2$$

$$= -\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\text{Sub}$$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - $$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - $$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - $$

$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - $$

Mathematica [A]
time = 1.80, size = 266, normalized size = 1.20

$$\frac{1}{8}a^2\left(\frac{16}{\sqrt{1-a^2x^2}} - \frac{16ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{8\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{2ax\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} - \tanh^{-1}(ax)^2\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right) + 12\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right) - 12\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right) + 8\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right) + 24\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 24\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 24\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 24\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right) - \tanh^{-1}(ax)^2\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right) + 4\tanh^{-1}(ax)\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*(16/Sqrt[1 - a^2*x^2] - (16*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (8*A
rcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]
^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*x]/2]^2 + 12*ArcTanh[
a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a
*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 24*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 24*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 24*PolyLog[3, -
E^(-ArcTanh[a*x])] - 24*PolyLog[3, E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Sech
[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A]
time = 0.74, size = 313, normalized size = 1.42

method	result

default	$-\frac{a^{2}\left(\operatorname{arctanh}(ax)^{2}-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^{2}+2\operatorname{arctanh}(ax)+2\right)a^{2}\sqrt{-(ax-1)}}{2ax+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)
-1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2-2*a^2*a
rctanh((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^
2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*
a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*ln(1+(a*x+
1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(
1/2))+3*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}(ax)}{x^{3}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

3.403 $\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]
```

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atanh}^3(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**m*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atanh}(a\,x)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^m*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

3.404 $\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=220

$$-\frac{6x}{a^3\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}}-\frac{6\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4}+\frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1}}{}$$

```
[Out] -6*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^4+6*I*arctanh(a*x)*p
olylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*I*arctanh(a*x)*polylog(2,I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^4+6*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*x/a^3/(-a^2*x^2+1)^(1
/2)+6*arctanh(a*x)/a^4/(-a^2*x^2+1)^(1/2)-3*x*arctanh(a*x)^2/a^3/(-a^2*x^2+
1)^(1/2)+arctanh(a*x)^3/a^4/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^3*(-a^2*x^2+1)^
(1/2)/a^4
```

Rubi [A]
time = 0.29, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6175, 6141, 6099, 4265, 2611, 2320, 6724, 6109, 197}

$$-\frac{6\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{6i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^4}+\frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{6x}{a^3\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*x)/(a^3*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^4*Sqrt[1 - a^2*x^2]) -
 (3*x*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) - (6*ArcTan[E^ArcTanh[a*x]]*A
rcTanh[a*x]^2)/a^4 + ArcTanh[a*x]^3/(a^4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^3)/a^4 + ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a
*x]])/a^4 - ((6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 - ((6*I)*
PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^4 + ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])
/a^4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
```

```
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^4} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{a^3} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$

$$= \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{a^4} - \frac{3\mathrm{Su}}{}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(}{a^4}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(}{a^4}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(}{a^4}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(}{a^4}$$

Mathematica [A]
time = 0.30, size = 249, normalized size = 1.13

$$\frac{6i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - 6i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right) + \frac{-6ax+6\tanh^{-1}(ax)-3ax\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^3-a^2x^2\tanh^{-1}(ax)^3+3i\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-3i\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+6i\sqrt{1-a^2x^2}\mathrm{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-6i\sqrt{1-a^2x^2}\mathrm{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*Po
lyLog[2, I/E^ArcTanh[a*x]] + (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^
```

```
2 + 2*ArcTanh[a*x]^3 - a^2*x^2*ArcTanh[a*x]^3 + (3*I)*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - (3*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, (-I)/E^A
rcTanh[a*x]] - (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, I/E^ArcTanh[a*x]])/Sqrt[1
 - a^2*x^2])/a^4
```

Maple [F]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \operatorname{arctanh}\left(a x\right)^{3}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
[Out] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \operatorname{atanh}^{3}{\left(a x \right)}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

3.405 $\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=246

$$-\frac{6}{a^3\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{2\mathrm{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a^3}+\frac{3i\,\mathrm{t}}{}$$

```
[Out] -2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a^3+3*I*arctanh(a*x)^2
*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*arctanh(a*x)^2*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-6*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a
^2*x^2+1)^(1/2))/a^3+6*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^3+6*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-6*I*polylog(4,I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^3-6/a^3/(-a^2*x^2+1)^(1/2)+6*x*arctanh(a*x)/a^2/
(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)^2/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^3
/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6175, 6099, 4265, 2611, 6744, 2320, 6724, 6109, 6105}

$$-\frac{2\tanh^{-1}(ax)^3\mathrm{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{3i\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\mathrm{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6i\tanh^{-1}(ax)\mathrm{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{6i\tanh^{-1}(ax)\mathrm{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{6i\mathrm{Li}_4\left(-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6i\mathrm{Li}_4\left(ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{6}{a^3\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -6/(a^3*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(a^2*Sqrt[1
- a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 + ((3*I)*ArcTan
h[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]^2*PolyL
og[2, I*E^ArcTanh[a*x]])/a^3 - ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTan
h[a*x]])/a^3 + ((6*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a^3 + ((6*
I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a^3 - ((6*I)*PolyLog[4, I*E^ArcTanh[a*x
]])/a^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6175

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^3} + \frac{6\int \ldots}{\ldots}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\ldots}\right.}{\ldots}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\ldots}\right.}{\ldots}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\ldots}\right.}{\ldots}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\ldots}\right.}{\ldots}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\ldots}\right.}{\ldots}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 541 vs. $2(246) = 492$.
time = 0.63, size = 541, normalized size = 2.20

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((7*I)*Pi^4 - 384/Sqrt[1 - a^2*x^2] - 8*Pi^3*ArcTanh[a*x] + (384*a*x*ArcTan
h[a*x])/Sqrt[1 - a^2*x^2] + (24*I)*Pi^2*ArcTanh[a*x]^2 - (192*ArcTanh[a*x]^
2)/Sqrt[1 - a^2*x^2] + 32*Pi*ArcTanh[a*x]^3 + (64*a*x*ArcTanh[a*x]^3)/Sqrt[
1 - a^2*x^2] - (16*I)*ArcTanh[a*x]^4 - 8*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + (
48*I)*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 96*Pi*ArcTanh[a*x]^2*Lo
g[1 + I/E^ArcTanh[a*x]] - (64*I)*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] -
 (48*I)*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] - 96*Pi*ArcTanh[a*x]^2*
Log[1 - I*E^ArcTanh[a*x]] + 8*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + (64*I)*ArcTa
nh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] - 8*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*
x])/4]] - (48*I)*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^ArcTanh[a*x]
] + (192*I)*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (48*I)*Pi^2*Po
lyLog[2, I*E^ArcTanh[a*x]] - 192*Pi*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x
]] - 192*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + (384*I)*ArcTanh[a*x]*PolyLog[
3, (-I)/E^ArcTanh[a*x]] - (384*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*
x]] + 192*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + (384*I)*PolyLog[4, (-I)/E^ArcTa
nh[a*x]] + (384*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/(64*a^3)
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \operatorname{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

3.406 $\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] -6*x/a/(-a^2*x^2+1)^(1/2)+6*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)-3*x*arctanh
(a*x)^2/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^3/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6141, 6109, 197}

$$-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^
2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{a}$$
$$= \frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{6\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{a}$$
$$= -\frac{6x}{a\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.48

$$\frac{-6ax+6\tanh^{-1}(ax)-3ax\tanh^{-1}(ax)^2+\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^2 + ArcTanh[a*x]^3)/(a^2*Sqrt
[1 - a^2*x^2])
```

Maple [A]
time = 0.65, size = 98, normalized size = 1.04

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^3-3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)a^2}+\frac{\left(\operatorname{arctanh}(ax)^3+3\operatorname{arctanh}(ax)^2+6\operatorname{arctan}\right.}{2(ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))^
(1/2)/(a*x-1)/a^2+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-
(a*x-1)*(a*x+1))^(1/2)/(a*x+1)/a^2
```

Maxima [A]
time = 0.25, size = 88, normalized size = 0.94

$$-\frac{3\,x\,\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,a}+\frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}\,a^2}-\frac{6\left(\frac{x}{\sqrt{-a^2x^2+1}}-\frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*x*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^3/(sqrt(-a^2*x^2
+ 1)*a^2) - 6*(x/sqrt(-a^2*x^2 + 1) - arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a))/
a
```

Fricas [A]
time = 0.38, size = 91, normalized size = 0.97

$$\frac{\sqrt{-a^2x^2+1}\left(6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-\log\left(-\frac{ax+1}{ax-1}\right)^3+48\,ax-24\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{8\,(a^4x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-a^2*x^2 + 1)*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - log(-(a*x + 1)/
(a*x - 1))^3 + 48*a*x - 24*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\,\mathrm{atanh}^3\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\mathrm{atanh}(a\,x)^3}{{(1-a^2\,x^2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

3.407 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}$$

```
[Out] -6/a/(-a^2*x^2+1)^(1/2)+6*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)^
2/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6109, 6105}

$$-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -6/(a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*ArcTan
h[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx&=-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}+6\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx\\&=-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.51

$$\frac{-6+6ax\tanh^{-1}(ax)-3\tanh^{-1}(ax)^2+ax\tanh^{-1}(ax)^3}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6 + 6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + a*x*ArcTanh[a*x]^3)/(a*Sqrt[1
 - a^2*x^2])
```

Maple [A]
time = 0.67, size = 56, normalized size = 0.64

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(\operatorname{arctanh}(ax)^3 ax+6ax\operatorname{arctanh}(ax)-3\operatorname{arctanh}(ax)^2-6\right)}{a(a^2x^2-1)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^3*a*x+6*a*x*arctanh(a*x)-3*arctanh(a*
x)^2-6)/(a^2*x^2-1)
```

Maxima [A]
time = 0.26, size = 86, normalized size = 0.98

$$\frac{x\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}+6a\left(\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,a}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2}\right)-\frac{3\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1) + 6*a*(x*arctanh(a*x)/(sqrt(-a^2*x^2 +
1)*a) - 1/(sqrt(-a^2*x^2 + 1)*a^2)) - 3*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*
a)
```

Fricas [A]
time = 0.37, size = 87, normalized size = 0.99

$$-\frac{\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^3+24\,ax\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(-\frac{ax+1}{ax-1}\right)^2-48\right)\sqrt{-a^2x^2+1}}{8\,(a^3x^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^3 + 24*a*x*log(-(a*x + 1)/(a*x - 1)) -
6*log(-(a*x + 1)/(a*x - 1))^2 - 48)*sqrt(-a^2*x^2 + 1)/(a^3*x^2 - a)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(3/2), x)
```

3.408 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3-3\tanh^{-}$$

```
[Out] -2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-3*arctanh(a*x)^2*poly
log(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arct
anh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*x/(-a^2*x^2+1)^(1/
2)+6*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*a*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2
)+arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6177, 6167, 4267, 2611, 6744, 2320, 6724, 6141, 6109, 197}

$$-\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-6\text{Li}_4\left(-e^{\tanh^{-1}(ax)}\right)+6\text{Li}_4\left(e^{\tanh^{-1}(ax)}\right)-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a*x)/Sqrt[1 - a^2*x^2] + (6*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*x*Ar
cTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2] - 2*ArcT
anh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^ArcTanh
[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyL
og[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 6*Poly
Log[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx+\text{Subst}\left(\int x^3\text{csch}(x)\,dx,x,\tanh^{-1}(ax)\right)\\
&= \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax\\
&= -\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}}\right.\\
&= -\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}}\right.\\
&= -\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}}\right.\\
&= -\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}\left(e^{\tanh^{-1}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 230, normalized size = 1.24

$$\frac{1}{8}\left(\pi^4-\frac{48ax}{\sqrt{1-a^2x^2}}+\frac{48\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{24ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{8\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)^4-8\tanh^{-1}(ax)^3\log\left(1+e^{-\tanh^{-1}(ax)}\right)+8\tanh^{-1}(ax)^3\log\left(1-e^{\tanh^{-1}(ax)}\right)+24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+48\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-48\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{\tanh^{-1}(ax)}\right)+48\text{PolyLog}\left(4,-e^{-\tanh^{-1}(ax)}\right)+48\text{PolyLog}\left(4,e^{\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (Pi^4 - (48*a*x)/Sqrt[1 - a^2*x^2] + (48*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] -
(24*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x]^3)/Sqrt[1 - a^2
```

```
*x^2] - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*
ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^(-
ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[
a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[
a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*x]])/
8
```

Maple [A]
time = 0.73, size = 305, normalized size = 1.65

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^{3}-3\operatorname{arctanh}(ax)^{2}+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^{3}+3\operatorname{arctanh}(ax)^{2}+6\operatorname{arctan}\right.}{2ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))^
(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-(a*x
-1)*(a*x+1))^(1/2)/(a*x+1)+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+
3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polyl
og(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-ar
ctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(2,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(
1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}{\left(ax \right)}}{x \left(- \left(ax - 1\right) \left(ax + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^3}{x\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(3/2)), x)
```

3.409 $\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-$$

```
[Out] -6*a*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-6*a*arctanh(a*x)*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(
a*x+1)/(-a^2*x^2+1)^(1/2))-6*a/(-a^2*x^2+1)^(1/2)+6*a^2*x*arctanh(a*x)/(-a^
2*x^2+1)^(1/2)-3*a*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x)^3/(
-a^2*x^2+1)^(1/2)-arctanh(a*x)^3*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.29, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6177, 6155, 6167, 4267, 2611, 2320, 6724, 6109, 6105}

$$-\frac{6a}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x}+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+6a\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-6a\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - 6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x
]^2 + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x
]*PolyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLo
g[3, E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
```

```
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{x}+(3a)\int \frac{\tanh^-}{x\sqrt{1-} \\
&= -\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a} \\
&= -\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan \\
&= -\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan \\
&= -\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan \\
&= -\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tan
\end{aligned}$$

Mathematica [A]
time = 1.79, size = 270, normalized size = 1.44

$$-\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{a^2x\tanh^{-1}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{4\sqrt{1-a^2x^2}}+3a\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right)-3a\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-6a\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+6a\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-6a\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3\sinh^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2
] - (a^2*x*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2)/(4*Sqrt[1 - a^2*x^2]) + 3
*a*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 3*a*ArcTanh[a*x]^2*Log[1 + E
^(-ArcTanh[a*x])] + 6*a*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*a*A
rcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 6*a*PolyLog[3, -E^(-ArcTanh[a*x
```

```
])] - 6*a*PolyLog[3, E^(-ArcTanh[a*x])] + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3
*Sinh[ArcTanh[a*x]/2]^2)/x
```

Maple [A]
time = 0.72, size = 282, normalized size = 1.51

method	result
default	$-\frac{a\left(\operatorname{arctanh}(ax)^3-3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}-\frac{\left(\operatorname{arctanh}(ax)^3+3\operatorname{arctanh}(ax)^2+6\operatorname{arctan}\right.}{2(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1)
)^(1/2)/(a*x-1)-1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*a*(-
(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^3/x+3*
a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(
2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}(ax)}{x^{2}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

3.410 $\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=360

$$-\frac{6a^3x}{\sqrt{1-a^2x^2}}+\frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}+\frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2}}{}$$

```
[Out] -3*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-6*a^2*arctanh(a*x
)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-9/2*a^2*arctanh(a*x)^2*polylog(2,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))+3*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3*a^2*polylog(2
,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+9*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))-9*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^
2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,(a*x+1)/(-a^2*x^2+
1)^(1/2))-6*a^3*x/(-a^2*x^2+1)^(1/2)+6*a^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-
3*a^3*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)+a^2*arctanh(a*x)^3/(-a^2*x^2+1)^(
1/2)-3/2*a*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^3*(-a^2*x^2
+1)^(1/2)/x^2
```

Rubi [A]
time = 0.67, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6177, 6173, 6155, 6165, 6167, 4267, 2611, 6744, 2320, 6724, 6141, 6109, 197}

$$3a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+9a^2\tanh^{-1}(ax)\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-9a^2\tanh^{-1}(ax)\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-9a^2\text{Li}_4\left(-e^{\tanh^{-1}(ax)}\right)+9a^2\text{Li}_4\left(e^{\tanh^{-1}(ax)}\right)+\frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}+\frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2}-3a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3-6a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{6a^3x}{\sqrt{1-a^2x^2}}-\frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a^3*x)/Sqrt[1 - a^2*x^2] + (6*a^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*
a^3*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^3)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (9*a^2*ArcTanh[a*x]^
2*PolyLog[2, -E^ArcTanh[a*x]])/2 + (9*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTa
nh[a*x]])/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*Poly
Log[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 9*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcT
anh[a*x]] - 9*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 9*a^2*PolyLog[4
, -E^ArcTanh[a*x]] + 9*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6177

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^2$$

$$= \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}(a}{\sqrt{1-a^2x^2}}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x}$$

Mathematica [A]
time = 5.99, size = 377, normalized size = 1.05

$$\frac{1}{16}a^2\left(3\pi^4 - \frac{96ax}{\sqrt{1-a^2x^2}} + \frac{96\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{48ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{16\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 6\tanh^{-1}(ax)^4 - \frac{6ax\tanh^{-1}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right) + 48\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right) - 48\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right) - 24\tanh^{-1}(ax)^3\log\left(1+e^{-\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^3\log\left(1-e^{\tanh^{-1}(ax)}\right) + 24\left(2+3\tanh^{-1}(ax)^2\right)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 48\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 72\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right) + 144\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 144\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{\tanh^{-1}(ax)}\right) + 144\text{PolyLog}\left(4,-e^{-\tanh^{-1}(ax)}\right) + 144\text{PolyLog}\left(4,e^{\tanh^{-1}(ax)}\right) - 2\tanh^{-1}(ax)^3\operatorname{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right) + 12\tanh^{-1}(ax)^2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*(3*Pi^4 - (96*a*x)/Sqrt[1 - a^2*x^2] + (96*ArcTanh[a*x])/Sqrt[1 - a^2*
x^2] - (48*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (16*ArcTanh[a*x]^3)/Sqrt
[1 - a^2*x^2] - 6*ArcTanh[a*x]^4 - (6*a*x*ArcTanh[a*x]^2*Csch[ArcTanh[a*x]/
2]^2)/Sqrt[1 - a^2*x^2] - 2*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2 + 48*ArcT
anh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] - 24*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]^3*Lo
```

```
g[1 - E^ArcTanh[a*x]] + 24*(2 + 3*ArcTanh[a*x]^2)*PolyLog[2, -E^(-ArcTanh[a
*x])] - 48*PolyLog[2, E^(-ArcTanh[a*x])] + 72*ArcTanh[a*x]^2*PolyLog[2, E^A
rcTanh[a*x]] + 144*ArcTanh[a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 144*ArcTan
h[a*x]*PolyLog[3, E^ArcTanh[a*x]] + 144*PolyLog[4, -E^(-ArcTanh[a*x])] + 14
4*PolyLog[4, E^ArcTanh[a*x]] - 2*ArcTanh[a*x]^3*Sech[ArcTanh[a*x]/2]^2 + 12
*ArcTanh[a*x]^2*Tanh[ArcTanh[a*x]/2]))/16
```

Maple [A]
time = 0.79, size = 482, normalized size = 1.34

method	result
default	$-\frac{a^{2}\left(\operatorname{arctanh}(ax)^{3}-3\operatorname{arctanh}(ax)^{2}+6\operatorname{arctanh}(ax)-6\right)\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^{3}+3\operatorname{arctanh}(ax)^{2}+6\operatorname{arctan}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+
1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*a^
2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x
)^2*(3*a*x+arctanh(a*x))/x^2+3/2*a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+
1)^(1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^
2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,(a*x+1
)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2
))-9/2*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*arct
anh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*polylog(4,-(a*x+1)/(-
a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2
*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))-3*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(3/2),x)
[Out] Integral(atanh(a*x)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\operatorname{atanh}(a\,x)}^3}{x^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

3.411 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Maple [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\,\mathrm{arctanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(x**m/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^m/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

3.412 $\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Defer[Int][x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 2.51, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Maple [A]
time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^2/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

3.413 $\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

```
[Out] Shi(arctanh(a*x))/a^2
```

Rubi [A]
time = 0.06, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6181, 3379}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}\\ &= \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 9, normalized size = 1.00

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(9) = 18$.
time = 0.65, size = 26, normalized size = 2.89

method	result	size
default	$-\frac{\text{expIntegral}(1,-\operatorname{arctanh}(ax))}{2a^2}+\frac{\text{expIntegral}(1,\operatorname{arctanh}(ax))}{2a^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] -1/2*Ei(1,-arctanh(a*x))/a^2+1/2*Ei(1,arctanh(a*x))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{x}{\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

3.414 $\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Chi}(\tanh^{-1}(ax))}{a}$$

[Out] Chi(arctanh(a*x))/a

Rubi [A]
time = 0.04, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6115, 3382}

$$\frac{\text{Chi}(\tanh^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] CoshIntegral[ArcTanh[a*x]]/a

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Chi}(\tanh^{-1}(ax))}{a}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 9, normalized size = 1.00

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Maple [A]
time = 0.63, size = 10, normalized size = 1.11

method	result	size
default	$\frac{\text{hyperbolicCosineIntegral}(\text{arctanh}(ax))}{a}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] Chi(arctanh(a*x))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

3.415 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Maple [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(x*atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

3.416 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(x**m/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^m/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

3.417 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{1}{a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\operatorname{Shi}\left(\tanh^{-1}(ax)\right)}{a^3}-\frac{\operatorname{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2},x\right)}{a^2}$$

[Out] Shi(arctanh(a*x))/a^3-1/a^3/arctanh(a*x)/(-a^2*x^2+1)^(1/2)-Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)/a^2

Rubi [A]

time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]

[Out] -(1/(a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a^3 - Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/a^2

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\,\tanh^{-1}(ax)}\,dx}{a}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\operatorname{Shi}\left(\tanh^{-1}(ax)\right)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}\,dx}{a^2}
\end{aligned}$$

Mathematica [A]

time = 2.18, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \operatorname{arctanh}\left(a x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-\left(a x-1\right)\left(a x+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}^{2}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^2/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

3.418 $\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=36

$$-\frac{x}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

```
[Out] Chi(arctanh(a*x))/a^2-x/a/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6153, 6115, 3382}

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx}{a}$$
$$= -\frac{x}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= -\frac{x}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.94

$$\frac{-\frac{ax}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \text{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]])/a^
2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(34) = 68.
time = 1.14, size = 90, normalized size = 2.50

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}}{2a^2(ax-1)\operatorname{arctanh}(ax)} - \frac{\text{expIntegral}(1,-\operatorname{arctanh}(ax))}{2a^2} + \frac{\sqrt{-(ax-1)(ax+1)}}{2\operatorname{arctanh}(ax)(ax+1)a^2} - \frac{\text{expIntegral}(1,\operatorname{arctanh}(ax))}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)-1/2*Ei(1,-arctanh(a*x
))/a^2+1/2*(-(a*x-1)*(a*x+1))^(1/2)/arctanh(a*x)/(a*x+1)/a^2-1/2*Ei(1,arcta
nh(a*x))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{{\mathrm{atanh}\left(a\,x\right)}^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

3.419 $\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] Shi(arctanh(a*x))/a-1/a/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6113, 6181, 3379}

$$\frac{\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + a\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 0.91

$$\frac{-\frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] (-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

Maple [A]
time = 1.25, size = 62, normalized size = 1.77

method	result	si
default	$\frac{\operatorname{arctanh}(ax)\,\text{hyperbolicSineIntegral}(\operatorname{arctanh}(ax))a^2x^2 - \text{hyperbolicSineIntegral}(\operatorname{arctanh}(ax))\operatorname{arctanh}(ax) + \sqrt{-a^2x^2+1}}{a\operatorname{arctanh}(ax)(a^2x^2-1)}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a*(arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2-Shi(arctanh(a*x))*arctanh(a*x)+
(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

3.420 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}+\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{\text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)},x\right)}{a}$$

```
[Out] Chi(arctanh(a*x))-a*x/arctanh(a*x)/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/
x/arctanh(a*x)-Unintegrable(1/x^2/arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)/a
```

Rubi [A]
time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) - Sqrt[1 - a^2*x^2]/(a*x*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]] - Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]), x]/a
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= a^2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx\\ &= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax}}{a}\\ &= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax}}{a}\\ &= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}+\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{\int -}{x}\end{aligned}$$

Mathematica [A]
time = 4.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \operatorname{arctanh}\left(a x\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(a x-1\right)\left(a x+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}^{2}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

3.421 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3),
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atanh}\left(a\,x\right)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(x^m/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

3.422 $\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a^3}-\frac{\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\right.}{a^2}$$

```
[Out] 1/2*Chi(arctanh(a*x))/a^3-1/2/a^3/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*x/a
^2/arctanh(a*x)/(-a^2*x^2+1)^(1/2)-Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctan
h(a*x)^3,x)/a^2
```

Rubi [A]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*a^2*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a^3) - Defer[Int][1/(Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x]^3), x]/a^2
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx = \frac{\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx}{a^2}$$

$$= -\frac{1}{2a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3}\,dx}{a^2}+\frac{\int \frac{}{(1-a^2x^2}}{}$$

$$= -\frac{1}{2a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\int \frac{}{(1-a^2x^2}}{}$$

$$= -\frac{1}{2a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{1}{2a^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh\right.}{2a}$$

Mathematica [A]
time = 4.65, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 2.64, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x^2/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

3.423 $\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=68

$$-\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}$$

```
[Out] 1/2*Shi(arctanh(a*x))/a^2-1/2*x/a/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2/a^2
/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6153, 6113, 6181, 3379}

$$\frac{\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - 1/(2*a^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]) + SinhIntegral[ArcTanh[a*x]]/(2*a^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
```

```
[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} + \frac{\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx}{2a}$$

$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{1}{2}\int \frac{}{(1-}$$

$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Shi}\left(\tanh\right.}{2a}$$

Mathematica [A]
time = 0.09, size = 43, normalized size = 0.63

$$\frac{-\frac{ax+\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} + \text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((a*x + ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + SinhIntegral[
ArcTanh[a*x]])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(58) = 116$.
time = 1.14, size = 154, normalized size = 2.26

method	result

default	$\frac{\sqrt{-(ax-1)(ax+1)}}{4a^2(ax-1)\operatorname{arctanh}(ax)^2}+\frac{\sqrt{-(ax-1)(ax+1)}}{4a^2(ax-1)\operatorname{arctanh}(ax)}-\frac{\operatorname{expIntegral}(1,-\operatorname{arctanh}(ax))}{4a^2}+\frac{\sqrt{-(ax-1)(ax+}}{4\operatorname{arctanh}(ax)^2(ax+1)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)^2+1/4*(-(a*x-1)*(a*x+
1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)-1/4*Ei(1,-arctanh(a*x))/a^2+1/4*(-(a*x-1
)*(a*x+1))^(1/2)/arctanh(a*x)^2/(a*x+1)/a^2-1/4*(-(a*x-1)*(a*x+1))^(1/2)/ar
ctanh(a*x)/(a*x+1)/a^2+1/4*Ei(1,arctanh(a*x))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atanh}(a\,x)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(x/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

3.424 $\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] 1/2*Chi(arctanh(a*x))/a-1/2/a/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*x/arcta
nh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6113, 6153, 6115, 3382}

$$-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTan
h[a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2} + \frac{1}{2}a\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{1}{2}\int \frac{}{(1-a^2}$$

$$= -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos}{}\right.}{}$$

$$= -\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)} + \frac{\text{Chi}\left(\tanh^{-1}\right.}{2a}$$

Mathematica [A]
time = 0.10, size = 44, normalized size = 0.68

$$\frac{-\frac{1+ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2} + \text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegra
l[ArcTanh[a*x]])/(2*a)
```

Maple [A]
time = 1.23, size = 86, normalized size = 1.32

method	result
default	$\frac{\operatorname{arctanh}(ax)^2 \operatorname{hyperbolicCosineIntegral}(\operatorname{arctanh}(ax))a^2x^2+\sqrt{-a^2x^2+1}\,ax\operatorname{arctanh}(ax)-\operatorname{hyperbolicCosineIntegral}(\operatorname{arctanh}(}{2a\operatorname{arctanh}(ax)^2(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+(-a^2*x^2+1)^(1/2)*a*x*arct
anh(a*x)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x)^
2/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

3.425 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{1}{2}\text{Shi}\left(\tanh^{-1}(ax)\right)-\frac{\text{Int}\left(\frac{}{x^2\sqrt{1-}}\right.}{}$$

```
[Out] 1/2*Shi(arctanh(a*x))-1/2*a*x/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2/(-a^2*x
^2+1)^(1/2)/arctanh(a*x)-1/2*(-a^2*x^2+1)^(1/2)/a/x/arctanh(a*x)^2-1/2*Unin
tegrable(1/x^2/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)/a
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*(a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - Sqrt[1 - a^2*x^2]/(2*a*x*Ar
cTanh[a*x]^2) - 1/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]) + SinhIntegral[ArcTanh
[a*x]]/2 - Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx = a^2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-}}}{2a}$$

$$= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(}$$

$$= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(}$$

$$= -\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(}$$

Mathematica [A]
time = 11.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

3.426 $\int x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=243

$$\frac{\sqrt{1-a^2x^2}}{16a^5}-\frac{7(1-a^2x^2)^{3/2}}{72a^5}+\frac{(1-a^2x^2)^{5/2}}{30a^5}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24a^2}+\frac{1}{6}x^5\sqrt{}$$

```
[Out] -7/72*(-a^2*x^2+1)^(3/2)/a^5+1/30*(-a^2*x^2+1)^(5/2)/a^5-1/8*arctan((-a*x+1
)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-1/16*I*polylog(2,-I*(-a*x+1)^(1/2)/
(a*x+1)^(1/2))/a^5+1/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+1/1
6*(-a^2*x^2+1)^(1/2)/a^5-1/16*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/24*x^
3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+1/6*x^5*arctanh(a*x)*(-a^2*x^2+1)^(1/
2)
```

Rubi [A]
time = 0.21, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6157, 6163, 272, 45, 267, 6097}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{8a^5}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24a^2}+\frac{(1-a^2x^2)^{5/2}}{30a^5}-\frac{7(1-a^2x^2)^{3/2}}{72a^5}+\frac{\sqrt{1-a^2x^2}}{16a^5}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^5) - (7*(1 - a^2*x^2)^(3/2))/(72*a^5) + (1 - a^2*x^
2)^(5/2)/(30*a^5) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^4) - (x^3*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/6 - (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^5) - ((I/16)*P
olyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + ((I/16)*PolyLog[2, (I*
Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{1}{6}\int \frac{x^4\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx - \frac{1}{6}a\int \frac{x^5}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24a^2} + \frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{\int \frac{x^2\tanh^{-1}(}{\sqrt{1-a^2}}}{8a^2}$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^4} - \frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24a^2} + \frac{1}{6}x^5\sqrt{1-a^2x}$$

$$= \frac{5\sqrt{1-a^2x^2}}{48a^5} - \frac{(1-a^2x^2)^{3/2}}{9a^5} + \frac{(1-a^2x^2)^{5/2}}{30a^5} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax}{16a^4}$$

$$= \frac{\sqrt{1-a^2x^2}}{16a^5} - \frac{7(1-a^2x^2)^{3/2}}{72a^5} + \frac{(1-a^2x^2)^{5/2}}{30a^5} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax}{16a^4}$$

Mathematica [A]
time = 0.48, size = 178, normalized size = 0.73

$$\frac{\sqrt{1-a^2x^2}\left(45+70(-1+a^2x^2)+24(-1+a^2x^2)^2+45ax\tanh^{-1}(ax)+210ax(-1+a^2x^2)\tanh^{-1}(ax)+120ax(-1+a^2x^2)^2\tanh^{-1}(ax)-\frac{45i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{720a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(45 + 70*(-1 + a^2*x^2) + 24*(-1 + a^2*x^2)^2 + 45*a*x*A
rcTanh[a*x] + 210*a*x*(-1 + a^2*x^2)*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^
2*ArcTanh[a*x] - ((45*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
 I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcT
anh[a*x]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

Maple [A]
time = 1.62, size = 195, normalized size = 0.80

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(120\,\mathrm{arctanh}(ax)a^5x^5+24a^4x^4-30a^3x^3\,\mathrm{arctanh}(ax)+22a^2x^2-45ax\,\mathrm{arctanh}(ax)-1)}{720a^5} - \frac{i\ln\left(1+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/720/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*a^5*x^5+24*a^4*x^4-30*
a^3*x^3*arctanh(a*x)+22*a^2*x^2-45*a*x*arctanh(a*x)-1)-1/16*I*ln(1+I*(a*x+1
```

```
)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(
1/2))*arctanh(a*x)/a^5-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/1
6*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{-(ax-1)(ax+1)}\ \operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
[Out] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

3.427 $\int x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=136

$$\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{x^3\sqrt{1-a^2x^2}}{20a}+\frac{11\text{ArcSin}(ax)}{120a^4}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{}$$

```
[Out] 11/120*arcsin(a*x)/a^4+1/24*x*(-a^2*x^2+1)^(1/2)/a^3+1/20*x^3*(-a^2*x^2+1)^
(1/2)/a-2/15*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/15*x^2*arctanh(a*x)*(-a^
2*x^2+1)^(1/2)/a^2+1/5*x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6157, 6163, 327, 222, 6141}

$$\frac{11\text{ArcSin}(ax)}{120a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{x^3\sqrt{1-a^2x^2}}{20a}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{15a^4}+\frac{x\sqrt{1-a^2x^2}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(24*a^3) + (x^3*Sqrt[1 - a^2*x^2])/(20*a) + (11*ArcSi
n[a*x])/(120*a^4) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^2*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/5
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{1}{5}\int \frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx - \frac{1}{5}a\int \frac{x^4}{\sqrt{1-a^2x^2}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{20a} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^2} + \frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3} + \frac{x^3\sqrt{1-a^2x^2}}{20a} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4} - \frac{x^2\sqrt{1-a^2}}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3} + \frac{x^3\sqrt{1-a^2x^2}}{20a} + \frac{11\sin^{-1}(ax)}{120a^4} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax}{15a^4}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.58

$$\frac{ax\sqrt{1-a^2x^2}\,(5+6a^2x^2)+11\text{ArcSin}(ax)+8\sqrt{1-a^2x^2}\,(-2-a^2x^2+3a^4x^4)\tanh^{-1}(ax)}{120a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(5 + 6*a^2*x^2) + 11*ArcSin[a*x] + 8*Sqrt[1 - a^2*x^
2]*(-2 - a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/(120*a^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.37, size = 120, normalized size = 0.88

method	result
default	$\frac{\sqrt{-\left(a x-1\right) \left(a x+1\right)}\, \left(24 a^{4} x^{4} \operatorname{arctanh}\left(a x\right)+6 a^{3} x^{3}-8 a^{2} x^{2} \operatorname{arctanh}\left(a x\right)+5 a x-16 \operatorname{arctanh}\left(a x\right)\right)}{120 a^{4}}+\frac{11 i \ln\left(\frac{a x+1}{\sqrt{-a^{2} x^{2}+1}}\right.}{120 a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*a^3*x^3-8*a^2
*x^2*arctanh(a*x)+5*a*x-16*arctanh(a*x))+11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)+I)/a^4-11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4
```

Maxima [A]
time = 0.46, size = 128, normalized size = 0.94

$$-\frac{1}{120}\,a\left(\frac{3\left(\frac{2\,(-a^2x^2+1)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{\arcsin(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{-a^2x^2+1}\,x+\frac{\arcsin(ax)}{a}\right)}{a^4}\right)-\frac{1}{15}\left(\frac{3\,(-a^2x^2+1)^{\frac{3}{2}}x^2}{a^2}+\frac{2\,(-a^2x^2+1)^{\frac{3}{2}}}{a^4}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*a*(3*(2*(-a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(-a^2*x^2 + 1)*x/a^2 - arcs
in(a*x)/a^3)/a^2 - 8*(sqrt(-a^2*x^2 + 1)*x + arcsin(a*x)/a)/a^4) - 1/15*(3*
(-a^2*x^2 + 1)^(3/2)*x^2/a^2 + 2*(-a^2*x^2 + 1)^(3/2)/a^4)*arctanh(a*x)
```

Fricas [A]
time = 0.41, size = 91, normalized size = 0.67

$$\frac{\left(6\,a^3x^3+5\,ax+4\left(3\,a^4x^4-a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}-22\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*((6*a^3*x^3 + 5*a*x + 4*(3*a^4*x^4 - a^2*x^2 - 2)*log(-(a*x + 1)/(a*x
 - 1)))*sqrt(-a^2*x^2 + 1) - 22*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

3.428 $\int x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{(1-a^2x^2)^{3/2}}{12a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\,\mathrm{t}}{4a^3}$$

```
[Out] -1/12*(-a^2*x^2+1)^(3/2)/a^3-1/4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arcta
nh(a*x)/a^3-1/8*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/8*I*poly
log(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/8*(-a^2*x^2+1)^(1/2)/a^3-1/8*x*
arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+1/4*x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6163, 267, 6097, 272, 45}

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a^3}-\frac{i\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}+\frac{i\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{(1-a^2x^2)^{3/2}}{12a^3}+\frac{\sqrt{1-a^2x^2}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3) - (1 - a^2*x^2)^(3/2)/(12*a^3) - (x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/4 - (ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^3) - ((I/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/8)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}\int\frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-\frac{1}{4}a\int\frac{x^3}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x}}}{8a^2}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{(1-a^2x^2)^{3/2}}{12a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2}$$

Mathematica [A]
time = 0.32, size = 160, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(1+2a^2x^2+3ax\tanh^{-1}(ax)+6ax(-1+a^2x^2)\tanh^{-1}(ax)-\frac{3i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}-\frac{3i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2 + 3*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^
2)*ArcTanh[a*x] - ((3*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((3*I)*(PolyLog[2, (-I)/E^ArcTanh[a
*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^3)
```

Maple [A]
time = 1.38, size = 175, normalized size = 0.90

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(6a^3x^3\,\text{arctanh}(ax)+2a^2x^2-3ax\,\text{arctanh}(ax)+1)}{24a^3}-\frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\text{arctanh}(ax)}{8a^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-3*a*x*a
rctanh(a*x)+1)-1/8*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/
8*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3-1/8*I*dilog(1+I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^
3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

3.429 $\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1-a^2x^2}}{6a}+\frac{\text{ArcSin}(ax)}{6a^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3a^2}$$

```
[Out] 1/6*arcsin(a*x)/a^2-1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/a^2+1/6*x*(-a^2*x^2
+1)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {6141, 201, 222}

$$\frac{\text{ArcSin}(ax)}{6a^2}+\frac{x\sqrt{1-a^2x^2}}{6a}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(6*a) + ArcSin[a*x]/(6*a^2) - ((1 - a^2*x^2)^(3/2)*Ar
cTanh[a*x])/(3*a^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = -\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3a^2} + \frac{\int \sqrt{1-a^2x^2}\,dx}{3a}$$
$$= \frac{x\sqrt{1-a^2x^2}}{6a} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{6a}$$
$$= \frac{x\sqrt{1-a^2x^2}}{6a} + \frac{\sin^{-1}(ax)}{6a^2} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3a^2}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.83

$$\frac{ax\sqrt{1-a^2x^2} + \mathrm{ArcSin}(ax) - 2(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x] - 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/
(6*a^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.36, size = 99, normalized size = 1.68

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\,\mathrm{arctanh}(ax)+ax-2\,\mathrm{arctanh}(ax))}{6a^2} + \frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{6a^2} - \frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{6a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-2*arctanh(a*x)
)+1/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^2-1/6*I*ln((a*x+1)/(-a^2*x^2+1)^
(1/2)-I)/a^2
```

Maxima [A]
time = 0.46, size = 50, normalized size = 0.85

$$-\frac{(-a^2x^2+1)^{\frac{3}{2}}\,\mathrm{artanh}\,(ax)}{3\,a^2} + \frac{\sqrt{-a^2x^2+1}\,x + \frac{\arcsin(ax)}{a}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

[Out] -1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/a^2 + 1/6*(sqrt(-a^2*x^2 + 1)*x + arcsin(a*x)/a)/a

Fricas [A]
time = 0.39, size = 72, normalized size = 1.22

$$\frac{\sqrt{-a^2x^2+1}\,\left(ax+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)-2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))) - 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

3.430 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=143

$$\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+$$

```
[Out] -arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-1/2*I*polylog(2,-I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)/a+1/2*(-a^2*x^2+1)^(1/2)/a+1/2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6089, 6097}

$$\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx = \frac{\sqrt{1-a^2x^2}}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) + \frac{1}{2}\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) - \frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}$$

Mathematica [A]
time = 0.18, size = 117, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(1+ax\tanh^{-1}(ax)-\frac{i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^Ar
cTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]]
- PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [A]
time = 4.12, size = 152, normalized size = 1.06

method	result
default	$\frac{\sqrt{-a^2x^2+1}\,(ax\,\mathrm{arctanh}(ax)+1)}{2a} - \frac{i\,\mathrm{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a} + \frac{i\,\mathrm{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(a*x*arctanh(a*x)+1)/a-1/2*I/a*arctanh(a*x)*ln(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

3.431 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=100

$$-\text{ArcSin}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{Pol}$$

```
[Out] -arcsin(a*x)-2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2
,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+(-a
^2*x^2+1)^(1/2)*arctanh(a*x)
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6157, 6165, 222}

$$\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\text{ArcSin}(ax)+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

```
[Out] -ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 2*ArcTanh[a*x]*ArcTanh[Sqrt
[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - Pol
yLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
```

```
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}\,dx = \sqrt{1-a^2x^2}\,\tanh^{-1}(ax) - a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx + \int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\sin^{-1}(ax) + \sqrt{1-a^2x^2}\,\tanh^{-1}(ax) - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.$$

Mathematica [A]
time = 0.09, size = 91, normalized size = 0.91

$$-2\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

```
[Out] -2*ArcTan[Tanh[ArcTanh[a*x]/2]] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + ArcTanh[
a*x]*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] +
 PolyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A]
time = 1.51, size = 113, normalized size = 1.13

method	result
default	$\sqrt{-(ax-1)(ax+1)}\ \operatorname{arctanh}(ax)-2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{dilog}\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)-2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-
dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^
(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x, x)
```

3.432 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+2a\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ia\text{PolyLog}\Bigg(2,-$$

```
[Out] 2*a*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)-a*arctanh((-a^2*x^2+1
)^(1/2))+I*a*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-I*a*polylog(2,I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))-arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6161, 6155, 272, 65, 214, 6097}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)+ia\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-ia\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + 2*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a
*x]]*ArcTanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + I*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - I*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]
]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + 2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax) + ia\mathrm{Li}_2\Bigg(-$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + 2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax) + ia\mathrm{Li}_2\Bigg(-$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + 2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax) + ia\mathrm{Li}_2\Bigg(-$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} + 2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax) - a\tanh^{-1}$$

Mathematica [A]
time = 0.26, size = 121, normalized size = 0.93

$$a\left(-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{ax}+i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I/E^A
rcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + Log[Tanh[ArcTanh[
a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x
]])
```

Maple [A]
time = 1.39, size = 188, normalized size = 1.45

method	result
default	$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{arctanh}(ax)}{x}+ia\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{arctanh}\left(ax\right)-ia\operatorname{dilog}\left(1-\frac{}{\sqrt{}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)/x+I*a*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1
/2))*arctanh(a*x)-I*a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+I*a*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))-I*a*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a
*x)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**2,x)

[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**2, x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^2,x)

[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^2, x)

3.433 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=136

$$-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

```
[Out] a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a^2*polylog(2,-(
-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)-1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6157, 6173, 270, 6165}

$$-\frac{1}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 + (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
```

```
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx-\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx-\frac{1}{2}a^2\\ &= -\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-a}}{\sqrt{1+a}}\right.\end{aligned}$$

Mathematica [A]
time = 0.87, size = 126, normalized size = 0.93

$$\frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)-4\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)+4\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)-4\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

```
[Out] (a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 4*Arc
Tanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] - 4*PolyLog[2, -E^(-ArcTanh[a*x])] + 4*PolyLog[2, E^(-ArcTanh[a*x])]
- ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A]
time = 1.48, size = 141, normalized size = 1.04

method	result

default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(ax+\operatorname{arctanh}(ax))}{2x^2}-\frac{a^2\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{a^2\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2-1/2*a^2*arctanh(a*x)*l
n(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))+1/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^3, x)
```

3.434 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3+1/6*a^3*arctanh((-a^2*x^2+1)^(1/2)
)-1/6*a*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6155, 272, 43, 65, 214}

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*(a*Sqrt[1 - a^2*x^2])/x^2 - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3)
 + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4}\,dx &= -\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= -\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}-\frac{1}{12}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,d\right.\\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{\right.}\\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 1.13

$$-\frac{ax\sqrt{1-a^2x^2}+2(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+a^3x^3\log(x)-a^3x^3\log\left(1+\sqrt{1-a^2x^2}\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*(a*x*Sqrt[1 - a^2*x^2] + 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x] + a^3*x^3*
Log[x] - a^3*x^3*Log[1 + Sqrt[1 - a^2*x^2]])/x^3
```

Maple [A]
time = 1.79, size = 96, normalized size = 1.37

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\operatorname{arctanh}(ax)-ax-2\operatorname{arctanh}(ax))}{6x^3}-\frac{a^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)}{6}+\frac{a^3\ln\left(1+\frac{ax}{\sqrt{-a^2}}\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x^
3-1/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+1/6*a^3*ln(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))
```

Maxima [A]
time = 0.47, size = 90, normalized size = 1.29

$$\frac{1}{6}\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}\,a^2-\frac{(-a^2x^2+1)^{\frac{3}{2}}}{x^2}\right)a-\frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a
^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a - 1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^
3
```

Fricas [A]
time = 0.36, size = 75, normalized size = 1.07

$$-\frac{a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,\left(ax-(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(a*x - (
a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\operatorname{atanh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^4,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^4, x)
```

3.435 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=191

$$-\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{a^3\sqrt{1-a^2x^2}}{24x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}+\frac{1}{4}a^4\tanh^{-1}(ax)\tanh$$

```
[Out] 1/4*a^4*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/8*a^4*polylog(
2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/8*a^4*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(
1/2))-1/12*a*(-a^2*x^2+1)^(1/2)/x^3-1/24*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*arcta
nh(a*x)*(-a^2*x^2+1)^(1/2)/x^4+1/8*a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.21, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6157, 6173, 277, 270, 6165}

$$-\frac{1}{8}a^4\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^4\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{4}a^4\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}-\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{a^3\sqrt{1-a^2x^2}}{24x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

```
[Out] -1/12*(a*Sqrt[1 - a^2*x^2])/x^3 - (a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*
x^2) + (a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 - (a^4*Pol
yLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 + (a^4*PolyLog[2, Sqrt[1 - a*x]/
Sqrt[1 + a*x]])/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
```

```
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^5}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^4}-\frac{1}{3}\int \frac{\tanh^{-1}(ax)}{x^5\sqrt{1-a^2x^2}}\,dx+\frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,d\\
&= -\frac{a\sqrt{1-a^2x^2}}{9x^3}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}-\frac{1}{12}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx-\frac{1}{4}a^2\\
&= -\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{2a^3\sqrt{1-a^2x^2}}{9x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x^2}}{8}\\
&= -\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{a^3\sqrt{1-a^2x^2}}{24x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x^2}}{8x}
\end{aligned}$$

Mathematica [A]
time = 1.15, size = 222, normalized size = 1.16

$$\frac{1}{192}a^4\left(-8\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-6\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{ax\text{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}-3\tanh^{-1}(ax)\text{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)-24\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)+24\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)-24\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+24\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+3\tanh^{-1}(ax)\text{sech}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{16\left(1-a^2x^2\right)^{3/2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{a^3x^3}+8\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

```
[Out] (a^4*(-8*Coth[ArcTanh[a*x]/2] - 6*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (a*
x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*ArcTanh[a*x]*Csch[ArcTanh[a
*x]/2]^4 - 24*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]*Log
[1 + E^(-ArcTanh[a*x])] - 24*PolyLog[2, -E^(-ArcTanh[a*x])] + 24*PolyLog[2,
 E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 3*ArcTanh[a*x
]*Sech[ArcTanh[a*x]/2]^4 - (16*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]^4)/
(a^3*x^3) + 8*Tanh[ArcTanh[a*x]/2]))/192
```

Maple [A]
time = 1.81, size = 164, normalized size = 0.86

method	result
default	$\frac{\sqrt{-\left(a x-1\right)\left(a x+1\right)}\,\left(-a^{3} x^{3}+3 a^{2} x^{2} \operatorname{arctanh}\left(a x\right)-2 a x-6 \operatorname{arctanh}\left(a x\right)\right)}{24 x^{4}}-\frac{a^{4} \operatorname{arctanh}\left(a x\right) \ln\left(1-\frac{a x+1}{\sqrt{-a^{2} x^{2}+1}}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-(a*x-1)*(a*x+1))^(1/2)*(-a^3*x^3+3*a^2*x^2*arctanh(a*x)-2*a*x-6*arct
anh(a*x))/x^4-1/8*a^4*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a^4
*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/8*a^4*arctanh(a*x)*ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))+1/8*a^4*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}\left(ax\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^5,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^5, x)
```

3.436 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=150

$$-\frac{a\sqrt{1-a^2x^2}}{20x^4}-\frac{a^3\sqrt{1-a^2x^2}}{24x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3}+\frac{2a^4\sqrt{1-a^2x^2}\tanh}{15x}$$

```
[Out] 11/120*a^5*arctanh((-a^2*x^2+1)^(1/2))-1/20*a*(-a^2*x^2+1)^(1/2)/x^4-1/24*a
^3*(-a^2*x^2+1)^(1/2)/x^2-1/5*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5+1/15*a^2*
arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3+2/15*a^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2
)/x
```

Rubi [A]
time = 0.26, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6157, 6173, 272, 44, 65, 214, 6155}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5}-\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3}+\frac{11}{120}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{2a^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x}-\frac{a^3\sqrt{1-a^2x^2}}{24x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

```
[Out] -1/20*(a*Sqrt[1 - a^2*x^2])/x^4 - (a^3*Sqrt[1 - a^2*x^2])/(24*x^2) - (Sqrt[
1 - a^2*x^2]*ArcTanh[a*x])/(5*x^5) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(
15*x^3) + (2*a^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*x) + (11*a^5*ArcTanh[S
qrt[1 - a^2*x^2]])/120
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^6}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^5} - \frac{1}{4}\int \frac{\tanh^{-1}(ax)}{x^6\sqrt{1-a^2x^2}}\,dx + \frac{1}{4}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} - \frac{1}{20}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx + \frac{1}{8}a\text{Subst}\bigg(\int \frac{}{x^3\sqrt{}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{16x^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} -$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3a^3\sqrt{1-a^2x^2}}{32x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2}}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x}}{15}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x}}{15}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x}}{15}$$

Mathematica [A]
time = 0.09, size = 104, normalized size = 0.69

$$\frac{1}{120}\left(-\frac{a\sqrt{1-a^2x^2}\,(6+5a^2x^2)}{x^4} + \frac{8\sqrt{1-a^2x^2}\,(-3+a^2x^2+2a^4x^4)\tanh^{-1}(ax)}{x^5} - 11a^5\log(x) + 11a^5\log\left(1+\sqrt{1-a^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

```
[Out] (-((a*Sqrt[1 - a^2*x^2]*(6 + 5*a^2*x^2))/x^4) + (8*Sqrt[1 - a^2*x^2]*(-3 +
a^2*x^2 + 2*a^4*x^4)*ArcTanh[a*x])/x^5 - 11*a^5*Log[x] + 11*a^5*Log[1 + Sqr
t[1 - a^2*x^2]])/120
```

Maple [A]
time = 1.84, size = 116, normalized size = 0.77

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(16a^4x^4\operatorname{arctanh}(ax)-5a^3x^3+8a^2x^2\operatorname{arctanh}(ax)-6ax-24\operatorname{arctanh}(ax))}{120x^5} + \frac{11a^5\ln\left(1+\frac{ax+}{\sqrt{-a^2x}}\right.}{120}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-(a*x-1)*(a*x+1))^(1/2)*(16*a^4*x^4*arctanh(a*x)-5*a^3*x^3+8*a^2*x^2
*arctanh(a*x)-6*a*x-24*arctanh(a*x))/x^5+11/120*a^5*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))-11/120*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Maxima [A]
time = 0.47, size = 204, normalized size = 1.36

$$\frac{1}{120}\left(3a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-3\sqrt{-a^2x^2+1}\,a^4+8\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}\,a^2-\frac{(-a^2x^2+1)^{\frac{3}{2}}}{x^2}\right)a^2-\frac{3(-a^2x^2+1)^{\frac{3}{2}}a^2}{x^2}-\frac{6(-a^2x^2+1)^{\frac{3}{2}}}{x^4}\right)a-\frac{1}{15}\left(\frac{2(-a^2x^2+1)^{\frac{3}{2}}a^2}{x^3}+\frac{3(-a^2x^2+1)^{\frac{3}{2}}}{x^5}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/120*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*sqrt(-a^2*x^2
+ 1)*a^4 + 8*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x
^2 + 1)*a^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a^2 - 3*(-a^2*x^2 + 1)^(3/2)*a^2/x^
2 - 6*(-a^2*x^2 + 1)^(3/2)/x^4)*a - 1/15*(2*(-a^2*x^2 + 1)^(3/2)*a^2/x^3 +
3*(-a^2*x^2 + 1)^(3/2)/x^5)*arctanh(a*x)
```

Fricas [A]
time = 0.42, size = 93, normalized size = 0.62

$$-\frac{11\,a^5x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(5\,a^3x^3+6\,ax-4\,(2\,a^4x^4+a^2x^2-3)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(11*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 + 6*a*x - 4
*(2*a^4*x^4 + a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x
^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**6, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^6,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^6, x)
```

3.437 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^7}\,dx$

Optimal. Leaf size=243

$$-\frac{a\sqrt{1-a^2x^2}}{30x^5}-\frac{11a^3\sqrt{1-a^2x^2}}{360x^3}+\frac{a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24x^4}+\frac{a^4\sqrt{}}{}$$

```
[Out] 1/8*a^6*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/16*a^6*polylog
(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/16*a^6*polylog(2,(-a*x+1)^(1/2)/(a*x+1)
^(1/2))-1/30*a*(-a^2*x^2+1)^(1/2)/x^5-11/360*a^3*(-a^2*x^2+1)^(1/2)/x^3+1/7
20*a^5*(-a^2*x^2+1)^(1/2)/x-1/6*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6+1/24*a^
2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4+1/16*a^4*arctanh(a*x)*(-a^2*x^2+1)^(1
/2)/x^2
```

Rubi [A]
time = 0.29, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6157, 6173, 277, 270, 6165}

$$-\frac{1}{16}a^6\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{16}a^6\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^6\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}-\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24x^4}+\frac{a^5\sqrt{1-a^2x^2}}{720x}+\frac{a^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16x^2}-\frac{11a^3\sqrt{1-a^2x^2}}{360x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```

```
[Out] -1/30*(a*Sqrt[1 - a^2*x^2])/x^5 - (11*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (a
^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6) +
(a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) + (a^4*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/(16*x^2) + (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]
)/8 - (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 + (a^6*PolyLog[2,
 Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^7}\,dx &= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5x^6} - \frac{1}{5}\int \frac{\tanh^{-1}(ax)}{x^7\sqrt{1-a^2x^2}}\,dx + \frac{1}{5}a\int \frac{1}{x^6\sqrt{1-a^2x^2}} \\ &= -\frac{a\sqrt{1-a^2x^2}}{25x^5} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6} - \frac{1}{30}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}\,dx - \frac{1}{6} \\ &= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{4a^3\sqrt{1-a^2x^2}}{75x^3} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{1-a^2}}{2} \\ &= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{11a^3\sqrt{1-a^2x^2}}{360x^3} - \frac{8a^5\sqrt{1-a^2x^2}}{75x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-}}{6x^6} \\ &= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{11a^3\sqrt{1-a^2x^2}}{360x^3} + \frac{a^5\sqrt{1-a^2x^2}}{720x} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(}{6x^6}\end{aligned}$$

Mathematica [A]
time = 2.31, size = 307, normalized size = 1.26

$$\frac{a^6\left(-76\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-90\tanh^{-1}(ax)\operatorname{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{26ax\operatorname{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}-90\tanh^{-1}(ax)\operatorname{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{3ax\operatorname{csch}^6\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}-15\tanh^{-1}(ax)\operatorname{csch}^6\left(\frac{1}{2}\tanh^{-1}(ax)\right)-360\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)+360\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)-360\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+360\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-90\tanh^{-1}(ax)\operatorname{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+90\tanh^{-1}(ax)\operatorname{sech}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)-15\tanh^{-1}(ax)\operatorname{sech}^6\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{416\left(1-a^2x^2\right)^{3/2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{a^3x^3}+76\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)+6\operatorname{sech}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)}{5760}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]

[Out] (a^6*(-76*Coth[ArcTanh[a*x]/2] - 90*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (26*a*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 90*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^4 - (3*a*x*Csch[ArcTanh[a*x]/2]^6)/Sqrt[1 - a^2*x^2] - 15*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^6 - 360*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 360*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] - 360*PolyLog[2, -E^(-ArcTanh[a*x])] + 360*PolyLog[2, E^(-ArcTanh[a*x])] - 90*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 90*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - 15*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^6 - (416*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]^4)/(a^3*x^3) + 76*Tanh[ArcTanh[a*x]/2] + 6*Sech[ArcTanh[a*x]/2]^4*Tanh[ArcTanh[a*x]/2]))/5760

Maple [A]
time = 1.92, size = 183, normalized size = 0.75

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(a^5x^5+45a^4x^4\operatorname{arctanh}(ax)-22a^3x^3+30a^2x^2\operatorname{arctanh}(ax)-24ax-120\operatorname{arctanh}(ax))}{720x^6}-\frac{a^6\operatorname{arctanh}(a}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x,method=_RETURNVERBOSE)

[Out] 1/720*(-(a*x-1)*(a*x+1))^(1/2)*(a^5*x^5+45*a^4*x^4*arctanh(a*x)-22*a^3*x^3+30*a^2*x^2*arctanh(a*x)-24*a*x-120*arctanh(a*x))/x^6-1/16*a^6*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**7, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^7,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^7, x)
```

3.438 $\int x^4 \sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=336

$$\frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\text{ArcSin}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{180a^3}+\frac{x^4\sqrt{}}{}$$

```
[Out] -19/360*arcsin(a*x)/a^5+1/8*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)
^2/a^5-1/8*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/8*
I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/8*I*polylog(3,
-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5-1/8*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(
1/2))/a^5+1/18*x*(-a^2*x^2+1)^(1/2)/a^4+1/60*x^3*(-a^2*x^2+1)^(1/2)/a^2-1/3
60*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^5+11/180*x^2*arctanh(a*x)*(-a^2*x^2+1)
^(1/2)/a^3+1/15*x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a-1/16*x*arctanh(a*x)^2
*(-a^2*x^2+1)^(1/2)/a^4-1/24*x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2+1/6*
x^5*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.97, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6161, 6163, 327, 222, 6141, 6099, 4265, 2611, 2320, 6724}

$$-\frac{19\text{ArcSin}(ax)}{360a^5}+\frac{\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{24a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{x\sqrt{1-a^2x^2}}{18a^4}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{16a^4}+\frac{11x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{180a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(18*a^4) + (x^3*Sqrt[1 - a^2*x^2])/(60*a^2) - (19*Arc
Sin[a*x])/(360*a^5) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(360*a^5) + (11*x^2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(180*a^3) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x])/(15*a) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(16*a^4) - (x^3*Sqrt[1
- a^2*x^2]*ArcTanh[a*x]^2)/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/6 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(8*a^5) - ((I/8)*ArcTanh[a*x]
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^5 + ((I/8)*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a^5 + ((I/8)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^5 - ((I/8)*P
olyLog[3, I*E^ArcTanh[a*x]])/a^5
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int \frac{x^6\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{4a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-\frac{5}{6}\int \frac{x^4\,\mathrm{t}}{\sqrt{}}$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a^3}+\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a}-\frac{3x\sqrt{1-a^2}}{}$$

$$= -\frac{x\sqrt{1-a^2x^2}}{12a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{13\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^5}+\frac{11x^2\sqrt{}}{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}+\frac{7\sin^{-1}(ax)}{6a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax}{360a^5}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax}{360a^5}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax}{360a^5}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax}{360a^5}$$

Mathematica [A]
time = 1.52, size = 268, normalized size = 0.80

$$\frac{\sqrt{1-a^2x^2}\left(90\tanh^{-1}(ax)+140(-1+a^2x^2)\tanh^{-1}(ax)+48(-1+a^2x^2)^2\tanh^{-1}(ax)+120ax(-1+a^2x^2)^2\tanh^{-1}(ax)^2+6ax(-1+a^2x^2)\left(2+35\tanh^{-1}(ax)^2\right)+ax\left(52+45\tanh^{-1}(ax)^2\right)-\frac{i\left(-76i\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+45\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-45\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+90\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-90\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+90\mathrm{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-90\mathrm{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{720a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(90*ArcTanh[a*x] + 140*(-1 + a^2*x^2)*ArcTanh[a*x] + 48*
(-1 + a^2*x^2)^2*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 6
*a*x*(-1 + a^2*x^2)*(2 + 35*ArcTanh[a*x]^2) + a*x*(52 + 45*ArcTanh[a*x]^2)
- (I*((-76*I)*ArcTan[Tanh[ArcTanh[a*x]/2]] + 45*ArcTanh[a*x]^2*Log[1 - I/E^
ArcTanh[a*x]] - 45*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 90*ArcTanh[a*
x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 90*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh
[a*x]] + 90*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 90*PolyLog[3, I/E^ArcTanh[a*x
]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

Maple [F]
time = 1.10, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arctanh}(ax)^2 \sqrt{-a^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
[Out] int(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{-(ax-1)(ax+1)}\, \operatorname{atanh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{atanh}(a x)^2 \sqrt{1 - a^2 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(x^4*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

3.439 $\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=281

$$\frac{11\sqrt{1-a^2x^2}}{60a^4}-\frac{(1-a^2x^2)^{3/2}}{30a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{11\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{30a^4}$$

```
[Out] -1/30*(-a^2*x^2+1)^(3/2)/a^4-11/30*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arc
tanh(a*x)/a^4-11/60*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+11/60*
I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+11/60*(-a^2*x^2+1)^(1/2)/a^
4+1/12*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^3+1/10*x^3*arctanh(a*x)*(-a^2*x^
2+1)^(1/2)/a-2/15*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4-1/15*x^2*arctanh(a*
x)^2*(-a^2*x^2+1)^(1/2)/a^2+1/5*x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.71, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6161, 6163, 267, 6097, 6141, 272, 45}

$$-\frac{11\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{30a^4}-\frac{11i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}+\frac{11i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{15a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{(1-a^2x^2)^{3/2}}{30a^4}+\frac{11\sqrt{1-a^2x^2}}{60a^4}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{15a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2])/(60*a^4) - (1 - a^2*x^2)^(3/2)/(30*a^4) + (x*Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(12*a^3) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(1
0*a) - (11*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(30*a^4) - (2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(15*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^2)/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/5 - (((11*I)/60)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (((11*I)/60)*PolyLog
[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx &= -\left(a^2\int\frac{x^5\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int\frac{x^3\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{3a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-\frac{4}{5}\int\frac{x^3\,\text{ta}}{\sqrt{1}}\\
&= -\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a^3}+\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{10a}-\frac{2\sqrt{1-a^2x^2}\,\text{t}}{3a}\\
&= -\frac{\sqrt{1-a^2x^2}}{3a^4}+\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{10a}-\\
&= \frac{\sqrt{1-a^2x^2}}{12a^4}+\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{10a}-\\
&= \frac{11\sqrt{1-a^2x^2}}{60a^4}-\frac{(1-a^2x^2)^{3/2}}{30a^4}+\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2}}{}
\end{aligned}$$

Mathematica [A]
time = 0.51, size = 175, normalized size = 0.62

$$\frac{\sqrt{1-a^2x^2}\left(11+11ax\tanh^{-1}(ax)+6ax(-1+a^2x^2)\tanh^{-1}(ax)+12(-1+a^2x^2)^2\tanh^{-1}(ax)^2+2(-1+a^2x^2)\left(1+10\tanh^{-1}(ax)^2\right)-\frac{11i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{60a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(11 + 11*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^2)*ArcTanh
[a*x] + 12*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*(1 + 10*ArcTa
nh[a*x]^2) - ((11*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E
^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[
a*x]]))/Sqrt[1 - a^2*x^2]))/(60*a^4)
```

Maple [A]
time = 0.70, size = 211, normalized size = 0.75

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(12a^4x^4\,\text{arctanh}(ax)^2+6a^3x^3\,\text{arctanh}(ax)-4a^2x^2\,\text{arctanh}(ax)^2+2a^2x^2+5ax\,\text{arctanh}(ax)-8\,\text{arctanh}\right.}{60a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(12*a^4*x^4*arctanh(a*x)^2+6*a^3*x^3*arct
anh(a*x)-4*a^2*x^2*arctanh(a*x)^2+2*a^2*x^2+5*a*x*arctanh(a*x)-8*arctanh(a*
x)^2+9)-11/60*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4+11/60*I
*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4-11/60*I*dilog(1+I*(a*x
+1)/(-a^2*x^2+1)^(1/2))/a^4+11/60*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
^4
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atanh}(a x)^2 \sqrt{1 - a^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(x^3*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

3.440 $\int x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=254

$$\frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\text{ArcSin}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6a}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2}$$

```
[Out] -1/6*arcsin(a*x)/a^3+1/4*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/
a^3-1/4*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/4*I*a
rctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/4*I*polylog(3,-I*
(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-1/4*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^3+1/12*x*(-a^2*x^2+1)^(1/2)/a^2+1/12*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a
^3+1/6*x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a-1/8*x*arctanh(a*x)^2*(-a^2*x^2
+1)^(1/2)/a^2+1/4*x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6161, 6163, 6141, 222, 6099, 4265, 2611, 2320, 6724, 327}

$$-\frac{\text{ArcSin}(ax)}{6a^3}+\frac{\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{4a^3}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{4a^3}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{x\sqrt{1-a^2x^2}}{12a^2}+\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6a}-\frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(12*a^2) - ArcSin[a*x]/(6*a^3) + (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x])/(12*a^3) + (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*a) - (x*Sqr
t[1 - a^2*x^2]*ArcTanh[a*x]^2)/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2)/4 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(4*a^3) - ((I/4)*ArcTanh[a
*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + ((I/4)*ArcTanh[a*x]*PolyLog[2, I
*E^ArcTanh[a*x]])/a^3 + ((I/4)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((I/4
)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
```

```
&& EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int \frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-\frac{3}{4}\int \frac{x^2 \text{ta}}{\sqrt{1}}$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3}+\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a}-\frac{x\sqrt{1-a^2x^2}}{8a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}+\frac{\sin^{-1}(ax)}{a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\text{ t}}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\text{ t}}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\text{ t}}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\text{ t}}{6a}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\sin^{-1}(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^2\sqrt{1-a^2x^2}\text{ t}}{6a}$$

Mathematica [A]
time = 0.77, size = 228, normalized size = 0.90

$$\frac{\sqrt{1-a^2x^2}\left(6\tanh^{-1}(ax)-4(1-a^2x^2)\tanh^{-1}(ax)-6ax(1-a^2x^2)\tanh^{-1}(ax)^2+ax\left(2+3\tanh^{-1}(ax)^2\right)-\frac{i\left(-8i\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+3\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+6\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6*ArcTanh[a*x] - 4*(1 - a^2*x^2)*ArcTanh[a*x] - 6*a*x*(
1 - a^2*x^2)*ArcTanh[a*x]^2 + a*x*(2 + 3*ArcTanh[a*x]^2) - (I*((-8*I)*ArcTa
n[Tanh[ArcTanh[a*x]/2]] + 3*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 3*Ar
cTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyLog[2, (-I)/E^A
rcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 6*PolyLog[3, (
-I)/E^ArcTanh[a*x]] - 6*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/
(24*a^3)
```

Maple [F]
time = 0.73, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arctanh}(ax)^2 \sqrt{-a^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-(ax-1)(ax+1)} \operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(a\,x)^2 \sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(x^2*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

3.441 $\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=175

$$\frac{\sqrt{1-a^2x^2}}{3a^2}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a}-\frac{2\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3a^2}-\frac{i\text{P}}{}$$

```
[Out] -2/3*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^2-1/3*(-a^2*x^2+1)
^(3/2)*arctanh(a*x)^2/a^2-1/3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/
a^2+1/3*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2+1/3*(-a^2*x^2+1)^(1
/2)/a^2+1/3*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A]

time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6141, 6089, 6097}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{3a^2}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{\sqrt{1-a^2x^2}}{3a^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a^2) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a) - (2*A
rcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^2) - ((1 - a^2*x^2)^(
3/2)*ArcTanh[a*x]^2)/(3*a^2) - ((I/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a^2 + ((I/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2\,dx = -\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3a^2} + \frac{2\int\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx}{3a}$$

$$= \frac{\sqrt{1-a^2x^2}}{3a^2} + \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3a^2} +$$

$$= \frac{\sqrt{1-a^2x^2}}{3a^2} + \frac{x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3a} - \frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(}{3a^2}$$

Mathematica [A]
time = 0.28, size = 135, normalized size = 0.77

$$\frac{\sqrt{1-a^2x^2}\left(1+ax\tanh^{-1}(ax)-(1-a^2x^2)\tanh^{-1}(ax)^2-\frac{i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (1 - a^2*x^2)*ArcTanh[a*x]^2 - (
I*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + P
olyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^
2*x^2]))/(3*a^2)
```

Maple [A]
time = 0.70, size = 175, normalized size = 1.00

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\left(a^2x^2\operatorname{arctanh}(ax)^2+ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2+1\right)}{3a^2} - \frac{i\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\operatorname{arctanh}(}{3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)-a
rctanh(a*x)^2+1)-1/3*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^2+
1/3*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^2-1/3*I*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+1/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atanh}(a x)^2 \sqrt{1 - a^2 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
[Out] int(x*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

3.442 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=158

$$-\frac{\text{ArcSin}(ax)}{a}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+\frac{\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a}-\frac{i\,\text{ta}}{}$$

```
[Out] -arcsin(a*x)/a+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-I*arctan
h(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*arctanh(a*x)*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+arctanh(a*x)*(-a^2*x^2+1)^(1/
2)/a+1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6091, 6099, 4265, 2611, 2320, 6724, 222}

$$\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{a}-\frac{\text{ArcSin}(ax)}{a}+\frac{\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2\,dx = \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2 + \text{Sub}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2 + \text{tan}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2 + \text{tan}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2 + \text{tan}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\ \tanh^{-1}(ax)^2 + \text{tan}$$

Mathematica [A]
time = 0.51, size = 187, normalized size = 1.18

$$\frac{\sqrt{1-a^2x^2}\left(2\tanh^{-1}(ax)+ax\tanh^{-1}(ax)^2-\frac{i\left(-4i\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [F]
time = 3.17, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \operatorname{arctanh}(ax)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{atanh}\left(a\,x\right)}^2\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

3.443 $\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=174

$$4\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-2\tanh^{-1}($$

```
[Out] 4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)-2*arctanh((a*x+1)/(-a^2
*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I*polylog(2
,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2
+1)^(1/2))+(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2
```

Rubi [A]
time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6161, 6167, 4267, 2611, 2320, 6724, 6141, 6097}

$$\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+4\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)+2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)+2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)-2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```

```
[Out] 4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x] + Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]^2 - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*Poly
Log[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + (2*I)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] - (2*I)*PolyLog[2, (I*Sqrt[
1 - a*x])/Sqrt[1 + a*x]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^A
rcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}\,dx = -\left(a^2\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-(2a)\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx+\text{Subst}\left(\int x^2\text{csch}(x)\,dx\right.$$
$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e\right.$$
$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e\right.$$
$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e\right.$$
$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e\right.$$

Mathematica [A]
time = 0.19, size = 203, normalized size = 1.17

$$\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2+\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right)+2i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-2i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+2i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
[Out] Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])
] + (2*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (2*I)*ArcTanh[a*x]*Log[1
 + I/E^ArcTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTan
h[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, E^(-Arc
Tanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a
*x])]
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)^2\sqrt{-a^2x^2+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x)
[Out] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x,x)
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x, x)
```

3.444 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-2a\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+2ia$$

```
[Out] -2*a*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-4*a*arctanh(a*x)*arc
tanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*I*a*arctanh(a*x)*polylog(2,-I*(a*x+1)/
(-a^2*x^2+1)^(1/2))-2*I*a*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/
2))+2*a*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/
2)/(a*x+1)^(1/2))-2*I*a*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I*a*poly
log(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.25, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6161, 6155, 6165, 6099, 4265, 2611, 2320, 6724}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-2a\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)+2a\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+2ia\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)-2ia\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)-2ia\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)+2ia\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)-4a\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 2*a*ArcTan[E^ArcTanh[a*x]]*ArcTan
h[a*x]^2 - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (2*I)*a*
ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (2*I)*a*ArcTanh[a*x]*PolyLog
[2, I*E^ArcTanh[a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*
a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (2*I)*a*PolyLog[3, (-I)*E^ArcTa
nh[a*x]] + (2*I)*a*PolyLog[3, I*E^ArcTanh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x^2}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-a\text{Subst}\left(\int x^2\text{sech}(x)\,dx,x,\tanh^{-1}(ax)\right)+ \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-4a\tanh \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-4a\tanh \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-4a\tanh \\
&= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x}-2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-4a\tanh
\end{aligned}
$$

Mathematica [A]
time = 0.52, size = 223, normalized size = 1.13

$$a\left(-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{ax}+2\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)+i\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-i\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+2i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+2i\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2i\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(a*x)) + 2*ArcTanh[a*x]*Log[1 - E^(
-ArcTanh[a*x])] + I*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*
x]^2*Log[1 + I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
+ 2*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^A
rcTanh[a*x]] - (2*I)*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] - 2*PolyLog[
2, E^(-ArcTanh[a*x])] + (2*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (2*I)*PolyL
og[3, I/E^ArcTanh[a*x]])
```

Maple [F]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)^2\sqrt{-a^2x^2+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}^{2}\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^2,x)
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^2, x)
```

3.445 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2-a^2\tanh^{-1}\left(\sqrt{1-\right.$$

```
[Out] a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-a^2*x^
2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arc
tanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)/(-a^
2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)*(-
a^2*x^2+1)^(1/2)/x-1/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.40, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6161, 6173, 6155, 272, 65, 214, 6167, 4267, 2611, 2320, 6724}

$$a^2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}(ax)\text{Li}_2\left(e^{\tanh^{-1}(ax)}\right)-a^2\text{Li}_3\left(-e^{\tanh^{-1}(ax)}\right)+a^2\text{Li}_3\left(e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) + a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] + a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] - a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] - a^2*PolyLog[3, -E^ArcTanh[a*x]] + a^2*Poly
Log[3, E^ArcTanh[a*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6167

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^3}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\\
&= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+2a^2\tanh^{-1}\left(e^{\tanh}\right.\\
&= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a^2\tanh^{-1}\left(e^{\tanh^-}\right.\\
&= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a^2\tanh^{-1}\left(e^{\tanh^-}\right.\\
&= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a^2\tanh^{-1}\left(e^{\tanh^-}\right.\\
&= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2}+a^2\tanh^{-1}\left(e^{\tanh^-}\right.
\end{aligned}$$

Mathematica [A]

time = 0.83, size = 188, normalized size = 1.25

$$\frac{1}{8}a^2\left(-4\tanh^{-1}(ax)\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\tanh^{-1}(ax)^2\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)-4\tanh^{-1}(ax)^2\log\left(1-e^{-\tanh^{-1}(ax)}\right)+4\tanh^{-1}(ax)^2\log\left(1+e^{-\tanh^{-1}(ax)}\right)+8\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)-8\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+8\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-8\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)+8\text{PolyLog}\left(3,e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^2\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+4\tanh^{-1}(ax)\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] (a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2 - 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]^2*Lo
g[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] - 8*ArcTanh[a*x]*Pol
yLog[2, -E^(-ArcTanh[a*x])] + 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])]
- 8*PolyLog[3, -E^(-ArcTanh[a*x])] + 8*PolyLog[3, E^(-ArcTanh[a*x])] - ArcT
anh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A]
time = 0.70, size = 231, normalized size = 1.53

method	result
default	$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{arctanh}(ax)(2ax+\text{arctanh}(ax))}{2x^2}-\frac{a^2\,\text{arctanh}(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-a^2\,\text{arctan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2-1/2*a^2
*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*
a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}^{2}\left(ax\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^2\,\sqrt{1-a^2\,x^2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^3, x)
```

3.446 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)^2/x^3+2/3*a^3*arctanh(a*x)*arctanh((-a
*x+1)^(1/2)/(a*x+1)^(1/2))-1/3*a^3*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))
+1/3*a^3*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/3*a^2*(-a^2*x^2+1)^(1/2)
/x-1/3*a*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6155, 6157, 6173, 270, 6165}

$$-\frac{1}{3}a^3\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{3}a^3\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{2}{3}a^3\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^2*Sqrt[1 - a^2*x^2])/x - (a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^2)
 - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2)/(3*x^3) + (2*a^3*ArcTanh[a*x]*ArcTa
nh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3 - (a^3*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
 + a*x])])/3 + (a^3*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
```

```
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{x^4}\,dx &= -\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^3}\,dx\\ &= -\frac{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3x^3}-\frac{1}{3}(2a)\int \frac{\tanh}{x^3\sqrt{1}}\\ &= -\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3x^3}\\ &= -\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^2}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}{3x^3}\end{aligned}$$

Mathematica [A]

time = 1.46, size = 177, normalized size = 1.05

$$-\frac{1}{3}a^3\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-\frac{(1-a^2x^2)^{3/2}\left(4\tanh^{-1}(ax)^2+2\left(-1+\cosh\left(2\tanh^{-1}(ax)\right)\right)-\frac{4a^3x^3\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)}{(1-a^2x^2)^{3/2}}+\tanh^{-1}(ax)\left(2\sinh\left(2\tanh^{-1}(ax)\right)+\frac{\left(\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\log\left(1+e^{-\tanh^{-1}(ax)}\right)\right)\left(-3ax+\sqrt{1-a^2x^2}\sinh\left(3\tanh^{-1}(ax)\right)\right)}{\sqrt{1-a^2x^2}}\right)\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^3*PolyLog[2, -E^(-ArcTanh[a*x])]) - ((1 - a^2*x^2)^(3/2)*(4*ArcTanh
[a*x]^2 + 2*(-1 + Cosh[2*ArcTanh[a*x]]) - (4*a^3*x^3*PolyLog[2, E^(-ArcTanh
[a*x])])/(1 - a^2*x^2)^(3/2) + ArcTanh[a*x]*(2*Sinh[2*ArcTanh[a*x]] + ((Log
[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])])*(-3*a*x + Sqrt[1 - a^
2*x^2]*Sinh[3*ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2])))/(12*x^3)
```

Maple [A]
time = 0.70, size = 171, normalized size = 1.01

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(a^2x^2\operatorname{arctanh}(ax)^2-a^2x^2-ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2\right)}{3x^3}-\frac{a^3\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2}}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2-a^2*x^2-a*x*arctanh(a*
x)-arctanh(a*x)^2)/x^3-1/3*a^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2)
)-1/3*a^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*a^3*arctanh(a*x)*ln(1+(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*a^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^4,x)
```

```
[Out] int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^4, x)
```

3.447 $\int x^4(1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=292

$$\frac{3\sqrt{1-a^2x^2}}{128a^5}+\frac{(1-a^2x^2)^{3/2}}{192a^5}-\frac{3(1-a^2x^2)^{5/2}}{80a^5}+\frac{(1-a^2x^2)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{128a^4}-\frac{x^3\sqrt{1-a^2x^2}\ \text{ta}}{64a^2}$$

```
[Out] 1/192*(-a^2*x^2+1)^(3/2)/a^5-3/80*(-a^2*x^2+1)^(5/2)/a^5+1/56*(-a^2*x^2+1)^
(7/2)/a^5-3/64*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-3/128*
I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+3/128*I*polylog(2,I*(-a*x+
1)^(1/2)/(a*x+1)^(1/2))/a^5+3/128*(-a^2*x^2+1)^(1/2)/a^5-3/128*x*arctanh(a*
x)*(-a^2*x^2+1)^(1/2)/a^4-1/64*x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+3/16
*x^5*arctanh(a*x)*(-a^2*x^2+1)^(1/2)-1/8*a^2*x^7*arctanh(a*x)*(-a^2*x^2+1)^
(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6161, 6157, 6163, 272, 45, 267, 6097}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{64a^5}-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{3}{16}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{64a^2}+\frac{(1-a^2x^2)^{7/2}}{56a^5}-\frac{3(1-a^2x^2)^{5/2}}{80a^5}+\frac{(1-a^2x^2)^{3/2}}{192a^5}+\frac{3\sqrt{1-a^2x^2}}{128a^5}-\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(128*a^5) + (1 - a^2*x^2)^(3/2)/(192*a^5) - (3*(1 - a
^2*x^2)^(5/2))/(80*a^5) + (1 - a^2*x^2)^(7/2)/(56*a^5) - (3*x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(128*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(64*a^2
) + (3*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 - (a^2*x^7*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x])/8 - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(64*
a^5) - (((3*I)/128)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (
((3*I)/128)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int x^4(1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx = -\left(a^2\int x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\right)+\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)$$

$$=\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{6}\int\frac{x^4}{\sqrt{}}$$

$$=-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24a^2}+\frac{3}{16}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{8}a^2x^7\sqrt{}$$

$$=-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{64a^2}+\frac{3}{16}x^5\sqrt{1-}$$

$$=-\frac{\sqrt{1-a^2x^2}}{48a^5}+\frac{(1-a^2x^2)^{3/2}}{72a^5}-\frac{(1-a^2x^2)^{5/2}}{24a^5}+\frac{(1-a^2x^2)^{7/2}}{56a^5}-\frac{3x\sqrt{}}{}$$

$$=-\frac{\sqrt{1-a^2x^2}}{384a^5}+\frac{(1-a^2x^2)^{3/2}}{72a^5}-\frac{3(1-a^2x^2)^{5/2}}{80a^5}+\frac{(1-a^2x^2)^{7/2}}{56a^5}-\frac{3x}{}$$

$$=\frac{3\sqrt{1-a^2x^2}}{128a^5}+\frac{(1-a^2x^2)^{3/2}}{192a^5}-\frac{3(1-a^2x^2)^{5/2}}{80a^5}+\frac{(1-a^2x^2)^{7/2}}{56a^5}-\frac{3x\sqrt{}}{}$$

Mathematica [A]
time = 0.95, size = 272, normalized size = 0.93

$$\frac{121\sqrt{1-a^2x^2}+218a^2x^2\sqrt{1-a^2x^2}+216a^4x^4\sqrt{1-a^2x^2}-240a^6x^6\sqrt{1-a^2x^2}-315ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)-210a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+2520a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-1680a^7x^7\sqrt{1-a^2x^2}\tanh^{-1}(ax)-315i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+315i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)-315i\,\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+315i\,\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{13440a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (121*Sqrt[1 - a^2*x^2] + 218*a^2*x^2*Sqrt[1 - a^2*x^2] + 216*a^4*x^4*Sqrt[1
 - a^2*x^2] - 240*a^6*x^6*Sqrt[1 - a^2*x^2] - 315*a*x*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x] - 210*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2520*a^5*x^5*Sqrt[
1 - a^2*x^2]*ArcTanh[a*x] - 1680*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (
315*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (315*I)*ArcTanh[a*x]*Log[1
+ I/E^ArcTanh[a*x]] - (315*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (315*I)*Pol
yLog[2, I/E^ArcTanh[a*x]])/(13440*a^5)
```

Maple [A]
time = 1.64, size = 215, normalized size = 0.74

method	result

default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(1680\,\mathrm{arctanh}(ax)a^7x^7+240a^6x^6-2520\,\mathrm{arctanh}(ax)a^5x^5-216a^4x^4+210a^3x^3\,\mathrm{arctanh}(ax)-218a^2x}{13440a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/13440/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(1680*arctanh(a*x)*a^7*x^7+240*a^6*x^
6-2520*arctanh(a*x)*a^5*x^5-216*a^4*x^4+210*a^3*x^3*arctanh(a*x)-218*a^2*x^
2+315*a*x*arctanh(a*x)-121)-3/128*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arct
anh(a*x)/a^5+3/128*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5-3/
128*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/128*I*dilog(1-I*(a*x+1)/(
-a^2*x^2+1)^(1/2))/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^6 - x^4)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(-(ax-1)(ax+1))^{\frac{3}{2}}\,\mathrm{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x**4*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\mathrm{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
[Out] int(x^4*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

3.448 $\int x^3(1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=186

$$\frac{3x\sqrt{1-a^2x^2}}{112a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{17\mathrm{ArcSin}(ax)}{560a^4}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{35a^4}-\frac{x^2\sqrt{1-a^2}}{}$$

```
[Out] 17/560*arcsin(a*x)/a^4+3/112*x*(-a^2*x^2+1)^(1/2)/a^3+23/840*x^3*(-a^2*x^2+
1)^(1/2)/a-1/42*a*x^5*(-a^2*x^2+1)^(1/2)-2/35*arctanh(a*x)*(-a^2*x^2+1)^(1/
2)/a^4-1/35*x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+8/35*x^4*arctanh(a*x)*(
-a^2*x^2+1)^(1/2)-1/7*a^2*x^6*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6161, 6157, 6163, 327, 222, 6141}

$$\frac{17\mathrm{ArcSin}(ax)}{560a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{35a^2}-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{8}{35}x^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{35a^4}+\frac{3x\sqrt{1-a^2x^2}}{112a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(112*a^3) + (23*x^3*Sqrt[1 - a^2*x^2])/(840*a) - (a
*x^5*Sqrt[1 - a^2*x^2])/42 + (17*ArcSin[a*x])/(560*a^4) - (2*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(35*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(35*a^2)
+ (8*x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/35 - (a^2*x^6*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])/7
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
```

```
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int x^3 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\, dx = -\left(a^2 \int x^5 \sqrt{1-a^2x^2} \tanh^{-1}(ax)\, dx\right) + \int x^3 \sqrt{1-a^2x^2} \tanh^{-1}(ax)\, d$$

$$= \frac{1}{5}x^4\sqrt{1-a^2x^2} \tanh^{-1}(ax) - \frac{1}{7}a^2x^6\sqrt{1-a^2x^2} \tanh^{-1}(ax) + \frac{1}{5}\int \frac{x^3 \text{ta}}{\sqrt{1}}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{20a} - \frac{1}{42}ax^5\sqrt{1-a^2x^2} - \frac{x^2\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{15a^2} + \frac{8}{35}x^4$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3} + \frac{23x^3\sqrt{1-a^2x^2}}{840a} - \frac{1}{42}ax^5\sqrt{1-a^2x^2} - \frac{2\sqrt{1-a^2x^2} \text{ ta}}{15a^4}$$

$$= \frac{3x\sqrt{1-a^2x^2}}{112a^3} + \frac{23x^3\sqrt{1-a^2x^2}}{840a} - \frac{1}{42}ax^5\sqrt{1-a^2x^2} + \frac{11\sin^{-1}(ax)}{120a^4} -$$

$$= \frac{3x\sqrt{1-a^2x^2}}{112a^3} + \frac{23x^3\sqrt{1-a^2x^2}}{840a} - \frac{1}{42}ax^5\sqrt{1-a^2x^2} + \frac{17\sin^{-1}(ax)}{560a^4} -$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.42

$$\frac{ax\sqrt{1-a^2x^2}\,(45+46a^2x^2-40a^4x^4)+51\text{ArcSin}(ax)-48(1-a^2x^2)^{5/2}\,(2+5a^2x^2)\tanh^{-1}(ax)}{1680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(45 + 46*a^2*x^2 - 40*a^4*x^4) + 51*ArcSin[a*x] - 48
*(1 - a^2*x^2)^(5/2)*(2 + 5*a^2*x^2)*ArcTanh[a*x])/(1680*a^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.37, size = 140, normalized size = 0.75

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(240\,\text{arctanh}(ax)a^6x^6+40a^5x^5-384a^4x^4\,\text{arctanh}(ax)-46a^3x^3+48a^2x^2\,\text{arctanh}(ax)-45ax+96\,\text{arc}}{1680a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(240*arctanh(a*x)*a^6*x^6+40*a^5*x^5-3
84*a^4*x^4*arctanh(a*x)-46*a^3*x^3+48*a^2*x^2*arctanh(a*x)-45*a*x+96*arctan
h(a*x))+17/560*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-17/560*I*ln((a*x+1)/(
-a^2*x^2+1)^(1/2)-I)/a^4
```

Maxima [A]
time = 0.46, size = 163, normalized size = 0.88

$$-\frac{1}{1680}a\left(\frac{5\left(\frac{8\left(-a^2x^2+1\right)^{\frac{5}{2}}x}{a^2}-\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{3\sqrt{-a^2x^2+1}\,x}{a^2}-\frac{3\arcsin(ax)}{a^3}\right)}{a^2}-\frac{12\left(2\left(-a^2x^2+1\right)^{\frac{3}{2}}x+3\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{a}\right)}{a^4}\right)-\frac{1}{35}\left(\frac{5\left(-a^2x^2+1\right)^{\frac{5}{2}}x^2}{a^2}+\frac{2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{a^4}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/1680*a*(5*(8*(-a^2*x^2 + 1)^(5/2)*x/a^2 - 2*(-a^2*x^2 + 1)^(3/2)*x/a^2 -
 3*sqrt(-a^2*x^2 + 1)*x/a^2 - 3*arcsin(a*x)/a^3)/a^2 - 12*(2*(-a^2*x^2 + 1)
^(3/2)*x + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a*x)/a)/a^4) - 1/35*(5*(-a^2*x
^2 + 1)^(5/2)*x^2/a^2 + 2*(-a^2*x^2 + 1)^(5/2)/a^4)*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 106, normalized size = 0.57

$$-\frac{\left(40\,a^5x^5-46\,a^3x^3-45\,ax+24\left(5\,a^6x^6-8\,a^4x^4+a^2x^2+2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+102\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{1680\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/1680*((40*a^5*x^5 - 46*a^3*x^3 - 45*a*x + 24*(5*a^6*x^6 - 8*a^4*x^4 + a^
2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 102*arctan((sqrt
(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(-(ax-1)\,(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atanh}(a x) \left(1 - a^2 x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(x^3*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

3.449 $\int x^2(1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=243

$$\frac{\sqrt{1-a^2x^2}}{16a^3}+\frac{(1-a^2x^2)^{3/2}}{72a^3}-\frac{(1-a^2x^2)^{5/2}}{30a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{6}a^2x$$

```
[Out] 1/72*(-a^2*x^2+1)^(3/2)/a^3-1/30*(-a^2*x^2+1)^(5/2)/a^3-1/8*arctan((-a*x+1)
^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/16*I*polylog(2,-I*(-a*x+1)^(1/2)/(
a*x+1)^(1/2))/a^3+1/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/16
*(-a^2*x^2+1)^(1/2)/a^3-1/16*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^2+7/24*x^3
*arctanh(a*x)*(-a^2*x^2+1)^(1/2)-1/6*a^2*x^5*arctanh(a*x)*(-a^2*x^2+1)^(1/2
)
```

Rubi [A]
time = 0.41, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6161, 6157, 6163, 267, 6097, 272, 45}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{8a^3}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^2}-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{7}{24}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{(1-a^2x^2)^{5/2}}{30a^3}+\frac{(1-a^2x^2)^{3/2}}{72a^3}+\frac{\sqrt{1-a^2x^2}}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^3) + (1 - a^2*x^2)^(3/2)/(72*a^3) - (1 - a^2*x^2)^(
5/2)/(30*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^2) + (7*x^3*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/24 - (a^2*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/6 -
 (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^3) - ((I/16)*PolyL
og[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6163

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)
*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[f^2*((m - 1)
/(c^2*m)), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && Gt
Q[m, 1]
```

Rubi steps

$$\int x^2 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\, dx = -\left(a^2 \int x^4 \sqrt{1-a^2x^2} \tanh^{-1}(ax)\, dx\right) + \int x^2 \sqrt{1-a^2x^2} \tanh^{-1}(ax)$$

$$= \frac{1}{4} x^3 \sqrt{1-a^2x^2} \tanh^{-1}(ax) - \frac{1}{6} a^2 x^5 \sqrt{1-a^2x^2} \tanh^{-1}(ax) + \frac{1}{4} \int \frac{x^2}{\sqrt{}}$$

$$= -\frac{x\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{8a^2} + \frac{7}{24} x^3 \sqrt{1-a^2x^2} \tanh^{-1}(ax) - \frac{1}{6} a^2 x^5 \sqrt{1}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a^3} - \frac{x\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{16a^2} + \frac{7}{24} x^3 \sqrt{1-a^2x^2} \tanh^{-1}(a$$

$$= \frac{\sqrt{1-a^2x^2}}{48a^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{36a^3} - \frac{\left(1-a^2x^2\right)^{5/2}}{30a^3} - \frac{x\sqrt{1-a^2x^2} \tanh^{-1}(a}{16a^2}$$

$$= \frac{\sqrt{1-a^2x^2}}{16a^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{72a^3} - \frac{\left(1-a^2x^2\right)^{5/2}}{30a^3} - \frac{x\sqrt{1-a^2x^2} \tanh^{-1}(a}{16a^2}$$

Mathematica [A]
time = 0.67, size = 224, normalized size = 0.92

$$\frac{31\sqrt{1-a^2x^2} + 38a^2x^2\sqrt{1-a^2x^2} - 24a^4x^4\sqrt{1-a^2x^2} - 45ax\sqrt{1-a^2x^2} \tanh^{-1}(ax) + 210a^3x^3\sqrt{1-a^2x^2} \tanh^{-1}(ax) - 120a^5x^5\sqrt{1-a^2x^2} \tanh^{-1}(ax) - 45i \tanh^{-1}(ax) \log\left(1-ie^{-\tanh^{-1}(ax)}\right) + 45i \tanh^{-1}(ax) \log\left(1+ie^{-\tanh^{-1}(ax)}\right) - 45i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) + 45i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{720a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (31*Sqrt[1 - a^2*x^2] + 38*a^2*x^2*Sqrt[1 - a^2*x^2] - 24*a^4*x^4*Sqrt[1 -
a^2*x^2] - 45*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 210*a^3*x^3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x] - 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (45*I)*Ar
cTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (45*I)*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (45*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (45*I)*PolyLog[2, I/E^
ArcTanh[a*x]])/(720*a^3)
```

Maple [A]
time = 1.38, size = 195, normalized size = 0.80

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(120 \operatorname{arctanh}(ax) a^5 x^5 + 24a^4x^4 - 210a^3x^3 \operatorname{arctanh}(ax) - 38a^2x^2 + 45ax \operatorname{arctanh}(ax) - 31)}{720a^3} - \frac{i \ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/720/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*a^5*x^5+24*a^4*x^4-21
0*a^3*x^3*arctanh(a*x)-38*a^2*x^2+45*a*x*arctanh(a*x)-31)-1/16*I*ln(1+I*(a*
x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^3-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+
1/16*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^4 - x^2)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}(-(ax-1)\,(ax+1))^{\frac{3}{2}}\operatorname{atanh}{(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(a x) \left(1 - a^2 x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
[Out] int(x^2*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

3.450 $\int x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=81

$$\frac{3x\sqrt{1-a^2x^2}}{40a}+\frac{x(1-a^2x^2)^{3/2}}{20a}+\frac{3\text{ArcSin}(ax)}{40a^2}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5a^2}$$

```
[Out] 1/20*x*(-a^2*x^2+1)^(3/2)/a+3/40*arcsin(a*x)/a^2-1/5*(-a^2*x^2+1)^(5/2)*arc
tanh(a*x)/a^2+3/40*x*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {6141, 201, 222}

$$\frac{3\text{ArcSin}(ax)}{40a^2}+\frac{x(1-a^2x^2)^{3/2}}{20a}+\frac{3x\sqrt{1-a^2x^2}}{40a}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(40*a) + (x*(1 - a^2*x^2)^(3/2))/(20*a) + (3*ArcSin
[a*x])/(40*a^2) - ((1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*a^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx = -\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5a^2} + \frac{\int (1-a^2x^2)^{3/2}\,dx}{5a}$$
$$= \frac{x(1-a^2x^2)^{3/2}}{20a} - \frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5a^2} + \frac{3\int \sqrt{1-a^2x^2}\,dx}{20a}$$
$$= \frac{3x\sqrt{1-a^2x^2}}{40a} + \frac{x(1-a^2x^2)^{3/2}}{20a} - \frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5a^2} + \frac{3\int \frac{}{\sqrt{1}}}{}$$
$$= \frac{3x\sqrt{1-a^2x^2}}{40a} + \frac{x(1-a^2x^2)^{3/2}}{20a} + \frac{3\sin^{-1}(ax)}{40a^2} - \frac{(1-a^2x^2)^{5/2}\tanh^{-1}(}{5a^2}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.75

$$\frac{ax(5-2a^2x^2)\sqrt{1-a^2x^2} + 3\text{ArcSin}(ax) - 8(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
[Out] (a*x*(5 - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] + 3*ArcSin[a*x] - 8*(1 - a^2*x^2)^(5
/2)*ArcTanh[a*x])/(40*a^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.35, size = 120, normalized size = 1.48

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(8a^4x^4\operatorname{arctanh}(ax)+2a^3x^3-16a^2x^2\operatorname{arctanh}(ax)-5ax+8\operatorname{arctanh}(ax))}{40a^2} + \frac{3i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+}}\right.}{40a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] -1/40/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)+2*a^3*x^3-16*a^2
*x^2*arctanh(a*x)-5*a*x+8*arctanh(a*x))+3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)+I)/a^2-3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

Maxima [A]
time = 0.46, size = 67, normalized size = 0.83

$$-\frac{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)}{5a^2} + \frac{2(-a^2x^2+1)^{\frac{3}{2}}x + 3\sqrt{-a^2x^2+1}\,x + \frac{3\arcsin(ax)}{a}}{40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/a^2 + 1/40*(2*(-a^2*x^2 + 1)^(3/2)*x
 + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a*x)/a)/a
```

Fricas [A]
time = 0.36, size = 90, normalized size = 1.11

$$-\frac{\left(2\,a^3x^3-5\,ax+4\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{40\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/40*((2*a^3*x^3 - 5*a*x + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1)))*sqrt(-a^2*x^2 + 1) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\mathrm{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(x*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

3.451 $\int (1-a^2x^2)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=189

$$\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{(1-a^2x^2)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)-\frac{3\text{ArcTan}\left(\frac{\sqrt{1-a}}{\sqrt{1+a}}\right.}{4a}$$

```
[Out] 1/12*(-a^2*x^2+1)^(3/2)/a+1/4*x*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-3/4*arctan(
(-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-3/8*I*polylog(2,-I*(-a*x+1)^(1
/2)/(a*x+1)^(1/2))/a+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3/8*
(-a^2*x^2+1)^(1/2)/a+3/8*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6089, 6097}

$$\frac{(1-a^2x^2)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{3\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a}-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax) + \frac{3}{4}\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax$$

$$= \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) + \frac{1}{4}x\left(1-a^2x^2\right.$$

$$= \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) + \frac{1}{4}x\left(1-a^2x^2\right.$$

Mathematica [A]
time = 0.42, size = 176, normalized size = 0.93

$$\frac{11\sqrt{1-a^2x^2} - 2a^2x^2\sqrt{1-a^2x^2} + 15ax\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) - 6a^3x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) - 9i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right) + 9i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right) - 9i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) + 9i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*
x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcTanh[
a*x]])/(24*a)
```

Maple [A]
time = 4.20, size = 173, normalized size = 0.92

method	result
default	$-\frac{\left(6a^3x^3\operatorname{arctanh}(ax)+2a^2x^2-15ax\operatorname{arctanh}(ax)-11\right)\sqrt{-a^2x^2+1}}{24a} - \frac{3i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a} + \frac{3i\operatorname{arctan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+1
)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a*a
rctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

3.452 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=144

$$-\frac{1}{6}ax\sqrt{1-a^2x^2}-\frac{7}{6}\text{ArcSin}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}(1-a^2x^2)^{3/2}\tanh^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}$$

```
[Out] -7/6*arcsin(a*x)+1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-2*arctanh(a*x)*arctanh
((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-pol
ylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/6*a*x*(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1
)^(1/2)*arctanh(a*x)
```

Rubi [A]
time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6161, 6157, 6165, 222, 6141, 201}

$$-\frac{1}{6}ax\sqrt{1-a^2x^2}+\frac{1}{3}(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{7}{6}\text{ArcSin}(ax)+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

```
[Out] -1/6*(a*x*Sqrt[1 - a^2*x^2]) - (7*ArcSin[a*x])/6 + Sqrt[1 - a^2*x^2]*ArcTan
h[a*x] + ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/3 - 2*ArcTanh[a*x]*ArcTanh[Sqrt
[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - Pol
yLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6141

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
```

```
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x}\,dx &= -\left(a^2\int x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}\,dx\\ &= \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}(1-a^2x^2)^{3/2}\tanh^{-1}(ax)-\frac{1}{3}a\int \sqrt{1-a^2x^2}\\ &= -\frac{1}{6}ax\sqrt{1-a^2x^2}-\sin^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}(1-a^2x^2)^{3}\\ &= -\frac{1}{6}ax\sqrt{1-a^2x^2}-\frac{7}{6}\sin^{-1}(ax)+\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{3}(1-a^2x^2\end{aligned}$$

Mathematica [A]

time = 0.16, size = 143, normalized size = 0.99

$$\frac{1}{6}\left(-ax\sqrt{1-a^2x^2}-14\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+8\sqrt{1-a^2x^2}\tanh^{-1}(ax)-2a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)+6\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+6\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

```
[Out] (-(a*x*Sqrt[1 - a^2*x^2]) - 14*ArcTan[Tanh[ArcTanh[a*x]/2]] + 8*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x] - 2*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*ArcTanh[
a*x]*Log[1 - E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 + 6*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*PolyLog[2, E^(-ArcTanh[a*x])])/6
```

Maple [A]
time = 1.51, size = 132, normalized size = 0.92

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\,\text{arctanh}(ax)+ax-8\,\text{arctanh}(ax))}{6}-\frac{7\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3}-\text{dilog}\left(1+\frac{}{\sqrt{}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-8*arctanh(a*x))-7
/3*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-a
rctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x,x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,{\left(1-a^2\,x^2\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x,x)
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x, x)
```

3.453 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=179

$$-\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+3a\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(a$$

```
[Out] 3*a*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)-a*arctanh((-a^2*x^2+1
)^(1/2))+3/2*I*a*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*I*a*polylog
(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a*(-a^2*x^2+1)^(1/2)-arctanh(a*x)*(-
a^2*x^2+1)^(1/2)/x-1/2*a^2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6161, 6155, 272, 65, 214, 6097, 6089}

$$-\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}+3a\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)+\frac{3}{2}ia\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}ia\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] -1/2*(a*Sqrt[1 - a^2*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (a^2*x*Sq
rt[1 - a^2*x^2]*ArcTanh[a*x])/2 + 3*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*A
rcTanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + ((3*I)/2)*a*PolyLog[2, ((-I)*S
qrt[1 - a*x])/Sqrt[1 + a*x]] - ((3*I)/2)*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqr
t[1 + a*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^2}\,dx = -\left(a^2\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx\right) + \int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2} - \frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) - \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx -$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{1}{2}a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax$$

Mathematica [A]
time = 0.43, size = 168, normalized size = 0.94

$$\frac{1}{2}\left(-a\sqrt{1-a^2x^2} - \frac{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - a^2x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax) + 3ia\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - 3ia\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right) + 2a\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right) + 3ia\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - 3ia\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] (-(a*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - a^2*x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x] + (3*I)*a*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]]
 - (3*I)*a*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 2*a*Log[Tanh[ArcTanh[a*
x]/2]] + (3*I)*a*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (3*I)*a*PolyLog[2, I/E^A
rcTanh[a*x]])/2
```

Maple [A]
time = 1.43, size = 205, normalized size = 1.15

method	result
default	$-\frac{(a^2x^2\operatorname{arctanh}(ax)+ax+2\operatorname{arctanh}(ax))\sqrt{-(ax-1)(ax+1)}}{2x} - a\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) + a\ln\left(\frac{a}{\sqrt{-a^2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a^2*x^2*arctanh(a*x)+a*x+2*arctanh(a*x))*(-(a*x-1)*(a*x+1))^(1/2)/x-a
*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+3/2*I*
a*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)-3/2*I*a*dilog(1-I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+3/2*I*a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*I*a*
ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="fricas")
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**2,x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^2,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^2, x)
```

3.454 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=168

$$-\frac{a\sqrt{1-a^2x^2}}{2x}+a^2\text{ArcSin}(ax)-a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+3a^2\tanh^{-1}(ax)\tanh^{-}$$

```
[Out] a^2*arcsin(a*x)+3*a^2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/
2*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/2*a^2*polylog(2,(-a*x+1)^(
1/2)/(a*x+1)^(1/2))-1/2*a*(-a^2*x^2+1)^(1/2)/x-a^2*arctanh(a*x)*(-a^2*x^2+1
)^(1/2)-1/2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6161, 6157, 6173, 270, 6165, 222}

$$a^2\text{ArcSin}(ax)-\frac{3}{2}a^2\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{3}{2}a^2\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2x^2}+3a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[1 - a^2*x^2])/x + a^2*ArcSin[a*x] - a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + 3*a^2*ArcTanh[a*x]*Ar
cTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt
[1 + a*x])])/2 + (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
```

```
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^3}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{x}+a^2\sin^{-1}(ax)-a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\mathrm{t}}{2x}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{2x}+a^2\sin^{-1}(ax)-a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\,\mathrm{t}}{2x}$$

Mathematica [A]
time = 0.67, size = 158, normalized size = 0.94

$$\frac{1}{8}a^2\left(16\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)-8\sqrt{1-a^2x^2}\tanh^{-1}(ax)-2\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)-12\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)+12\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)-12\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+12\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] (a^2*(16*ArcTan[Tanh[ArcTanh[a*x]/2]] - 8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] -
2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 12*ArcTanh[a
*x]*Log[1 - E^(-ArcTanh[a*x])] + 12*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 - 12*PolyLog[2, -E^(-ArcTanh[a*x])] + 12*PolyLog[2, E^(-ArcTanh[a*x])] - A
rcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A]
time = 1.47, size = 145, normalized size = 0.86

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\operatorname{arctanh}(ax)+ax+\operatorname{arctanh}(ax))}{2x^2}+2a^2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{3a^2\operatorname{dilog}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x+arctanh(a*x))/x^2
+2*a^2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*dilog(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))+3/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*di
log((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^3,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^3, x)
```

3.455 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(a$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3-2*a^3*arctan((-a*x+1)^(1/2)/(a*x+1
)^(1/2))*arctanh(a*x)+7/6*a^3*arctanh((-a^2*x^2+1)^(1/2))-I*a^3*polylog(2,-
I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))+I*a^3*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))-1/6*a*(-a^2*x^2+1)^(1/2)/x^2+a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6161, 6155, 272, 43, 65, 214, 6097}

$$-2a^3\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)-ia^3\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)+ia^3\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{7}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/6*(a*Sqrt[1 - a^2*x^2])/x^2 + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (
(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3) - 2*a^3*ArcTan[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]*ArcTanh[a*x] + (7*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6 - I*a^3*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + I*a^3*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rubi steps

$$\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^4}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}\,dx\right) + \int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^4}\,dx$$
$$= -\frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3} + \frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx - a^2\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}$$
$$= \frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3} - 2a^3\tan^{-1}\left(\frac{\sqrt{1}}{\sqrt{1}}\right.$$
$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2} + \frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2} + \frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2} + \frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x} - \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{3x^3}$$

Mathematica [A]
time = 0.87, size = 199, normalized size = 1.05

$$-a^3\left(-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{ax} + i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right) - i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right) + \log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right) + i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right) - \frac{(1-a^2x^2)^{3/2}\left(8\tanh^{-1}(ax) + 2\sinh\left(2\tanh^{-1}(ax)\right) + \log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)\left(-\frac{3ax}{\sqrt{1-a^2x^2}} + \sinh\left(3\tanh^{-1}(ax)\right)\right)\right)}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a^3*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I
/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + Log[Tanh[ArcT
anh[a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh
[a*x]])) - ((1 - a^2*x^2)^(3/2)*(8*ArcTanh[a*x] + 2*Sinh[2*ArcTanh[a*x]] +
Log[Tanh[ArcTanh[a*x]/2]]*((-3*a*x)/Sqrt[1 - a^2*x^2] + Sinh[3*ArcTanh[a*x]
])))/(24*x^3)
```

Maple [A]
time = 1.42, size = 220, normalized size = 1.16

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(8a^2x^2\operatorname{arctanh}(ax)-ax-2\operatorname{arctanh}(ax))}{6x^3} + \frac{7a^3\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{6} - \frac{7a^3\ln\left(\frac{ax-}{\sqrt{-a^2x}}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x^
3+7/6*a^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-7/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^
(1/2)-1)-I*a^3*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)+I*a^3*dilog(
1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I*a^3*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))
+I*a^3*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^4,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^4, x)
```

3.456 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=191

$$-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-\frac{3}{4}a^4\tanh^{-1}(ax)\,\mathrm{ta}$$

```
[Out] -3/4*a^4*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/8*a^4*polylog
(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/8*a^4*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^
(1/2))-1/12*a*(-a^2*x^2+1)^(1/2)/x^3+11/24*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*arc
tanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4+5/8*a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^
2
```

Rubi [A]
time = 0.39, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6161, 6157, 6173, 277, 270, 6165}

$$\frac{3}{8}a^4\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{8}a^4\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{4}a^4\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{11a^3\sqrt{1-a^2x^2}}{24x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -1/12*(a*Sqrt[1 - a^2*x^2])/x^3 + (11*a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (5*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]
)/(8*x^2) - (3*a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + (
3*a^4*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 - (3*a^4*PolyLog[2, Sqr
t[1 - a*x]/Sqrt[1 + a*x]])/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
```

```
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^5}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^3}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^5}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^4}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^2}-\frac{1}{3}\int \frac{\tanh^{-1}(ax}{x^5\sqrt{1-a^2x}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{9x^3}+\frac{a^3\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x}}{2}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{5a^3\sqrt{1-a^2x^2}}{18x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-a}}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-}}{}$$

Mathematica [A]
time = 2.90, size = 282, normalized size = 1.48

$$\frac{1}{192}a\left(40a^3\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)+18a^3\tanh^{-1}(ax)\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{a^4x\,\text{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}-3a^3\tanh^{-1}(ax)\text{csch}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)+72a^3\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-72a^3\tanh^{-1}(ax)\log\left(1+e^{-\tanh^{-1}(ax)}\right)+72a^3\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-72a^3\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+18a^3\tanh^{-1}(ax)\text{sech}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)+3a^3\tanh^{-1}(ax)\text{sech}^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)-\frac{16\sqrt{1-a^2x^2}\,\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{x^3}+\frac{16a^2\sqrt{1-a^2x^2}\,\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{x}-40a^3\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] (a*(40*a^3*Coth[ArcTanh[a*x]/2] + 18*a^3*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^
2 - (a^4*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*a^3*ArcTanh[a*x]*C
sch[ArcTanh[a*x]/2]^4 + 72*a^3*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 72
*a^3*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 72*a^3*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 72*a^3*PolyLog[2, E^(-ArcTanh[a*x])] + 18*a^3*ArcTanh[a*x]*Sech
[ArcTanh[a*x]/2]^2 + 3*a^3*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - (16*Sqrt[1
 - a^2*x^2]*Sinh[ArcTanh[a*x]/2]^4)/x^3 + (16*a^2*Sqrt[1 - a^2*x^2]*Sinh[Ar
cTanh[a*x]/2]^4)/x - 40*a^3*Tanh[ArcTanh[a*x]/2]))/192
```

Maple [A]
time = 1.41, size = 164, normalized size = 0.86

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(11a^3x^3+15a^2x^2\,\text{arctanh}(ax)-2ax-6\,\text{arctanh}(ax))}{24x^4}+\frac{3a^4\,\text{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-(a*x-1)*(a*x+1))^(1/2)*(11*a^3*x^3+15*a^2*x^2*arctanh(a*x)-2*a*x-6*a
rctanh(a*x))/x^4+3/8*a^4*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*
a^4*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*a^4*arctanh(a*x)*ln(1+(a*x+1)
/(-a^2*x^2+1)^(1/2))-3/8*a^4*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^5,x)
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^5, x)
```

3.457 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=94

$$\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a(1-a^2x^2)^{3/2}}{20x^4}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/20*a*(-a^2*x^2+1)^(3/2)/x^4-1/5*(-a^2*x^2+1)^(5/2)*arctanh(a*x)/x^5-3/40
*a^5*arctanh((-a^2*x^2+1)^(1/2))+3/40*a^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]

time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6155, 272, 43, 65, 214}

$$-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{a(1-a^2x^2)^{3/2}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - (a*(1 - a^2*x^2)^(3/2))/(20*x^4) - ((1
 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]]
)/40
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6155

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Dist[b*c*(p/(m + 1)), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^6}\,dx &= -\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{5}a\int \frac{(1-a^2x^2)^{3/2}}{x^5}\,dx \\
&= -\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{10}a\text{Subst}\left(\int \frac{(1-a^2x)^{3/2}}{x^3}\,dx,x,x^2\right) \\
&= -\frac{a(1-a^2x^2)^{3/2}}{20x^4}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{1}{40}(3a^3)\,\text{Subst}\left(\int \frac{\sqrt{1-a}}{x^2}\right. \\
&= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a(1-a^2x^2)^{3/2}}{20x^4}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{80}(3a^5)\,\text{S} \\
&= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a(1-a^2x^2)^{3/2}}{20x^4}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{1}{40}(3a^3)\,\text{S} \\
&= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a(1-a^2x^2)^{3/2}}{20x^4}-\frac{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\tanh
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 1.11

$$\left(-\frac{a}{20x^4}+\frac{a^3}{8x^2}\right)\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\,(-1+a^2x^2)^2\tanh^{-1}(ax)}{5x^5}+\frac{3}{40}a^5\log(x)-\frac{3}{40}a^5\log\left(1+\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] (-1/20*a/x^4 + a^3/(8*x^2))*Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*(-1 + a^
2*x^2)^2*ArcTanh[a*x])/(5*x^5) + (3*a^5*Log[x])/40 - (3*a^5*Log[1 + Sqrt[1
- a^2*x^2]])/40
```

Maple [A]
time = 1.44, size = 116, normalized size = 1.23

method	result
default	$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(8a^{4}x^{4}\operatorname{arctanh}\left(ax\right)-5a^{3}x^{3}-16a^{2}x^{2}\operatorname{arctanh}\left(ax\right)+2ax+8\operatorname{arctanh}\left(ax\right)\right)}{40x^{5}}+\frac{3a^{5}\ln\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}}}\right.}{40}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)-5*a^3*x^3-16*a^2*x^2
*arctanh(a*x)+2*a*x+8*arctanh(a*x))/x^5+3/40*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1
/2)-1)-3/40*a^5*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.47, size = 126, normalized size = 1.34

$$\frac{1}{40}\left(\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}a^{4}-3\,a^{4}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+3\sqrt{-a^{2}x^{2}+1}\,a^{4}+\frac{\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}a^{2}}{x^{2}}-\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{4}}\right)a-\frac{\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}\operatorname{artanh}\left(ax\right)}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/40*((-a^2*x^2 + 1)^(3/2)*a^4 - 3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/
abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^4 + (-a^2*x^2 + 1)^(5/2)*a^2/x^2 - 2*(-a^2
*x^2 + 1)^(5/2)/x^4)*a - 1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/x^5
```

Fricas [A]
time = 0.40, size = 93, normalized size = 0.99

$$\frac{3\,a^{5}x^{5}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+\left(5\,a^{3}x^{3}-2\,ax-4\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^{2}x^{2}+1}}{40\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(3*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 - 2*a*x - 4*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**6,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**6, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^6,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^6, x)
```

3.458 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^7}\,dx$

Optimal. Leaf size=243

$$-\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}+\frac{31a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}+\frac{7a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24x^4}$$

```
[Out] -1/8*a^6*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/16*a^6*polylo
g(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/16*a^6*polylog(2,(-a*x+1)^(1/2)/(a*x+1
)^(1/2))-1/30*a*(-a^2*x^2+1)^(1/2)/x^5+19/360*a^3*(-a^2*x^2+1)^(1/2)/x^3+31
/720*a^5*(-a^2*x^2+1)^(1/2)/x-1/6*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6+7/24*
a^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4-1/16*a^4*arctanh(a*x)*(-a^2*x^2+1)^
(1/2)/x^2
```

Rubi [A]
time = 0.55, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6161, 6157, 6173, 277, 270, 6165}

$$\frac{1}{16}a^6\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{16}a^6\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{8}a^6\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}-\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{7a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{24x^4}+\frac{31a^5\sqrt{1-a^2x^2}}{720x}-\frac{a^4\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{16x^2}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

```
[Out] -1/30*(a*Sqrt[1 - a^2*x^2])/x^5 + (19*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (3
1*a^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6)
 + (7*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) - (a^4*Sqrt[1 - a^2*x^2]
*ArcTanh[a*x])/(16*x^2) - (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 +
a*x]])/8 + (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 - (a^6*PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6157

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTanh[c*x])/
Sqrt[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6161

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6165

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6173

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Dist[c^2*((
m + 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e
*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^7}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^5}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{x^7}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{5x^6}+\frac{a^2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{3x^4}-\frac{1}{5}\int \frac{\tanh^{-1}(a}{x^7\sqrt{1-a^2}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{25x^5}+\frac{a^3\sqrt{1-a^2x^2}}{9x^3}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{6x^6}+\frac{a^2\sqrt{1-a^2}}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{3a^3\sqrt{1-a^2x^2}}{100x^3}+\frac{2a^5\sqrt{1-a^2x^2}}{9x}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-}}{6x^6}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{13a^5\sqrt{1-a^2x^2}}{200x}-\frac{\sqrt{1-a^2x^2}\,\tan}{6x^6}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}+\frac{31a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\,\tan}{6x^6}$$

Mathematica [**A**]
time = 4.46, size = 474, normalized size = 1.95

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

```
[Out] (82*a^7*x^4*Csch[ArcTanh[a*x]/2]^2 + 90*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a
*x]*Csch[ArcTanh[a*x]/2]^2 + 4*a^7*x^4*Csch[ArcTanh[a*x]/2]^4 - 3*a^7*x^4*C
sch[ArcTanh[a*x]/2]^6 - 15*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Csch[ArcT
anh[a*x]/2]^6 + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-ArcT
anh[a*x])] - 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh
[a*x])] + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 36
0*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, E^(-ArcTanh[a*x])] + 328*a^5*x^2*(-1
 + a^2*x^2)*Sinh[ArcTanh[a*x]/2]^2 + 360*a^4*x*(1 - a^2*x^2)^(3/2)*ArcTanh[
a*x]*Sinh[ArcTanh[a*x]/2]^2 + 64*a^3*Sinh[ArcTanh[a*x]/2]^4 - 128*a^5*x^2*S
inh[ArcTanh[a*x]/2]^4 + 64*a^7*x^4*Sinh[ArcTanh[a*x]/2]^4 - (192*a*(-1 + a^
2*x^2)^3*Sinh[ArcTanh[a*x]/2]^6)/x^2 - (960*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x
]*Sinh[ArcTanh[a*x]/2]^6)/x^3)/(5760*x^3*Sqrt[1 - a^2*x^2])
```

Maple [**A**]
time = 1.57, size = 184, normalized size = 0.76

method	result

default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(-31a^5x^5+45a^4x^4\operatorname{arctanh}(ax)-38a^3x^3-210a^2x^2\operatorname{arctanh}(ax)+24ax+120\operatorname{arctanh}(ax))}{720x^6}+\frac{a^6 \text{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x,method=_RETURNVERBOSE)

[Out] -1/720*(-(a*x-1)*(a*x+1))^(1/2)*(-31*a^5*x^5+45*a^4*x^4*arctanh(a*x)-38*a^3*x^3-210*a^2*x^2*arctanh(a*x)+24*a*x+120*arctanh(a*x))/x^6+1/16*a^6*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="maxima")

[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^7, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="fricas")

[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**7,x)

[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**7, x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^7,x)
```

```
[Out] int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^7, x)
```

3.459 $\int (1-a^2x^2)^{5/2} \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=233

$$\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{5(1-a^2x^2)^{3/2}}{72a}+\frac{(1-a^2x^2)^{5/2}}{30a}+\frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{5}{24}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+\frac{1}{6}$$

```
[Out] 5/72*(-a^2*x^2+1)^(3/2)/a+1/30*(-a^2*x^2+1)^(5/2)/a+5/24*x*(-a^2*x^2+1)^(3/
2)*arctanh(a*x)+1/6*x*(-a^2*x^2+1)^(5/2)*arctanh(a*x)-5/8*arctan((-a*x+1)^(
1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-5/16*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+
1)^(1/2))/a+5/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+5/16*(-a^2*x
^2+1)^(1/2)/a+5/16*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6089, 6097}

$$\frac{(1-a^2x^2)^{5/2}}{30a}+\frac{5(1-a^2x^2)^{3/2}}{72a}+\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{1}{6}x(1-a^2x^2)^{5/2}\tanh^{-1}(ax)+\frac{5}{24}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+\frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{5\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{8a}-\frac{5i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{5i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

```
[Out] (5*Sqrt[1 - a^2*x^2])/(16*a) + (5*(1 - a^2*x^2)^(3/2))/(72*a) + (1 - a^2*x^
2)^(5/2)/(30*a) + (5*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 + (5*x*(1 - a^2*x
^2)^(3/2)*ArcTanh[a*x])/24 + (x*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/6 - (5*Ar
cTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a) - (((5*I)/16)*PolyLog
[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((5*I)/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
```

```
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+\frac{5}{6}\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}$$

$$=\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5}{24}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{1}{6}x(1-$$

$$=\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}$$

$$=\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}$$

Mathematica [A]
time = 0.84, size = 224, normalized size = 0.96

$$\frac{299\sqrt{1-a^2x^2}-98a^2x^2\sqrt{1-a^2x^2}+24a^4x^4\sqrt{1-a^2x^2}+495ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)-390a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+120a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-225i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+225i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)-225i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+225i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{720a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

```
[Out] (299*Sqrt[1 - a^2*x^2] - 98*a^2*x^2*Sqrt[1 - a^2*x^2] + 24*a^4*x^4*Sqrt[1 -
 a^2*x^2] + 495*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 390*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[a*x] + 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (225*I)
*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (225*I)*ArcTanh[a*x]*Log[1 + I/E^
ArcTanh[a*x]] - (225*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (225*I)*PolyLog[2
, I/E^ArcTanh[a*x]])/(720*a)
```

Maple [A]
time = 5.23, size = 193, normalized size = 0.83

method	result
default	$\frac{\left(120\,\text{arctanh}(ax)a^5x^5+24a^4x^4-390a^3x^3\,\text{arctanh}(ax)-98a^2x^2+495ax\,\text{arctanh}(ax)+299\right)\sqrt{-a^2x^2+1}}{720a}-\frac{5i\,\text{arctanh}(ax)\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(5/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/720*(120*arctanh(a*x)*a^5*x^5+24*a^4*x^4-390*a^3*x^3*arctanh(a*x)-98*a^2*
x^2+495*a*x*arctanh(a*x)+299)*(-a^2*x^2+1)^(1/2)/a-5/16*I/a*arctanh(a*x)*ln
```

```
(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))-5/16*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*dilo
g(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^(5/2)*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="fricas")
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(5/2)*atanh(a*x),x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(5/2), x)
```

3.460 $\int (1-a^2x^2)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=189

$$\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{(1-a^2x^2)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{4}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)-\frac{3\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{4a}$$

```
[Out] 1/12*(-a^2*x^2+1)^(3/2)/a+1/4*x*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-3/4*arctan(
(-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-3/8*I*polylog(2,-I*(-a*x+1)^(1
/2)/(a*x+1)^(1/2))/a+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3/8*
(-a^2*x^2+1)^(1/2)/a+3/8*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6089, 6097}

$$\frac{(1-a^2x^2)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{3\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a}-\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int (1-a^2x^2)^{3/2}\tanh^{-1}(ax)\,dx &= \frac{(1-a^2x^2)^{3/2}}{12a}+\frac{1}{4}x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)+\frac{3}{4}\int \sqrt{1-a^2x^2}\,\tanh^{-1}(a\\ &= \frac{3\sqrt{1-a^2x^2}}{8a}+\frac{(1-a^2x^2)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{4}x(1-a^2x\\ &= \frac{3\sqrt{1-a^2x^2}}{8a}+\frac{(1-a^2x^2)^{3/2}}{12a}+\frac{3}{8}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)+\frac{1}{4}x(1-a^2x\end{aligned}$$

Mathematica [A]
time = 0.06, size = 176, normalized size = 0.93

$$\frac{11\sqrt{1-a^2x^2}-2a^2x^2\sqrt{1-a^2x^2}+15ax\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-6a^3x^3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-9i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+9i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)-9i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+9i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*
x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcTanh[
a*x]])/(24*a)
```

Maple [A]
time = 0.00, size = 173, normalized size = 0.92

method	result
default	$-\frac{(6a^3x^3\operatorname{arctanh}(ax)+2a^2x^2-15ax\operatorname{arctanh}(ax)-11)\sqrt{-a^2x^2+1}}{24a}-\frac{3i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a}+\frac{3i\operatorname{arct}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+1
)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a*a
rctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)\,{\left(1-a^2\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

3.461 $\int \sqrt{1-a^2x^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+$$

```
[Out] -arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-1/2*I*polylog(2,-I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)/a+1/2*(-a^2*x^2+1)^(1/2)/a+1/2*x*arctanh(a*x)*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6089, 6097}

$$\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)-\frac{\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{2}\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}$$

Mathematica [A]
time = 0.05, size = 117, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(1+ax\tanh^{-1}(ax)-\frac{i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^Ar
cTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]]
- PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [A]
time = 0.00, size = 152, normalized size = 1.06

method	result
default	$\frac{\sqrt{-a^2x^2+1}\,(ax\,\mathrm{arctanh}(ax)+1)}{2a}-\frac{i\,\mathrm{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{i\,\mathrm{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(a*x*arctanh(a*x)+1)/a-1/2*I/a*arctanh(a*x)*ln(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
[Out] int(atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

3.462 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{9a\,(1-a^2x^2)^{3/2}}-\frac{2}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\,(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}$$

```
[Out] -1/9/a/(-a^2*x^2+1)^(3/2)+1/3*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-2/3/a/(-a^2
*x^2+1)^(1/2)+2/3*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6107, 6105}

$$-\frac{2}{3a\sqrt{1-a^2x^2}}-\frac{1}{9a\,(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\,(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a*(1 - a^2*x^2)^(3/2)) - 2/(3*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x
])/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*Sqrt[1 - a^2*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{5/2}}\,dx &= -\frac{1}{9a\,(1-a^2x^2)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3\,(1-a^2x^2)^{3/2}}+\frac{2}{3}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx\\ &= -\frac{1}{9a\,(1-a^2x^2)^{3/2}}-\frac{2}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\,(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 49, normalized size = 0.55

$$-\frac{7-6a^2x^2+(-9ax+6a^3x^3)\tanh^{-1}(ax)}{9a(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -1/9*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x])/(a*(1 - a^2*x^2)^(
3/2))
```

Maple [**A**]
time = 1.95, size = 59, normalized size = 0.66

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,(6a^3x^3\,\mathrm{arctanh}(ax)-6a^2x^2-9ax\,\mathrm{arctanh}(ax)+7)}{9a(a^2x^2-1)^2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a*(-a^2*x^2+1)^(1/2)*(6*a^3*x^3*arctanh(a*x)-6*a^2*x^2-9*a*x*arctanh(a
*x)+7)/(a^2*x^2-1)^2
```

Maxima [**A**]
time = 0.26, size = 74, normalized size = 0.83

$$-\frac{1}{9}\,a\left(\frac{6}{\sqrt{-a^2x^2+1}\,a^2}+\frac{1}{(-a^2x^2+1)^{\frac{3}{2}}a^2}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2x^2+1}}+\frac{x}{(-a^2x^2+1)^{\frac{3}{2}}}\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(6/(sqrt(-a^2*x^2 + 1)*a^2) + 1/((-a^2*x^2 + 1)^(3/2)*a^2)) + 1/3*(2
*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)
```

Fricas [**A**]
time = 0.37, size = 73, normalized size = 0.82

$$\frac{\left(12\,a^2x^2-3\,(2\,a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)\sqrt{-a^2x^2+1}}{18\,(a^5x^4-2\,a^3x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 14)*sq
rt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [A]
time = 0.46, size = 90, normalized size = 1.01

$$-\frac{\left(2\,a^2x^2-3\right)\sqrt{-a^2x^2+1}\,x\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(a^2x^2-1\right)^2}-\frac{6\,a^2x^2-7}{9\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*x^2 - 3)*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^
2 - 1)^2 - 1/9*(6*a^2*x^2 - 7)/((a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(1-a^2\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(5/2), x)
```

3.463 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{1}{25a\,(1-a^2x^2)^{5/2}}-\frac{4}{45a\,(1-a^2x^2)^{3/2}}-\frac{8}{15a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{5\,(1-a^2x^2)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15\,(1-a^2x^2)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15\sqrt{1-a^2x^2}}$$

```
[Out] -1/25/a/(-a^2*x^2+1)^(5/2)-4/45/a/(-a^2*x^2+1)^(3/2)+1/5*x*arctanh(a*x)/(-a
^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-8/15/a/(-a^2*x^2+1)^
(1/2)+8/15*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6107, 6105}

$$-\frac{8}{15a\sqrt{1-a^2x^2}}-\frac{4}{45a\,(1-a^2x^2)^{3/2}}-\frac{1}{25a\,(1-a^2x^2)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)}{15\,(1-a^2x^2)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\,(1-a^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] -1/25*1/(a*(1 - a^2*x^2)^(5/2)) - 4/(45*a*(1 - a^2*x^2)^(3/2)) - 8/(15*a*Sq
rt[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[
a*x])/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*Sqrt[1 - a^2*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{7/2}}\,dx = -\frac{1}{25a(1-a^2x^2)^{5/2}} + \frac{x\tanh^{-1}(ax)}{5(1-a^2x^2)^{5/2}} + \frac{4}{5}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{5/2}}\,dx$$
$$= -\frac{1}{25a(1-a^2x^2)^{5/2}} - \frac{4}{45a(1-a^2x^2)^{3/2}} + \frac{x\tanh^{-1}(ax)}{5(1-a^2x^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15(1-a^2x^2)^{3/2}} + \frac{8}{15}\int$$
$$= -\frac{1}{25a(1-a^2x^2)^{5/2}} - \frac{4}{45a(1-a^2x^2)^{3/2}} - \frac{8}{15a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{5(1-a^2x^2)^{5/2}} + \frac{4x\tan}{15(1-}$$

Mathematica [A]
time = 0.05, size = 65, normalized size = 0.49

$$\frac{-149+260a^2x^2-120a^4x^4+15ax(15-20a^2x^2+8a^4x^4)\tanh^{-1}(ax)}{225a(1-a^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-149 + 260*a^2*x^2 - 120*a^4*x^4 + 15*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*Ar
cTanh[a*x])/(225*a*(1 - a^2*x^2)^(5/2))
```

Maple [A]
time = 1.81, size = 79, normalized size = 0.59

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,(120\,\mathrm{arctanh}(ax)a^5x^5-120a^4x^4-300a^3x^3\,\mathrm{arctanh}(ax)+260a^2x^2+225ax\,\mathrm{arctanh}(ax)-149)}{225a(a^2x^2-1)^3}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/225/a*(-a^2*x^2+1)^(1/2)*(120*arctanh(a*x)*a^5*x^5-120*a^4*x^4-300*a^3*x
^3*arctanh(a*x)+260*a^2*x^2+225*a*x*arctanh(a*x)-149)/(a^2*x^2-1)^3
```

Maxima [A]
time = 0.27, size = 108, normalized size = 0.81

$$-\frac{1}{225}a\left(\frac{120}{\sqrt{-a^2x^2+1}\,a^2}+\frac{20}{(-a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{9}{(-a^2x^2+1)^{\frac{5}{2}}a^2}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{3\,x}{(-a^2x^2+1)^{\frac{5}{2}}}\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

[Out] -1/225*a*(120/(sqrt(-a^2*x^2 + 1)*a^2) + 20/((-a^2*x^2 + 1)^(3/2)*a^2) + 9/((-a^2*x^2 + 1)^(5/2)*a^2)) + 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) + 3*x/(-a^2*x^2 + 1)^(5/2))*arctanh(a*x)

Fricas [A]
time = 0.39, size = 99, normalized size = 0.74

$$\frac{\left(240\,a^4x^4-520\,a^2x^2-15\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+298\right)\sqrt{-a^2x^2+1}}{450\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*log(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(ax\right)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

Giac [A]
time = 0.44, size = 114, normalized size = 0.86

$$-\frac{\sqrt{-a^2x^2+1}\,(4\,(2\,a^4x^2-5\,a^2)x^2+15)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^2x^2-1\right)^3}+\frac{20\,a^2x^2-120\left(a^2x^2-1\right)^2-29}{225\left(a^2x^2-1\right)^2\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a^2*x^2 + 1)*(4*(2*a^4*x^2 - 5*a^2)*x^2 + 15)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1)^3 + 1/225*(20*a^2*x^2 - 120*(a^2*x^2 - 1)^2 - 29)/((a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{atanh}(a\,x)}{{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(7/2),x)
```

```
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(7/2), x)
```

3.464 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{16}{35a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tanh^{-1}(}{35\left(1-a^2x\right.}$$

```
[Out] -1/49/a/(-a^2*x^2+1)^(7/2)-6/175/a/(-a^2*x^2+1)^(5/2)-8/105/a/(-a^2*x^2+1)^
(3/2)+1/7*x*arctanh(a*x)/(-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)/(-a^2*x^2+1
)^(5/2)+8/35*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-16/35/a/(-a^2*x^2+1)^(1/2)+1
6/35*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6107, 6105}

$$-\frac{16}{35a\sqrt{1-a^2x^2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] -1/49*1/(a*(1 - a^2*x^2)^(7/2)) - 6/(175*a*(1 - a^2*x^2)^(5/2)) - 8/(105*a*
(1 - a^2*x^2)^(3/2)) - 16/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(7*(1
 - a^2*x^2)^(7/2)) + (6*x*ArcTanh[a*x])/(35*(1 - a^2*x^2)^(5/2)) + (8*x*Arc
Tanh[a*x])/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*Sqrt[1 - a^2*
x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{9/2}}\,dx = -\frac{1}{49a\,(1-a^2x^2)^{7/2}} + \frac{x\tanh^{-1}(ax)}{7\,(1-a^2x^2)^{7/2}} + \frac{6}{7}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{7/2}}\,dx$$

$$= -\frac{1}{49a\,(1-a^2x^2)^{7/2}} - \frac{6}{175a\,(1-a^2x^2)^{5/2}} + \frac{x\tanh^{-1}(ax)}{7\,(1-a^2x^2)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35\,(1-a^2x^2)^{5/2}} + \frac{24}{35}$$

$$= -\frac{1}{49a\,(1-a^2x^2)^{7/2}} - \frac{6}{175a\,(1-a^2x^2)^{5/2}} - \frac{8}{105a\,(1-a^2x^2)^{3/2}} + \frac{x\tanh^{-1}(ax)}{7\,(1-a^2x^2)^{7/2}} +$$

$$= -\frac{1}{49a\,(1-a^2x^2)^{7/2}} - \frac{6}{175a\,(1-a^2x^2)^{5/2}} - \frac{8}{105a\,(1-a^2x^2)^{3/2}} - \frac{16}{35a\sqrt{1-a^2x^2}} +$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.46

$$\frac{-2161+5726a^2x^2-5320a^4x^4+1680a^6x^6-105ax(-35+70a^2x^2-56a^4x^4+16a^6x^6)\tanh^{-1}(ax)}{3675a\,(1-a^2x^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6 - 105*a*x*(-35 + 70*a^2
*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x])/(3675*a*(1 - a^2*x^2)^(7/2))
```

Maple [A]
time = 1.81, size = 99, normalized size = 0.56

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,(1680\,\mathrm{arctanh}(ax)a^7x^7-1680a^6x^6-5880\,\mathrm{arctanh}(ax)a^5x^5+5320a^4x^4+7350a^3x^3\,\mathrm{arctanh}(ax)-5726a^2x^2-367}{3675a(a^2x^2-1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3675/a*(-a^2*x^2+1)^(1/2)*(1680*arctanh(a*x)*a^7*x^7-1680*a^6*x^6-5880*a
rctanh(a*x)*a^5*x^5+5320*a^4*x^4+7350*a^3*x^3*arctanh(a*x)-5726*a^2*x^2-367
5*a*x*arctanh(a*x)+2161)/(a^2*x^2-1)^4
```

Maxima [A]
time = 0.26, size = 140, normalized size = 0.79

$$-\frac{1}{3675}a\left(\frac{1680}{\sqrt{-a^2x^2+1}\,a^2}+\frac{280}{(-a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{126}{(-a^2x^2+1)^{\frac{5}{2}}a^2}+\frac{75}{(-a^2x^2+1)^{\frac{7}{2}}a^2}\right)+\frac{1}{35}\left(\frac{16\,x}{\sqrt{-a^2x^2+1}}+\frac{8\,x}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{6\,x}{(-a^2x^2+1)^{\frac{5}{2}}}+\frac{5\,x}{(-a^2x^2+1)^{\frac{7}{2}}}\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3675*a*(1680/(sqrt(-a^2*x^2 + 1)*a^2) + 280/((-a^2*x^2 + 1)^(3/2)*a^2) +
 126/((-a^2*x^2 + 1)^(5/2)*a^2) + 75/((-a^2*x^2 + 1)^(7/2)*a^2)) + 1/35*(16
*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 + 1)^(5/2)
 + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)
```

Fricas [**A**]
time = 0.37, size = 121, normalized size = 0.68

$$\frac{\left(3360\,a^{6}x^{6}-10640\,a^{4}x^{4}+11452\,a^{2}x^{2}-105\left(16\,a^{7}x^{7}-56\,a^{5}x^{5}+70\,a^{3}x^{3}-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-4322\right)\sqrt{-a^{2}x^{2}+1}}{7350\left(a^{9}x^{8}-4\,a^{7}x^{6}+6\,a^{5}x^{4}-4\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7350*(3360*a^6*x^6 - 10640*a^4*x^4 + 11452*a^2*x^2 - 105*(16*a^7*x^7 - 56
*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 4322)*sqrt(-a^2
*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(ax\right)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

Giac [**A**]
time = 0.43, size = 138, normalized size = 0.78

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,(2\,(4\,(2\,a^{6}x^{2}-7\,a^{4})x^{2}+35\,a^{2})x^{2}-35)x\log\left(-\frac{ax+1}{ax-1}\right)}{70\left(a^{2}x^{2}-1\right)^{4}}-\frac{126\,a^{2}x^{2}+1680\left(a^{2}x^{2}-1\right)^{3}-280\left(a^{2}x^{2}-1\right)^{2}-201}{3675\left(a^{2}x^{2}-1\right)^{3}\sqrt{-a^{2}x^{2}+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] -1/70*sqrt(-a^2*x^2 + 1)*(2*(4*(2*a^6*x^2 - 7*a^4)*x^2 + 35*a^2)*x^2 - 35)*
x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1)^4 - 1/3675*(126*a^2*x^2 + 1680*(a
^2*x^2 - 1)^3 - 280*(a^2*x^2 - 1)^2 - 201)/((a^2*x^2 - 1)^3*sqrt(-a^2*x^2 +
 1)*a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atanh}(a\,x)}{{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(1 - a^2*x^2)^(9/2),x)
[Out] int(atanh(a*x)/(1 - a^2*x^2)^(9/2), x)
```

3.465 $\int (c-a^2cx^2)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=291

$$\frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{(c-a^2cx^2)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x(c-a^2cx^2)^{3/2}\tanh^{-1}(ax)-\frac{3c^2\sqrt{1-a^2x^2}}{}$$

```
[Out] 1/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*x*(-a^2*c*x^2+c)^(3/2)*arctanh(a*x)-3/4*c^2
*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/(-a
^2*c*x^2+c)^(1/2)-3/8*I*c^2*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^
2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+3/8*I*c^2*polylog(2,I*(-a*x+1)^(1/2)/
(a*x+1)^(1/2))*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+3/8*c*(-a^2*c*x^2+
c)^(1/2)/a+3/8*c*x*arctanh(a*x)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6089, 6101, 6097}

$$-\frac{3c^2\sqrt{1-a^2x^2}\,\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a\sqrt{c-a^2cx^2}}-\frac{3ic^2\sqrt{1-a^2x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}+\frac{3ic^2\sqrt{1-a^2x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}+\frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{(c-a^2cx^2)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x(c-a^2cx^2)^{3/2}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*c*Sqrt[c - a^2*c*x^2])/(8*a) + (c - a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sqr
t[c - a^2*c*x^2]*ArcTanh[a*x])/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x])/4
 - (3*c^2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x
])/(4*a*Sqrt[c - a^2*c*x^2]) - (((3*I)/8)*c^2*Sqrt[1 - a^2*x^2]*PolyLog[2,
((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (((3*I)/8)*c
^2*Sqrt[1 - a^2*x^2]*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c
 - a^2*c*x^2])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
```

```
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6101

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)\,dx = \frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)+\frac{1}{4}(3c)\int \sqrt{c-a^2cx^2}\ \text{t}$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x(c$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x(c$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x(c$$

Mathematica [A]
time = 0.46, size = 206, normalized size = 0.71

$$-\frac{c\sqrt{c-a^2cx^2}\left(-11\sqrt{1-a^2x^2}+2a^2x^2\sqrt{1-a^2x^2}-15ax\sqrt{1-a^2x^2}\tanh^{-1}(ax)+6a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+9i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-9i\tanh^{-1}(ax)\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+9i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-9i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{24a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] -1/24*(c*Sqrt[c - a^2*c*x^2]*(-11*Sqrt[1 - a^2*x^2] + 2*a^2*x^2*Sqrt[1 - a^
2*x^2] - 15*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*a^3*x^3*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x] + (9*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (9*I)*ArcTa
nh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] -
 (9*I)*PolyLog[2, I/E^ArcTanh[a*x]]))/(a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 2.62, size = 345, normalized size = 1.19

method	result
default	$-\frac{c\sqrt{-\left(ax-1\right)\left(ax+1\right)c}\,\left(6a^{3}x^{3}\operatorname{arctanh}\left(ax\right)+2a^{2}x^{2}-15ax\operatorname{arctanh}\left(ax\right)-11\right)}{24a}+\frac{3ic\sqrt{-a^{2}x^{2}+1}\,\sqrt{-\left(ax-1\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*c/a*(-(a*x-1)*(a*x+1)*c)^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a
*x*arctanh(a*x)-11)+3/8*I*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*
(a*x+1)*c)^(1/2)*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*c/a/
(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*arctanh(a*x)*
ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*
x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I
*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1-
I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arctanh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)*(c - a^2*c*x^2)^(3/2), x)
```

3.466 $\int \sqrt{c-a^2cx^2}\,\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\,\tanh^{-1}(ax)-\frac{c\sqrt{1-a^2x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{ic\sqrt{1-a^2x^2}\,\mathrm{Pol}}{2a\sqrt{}}$$

```
[Out] -c*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/(
-a^2*c*x^2+c)^(1/2)-1/2*I*c*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^
2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+1/2*I*c*polylog(2,I*(-a*x+1)^(1/2)/(a
*x+1)^(1/2))*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*c*x^2+c)^(
1/2)/a+1/2*x*arctanh(a*x)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6089, 6101, 6097}

$$-\frac{c\sqrt{1-a^2x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{ic\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{ic\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\,\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a) + (x*Sqrt[c - a^2*c*x^2]*ArcTanh[a*x])/2 - (c*Sqr
t[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*Sqrt[c
- a^2*c*x^2]) - ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLo
g[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
```

```
0]
```

Rule 6101

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\tanh^{-1}(ax)\,dx = \frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{2}c\int\frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx$$

$$=\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{\left(c\sqrt{1-a^2x^2}\right)\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x}}}{2\sqrt{c-a^2cx^2}}$$

$$=\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)-\frac{c\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.22, size = 119, normalized size = 0.51

$$\frac{\sqrt{c(1-a^2x^2)}\left(1+ax\tanh^{-1}(ax)-\frac{i\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c*(1 - a^2*x^2)]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/
E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*
x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [A]
time = 1.83, size = 319, normalized size = 1.36

method	result
default	$\frac{(ax\,\mathrm{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)c}}{2a}+\frac{i\sqrt{-a^2x^2+1}\,\sqrt{-(ax-1)(ax+1)c}\,\mathrm{arctanh}(ax)\ln\left(1+\right.}{2a(ax+1)(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a+1/2*I/a/(a*x+1)*(-a^2
*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*arctanh(a*x)*ln(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))-1/2*I/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*
(a*x+1)*c)^(1/2)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a/(a
*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1+I*(a*x+
1)/(-a^2*x^2+1)^(1/2))-1/2*I/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)
*(a*x+1)*c)^(1/2)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(ax-1\right)\left(ax+1\right)}\ \operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(a\,x)\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)*(c - a^2*c*x^2)^(1/2), x)
```

3.467 $\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=182

$$-\frac{2\sqrt{1-a^2x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{i\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\mathrm{Poly}}{a\sqrt{c}}$$

```
[Out] -2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/(
-a^2*c*x^2+c)^(1/2)-I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*x^2+
1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)+I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))
*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6101, 6097}

$$-\frac{2\sqrt{1-a^2x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{i\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*
Sqrt[c - a^2*c*x^2]) - (I*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])
/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (I*Sqrt[1 - a^2*x^2]*PolyLog[2,
(I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6101

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{2\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}} - \frac{i\sqrt{1-a^2x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} +$$

Mathematica [A]
time = 0.08, size = 109, normalized size = 0.60

$$-\frac{i\sqrt{c(1-a^2x^2)}\left(\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((-I)*Sqrt[c*(1 - a^2*x^2)]*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[
1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^A
rcTanh[a*x]]))/(a*c*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 1.60, size = 302, normalized size = 1.66

method	result
default	$\frac{i\sqrt{-a^2x^2+1}\,\sqrt{-(ax-1)(ax+1)c}\,\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{(ax-1)(ax+1)ac} - \frac{i\sqrt{-a^2x^2+1}\,\sqrt{-(ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1)*c)^(1/2)*arctanh(a*x)*ln(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+1)/a/c-I*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*
x+1)*c)^(1/2)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+
1)/a/c+I*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+1)/a/c-I*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1
)*c)^(1/2)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+1)/a/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arctanh(a*x)/(a^2*c*x^2 - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(1/2), x)
```

3.468 $\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{ac\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}$$

```
[Out] -1/a/c/(-a^2*c*x^2+c)^(1/2)+x*arctanh(a*x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6105}

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
[Out] -(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a*x])/(c*Sqrt[c - a^2*c*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{1}{ac\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.90

$$\frac{\sqrt{c-a^2cx^2}\left(1-ax\tanh^{-1}(ax)\right)}{ac^2\left(-1+a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[c - a^2*c*x^2]*(1 - a*x*ArcTanh[a*x]))/(a*c^2*(-1 + a^2*x^2))
```

Maple [A]
time = 1.60, size = 74, normalized size = 1.54

method	result	size
default	$-\frac{(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)c}}{2a(ax-1)c^2}-\frac{(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)c}}{2a(ax+1)c^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^2-1/2*(arctanh
(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(44) = 88$.
time = 0.47, size = 90, normalized size = 1.88

$$-\frac{a^2\left(\frac{\sqrt{-a^2cx^2+c}}{a^4cx+a^3c}-\frac{\sqrt{-a^2cx^2+c}}{a^4cx-a^3c}\right)}{2\,c}+\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(sqrt(-a^2*c*x^2 + c)/(a^4*c*x + a^3*c) - sqrt(-a^2*c*x^2 + c)/(a^
4*c*x - a^3*c))/c + x*arctanh(a*x)/(sqrt(-a^2*c*x^2 + c)*c)
```

Fricas [A]
time = 0.36, size = 54, normalized size = 1.12

$$-\frac{\sqrt{-a^2cx^2+c}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\left(a^3c^2x^2-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*c^2*x^2
- a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}(ax)}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.45, size = 70, normalized size = 1.46

$$-\frac{\sqrt{-a^2cx^2+c}\,x\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^2cx^2-c\right)c}-\frac{1}{\sqrt{-a^2cx^2+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1))/((a^2*c*x^2 - c)*c) -
 1/(sqrt(-a^2*c*x^2 + c)*a*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atanh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(3/2), x)
```

3.469 $\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}$$

```
[Out] -1/9/a/c/(-a^2*c*x^2+c)^(3/2)+1/3*x*arctanh(a*x)/c/(-a^2*c*x^2+c)^(3/2)-2/3
/a/c^2/(-a^2*c*x^2+c)^(1/2)+2/3*x*arctanh(a*x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6107, 6105}

$$-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}-\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a*c*(c - a^2*c*x^2)^(3/2)) - 2/(3*a*c^2*Sqrt[c - a^2*c*x^2]) + (x*A
rcTanh[a*x])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c
 - a^2*c*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{1}{9ac\,(c-a^2cx^2)^{3/2}} + \frac{x\tanh^{-1}(ax)}{3c\,(c-a^2cx^2)^{3/2}} + \frac{2\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{3/2}}\,dx}{3c}$$
$$= -\frac{1}{9ac\,(c-a^2cx^2)^{3/2}} - \frac{2}{3ac^2\sqrt{c-a^2cx^2}} + \frac{x\tanh^{-1}(ax)}{3c\,(c-a^2cx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.05, size = 64, normalized size = 0.61

$$-\frac{\sqrt{c-a^2cx^2}\,\left(7-6a^2x^2+\left(-9ax+6a^3x^3\right)\tanh^{-1}(ax)\right)}{9ac^3\left(-1+a^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/9*(Sqrt[c - a^2*c*x^2]*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x
]))/(a*c^3*(-1 + a^2*x^2)^2)
```

Maple [A]
time = 1.60, size = 160, normalized size = 1.52

method	result
default	$\frac{(ax+1)(3\,\mathrm{arctanh}(ax)-1)\sqrt{-\left(ax-1\right)\left(ax+1\right)c}}{72a(ax-1)^2c^3} - \frac{3(\mathrm{arctanh}(ax)-1)\sqrt{-\left(ax-1\right)\left(ax+1\right)c}}{8a\,c^3(ax-1)} - \frac{3(\mathrm{arctanh}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(a*x+1)*(3*arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)^2/c^3-
3/8*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/c^3/(a*x-1)-3/8*(arctanh(
a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^3+1/72*(a*x-1)*(1+3*arctanh(
a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)^2/c^3
```

Maxima [A]
time = 0.28, size = 90, normalized size = 0.86

$$-\frac{1}{9}\,a\left(\frac{6}{\sqrt{-a^2cx^2+c}\,a^2c^2}+\frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2c}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2cx^2+c}\,c^2}+\frac{x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}c}\right)\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(6/(sqrt(-a^2*c*x^2 + c)*a^2*c^2) + 1/((-a^2*c*x^2 + c)^(3/2)*a^2*c)
) + 1/3*(2*x/(sqrt(-a^2*c*x^2 + c)*c^2) + x/((-a^2*c*x^2 + c)^(3/2)*c))*arc
tanh(a*x)
```

Fricas [A]
time = 0.38, size = 84, normalized size = 0.80

$$\frac{\sqrt{-a^2cx^2+c}\left(12\,a^2x^2-3\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)}{18\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a^2*c*x^2 + c)*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1
)/(a*x - 1)) - 14)/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(ax\right)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [A]
time = 0.47, size = 111, normalized size = 1.06

$$-\frac{\sqrt{-a^2cx^2+c}\left(\frac{2\,a^2x^2}{c}-\frac{3}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(a^2cx^2-c\right)^2}-\frac{6\,a^2cx^2-7\,c}{9\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*c*x^2 + c)*(2*a^2*x^2/c - 3/c)*x*log(-(a*x + 1)/(a*x - 1))/(
a^2*c*x^2 - c)^2 - 1/9*(6*a^2*c*x^2 - 7*c)/((a^2*c*x^2 - c)*sqrt(-a^2*c*x^2
 + c)*a*c^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{atanh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(5/2), x)
```

3.470 $\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8}{15}$$

```
[Out] -1/25/a/c/(-a^2*c*x^2+c)^(5/2)-4/45/a/c^2/(-a^2*c*x^2+c)^(3/2)+1/5*x*arctan
h(a*x)/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arctanh(a*x)/c^2/(-a^2*c*x^2+c)^(3/2)-
8/15/a/c^3/(-a^2*c*x^2+c)^(1/2)+8/15*x*arctanh(a*x)/c^3/(-a^2*c*x^2+c)^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6107, 6105}

$$-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/25*1/(a*c*(c - a^2*c*x^2)^(5/2)) - 4/(45*a*c^2*(c - a^2*c*x^2)^(3/2)) -
8/(15*a*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcTanh[a*x])/(5*c*(c - a^2*c*x^2)^(5
/2)) + (4*x*ArcTanh[a*x])/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcTanh[a*x
])/(15*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{1}{25ac\,(c-a^2cx^2)^{5/2}} + \frac{x\tanh^{-1}(ax)}{5c\,(c-a^2cx^2)^{5/2}} + \frac{4\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{5/2}}\,dx}{5c}$$

$$= -\frac{1}{25ac\,(c-a^2cx^2)^{5/2}} - \frac{4}{45ac^2\,(c-a^2cx^2)^{3/2}} + \frac{x\tanh^{-1}(ax)}{5c\,(c-a^2cx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c-a^2cx^2)^{3/}}$$

$$= -\frac{1}{25ac\,(c-a^2cx^2)^{5/2}} - \frac{4}{45ac^2\,(c-a^2cx^2)^{3/2}} - \frac{8}{15ac^3\sqrt{c-a^2cx^2}} + \frac{x\tanh^{-1}(ax)}{5c\,(c-a^2cx^2)^{5/2}}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.51

$$\frac{\sqrt{c-a^2cx^2}\,\left(149-260a^2x^2+120a^4x^4-15ax(15-20a^2x^2+8a^4x^4)\tanh^{-1}(ax)\right)}{225ac^4\,(-1+a^2x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(149 - 260*a^2*x^2 + 120*a^4*x^4 - 15*a*x*(15 - 20*a^2
*x^2 + 8*a^4*x^4)*ArcTanh[a*x]))/(225*a*c^4*(-1 + a^2*x^2)^3)
```

Maple [A]
time = 1.63, size = 250, normalized size = 1.59

method	result
default	$-\frac{(ax+1)^2(-1+5\,\mathrm{arctanh}(ax))\sqrt{-(ax-1)(ax+1)c}}{800a(ax-1)^3c^4} + \frac{5(ax+1)(3\,\mathrm{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)c}}{288a\,c^4(ax-1)^2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/800*(a*x+1)^2*(-1+5*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)^3
/c^4+5/288*(a*x+1)*(3*arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/c^4/(a*x
-1)^2-5/16*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/c^4/(a*x-1)-5/16*(
arctanh(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^4+5/288*(a*x-1)*(1+3
*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)^2/c^4-1/800*(a*x-1)^2*(
1+5*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)^3/a/c^4
```

Maxima [A]
time = 0.28, size = 132, normalized size = 0.84

$$-\frac{1}{225}a\left(\frac{120}{\sqrt{-a^2cx^2+c}\,a^2c^3} + \frac{20}{(-a^2cx^2+c)^{\frac{3}{2}}a^2c^2} + \frac{9}{(-a^2cx^2+c)^{\frac{5}{2}}a^2c}\right) + \frac{1}{15}\left(\frac{8x}{\sqrt{-a^2cx^2+c}\,c^3} + \frac{4x}{(-a^2cx^2+c)^{\frac{3}{2}}c^2} + \frac{3x}{(-a^2cx^2+c)^{\frac{5}{2}}c}\right)\mathrm{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/225*a*(120/(sqrt(-a^2*c*x^2 + c)*a^2*c^3) + 20/((-a^2*c*x^2 + c)^(3/2)*a
^2*c^2) + 9/((-a^2*c*x^2 + c)^(5/2)*a^2*c)) + 1/15*(8*x/(sqrt(-a^2*c*x^2 +
c)*c^3) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)*c)
)*arctanh(a*x)
```

Fricas [A]
time = 0.43, size = 112, normalized size = 0.71

$$\frac{\left(240\,a^4x^4-520\,a^2x^2-15\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+298\right)\sqrt{-a^2cx^2+c}}{450\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*log
(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*c*x^2 + c)/(a^7*c^4*x^6 - 3*a^5*c^4
*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(ax\right)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Giac [A]
time = 0.47, size = 149, normalized size = 0.95

$$-\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2\,a^4x^2}{c}-\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^2cx^2-c\right)^3}-\frac{120\left(a^2cx^2-c\right)^2-20\left(a^2cx^2-c\right)c+9\,c^2}{225\left(a^2cx^2-c\right)^2\sqrt{-a^2cx^2+c}\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c - 5*a^2/c)*x^2 + 15/c)*x*log(-(a
*x + 1)/(a*x - 1))/(a^2*c*x^2 - c)^3 - 1/225*(120*(a^2*c*x^2 - c)^2 - 20*(a
^2*c*x^2 - c)*c + 9*c^2)/((a^2*c*x^2 - c)^2*sqrt(-a^2*c*x^2 + c)*a*c^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c - a^2*c*x^2)^(7/2),x)
[Out] int(atanh(a*x)/(c - a^2*c*x^2)^(7/2), x)
```

3.471 $\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=158

$$-\frac{\text{ArcSin}(ax)}{a}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+\frac{\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a}-\frac{i$$

```
[Out] -arcsin(a*x)/a+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-I*arctan
h(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*arctanh(a*x)*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+arctanh(a*x)*(-a^2*x^2+1)^(1/
2)/a+1/2*x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6091, 6099, 4265, 2611, 2320, 6724, 222}

$$\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a}-\frac{\text{ArcSin}(ax)}{a}+\frac{\tanh^{-1}(ax)^2\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 \, dx = \frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 + \frac{\mathrm{Su}}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 + \frac{\mathrm{ta}}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 + \frac{\mathrm{ta}}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 + \frac{\mathrm{ta}}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2} \tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2} \tanh^{-1}(ax)^2 + \frac{\mathrm{ta}}{}$$

Mathematica [A]
time = 0.07, size = 187, normalized size = 1.18

$$\frac{\sqrt{1-a^2x^2}\left(2\tanh^{-1}(ax)+ax\tanh^{-1}(ax)^2-\frac{i\left(-4i\text{ArcTan}\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)+\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)^2\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\ \operatorname{arctanh}(ax)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

3.472 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2x}{27(1-a^2x^2)^{3/2}}+\frac{40x}{27\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a(1-a^2x^2)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}$$

```
[Out] 2/27*x/(-a^2*x^2+1)^(3/2)-2/9*arctanh(a*x)/a/(-a^2*x^2+1)^(3/2)+1/3*x*arcta
nh(a*x)^2/(-a^2*x^2+1)^(3/2)+40/27*x/(-a^2*x^2+1)^(1/2)-4/3*arctanh(a*x)/a/
(-a^2*x^2+1)^(1/2)+2/3*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6111, 6109, 197, 198}

$$\frac{40x}{27\sqrt{1-a^2x^2}}+\frac{2x}{27(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2*x)/(27*(1 - a^2*x^2)^(3/2)) + (40*x)/(27*Sqrt[1 - a^2*x^2]) - (2*ArcTanh
[a*x])/(9*a*(1 - a^2*x^2)^(3/2)) - (4*ArcTanh[a*x])/(3*a*Sqrt[1 - a^2*x^2])
 + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^2)/(3*Sqr
t[1 - a^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
```

```
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = -\frac{2\tanh^{-1}(ax)}{9a\,(1-a^2x^2)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{3\,(1-a^2x^2)^{3/2}}+\frac{2}{9}\int \frac{1}{(1-a^2x^2)^{5/2}}\,dx+\frac{2}{3}\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$$
$$=\frac{2x}{27\,(1-a^2x^2)^{3/2}}-\frac{2\tanh^{-1}(ax)}{9a\,(1-a^2x^2)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3\,(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(a}{3\sqrt{1-a^2x}}$$
$$=\frac{2x}{27\,(1-a^2x^2)^{3/2}}+\frac{40x}{27\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a\,(1-a^2x^2)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax}{3\,(1-a^2x^2)^{3}}$$

Mathematica [A]
time = 0.06, size = 70, normalized size = 0.50

$$\frac{42ax-40a^3x^3+6(-7+6a^2x^2)\tanh^{-1}(ax)-9ax(-3+2a^2x^2)\tanh^{-1}(ax)^2}{27a\,(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (42*a*x - 40*a^3*x^3 + 6*(-7 + 6*a^2*x^2)*ArcTanh[a*x] - 9*a*x*(-3 + 2*a^2*
x^2)*ArcTanh[a*x]^2)/(27*a*(1 - a^2*x^2)^(3/2))
```

Maple [A]
time = 0.70, size = 84, normalized size = 0.60

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(18\,\mathrm{arctanh}(ax)^2a^3x^3+40a^3x^3-36a^2x^2\,\mathrm{arctanh}(ax)-27\,\mathrm{arctanh}(ax)^2ax-42ax+42\,\mathrm{arctanh}(ax)\right)}{27a(a^2x^2-1)^2}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^2*a^3*x^3+40*a^3*x^3-36*a^2*x^2
*arctanh(a*x)-27*arctanh(a*x)^2*a*x-42*a*x+42*arctanh(a*x))/(a^2*x^2-1)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(115) = 230$.
time = 0.52, size = 304, normalized size = 2.19

$$\frac{1}{3}\left(\frac{2x}{\sqrt{-a^2x^2+1}}+\frac{x}{(-a^2x^2+1)^{\frac{3}{2}}}\right)\operatorname{artanh}(ax)^2+\frac{1}{27}a\left(\frac{\frac{2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2x+\sqrt{-a^2x^2+1}\,a}}{a}+\frac{\frac{2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2x-\sqrt{-a^2x^2+1}\,a}}{a}-\frac{18\sqrt{-a^2x^2+1}}{(a^2x+a)a}-\frac{18\sqrt{-a^2x^2+1}}{(a^2x-a)a}-\frac{18\log(ax+1)}{\sqrt{-a^2x^2+1}\,a^2}+\frac{18\log(-ax+1)}{\sqrt{-a^2x^2+1}\,a^2}-\frac{3\log(ax+1)}{(-a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{3\log(-ax+1)}{(-a^2x^2+1)^{\frac{3}{2}}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)^2 + 1/27
*a*((2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 +
 1)*a))/a + (2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a
^2*x^2 + 1)*a))/a - 18*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 18*sqrt(-a^2*x^
2 + 1)/((a^2*x - a)*a) - 18*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 18*log(
-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 3*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a
^2) + 3*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2))
```

Fricas [A]
time = 0.36, size = 105, normalized size = 0.76

$$-\frac{\left(160\,a^3x^3+9\,(2\,a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-168\,ax-12\,(6\,a^2x^2-7)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{108\,(a^5x^4-2\,a^3x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/108*(160*a^3*x^3 + 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 1
68*a*x - 12*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(
a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^2}{{\left(1-a^2\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(5/2), x)
```

3.473 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=208

$$\frac{2x}{125\left(1-a^2x^2\right)^{5/2}}+\frac{272x}{3375\left(1-a^2x^2\right)^{3/2}}+\frac{4144x}{3375\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}}-\frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{16\tanh^{-1}}{15a\sqrt{1-}}$$

```
[Out] 2/125*x/(-a^2*x^2+1)^(5/2)+272/3375*x/(-a^2*x^2+1)^(3/2)-2/25*arctanh(a*x)/
a/(-a^2*x^2+1)^(5/2)-8/45*arctanh(a*x)/a/(-a^2*x^2+1)^(3/2)+1/5*x*arctanh(a
*x)^2/(-a^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2)+4144/3375
*x/(-a^2*x^2+1)^(1/2)-16/15*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+8/15*x*arctan
h(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6111, 6109, 197, 198}

$$\frac{4144x}{3375\sqrt{1-a^2x^2}}+\frac{272x}{3375\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{125\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^2}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^2}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}}-\frac{16\tanh^{-1}(ax)}{15a\sqrt{1-a^2x^2}}-\frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (2*x)/(125*(1 - a^2*x^2)^(5/2)) + (272*x)/(3375*(1 - a^2*x^2)^(3/2)) + (414
4*x)/(3375*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(25*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x])/(45*a*(1 - a^2*x^2)^(3/2)) - (16*ArcTanh[a*x])/(15*a*Sq
rt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTan
h[a*x]^2)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^2)/(15*Sqrt[1 - a^2*
x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
```

```
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx &= -\frac{2\tanh^{-1}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{x\tanh^{-1}(ax)^2}{5(1-a^2x^2)^{5/2}}+\frac{2}{25}\int\frac{1}{(1-a^2x^2)^{7/2}}\,dx+\frac{4}{5}\int\frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{5/2}}\,d\cdots\\ &=\frac{2x}{125(1-a^2x^2)^{5/2}}-\frac{2\tanh^{-1}(ax)}{25a(1-a^2x^2)^{5/2}}-\frac{8\tanh^{-1}(ax)}{45a(1-a^2x^2)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{5(1-a^2x^2)^{5/2}}+\frac{4x\tan\cdots}{15(1-\cdots}\\ &=\frac{2x}{125(1-a^2x^2)^{5/2}}+\frac{272x}{3375(1-a^2x^2)^{3/2}}-\frac{2\tanh^{-1}(ax)}{25a(1-a^2x^2)^{5/2}}-\frac{8\tanh^{-1}(ax)}{45a(1-a^2x^2)^{3/2}}-\frac{16\cdots}{15a\cdots}\\ &=\frac{2x}{125(1-a^2x^2)^{5/2}}+\frac{272x}{3375(1-a^2x^2)^{3/2}}+\frac{4144x}{3375\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{25a(1-a^2x^2)^{5/2}}-\frac{8\cdots}{45a\cdots}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.45

$$\frac{4470ax-8560a^3x^3+4144a^5x^5-30(149-260a^2x^2+120a^4x^4)\tanh^{-1}(ax)+225ax(15-20a^2x^2+8a^4x^4)\tanh^{-1}(ax)^2}{3375a\,(1-a^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (4470*a*x - 8560*a^3*x^3 + 4144*a^5*x^5 - 30*(149 - 260*a^2*x^2 + 120*a^4*x
^4)*ArcTanh[a*x] + 225*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^2)/(3
375*a*(1 - a^2*x^2)^(5/2))
```

Maple [A]
time = 0.71, size = 118, normalized size = 0.57

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(1800\,\mathrm{arctanh}(ax)^2a^5x^5+4144a^5x^5-3600a^4x^4\,\mathrm{arctanh}(ax)-4500\,\mathrm{arctanh}(ax)^2a^3x^3-8560a^3x^3+7800a^2x^2\,\mathrm{ar}\right.}{3375a(a^2x^2-1)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3375/a*(-a^2*x^2+1)^(1/2)*(1800*arctanh(a*x)^2*a^5*x^5+4144*a^5*x^5-3600
*a^4*x^4*arctanh(a*x)-4500*arctanh(a*x)^2*a^3*x^3-8560*a^3*x^3+7800*a^2*x^2
*arctanh(a*x)+3375*arctanh(a*x)^2*a*x+4470*a*x-4470*arctanh(a*x))/(a^2*x^2-
1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(172) = 344$.
time = 0.52, size = 514, normalized size = 2.47

$$\frac{1}{15}\left(\frac{8x}{\sqrt{-a^2x^2+1}}+\frac{4x}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{3x}{(-a^2x^2+1)^{\frac{5}{2}}}\right)\mathrm{artanh}\,(ax)^2+\frac{1}{3375}a\left(\frac{9\left(\frac{8x}{\sqrt{-a^2x^2+1}}+\frac{4x}{(-a^2x^2+1)^{\frac{3}{2}}}-\frac{3}{(-a^2x^2+1)^{\frac{3}{2}}a^2x+(-a^2x^2+1)^{\frac{3}{2}}a}\right)}{a}+\frac{9\left(\frac{8x}{\sqrt{-a^2x^2+1}}+\frac{4x}{(-a^2x^2+1)^{\frac{3}{2}}}-\frac{3}{(-a^2x^2+1)^{\frac{3}{2}}a^2x-(-a^2x^2+1)^{\frac{3}{2}}a}\right)}{a}+\frac{100\left(\frac{2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2x+\sqrt{-a^2x^2+1}\,a}\right)}{a}+\frac{100\left(\frac{2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,a^2x-\sqrt{-a^2x^2+1}\,a}\right)}{a}-\frac{1800\sqrt{-a^2x^2+1}}{(a^2x+a)a}-\frac{1800\sqrt{-a^2x^2+1}}{(a^2x-a)a}-\frac{1800\log(ax+1)}{\sqrt{-a^2x^2+1}\,a^2}+\frac{1800\log(-ax+1)}{\sqrt{-a^2x^2+1}\,a^2}-\frac{300\log(ax+1)}{(-a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{300\log(-ax+1)}{(-a^2x^2+1)^{\frac{3}{2}}a^2}-\frac{135\log(ax+1)}{(-a^2x^2+1)^{\frac{5}{2}}a^2}+\frac{135\log(-ax+1)}{(-a^2x^2+1)^{\frac{5}{2}}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) + 3*x/(-a^2*x^2 + 1
)^(5/2))*arctanh(a*x)^2 + 1/3375*a*(9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x
^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a^2*x^2 + 1)^(3/2)*a))/a
+ 9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^
(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 100*(2*x/sqrt(-a^2*x^2 + 1) - 1/
(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a))/a + 100*(2*x/sqrt(-a^2*x
^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a^2*x^2 + 1)*a))/a - 1800*sqr
t(-a^2*x^2 + 1)/((a^2*x + a)*a) - 1800*sqrt(-a^2*x^2 + 1)/((a^2*x - a)*a) -
 1800*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 1800*log(-a*x + 1)/(sqrt(-a^2
*x^2 + 1)*a^2) - 300*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) + 300*log(-a*x
 + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) - 135*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a
^2) + 135*log(-a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2))
```

Fricas [A]
time = 0.36, size = 139, normalized size = 0.67

$$-\frac{\left(16576\,a^5x^5-34240\,a^3x^3+225\,(8\,a^5x^5-20\,a^3x^3+15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2+17880\,ax-60\,(120\,a^4x^4-260\,a^2x^2+149)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{13500\,(a^7x^6-3\,a^5x^4+3\,a^3x^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/13500*(16576*a^5*x^5 - 34240*a^3*x^3 + 225*(8*a^5*x^5 - 20*a^3*x^3 + 15*
a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 17880*a*x - 60*(120*a^4*x^4 - 260*a^2*x^
```

```
2 + 149)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4
 + 3*a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(7/2),x)
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^2}{{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(7/2),x)
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(7/2), x)
```

3.474 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=277

$$\frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{413312x}{385875\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{1}{175}$$

```
[Out] 2/343*x/(-a^2*x^2+1)^(7/2)+888/42875*x/(-a^2*x^2+1)^(5/2)+30256/385875*x/(-
a^2*x^2+1)^(3/2)-2/49*arctanh(a*x)/a/(-a^2*x^2+1)^(7/2)-12/175*arctanh(a*x)
/a/(-a^2*x^2+1)^(5/2)-16/105*arctanh(a*x)/a/(-a^2*x^2+1)^(3/2)+1/7*x*arctan
h(a*x)^2/(-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)^2/(-a^2*x^2+1)^(5/2)+8/35*x
*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2)+413312/385875*x/(-a^2*x^2+1)^(1/2)-32/35
*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+16/35*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2
)
```

Rubi [A]

time = 0.16, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6111, 6109, 197, 198}

$$\frac{413312x}{385875\sqrt{1-a^2x^2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^2}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)^2}{35\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(ax)^2}{35\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)^2}{7\left(1-a^2x^2\right)^{7/2}}-\frac{32\tanh^{-1}(ax)}{35a\sqrt{1-a^2x^2}}-\frac{16\tanh^{-1}(ax)}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{12\tanh^{-1}(ax)}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (2*x)/(343*(1 - a^2*x^2)^(7/2)) + (888*x)/(42875*(1 - a^2*x^2)^(5/2)) + (30
256*x)/(385875*(1 - a^2*x^2)^(3/2)) + (413312*x)/(385875*Sqrt[1 - a^2*x^2])
 - (2*ArcTanh[a*x])/(49*a*(1 - a^2*x^2)^(7/2)) - (12*ArcTanh[a*x])/(175*a*(
1 - a^2*x^2)^(5/2)) - (16*ArcTanh[a*x])/(105*a*(1 - a^2*x^2)^(3/2)) - (32*A
rcTanh[a*x])/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(7*(1 - a^2*x^2)
^(7/2)) + (6*x*ArcTanh[a*x]^2)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]
^2)/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x]^2)/(35*Sqrt[1 - a^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx &= -\frac{2\tanh^{-1}(ax)}{49a(1-a^2x^2)^{7/2}}+\frac{x\tanh^{-1}(ax)^2}{7(1-a^2x^2)^{7/2}}+\frac{2}{49}\int\frac{1}{(1-a^2x^2)^{9/2}}\,dx+\frac{6}{7}\int\frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx \\
&= \frac{2x}{343(1-a^2x^2)^{7/2}}-\frac{2\tanh^{-1}(ax)}{49a(1-a^2x^2)^{7/2}}-\frac{12\tanh^{-1}(ax)}{175a(1-a^2x^2)^{5/2}}+\frac{x\tanh^{-1}(ax)^2}{7(1-a^2x^2)^{7/2}}+\frac{6x\tanh \ldots}{35(1 \ldots} \\
&= \frac{2x}{343(1-a^2x^2)^{7/2}}+\frac{888x}{42875(1-a^2x^2)^{5/2}}-\frac{2\tanh^{-1}(ax)}{49a(1-a^2x^2)^{7/2}}-\frac{12\tanh^{-1}(ax)}{175a(1-a^2x^2)^{5/2}}-\ldots \\
&= \frac{2x}{343(1-a^2x^2)^{7/2}}+\frac{888x}{42875(1-a^2x^2)^{5/2}}+\frac{30256x}{385875(1-a^2x^2)^{3/2}}-\frac{2\tanh^{-1}(ax)}{49a(1-a^2x^2)^{7/2}}-\ldots \\
&= \frac{2x}{343(1-a^2x^2)^{7/2}}+\frac{888x}{42875(1-a^2x^2)^{5/2}}+\frac{30256x}{385875(1-a^2x^2)^{3/2}}+\frac{413312x}{385875\sqrt{1-a^2x^2}}\ldots
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 120, normalized size = 0.43

$$\frac{2ax(226905-654220a^2x^2+635096a^4x^4-206656a^6x^6)+210(-2161+5726a^2x^2-5320a^4x^4+1680a^6x^6)\tanh^{-1}(ax)-11025ax(-35+70a^2x^2-56a^4x^4+16a^6x^6)\tanh^{-1}(ax)^2}{385875a(1-a^2x^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (2*a*x*(226905 - 654220*a^2*x^2 + 635096*a^4*x^4 - 206656*a^6*x^6) + 210*(-
2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*x] - 11025*a*x
```

```
*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]^2)/(385875*a*(1
- a^2*x^2)^(7/2))
```

Maple [A]
time = 0.70, size = 152, normalized size = 0.55

method	result
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\,\left(176400 \operatorname{arctanh}\left(a x\right)^{2} a^{7} x^{7}+413312 a^{7} x^{7}-352800 \operatorname{arctanh}\left(a x\right) a^{6} x^{6}-617400 \operatorname{arctanh}\left(a x\right)^{2} a^{5} x^{5}-1270192 a^{5} x^{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/385875/a*(-a^2*x^2+1)^(1/2)*(176400*arctanh(a*x)^2*a^7*x^7+413312*a^7*x^
7-352800*arctanh(a*x)*a^6*x^6-617400*arctanh(a*x)^2*a^5*x^5-1270192*a^5*x^5
+1117200*a^4*x^4*arctanh(a*x)+771750*arctanh(a*x)^2*a^3*x^3+1308440*a^3*x^3
-1202460*a^2*x^2*arctanh(a*x)-385875*arctanh(a*x)^2*a*x-453810*a*x+453810*a
rctanh(a*x))/(a^2*x^2-1)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(229) = 458$.
time = 0.54, size = 751, normalized size = 2.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/35*(16*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 +
1)^(5/2) + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)^2 + 1/385875*a*(225*(16*x
/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a^
2*x + (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 225*(16*x/sqr
t(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a^2*x
- (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 882*(8*x/sqrt(-a^
2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a
^2*x^2 + 1)^(3/2)*a))/a + 882*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^
(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 9800*(
2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a
))/a + 9800*(2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a
^2*x^2 + 1)*a))/a - 176400*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 176400*sqrt
(-a^2*x^2 + 1)/((a^2*x - a)*a) - 176400*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^
2) + 176400*log(-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 29400*log(a*x + 1)/((-
a^2*x^2 + 1)^(3/2)*a^2) + 29400*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) -
13230*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2) + 13230*log(-a*x + 1)/((-a^2*
x^2 + 1)^(5/2)*a^2) - 7875*log(a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2) + 7875*l
og(-a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2))
```

Fricas [A]
time = 0.39, size = 169, normalized size = 0.61

$$-\frac{\left(1653248\,a^{7}x^{7}-5080768\,a^{5}x^{5}+5233760\,a^{3}x^{3}+11025\left(16\,a^{7}x^{7}-56\,a^{5}x^{5}+70\,a^{3}x^{3}-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-1815240\,ax-420\left(1680\,a^{6}x^{6}-5320\,a^{4}x^{4}+5726\,a^{2}x^{2}-2161\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^{2}x^{2}+1}}{1543500\left(a^{9}x^{8}-4\,a^{7}x^{6}+6\,a^{5}x^{4}-4\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/1543500*(1653248*a^7*x^7 - 5080768*a^5*x^5 + 5233760*a^3*x^3 + 11025*(16
*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1))^2 -
1815240*a*x - 420*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(-
(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 -
 4*a^3*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}\left(ax\right)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atanh}\left(a\,x\right)}^2}{{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^2/(1 - a^2*x^2)^(9/2),x)
```

```
[Out] int(atanh(a*x)^2/(1 - a^2*x^2)^(9/2), x)
```

3.475 $\int \sqrt{1-a^2x^2}\, \tanh^{-1}(ax)^3\, dx$

Optimal. Leaf size=302

$$\frac{6\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}+\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3+\frac{\text{ArcTan}\left(e^{\tanh}\right.}{}$$

```
[Out] 6*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a+arctan((a*x+1)/(-a^2*
x^2+1)^(1/2))*arctanh(a*x)^3/a-3/2*I*arctanh(a*x)^2*polylog(2,-I*(a*x+1)/(-
a^2*x^2+1)^(1/2))/a+3/2*I*arctanh(a*x)^2*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(
1/2))/a+3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-3*I*polylog(2,I*(-
a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a^2*x
^2+1)^(1/2))/a-3*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-3
*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+3*I*polylog(4,I*(a*x+1)/(-a^2
*x^2+1)^(1/2))/a+3/2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/a+1/2*x*arctanh(a*x)
^3*(-a^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6091, 6099, 4265, 2611, 6744, 2320, 6724, 6097}

$$\frac{1}{2}x\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^3+\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}{2a}+\frac{\tanh^{-1}(ax)^3\text{ArcTan}\left(e^{\tanh^{-1}(ax)}\right)}{a}+\frac{6\text{ArcTan}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}+\frac{3i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{2a}+\frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tanh^{-1}(ax)}\right)}{2a}+\frac{3i\tanh^{-1}(ax)\text{Li}_3\left(-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{3i\tanh^{-1}(ax)\text{Li}_3\left(ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{3i\text{Li}_4\left(-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\text{Li}_4\left(ie^{\tanh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

```
[Out] (6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a + (3*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/2 + (ArcTan
[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, (
-I)*E^ArcTanh[a*x]])/a + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[a
*x]])/a + ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a - ((3*I)
*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + ((3*I)*ArcTanh[a*x]*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a - ((3*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[
a*x]])/a - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*PolyLog[4, I*
E^ArcTanh[a*x]])/a
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6091

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a
+ b*ArcTanh[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^3\,dx = \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3 + \frac{1}{2}\int \frac{\tanh^{-1}(a}{\sqrt{1-a^2}}$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-}$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-}$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-}$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-}$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-}$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-}$$

Mathematica [A]
time = 2.46, size = 569, normalized size = 1.88

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 + (1
92*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 + (64*I)*
a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 - 384*ArcTanh[a*x]
*Log[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + 384*Arc
Tanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh
[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTan
h[a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*Ar
cTanh[a*x] - 4*ArcTanh[a*x]^2)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + 384*PolyLo
g[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]]
 - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2
, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcT
anh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)
*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4,
 (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

Maple [**F**]
time = 1.08, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} x^{2}+1}\, \operatorname{arctanh}\left(a x\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{atanh}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{atanh}\left(a\,x\right)}^3\,\sqrt{1-a^2\,x^2}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3*(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)^3*(1 - a^2*x^2)^(1/2), x)
```

3.476 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{2}{27a\,(1-a^2x^2)^{3/2}}-\frac{40}{9a\sqrt{1-a^2x^2}}+\frac{2x\tanh^{-1}(ax)}{9\,(1-a^2x^2)^{3/2}}+\frac{40x\tanh^{-1}(ax)}{9\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\,(1-a^2x^2)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x}{3}$$

```
[Out] -2/27/a/(-a^2*x^2+1)^(3/2)+2/9*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-1/3*arctan
h(a*x)^2/a/(-a^2*x^2+1)^(3/2)+1/3*x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2)-40/9/
a/(-a^2*x^2+1)^(1/2)+40/9*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)^
2/a/(-a^2*x^2+1)^(1/2)+2/3*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6111, 6109, 6105, 6107}

$$-\frac{40}{9a\sqrt{1-a^2x^2}}-\frac{2}{27a\,(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)^3}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{3\,(1-a^2x^2)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\,(1-a^2x^2)^{3/2}}+\frac{40x\tanh^{-1}(ax)}{9\sqrt{1-a^2x^2}}+\frac{2x\tanh^{-1}(ax)}{9\,(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -2/(27*a*(1 - a^2*x^2)^(3/2)) - 40/(9*a*Sqrt[1 - a^2*x^2]) + (2*x*ArcTanh[a
*x])/(9*(1 - a^2*x^2)^(3/2)) + (40*x*ArcTanh[a*x])/(9*Sqrt[1 - a^2*x^2]) -
ArcTanh[a*x]^2/(3*a*(1 - a^2*x^2)^(3/2)) - (2*ArcTanh[a*x]^2)/(a*Sqrt[1 - a
^2*x^2]) + (x*ArcTanh[a*x]^3)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^3
)/(3*Sqrt[1 - a^2*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
```

```
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = -\frac{\tanh^{-1}(ax)^2}{3a\,(1-a^2x^2)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{3\,(1-a^2x^2)^{3/2}}+\frac{2}{3}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{5/2}}\,dx+\frac{2}{3}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

$$= -\frac{2}{27a\,(1-a^2x^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{9\,(1-a^2x^2)^{3/2}}-\frac{\tanh^{-1}(ax)^2}{3a\,(1-a^2x^2)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}}{3\,(1-a}$$

$$= -\frac{2}{27a\,(1-a^2x^2)^{3/2}}-\frac{40}{9a\sqrt{1-a^2x^2}}+\frac{2x\tanh^{-1}(ax)}{9\,(1-a^2x^2)^{3/2}}+\frac{40x\tanh^{-1}(ax)}{9\sqrt{1-a^2x^2}}-\frac{\tanh^{-}}{3a\,(1-}$$

Mathematica [A]
time = 0.06, size = 87, normalized size = 0.46

$$\frac{-122+120a^2x^2-6ax(-21+20a^2x^2)\tanh^{-1}(ax)+9(-7+6a^2x^2)\tanh^{-1}(ax)^2-9ax(-3+2a^2x^2)\tanh^{-1}(ax)^3}{27a\,(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-122 + 120*a^2*x^2 - 6*a*x*(-21 + 20*a^2*x^2)*ArcTanh[a*x] + 9*(-7 + 6*a^2
*x^2)*ArcTanh[a*x]^2 - 9*a*x*(-3 + 2*a^2*x^2)*ArcTanh[a*x]^3)/(27*a*(1 - a^
2*x^2)^(3/2))
```

Maple [A]
time = 0.68, size = 105, normalized size = 0.55

method	result

default	$-\frac{\sqrt{-a^2x^2+1}\,\left(18\,\mathrm{arctanh}(ax)^3a^3x^3+120a^3x^3\,\mathrm{arctanh}(ax)-54a^2x^2\,\mathrm{arctanh}(ax)^2-27\,\mathrm{arctanh}(ax)^3ax-120a^2x^2-126ax\,\mathrm{arcta}\right.}{27a(a^2x^2-1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^3*a^3*x^3+120*a^3*x^3*arctanh(a
*x)-54*a^2*x^2*arctanh(a*x)^2-27*arctanh(a*x)^3*a*x-120*a^2*x^2-126*a*x*arc
tanh(a*x)+63*arctanh(a*x)^2+122)/(a^2*x^2-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

Fricas [A]
time = 0.42, size = 134, normalized size = 0.70

$$\frac{\left(960\,a^2x^2-9\,(2\,a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3+18\,(6\,a^2x^2-7)\log\left(-\frac{ax+1}{ax-1}\right)^2-24\,(20\,a^3x^3-21\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-976\right)\sqrt{-a^2x^2+1}}{216\,(a^5x^4-2\,a^3x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/216*(960*a^2*x^2 - 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 18
*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1))^2 - 24*(20*a^3*x^3 - 21*a*x)*log
(-(a*x + 1)/(a*x - 1)) - 976)*sqrt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(5/2), x)
```

3.477 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{7/2}} \, dx$

Optimal. Leaf size=289

$$-\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{4144}{1125a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}}+\frac{272x\tanh^{-1}(ax)}{1125\left(1-a^2x^2\right)^{3/2}}+\frac{4144}{112}$$

```
[Out] -6/625/a/(-a^2*x^2+1)^(5/2)-272/3375/a/(-a^2*x^2+1)^(3/2)+6/125*x*arctanh(a
*x)/(-a^2*x^2+1)^(5/2)+272/1125*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)-3/25*arct
anh(a*x)^2/a/(-a^2*x^2+1)^(5/2)-4/15*arctanh(a*x)^2/a/(-a^2*x^2+1)^(3/2)+1/
5*x*arctanh(a*x)^3/(-a^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)^3/(-a^2*x^2+1)^(3
/2)-4144/1125/a/(-a^2*x^2+1)^(1/2)+4144/1125*x*arctanh(a*x)/(-a^2*x^2+1)^(1
/2)-8/5*arctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+8/15*x*arctanh(a*x)^3/(-a^2*x^2
+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6111, 6109, 6105, 6107}

$$-\frac{4144}{1125a\sqrt{1-a^2x^2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^3}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^3}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}}-\frac{8\tanh^{-1}(ax)^2}{5a\sqrt{1-a^2x^2}}-\frac{4\tanh^{-1}(ax)^2}{15a\left(1-a^2x^2\right)^{3/2}}-\frac{3\tanh^{-1}(ax)^2}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{4144x\tanh^{-1}(ax)}{1125\sqrt{1-a^2x^2}}+\frac{272x\tanh^{-1}(ax)}{1125\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] -6/(625*a*(1 - a^2*x^2)^(5/2)) - 272/(3375*a*(1 - a^2*x^2)^(3/2)) - 4144/(1
125*a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(125*(1 - a^2*x^2)^(5/2)) + (
272*x*ArcTanh[a*x])/(1125*(1 - a^2*x^2)^(3/2)) + (4144*x*ArcTanh[a*x])/(112
5*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(25*a*(1 - a^2*x^2)^(5/2)) - (4*A
rcTanh[a*x]^2)/(15*a*(1 - a^2*x^2)^(3/2)) - (8*ArcTanh[a*x]^2)/(5*a*Sqrt[1
- a^2*x^2]) + (x*ArcTanh[a*x]^3)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*x
]^3)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^3)/(15*Sqrt[1 - a^2*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
```

```
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx &= -\frac{3\tanh^{-1}(ax)^2}{25a\,(1-a^2x^2)^{5/2}}+\frac{x\tanh^{-1}(ax)^3}{5\,(1-a^2x^2)^{5/2}}+\frac{6}{25}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{7/2}}\,dx+\frac{4}{5}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{5/2}} \\
&= -\frac{6}{625a\,(1-a^2x^2)^{5/2}}+\frac{6x\tanh^{-1}(ax)}{125\,(1-a^2x^2)^{5/2}}-\frac{3\tanh^{-1}(ax)^2}{25a\,(1-a^2x^2)^{5/2}}-\frac{4\tanh^{-1}(ax)^2}{15a\,(1-a^2x^2)^{3/2}}+ \\
&= -\frac{6}{625a\,(1-a^2x^2)^{5/2}}-\frac{272}{3375a\,(1-a^2x^2)^{3/2}}+\frac{6x\tanh^{-1}(ax)}{125\,(1-a^2x^2)^{5/2}}+\frac{272x\tanh^{-1}(ax)}{1125\,(1-a^2x^2)^{3/2}} \\
&= -\frac{6}{625a\,(1-a^2x^2)^{5/2}}-\frac{272}{3375a\,(1-a^2x^2)^{3/2}}-\frac{4144}{1125a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{125\,(1-a^2x^2)^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 119, normalized size = 0.41

$$\frac{-63682+125680a^2x^2-62160a^4x^4+30ax(2235-4280a^2x^2+2072a^4x^4)\tanh^{-1}(ax)-225(149-260a^2x^2+120a^4x^4)\tanh^{-1}(ax)^2+1125ax(15-20a^2x^2+8a^4x^4)\tanh^{-1}(ax)^3}{16875a\,(1-a^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-63682 + 125680*a^2*x^2 - 62160*a^4*x^4 + 30*a*x*(2235 - 4280*a^2*x^2 + 20
72*a^4*x^4)*ArcTanh[a*x] - 225*(149 - 260*a^2*x^2 + 120*a^4*x^4)*ArcTanh[a*
```

```
x]^2 + 1125*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^3)/(16875*a*(1 -
 a^2*x^2)^(5/2))
```

Maple [A]
time = 0.71, size = 153, normalized size = 0.53

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(9000\operatorname{arctanh}(ax)^3a^5x^5+62160\operatorname{arctanh}(ax)a^5x^5-27000a^4x^4\operatorname{arctanh}(ax)^2-22500\operatorname{arctanh}(ax)^3a^3x^3-62160a^4\right.}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16875/a*(-a^2*x^2+1)^(1/2)*(9000*arctanh(a*x)^3*a^5*x^5+62160*arctanh(a*
x)*a^5*x^5-27000*a^4*x^4*arctanh(a*x)^2-22500*arctanh(a*x)^3*a^3*x^3-62160*
a^4*x^4-128400*a^3*x^3*arctanh(a*x)+58500*a^2*x^2*arctanh(a*x)^2+16875*arct
anh(a*x)^3*a*x+125680*a^2*x^2+67050*a*x*arctanh(a*x)-33525*arctanh(a*x)^2-6
3682)/(a^2*x^2-1)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)
```

Fricas [A]
time = 0.37, size = 176, normalized size = 0.61

$$\frac{\left(497280\,a^4x^4-1005440\,a^2x^2-1125\,(8\,a^5x^5-20\,a^3x^3+15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3+450\,(120\,a^4x^4-260\,a^2x^2+149)\log\left(-\frac{ax+1}{ax-1}\right)^2-120\,(2072\,a^5x^5-4280\,a^3x^3+2235\,ax)\log\left(-\frac{ax+1}{ax-1}\right)+509456\right)\sqrt{-a^2x^2+1}}{135000\,(a^7x^6-3\,a^5x^4+3\,a^3x^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/135000*(497280*a^4*x^4 - 1005440*a^2*x^2 - 1125*(8*a^5*x^5 - 20*a^3*x^3 +
 15*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 450*(120*a^4*x^4 - 260*a^2*x^2 + 149
)*log(-(a*x + 1)/(a*x - 1))^2 - 120*(2072*a^5*x^5 - 4280*a^3*x^3 + 2235*a*x
)*log(-(a*x + 1)/(a*x - 1)) + 509456)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x
^4 + 3*a^3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}(a\,x)^3}{{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(7/2),x)
```

```
[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(7/2), x)
```

3.478 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{413312}{128625a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax}{343\left(1-a^2x^2\right)}$$

```
[Out] -6/2401/a/(-a^2*x^2+1)^(7/2)-2664/214375/a/(-a^2*x^2+1)^(5/2)-30256/385875/
a/(-a^2*x^2+1)^(3/2)+6/343*x*arctanh(a*x)/(-a^2*x^2+1)^(7/2)+2664/42875*x*a
rctanh(a*x)/(-a^2*x^2+1)^(5/2)+30256/128625*x*arctanh(a*x)/(-a^2*x^2+1)^(3/
2)-3/49*arctanh(a*x)^2/a/(-a^2*x^2+1)^(7/2)-18/175*arctanh(a*x)^2/a/(-a^2*x
^2+1)^(5/2)-8/35*arctanh(a*x)^2/a/(-a^2*x^2+1)^(3/2)+1/7*x*arctanh(a*x)^3/(
-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)^3/(-a^2*x^2+1)^(5/2)+8/35*x*arctanh(a
*x)^3/(-a^2*x^2+1)^(3/2)-413312/128625/a/(-a^2*x^2+1)^(1/2)+413312/128625*x
*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-48/35*arctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+
16/35*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6111, 6109, 6105, 6107}

$$-\frac{413312}{128625a\sqrt{1-a^2x^2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^3}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)^3}{35\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(ax)^3}{35\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)^3}{7\left(1-a^2x^2\right)^{7/2}}-\frac{48\tanh^{-1}(ax)^2}{35a\sqrt{1-a^2x^2}}-\frac{8\tanh^{-1}(ax)^2}{35a\left(1-a^2x^2\right)^{3/2}}-\frac{18\tanh^{-1}(ax)^2}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{3\tanh^{-1}(ax)^2}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{413312x\tanh^{-1}(ax)}{128625\sqrt{1-a^2x^2}}+\frac{30256x\tanh^{-1}(ax)}{128625\left(1-a^2x^2\right)^{3/2}}+\frac{2664x\tanh^{-1}(ax)}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{6x\tanh^{-1}(ax)}{343\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] -6/(2401*a*(1 - a^2*x^2)^(7/2)) - 2664/(214375*a*(1 - a^2*x^2)^(5/2)) - 302
56/(385875*a*(1 - a^2*x^2)^(3/2)) - 413312/(128625*a*Sqrt[1 - a^2*x^2]) + (
6*x*ArcTanh[a*x])/(343*(1 - a^2*x^2)^(7/2)) + (2664*x*ArcTanh[a*x])/(42875*
(1 - a^2*x^2)^(5/2)) + (30256*x*ArcTanh[a*x])/(128625*(1 - a^2*x^2)^(3/2))
+ (413312*x*ArcTanh[a*x])/(128625*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(
49*a*(1 - a^2*x^2)^(7/2)) - (18*ArcTanh[a*x]^2)/(175*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x]^2)/(35*a*(1 - a^2*x^2)^(3/2)) - (48*ArcTanh[a*x]^2)/(35*
a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(7*(1 - a^2*x^2)^(7/2)) + (6*x*Ar
cTanh[a*x]^3)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]^3)/(35*(1 - a^2*
x^2)^(3/2)) + (16*x*ArcTanh[a*x]^3)/(35*Sqrt[1 - a^2*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(
3/2), x], x] + Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q +
1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q
 + 1)*((a + b*ArcTanh[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx &= -\frac{3\tanh^{-1}(ax)^2}{49a\,(1-a^2x^2)^{7/2}}+\frac{x\tanh^{-1}(ax)^3}{7\,(1-a^2x^2)^{7/2}}+\frac{6}{49}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{9/2}}\,dx+\frac{6}{7}\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{7/2}}\\ &= -\frac{6}{2401a\,(1-a^2x^2)^{7/2}}+\frac{6x\tanh^{-1}(ax)}{343\,(1-a^2x^2)^{7/2}}-\frac{3\tanh^{-1}(ax)^2}{49a\,(1-a^2x^2)^{7/2}}-\frac{18\tanh^{-1}(ax)^2}{175a\,(1-a^2x^2)^{5/2}}\\ &= -\frac{6}{2401a\,(1-a^2x^2)^{7/2}}-\frac{2664}{214375a\,(1-a^2x^2)^{5/2}}+\frac{6x\tanh^{-1}(ax)}{343\,(1-a^2x^2)^{7/2}}+\frac{2664x\tanh^{-1}}{42875\,(1-a^2x}\\ &= -\frac{6}{2401a\,(1-a^2x^2)^{7/2}}-\frac{2664}{214375a\,(1-a^2x^2)^{5/2}}-\frac{30256}{385875a\,(1-a^2x^2)^{3/2}}+\frac{6x\tanh}{343\,(1-a}\\ &= -\frac{6}{2401a\,(1-a^2x^2)^{7/2}}-\frac{2664}{214375a\,(1-a^2x^2)^{5/2}}-\frac{30256}{385875a\,(1-a^2x^2)^{3/2}}-\frac{413}{128625a\sqrt{}}\end{aligned}$$

Mathematica [A]

time = 0.09, size = 151, normalized size = 0.39

$$\frac{-44658302+132479032a^2x^2-131252240a^4x^4+43397760a^6x^6-210ax(-226905+654220a^2x^2-635096a^4x^4+206656a^6x^6)\tanh^{-1}(ax)+11025(-2161+5726a^2x^2-5320a^4x^4+1680a^6x^6)\tanh^{-1}(ax)^2-385875ax(-35+70a^2x^2-56a^4x^4+16a^6x^6)\tanh^{-1}(ax)^3}{13505625a\,(1-a^2x^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-44658302 + 132479032*a^2*x^2 - 131252240*a^4*x^4 + 43397760*a^6*x^6 - 210
*a*x*(-226905 + 654220*a^2*x^2 - 635096*a^4*x^4 + 206656*a^6*x^6)*ArcTanh[a
*x] + 11025*(-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*
x]^2 - 385875*a*x*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]
^3)/(13505625*a*(1 - a^2*x^2)^(7/2))
```

Maple [A]
time = 0.73, size = 201, normalized size = 0.52

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(6174000\,\mathrm{arctanh}(ax)^3a^7x^7+43397760\,\mathrm{arctanh}(ax)a^7x^7-18522000\,\mathrm{arctanh}(ax)^2a^6x^6-21609000\,\mathrm{arctanh}(ax)^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/13505625/a*(-a^2*x^2+1)^(1/2)*(6174000*arctanh(a*x)^3*a^7*x^7+43397760*a
rctanh(a*x)*a^7*x^7-18522000*arctanh(a*x)^2*a^6*x^6-21609000*arctanh(a*x)^3
*a^5*x^5-43397760*a^6*x^6-133370160*arctanh(a*x)*a^5*x^5+58653000*a^4*x^4*a
rctanh(a*x)^2+27011250*arctanh(a*x)^3*a^3*x^3+131252240*a^4*x^4+137386200*a
^3*x^3*arctanh(a*x)-63129150*a^2*x^2*arctanh(a*x)^2-13505625*arctanh(a*x)^3
*a*x-132479032*a^2*x^2-47650050*a*x*arctanh(a*x)+23825025*arctanh(a*x)^2+44
658302)/(a^2*x^2-1)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

Fricas [A]
time = 0.39, size = 214, normalized size = 0.56

$$\frac{\left(347182080\,a^6x^6-1050017920\,a^4x^4+1059832256\,a^2x^2-385875\,(16\,a^7x^7-56\,a^5x^5+70\,a^3x^3-35\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3+22050\,(1680\,a^6x^6-5320\,a^4x^4+5726\,a^2x^2-2161)\log\left(-\frac{ax+1}{ax-1}\right)^2-840\,(206656\,a^7x^7-635096\,a^5x^5+654220\,a^3x^3-226905\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-357266416\right)\sqrt{-a^2x^2+1}}{108045000\,(a^9x^8-4\,a^7x^6+6\,a^5x^4-4\,a^3x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

[Out] 1/108045000*(347182080*a^6*x^6 - 1050017920*a^4*x^4 + 1059832256*a^2*x^2 - 385875*(16*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 22050*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(-(a*x + 1)/(a*x - 1))^2 - 840*(206656*a^7*x^7 - 635096*a^5*x^5 + 654220*a^3*x^3 - 226905*a*x)*log(-(a*x + 1)/(a*x - 1)) - 357266416)*sqrt(-a^2*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(9/2),x)

[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(9/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="giac")

[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)^3}{\left(1-a^2\,x^2\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a*x)^3/(1 - a^2*x^2)^(9/2),x)

[Out] int(atanh(a*x)^3/(1 - a^2*x^2)^(9/2), x)

3.479 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)} \, dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)} \, dx$$

Mathematica [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]
```

```
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Maple [A]
time = 8.50, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1 - a^2 x^2}}{\operatorname{atanh}(a x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/atanh(a*x),x)
[Out] int((1 - a^2*x^2)^(1/2)/atanh(a*x), x)
```

3.480 $\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}\,dx=\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Maple [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

```
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.481 $\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] Chi(arctanh(a*x))/a
```

Rubi [A]

time = 0.04, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6115, 3382}

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 9, normalized size = 1.00

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Maple [A]
time = 0.00, size = 10, normalized size = 1.11

method	result	size
default	$\frac{\mathrm{hyperbolicCosineIntegral}(\mathrm{arctanh}(ax))}{a}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] Chi(arctanh(a*x))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

3.482 $\int \frac{1}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

```
[Out] 3/4*Chi(arctanh(a*x))/a+1/4*Chi(3*arctanh(a*x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {6115, 3393, 3382}

$$\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

```
[Out] (3*CoshIntegral[ArcTanh[a*x]])/(4*a) + CoshIntegral[3*ArcTanh[a*x]]/(4*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a} + \frac{3\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a}$$
$$= \frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{4a} + \frac{\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Mathematica [A]
time = 0.05, size = 22, normalized size = 0.81

$$\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)+\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
[Out] (3*CoshIntegral[ArcTanh[a*x]] + CoshIntegral[3*ArcTanh[a*x]])/(4*a)
```

Maple [A]
time = 0.88, size = 21, normalized size = 0.78

method	result	size
default	$\frac{3\,\text{hyperbolicCosineIntegral}(\text{arctanh}(ax))+\text{hyperbolicCosineIntegral}(3\,\text{arctanh}(ax))}{4a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/4*(3*Chi(arctanh(a*x))+Chi(3*arctanh(a*x)))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x),x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(5/2)),x)
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(5/2)), x)
```

3.483 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$\frac{5\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\mathrm{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] 5/8*Chi(arctanh(a*x))/a+5/16*Chi(3*arctanh(a*x))/a+1/16*Chi(5*arctanh(a*x))
/a
```

Rubi [A]
time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6115, 3393, 3382}

$$\frac{5\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\mathrm{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

```
[Out] (5*CoshIntegral[ArcTanh[a*x]])/(8*a) + (5*CoshIntegral[3*ArcTanh[a*x]])/(16
*a) + CoshIntegral[5*ArcTanh[a*x]]/(16*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x}+\frac{5\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{5\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \tanh^{-1}(ax\right.}{16a}$$

$$= \frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a} + \frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a} + \frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 0.76

$$\frac{10\text{Chi}\left(\tanh^{-1}(ax)\right)+5\text{Chi}\left(3\tanh^{-1}(ax)\right)+\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
[Out] (10*CoshIntegral[ArcTanh[a*x]] + 5*CoshIntegral[3*ArcTanh[a*x]] + CoshInteg
ral[5*ArcTanh[a*x]])/(16*a)
```

Maple [A]
time = 0.89, size = 30, normalized size = 0.73

method	result
default	$\frac{10\,\text{hyperbolicCosineIntegral}(\text{arctanh}(ax))+5\,\text{hyperbolicCosineIntegral}(3\,\text{arctanh}(ax))+\text{hyperbolicCosineIntegral}(5\,\text{arctanh}(ax))}{16a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/16*(10*Chi(arctanh(a*x))+5*Chi(3*arctanh(a*x))+Chi(5*arctanh(a*x)))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(7/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(7/2)), x)
```

3.484 $\int \frac{1}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

```
[Out] 35/64*Chi(arctanh(a*x))/a+21/64*Chi(3*arctanh(a*x))/a+7/64*Chi(5*arctanh(a*
x))/a+1/64*Chi(7*arctanh(a*x))/a
```

Rubi [A]
time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6115, 3393, 3382}

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

```
[Out] (35*CoshIntegral[ArcTanh[a*x]])/(64*a) + (21*CoshIntegral[3*ArcTanh[a*x]])/
(64*a) + (7*CoshIntegral[5*ArcTanh[a*x]])/(64*a) + CoshIntegral[7*ArcTanh[a
*x]]/(64*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^7(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x}+\frac{21\cosh(3x)}{64x}+\frac{7\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a} + \frac{7\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a}$$
$$= \frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a} + \frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a} + \frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a} + \frac{\text{Ch}}{}$$

Mathematica [A]
time = 0.05, size = 40, normalized size = 0.73

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)+21\text{Chi}\left(3\tanh^{-1}(ax)\right)+7\text{Chi}\left(5\tanh^{-1}(ax)\right)+\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
[Out] (35*CoshIntegral[ArcTanh[a*x]] + 21*CoshIntegral[3*ArcTanh[a*x]] + 7*CoshIn
tegral[5*ArcTanh[a*x]] + CoshIntegral[7*ArcTanh[a*x]])/(64*a)
```

Maple [A]
time = 0.89, size = 39, normalized size = 0.71

method	result
default	$\frac{35\,\text{hyperbolicCosineIntegral}(\text{arctanh}(ax))+21\,\text{hyperbolicCosineIntegral}(3\,\text{arctanh}(ax))+7\,\text{hyperbolicCosineIntegral}(5\,\text{arctanh}(ax))+}{64a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/64*(35*Chi(arctanh(a*x))+21*Chi(3*arctanh(a*x))+7*Chi(5*arctanh(a*x))+Chi
(7*arctanh(a*x)))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)*(1 - a^2*x^2)^(9/2)),x)
```

```
[Out] int(1/(atanh(a*x)*(1 - a^2*x^2)^(9/2)), x)
```

3.485 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Maple [A]
time = 8.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^2,x)
[Out] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^2, x)
```

3.486 $\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx = \int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(1/2)), x)
```

3.487 $\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] Shi(arctanh(a*x))/a-1/a/arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6113, 6181, 3379}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + a\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.91

$$\frac{-\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

Maple [A]
time = 0.00, size = 62, normalized size = 1.77

method	result	si
default	$\frac{\operatorname{arctanh}(ax)\,\text{hyperbolicSineIntegral}(\operatorname{arctanh}(ax))a^2x^2-\text{hyperbolicSineIntegral}(\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)+\sqrt{-a^2x^2+1}}{a\operatorname{arctanh}(ax)(a^2x^2-1)}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2-Shi(arctanh(a*x))*arctanh(a*x)+
(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\mathrm{atanh}\left(a\,x\right)}^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

3.488 $\int \frac{1}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\mathrm{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

```
[Out] -1/a/(-a^2*x^2+1)^(3/2)/arctanh(a*x)+3/4*Shi(arctanh(a*x))/a+3/4*Shi(3*arct
anh(a*x))/a
```

Rubi [A]
time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6113, 6181, 5556, 3379}

$$-\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\mathrm{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])) + (3*SinhIntegral[ArcTanh[a*x]])/
(4*a) + (3*SinhIntegral[3*ArcTanh[a*x]])/(4*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx &= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+(3a)\int \frac{x}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}\,dx\\ &= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x}+\frac{\sinh(3x)}{4x}\right)dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{S}}{}\\ &= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.87

$$\frac{-\frac{4}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+3\left(\text{Shi}\left(\tanh^{-1}(ax)\right)+\text{Shi}\left(3\tanh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
[Out] (-4/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]) + 3*(SinhIntegral[ArcTanh[a*x]] + Si
nhIntegral[3*ArcTanh[a*x]]))/(4*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.
time = 2.56, size = 120, normalized size = 2.31

method	result

default	$\frac{3\operatorname{arctanh}(ax)\,\mathrm{hyperbolicSineIntegral}(\operatorname{arctanh}(ax))a^2x^2+3\operatorname{arctanh}(ax)\,\mathrm{hyperbolicSineIntegral}(3\operatorname{arctanh}(ax))a^2x^2-\cosh(3\operatorname{arctanh}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(3*arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2+3*arctanh(a*x)*Shi(3*arctan
h(a*x))*a^2*x^2-cosh(3*arctanh(a*x))*a^2*x^2-3*Shi(arctanh(a*x))*arctanh(a*
x)-3*Shi(3*arctanh(a*x))*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a
*x)))/arctanh(a*x)/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)^2\,{\left(1-a^2\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(5/2)), x)
```

3.489 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{5\text{Shi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -1/a/(-a^2*x^2+1)^(5/2)/arctanh(a*x)+5/8*Shi(arctanh(a*x))/a+15/16*Shi(3*ar
ctanh(a*x))/a+5/16*Shi(5*arctanh(a*x))/a
```

Rubi [A]
time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6113, 6181, 5556, 3379}

$$-\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{5\text{Shi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])) + (5*SinhIntegral[ArcTanh[a*x]])/
(8*a) + (15*SinhIntegral[3*ArcTanh[a*x]])/(16*a) + (5*SinhIntegral[5*ArcTan
h[a*x]])/(16*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2}\,dx &= -\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+(5a)\int \frac{x}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)}\,dx\\ &= -\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x}+\frac{3\sinh(3x)}{16x}+\frac{\sinh(5x)}{16x}\right) d\right.}{a}\\ &= -\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{16a}+\frac{5S}{}\\ &= -\frac{1}{a(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 56, normalized size = 0.85

$$\frac{-\frac{16}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+5\left(2\text{Shi}\left(\tanh^{-1}(ax)\right)+3\text{Shi}\left(3\tanh^{-1}(ax)\right)+\text{Shi}\left(5\tanh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
[Out] (-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]) + 5*(2*SinhIntegral[ArcTanh[a*x]] +
 3*SinhIntegral[3*ArcTanh[a*x]] + SinhIntegral[5*ArcTanh[a*x]]))/(16*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(58) = 116.
time = 2.97, size = 176, normalized size = 2.67

method	result

default	$\frac{5 \operatorname{arctanh}(ax) \operatorname{hyperbolicSineIntegral}(5 \operatorname{arctanh}(ax)) a^2 x^2 + 10 \operatorname{arctanh}(ax) \operatorname{hyperbolicSineIntegral}(\operatorname{arctanh}(ax)) a^2 x^2 + 15 \operatorname{arctanh}(a}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/16/a*(5*arctanh(a*x)*Shi(5*arctanh(a*x))*a^2*x^2+10*arctanh(a*x)*Shi(arct
anh(a*x))*a^2*x^2+15*arctanh(a*x)*Shi(3*arctanh(a*x))*a^2*x^2-cosh(5*arctan
h(a*x))*a^2*x^2-5*cosh(3*arctanh(a*x))*a^2*x^2-5*Shi(5*arctanh(a*x))*arctan
h(a*x)-10*Shi(arctanh(a*x))*arctanh(a*x)-15*Shi(3*arctanh(a*x))*arctanh(a*x
)+cosh(5*arctanh(a*x))+10*(-a^2*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x)))/arctan
h(a*x)/(a^2*x^2-1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="maxima")

[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
1)*arctanh(a*x)^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**2,x)

[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)**2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(7/2)),x)
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(7/2)), x)
```

3.490 $\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{63\mathrm{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{35\mathrm{Shi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{7\mathrm{Shi}(7\,\mathrm{ta}}{6}$$

```
[Out] -1/a/(-a^2*x^2+1)^(7/2)/arctanh(a*x)+35/64*Shi(arctanh(a*x))/a+63/64*Shi(3*
arctanh(a*x))/a+35/64*Shi(5*arctanh(a*x))/a+7/64*Shi(7*arctanh(a*x))/a
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6113, 6181, 5556, 3379}

$$-\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{63\mathrm{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{35\mathrm{Shi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{7\mathrm{Shi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x])) + (35*SinhIntegral[ArcTanh[a*x]])
/(64*a) + (63*SinhIntegral[3*ArcTanh[a*x]])/(64*a) + (35*SinhIntegral[5*Arc
Tanh[a*x]])/(64*a) + (7*SinhIntegral[7*ArcTanh[a*x]])/(64*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + (7a)\int \frac{x}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64x} + \frac{9\sinh(3x)}{64x} + \frac{5\sinh(5x)}{64x} -\right.\right.}{a}$$

$$= -\frac{1}{a(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a} + \frac{35}{-}$$

$$= -\frac{1}{a(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a} + \frac{63\text{Shi}\left(3\tanh^{-1}(ax\right.}{64a}$$

Mathematica [A]
time = 0.16, size = 65, normalized size = 0.81

$$\frac{-\frac{64}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + 7\left(5\text{Shi}\left(\tanh^{-1}(ax)\right) + 9\text{Shi}\left(3\tanh^{-1}(ax)\right) + 5\text{Shi}\left(5\tanh^{-1}(ax)\right) + \text{Shi}\left(7\tanh^{-1}(ax)\right)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-64/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]) + 7*(5*SinhIntegral[ArcTanh[a*x]] +
 9*SinhIntegral[3*ArcTanh[a*x]] + 5*SinhIntegral[5*ArcTanh[a*x]] + SinhInte
gral[7*ArcTanh[a*x]]))/(64*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(70) = 140.
time = 2.94, size = 232, normalized size = 2.90

method	result

default	$\frac{7 \operatorname{arctanh}(ax) \operatorname{hyperbolicSineIntegral}(7 \operatorname{arctanh}(ax)) a^2 x^2 + 35 \operatorname{arctanh}(ax) \operatorname{hyperbolicSineIntegral}(5 \operatorname{arctanh}(ax)) a^2 x^2 + 35 \operatorname{arctanh}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/64/a*(7*arctanh(a*x)*Shi(7*arctanh(a*x))*a^2*x^2+35*arctanh(a*x)*Shi(5*arctanh(a*x))*a^2*x^2+35*arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2+63*arctanh(a*x)*Shi(3*arctanh(a*x))*a^2*x^2-cosh(7*arctanh(a*x))*a^2*x^2-7*cosh(5*arctanh(a*x))*a^2*x^2-21*cosh(3*arctanh(a*x))*a^2*x^2-7*Shi(7*arctanh(a*x))*arctanh(a*x)-35*Shi(5*arctanh(a*x))*arctanh(a*x)-35*Shi(arctanh(a*x))*arctanh(a*x)-63*Shi(3*arctanh(a*x))*arctanh(a*x)+cosh(7*arctanh(a*x))+7*cosh(5*arctanh(a*x))+35*(-a^2*x^2+1)^(1/2)+21*cosh(3*arctanh(a*x)))/arctanh(a*x)/(a^2*x^2-1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="maxima")

[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="fricas")

[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**2,x)

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(9/2)),x)
[Out] int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(9/2)), x)
```

3.491 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Maple [A]
time = 5.94, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^3,x)
[Out] int((1 - a^2*x^2)^(1/2)/atanh(a*x)^3, x)
```

3.492 $\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx = \int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(1/2)), x)
```

3.493 $\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] 1/2*Chi(arctanh(a*x))/a-1/2/a/arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*x/arcta
nh(a*x)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6113, 6153, 6115, 3382}

$$-\frac{x}{2\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)^2}+\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTan
h[a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6153

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} + \frac{1}{2}a\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{1}{2}\int \frac{}{(1-a^2x}$$

$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(}{x}\right.}{}$$

$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Chi}\left(\tanh^{-1}(\right.}{2a}$$

Mathematica [A]
time = 0.08, size = 44, normalized size = 0.68

$$\frac{-\frac{1+ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} + \text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegra
l[ArcTanh[a*x]])/(2*a)
```

Maple [A]
time = 0.00, size = 86, normalized size = 1.32

method	result
default	$\frac{\operatorname{arctanh}(ax)^2 \operatorname{hyperbolicCosineIntegral}(\operatorname{arctanh}(ax))a^2x^2+\sqrt{-a^2x^2+1}\,ax\operatorname{arctanh}(ax)-\operatorname{hyperbolicCosineIntegral}(\operatorname{arctanh}(ax}{2a\operatorname{arctanh}(ax)^2(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+(-a^2*x^2+1)^(1/2)*a*x*arct
anh(a*x)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x)^
2/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

3.494 $\int \frac{1}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{9\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2-3/2*x/(-a^2*x^2+1)^(3/2)/arctanh(a
*x)+3/8*Chi(arctanh(a*x))/a+9/8*Chi(3*arctanh(a*x))/a
```

Rubi [A]
time = 0.25, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6113, 6179, 6181, 5556, 3382, 6115, 3393}

$$-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{3\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{9\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) - (3*x)/(2*(1 - a^2*x^2)^(3/2
)*ArcTanh[a*x]) + (3*CoshIntegral[ArcTanh[a*x]])/(8*a) + (9*CoshIntegral[3*
ArcTanh[a*x]])/(8*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule steps

$$\int \frac{1}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} + \frac{1}{2}(3a)\int \frac{x}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} - \frac{3x}{2\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)} + \frac{3}{2}\int \frac{}{(1-}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} - \frac{3x}{2\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)} + \frac{3\mathrm{Subst}\big(}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} - \frac{3x}{2\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)} + \frac{3\mathrm{Subst}\big(}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} - \frac{3x}{2\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)} + \frac{3\mathrm{Subst}\big(}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} - \frac{3x}{2\,(1-a^2x^2)^{3/2}\tanh^{-1}(ax)} + \frac{3\mathrm{Chi}\big(\mathrm{ta}}{}$$

Mathematica [A]
time = 0.15, size = 56, normalized size = 0.71

$$\frac{-\frac{4(1+3ax\tanh^{-1}(ax))}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2} + 3\mathrm{Chi}\big(\tanh^{-1}(ax)\big) + 9\mathrm{Chi}\big(3\tanh^{-1}(ax)\big)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

```
[Out] ((-4*(1 + 3*a*x*ArcTanh[a*x]))/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) + 3*Cos
hIntegral[ArcTanh[a*x]] + 9*CoshIntegral[3*ArcTanh[a*x]])/(8*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(67) = 134.
time = 3.42, size = 180, normalized size = 2.28

method	result
default	$\frac{9\,\mathrm{hyperbolicCosineIntegral}(3\,\mathrm{arctanh}(ax))\,\mathrm{arctanh}(ax)^2a^2x^2+3\,\mathrm{arctanh}(ax)^2\,\mathrm{hyperbolicCosineIntegral}(\mathrm{arctanh}(ax))a^2x^2-3\,\mathrm{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(9*Chi(3*arctanh(a*x))*arctanh(a*x)^2*a^2*x^2+3*arctanh(a*x)^2*Chi(ar
ctanh(a*x))*a^2*x^2-3*arctanh(a*x)*sinh(3*arctanh(a*x))*a^2*x^2-cosh(3*arct
```

```
anh(a*x))*a^2*x^2+3*(-a^2*x^2+1)^(1/2)*a*x*arctanh(a*x)-9*Chi(3*arctanh(a*x
))*arctanh(a*x)^2-3*Chi(arctanh(a*x))*arctanh(a*x)^2+3*sinh(3*arctanh(a*x))
*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a*x)))/arctanh(a*x)^2/(a^
2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(5/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(5/2)), x)
```

3.495 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{16a}+\frac{45\text{Chi}\left(3\tanh^{-1}(ax)\right)}{32a}+\frac{25}{}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2-5/2*x/(-a^2*x^2+1)^(5/2)/arctanh(a
*x)+5/16*Chi(arctanh(a*x))/a+45/32*Chi(3*arctanh(a*x))/a+25/32*Chi(5*arctan
h(a*x))/a
```

Rubi [A]
time = 0.28, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6113, 6179, 6181, 5556, 3382, 6115, 3393}

$$-\frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)}-\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2}+\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{16a}+\frac{45\text{Chi}\left(3\tanh^{-1}(ax)\right)}{32a}+\frac{25\text{Chi}\left(5\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (5*x)/(2*(1 - a^2*x^2)^(5/2
)*ArcTanh[a*x]) + (5*CoshIntegral[ArcTanh[a*x]])/(16*a) + (45*CoshIntegral[
3*ArcTanh[a*x]])/(32*a) + (25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} + \frac{1}{2}(5a)\int \frac{x}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} - \frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)} + \frac{5}{2}\int \frac{}{(1-}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} - \frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)} + \frac{5\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} - \frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)} + \frac{5\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} - \frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)} + \frac{5\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} - \frac{5x}{2\,(1-a^2x^2)^{5/2}\tanh^{-1}(ax)} + \frac{5\mathrm{Chi}\big(\tan}{16}$$

Mathematica [A]
time = 0.17, size = 79, normalized size = 0.85

$$\frac{-\frac{16}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)^2} - \frac{80ax}{(1-a^2x^2)^{5/2}\tanh^{-1}(ax)} + 10\mathrm{Chi}\big(\tanh^{-1}(ax)\big) + 45\mathrm{Chi}\big(3\tanh^{-1}(ax)\big) + 25\mathrm{Chi}\big(5\tanh^{-1}(ax)\big)}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (80*a*x)/((1 - a^2*x^2)^(5/2)*A
rcTanh[a*x]) + 10*CoshIntegral[ArcTanh[a*x]] + 45*CoshIntegral[3*ArcTanh[a*
x]] + 25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(79) = 158$.
time = 2.86, size = 272, normalized size = 2.92

method	result
default	$\frac{45\,\mathrm{hyperbolicCosineIntegral}(3\,\mathrm{arctanh}(ax))\,\mathrm{arctanh}(ax)^2a^2x^2+25\,\mathrm{hyperbolicCosineIntegral}(5\,\mathrm{arctanh}(ax))\,\mathrm{arctanh}(ax)^2a^2x^2+10\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a*(45*Chi(3*arctanh(a*x))*arctanh(a*x)^2*a^2*x^2+25*Chi(5*arctanh(a*x)
)*arctanh(a*x)^2*a^2*x^2+10*arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2-15*arc
```

```
tanh(a*x)*sinh(3*arctanh(a*x))*a^2*x^2-5*arctanh(a*x)*sinh(5*arctanh(a*x))*
a^2*x^2-cosh(5*arctanh(a*x))*a^2*x^2-5*cosh(3*arctanh(a*x))*a^2*x^2+10*(-a^
2*x^2+1)^(1/2)*a*x*arctanh(a*x)-45*Chi(3*arctanh(a*x))*arctanh(a*x)^2-25*Ch
i(5*arctanh(a*x))*arctanh(a*x)^2-10*Chi(arctanh(a*x))*arctanh(a*x)^2+15*sin
h(3*arctanh(a*x))*arctanh(a*x)+5*sinh(5*arctanh(a*x))*arctanh(a*x)+cosh(5*a
rctanh(a*x))+10*(-a^2*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x)))/arctanh(a*x)^2/(
a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(7/2)),x)
```

```
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(7/2)), x)
```

3.496 $\int \frac{1}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{128a}+\frac{189\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{128a}$$

```
[Out] -1/2/a/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2-7/2*x/(-a^2*x^2+1)^(7/2)/arctanh(a
*x)+35/128*Chi(arctanh(a*x))/a+189/128*Chi(3*arctanh(a*x))/a+175/128*Chi(5*
arctanh(a*x))/a+49/128*Chi(7*arctanh(a*x))/a
```

Rubi [A]
time = 0.30, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6113, 6179, 6181, 5556, 3382, 6115, 3393}

$$-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{128a}+\frac{189\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{128a}+\frac{175\mathrm{Chi}\left(5\tanh^{-1}(ax)\right)}{128a}+\frac{49\mathrm{Chi}\left(7\tanh^{-1}(ax)\right)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (7*x)/(2*(1 - a^2*x^2)^(7/2
)*ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/(128*a) + (189*CoshIntegr
al[3*ArcTanh[a*x]])/(128*a) + (175*CoshIntegral[5*ArcTanh[a*x]])/(128*a) +
(49*CoshIntegral[7*ArcTanh[a*x]])/(128*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6113

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6115

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 6179

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6181

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sinh[x]^m
/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} + \frac{1}{2}(7a)\int \frac{x}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} - \frac{7x}{2\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7}{2}\int \frac{}{(1}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} - \frac{7x}{2\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7\text{Subst}\big(}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} - \frac{7x}{2\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7\text{Subst}\big(}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} - \frac{7x}{2\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{7\text{Subst}\big(}{}$$

$$= -\frac{1}{2a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} - \frac{7x}{2\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{35\text{Chi}\big(\text{t}}{}$$

Mathematica [A]
time = 0.30, size = 99, normalized size = 0.93

$$\frac{1}{128}\left(-\frac{64}{a\,(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2} - \frac{448x}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)} + \frac{35\text{Chi}\big(\tanh^{-1}(ax)\big)}{a} + \frac{189\text{Chi}\big(3\tanh^{-1}(ax)\big)}{a} + \frac{175\text{Chi}\big(5\tanh^{-1}(ax)\big)}{a} + \frac{49\text{Chi}\big(7\tanh^{-1}(ax)\big)}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-64/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (448*x)/((1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/a + (189*CoshIntegral[3*Arc
Tanh[a*x]])/a + (175*CoshIntegral[5*ArcTanh[a*x]])/a + (49*CoshIntegral[7*A
rcTanh[a*x]])/a)/128
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(91) = 182.
time = 4.01, size = 364, normalized size = 3.40

method	result
default	$\frac{189\,\text{hyperbolicCosineIntegral}(3\,\text{arctanh}(ax))\,\text{arctanh}(ax)^2a^2x^2+175\,\text{hyperbolicCosineIntegral}(5\,\text{arctanh}(ax))\,\text{arctanh}(ax)^2a^2x^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/a*(189*Chi(3*arctanh(a*x))*arctanh(a*x)^2*a^2*x^2+175*Chi(5*arctanh(a
*x))*arctanh(a*x)^2*a^2*x^2+49*Chi(7*arctanh(a*x))*arctanh(a*x)^2*a^2*x^2+3
5*arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2-63*arctanh(a*x)*sinh(3*arctanh(a
*x))*a^2*x^2-7*arctanh(a*x)*sinh(7*arctanh(a*x))*a^2*x^2-35*arctanh(a*x)*si
nh(5*arctanh(a*x))*a^2*x^2-cosh(7*arctanh(a*x))*a^2*x^2-7*cosh(5*arctanh(a*
x))*a^2*x^2-21*cosh(3*arctanh(a*x))*a^2*x^2+35*(-a^2*x^2+1)^(1/2)*a*x*arcta
nh(a*x)-189*Chi(3*arctanh(a*x))*arctanh(a*x)^2-175*Chi(5*arctanh(a*x))*arct
anh(a*x)^2-49*Chi(7*arctanh(a*x))*arctanh(a*x)^2-35*Chi(arctanh(a*x))*arcta
nh(a*x)^2+63*sinh(3*arctanh(a*x))*arctanh(a*x)+7*sinh(7*arctanh(a*x))*arcta
nh(a*x)+35*sinh(5*arctanh(a*x))*arctanh(a*x)+cosh(7*arctanh(a*x))+7*cosh(5*
arctanh(a*x))+35*(-a^2*x^2+1)^(1/2)+21*cosh(3*arctanh(a*x)))/arctanh(a*x)^2
/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\operatorname{atanh}^{3}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**3,x)
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atanh}(a\,x)^3\,\left(1-a^2\,x^2\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(9/2)),x)
[Out] int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(9/2)), x)
```

3.497 $\int \frac{(d+ex)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=122

$$\frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c^2}+\frac{be\left(a+b\tanh^{-1}(cx)\right)\text{PolyLo}}{c^2}$$

```
[Out] 1/3*d*(a+b*arctanh(c*x))^3/b/c-1/3*e*(a+b*arctanh(c*x))^3/b/c^2+e*(a+b*arct
anh(c*x))^2*ln(2/(-c*x+1))/c^2+b*e*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1
))/c^2-1/2*b^2*e*polylog(3,1-2/(-c*x+1))/c^2
```

Rubi [A]
time = 0.24, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6195, 6095, 6131, 6055, 6205, 6745}

$$\frac{be\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^2}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc}-\frac{b^2e\text{Li}_3\left(1-\frac{2}{1-cx}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

```
[Out] (d*(a + b*ArcTanh[c*x])^3)/(3*b*c) - (e*(a + b*ArcTanh[c*x])^3)/(3*b*c^2) +
 (e*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^2 + (b*e*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 - c*x)])/c^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2
*c^2)
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}+\frac{ex\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx+e\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-cx}\,d}{c}$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)}{c^2}$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)}{c^2}$$

$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\left(a+b\tanh^{-1}(cx)\right)}{c^2}$$

Mathematica [A]

time = 0.21, size = 193, normalized size = 1.58

$$\frac{6abcd\tanh^{-1}(cx)^2+6abe\tanh^{-1}(cx)^2+2b^2cd\tanh^{-1}(cx)^3+2b^2e\tanh^{-1}(cx)^3+12abe\tanh^{-1}(cx)\log\left(1+e^{-2\tanh^{-1}(cx)}\right)+6b^2e\tanh^{-1}(cx)^2\log\left(1+e^{-2\tanh^{-1}(cx)}\right)-3a^2cd\log(1-cx)-3a^2e\log(1-cx)+3a^2cd\log(1+cx)-3a^2e\log(1+cx)-6be\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-3b^2e\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

```
[Out] (6*a*b*c*d*ArcTanh[c*x]^2 + 6*a*b*e*ArcTanh[c*x]^2 + 2*b^2*c*d*ArcTanh[c*x]
^3 + 2*b^2*e*ArcTanh[c*x]^3 + 12*a*b*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*b^2*e*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 3*a^2*c*d*Log
[1 - c*x] - 3*a^2*e*Log[1 - c*x] + 3*a^2*c*d*Log[1 + c*x] - 3*a^2*e*Log[1 +
 c*x] - 6*b*e*(a + b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 3*b^2
*e*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(6*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 22.33, size = 1806, normalized size = 14.80

method	result	size
derivativedivides	Expression too large to display	1806
default	Expression too large to display	1806

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/4*I*b^2/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh
(c*x)^2*e-1/2*a^2/c*ln(c*x-1)*e-1/2*a^2/c*ln(c*x+1)*e-1/2*b^2*arctanh(c*x)^
2*ln(c*x-1)*d+1/2*b^2*arctanh(c*x)^2*ln(c*x+1)*d-1/2*b^2/c*e*polylog(3,-(c*
x+1)^2/(-c^2*x^2+1))-1/3*b^2/c*arctanh(c*x)^3*e-1/4*a*b*ln(c*x+1)^2*d-1/4*a
*b*ln(c*x-1)^2*d-b^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*d-1/2*a^
2*ln(c*x-1)*d+1/2*a^2*ln(c*x+1)*d+1/3*b^2*arctanh(c*x)^3*d-1/4*a*b/c*ln(c*x
-1)^2*e-a*b*arctanh(c*x)*ln(c*x-1)*d+a*b*arctanh(c*x)*ln(c*x+1)*d-1/2*a*b*l
n(1/2*c*x+1/2)*ln(-1/2*c*x+1/2)*d+1/2*a*b*ln(-1/2*c*x+1/2)*ln(c*x+1)*d+a*b/
c*dilog(1/2*c*x+1/2)*e+1/4*a*b/c*ln(c*x+1)^2*e+1/2*a*b*ln(1/2*c*x+1/2)*ln(c
*x-1)*d-1/2*b^2/c*arctanh(c*x)^2*ln(c*x-1)*e-1/2*b^2/c*arctanh(c*x)^2*ln(c*
x+1)*e+b^2/c*ln((c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*e+b^2/c*ln(2)*ar
ctanh(c*x)^2*e+b^2/c*e*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*
I*b^2*Pi*arctanh(c*x)^2*d-a*b/c*arctanh(c*x)*ln(c*x+1)*e+1/2*a*b/c*ln(1/2*c
*x+1/2)*ln(-1/2*c*x+1/2)*e-1/2*a*b/c*ln(-1/2*c*x+1/2)*ln(c*x+1)*e+1/2*a*b/c
*ln(1/2*c*x+1/2)*ln(c*x-1)*e+1/2*I*b^2/c*Pi*arctanh(c*x)^2*e-1/4*I*b^2*Pi*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*d+
1/2*I*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*d-1/4*I*b^
2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2*d-1/2*I*b^2*Pi*csgn(I/(
1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*d+1/4*I*b^2/c*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*e+1/2*I*b^2/c*P
i*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2*e+1/4*I*b^2/c*Pi*csgn
(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2*e-1/2*I*b^2/c*Pi*csgn(I/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2*e-1/4*I*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x
```

```
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arct
anh(c*x)^2*d+1/4*I*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2*d-1/2*I*b^2*Pi*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^
2*d-1/4*I*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^
(1/2))^2*arctanh(c*x)^2*d+1/4*I*b^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c
^2*x^2-1))*arctanh(c*x)^2*d+1/4*I*b^2/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+
(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^
2*e-1/4*I*b^2/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2*e+1/2*I*b^2/c*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*e+1
/4*I*b^2/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/
2))^2*arctanh(c*x)^2*e-a*b/c*arctanh(c*x)*ln(c*x-1)*e)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] a*b*d*(log(c*x + 1)/c - log(c*x - 1)/c)*arctanh(c*x) + 1/2*a^2*d*(log(c*x +
 1)/c - log(c*x - 1)/c) - 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1)
 + log(c*x - 1)^2)*a*b*d/c - 1/2*a^2*e*log(c^2*x^2 - 1)/c^2 + 1/24*(3*(b^2*
c*d - b^2*e)*log(c*x + 1)*log(-c*x + 1)^2 - (b^2*c*d + b^2*e)*log(-c*x + 1)
^3)/c^2 + integrate(-1/4*(4*a*b*c*x*e*log(c*x + 1) + (b^2*c*x*e + b^2*c*d)*
log(c*x + 1)^2 - (4*a*b*c*x*e + (b^2*c*d + b^2*e - (b^2*c^2*d - 3*b^2*c*e)*
x)*log(c*x + 1))*log(-c*x + 1))/(c^3*x^2 - c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x*e + a^2*d + (b^2*x*e + b^2*d)*arctanh(c*x)^2 + 2*(a*b*x*e
+ a*b*d)*arctanh(c*x))/(c^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2 d}{c^2x^2-1}\,dx - \int \frac{a^2 ex}{c^2x^2-1}\,dx - \int \frac{b^2 d\operatorname{atanh}^2(cx)}{c^2x^2-1}\,dx - \int \frac{2abd\operatorname{atanh}(cx)}{c^2x^2-1}\,dx - \int \frac{b^2 ex\operatorname{atanh}^2(cx)}{c^2x^2-1}\,dx - \int \frac{2abex\operatorname{atanh}(cx)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2*d/(c**2*x**2 - 1), x) - Integral(a**2*e*x/(c**2*x**2 - 1), x
) - Integral(b**2*d*atanh(c*x)**2/(c**2*x**2 - 1), x) - Integral(2*a*b*d*at
anh(c*x)/(c**2*x**2 - 1), x) - Integral(b**2*e*x*atanh(c*x)**2/(c**2*x**2 -
 1), x) - Integral(2*a*b*e*x*atanh(c*x)/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(e*x + d)*(b*arctanh(c*x) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a+b\,\mathrm{atanh}(c\,x))^2\,(d+e\,x)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*atanh(c*x))^2*(d + e*x))/(c^2*x^2 - 1),x)
```

```
[Out] int(-((a + b*atanh(c*x))^2*(d + e*x))/(c^2*x^2 - 1), x)
```

3.498 $\int (c+dx^2)^4 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=245

$$\frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d^2)\,x^4}{1260a^5}+\frac{d^3(36a^2c+7d)\,x^6}{378a^3}+\frac{d^4x^8}{72a}+$$

```
[Out] 1/630*d*(420*a^6*c^3+378*a^4*c^2*d+180*a^2*c*d^2+35*d^3)*x^2/a^7+1/1260*d^2
*(378*a^4*c^2+180*a^2*c*d+35*d^2)*x^4/a^5+1/378*d^3*(36*a^2*c+7*d)*x^6/a^3+
1/72*d^4*x^8/a+c^4*x*arctanh(a*x)+4/3*c^3*d*x^3*arctanh(a*x)+6/5*c^2*d^2*x^
5*arctanh(a*x)+4/7*c*d^3*x^7*arctanh(a*x)+1/9*d^4*x^9*arctanh(a*x)+1/630*(3
15*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*ln(-a^2*x^2+
1)/a^9
```

Rubi [A]
time = 0.14, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 6123, 1824, 266}

$$\frac{d^3x^6(36a^2c+7d)}{378a^3}+\frac{d^2x^4(378a^4c^2+180a^2cd+35d^2)}{1260a^5}+\frac{dx^2(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)}{630a^7}+\frac{(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)\log(1-a^2x^2)}{630a^9}+c^4x\tanh^{-1}(ax)+\frac{4}{3}c^3dx^3\tanh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tanh^{-1}(ax)+\frac{4}{7}cd^3x^7\tanh^{-1}(ax)+\frac{1}{9}d^4x^9\tanh^{-1}(ax)+\frac{d^4x^8}{72a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcTanh[a*x],x]
```

```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcTanh[a*x] + (4*c^3*d*x^
3*ArcTanh[a*x])/3 + (6*c^2*d^2*x^5*ArcTanh[a*x])/5 + (4*c*d^3*x^7*ArcTanh[a
*x])/7 + (d^4*x^9*ArcTanh[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^2\right)^4 \tanh^{-1}(ax)\,dx &= c^4x\tanh^{-1}(ax)+\frac{4}{3}c^3dx^3\tanh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tanh^{-1}(ax)+\frac{4}{7}cd^3x^7\tanh^{-1}\\ &= c^4x\tanh^{-1}(ax)+\frac{4}{3}c^3dx^3\tanh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tanh^{-1}(ax)+\frac{4}{7}cd^3x^7\tanh^{-1}\\ &= \frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d^2}{1260a^5}\\ &= \frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d^2}{1260a^5}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 213, normalized size = 0.87

$$\frac{a^2dx^2(420d^3+30a^2d^2(72c+7dx^2)+4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240cd^2x^4+35d^3x^6))+24a^9x(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8)\tanh^{-1}(ax)+12(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)\log\left(1-a^2x^2\right)}{7560a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 270
*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*
d^3*x^6 + 35*d^4*x^8)*ArcTanh[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*
a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

Maple [A]
time = 0.90, size = 301, normalized size = 1.23

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)c^4ax+\frac{4a\operatorname{arctanh}(ax)c^3d\,x^3}{3}+\frac{6a\operatorname{arctanh}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arctanh}(ax)c\,d^3x^7}{7}+\frac{a\operatorname{arctanh}(ax)d^4x^9}{9}-\frac{\left(-315a^8c^4-420a^6\right.}{}}{}$
default	$\frac{\operatorname{arctanh}(ax)c^4ax+\frac{4a\operatorname{arctanh}(ax)c^3d\,x^3}{3}+\frac{6a\operatorname{arctanh}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arctanh}(ax)c\,d^3x^7}{7}+\frac{a\operatorname{arctanh}(ax)d^4x^9}{9}-\frac{\left(-315a^8c^4-420a^6\right.}{}}{}$

risch	$\left(\frac{1}{18}d^4x^9+\frac{2}{7}d^3c\,x^7+\frac{3}{5}c^2d^2x^5+\frac{2}{3}d\,c^3x^3+\frac{1}{2}c^4x\right)\ln\left(ax+1\right)-\frac{d^4x^9\ln(-ax+1)}{18}-\frac{2c\,d^3x^7\ln(-$
meijerg	$-\frac{d^4\left(-\frac{a^2x^2\left(15a^6x^6+20a^4x^4+30a^2x^2+60\right)}{270}+\frac{2a^{10}x^{10}\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{9\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{9}\right)}{4a^9}+\frac{d^3c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arctanh(a*x)*c^4*a*x+4/3*a*arctanh(a*x)*c^3*d*x^3+6/5*a*arctanh(a*x)*c
^2*d^2*x^5+4/7*a*arctanh(a*x)*c*d^3*x^7+1/9*a*arctanh(a*x)*d^4*x^9-1/315/a^
8*(1/2*(-315*a^8*c^4-420*a^6*c^3*d-378*a^4*c^2*d^2-180*a^2*c*d^3-35*d^4)*ln
(a*x+1)-1/2*(315*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4
)*ln(a*x-1)-35/6*d^4*a^6*x^6-35/2*d^4*a^2*x^2-35/4*d^4*a^4*x^4-35/8*d^4*a^8
*x^8-45*a^6*c*d^3*x^4-90*a^4*c*d^3*x^2-30*c*a^8*d^3*x^6-189/2*c^2*a^8*d^2*x
^4-210*c^3*a^8*d*x^2-189*c^2*a^6*d^2*x^2))
```

Maxima [A]

time = 0.26, size = 276, normalized size = 1.13

$$\frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3+7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)x^2}{a^8}+\frac{12\,(315\,a^8c^4+420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)\log\left(ax+1\right)}{a^{10}}+\frac{12\,(315\,a^8c^4+420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)\log\left(ax-1\right)}{a^{10}}\right)+\frac{1}{315}\,(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x)\,\mathrm{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315*a^8
*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x -
1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*
x^3 + 315*c^4*x)*arctanh(a*x)
```

Fricas [A]

time = 0.35, size = 248, normalized size = 1.01

$$\frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3+7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2+180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d+378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4)x^2+12\,(315\,a^8c^4+420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)\log\left(a^2x^2-1\right)+12\,(35\,a^9d^4x^9+180\,a^9cd^3x^7+378\,a^9c^2d^2x^5+420\,a^9c^3dx^3+315\,a^9c^4x)\log\left(-\frac{ax+1}{ax-1}\right)}{7560\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 37
8*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d^4*x
```

```
^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*
c^4*x)*log(-(a*x + 1)/(a*x - 1)))/a^9
```

Sympy [A]
time = 0.85, size = 372, normalized size = 1.52

$$\begin{cases} c^4x\operatorname{atanh}(ax) + \frac{4c^3dx^3\operatorname{atanh}(ax)}{3} + \frac{6c^2d^2x^5\operatorname{atanh}(ax)}{5} + \frac{4cd^3x^7\operatorname{atanh}(ax)}{7} + \frac{d^4x^9\operatorname{atanh}(ax)}{9} + \frac{c^4\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^4\operatorname{atanh}(ax)}{a} + \frac{2c^3dx^2}{3a} + \frac{3c^2d^2x^4}{10a} + \frac{2cd^3x^6}{21a} + \frac{d^4x^8}{72a} + \frac{4c^3d\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{4c^3d\operatorname{atanh}(ax)}{3a^3} + \frac{3c^2d^2x^2}{5a^3} + \frac{cd^3x^4}{7a^3} + \frac{d^4x^6}{54a^3} + \frac{6c^2d^2\log\left(x-\frac{1}{a}\right)}{5a^5} + \frac{6c^2d^2\operatorname{atanh}(ax)}{5a^5} + \frac{2cd^3x^2}{7a^5} + \frac{d^4x^4}{36a^5} + \frac{4cd^3\log\left(x-\frac{1}{a}\right)}{7a^7} + \frac{4cd^3\operatorname{atanh}(ax)}{7a^7} + \frac{d^4x^2}{18a^7} + \frac{d^4\log\left(x-\frac{1}{a}\right)}{9a^9} + \frac{d^4\operatorname{atanh}(ax)}{9a^9} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*atanh(a*x),x)
```

```
[Out] Piecewise((c**4*x*atanh(a*x) + 4*c**3*d*x**3*atanh(a*x)/3 + 6*c**2*d**2*x**
5*atanh(a*x)/5 + 4*c*d**3*x**7*atanh(a*x)/7 + d**4*x**9*atanh(a*x)/9 + c**4
*log(x - 1/a)/a + c**4*atanh(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**
4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1/a)/
(3*a**3) + 4*c**3*d*atanh(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**
3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a**5) +
 6*c**2*d**2*atanh(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**4/(36*a
**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*atanh(a*x)/(7*a**7) + d**4
*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*atanh(a*x)/(9*a**9), Ne
(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1471 vs. $2(227) = 454$.
time = 0.46, size = 1471, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/945*a*(3*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 +
 35*d^4)*log(abs(-a*x - 1)/abs(a*x - 1))/a^10 - 3*(315*a^8*c^4 + 420*a^6*c^
3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(abs(-(a*x + 1)/(a*x - 1
) + 1))/a^10 + 8*(3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35*d
^4)*(a*x + 1)^7/(a*x - 1)^7 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*
d^3 + 7*d^4)*(a*x + 1)^6/(a*x - 1)^6 + (4725*a^6*c^3*d + 6237*a^4*c^2*d^2 +
 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^5/(a*x - 1)^5 - 2*(3150*a^6*c^3*d + 39
69*a^4*c^2*d^2 + 2340*a^2*c*d^3 + 455*d^4)*(a*x + 1)^4/(a*x - 1)^4 + (4725*
a^6*c^3*d + 6237*a^4*c^2*d^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^3/(a*x -
 1)^3 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*d^3 + 7*d^4)*(a*x + 1)
^2/(a*x - 1)^2 + 3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35*d^
4)*(a*x + 1)/(a*x - 1))/(a^10*((a*x + 1)/(a*x - 1) - 1)^8) + 3*(315*(a*x +
1)^8*a^8*c^4/(a*x - 1)^8 - 2520*(a*x + 1)^7*a^8*c^4/(a*x - 1)^7 + 8820*(a*x
 + 1)^6*a^8*c^4/(a*x - 1)^6 - 17640*(a*x + 1)^5*a^8*c^4/(a*x - 1)^5 + 22050
*(a*x + 1)^4*a^8*c^4/(a*x - 1)^4 - 17640*(a*x + 1)^3*a^8*c^4/(a*x - 1)^3 +
```

```
8820*(a*x + 1)^2*a^8*c^4/(a*x - 1)^2 - 2520*(a*x + 1)*a^8*c^4/(a*x - 1) + 3
15*a^8*c^4 + 1260*(a*x + 1)^8*a^6*c^3*d/(a*x - 1)^8 - 7560*(a*x + 1)^7*a^6*
c^3*d/(a*x - 1)^7 + 19320*(a*x + 1)^6*a^6*c^3*d/(a*x - 1)^6 - 27720*(a*x +
1)^5*a^6*c^3*d/(a*x - 1)^5 + 25200*(a*x + 1)^4*a^6*c^3*d/(a*x - 1)^4 - 1596
0*(a*x + 1)^3*a^6*c^3*d/(a*x - 1)^3 + 7560*(a*x + 1)^2*a^6*c^3*d/(a*x - 1)^
2 - 2520*(a*x + 1)*a^6*c^3*d/(a*x - 1) + 420*a^6*c^3*d + 1890*(a*x + 1)^8*a
^4*c^2*d^2/(a*x - 1)^8 - 7560*(a*x + 1)^7*a^4*c^2*d^2/(a*x - 1)^7 + 15120*(
a*x + 1)^6*a^4*c^2*d^2/(a*x - 1)^6 - 22680*(a*x + 1)^5*a^4*c^2*d^2/(a*x - 1
)^5 + 24948*(a*x + 1)^4*a^4*c^2*d^2/(a*x - 1)^4 - 16632*(a*x + 1)^3*a^4*c^2
*d^2/(a*x - 1)^3 + 6048*(a*x + 1)^2*a^4*c^2*d^2/(a*x - 1)^2 - 1512*(a*x + 1
)*a^4*c^2*d^2/(a*x - 1) + 378*a^4*c^2*d^2 + 1260*(a*x + 1)^8*a^2*c*d^3/(a*x
 - 1)^8 - 2520*(a*x + 1)^7*a^2*c*d^3/(a*x - 1)^7 + 7560*(a*x + 1)^6*a^2*c*d
^3/(a*x - 1)^6 - 12600*(a*x + 1)^5*a^2*c*d^3/(a*x - 1)^5 + 10080*(a*x + 1)^
4*a^2*c*d^3/(a*x - 1)^4 - 7560*(a*x + 1)^3*a^2*c*d^3/(a*x - 1)^3 + 3960*(a*
x + 1)^2*a^2*c*d^3/(a*x - 1)^2 - 360*(a*x + 1)*a^2*c*d^3/(a*x - 1) + 180*a^
2*c*d^3 + 315*(a*x + 1)^8*d^4/(a*x - 1)^8 + 2940*(a*x + 1)^6*d^4/(a*x - 1)^
6 + 4410*(a*x + 1)^4*d^4/(a*x - 1)^4 + 1260*(a*x + 1)^2*d^4/(a*x - 1)^2 + 3
5*d^4)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(
a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^10*((a*x +
 1)/(a*x - 1) - 1)^9))
```

Mupad [B]
time = 1.28, size = 288, normalized size = 1.18

$$x^2\left(\frac{\frac{\frac{d^4}{9a^3}+\frac{4cd^3}{7a}}{a^2}+\frac{6c^2d^2}{5a}}{2a^2}+\frac{2c^3d}{3a}\right)+x^6\left(\frac{d^4}{54a^3}+\frac{2cd^3}{21a}\right)+\ln(ax+1)\left(\frac{c^4x}{2}+\frac{2c^3dx^3}{3}+\frac{3c^2d^2x^5}{5}+\frac{2cd^3x^7}{7}+\frac{d^4x^9}{18}\right)-\ln(1-ax)\left(\frac{c^4x}{2}+\frac{2c^3dx^3}{3}+\frac{3c^2d^2x^5}{5}+\frac{2cd^3x^7}{7}+\frac{d^4x^9}{18}\right)+x^4\left(\frac{\frac{d^4}{9a^3}+\frac{4cd^3}{7a}}{4a^2}+\frac{3c^2d^2}{10a}\right)+\frac{\ln(a^2x^2-1)\,(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)}{630a^9}+\frac{d^4x^8}{72a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^4,x)
```

```
[Out] x^2*(((d^4/(9*a^3) + (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a))/(2*a^2) + (2
*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) + (2*c*d^3)/(21*a)) + log(a*x + 1)*((c^4
*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^5)/
5) - log(1 - a*x)*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^
7)/7 + (3*c^2*d^2*x^5)/5) + x^4*((d^4/(9*a^3) + (4*c*d^3)/(7*a))/(4*a^2) +
(3*c^2*d^2)/(10*a)) + (log(a^2*x^2 - 1)*(35*d^4 + 315*a^8*c^4 + 180*a^2*c*d
^3 + 420*a^6*c^3*d + 378*a^4*c^2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

3.499 $\int (c+dx^2)^3 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=169

$$\frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5}+\frac{d^2(21a^2c+5d)\,x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}($$

```
[Out] 1/70*d*(35*a^4*c^2+21*a^2*c*d+5*d^2)*x^2/a^5+1/140*d^2*(21*a^2*c+5*d)*x^4/a
^3+1/42*d^3*x^6/a+c^3*x*arctanh(a*x)+c^2*d*x^3*arctanh(a*x)+3/5*c*d^2*x^5*a
rctanh(a*x)+1/7*d^3*x^7*arctanh(a*x)+1/70*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c
*d^2+5*d^3)*ln(-a^2*x^2+1)/a^7
```

Rubi [A]

time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 6123, 1824, 266}

$$\frac{d^2x^4(21a^2c+5d)}{140a^3}+\frac{dx^2(35a^4c^2+21a^2cd+5d^2)}{70a^5}+\frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log(1-a^2x^2)}{70a^7}+c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(ax)+\frac{d^3x^6}{42a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcTanh[a*x] + c^2*d*x^3*ArcTanh[
a*x] + (3*c*d^2*x^5*ArcTanh[a*x])/5 + (d^3*x^7*ArcTanh[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \tanh^{-1}(ax)\,dx = c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(a\ldots$$

$$= c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(a\ldots$$

$$= \frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5}+\frac{d^2(21a^2c+5d)\,x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)\ldots$$

$$= \frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5}+\frac{d^2(21a^2c+5d)\,x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)\ldots$$

Mathematica [A]
time = 0.05, size = 150, normalized size = 0.89

$$\frac{a^2dx^2(30d^2+3a^2d(42c+5dx^2)+a^4(210c^2+63cdx^2+10d^2x^4))+12a^7x(35c^3+35c^2dx^2+21cd^2x^4+5d^3x^6)\tanh^{-1}(ax)+6(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log\left(1-a^2x^2\right)}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcTanh[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1
- a^2*x^2])/(420*a^7)
```

Maple [A]
time = 0.57, size = 211, normalized size = 1.25

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)c^3ax+a\operatorname{arctanh}(ax)c^2dx^3+\frac{3a\operatorname{arctanh}(ax)cd^2x^5}{5}+\frac{a\operatorname{arctanh}(ax)d^3x^7}{7}-\frac{\left(-35a^6c^3-35a^4c^2d-21a^2cd^2-5d^3\right)\ln(\ldots}{2}\ldots}{a}$
default	$\frac{\operatorname{arctanh}(ax)c^3ax+a\operatorname{arctanh}(ax)c^2dx^3+\frac{3a\operatorname{arctanh}(ax)cd^2x^5}{5}+\frac{a\operatorname{arctanh}(ax)d^3x^7}{7}-\frac{\left(-35a^6c^3-35a^4c^2d-21a^2cd^2-5d^3\right)\ln(\ldots}{2}\ldots}{a}$
risch	$\left(\frac{1}{14}d^3x^7+\frac{3}{10}cd^2x^5+\frac{1}{2}c^2dx^3+\frac{1}{2}xc^3\right)\ln(ax+1)-\frac{d^3x^7\ln(-ax+1)}{14}-\frac{3cd^2x^5\ln(-ax+1)}{10}+\frac{d^3\ldots}{42\ldots}$
meijerg	$\frac{d^3\left(\frac{a^2x^2\left(4a^4x^4+6a^2x^2+12\right)}{42}-\frac{2a^8x^8\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(-a^2x^2+1\right)}{7}\right)}{4a^7}-\frac{3d^2c\left(-\frac{a^2x^2\left(3a^2x^2\ldots\right)}{15}\ldots\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arctanh(a*x)*c^3*a*x+a*arctanh(a*x)*c^2*d*x^3+3/5*a*arctanh(a*x)*c*d^2
*x^5+1/7*a*arctanh(a*x)*d^3*x^7-1/35/a^6*(1/2*(-35*a^6*c^3-35*a^4*c^2*d-21*
a^2*c*d^2-5*d^3)*ln(a*x+1)-1/2*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c*d^2+5*d^3)
*ln(a*x-1)-5/4*a^4*d^3*x^4-5/2*d^3*x^2*a^2-21/4*a^6*c*d^2*x^4-35/2*a^6*c^2*
d*x^2-21/2*a^4*c*d^2*x^2-5/6*a^6*d^3*x^6))
```

Maxima [A]
time = 0.26, size = 198, normalized size = 1.17

$$\frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\,(21\,a^4cd^2+5\,a^2d^3)x^4+6\,(35\,a^4c^2d+21\,a^2cd^2+5\,d^3)x^2}{a^6}+\frac{6\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log(ax+1)}{a^8}+\frac{6\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log(ax-1)}{a^8}\right)+\frac{1}{35}\,(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x)\,\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*
d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*
x^3 + 35*c^3*x)*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 178, normalized size = 1.05

$$\frac{10\,a^6d^3x^6+3\,(21\,a^6cd^2+5\,a^4d^3)x^4+6\,(35\,a^6c^2d+21\,a^4cd^2+5\,a^2d^3)x^2+6\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log(a^2x^2-1)+6\,(5\,a^7d^3x^7+21\,a^7cd^2x^5+35\,a^7c^2dx^3+35\,a^7c^3x)\log\left(-\frac{ax+1}{ax-1}\right)}{420\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
+ 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d
^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7
*c^2*d*x^3 + 35*a^7*c^3*x)*log(-(a*x + 1)/(a*x - 1)))/a^7
```

Sympy [A]
time = 0.52, size = 245, normalized size = 1.45

$$\begin{cases} c^3x\,\operatorname{atanh}(ax)+c^2dx^3\,\operatorname{atanh}(ax)+\frac{3cd^2x^5\,\operatorname{atanh}(ax)}{5}+\frac{d^3x^7\,\operatorname{atanh}(ax)}{7}+\frac{c^3\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^3\,\operatorname{atanh}(ax)}{a}+\frac{c^2dx^2}{2a}+\frac{3cd^2x^4}{20a}+\frac{d^3x^6}{42a}+\frac{c^2d\log\left(x-\frac{1}{a}\right)}{a^3}+\frac{c^2d\,\operatorname{atanh}(ax)}{a^3}+\frac{3cd^2x^2}{10a^3}+\frac{d^3x^4}{28a^3}+\frac{3cd^2\log\left(x-\frac{1}{a}\right)}{5a^5}+\frac{3cd^2\,\operatorname{atanh}(ax)}{5a^5}+\frac{d^3x^2}{14a^5}+\frac{d^3\log\left(x-\frac{1}{a}\right)}{7a^7}+\frac{d^3\,\operatorname{atanh}(ax)}{7a^7} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*atanh(a*x),x)
```

[Out] Piecewise((c**3*x*atanh(a*x) + c**2*d*x**3*atanh(a*x) + 3*c*d**2*x**5*atanh(a*x)/5 + d**3*x**7*atanh(a*x)/7 + c**3*log(x - 1/a)/a + c**3*atanh(a*x)/a + c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*log(x - 1/a)/a**3 + c**2*d*atanh(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3*x**4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*atanh(a*x)/(5*a**5) + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*atanh(a*x)/(7*a**7), Ne(a, 0)), (0, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 930 vs. 2(157) = 314.

time = 0.42, size = 930, normalized size = 5.50

$$\frac{1}{105}\,a\left(\frac{3\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^8}-\frac{3\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^8}+\frac{2\left(\frac{3\,(35\,a^4c^2d+42\,a^2cd^2+15\,d^3)(ax+1)^5}{(ax-1)^5}-\frac{6\,(70\,a^4c^2d+63\,a^2cd^2+15\,d^3)(ax+1)^4}{(ax-1)^4}+\frac{2\,(315\,a^4c^2d+252\,a^2cd^2+85\,d^3)(ax+1)^3}{(ax-1)^3}-\frac{6\,(70\,a^4c^2d+63\,a^2cd^2+15\,d^3)(ax+1)^2}{(ax-1)^2}+\frac{3\,(35\,a^4c^2d+42\,a^2cd^2+15\,d^3)(ax+1)}{ax-1}\right)}{a^8\left(\frac{ax+1}{ax-1}-1\right)^6}+\frac{3\left(\frac{35\,(ax+1)^6a^6c^3}{(ax-1)^6}-\frac{210\,(ax+1)^5a^6c^3}{(ax-1)^5}+\frac{525\,(ax+1)^4a^6c^3}{(ax-1)^4}-\frac{700\,(ax+1)^3a^6c^3}{(ax-1)^3}+\frac{525\,(ax+1)^2a^6c^3}{(ax-1)^2}-\frac{210\,(ax+1)a^6c^3}{ax-1}+35\,a^6c^3+\frac{105\,(ax+1)^6a^4c^2d}{(ax-1)^6}-\frac{420\,(ax+1)^5a^4c^2d}{(ax-1)^5}+\frac{665\,(ax+1)^4a^4c^2d}{(ax-1)^4}-\frac{560\,(ax+1)^3a^4c^2d}{(ax-1)^3}+\frac{315\,(ax+1)^2a^4c^2d}{(ax-1)^2}-\frac{140\,(ax+1)a^4c^2d}{ax-1}+35\,a^4c^2d+\frac{105\,(ax+1)^6a^2cd^2}{(ax-1)^6}-\frac{210\,(ax+1)^5a^2cd^2}{(ax-1)^5}+\frac{315\,(ax+1)^4a^2cd^2}{(ax-1)^4}-\frac{420\,(ax+1)^3a^2cd^2}{(ax-1)^3}+\frac{231\,(ax+1)^2a^2cd^2}{(ax-1)^2}-\frac{42\,(ax+1)a^2cd^2}{ax-1}+21\,a^2cd^2+\frac{35\,(ax+1)^6d^3}{(ax-1)^6}+\frac{175\,(ax+1)^4d^3}{(ax-1)^4}+\frac{105\,(ax+1)^2d^3}{(ax-1)^2}+5\,d^3\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^8\left(\frac{ax+1}{ax-1}-1\right)^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="giac")

[Out] 1/105*a*(3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(-a*x - 1)/abs(a*x - 1))/a^8 - 3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(-(a*x + 1)/(a*x - 1) + 1))/a^8 + 2*(3*(35*a^4*c^2*d + 42*a^2*c*d^2 + 15*d^3)*(a*x + 1)^5/(a*x - 1)^5 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^3)*(a*x + 1)^4/(a*x - 1)^4 + 2*(315*a^4*c^2*d + 252*a^2*c*d^2 + 85*d^3)*(a*x + 1)^3/(a*x - 1)^3 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^3)*(a*x + 1)^2/(a*x - 1)^2 + 3*(35*a^4*c^2*d + 42*a^2*c*d^2 + 15*d^3)*(a*x + 1)/(a*x - 1))/(a^8*((a*x + 1)/(a*x - 1) - 1)^6) + 3*(35*(a*x + 1)^6*a^6*c^3/(a*x - 1)^6 - 210*(a*x + 1)^5*a^6*c^3/(a*x - 1)^5 + 525*(a*x + 1)^4*a^6*c^3/(a*x - 1)^4 - 700*(a*x + 1)^3*a^6*c^3/(a*x - 1)^3 + 525*(a*x + 1)^2*a^6*c^3/(a*x - 1)^2 - 210*(a*x + 1)*a^6*c^3/(a*x - 1) + 35*a^6*c^3 + 105*(a*x + 1)^6*a^4*c^2*d/(a*x - 1)^6 - 420*(a*x + 1)^5*a^4*c^2*d/(a*x - 1)^5 + 665*(a*x + 1)^4*a^4*c^2*d/(a*x - 1)^4 - 560*(a*x + 1)^3*a^4*c^2*d/(a*x - 1)^3 + 315*(a*x + 1)^2*a^4*c^2*d/(a*x - 1)^2 - 140*(a*x + 1)*a^4*c^2*d/(a*x - 1) + 35*a^4*c^2*d + 105*(a*x + 1)^6*a^2*c*d^2/(a*x - 1)^6 - 210*(a*x + 1)^5*a^2*c*d^2/(a*x - 1)^5 + 315*(a*x + 1)^4*a^2*c*d^2/(a*x - 1)^4 - 420*(a*x + 1)^3*a^2*c*d^2/(a*x - 1)^3 + 231*(a*x + 1)^2*a^2*c*d^2/(a*x - 1)^2 - 42*(a*x + 1)*a^2*c*d^2/(a*x - 1) + 21*a^2*c*d^2 + 35*(a*x + 1)^6*d^3/(a*x - 1)^6 + 175*(a*x + 1)^4*d^3/(a*x - 1)^4 + 105*(a*x + 1)^2*d^3/(a*x - 1)^2 + 5*d^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^8*((a*x + 1)/(a*x - 1) - 1)^7))

Mupad [B]

time = 1.38, size = 190, normalized size = 1.12

$$c^3\,x\,\mathrm{atanh}(a\,x)+\frac{d^3\,x^7\,\mathrm{atanh}(a\,x)}{7}+\frac{c^3\,\ln\left(a^2\,x^2-1\right)}{2\,a}+\frac{d^3\,\ln\left(a^2\,x^2-1\right)}{14\,a^7}+\frac{d^3\,x^6}{42\,a}+\frac{d^3\,x^4}{28\,a^3}+\frac{d^3\,x^2}{14\,a^5}+\frac{c^2\,d\,\ln\left(a^2\,x^2-1\right)}{2\,a^3}+\frac{3\,c\,d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5}+\frac{c^2\,d\,x^2}{2\,a}+\frac{3\,c\,d^2\,x^4}{20\,a}+\frac{3\,c\,d^2\,x^2}{10\,a^3}+c^2\,d\,x^3\,\mathrm{atanh}(a\,x)+\frac{3\,c\,d^2\,x^5\,\mathrm{atanh}(a\,x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^3,x)

[Out] c^3*x*atanh(a*x) + (d^3*x^7*atanh(a*x))/7 + (c^3*log(a^2*x^2 - 1))/(2*a) +
(d^3*log(a^2*x^2 - 1))/(14*a^7) + (d^3*x^6)/(42*a) + (d^3*x^4)/(28*a^3) + (
d^3*x^2)/(14*a^5) + (c^2*d*log(a^2*x^2 - 1))/(2*a^3) + (3*c*d^2*log(a^2*x^2
 - 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) + (3*c*d^2*x^2)/
(10*a^3) + c^2*d*x^3*atanh(a*x) + (3*c*d^2*x^5*atanh(a*x))/5
```

3.500 $\int (c+dx^2)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=110

$$\frac{d(10a^2c+3d)\,x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)+\frac{(15a^4c^2+10a^2cd+3d^2)}{30a^5}$$

```
[Out] 1/30*d*(10*a^2*c+3*d)*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arctanh(a*x)+2/3*c*d*x^3
*arctanh(a*x)+1/5*d^2*x^5*arctanh(a*x)+1/30*(15*a^4*c^2+10*a^2*c*d+3*d^2)*l
n(-a^2*x^2+1)/a^5
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {200, 6123, 1608, 1261, 712}

$$\frac{dx^2(10a^2c+3d)}{30a^3}+\frac{(15a^4c^2+10a^2cd+3d^2)\log(1-a^2x^2)}{30a^5}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)+\frac{d^2x^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcTanh[a*x] +
 (2*c*d*x^3*ArcTanh[a*x])/3 + (d^2*x^5*ArcTanh[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^2 \tanh^{-1}(ax)\,dx = c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-a\int \frac{c^2x+\frac{2}{3}cdx^3}{1-a^2x}$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-a\int \frac{x\left(c^2+\frac{2}{3}cdx\right.}{1-a^2}$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int \frac{c^2+}{}\right.$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int \left(-\right.\right.$$

$$= \frac{d(10a^2c+3d)\,x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.89

$$\frac{a^2dx^2(6d+a^2(20c+3dx^2))+4a^5x(15c^2+10cdx^2+3d^2x^4)\tanh^{-1}(ax)+(30a^4c^2+20a^2cd+6d^2)\log\left(1-a^2x^2\right)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3*
d^2*x^4)*ArcTanh[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2])
/(60*a^5)
```

Maple [A]
time = 1.32, size = 137, normalized size = 1.25

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)c^2ax+\frac{2a\operatorname{arctanh}(ax)cd\,x^3}{3}+\frac{a\operatorname{arctanh}(ax)d^2x^5}{5}-\frac{-5a^4cd\,x^2-\frac{3a^4d^2x^4}{4}-\frac{3d^2x^2a^2}{2}-\frac{\left(15a^4c^2+10a^2dc+3d^2\right)\ln(ax-1)}{2}}{15a^4}}{a}$
default	$\frac{\operatorname{arctanh}(ax)c^2ax+\frac{2a\operatorname{arctanh}(ax)cd\,x^3}{3}+\frac{a\operatorname{arctanh}(ax)d^2x^5}{5}-\frac{-5a^4cd\,x^2-\frac{3a^4d^2x^4}{4}-\frac{3d^2x^2a^2}{2}-\frac{\left(15a^4c^2+10a^2dc+3d^2\right)\ln(ax-1)}{2}}{15a^4}}{a}$
risch	$\left(\frac{1}{10}d^2x^5+\frac{1}{3}cd\,x^3+\frac{1}{2}x\,c^2\right)\ln(ax+1)-\frac{d^2x^5\ln(-ax+1)}{10}-\frac{cd\,x^3\ln(-ax+1)}{3}+\frac{d^2x^4}{20a}-\frac{c^2x\ln(-ax}{2}$
meijerg	$-\frac{d^2\left(-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\frac{2a^6x^6\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{5\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{5}\right)}{4a^5}+\frac{dc\left(\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arctanh(a*x)*c^2*a*x+2/3*a*arctanh(a*x)*c*d*x^3+1/5*a*arctanh(a*x)*d^2
*x^5-1/15/a^4*(-5*a^4*c*d*x^2-3/4*a^4*d^2*x^4-3/2*d^2*x^2*a^2-1/2*(15*a^4*c
^2+10*a^2*c*d+3*d^2)*ln(a*x-1)+1/2*(-15*a^4*c^2-10*a^2*c*d-3*d^2)*ln(a*x+1)
))
```

Maxima [A]
time = 0.26, size = 131, normalized size = 1.19

$$\frac{1}{60}a\left(\frac{3a^2d^2x^4+2\,(10\,a^2cd+3\,d^2)x^2}{a^4}+\frac{2\,(15\,a^4c^2+10\,a^2cd+3\,d^2)\log(ax+1)}{a^6}+\frac{2\,(15\,a^4c^2+10\,a^2cd+3\,d^2)\log(ax-1)}{a^6}\right)+\frac{1}{15}\left(3\,d^2x^5+10\,cdx^3+15\,c^2x\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*
log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arctanh(a*x)
```

Fricas [A]
time = 0.38, size = 119, normalized size = 1.08

$$\frac{3\,a^4d^2x^4+2\,(10\,a^4cd+3\,a^2d^2)x^2+2\,(15\,a^4c^2+10\,a^2cd+3\,d^2)\log\left(a^2x^2-1\right)+2\,(3\,a^5d^2x^5+10\,a^5cdx^3+15\,a^5c^2x)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*a
^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a
^5*c^2*x)*log(-(a*x + 1)/(a*x - 1)))/a^5
```

Sympy [A]

time = 0.36, size = 155, normalized size = 1.41

$$\begin{cases} c^2x\operatorname{atanh}(ax) + \frac{2cdx^3\operatorname{atanh}(ax)}{3} + \frac{d^2x^5\operatorname{atanh}(ax)}{5} + \frac{c^2\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^2\operatorname{atanh}(ax)}{a} + \frac{cdx^2}{3a} + \frac{d^2x^4}{20a} + \frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{2cd\operatorname{atanh}(ax)}{3a^3} + \frac{d^2x^2}{10a^3} + \frac{d^2\log\left(x-\frac{1}{a}\right)}{5a^5} + \frac{d^2\operatorname{atanh}(ax)}{5a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*atanh(a*x),x)
```

```
[Out] Piecewise((c**2*x*atanh(a*x) + 2*c*d*x**3*atanh(a*x)/3 + d**2*x**5*atanh(a*
x)/5 + c**2*log(x - 1/a)/a + c**2*atanh(a*x)/a + c*d*x**2/(3*a) + d**2*x**4
/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*atanh(a*x)/(3*a**3) + d**2*x*
*2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*atanh(a*x)/(5*a**5), Ne(a,
 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(100) = 200.

time = 0.42, size = 527, normalized size = 4.79

$$\frac{1}{15}a\left(\frac{(15a^4c^2+10a^2cd+3d^2)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6} - \frac{(15a^4c^2+10a^2cd+3d^2)\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6} + \frac{4\left(\frac{(5a^2cd+3d^2)(ax+1)^3}{(ax-1)^3} - \frac{(10a^2cd+3d^2)(ax+1)^2}{(ax-1)^2} + \frac{(5a^2cd+3d^2)(ax+1)}{ax-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^4} + \frac{\left(\frac{15(ax+1)^4a^4c^2}{(ax-1)^4} - \frac{60(ax+1)^3a^4c^2}{(ax-1)^3} + \frac{90(ax+1)^2a^4c^2}{(ax-1)^2} - \frac{60(ax+1)a^4c^2}{ax-1} + 15a^4c^2 + \frac{30(ax+1)^4a^2cd}{(ax-1)^4} - \frac{60(ax+1)^3a^2cd}{(ax-1)^3} + \frac{40(ax+1)^2a^2cd}{(ax-1)^2} - \frac{20(ax+1)a^2cd}{ax-1} + 10a^2cd + \frac{15(ax+1)^4d^2}{(ax-1)^4} + \frac{30(ax+1)^2d^2}{(ax-1)^2} + 3d^2\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(-a*x - 1)/abs(a*x - 1))/a
^6 - (15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(-(a*x + 1)/(a*x - 1) + 1))/a
^6 + 4*((5*a^2*c*d + 3*d^2)*(a*x + 1)^3/(a*x - 1)^3 - (10*a^2*c*d + 3*d^2)*
(a*x + 1)^2/(a*x - 1)^2 + (5*a^2*c*d + 3*d^2)*(a*x + 1)/(a*x - 1))/(a^6*((a
*x + 1)/(a*x - 1) - 1)^4) + (15*(a*x + 1)^4*a^4*c^2/(a*x - 1)^4 - 60*(a*x +
 1)^3*a^4*c^2/(a*x - 1)^3 + 90*(a*x + 1)^2*a^4*c^2/(a*x - 1)^2 - 60*(a*x +
1)*a^4*c^2/(a*x - 1) + 15*a^4*c^2 + 30*(a*x + 1)^4*a^2*c*d/(a*x - 1)^4 - 60
*(a*x + 1)^3*a^2*c*d/(a*x - 1)^3 + 40*(a*x + 1)^2*a^2*c*d/(a*x - 1)^2 - 20*
(a*x + 1)*a^2*c*d/(a*x - 1) + 10*a^2*c*d + 15*(a*x + 1)^4*d^2/(a*x - 1)^4 +
 30*(a*x + 1)^2*d^2/(a*x - 1)^2 + 3*d^2)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/
(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B]

time = 1.09, size = 118, normalized size = 1.07

$$c^2\,x\,\operatorname{atanh}(a\,x) + \frac{d^2\,x^5\,\operatorname{atanh}(a\,x)}{5} + \frac{c^2\,\ln\left(a^2\,x^2-1\right)}{2\,a} + \frac{d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5} + \frac{d^2\,x^4}{20\,a} + \frac{d^2\,x^2}{10\,a^3} + \frac{2\,c\,d\,x^3\,\operatorname{atanh}(a\,x)}{3} + \frac{c\,d\,\ln\left(a^2\,x^2-1\right)}{3\,a^3} + \frac{c\,d\,x^2}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^2,x)
```

```
[Out] c^2*x*atanh(a*x) + (d^2*x^5*atanh(a*x))/5 + (c^2*log(a^2*x^2 - 1))/(2*a) +
(d^2*log(a^2*x^2 - 1))/(10*a^5) + (d^2*x^4)/(20*a) + (d^2*x^2)/(10*a^3) + (
2*c*d*x^3*atanh(a*x))/3 + (c*d*log(a^2*x^2 - 1))/(3*a^3) + (c*d*x^2)/(3*a)
```

3.501 $\int (c + dx^2) \tanh^{-1}(ax) \, dx$

Optimal. Leaf size=57

$$\frac{dx^2}{6a} + cx\tanh^{-1}(ax) + \frac{1}{3}dx^3\tanh^{-1}(ax) + \frac{(3a^2c + d)\log(1 - a^2x^2)}{6a^3}$$

```
[Out] 1/6*d*x^2/a+c*x*arctanh(a*x)+1/3*d*x^3*arctanh(a*x)+1/6*(3*a^2*c+d)*ln(-a^2
*x^2+1)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6123, 1607, 455, 45}

$$\frac{(3a^2c + d)\log(1 - a^2x^2)}{6a^3} + cx\tanh^{-1}(ax) + \frac{1}{3}dx^3\tanh^{-1}(ax) + \frac{dx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcTanh[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
```

```
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx^2)\tanh^{-1}(ax)\,dx &= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-a\int \frac{cx+\frac{dx^3}{3}}{1-a^2x^2}\,dx\\
&= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-a\int \frac{x\left(c+\frac{dx^2}{3}\right)}{1-a^2x^2}\,dx\\
&= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int \frac{c+\frac{dx}{3}}{1-a^2x}\,dx,x,x^2\right)\\
&= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int \left(-\frac{d}{3a^2}+\frac{-3a^2c-d}{3a^2\left(-1+a^2x\right)}\right)\right.\\
&= \frac{dx^2}{6a}+cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)+\frac{(3a^2c+d)\log(1-a^2x^2)}{6a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.21

$$\frac{dx^2}{6a}+cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)+\frac{c\log(1-a^2x^2)}{2a}+\frac{d\log(1-a^2x^2)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcTanh[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + (c*Log[1 - a^2*
x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

Maple [A]
time = 0.14, size = 74, normalized size = 1.30

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)acx+\frac{a\operatorname{arctanh}(ax)dx^3}{3}-\frac{-\frac{dx^2a^2}{2}-\frac{(3a^2c+d)\ln(ax-1)}{2}+\frac{(-3a^2c-d)\ln(ax+1)}{2}}{3a^2}}{a}$
default	$\frac{\operatorname{arctanh}(ax)acx+\frac{a\operatorname{arctanh}(ax)dx^3}{3}-\frac{-\frac{dx^2a^2}{2}-\frac{(3a^2c+d)\ln(ax-1)}{2}+\frac{(-3a^2c-d)\ln(ax+1)}{2}}{3a^2}}{a}$
risch	$\left(\frac{1}{6}dx^3+\frac{1}{2}cx\right)\ln(ax+1)-\frac{dx^3\ln(-ax+1)}{6}-\frac{cx\ln(-ax+1)}{2}+\frac{dx^2}{6a}+\frac{\ln(a^2x^2-1)c}{2a}+\frac{\ln(a^2x^2-1)d}{6a^3}$

meijerg	$\frac{d\left(\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(-a^2x^2+1)}{3}\right)}{4a^3}-\frac{c\left(\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{\right.\right.}{\sqrt{a^2x^2}}\right.}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arctanh(a*x)*a*c*x+1/3*a*arctanh(a*x)*d*x^3-1/3/a^2*(-1/2*d*x^2*a^2-1/
2*(3*a^2*c+d)*ln(a*x-1)+1/2*(-3*a^2*c-d)*ln(a*x+1)))
```

Maxima [A]
time = 0.25, size = 65, normalized size = 1.14

$$\frac{1}{6}a\left(\frac{dx^2}{a^2}+\frac{(3a^2c+d)\log(ax+1)}{a^4}+\frac{(3a^2c+d)\log(ax-1)}{a^4}\right)+\frac{1}{3}\left(dx^3+3cx\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x -
 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arctanh(a*x)
```

Fricas [A]
time = 0.36, size = 65, normalized size = 1.14

$$\frac{a^2dx^2+(3a^2c+d)\log(a^2x^2-1)+(a^3dx^3+3a^3cx)\log\left(-\frac{ax+1}{ax-1}\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*l
og(-(a*x + 1)/(a*x - 1)))/a^3
```

Sympy [A]
time = 0.56, size = 73, normalized size = 1.28

$$\begin{cases} cx\operatorname{atanh}(ax)+\frac{dx^3\operatorname{atanh}(ax)}{3}+\frac{c\log\left(x-\frac{1}{a}\right)}{a}+\frac{c\operatorname{atanh}(ax)}{a}+\frac{dx^2}{6a}+\frac{d\log\left(x-\frac{1}{a}\right)}{3a^3}+\frac{d\operatorname{atanh}(ax)}{3a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*atanh(a*x),x)
```

```
[Out] Piecewise((c*x*atanh(a*x) + d*x**3*atanh(a*x)/3 + c*log(x - 1/a)/a + c*atan
h(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*atanh(a*x)/(3*a**3),
Ne(a, 0)), (0, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(51) = 102$.

time = 0.43, size = 266, normalized size = 4.67

$$\frac{1}{3}\,a\left(\frac{\left(3\,a^{2}c+d\right)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^{4}}-\frac{\left(3\,a^{2}c+d\right)\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^{4}}+\frac{2\,(ax+1)d}{(ax-1)a^{4}\left(\frac{ax+1}{ax-1}-1\right)^{2}}+\frac{\left(\frac{3\,(ax+1)^{2}a^{2}c}{(ax-1)^{2}}-\frac{6\,(ax+1)a^{2}c}{ax-1}+3\,a^{2}c+\frac{3\,(ax+1)^{2}d}{(ax-1)^{2}}+d\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^{4}\left(\frac{ax+1}{ax-1}-1\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + d)*log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - (3*a^2*c + d)*lo
g(abs(-(a*x + 1)/(a*x - 1) + 1))/a^4 + 2*(a*x + 1)*d/((a*x - 1)*a^4*((a*x +
 1)/(a*x - 1) - 1)^2) + (3*(a*x + 1)^2*a^2*c/(a*x - 1)^2 - 6*(a*x + 1)*a^2*
c/(a*x - 1) + 3*a^2*c + 3*(a*x + 1)^2*d/(a*x - 1)^2 + d)*log(-(a*((a*x + 1)
/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) +
1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^3))
```

Mupad [B]

time = 0.94, size = 60, normalized size = 1.05

$$\frac{\frac{d\,\ln\left(a^{2}\,x^{2}-1\right)}{6}+a^{2}\left(\frac{c\,\ln\left(a^{2}\,x^{2}-1\right)}{2}+\frac{d\,x^{2}}{6}\right)}{a^{3}}+\frac{d\,x^{3}\,\mathrm{atanh}(a\,x)}{3}+c\,x\,\mathrm{atanh}(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2),x)
```

```
[Out] ((d*log(a^2*x^2 - 1))/6 + a^2*((c*log(a^2*x^2 - 1))/2 + (d*x^2)/6))/a^3 + (
d*x^3*atanh(a*x))/3 + c*x*atanh(a*x)
```

3.502 $\int \frac{\tanh^{-1}(ax)}{c+dx^2} \, dx$

Optimal. Leaf size=429

$$-\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{\right.}}{a\sqrt{-c}-\sqrt{}}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

```
[Out] -1/4*ln(-a*x+1)*ln(a*((-c)^(1/2)-x*d^(1/2))/(a*(-c)^(1/2)-d^(1/2)))/(-c)^(1
/2)/d^(1/2)+1/4*ln(a*x+1)*ln(a*((-c)^(1/2)-x*d^(1/2))/(a*(-c)^(1/2)+d^(1/2)
))/(-c)^(1/2)/d^(1/2)-1/4*ln(a*x+1)*ln(a*((-c)^(1/2)+x*d^(1/2))/(a*(-c)^(1/
2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln(-a*x+1)*ln(a*((-c)^(1/2)+x*d^(1/2))/
(a*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(-a*x+1)*d^(1/2)/
(a*(-c)^(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(a*x+1)*d^(1/2)/(
a*(-c)^(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,(-a*x+1)*d^(1/2)/(a
*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,(a*x+1)*d^(1/2)/(a*(
-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6119, 2456, 2441, 2440, 2438}

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}(1-ax)}{\sqrt{-c}\,a+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}(ax+1)}{\sqrt{-c}\,a+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(ax+1)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(ax+1)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2),x]
```

```
[Out] -1/4*(Log[1 - a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] - Sqrt[d])])/
(Sqrt[-c]*Sqrt[d]) + (Log[1 + a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-
c] + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - (Log[1 + a*x]*Log[(a*(Sqrt[-c] + Sqr
t[d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 - a*x]*Log[
(a*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) -
PolyLog[2, -((Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[
d]) + PolyLog[2, (Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt[-c]*Sq
rt[d]) - PolyLog[2, -((Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[
-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sq
rt[-c]*Sqrt[d])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6119

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-ax)}{c+dx^2}\,dx\right) + \frac{1}{2}\int \frac{\log(1+ax)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\sqrt{-c}\,\log(1-ax)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)} + \frac{\sqrt{-c}\,\log(1-ax)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx\right) + \frac{1}{2}\int \left(\frac{\sqrt{-c}\,\log(1+\cdots}{2c\left(\sqrt{-c}-\cdots\right.}\right.$$

$$= \frac{\int \frac{\log(1-ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}} + \frac{\int \frac{\log(1-ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}} - \frac{\int \frac{\log(1+ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}} - \frac{\int \frac{\log(1+ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} - \frac{\log(1\cdots}{\cdots}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} - \frac{\log(1\cdots}{\cdots}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} - \frac{\log(1\cdots}{\cdots}$$

__Mathematica__ [C] Result contains complex when optimal does not.
time = 0.96, size = 662, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2),x]
```

```
[Out] -1/4*(a*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*
c*d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c)
+ d)/(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d +
Sqrt[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (ArcC
os[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*
c*(d + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))]
 + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
 + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c
+ d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] +
```

```
(ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] +
ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/
(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(
-PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x)
)/((a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((-(a^2*c) + d +
 (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + S
qrt[a^2*c*d]*x))])))/Sqrt[a^2*c*d]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(325) = 650$.
time = 5.25, size = 1002, normalized size = 2.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)*a^2/d/(a^4*c
^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c
)^(1/2)+d))*arctanh(a*x)-1/2*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(
1/2)*d)*a^2/d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2+1/4*((-a^2*d*c)^(1/2)*
a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)*a^2/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2
,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))-(a^2*c-2*(
-a^2*d*c)^(1/2)-d)/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x
^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*arctanh(a*x)*a^2-1/2*((-a^2*d*c)^(1/2)
*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+
d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*arctanh(a*x)+(a^2*
c-2*(-a^2*d*c)^(1/2)-d)/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*a^2+1/2*((-a
^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c/(a^4*c^2+2*a^2*c*d+d^2)
*arctanh(a*x)^2-1/2*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/(a^4*c^2+2*a^2*c*d+d^2)*po
lylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*a^2
-1/4*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c/(a^4*c^2+2*a^2
*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(
1/2)+d))-1/2*(-a^2*d*c)^(1/2)/c/d*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a
^2*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d))+1/2*(-a^2*d*c)^(1/2)/c/d*arctanh(a
*x)^2-1/4*(-a^2*d*c)^(1/2)/c/d*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(
-a^2*c+2*(-a^2*d*c)^(1/2)+d)))
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.54, size = 406, normalized size = 0.95

$$\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)\operatorname{artanh}(ax)}{\sqrt{cd}}+\frac{\left(\arctan\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d},\frac{adx+d}{a^2c+d}\right)-\arctan\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d},-\frac{adx-d}{a^2c+d}\right)\right)\log\left(dx^2+c\right)-\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2+2\,adx+d}{a^2c+d}\right)+\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2-2\,adx+d}{a^2c+d}\right)-i\,\mathrm{Li}_2\left(\frac{a^2c+adx-(i\,a^2x-i\,a)\sqrt{c}\sqrt{d}}{a^2c+2i\,a\sqrt{c}\sqrt{d}-d}\right)-i\,\mathrm{Li}_2\left(\frac{a^2c-adx+(i\,a^2x+i\,a)\sqrt{c}\sqrt{d}}{a^2c+2i\,a\sqrt{c}\sqrt{d}-d}\right)+i\,\mathrm{Li}_2\left(\frac{a^2c+adx+(i\,a^2x-i\,a)\sqrt{c}\sqrt{d}}{a^2c-2i\,a\sqrt{c}\sqrt{d}-d}\right)+i\,\mathrm{Li}_2\left(\frac{a^2c-adx-(i\,a^2x+i\,a)\sqrt{c}\sqrt{d}}{a^2c-2i\,a\sqrt{c}\sqrt{d}-d}\right)}{4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] arctan(d*x/sqrt(c*d))*arctanh(a*x)/sqrt(c*d) + 1/4*((arctan2((a^2*x + a)*sq
rt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - arctan2((a^2*x - a)*s
```

```
qrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c) - arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) + arctan(
sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - I*dilog((a^
2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(
d) - d)) - I*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c
 + 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*s
qrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c - a*d
*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)))
/sqrt(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="fricas")
[Out] integral(arctanh(a*x)/(d*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c),x)
[Out] Integral(atanh(a*x)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(arctanh(a*x)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2),x)
[Out] int(atanh(a*x)/(c + d*x^2), x)
```

3.503 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=590

$$\frac{x\tanh^{-1}(ax)}{2c\,(c+dx^2)}+\frac{\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}}+\frac{i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\log\left(-\frac{\sqrt{d}\,(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\text{l}}{8c^{3/2}\sqrt{}}$$

```
[Out] 1/2*x*arctanh(a*x)/c/(d*x^2+c)+1/4*a*ln(-a^2*x^2+1)/c/(a^2*c+d)-1/4*a*ln(d*
x^2+c)/c/(a^2*c+d)+1/2*arctan(x*d^(1/2)/c^(1/2))*arctanh(a*x)/c^(3/2)/d^(1/
2)-1/8*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2
))/c^(3/2)/d^(1/2)+1/8*I*ln((-a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1-I*
x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)-1/8*I*ln(-(-a*x+1)*d^(1/2)/(I*a*c^(1/2)-
d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*ln((a*x+1)*d^(1/2
)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*po
lylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1/2)-1/8*
I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)/d^(1/2)+
1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1
/2)-1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)/
d^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {205, 211, 6123, 6857, 531, 455, 36, 31, 5028, 2456, 2441, 2440, 2438}

$$\frac{a\log(1-a^2x^2)}{4c(a^2c+d)}-\frac{a\log(c+dx^2)}{4c(a^2c+d)}+\frac{\tanh^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}\,x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}\,x\right)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{a\left(i\sqrt{d}\,x+\sqrt{c}\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{Li}_2\left(\frac{a\left(i\sqrt{d}\,x+\sqrt{c}\right)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(1-ax)}{\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(ax+1)}{-\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(1-ax)}{-\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(ax+1)}{\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}+\frac{x\tanh^{-1}(ax)}{2c\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (x*ArcTanh[a*x])/(2*c*(c + d*x^2)) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTanh[a
*x])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/
```

```
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2
)*Sqrt[d])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - a\int \frac{\frac{x}{2c(c+dx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - a\int \left(-\frac{x}{2c(-1+ax)(1+ax)\left(c+dx^2\right)}\right.$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(-1+ax)(1+ax)(c+dx^2)}\,dx}{2c} + \frac{a\int \frac{\tan^{-1}\left(\cdots\right)}{-1+\cdots}}{2c^{3/2}\cdots}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(-1+a^2x^2)(c+dx^2)}\,dx}{2c} + \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{\cdots}}{\cdots}\right)}{-1+a^2\cdots}}{4c^{3/2}\sqrt{\cdots}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \frac{1}{(-1+a^2x)(c+dx)}\,dx, x, x^2\right)}{4c} + \cdots$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{1-ax}\,dx}{8c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\cdots}{\cdots}\right)}{1+\cdots}}{8c^{3/2}\sqrt{\cdots}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \cdots$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \cdots$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \cdots$$

Mathematica [A]
time = 5.03, size = 746, normalized size = 1.26

$$\frac{a\left(-\frac{2\log\left(1+\frac{(a^2c+d)\cosh\left(2\tanh^{-1}(ax)\right)}{a^2c-d}\right)}{a^2c+d}+\frac{2i\,\mathrm{ArcCos}\left(\frac{-a^2c+d}{a^2c+d}\right)\mathrm{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)-4\,\mathrm{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)\tanh^{-1}(ax)+\left(\mathrm{ArcCos}\left(\frac{-a^2c+d}{a^2c+d}\right)+2\,\mathrm{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\log\left(\frac{2iac\left(id+\sqrt{a^2cd}\right)(-1+ax)}{(a^2c+d)\left(ac+i\sqrt{a^2cd}\,x\right)}\right)+\left(\mathrm{ArcCos}\left(\frac{-a^2c+d}{a^2c+d}\right)-2\,\mathrm{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\log\left(\frac{2ac\left(d+i\sqrt{a^2cd}\right)(1+ax)}{(a^2c+d)\left(ac+i\sqrt{a^2cd}\,x\right)}\right)-\left(\mathrm{ArcCos}\left(\frac{-a^2c+d}{a^2c+d}\right)+2\left(\mathrm{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+\mathrm{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{a^2cd}}{\sqrt{a^2c+d}\,e^{\tanh^{-1}(ax)}\sqrt{a^2c-d+(a^2c+d)\cosh\left(2\tanh^{-1}(ax)\right)}}\right)-\left(\mathrm{ArcCos}\left(\frac{-a^2c+d}{a^2c+d}\right)-2\left(\mathrm{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+\mathrm{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{a^2cd}\,e^{\tanh^{-1}(ax)}}{\sqrt{a^2c+d}\sqrt{a^2c-d+(a^2c+d)\cosh\left(2\tanh^{-1}(ax)\right)}}\right)+i\left(\mathrm{PolyLog}\left(2,\frac{\left(-a^2c+d-2i\sqrt{a^2cd}\right)\left(iac+\sqrt{a^2cd}\,x\right)}{(a^2c+d)\left(-iac+\sqrt{a^2cd}\,x\right)}\right)-\mathrm{PolyLog}\left(2,\frac{\left(-a^2c+d+2i\sqrt{a^2cd}\right)\left(iac+\sqrt{a^2cd}\,x\right)}{(a^2c+d)\left(-iac+\sqrt{a^2cd}\,x\right)}\right)\right)}{\sqrt{a^2cd}}+\frac{4\tanh^{-1}(ax)\sinh\left(2\tanh^{-1}(ax)\right)}{a^2c-d+(a^2c+d)\cosh\left(2\tanh^{-1}(ax)\right)}\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (a*((-2*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)])/(a^2*c + d
) + ((2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]
 - 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] + (ArcCos[(-(a^2*c) + d)/
(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sqrt[
a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (ArcCos[(-
(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*c*(d
+ I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (A
rcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + Ar
cTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*
E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] - (ArcC
os[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTa
n[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt
[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyL
og[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^
2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] - PolyLog[2, ((-(a^2*c) + d + (2*I)
*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^
2*c*d]*x))]))/Sqrt[a^2*c*d] + (4*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/(a^2*c
- d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]])))/(8*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2226 vs. $2(430) = 860$.
time = 6.23, size = 2227, normalized size = 3.77

method	result	size
risch	Expression too large to display	1951
derivativedivides	Expression too large to display	2227
default	Expression too large to display	2227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/c/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*
a^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*d*
arctanh(a*x)+1/4*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)*a^4/
d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(
-a^2*c-2*(-a^2*d*c)^(1/2)+d))*arctanh(a*x)-1/4*(-a^2*d*c)^(1/2)/c*a^2/d/(a^
```

```
2*c+d)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*d
*c)^(1/2)+d))+1/2*a^2*arctanh(a*x)*(a^2*c-a*d*x)*(a*x-1)/c/(a^2*c+d)/(a^2*d
*x^2+a^2*c)-1/4*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/c/(a^2*c+d)/(a^4*c^2+2*a^2*c*d
+d^2)*a^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(
1/2)+d))*d+1/2*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/c/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+
d^2)*a^2*d*arctanh(a*x)^2-1/4*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^
(1/2)*d)/(a^2*c+d)/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(
-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*arctanh(a*x)-1/4*(a^2*c-2*(-a^2*
d*c)^(1/2)-d)/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)
^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*a^4+1/2*(a^2*c-2*(-a^2*d*c)^
(1/2)-d)/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*a^4-1/4*(d*c)^(1/
2)/d*a^5*arctan(1/4*(2*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c
)^(1/2))/(a^2*c+d)^2-1/4*(d*c)^(1/2)/c^2*a*arctan(1/4*(2*(a^2*c+d)*(a*x+1)^
2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^2*c+d)+1/(a^2*c+d)^2/c*a^2*d*
ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4/(a^2*c+d)^2/c*a^2*d*ln(a^2*c*(a*x+1)^4/(
-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2
*c-2*d*(a*x+1)^2/(-a^2*x^2+1)+d)+1/(a^2*c+d)^2*a^4*ln((a*x+1)/(-a^2*x^2+1)^
(1/2))-1/4/(a^2*c+d)^2*a^4*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1
)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2*c-2*d*(a*x+1)^2/(-a^2*x^2+1
)+d)-1/8*(-a^2*d*c)^(1/2)/(a^2*c+d)/c^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2
*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d))+1/4*(-a^2*d*c)^(1/2)/(a^2*c+d)/c^2*a
rctanh(a*x)^2+1/4*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/(a^
2*c+d)/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2-1/8*(-a^2*d*c)^(1/2)/c*
a^2/d/(a^2*c+d)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*
d*c)^(1/2)+d))+1/4*(-a^2*d*c)^(1/2)/c*a^2/d/(a^2*c+d)*arctanh(a*x)^2+1/4*(d
*c)^(1/2)/d/c*a^3*arctan(1/4*(2*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*
d)/a/(d*c)^(1/2))/(a^2*c+d)-1/2*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/(a^2*c+d)/(a^4
*c^2+2*a^2*c*d+d^2)*a^4*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a
^2*d*c)^(1/2)+d))*arctanh(a*x)+1/8*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*
d*c)^(1/2)*d)*a^4/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(
a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))-1/4*((-a^2*d*c)^(1/2)*
a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)*a^4/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)
*arctanh(a*x)^2+1/4*(d*c)^(1/2)/c^2*d*a*arctan(1/4*(2*(a^2*c+d)*(a*x+1)^2/(
-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^2*c+d)^2-1/8*((-a^2*d*c)^(1/2)*a
^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/(a^2*c+d)/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)*
polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))-1
/4*(-a^2*d*c)^(1/2)/(a^2*c+d)/c^2*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a
^2*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d)))
```

Maxima [A]
time = 0.53, size = 550, normalized size = 0.93

$$\frac{1}{2}\left(\frac{x}{cdx^2+c^2}+\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}\right)\operatorname{artanh}(ax)-\frac{\left(2acd\log(dx^2+c)-2acd\log(ax+1)-2acd\log(ax-1)+\left((a^2c+d)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2+2adx+d}{a^2c+d}\right)-(a^2c+d)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2-2adx+d}{a^2c+d}\right)+(ia^2c+id)\mathrm{Li}_2\left(\frac{a^2c+adx-(ia^2x-ia)\sqrt{c}\sqrt{d}}{a^2c+2ia\sqrt{c}\sqrt{d}-d}\right)+(ia^2c+id)\mathrm{Li}_2\left(\frac{a^2c-adx+(ia^2x+ia)\sqrt{c}\sqrt{d}}{a^2c+2ia\sqrt{c}\sqrt{d}-d}\right)+(-ia^2c-id)\mathrm{Li}_2\left(\frac{a^2c+adx+(ia^2x-ia)\sqrt{c}\sqrt{d}}{a^2c-2ia\sqrt{c}\sqrt{d}-d}\right)+(-ia^2c-id)\mathrm{Li}_2\left(\frac{a^2c-adx-(ia^2x+ia)\sqrt{c}\sqrt{d}}{a^2c-2ia\sqrt{c}\sqrt{d}-d}\right)-\left((a^2c+d)\arctan\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d},\frac{adx+d}{a^2c+d}\right)-(a^2c+d)\arctan\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d},-\frac{adx-d}{a^2c+d}\right)\right)\log(dx^2+c)\right)\sqrt{c}\sqrt{d}\right)a}{8(a^3c^3d+ac^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arctanh(a*x)
- 1/8*(2*a*c*d*log(d*x^2 + c) - 2*a*c*d*log(a*x + 1) - 2*a*c*d*log(a*x - 1)
 + ((a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^
2*c + d)) - (a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x
+ d)/(a^2*c + d)) + (I*a^2*c + I*d)*dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*
sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (I*a^2*c + I*d)*dil
og((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)
*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*sq
rt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilo
g((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*
sqrt(d) - d)) - ((a^2*c + d)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*c + d
), (a*d*x + d)/(a^2*c + d)) - (a^2*c + d)*arctan2((a^2*x - a)*sqrt(c)*sqrt(
d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d))
*a/(a^3*c^3*d + a*c^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^2,x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^2, x)
```

3.504 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=657

$$\frac{a}{8c(a^2c+d)(c+dx^2)}+\frac{x\tanh^{-1}(ax)}{4c(c+dx^2)^2}+\frac{3x\tanh^{-1}(ax)}{8c^2(c+dx^2)}+\frac{3\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)}{32c^{5/2}}$$

```
[Out] 1/8*a/c/(a^2*c+d)/(d*x^2+c)+1/4*x*arctanh(a*x)/c/(d*x^2+c)^2+3/8*x*arctanh(
a*x)/c^2/(d*x^2+c)+1/16*a*(5*a^2*c+3*d)*ln(-a^2*x^2+1)/c^2/(a^2*c+d)^2-1/16
*a*(5*a^2*c+3*d)*ln(d*x^2+c)/c^2/(a^2*c+d)^2+3/8*arctan(x*d^(1/2)/c^(1/2))*
arctanh(a*x)/c^(5/2)/d^(1/2)-3/32*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2
)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32*I*ln((-a*x+1)*d^(1/2)/(I
*a*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-(
-a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^
(1/2)+3/32*I*ln((a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(
1/2))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I
*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1
/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a
*c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2
))/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {205, 211, 6123, 6857, 585, 78, 5028, 2456, 2441, 2440, 2438}

$$\frac{a(5a^2c+3d)\log(1-a^2x^2)}{16c^2(a^2c+d)^2}-\frac{a(5a^2c+3d)\log(c+dx^2)}{16c^2(a^2c+d)^2}+\frac{a}{8c(a^2c+d)(c+dx^2)}+\frac{3\tanh^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3i\text{Li}_2\left(\frac{a(\sqrt{c}-i\sqrt{d}\,x)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{Li}_2\left(\frac{a(\sqrt{c}-i\sqrt{d}\,x)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{Li}_2\left(\frac{a(i\sqrt{d}\,x+\sqrt{c})}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{Li}_2\left(\frac{a(i\sqrt{d}\,x+\sqrt{c})}{\sqrt{c}\,a+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(1-ax)}{\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(ax+1)}{-\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(1-ax)}{-\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(ax+1)}{\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3x\tanh^{-1}(ax)}{8c^2(c+dx^2)}+\frac{x\tanh^{-1}(ax)}{4c(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcTanh[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcTanh[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTa
nh[a*x])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
```

```
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^3}\,dx = \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} - a\int \frac{\frac{x}{4c(c+dx^2)^2} + \frac{}{8c^2(c}}{1}$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} - a\int \left(-\frac{x(5c}{8c^2\left(-1+a\right.}\right.$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\int \frac{x\left(5c+3dx^2\right)}{(-1+a^2x^2)(c+dx^2)^2}}{8c^2}$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \frac{5c+3dx}{(-1+a^2x)(c}\right.}{16c^2}$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \left(\frac{a^2\left(5a^2c\right.}{(a^2c+d)^2(}\right.\right.}{}$$

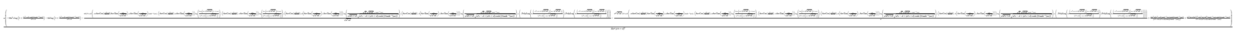
$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax}{8c^{5/2}\sqrt{d}}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax}{8c^{5/2}\sqrt{d}}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax}{8c^{5/2}\sqrt{d}}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\tanh^{-1}(ax}{8c^{5/2}\sqrt{d}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1541 vs. 2(657) = 1314.
time = 9.22, size = 1541, normalized size = 2.35



Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

```
[Out] (a*(-10*a^2*c*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)] - 6*d
*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)] - (3*d*(a^2*c + d)
*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] +
 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c) + d)/(a
^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sqrt[a^
2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (ArcCos[(-(a
^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*c*(d +
I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (Arc
Cos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcT
an[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^
ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + (ArcCos
[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[
(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt[a
^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(-PolyLo
g[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2
*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((-(a^2*c) + d + (2*I)*
Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^2
*c*d]*x))])))/Sqrt[a^2*c*d] - (3*Sqrt[a^2*c*d]*(a^2*c + d)*((-2*I)*ArcCos[(
-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] + 4*ArcTan[(a*c)/(
Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*Ar
cTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sqrt[a^2*c*d])*(-1 + a*x
))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (ArcCos[(-(a^2*c) + d)/(a^2*c
 + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*c*(d + I*Sqrt[a^2*c*d])*
(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (ArcCos[(-(a^2*c) + d
)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*d*x)/Sqrt[a
^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcTanh[a*x]*Sqrt
[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + (ArcCos[(-(a^2*c) + d)/(
a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*d*x)/Sqrt[a^2*
c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt[a^2*c + d]*Sqrt[a^
2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(-PolyLog[2, ((-(a^2*c) +
 d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c
 + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((-(a^2*c) + d + (2*I)*Sqrt[a^2*c*d])*(I
*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))])))/d +
 (16*a^2*c*d*(a^2*c + d)*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/(a^2*c - d + (a
^2*c + d)*Cosh[2*ArcTanh[a*x]])^2 + (8*a^2*c*d + 4*(5*a^4*c^2 + 8*a^2*c*d +
```

```
 3*d^2)*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/(a^2*c - d + (a^2*c + d)*Cosh[2*
ArcTanh[a*x]])))/(32*c^2*(a^2*c + d)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4046 vs. $2(493) = 986$.
time = 4.70, size = 4047, normalized size = 6.16

method	result	size
derivativedivides	Expression too large to display	4047
default	Expression too large to display	4047
risch	Expression too large to display	4564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-3/16*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c^2/(a^4*c
^2+2*a^2*c*d+d^2)^2*a^2*d*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(
-a^2*d*c)^(1/2)+d))*arctanh(a*x)+5/16*(d*c)^(1/2)/c^2*d*a^3*arctan(1/4*(2*(
a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^2*c+d)/(a^4*
c^2+2*a^2*c*d+d^2)-3/8*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/c^2/(a^4*c^2+2*a^2*c*d+
d^2)^2*a^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)
+d))*d^2*arctanh(a*x)+3/16*(d*c)^(1/2)*d^2/c^3*a*arctan(1/4*(2*(a^2*c+d)*(a
*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^2*c+d)/(a^4*c^2+2*a^2*c
*d+d^2)-3/4*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/c/(a^4*c^2+2*a^2*c*d+d^2)^2*a^4*d*
ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*arctan
h(a*x)-3/16*(-a^2*d*c)^(1/2)/c*a^4/d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)*l
n(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d))+3/16*((
-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c^3/(a^4*c^2+2*a^2*c*d+
d^2)^2*d^2*arctanh(a*x)^2-3/8*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/(a^4*c^2+2*a^2*c
*d+d^2)^2*a^6*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1
/2)+d))*arctanh(a*x)+3/32*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2
)*d)*a^6/d/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^
2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))-3/16*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-
(-a^2*d*c)^(1/2)*d)*a^6/d/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2-3/16*(-a
^2*d*c)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*a^2*polylog(2,(a^2*c+d)*(a*x+1)^2
/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d))+3/8*(-a^2*d*c)^(1/2)/c^2/(a^4*
c^2+2*a^2*c*d+d^2)*a^2*arctanh(a*x)^2-1/8*(d*c)^(1/2)/c^2*a^3*arctan(1/4*(2
*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^4*c^2+2*a^
2*c*d+d^2)+3/32*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c*a^4
/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2
*c-2*(-a^2*d*c)^(1/2)+d))-3/16*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)
^(1/2)*d)/c*a^4/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2-3/32*(-a^2*d*c)^(1
/2)/c^3/(a^4*c^2+2*a^2*c*d+d^2)*d*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1
)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d))+3/16*(-a^2*d*c)^(1/2)/c^3/(a^4*c^2+2*a^2*c
*d+d^2)*d*arctanh(a*x)^2-3/16*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/(a^4*c^2+2*a^2*c
```

```
*d+d^2)^2*a^6*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*
c)^(1/2)+d))+3/8*(a^2*c-2*(-a^2*d*c)^(1/2)-d)/(a^4*c^2+2*a^2*c*d+d^2)^2*a^6
*arctanh(a*x)^2-5/16/(a^4*c^2+2*a^2*c*d+d^2)*a^6/(a^2*c+d)*ln(a^2*c*(a*x+1)
^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2
+a^2*c-2*d*(a*x+1)^2/(-a^2*x^2+1)+d)+5/4/(a^4*c^2+2*a^2*c*d+d^2)*a^6/(a^2*c
+d)*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/16*(-a^2*d*c)^(1/2)/c^3/(a^4*c^2+2*a^2
*c*d+d^2)*d*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-
a^2*d*c)^(1/2)+d))+3/16*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*
d)/c*a^4/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-
a^2*c-2*(-a^2*d*c)^(1/2)+d))*arctanh(a*x)-3/32*((-a^2*d*c)^(1/2)*a^2*c+2*a^
2*d*c-(-a^2*d*c)^(1/2)*d)/c^2/(a^4*c^2+2*a^2*c*d+d^2)^2*a^2*d*polylog(2,(a^
2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))+3/16*((-a^2*d*
c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)/c^2/(a^4*c^2+2*a^2*c*d+d^2)^2*
a^2*d*arctanh(a*x)^2-3/8*(-a^2*d*c)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*a^2*a
rctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2
)+d))+3/16*((-a^2*d*c)^(1/2)*a^2*c+2*a^2*d*c-(-a^2*d*c)^(1/2)*d)*a^6/d/(a^4
*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2
*d*c)^(1/2)+d))*arctanh(a*x)+1/8*a^2*(5*a^6*c^3*arctanh(a*x)+3*arctanh(a*x)
*a^6*c^2*d*x^2-7*arctanh(a*x)*a^5*c^2*d*x-5*arctanh(a*x)*a^5*c*d^2*x^3+3*a^
4*c^2*d*arctanh(a*x)+arctanh(a*x)*a^4*c*d^2*x^2-c^2*a^5*d*x-a^5*c*d^2*x^3-5
*arctanh(a*x)*a^3*c*d^2*x-3*arctanh(a*x)*d^3*a^3*x^3-a^4*c^2*d-a^4*c*d^2*x^
2)*(a*x-1)/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2-3/16*(a^2*c-2*(-
a^2*d*c)^(1/2)-d)/c^2/(a^4*c^2+2*a^2*c*d+d^2)^2*a^2*polylog(2,(a^2*c+d)*(a*
x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))*d^2+3/8*(a^2*c-2*(-a^2*d
*c)^(1/2)-d)/c^2/(a^4*c^2+2*a^2*c*d+d^2)^2*a^2*d^2*arctanh(a*x)^2-3/8*(a^2*
c-2*(-a^2*d*c)^(1/2)-d)/c/(a^4*c^2+2*a^2*c*d+d^2)^2*a^4*d*polylog(2,(a^2*c+
d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*d*c)^(1/2)+d))+3/4*(a^2*c-2*(-a^2
*d*c)^(1/2)-d)/c/(a^4*c^2+2*a^2*c*d+d^2)^2*a^4*d*arctanh(a*x)^2-5/16*(d*c)^
(1/2)/d*a^7*arctan(1/4*(2*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(
d*c)^(1/2))/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)-3/16*(d*c)^(1/2)*d/c^3*a*arct
an(1/4*(2*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^4
*c^2+2*a^2*c*d+d^2)-3/16*(d*c)^(1/2)/c*a^5*arctan(1/4*(2*(a^2*c+d)*(a*x+1)^
2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(d*c)^(1/2))/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2
)-3/32*(-a^2*d*c)^(1/2)/c*a^4/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)
*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*d*c)^(1/2)+d))+3/16*(-a^2*d*c)^(1/2
)/c*a^4/d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2+5/16*(d*c)^(1/2)/d/c*a^5*a
rctan(1/4*(2*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2...
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(463) = 926$.

time = 0.57, size = 1087, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*d*x^3 + 5*c*x)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4) + 3*arctan(d*x/sqr
t(c*d))/(sqrt(c*d)*c^2))*arctanh(a*x) + 1/32*(4*a^3*c^3*d + 4*a*c^2*d^2 - 3
*((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) - (a^4*c^
3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(sqrt(
d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - (-I*a^4*c^3 - 2*
I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d - 2*I*a^2*c*d^2 - I*d^3)*x^2)*dilog((
a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqr
t(d) - d)) - (-I*a^4*c^3 - 2*I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d - 2*I*a^
2*c*d^2 - I*d^3)*x^2)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d
))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) - (I*a^4*c^3 + 2*I*a^2*c^2*d + I*c*
d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2 + I*d^3)*x^2)*dilog((a^2*c + a*d*x + (I*
a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) - (I*a^4
*c^3 + 2*I*a^2*c^2*d + I*c*d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2 + I*d^3)*x^2)
*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqr
t(c)*sqrt(d) - d)) - ((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c
*d^2 + d^3)*x^2)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*c + d), (a*d*x +
d)/(a^2*c + d)) - (a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2
 + d^3)*x^2)*arctan2((a^2*x - a)*sqrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/
(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d) - 2*(5*a^3*c^3*d + 3*a*c^2*d^
2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*log(d*x^2 + c) + 2*(5*a^3*c^3*d + 3*a*
c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*log(a*x + 1) + 2*(5*a^3*c^3*d +
3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*log(a*x - 1))*a/(a^5*c^6*d +
 2*a^3*c^5*d^2 + a*c^4*d^3 + (a^5*c^5*d^2 + 2*a^3*c^4*d^3 + a*c^3*d^4)*x^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="giac")
[Out] integrate(arctanh(a*x)/(d*x^2 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^3,x)
[Out] int(atanh(a*x)/(c + d*x^2)^3, x)
```

3.505 $\int \frac{1}{(a-ax^2)(b-2b\tanh^{-1}(x))}\,dx$

Optimal. Leaf size=17

$$-\frac{\log(1-2\tanh^{-1}(x))}{2ab}$$

```
[Out] -1/2*ln(1-2*arctanh(x))/a/b
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6093}

$$-\frac{\log(1-2\tanh^{-1}(x))}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

```
[Out] -1/2*Log[1 - 2*ArcTanh[x]]/(a*b)
```

Rule 6093

```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(a-ax^2)(b-2b\tanh^{-1}(x))}\,dx = -\frac{\log(1-2\tanh^{-1}(x))}{2ab}$$

Mathematica [A]
time = 0.05, size = 17, normalized size = 1.00

$$-\frac{\log(-1+2\tanh^{-1}(x))}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

```
[Out] -1/2*Log[-1 + 2*ArcTanh[x]]/(a*b)
```

Maple [A]
time = 1.08, size = 19, normalized size = 1.12

method	result	size
default	$-\frac{\ln(2b\operatorname{arctanh}(x)-b)}{2ab}$	19
risch	$-\frac{\ln(-\ln(1-x)+\ln(1+x)-1)}{2ab}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2/a*ln(2*b*arctanh(x)-b)/b
```

Maxima [A]
time = 0.26, size = 23, normalized size = 1.35

$$-\frac{\log(-\log(x+1)+\log(-x+1)+1)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="maxima")
[Out] -1/2*log(-log(x + 1) + log(-x + 1) + 1)/(a*b)
```

Fricas [A]
time = 0.34, size = 22, normalized size = 1.29

$$-\frac{\log\left(\log\left(-\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="fricas")
[Out] -1/2*log(log(-(x + 1)/(x - 1)) - 1)/(a*b)
```

Sympy [A]
time = 0.24, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{atanh}(x)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x**2+a)/(b-2*b*atanh(x)),x)
[Out] -log(atanh(x) - 1/2)/(2*a*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.41, size = 48, normalized size = 2.82

$$-\frac{\log\left(\frac{1}{4}\pi^2(\operatorname{sgn}(x-1)\operatorname{sgn}(-x-1)-1)^2+\left(\log\left(\frac{|-x-1|}{|x-1|}\right)-1\right)^2\right)}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(1/4*pi^2*(sgn(x - 1)*sgn(-x - 1) - 1)^2 + (log(abs(-x - 1)/abs(x -
 1)) - 1)^2)/(a*b)
```

Mupad [B]
time = 0.98, size = 15, normalized size = 0.88

$$-\frac{\ln\left(2\,\mathrm{atanh}(x)-1\right)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x^2)*(b - 2*b*atanh(x))),x)
```

```
[Out] -log(2*atanh(x) - 1)/(2*a*b)
```

3.506 $\int \frac{\tanh^{-1}(bx)}{1-x^2} \, dx$

Optimal. Leaf size=171

$$\frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)+\frac{1}{4}\log\left(-\frac{b(1+x)}{1-b}\right.$$

```
[Out] 1/4*ln(-b*(1-x)/(1-b))*ln(-b*x+1)-1/4*ln(b*(1+x)/(1+b))*ln(-b*x+1)-1/4*ln(b
*(1-x)/(1+b))*ln(b*x+1)+1/4*ln(-b*(1+x)/(1-b))*ln(b*x+1)+1/4*polylog(2,(-b*
x+1)/(1-b))-1/4*polylog(2,(-b*x+1)/(1+b))+1/4*polylog(2,(b*x+1)/(1-b))-1/4*
polylog(2,(b*x+1)/(1+b))
```

Rubi [A]
time = 0.17, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6119, 2456, 2441, 2440, 2438}

$$\frac{1}{4}\text{Li}_2\left(\frac{1-bx}{1-b}\right)-\frac{1}{4}\text{Li}_2\left(\frac{1-bx}{b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{bx+1}{1-b}\right)-\frac{1}{4}\text{Li}_2\left(\frac{bx+1}{b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(x+1)}{b+1}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{b+1}\right)\log(bx+1)+\frac{1}{4}\log\left(-\frac{b(x+1)}{1-b}\right)\log(bx+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - b))]*Log[1 - b*x])/4 - (Log[(b*(1 + x))/(1 + b)]*Lo
g[1 - b*x])/4 - (Log[(b*(1 - x))/(1 + b)]*Log[1 + b*x])/4 + (Log[-((b*(1 +
x))/(1 - b))]*Log[1 + b*x])/4 + PolyLog[2, (1 - b*x)/(1 - b)]/4 - PolyLog[2
, (1 - b*x)/(1 + b)]/4 + PolyLog[2, (1 + b*x)/(1 - b)]/4 - PolyLog[2, (1 +
b*x)/(1 + b)]/4
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6119

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(bx)}{1-x^2}\,dx &= -\left(\frac{1}{2}\int \frac{\log(1-bx)}{1-x^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+bx)}{1-x^2}\,dx\\
&= -\left(\frac{1}{2}\int \left(\frac{\log(1-bx)}{2(1-x)}+\frac{\log(1-bx)}{2(1+x)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+bx)}{2(1-x)}+\frac{\log(1+bx)}{2(1+x)}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\log(1-bx)}{1-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-bx)}{1+x}\,dx+\frac{1}{4}\int \frac{\log(1+bx)}{1-x}\,dx+\frac{1}{4}\int \frac{\log(1+}{1+x}\\
&= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\text{lo}\\
&= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\text{lo}\\
&= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\text{lo}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.65, size = 576, normalized size = 3.37

$$-\frac{b\left(2i\,\text{ArcCos}\left(\frac{1+b^2}{1-b^2}\right)\text{ArcTan}\left(\frac{bx}{\sqrt{-b^2}}\right)-4\text{ArcTan}\left(\frac{\sqrt{-b^2}}{bx}\right)\tanh^{-1}(bx)-\left(\text{ArcCos}\left(\frac{1+b^2}{1-b^2}\right)-2\text{ArcTan}\left(\frac{bx}{\sqrt{-b^2}}\right)\right)\log\left(\frac{2b\left(-i+\sqrt{-b^2}\right)(-1+bx)}{(-1+b^2)\left(-ib+\sqrt{-b^2}\,x\right)}\right)-\left(\text{ArcCos}\left(\frac{1+b^2}{1-b^2}\right)+2\text{ArcTan}\left(\frac{bx}{\sqrt{-b^2}}\right)\right)\log\left(\frac{2b\left(i+\sqrt{-b^2}\right)(1+bx)}{(-1+b^2)\left(-ib+\sqrt{-b^2}\,x\right)}\right)+\left(\text{ArcCos}\left(\frac{1+b^2}{1-b^2}\right)-2\left(\text{ArcTan}\left(\frac{\sqrt{-b^2}}{bx}\right)+\text{ArcTan}\left(\frac{bx}{\sqrt{-b^2}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{-b^2}\,e^{-\tanh^{-1}(bx)}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)\cosh\left(2\tanh^{-1}(bx)\right)}}\right)+\left(\text{ArcCos}\left(\frac{1+b^2}{1-b^2}\right)+2\left(\text{ArcTan}\left(\frac{\sqrt{-b^2}}{bx}\right)+\text{ArcTan}\left(\frac{bx}{\sqrt{-b^2}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{-b^2}\,e^{\tanh^{-1}(bx)}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)\cosh\left(2\tanh^{-1}(bx)\right)}}\right)+i\left(\text{PolyLog}\left(2,\frac{\left(1+b^2-2i\sqrt{-b^2}\right)\left(b-i\sqrt{-b^2}\,x\right)}{(-1+b^2)\left(b+i\sqrt{-b^2}\,x\right)}\right)-\text{PolyLog}\left(2,\frac{\left(1+b^2+2i\sqrt{-b^2}\right)\left(b-i\sqrt{-b^2}\,x\right)}{(-1+b^2)\left(b+i\sqrt{-b^2}\,x\right)}\right)\right)\right)}{4\sqrt{-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[b*x]/(1 - x^2),x]
```

```
[Out] -1/4*(b*((2*I)*ArcCos[(1 + b^2)/(1 - b^2)]*ArcTan[(b*x)/Sqrt[-b^2]] - 4*Arc
Tan[Sqrt[-b^2]/(b*x)]*ArcTanh[b*x] - (ArcCos[(1 + b^2)/(1 - b^2)] - 2*ArcTa
n[(b*x)/Sqrt[-b^2]])*Log[(2*b*(-I + Sqrt[-b^2])*(-1 + b*x))/((-1 + b^2)*((-
I)*b + Sqrt[-b^2]*x))] - (ArcCos[(1 + b^2)/(1 - b^2)] + 2*ArcTan[(b*x)/Sqrt
```

```
[-b^2]])*Log[(2*b*(I + Sqrt[-b^2])*(1 + b*x))/((-1 + b^2)*((-I)*b + Sqrt[-b
^2]*x))] + (ArcCos[(1 + b^2)/(1 - b^2)] - 2*(ArcTan[Sqrt[-b^2]/(b*x)] + Arc
Tan[(b*x)/Sqrt[-b^2]]))*Log[(Sqrt[2]*Sqrt[-b^2])/(Sqrt[-1 + b^2]*E^ArcTanh[
b*x]*Sqrt[1 + b^2 + (-1 + b^2)*Cosh[2*ArcTanh[b*x]]])] + (ArcCos[(1 + b^2)/
(1 - b^2)] + 2*(ArcTan[Sqrt[-b^2]/(b*x)] + ArcTan[(b*x)/Sqrt[-b^2]]))*Log[(
Sqrt[2]*Sqrt[-b^2]*E^ArcTanh[b*x])/(Sqrt[-1 + b^2]*Sqrt[1 + b^2 + (-1 + b^2
)*Cosh[2*ArcTanh[b*x]]])] + I*(PolyLog[2, ((1 + b^2 - (2*I)*Sqrt[-b^2])*(b
- I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x))] - PolyLog[2, ((1 + b^
2 + (2*I)*Sqrt[-b^2])*(b - I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x
))])))/Sqrt[-b^2]
```

Maple [A]
time = 6.54, size = 221, normalized size = 1.29

method	result
risch	$\frac{\ln(-bx+1)\ln\left(\frac{-bx+b}{-1+b}\right)}{4}+\frac{\mathrm{dilog}\left(\frac{-bx+b}{-1+b}\right)}{4}-\frac{\ln(-bx+1)\ln\left(\frac{-bx-b}{-b-1}\right)}{4}-\frac{\mathrm{dilog}\left(\frac{-bx-b}{-b-1}\right)}{4}+\frac{\ln(bx+1)\ln\left(\frac{bx+b}{-1+b}\right)}{4}+$
derivativedivides	$-\frac{\mathrm{arctanh}(bx)b\ln(-bx+b)}{2}+\frac{\mathrm{arctanh}(bx)b\ln(bx+b)}{2}-b^2\left(\frac{\mathrm{dilog}\left(\frac{-bx+1}{1-b}\right)}{2b}+\frac{\ln(-bx+b)\ln\left(\frac{-bx+1}{1-b}\right)}{2b}-\frac{\mathrm{dilog}\left(\frac{-bx-1}{-b-1}\right)}{2b}-\frac{\ln(-bx+b)\ln(}{2b}\right.$ over b
default	$-\frac{\mathrm{arctanh}(bx)b\ln(-bx+b)}{2}+\frac{\mathrm{arctanh}(bx)b\ln(bx+b)}{2}-b^2\left(\frac{\mathrm{dilog}\left(\frac{-bx+1}{1-b}\right)}{2b}+\frac{\ln(-bx+b)\ln\left(\frac{-bx+1}{1-b}\right)}{2b}-\frac{\mathrm{dilog}\left(\frac{-bx-1}{-b-1}\right)}{2b}-\frac{\ln(-bx+b)\ln(}{2b}\right.$ over b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*arctanh(b*x)*b*ln(-b*x+b)+1/2*arctanh(b*x)*b*ln(b*x+b)-1/2*b^2*(1
/2/b*dilog((-b*x+1)/(1-b))+1/2/b*ln(-b*x+b)*ln((-b*x+1)/(1-b))-1/2/b*dilog(
(-b*x-1)/(-b-1))-1/2/b*ln(-b*x+b)*ln((-b*x-1)/(-b-1))-1/2/b*dilog((b*x-1)/(
-b-1))-1/2/b*ln(b*x+b)*ln((b*x-1)/(-b-1))+1/2/b*dilog((b*x+1)/(1-b))+1/2/b*
ln(b*x+b)*ln((b*x+1)/(1-b))))
```

Maxima [A]
time = 0.26, size = 180, normalized size = 1.05

$$\frac{1}{4}b\left(\frac{\log(x+1)\log\left(-\frac{bx+b}{b+1}+1\right)+\mathrm{Li}_2\left(\frac{bx+b}{b+1}\right)}{b}+\frac{\log(x-1)\log\left(\frac{bx-b}{b+1}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{b+1}\right)}{b}-\frac{\log(x+1)\log\left(-\frac{bx+b}{b-1}+1\right)+\mathrm{Li}_2\left(\frac{bx+b}{b-1}\right)}{b}-\frac{\log(x-1)\log\left(\frac{bx-b}{b-1}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{b-1}\right)}{b}\right)+\frac{1}{2}\left(\log(x+1)-\log(x-1)\right)\mathrm{artanh}(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(x + 1)*log(-(b*x + b)/(b + 1) + 1) + dilog((b*x + b)/(b + 1)))/
b + (log(x - 1)*log((b*x - b)/(b + 1) + 1) + dilog(-(b*x - b)/(b + 1)))/b -
 (log(x + 1)*log(-(b*x + b)/(b - 1) + 1) + dilog((b*x + b)/(b - 1)))/b - (l
og(x - 1)*log((b*x - b)/(b - 1) + 1) + dilog(-(b*x - b)/(b - 1)))/b) + 1/2*
(log(x + 1) - log(x - 1))*arctanh(b*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="fricas")
[Out] integral(-arctanh(b*x)/(x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}{\left(bx \right)}}{x^{2} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x)/(-x**2+1),x)
[Out] -Integral(atanh(b*x)/(x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="giac")
[Out] integrate(-arctanh(b*x)/(x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\mathrm{atanh}(b\,x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(b*x)/(x^2 - 1),x)
[Out] -int(atanh(b*x)/(x^2 - 1), x)
```

3.507 $\int \frac{\tanh^{-1}(a+bx)}{1-x^2} \, dx$

Optimal. Leaf size=203

$$\frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+a+b}\right)\log(1+a+bx)+\frac{1}{4}\,\text{lo}$$

```
[Out] 1/4*ln(-b*(1-x)/(1-a-b))*ln(-b*x-a+1)-1/4*ln(b*(1+x)/(1-a+b))*ln(-b*x-a+1)-
1/4*ln(b*(1-x)/(1+a+b))*ln(b*x+a+1)+1/4*ln(-b*(1+x)/(1+a-b))*ln(b*x+a+1)+1/
4*polylog(2,(-b*x-a+1)/(1-a-b))-1/4*polylog(2,(-b*x-a+1)/(1-a+b))+1/4*polyl
og(2,(b*x+a+1)/(1+a-b))-1/4*polylog(2,(b*x+a+1)/(1+a+b))
```

Rubi [A]
time = 0.20, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6250, 2456, 2441, 2440, 2438}

$$\frac{1}{4}\text{Li}_2\left(\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{Li}_2\left(\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{a+bx+1}{a+b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{-a-b+1}\right)\log(-a-bx+1)-\frac{1}{4}\log\left(\frac{b(x+1)}{-a+b+1}\right)\log(-a-bx+1)-\frac{1}{4}\log\left(\frac{b(1-x)}{a+b+1}\right)\log(a+bx+1)+\frac{1}{4}\log\left(-\frac{b(x+1)}{a-b+1}\right)\log(a+bx+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{1-x^2}\,dx &= -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{1-x^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{1-x^2}\,dx \\
&= -\left(\frac{1}{2}\int \left(\frac{\log(1-a-bx)}{2(1-x)}+\frac{\log(1-a-bx)}{2(1+x)}\right)\,dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{2(1-x)}+\text{lo}\right. \\
&= -\left(\frac{1}{4}\int \frac{\log(1-a-bx)}{1-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-a-bx)}{1+x}\,dx+\frac{1}{4}\int \frac{\log(1+a+bx)}{1-x}\,dx \\
&= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log \\
&= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log \\
&= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 203, normalized size = 1.00

$$\frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+a+b}\right)\log(1+a+bx)+\frac{1}{4}\log\left(-\frac{b(1+x)}{1+a-b}\right)\log(1+a+bx)+\frac{1}{4}\text{PolyLog}\left(2,\frac{1-a-bx}{1-a-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1-a-bx}{1-a+b}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{1+a+bx}{1+a-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1+a+bx}{1+a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
```

```
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

Maple [A]
time = 12.85, size = 278, normalized size = 1.37

method	result
risch	$\frac{\ln(-bx-a+1)\ln\left(\frac{-bx+b}{b-1+a}\right)}{4}+\frac{\mathrm{dilog}\left(\frac{-bx+b}{b-1+a}\right)}{4}-\frac{\ln(-bx-a+1)\ln\left(\frac{-bx-b}{-b-1+a}\right)}{4}-\frac{\mathrm{dilog}\left(\frac{-bx-b}{-b-1+a}\right)}{4}+\frac{\ln(bx+a+1)\ln\left(\right.}{4}$
derivativedivides	$\frac{-\frac{\mathrm{arctanh}(bx+a)b\ln(-bx+b)}{2}+\frac{\mathrm{arctanh}(bx+a)b\ln(-bx-b)}{2}+\frac{b^2\left(\frac{\mathrm{dilog}\left(\frac{-bx-a+1}{1-a+b}\right)}{2b}+\frac{\ln(-bx-b)\ln\left(\frac{-bx-a+1}{1-a+b}\right)}{2b}-\frac{\mathrm{dilog}\left(\frac{-bx-a-1}{-1+b-a}\right)}{2b}-\right.}{}}{b}$
default	$\frac{-\frac{\mathrm{arctanh}(bx+a)b\ln(-bx+b)}{2}+\frac{\mathrm{arctanh}(bx+a)b\ln(-bx-b)}{2}+\frac{b^2\left(\frac{\mathrm{dilog}\left(\frac{-bx-a+1}{1-a+b}\right)}{2b}+\frac{\ln(-bx-b)\ln\left(\frac{-bx-a+1}{1-a+b}\right)}{2b}-\frac{\mathrm{dilog}\left(\frac{-bx-a-1}{-1+b-a}\right)}{2b}-\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*arctanh(b*x+a)*b*ln(-b*x+b)+1/2*arctanh(b*x+a)*b*ln(-b*x-b)+1/2*b
^2*(1/2/b*dilog((-b*x-a+1)/(1-a+b))+1/2/b*ln(-b*x-b)*ln((-b*x-a+1)/(1-a+b))
-1/2/b*dilog((-b*x-a-1)/(-1+b-a))-1/2/b*ln(-b*x-b)*ln((-b*x-a-1)/(-1+b-a))+
1/2/b*dilog((-b*x-a-1)/(-1-b-a))+1/2/b*ln(-b*x+b)*ln((-b*x-a-1)/(-1-b-a))-1
/2/b*dilog((-b*x-a+1)/(1-a-b))-1/2/b*ln(-b*x+b)*ln((-b*x-a+1)/(1-a-b))))
```

Maxima [A]
time = 0.26, size = 198, normalized size = 0.98

$$\frac{1}{4}b\left(\frac{\log(x-1)\log\left(\frac{bx-b}{a+b+1}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{a+b+1}\right)}{b}-\frac{\log(x-1)\log\left(\frac{bx-b}{a+b-1}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{a+b-1}\right)}{b}-\frac{\log(x+1)\log\left(\frac{bx+b}{a-b+1}+1\right)+\mathrm{Li}_2\left(-\frac{bx+b}{a-b+1}\right)}{b}+\frac{\log(x+1)\log\left(\frac{bx+b}{a-b-1}+1\right)+\mathrm{Li}_2\left(-\frac{bx+b}{a-b-1}\right)}{b}\right)+\frac{1}{2}(\log(x+1)-\log(x-1))\,\mathrm{artanh}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(x - 1)*log((b*x - b)/(a + b + 1) + 1) + dilog(-(b*x - b)/(a + b
 + 1)))/b - (log(x - 1)*log((b*x - b)/(a + b - 1) + 1) + dilog(-(b*x - b)/(
a + b - 1)))/b - (log(x + 1)*log((b*x + b)/(a - b + 1) + 1) + dilog(-(b*x +
 b)/(a - b + 1)))/b + (log(x + 1)*log((b*x + b)/(a - b - 1) + 1) + dilog(-(
b*x + b)/(a - b - 1)))/b) + 1/2*(log(x + 1) - log(x - 1))*arctanh(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="fricas")
[Out] integral(-arctanh(b*x + a)/(x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}(a+bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(-x**2+1),x)
[Out] -Integral(atanh(a + b*x)/(x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="giac")
[Out] integrate(-arctanh(b*x + a)/(x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\operatorname{atanh}(a+b\,x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(a + b*x)/(x^2 - 1),x)
[Out] -int(atanh(a + b*x)/(x^2 - 1), x)
```

3.508 $\int \frac{\tanh^{-1}(x)}{a+bx}\,dx$

Optimal. Leaf size=86

$$-\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1+x}\right)}{2b}-\frac{\text{PolyLog}\left(2,1-\frac{2(a+bx)}{(a+b)(1+x)}\right)}{2b}$$

```
[Out] -arctanh(x)*ln(2/(1+x))/b+arctanh(x)*ln(2*(b*x+a)/(a+b)/(1+x))/b+1/2*polylo
g(2,1-2/(1+x))/b-1/2*polylog(2,1-2*(b*x+a)/(a+b)/(1+x))/b
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6057, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(x+1)}\right)}{2b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(x+1)(a+b)}\right)}{b}+\frac{\text{Li}_2\left(1-\frac{2}{x+1}\right)}{2b}-\frac{\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x),x]
```

```
[Out] -((ArcTanh[x]*Log[2/(1 + x)])/b) + (ArcTanh[x]*Log[(2*(a + b*x))/((a + b)*(
1 + x))])/b + PolyLog[2, 1 - 2/(1 + x)]/(2*b) - PolyLog[2, 1 - (2*(a + b*x)
)/((a + b)*(1 + x))]/(2*b)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{a+bx}\,dx &= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b}+\frac{\int\frac{\log\left(\frac{2}{1+x}\right)}{1-x^2}\,dx}{b}-\frac{\int\frac{\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{1-x^2}\,d}{b}\\ &= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b}-\frac{\mathrm{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(1+x)}\right)}{2b}+\frac{\mathrm{Subst}\left(\int \mathrm{l}\right.}{}\\ &= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b}+\frac{\mathrm{Li}_2\left(1-\frac{2}{1+x}\right)}{2b}-\frac{\mathrm{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(1+}\right.}{2b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 260, normalized size = 3.02

$$\frac{-\pi^2+4\tanh^{-1}\left(\frac{a}{b}\right)^2+4i\pi\tanh^{-1}(x)+8\tanh^{-1}\left(\frac{a}{b}\right)\tanh^{-1}(x)+8\tanh^{-1}(x)^2-4i\pi\log\left(1+e^{2\tanh^{-1}(x)}\right)-8\tanh^{-1}(x)\log\left(1+e^{2\tanh^{-1}(x)}\right)+8\tanh^{-1}\left(\frac{a}{b}\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)}\right)+8\tanh^{-1}(x)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)}\right)+4i\pi\log\left(\frac{2}{\sqrt{1-x^2}}\right)+8\tanh^{-1}(x)\log\left(\frac{2}{\sqrt{1-x^2}}\right)+4\tanh^{-1}(x)\log\left(1-x^2\right)+8\tanh^{-1}(x)\log\left(i\sinh\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)\right)-8\tanh^{-1}\left(\frac{a}{b}\right)\log\left(2i\sinh\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)\right)-8\tanh^{-1}(x)\log\left(2i\sinh\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)\right)-4\mathrm{PolyLog}\left(2,-e^{2\tanh^{-1}(x)}\right)-4\mathrm{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a + b*x),x]
```

```
[Out] (-Pi^2 + 4*ArcTanh[a/b]^2 + (4*I)*Pi*ArcTanh[x] + 8*ArcTanh[a/b]*ArcTanh[x]
 + 8*ArcTanh[x]^2 - (4*I)*Pi*Log[1 + E^(2*ArcTanh[x])] - 8*ArcTanh[x]*Log[1
 + E^(2*ArcTanh[x])] + 8*ArcTanh[a/b]*Log[1 - E^(-2*(ArcTanh[a/b] + ArcTanh
[x]))] + 8*ArcTanh[x]*Log[1 - E^(-2*(ArcTanh[a/b] + ArcTanh[x]))] + (4*I)*P
i*Log[2/Sqrt[1 - x^2]] + 8*ArcTanh[x]*Log[2/Sqrt[1 - x^2]] + 4*ArcTanh[x]*L
og[1 - x^2] + 8*ArcTanh[x]*Log[I*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 8*ArcTa
nh[a/b]*Log[(2*I)*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 8*ArcTanh[x]*Log[(2*I)
*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 4*PolyLog[2, -E^(2*ArcTanh[x])] - 4*Pol
yLog[2, E^(-2*(ArcTanh[a/b] + ArcTanh[x]))])/(8*b)
```

Maple [A]
time = 1.25, size = 106, normalized size = 1.23

method	result	size
default	$\frac{\ln(bx+a)\operatorname{arctanh}(x)}{b}-\frac{-\frac{\left(\operatorname{dilog}\left(\frac{bx-b}{-a-b}\right)+\ln(bx+a)\ln\left(\frac{bx-b}{-a-b}\right)\right)b}{2}+\frac{\left(\operatorname{dilog}\left(\frac{bx+b}{b-a}\right)+\ln(bx+a)\ln\left(\frac{bx+b}{b-a}\right)\right)b}{2}}{b^2}$	106

risch	$-\frac{\operatorname{dilog}\left(\frac{(1-x)b-a-b}{-a-b}\right)}{2b}-\frac{\ln(1-x)\ln\left(\frac{(1-x)b-a-b}{-a-b}\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{(1+x)b+a-b}{a-b}\right)}{2b}+\frac{\ln(1+x)\ln\left(\frac{(1+x)b+a-b}{a-b}\right)}{2b}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b*arctanh(x)-1/b^2*(-1/2*(dilog((b*x-b)/(-a-b))+ln(b*x+a)*ln((b*x
-b)/(-a-b)))*b+1/2*(dilog((b*x+b)/(b-a))+ln(b*x+a)*ln((b*x+b)/(b-a)))*b)
```

Maxima [A]
time = 0.26, size = 119, normalized size = 1.38

$$-\frac{(\log(x+1)-\log(x-1))\log(bx+a)}{2b}+\frac{\operatorname{artanh}(x)\log(bx+a)}{b}-\frac{\log(x-1)\log\left(\frac{bx-b}{a+b}+1\right)+\operatorname{Li}_2\left(-\frac{bx-b}{a+b}\right)}{2b}+\frac{\log(x+1)\log\left(\frac{bx+b}{a-b}+1\right)+\operatorname{Li}_2\left(-\frac{bx+b}{a-b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(x + 1) - log(x - 1))*log(b*x + a)/b + arctanh(x)*log(b*x + a)/b -
 1/2*(log(x - 1)*log((b*x - b)/(a + b) + 1) + dilog(-(b*x - b)/(a + b)))/b
+ 1/2*(log(x + 1)*log((b*x + b)/(a - b) + 1) + dilog(-(b*x + b)/(a - b)))/b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}(x)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(b*x+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a + b*x),x)
```

```
[Out] int(atanh(x)/(a + b*x), x)
```

3.509 $\int \frac{\tanh^{-1}(x)}{a+bx^2} dx$

Optimal. Leaf size=397

$$-\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(1-$$

```
[Out] -1/4*ln(1-x)*ln(((-a)^(1/2)-x*b^(1/2))/((-a)^(1/2)-b^(1/2)))/(-a)^(1/2)/b^(
1/2)+1/4*ln(1+x)*ln(((-a)^(1/2)-x*b^(1/2))/((-a)^(1/2)+b^(1/2)))/(-a)^(1/2)
/b^(1/2)-1/4*ln(1+x)*ln(((-a)^(1/2)+x*b^(1/2))/((-a)^(1/2)-b^(1/2)))/(-a)^(
1/2)/b^(1/2)+1/4*ln(1-x)*ln(((-a)^(1/2)+x*b^(1/2))/((-a)^(1/2)+b^(1/2)))/(-
a)^(1/2)/b^(1/2)-1/4*polylog(2,-(1-x)*b^(1/2)/((-a)^(1/2)-b^(1/2)))/(-a)^(1
/2)/b^(1/2)-1/4*polylog(2,-(1+x)*b^(1/2)/((-a)^(1/2)-b^(1/2)))/(-a)^(1/2)/b
^(1/2)+1/4*polylog(2,(1-x)*b^(1/2)/((-a)^(1/2)+b^(1/2)))/(-a)^(1/2)/b^(1/2)
+1/4*polylog(2,(1+x)*b^(1/2)/((-a)^(1/2)+b^(1/2)))/(-a)^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6119, 2456, 2441, 2440, 2438}

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{b}\,(1-x)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\text{Li}_2\left(\frac{\sqrt{b}\,(1-x)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\text{Li}_2\left(-\frac{\sqrt{b}\,(x+1)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\text{Li}_2\left(\frac{\sqrt{b}\,(x+1)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(x+1)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{\log(x+1)\log\left(\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{\log(1-x)\log\left(\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x^2),x]
```

```
[Out] -1/4*(Log[1 - x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] - Sqrt[b])])/(Sqrt[-a
]*Sqrt[b]) + (Log[1 + x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] + Sqrt[b])])/
(4*Sqrt[-a]*Sqrt[b]) - (Log[1 + x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt[-a] - S
qrt[b])])/(4*Sqrt[-a]*Sqrt[b]) + (Log[1 - x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sq
rt[-a] + Sqrt[b])])/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 - x))/(
Sqrt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 - x))/(
Sqrt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 + x))/
(Sqrt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 + x))/
(Sqrt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6119

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-x)}{a+bx^2}\,dx\right) + \frac{1}{2}\int \frac{\log(1+x)}{a+bx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\sqrt{-a}\,\log(1-x)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\log(1-x)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx\right) + \frac{1}{2}\int \left(\frac{\sqrt{-a}\,\log(1+\right.}{2a\left(\sqrt{-a}-\sqrt{b}\right.}$$

$$= \frac{\int \frac{\log(1-x)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}} + \frac{\int \frac{\log(1-x)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}} - \frac{\int \frac{\log(1+x)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}} - \frac{\int \frac{\log(1+x)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}} + \frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}} - \frac{\log(1+x)\log\left(\frac{\sqrt{-}}{\sqrt{-}}\right.}{4\sqrt{-a}\,\sqrt{b}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}} + \frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}} - \frac{\log(1+x)\log\left(\frac{\sqrt{-}}{\sqrt{-}}\right.}{4\sqrt{-a}\,\sqrt{b}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}} + \frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\,\sqrt{b}} - \frac{\log(1+x)\log\left(\frac{\sqrt{-}}{\sqrt{-}}\right.}{4\sqrt{-a}\,\sqrt{b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 485, normalized size = 1.22

$$-\frac{-2i\,\mathrm{ArcCos}\left(\frac{-a+b}{a+b}\right)\mathrm{ArcTan}\left(\frac{bx}{\sqrt{ab}}\right) + 4\,\mathrm{ArcTan}\left(\frac{a}{\sqrt{ab}\,x}\right)\tanh^{-1}(x) - \left(\mathrm{ArcCos}\left(\frac{-a+b}{a+b}\right) + 2\,\mathrm{ArcTan}\left(\frac{bx}{\sqrt{ab}}\right)\right)\log\left(\frac{2ia\left(ib+\sqrt{ab}\right)(-1+x)}{(a+b)\left(a+i\sqrt{ab}\,x\right)}\right) - \left(\mathrm{ArcCos}\left(\frac{-a+b}{a+b}\right) - 2\,\mathrm{ArcTan}\left(\frac{bx}{\sqrt{ab}}\right)\right)\log\left(\frac{2a\left(b+i\sqrt{ab}\right)(1+x)}{(a+b)\left(a+i\sqrt{ab}\,x\right)}\right) + \left(\mathrm{ArcCos}\left(\frac{-a+b}{a+b}\right) + 2\left(\mathrm{ArcTan}\left(\frac{a}{\sqrt{ab}\,x}\right) + \mathrm{ArcTan}\left(\frac{bx}{\sqrt{ab}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{ab}\,e^{-\tanh^{-1}(x)}}{\sqrt{a+b}\sqrt{a-b+(a+b)\cosh\left(2\tanh^{-1}(x)\right)}}\right) + \left(\mathrm{ArcCos}\left(\frac{-a+b}{a+b}\right) - 2\left(\mathrm{ArcTan}\left(\frac{a}{\sqrt{ab}\,x}\right) + \mathrm{ArcTan}\left(\frac{bx}{\sqrt{ab}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{ab}\,e^{\tanh^{-1}(x)}}{\sqrt{a+b}\sqrt{a-b+(a+b)\cosh\left(2\tanh^{-1}(x)\right)}}\right) + i\left(-\mathrm{PolyLog}\left(2,\frac{\left(-a+b-2i\sqrt{ab}\right)\left(ia+\sqrt{ab}\,x\right)}{(a+b)\left(-ia+\sqrt{ab}\,x\right)}\right) + \mathrm{PolyLog}\left(2,\frac{\left(-a+b+2i\sqrt{ab}\right)\left(ia+\sqrt{ab}\,x\right)}{(a+b)\left(-ia+\sqrt{ab}\,x\right)}\right)\right)}{4\sqrt{ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a + b*x^2),x]
```

```
[Out] -1/4*((-2*I)*ArcCos[(-a + b)/(a + b)]*ArcTan[(b*x)/Sqrt[a*b]] + 4*ArcTan[a/
(Sqrt[a*b]*x)]*ArcTanh[x] - (ArcCos[(-a + b)/(a + b)] + 2*ArcTan[(b*x)/Sqrt
[a*b]])*Log[((2*I)*a*(I*b + Sqrt[a*b])*(-1 + x))/((a + b)*(a + I*Sqrt[a*b]*
x))] - (ArcCos[(-a + b)/(a + b)] - 2*ArcTan[(b*x)/Sqrt[a*b]])*Log[(2*a*(b +
 I*Sqrt[a*b])*(1 + x))/((a + b)*(a + I*Sqrt[a*b]*x))] + (ArcCos[(-a + b)/(a
 + b)] + 2*(ArcTan[a/(Sqrt[a*b]*x)] + ArcTan[(b*x)/Sqrt[a*b]]))*Log[(Sqrt[2
]*Sqrt[a*b])/(Sqrt[a + b]*E^ArcTanh[x]*Sqrt[a - b + (a + b)*Cosh[2*ArcTanh[
x]]])] + (ArcCos[(-a + b)/(a + b)] - 2*(ArcTan[a/(Sqrt[a*b]*x)] + ArcTan[(b
*x)/Sqrt[a*b]]))*Log[(Sqrt[2]*Sqrt[a*b]*E^ArcTanh[x])/(Sqrt[a + b]*Sqrt[a -
 b + (a + b)*Cosh[2*ArcTanh[x]]])] + I*(-PolyLog[2, ((-a + b - (2*I)*Sqrt[a
*b])*(I*a + Sqrt[a*b]*x))/((a + b)*((-I)*a + Sqrt[a*b]*x))] + PolyLog[2, ((
```

```
-a + b + (2*I)*Sqrt[a*b])*(I*a + Sqrt[a*b]*x))/((a + b)*((-I)*a + Sqrt[a*b]
*x))]]))/Sqrt[a*b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(293) = 586$.

time = 3.60, size = 679, normalized size = 1.71

method	result
risch	$\frac{\ln(1-x)\left(\ln\left(\frac{-(1-x)b+\sqrt{-ab}+b}{\sqrt{-ab}+b}\right)-\ln\left(\frac{(1-x)b+\sqrt{-ab}-b}{-b+\sqrt{-ab}}\right)\right)}{4\sqrt{-ab}}+\frac{\operatorname{dilog}\left(\frac{-(1-x)b+\sqrt{-ab}+b}{\sqrt{-ab}+b}\right)}{4\sqrt{-ab}}-\frac{\operatorname{dilog}\left(\frac{(1-x)b+\sqrt{-ab}\cdots}{-b+\sqrt{-ab}\cdots}\right)}{4\sqrt{-ab}}$
default	$-\frac{\left(-2\sqrt{-ab}+a-b\right)\ln\left(1-\frac{(a+b)(1+x)^2}{\left(-x^2+1\right)\left(-2\sqrt{-ab}-a+b\right)}\right)\operatorname{arctanh}(x)}{a^2+2ab+b^2}+\frac{\left(2ab+\sqrt{-ab}\,a-\sqrt{-ab}\,b\right)\ln\left(1-\frac{(a\cdots}{\left(-x^2+1\right)\left(-\cdots\right.}\right.}{2b(a^2+2ab+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-2*(-a*b)^(1/2)+a-b)/(a^2+2*a*b+b^2)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*
b)^(1/2)-a+b))*arctanh(x)+1/2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/b/(a^2+
2*a*b+b^2)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*arctanh(x)-1/
2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/a/(a^2+2*a*b+b^2)*ln(1-(a+b)*(1+x)^
2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*arctanh(x)+(-2*(-a*b)^(1/2)+a-b)/(a^2+2*a
*b+b^2)*arctanh(x)^2-1/2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/b/(a^2+2*a*b
+b^2)*arctanh(x)^2+1/2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/a/(a^2+2*a*b+b
^2)*arctanh(x)^2-1/2*(-2*(-a*b)^(1/2)+a-b)/(a^2+2*a*b+b^2)*polylog(2,(a+b)*
(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))+1/4*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1
/2)*b)/b/(a^2+2*a*b+b^2)*polylog(2,(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-
a+b))-1/4*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/a/(a^2+2*a*b+b^2)*polylog(2
,(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))-1/2*(-a*b)^(1/2)/a/b*arctanh
(x)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(2*(-a*b)^(1/2)-a+b))+1/2*(-a*b)^(1/2)/a/b*
arctanh(x)^2-1/4*(-a*b)^(1/2)/a/b*polylog(2,(a+b)*(1+x)^2/(-x^2+1)/(2*(-a*b
)^(1/2)-a+b))
```

Maxima [C] Result contains complex when optimal does not.

time = 0.52, size = 304, normalized size = 0.77

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{artanh}(x)}{\sqrt{ab}}+\frac{\left(\arctan\left(\frac{\sqrt{a}\sqrt{b}(x+1)}{a+b},\frac{bx+b}{a+b}\right)-\arctan\left(\frac{\sqrt{a}\sqrt{b}(x-1)}{a+b},-\frac{bx-b}{a+b}\right)\right)\log\left(bx^2+a\right)-\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{bx^2+2bx+b}{a+b}\right)+\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{bx^2-2bx+b}{a+b}\right)-i\operatorname{Li}_2\left(-\frac{bx-\sqrt{a}\sqrt{b}(ix+i)-a}{a+2i\sqrt{a}\sqrt{b}-b}\right)-i\operatorname{Li}_2\left(\frac{bx-\sqrt{a}\sqrt{b}(ix-i)+a}{a+2i\sqrt{a}\sqrt{b}-b}\right)+i\operatorname{Li}_2\left(-\frac{bx+\sqrt{a}\sqrt{b}(ix+i)-a}{a-2i\sqrt{a}\sqrt{b}-b}\right)+i\operatorname{Li}_2\left(\frac{bx+\sqrt{a}\sqrt{b}(ix-i)+a}{a-2i\sqrt{a}\sqrt{b}-b}\right)}{4\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))*arctanh(x)/sqrt(a*b) + 1/4*((arctan2(sqrt(a)*sqrt(b)*
(x + 1)/(a + b), (b*x + b)/(a + b)) - arctan2(sqrt(a)*sqrt(b)*(x - 1)/(a +
```

```
b), -(b*x - b)/(a + b)))*log(b*x^2 + a) - arctan(sqrt(b)*x/sqrt(a))*log((b*
x^2 + 2*b*x + b)/(a + b)) + arctan(sqrt(b)*x/sqrt(a))*log((b*x^2 - 2*b*x +
b)/(a + b)) - I*dilog(-(b*x - sqrt(a)*sqrt(b)*(I*x + I) - a)/(a + 2*I*sqrt(
a)*sqrt(b) - b)) - I*dilog((b*x - sqrt(a)*sqrt(b)*(I*x - I) + a)/(a + 2*I*s
qrt(a)*sqrt(b) - b)) + I*dilog(-(b*x + sqrt(a)*sqrt(b)*(I*x + I) - a)/(a -
2*I*sqrt(a)*sqrt(b) - b)) + I*dilog((b*x + sqrt(a)*sqrt(b)*(I*x - I) + a)/(
a - 2*I*sqrt(a)*sqrt(b) - b)))/sqrt(a*b)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(arctanh(x)/(b*x^2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(b*x**2+a),x)
[Out] Integral(atanh(x)/(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(arctanh(x)/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(x)}{b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a + b*x^2),x)
[Out] int(atanh(x)/(a + b*x^2), x)
```

3.510 $\int \frac{\tanh^{-1}(x)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=258

$$\frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left(2,1-\frac{2}{(b\cdots}\right.}{2\sqrt{b\cdots}}$$

```
[Out] arctanh(x)*ln(2*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1+x)/(b+2*c-(-4*a*c+b^2)^(1/2
)))/(-4*a*c+b^2)^(1/2)-arctanh(x)*ln(2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(1+x)/(
b+2*c+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)-1/2*polylog(2,1-2*(b+2*c*x-(-
4*a*c+b^2)^(1/2))/(1+x)/(b+2*c-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+1/2*
polylog(2,1-2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(1+x)/(b+2*c+(-4*a*c+b^2)^(1/2))
)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {632, 212, 6860, 6057, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(1-\frac{2\left(b+2cx-\sqrt{b^2-4ac}\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(x+1)}\right)}{2\sqrt{b^2-4ac}}+\frac{\text{Li}_2\left(1-\frac{2\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(x+1)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tanh^{-1}(x)\log\left(\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTanh[x]*Log[(2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2 -
4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c] - (ArcTanh[x]*Log[(2*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c]
 - PolyLog[2, 1 - (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2
- 4*a*c])*(1 + x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 - (2*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))]/(2*Sqrt[b^2 -
4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\tanh^{-1}(x)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\tanh^{-1}(x)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)} \right) dx$$

$$= \frac{(2c)\int \frac{\tanh^{-1}(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\tanh^{-1}(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} -$$

$$= \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.17, size = 874, normalized size = 3.39

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[-b^2 + 4*a*c]*(b*(Sqrt[(c*(a + b + c))/(-b^2 + 4*a*c)]*E^(I*ArcTan
[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]) - Sqrt[(c*(a - b + c))/(-b^2 + 4*a*c)]*E^(
I*ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]])) - 2*c*(-1 + Sqrt[(c*(a + b + c))/
(-b^2 + 4*a*c)]*E^(I*ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]) + Sqrt[(c*(a -
b + c))/(-b^2 + 4*a*c)]*E^(I*ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]])))*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]^2)/(b^2 - 4*c^2) + 2*ArcTan[(b + 2*c*x)/S
qrt[-b^2 + 4*a*c]]*((-I)*ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + I*ArcTan[(
-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + 2*ArcTanh[x] + Log[1 - E^((2*I)*(ArcTan[(-b
 - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]))] - L
og[1 - E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]]))]) + 2*(ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]*(Log[1
- E^((2*I)*(ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]]))] - Log[Sin[ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]]]) + ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]]
*(-Log[1 - E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*
c*x)/Sqrt[-b^2 + 4*a*c]]))] + Log[Sin[ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]]
 + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]]])) - I*PolyLog[2, E^((2*I)*(ArcT
```

```
an[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
)] + I*PolyLog[2, E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]))])/(2*Sqrt[-b^2 + 4*a*c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(230) = 460$.
time = 4.69, size = 937, normalized size = 3.63

method	result
risch	$\frac{\ln(1-x)\left(\ln\left(\frac{-2c(1-x)+\sqrt{-4ac+b^2}+b+2c}{b+2c+\sqrt{-4ac+b^2}}\right)-\ln\left(\frac{2c(1-x)+\sqrt{-4ac+b^2}-b-2c}{-b-2c+\sqrt{-4ac+b^2}}\right)\right)}{2\sqrt{-4ac+b^2}}+\frac{\mathrm{dilog}\left(\frac{-2c(1-x)+\sqrt{-4ac+b^2}}{b+2c+\sqrt{-4ac+b^2}}\right.}{2\sqrt{-4ac+b^2}}$
default	$-\frac{\sqrt{-4ac+b^2}\,\mathrm{arctanh}(x)\ln\left(1-\frac{(a+b+c)(1+x)^2}{(-x^2+1)\left(\sqrt{-4ac+b^2}-a+c\right)}\right)}{4ac-b^2}+\frac{\sqrt{-4ac+b^2}\,\mathrm{arctanh}(x)^2}{4ac-b^2}-\frac{\sqrt{-4ac+b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*arctanh(x)*ln(1-(a+b+c)*(1+x)^2/(-x^2+1)/((
-4*a*c+b^2)^(1/2)-a+c))+(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*arctanh(x)^2-1/2*(-4
*a*c+b^2)^(1/2)/(4*a*c-b^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/((-4*a*c+b^2
)^(1/2)-a+c))-(-(-4*a*c+b^2)^(1/2)+a-c)/(a^2+2*a*c-b^2+c^2)*ln(1-(a+b+c)*(1
+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*arctanh(x)+(4*a*c-b^2+(-4*a*c+b^2
)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*ln(1-(a+b+c
)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a*arctanh(x)-(4*a*c-b^2+(-4*a
*c+b^2)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*ln(1-
(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*c*arctanh(x)+(-(-4*a*c+
b^2)^(1/2)+a-c)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2-(4*a*c-b^2+(-4*a*c+b^2)^(1
/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*a*arctanh(x)^2+
(4*a*c-b^2+(-4*a*c+b^2)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*
c-b^2+c^2)*c*arctanh(x)^2-1/2*(-(-4*a*c+b^2)^(1/2)+a-c)/(a^2+2*a*c-b^2+c^2)
*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))+1/2*(4*a*c-b
^2+(-4*a*c+b^2)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^
2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a-1/2*(4*a
*c-b^2+(-4*a*c+b^2)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^
2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}(x)}{c\,x^2 + b\,x + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a + b*x + c*x^2),x)
```

```
[Out] int(atanh(x)/(a + b*x + c*x^2), x)
```

3.511 $\int \sqrt{c+dx^2}\, \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\sqrt{c+dx^2}\, \tanh^{-1}(ax), x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx^2}\, \tanh^{-1}(ax)\, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2}\, \tanh^{-1}(ax)\, dx = \int \sqrt{c+dx^2}\, \tanh^{-1}(ax)\, dx$$

Mathematica [A]
time = 4.27, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2}\, \tanh^{-1}(ax)\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
[Out] Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Maple [A]
time = 5.49, size = 0, normalized size = 0.00

$$\int \sqrt{d\,x^2+c}\, \operatorname{arctanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arctanh(a*x),x)
[Out] int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*atanh(a*x),x)
[Out] Integral(sqrt(c + d*x**2)*atanh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{atanh}(a\,x)\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(atanh(a*x)*(c + d*x^2)^(1/2), x)
```

3.512 $\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
[Out] Defer[Int][ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx = \int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

Mathematica [A]
time = 2.79, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A]
time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{d\,x^2+c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(ax\right)}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(1/2),x)
[Out] int(atanh(a*x)/(c + d*x^2)^(1/2), x)
```

3.513 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

```
[Out] -arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c/(a^2*c+d)^(1/2)+x*arctanh(a*x
)/c/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {197, 6123, 12, 455, 65, 214}

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcTanh[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx &= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\,(1-a^2x^2)\,\sqrt{c+dx^2}}\,dx \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{(1-a^2x^2)\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{1}{(1-a^2x)\sqrt{c+dx}}\,dx, x, x^2\right)}{2c} \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd} \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 119, normalized size = 1.92

$$\frac{\frac{2x\tanh^{-1}(ax)}{\sqrt{c+dx^2}} + \frac{\log(1-ax)+\log(1+ax)-\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)-\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{\sqrt{a^2c+d}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcTanh[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a*
c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c + d
]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

Maple [F]
time = 3.06, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(a x\right)}{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(54) = 108.
time = 0.28, size = 153, normalized size = 2.47

$$\frac{a^{2}\left(\frac{\operatorname{arsinh}\left(-\frac{2\,a^{2}c}{\sqrt{cd}\,|2\,a^{2}x+2\,a|}+\frac{2\,adx}{\sqrt{cd}\,|2\,a^{2}x+2\,a|}\right)}{a^{3}\sqrt{c+\frac{d}{a^{2}}}}-\frac{\operatorname{arsinh}\left(\frac{2\,a^{2}c}{\sqrt{cd}\,|2\,a^{2}x-2\,a|}+\frac{2\,adx}{\sqrt{cd}\,|2\,a^{2}x-2\,a|}\right)}{a^{3}\sqrt{c+\frac{d}{a^{2}}}}\right)}{2\,c}+\frac{x\operatorname{artanh}\left(ax\right)}{\sqrt{dx^{2}+c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(arcsinh(-2*a^2*c/(sqrt(c*d)*abs(2*a^2*x + 2*a)) + 2*a*d*x/(sqrt(c*
d)*abs(2*a^2*x + 2*a)))/(a^3*sqrt(c + d/a^2)) - arcsinh(2*a^2*c/(sqrt(c*d)*
abs(2*a^2*x - 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x - 2*a)))/(a^3*sqrt(c +
 d/a^2)))/c + x*arctanh(a*x)/(sqrt(d*x^2 + c)*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(54) = 108.
time = 0.41, size = 356, normalized size = 5.74

$$\left[\frac{2\left(a^{2}c+d\right)\sqrt{dx^{2}+c}\,x\log\left(-\frac{ax+1}{ax-1}\right)+\sqrt{a^{2}c+d}\,\left(dx^{2}+c\right)\log\left(\frac{a^{4}d^{2}x^{4}+8\,a^{4}c^{2}+8\,a^{2}cd+2\left(4\,a^{4}cd+3\,a^{2}d^{2}\right)x^{2}-4\left(a^{3}dx^{2}+2\,a^{3}c+ad\right)\sqrt{a^{2}c+d}\,\sqrt{dx^{2}+c}+d^{2}}{a^{4}x^{4}-2\,a^{2}x^{2}+1}\right)}{4\left(a^{2}c^{3}+c^{2}d+\left(a^{2}c^{2}d+cd^{2}\right)x^{2}\right)},\ \frac{\left(a^{2}c+d\right)\sqrt{dx^{2}+c}\,x\log\left(-\frac{ax+1}{ax-1}\right)+\sqrt{-a^{2}c-d}\,\left(dx^{2}+c\right)\arctan\left(\frac{\left(a^{2}dx^{2}+2\,a^{2}c+d\right)\sqrt{-a^{2}c-d}\,\sqrt{dx^{2}+c}}{2\left(a^{3}c^{2}+acd+\left(a^{3}cd+ad^{2}\right)x^{2}\right)}\right)}{2\left(a^{2}c^{3}+c^{2}d+\left(a^{2}c^{2}d+cd^{2}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + sqrt(a^2*
c + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d
```

```
+ 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2
 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d + c*
d^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + s
qrt(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*
c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(a^2*c^3
 + c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [A]
time = 0.43, size = 71, normalized size = 1.15

$$\frac{x\log\left(-\frac{ax+1}{ax-1}\right)}{2\sqrt{dx^2+c}\,c}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-d}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*log(-(a*x + 1)/(a*x - 1))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 + c
)*a/sqrt(-a^2*c - d))/(sqrt(-a^2*c - d)*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(3/2), x)
```

3.514 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}-\frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\,(a^2c+d)^{3/2}}$$

```
[Out] 1/3*x*arctanh(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c+2*d)*arctanh(a*(d*x^2+c)^
(1/2)/(a^2*c+d)^(1/2))/c^2/(a^2*c+d)^(3/2)+1/3*a/c/(a^2*c+d)/(d*x^2+c)^(1/2
)+2/3*x*arctanh(a*x)/c^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 6123, 6820, 12, 585, 79, 65, 214}

$$-\frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\,(a^2c+d)^{3/2}}+\frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{\frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x(3c+2dx^2)}{3c^2\,(1-a^2x^2)\,(c+dx^2)^{3/2}}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x(3c+2dx^2)}{(1-a^2x^2)(c+dx^2)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{3c+2dx}{(1-a^2x)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d))\,\text{Subst}\left(\int \ldots\right.}{6c^2\,(a^2 \ldots}$$

$$= \frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d))\,\text{Subst}\left(\int \ldots\right.}{3c^2d\,(\ldots}$$

$$= \frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c \ldots}}{\sqrt{a^2 \ldots}}\right.}{3c^2\,(a^2c+d)^{3/2}}$$

Mathematica [A]
time = 0.21, size = 226, normalized size = 1.77

$$\frac{\frac{2ac}{(a^2c+d)\sqrt{c+dx^2}} + \frac{2x(3c+2dx^2)\tanh^{-1}(ax)}{(c+dx^2)^{3/2}} + \frac{(3a^2c+2d)\log(1-ax)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d)\log(1+ax)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d)\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d)\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{(a^2c+d)^{3/2}}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcTanh[a*x])
/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + ((3
*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c -
d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d
)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))/(6*c
^2)
```

Maple [F]
time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(108) = 216$.
time = 0.48, size = 223, normalized size = 1.74

$$\frac{1}{6}\,a\left(\frac{\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^2c^2+cd)\sqrt{a^2c+d}}+\frac{2d}{(a^2c^2+cd)\sqrt{dx^2+c}}}{d}+\frac{2\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\sqrt{a^2c+d}\,ac^2}\right)+\frac{1}{3}\left(\frac{2x}{\sqrt{dx^2+c}\,c^2}+\frac{x}{(dx^2+c)^{\frac{3}{2}}c}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*
a^2 + sqrt(a^2*c + d)*a))/((a^2*c^2 + c*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^2
 + c*d)*sqrt(d*x^2 + c)))/d + 2*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*
a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^2)) + 1/
3*(2*x/(sqrt(d*x^2 + c)*c^2) + x/((d*x^2 + c)^(3/2)*c))*arctanh(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(108) = 216$.
time = 0.38, size = 730, normalized size = 5.70

$$\left[\frac{(3a^2c^3+(3a^2cd^2+2d^3)x^4+2c^2d+2(3a^2c^2d+2cd^2)x^2)\sqrt{a^2c+d}\,\log\left(\frac{a^4d^2x^4+8a^4c^2+8a^2cd+2(4a^4cd+3a^2d^2)x^2-4(a^3dx^2+2a^3c+ad)\sqrt{a^2c+d}\,\sqrt{dx^2+c}+d^2}{a^4x^4-2a^2x^2+1}\right)+2\left(2a^3c^3+2ac^2d+2(a^3c^2d+acd^2)x^2+(2(a^4c^2d+2a^2cd^2+d^3)x^3+3(a^4c^3+2a^2c^2d+cd^2)x)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{dx^2+c}}{12(a^4c^6+2a^2c^5d+c^4d^2+(a^4c^4d^2+2a^2c^3d^3+c^2d^4)x^4+2(a^4c^5d+2a^2c^4d^2+c^3d^3)x^2)},\ \frac{(3a^2c^3+(3a^2cd^2+2d^3)x^4+2c^2d+2(3a^2c^2d+2cd^2)x^2)\sqrt{-a^2c-d}\,\arctan\left(\frac{(a^2dx^2+2a^2c+d)\sqrt{-a^2c-d}\,\sqrt{dx^2+c}}{2(a^3c^2+acd+(a^3cd+ad^2)x^2)}\right)+\left(2a^3c^3+2ac^2d+2(a^3c^2d+acd^2)x^2+(2(a^4c^2d+2a^2cd^2+d^3)x^3+3(a^4c^3+2a^2c^2d+cd^2)x)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{dx^2+c}}{6(a^4c^6+2a^2c^5d+c^4d^2+(a^4c^4d^2+2a^2c^3d^3+c^2d^4)x^4+2(a^4c^5d+2a^2c^4d^2+c^3d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*
(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a*c^2
*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 +
 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2
 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^
2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a^2*c^3
+ (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt
(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x
^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 + 2*a*c^2*d
 + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3
*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 +
```

```
c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*
d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [A]
time = 0.43, size = 135, normalized size = 1.05

$$\frac{1}{3}\,a\left(\frac{(3\,a^2c+2\,d)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{(a^2c^3+c^2d)\sqrt{-a^2c-d}\,a}+\frac{1}{(a^2c^2+cd)\sqrt{dx^2+c}}\right)+\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^2*c^3
 + c^2*d)*sqrt(-a^2*c - d)*a) + 1/((a^2*c^2 + c*d)*sqrt(d*x^2 + c))) + 1/6*
x*(2*d*x^2/c^2 + 3/c)*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a\,x)}{(d\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(5/2), x)
```

3.515 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}+\frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

```
[Out] 1/15*a/c/(a^2*c+d)/(d*x^2+c)^(3/2)+1/5*x*arctanh(a*x)/c/(d*x^2+c)^(5/2)+4/1
5*x*arctanh(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2+20*a^2*c*d+8*d^2)*arc
tanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c^3/(a^2*c+d)^(5/2)+1/15*a*(7*a^2*c
+4*d)/c^2/(a^2*c+d)^2/(d*x^2+c)^(1/2)+8/15*x*arctanh(a*x)/c^3/(d*x^2+c)^(1/
2)
```

Rubi [A]

time = 0.72, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {198, 197, 6123, 6820, 12, 6847, 911, 1275, 214}

$$\frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}-\frac{\left(15a^4c^2+20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3\left(a^2c+d\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcTanh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}}}{1-a^2x^2}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x(15c^2+20cdx^2+8d^2x}{15c^3\,(1-a^2x^2)\,(c+dx^2)}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1-a^2x^2)(c+dx^2)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1-a^2x)(c+dx)^{5/2}}\,dx,\right.}{30c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)}\,dx,\right.}{15c^3d}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \left(\frac{3c^2d}{(a^2c+d)x^4} + \frac{cd(7a^2c}{(a^2c+d)}\right.\right.}{}$$

$$= \frac{a}{15c\,(a^2c+d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2\,(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}}{15c^2\,(c+}$$

$$= \frac{a}{15c\,(a^2c+d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2\,(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\tanh^{-1}}{15c^2\,(c+}$$

Mathematica [A]
time = 0.50, size = 329, normalized size = 1.64

$$\frac{2ac\sqrt{a^2c+d}\,(c+dx^2)\,(d(5c+4dx^2)+a^2c(8c+7dx^2)) + 2(a^2c+d)^{5/2}\,x(15c^2+20cdx^2+8d^2x^4)\tanh^{-1}(ax) + (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log(1-ax) + (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log(1+ax) - (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right) - (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{30c^3\,(a^2c+d)^{5/2}\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x^
2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTanh[a*x]
+ (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15*a^
4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^2 +
20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sqrt[
c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c +
 d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(c + d*x
^2)^(5/2))
```

Maple [F]
time = 3.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{(d\,x^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(172) = 344$.
time = 0.48, size = 401, normalized size = 2.00

$$\frac{1}{30}a\left(\frac{\frac{3a^3d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^4c^3+2a^2c^2d+cd^2)\sqrt{a^2c+d}}+\frac{2(3(dx^2+c)a^2d+a^2cd+d^2)}{(a^4c^3+2a^2c^2d+cd^2)(dx^2+c)^{\frac{3}{2}}}}{d}+\frac{4\left(\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^2c^3+c^2d)\sqrt{a^2c+d}}+\frac{2d}{(a^2c^3+c^2d)\sqrt{dx^2+c}}\right)}{d}+\frac{8\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\sqrt{a^2c+d}\,ac^3}\right)+\frac{1}{15}\left(\frac{8x}{\sqrt{dx^2+c}\,c^3}+\frac{4x}{(dx^2+c)^{\frac{3}{2}}c^2}+\frac{3x}{(dx^2+c)^{\frac{5}{2}}c}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*a*((3*a^3*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
+ c)*a^2 + sqrt(a^2*c + d)*a))/((a^4*c^3 + 2*a^2*c^2*d + c*d^2)*sqrt(a^2*c
+ d)) + 2*(3*(d*x^2 + c)*a^2*d + a^2*c*d + d^2)/((a^4*c^3 + 2*a^2*c^2*d + c
*d^2)*(d*x^2 + c)^(3/2)))/d + 4*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c
+ d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^3 + c^2*d)*sqrt(
a^2*c + d)) + 2*d/((a^2*c^3 + c^2*d)*sqrt(d*x^2 + c)))/d + 8*log((sqrt(d*x^
2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/
(sqrt(a^2*c + d)*a*c^3)) + 1/15*(8*x/(sqrt(d*x^2 + c)*c^3) + 4*x/((d*x^2 +
c)^(3/2)*c^2) + 3*x/((d*x^2 + c)^(5/2)*c))*arctanh(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(172) = 344$.
time = 0.52, size = 1280, normalized size = 6.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
```

```
 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x
^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 +
 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2
*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(
a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^
4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2
 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*
a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3
 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*
d^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 2
0*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 +
 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^
2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 +
 c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d
 + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5
*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2
*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d
^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*l
og(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c
^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*
x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^
6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(ax \right)}}{\left(c + dx^{2}\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(7/2), x)
```

Giac [**A**]
time = 0.44, size = 218, normalized size = 1.09

$$\frac{1}{15}\,a\left(\frac{(15\,a^4c^2+20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{(a^4c^5+2\,a^2c^4d+c^3d^2)\sqrt{-a^2c-d}\,a}+\frac{7\,(dx^2+c)a^2c+a^2c^2+4\,(dx^2+c)d+cd}{(a^4c^4+2\,a^2c^3d+c^2d^2)(dx^2+c)^{\frac{3}{2}}}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\,(dx^2+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^
2*c - d))/((a^4*c^5 + 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c - d)*a) + (7*(d*x^
```

```
2 + c)*a^2*c + a^2*c^2 + 4*(d*x^2 + c)*d + c*d)/((a^4*c^4 + 2*a^2*c^3*d + c
^2*d^2)*(d*x^2 + c)^(3/2))) + 1/30*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)
*x*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(5/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{\left(d\,x^2+c\right)^{7/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(7/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(7/2), x)
```

3.516 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=283

$$\frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}}+\frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}}+\frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}+\frac{6x}{35c^2}$$

```
[Out] 1/35*a/c/(a^2*c+d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c+6*d)/c^2/(a^2*c+d)^2/(
d*x^2+c)^(3/2)+1/7*x*arctanh(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arctanh(a*x)/c^2
/(d*x^2+c)^(5/2)+8/35*x*arctanh(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^3+7
0*a^4*c^2*d+56*a^2*c*d^2+16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))
/c^4/(a^2*c+d)^(7/2)+1/35*a*(19*a^4*c^2+22*a^2*c*d+8*d^2)/c^3/(a^2*c+d)^3/(
d*x^2+c)^(1/2)+16/35*x*arctanh(a*x)/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 6123, 6820, 12, 6847, 1633, 65, 214}

$$\frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}}+\frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}}+\frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}-\frac{(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\,(a^2c+d)^{7/2}}+\frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\tanh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}}+\frac{6x\tanh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcTanh[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTanh[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{}{7c(c+\cdots}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x(35\cdots}{}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x(35c^3\cdots}{}}{}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\text{Subst}\Big(\cdots}{}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\text{Subst}\Bigg(\cdots}{}$$

$$= \frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}\cdots$$

$$= \frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}\cdots$$

$$= \frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}\cdots$$

Mathematica [A]
time = 0.99, size = 431, normalized size = 1.52

$$\frac{2ac\sqrt{a^2c+d}\,(c+dx^2)\left(3c^2(a^2c+d)^2+c(a^2c+d)\,(11a^2c+6d)\,(c+dx^2)+3(19a^4c^2+22a^2cd+8d^2)\,(c+dx^2)^2\right)+6(a^2c+d)^{7/2}x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)\tanh^{-1}(ax)+3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log(1-ax)+3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log(1+ax)-3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)-3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{210c^4\,(a^2c+d)^{7/2}\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(11
*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^2)
^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*
x^6)*ArcTanh[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(
c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2
 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^2*d +
56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sq
```

```
rt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(c +
 d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(210*c^4*(a
^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

Maple [F]
time = 3.30, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(ax)}{(d\,x^2+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(247) = 494.
time = 0.49, size = 639, normalized size = 2.26

$$\frac{1}{210}\,a\left(\frac{\frac{15\,a^5 d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^6c^4+3a^4c^3d+3a^2c^2d^2+cd^3)\sqrt{a^2c+d}}+\frac{2\left(15\,(dx^2+c)^2a^4d+3a^4c^2d+6a^2cd^2+3d^3+5\,(a^4cd+a^2d^2)(dx^2+c)\right)}{(a^6c^4+3a^4c^3d+3a^2c^2d^2+cd^3)(dx^2+c)^{\frac{5}{2}}}}{d}+\frac{6\left(\frac{3\,a^3 d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^4c^4+2a^2c^3d+c^2d^2)\sqrt{a^2c+d}}+\frac{2\,(3\,(dx^2+c)a^2d+a^2cd+d^2)}{(a^4c^4+2a^2c^3d+c^2d^2)(dx^2+c)^{\frac{3}{2}}}\right)}{d}+\frac{24\left(\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^2c^4+c^3d)\sqrt{a^2c+d}}+\frac{2\,d}{(a^2c^4+c^3d)\sqrt{dx^2+c}}\right)}{d}+\frac{48\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\sqrt{a^2c+d}\,ac^4}\right)+\frac{1}{35}\left(\frac{16\,x}{\sqrt{dx^2+c}\,c^4}+\frac{8\,x}{(dx^2+c)^{\frac{3}{2}}c^3}+\frac{6\,x}{(dx^2+c)^{\frac{5}{2}}c^2}+\frac{5\,x}{(dx^2+c)^{\frac{7}{2}}c}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/210*a*((15*a^5*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^
2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^6*c^4 + 3*a^4*c^3*d + 3*a^2*c^2*d^2 +
c*d^3)*sqrt(a^2*c + d)) + 2*(15*(d*x^2 + c)^2*a^4*d + 3*a^4*c^2*d + 6*a^2*c
*d^2 + 3*d^3 + 5*(a^4*c*d + a^2*d^2)*(d*x^2 + c))/((a^6*c^4 + 3*a^4*c^3*d +
 3*a^2*c^2*d^2 + c*d^3)*(d*x^2 + c)^(5/2)))/d + 6*(3*a^3*d*log((sqrt(d*x^2
+ c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((
a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*sqrt(a^2*c + d)) + 2*(3*(d*x^2 + c)*a^2*d
+ a^2*c*d + d^2)/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*(d*x^2 + c)^(3/2)))/d +
 24*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2
 + sqrt(a^2*c + d)*a))/((a^2*c^4 + c^3*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^4
+ c^3*d)*sqrt(d*x^2 + c)))/d + 48*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d
)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^4)) +
1/35*(16*x/(sqrt(d*x^2 + c)*c^4) + 8*x/((d*x^2 + c)^(3/2)*c^3) + 6*x/((d*x^
2 + c)^(5/2)*c^2) + 5*x/((d*x^2 + c)^(7/2)*c))*arctanh(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(247) = 494.
time = 0.56, size = 2006, normalized size = 7.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2
+ 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6
*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(18
2*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2*(
196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3*(
16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7 +
56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*x^
5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d^5
)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d^4
)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d +
 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d
^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^6
*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d + 4
*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35
*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5
 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3
*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3
+ 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*
a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a
^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*
d^2)*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3
+ 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*
(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 +
2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 +
3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7
 + 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)
*x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*
d^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*
d^4)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*
d + 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5
 + 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^
8*d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*
a^6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d
+ 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(9/2), x)
```

Giac [A]
time = 0.44, size = 349, normalized size = 1.23

$$\frac{1}{105}\,a\left(\frac{3\,(35\,a^6c^3+70\,a^4c^2d+56\,a^2cd^2+16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{(a^6c^7+3\,a^4c^6d+3\,a^2c^5d^2+c^4d^3)\sqrt{-a^2c-d}\,a}+\frac{57\,(dx^2+c)^2a^4c^2+11\,(dx^2+c)a^4c^3+3\,a^4c^4+66\,(dx^2+c)^2a^2cd+17\,(dx^2+c)a^2c^2d+6\,a^2c^3d+24\,(dx^2+c)^2d^2+6\,(dx^2+c)cd^2+3\,c^2d^2}{(a^6c^6+3\,a^4c^5d+3\,a^2c^4d^2+c^3d^3)(dx^2+c)^{\frac{5}{2}}}\right)+\frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4}+\frac{7\,d^2}{c^3}\right)+\frac{35\,d}{c^2}\right)x^2+\frac{35}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{70\,(dx^2+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + c^
4*d^3)*sqrt(-a^2*c - d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^4
*c^3 + 3*a^4*c^4 + 66*(d*x^2 + c)^2*a^2*c*d + 17*(d*x^2 + c)*a^2*c^2*d + 6*
a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*c
^6 + 3*a^4*c^5*d + 3*a^2*c^4*d^2 + c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/70*(2*(
4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*log(-(a*x + 1)/
(a*x - 1))/(d*x^2 + c)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a\,x)}{{\left(d\,x^2+c\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(atanh(a*x)/(c + d*x^2)^(9/2), x)
```

3.517 $\int \sqrt{a-ax^2} \tanh^{-1}(x) \, dx$

Optimal. Leaf size=186

$$\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\,\text{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2\right.}{2\sqrt{a-ax^2}}$$

```
[Out] -a*arctan((1-x)^(1/2)/(1+x)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/
2)-1/2*I*a*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^
(1/2)+1/2*I*a*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a
)^(1/2)+1/2*(-a*x^2+a)^(1/2)+1/2*x*arctanh(x)*(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6089, 6101, 6097}

$$-\frac{a\sqrt{1-x^2}\,\text{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcTanh[x])/2 - (a*Sqrt[1 - x^2]*Arc
Tan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```

Rule 6089

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcTanh[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6101

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-ax^2}\,\tanh^{-1}(x)\,dx &= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)+\frac{1}{2}a\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx\\ &= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)+\frac{\left(a\sqrt{1-x^2}\right)\int \frac{\tanh^{-1}(x)}{\sqrt{1-x^2}}\,dx}{2\sqrt{a-ax^2}}\\ &= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\,\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-}}{\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 97, normalized size = 0.52

$$\frac{1}{2}\sqrt{a\left(1-x^2\right)}\left(1+x\tanh^{-1}(x)-\frac{i\left(\tanh^{-1}(x)\left(\log\left(1-ie^{-\tanh^{-1}(x)}\right)-\log\left(1+ie^{-\tanh^{-1}(x)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(x)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(x)}\right)\right)}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

```
[Out] (Sqrt[a*(1 - x^2)]*(1 + x*ArcTanh[x] - (I*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[
x]] - Log[1 + I/E^ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2,
 I/E^ArcTanh[x]]))/Sqrt[1 - x^2]))/2
```

Maple [A]
time = 4.00, size = 229, normalized size = 1.23

method	result
default	$\frac{(x\,\text{arctanh}(x)+1)\sqrt{-(x-1)(1+x)a}}{2}+\frac{i\sqrt{-(x-1)(1+x)a}\,\sqrt{-x^2+1}\,\text{arctanh}(x)\ln\left(1+\frac{i(1+x)}{\sqrt{-x^2+1}}\right.}{2(1+x)(x-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+a)^(1/2)*arctanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x*arctanh(x)+1)*(-(x-1)*(1+x)*a)^(1/2)+1/2*I*(-(x-1)*(1+x)*a)^(1/2)/(1
+x)*(-x^2+1)^(1/2)/(x-1)*arctanh(x)*ln(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(-(x
-1)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(x-1)*arctanh(x)*ln(1-I*(1+x)/(-x^2
+1)^(1/2))+1/2*I*(-(x-1)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(x-1)*dilog(1+
I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(-(x-1)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(
x-1)*dilog(1-I*(1+x)/(-x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="maxima")
[Out] integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="fricas")
[Out] integral(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a(x-1)(x+1)}\,\operatorname{atanh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+a)**(1/2)*atanh(x),x)
[Out] Integral(sqrt(-a*(x - 1)*(x + 1))*atanh(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="giac")
[Out] integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(x)\,\sqrt{a-a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)*(a - a*x^2)^(1/2),x)
[Out] int(atanh(x)*(a - a*x^2)^(1/2), x)
```

3.518 $\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{2\sqrt{1-x^2}\,\text{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i \ldots}{}\right)}{\sqrt{a-ax^2}}$$

```
[Out] -2*arctan((1-x)^(1/2)/(1+x)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/
2)-I*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)+
I*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6101, 6097}

$$-\frac{2\sqrt{1-x^2}\,\text{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (-2*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6101

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{\tanh^{-1}(x)}{\sqrt{1-x^2}}\,dx}{\sqrt{a-ax^2}}$$
$$= -\frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}} - \frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2}\,\mathrm{L}}{\sqrt{a}}$$

Mathematica [A]
time = 0.08, size = 90, normalized size = 0.62

$$-\frac{i\sqrt{a(1-x^2)}\left(\tanh^{-1}(x)\left(\log\left(1-ie^{-\tanh^{-1}(x)}\right)-\log\left(1+ie^{-\tanh^{-1}(x)}\right)\right)+\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(x)}\right)-\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(x)}\right)\right)}{a\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

```
[Out] ((-I)*Sqrt[a*(1 - x^2)]*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[x]] - Log[1 + I/E^
ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2, I/E^ArcTanh[x]]))
/(a*Sqrt[1 - x^2])
```

Maple [A]
time = 1.84, size = 210, normalized size = 1.46

method	result
default	$\frac{i\sqrt{-(x-1)(1+x)a}\,\sqrt{-x^2+1}\,\operatorname{arctanh}(x)\ln\left(1+\frac{i(1+x)}{\sqrt{-x^2+1}}\right)}{a(x^2-1)} - \frac{i\sqrt{-(x-1)(1+x)a}\,\sqrt{-x^2+}}{a(x^2-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-(x-1)*(1+x)*a)^(1/2)*(-x^2+1)^(1/2)*arctanh(x)*ln(1+I*(1+x)/(-x^2+1)^(1
/2))/a/(x^2-1)-I*(-(x-1)*(1+x)*a)^(1/2)*(-x^2+1)^(1/2)*arctanh(x)*ln(1-I*(1
+x)/(-x^2+1)^(1/2))/a/(x^2-1)+I*(-(x-1)*(1+x)*a)^(1/2)*(-x^2+1)^(1/2)*dilog
(1+I*(1+x)/(-x^2+1)^(1/2))/a/(x^2-1)-I*(-(x-1)*(1+x)*a)^(1/2)*(-x^2+1)^(1/2
)*dilog(1-I*(1+x)/(-x^2+1)^(1/2))/a/(x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x^2 + a)*arctanh(x)/(a*x^2 - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(1/2),x)
```

```
[Out] Integral(atanh(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{\sqrt{a-a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(1/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(1/2), x)
```

3.519 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}$$

```
[Out] -1/a/(-a*x^2+a)^(1/2)+x*arctanh(x)/a/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6105}

$$\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(3/2),x]
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx=-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2}\,\left(1-x\tanh^{-1}(x)\right)}{a^2\left(-1+x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(3/2),x]
[Out] (Sqrt[a - a*x^2]*(1 - x*ArcTanh[x]))/(a^2*(-1 + x^2))
```

Maple [A]
time = 1.82, size = 52, normalized size = 1.41

method	result	size
risch	$\frac{x\ln(1+x)}{2a\sqrt{-a\left(x^2-1\right)}}-\frac{x\ln(1-x)+2}{2a\sqrt{-a\left(x^2-1\right)}}$	47
default	$-\frac{(\operatorname{arctanh}(x)-1)\sqrt{-\left(x-1\right)\left(1+x\right)a}}{2a^2(x-1)}-\frac{(1+\operatorname{arctanh}(x))\sqrt{-\left(x-1\right)\left(1+x\right)a}}{2a^2(1+x)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctanh(x)-1)*(-(x-1)*(1+x)*a)^(1/2)/a^2/(x-1)-1/2*(1+arctanh(x))*(-(
x-1)*(1+x)*a)^(1/2)/a^2/(1+x)
```

Maxima [A]
time = 0.47, size = 63, normalized size = 1.70

$$\frac{x\operatorname{artanh}\left(x\right)}{\sqrt{-ax^2+a}\,a}-\frac{\frac{\sqrt{-ax^2+a}}{ax+a}-\frac{\sqrt{-ax^2+a}}{ax-a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(x)/(sqrt(-a*x^2 + a)*a) - 1/2*(sqrt(-a*x^2 + a)/(a*x + a) - sqrt(
-a*x^2 + a)/(a*x - a))/a
```

Fricas [A]
time = 0.36, size = 42, normalized size = 1.14

$$-\frac{\sqrt{-ax^2+a}\left(x\log\left(-\frac{x+1}{x-1}\right)-2\right)}{2\left(a^2x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*(x*log(-(x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(x\right)}{\left(-a\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(3/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

Giac [A]
time = 0.41, size = 54, normalized size = 1.46

$$-\frac{\sqrt{-ax^2+a}\,x\log\left(-\frac{x+1}{x-1}\right)}{2\left(ax^2-a\right)a}-\frac{1}{\sqrt{-ax^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*x*log(-(x + 1)/(x - 1))/((a*x^2 - a)*a) - 1/(sqrt(-a*
x^2 + a)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\mathrm{atanh}(x)}{\left(a-a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(3/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(3/2), x)
```

3.520 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}$$

```
[Out] -1/9/a/(-a*x^2+a)^(3/2)+1/3*x*arctanh(x)/a/(-a*x^2+a)^(3/2)-2/3/a^2/(-a*x^2
+a)^(1/2)+2/3*x*arctanh(x)/a^2/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6107, 6105}

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcTanh[x])/(
3*a*(a - a*x^2)^(3/2)) + (2*x*ArcTanh[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.54

$$-\frac{\sqrt{a-ax^2}\left(7-6x^2+\left(-9x+6x^3\right)\tanh^{-1}(x)\right)}{9a^3\left(-1+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/9*(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcTanh[x]))/(a^3*(-1 + x
^2)^2)
```

Maple [A]
time = 1.81, size = 112, normalized size = 1.35

method	result
risch	$\frac{x(2x^2-3)\ln(1+x)}{6a^2(x^2-1)\sqrt{-a\left(x^2-1\right)}}-\frac{6x^3\ln(1-x)+12x^2-9x\ln(1-x)-14}{18a^2(x^2-1)\sqrt{-a\left(x^2-1\right)}}$
default	$\frac{(1+x)(-1+3\,\mathrm{arctanh}(x))\sqrt{-(x-1)(1+x)\,a}}{72(x-1)^2a^3}-\frac{3(\mathrm{arctanh}(x)-1)\sqrt{-(x-1)(1+x)\,a}}{8a^3(x-1)}-\frac{3(1+\mathrm{arctanh}(x))\sqrt{}}{8(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(1+x)*(-1+3*arctanh(x))*(-(x-1)*(1+x)*a)^(1/2)/(x-1)^2/a^3-3/8*(arctan
h(x)-1)*(-(x-1)*(1+x)*a)^(1/2)/a^3/(x-1)-3/8*(1+arctanh(x))*(-(x-1)*(1+x)*a
)^(1/2)/(1+x)/a^3+1/72*(x-1)*(1+3*arctanh(x))*(-(x-1)*(1+x)*a)^(1/2)/(1+x)^
2/a^3
```

Maxima [A]
time = 0.26, size = 67, normalized size = 0.81

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{-ax^2+a}\,a^2}+\frac{x}{\left(-ax^2+a\right)^{\frac{3}{2}}a}\right)\mathrm{artanh}\left(x\right)-\frac{2}{3\sqrt{-ax^2+a}\,a^2}-\frac{1}{9\left(-ax^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arctanh(x) - 2/
3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 0.75

$$\frac{\sqrt{-ax^2+a}\left(12\,x^2-3\left(2\,x^3-3\,x\right)\log\left(-\frac{x+1}{x-1}\right)-14\right)}{18\left(a^3x^4-2\,a^3x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log(-(x + 1)/(x - 1)) - 14)
/(a^3*x^4 - 2*a^3*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(5/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

Giac [A]
time = 0.44, size = 86, normalized size = 1.04

$$-\frac{\sqrt{-ax^2+a}\,x\left(\frac{2x^2}{a}-\frac{3}{a}\right)\log\left(-\frac{x+1}{x-1}\right)}{6\left(ax^2-a\right)^2}-\frac{6\,ax^2-7\,a}{9\left(ax^2-a\right)\sqrt{-ax^2+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a*x^2 + a)*x*(2*x^2/a - 3/a)*log(-(x + 1)/(x - 1))/(a*x^2 - a)^2
 - 1/9*(6*a*x^2 - 7*a)/((a*x^2 - a)*sqrt(-a*x^2 + a)*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{\left(a-a\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(5/2),x)
```

```
[Out] int(atanh(x)/(a - a*x^2)^(5/2), x)
```

3.521 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\tanh^{-1}(x)}{15a^3\sqrt{a-ax^2}}$$

```
[Out] -1/25/a/(-a*x^2+a)^(5/2)-4/45/a^2/(-a*x^2+a)^(3/2)+1/5*x*arctanh(x)/a/(-a*x
^2+a)^(5/2)+4/15*x*arctanh(x)/a^2/(-a*x^2+a)^(3/2)-8/15/a^3/(-a*x^2+a)^(1/2
)+8/15*x*arctanh(x)/a^3/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6107, 6105}

$$-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{8x\tanh^{-1}(x)}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] -1/25*1/(a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sq
rt[a - a*x^2]) + (x*ArcTanh[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcTanh[x])/
(15*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcTanh[x])/(15*a^3*Sqrt[a - a*x^2])
```

Rule 6105

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{1}{25a(a-ax^2)^{5/2}} + \frac{x\tanh^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{5/2}}\,dx}{5a}$$

$$= -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{x\tanh^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x\tanh^{-1}(x)}{15a^2(a-ax^2)^{3/2}} + \frac{8\int \frac{\ }{(}}{}$$

$$= -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a-ax^2}} + \frac{x\tanh^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x\,\mathrm{ta}}{15a^2(}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2}\left(149-260x^2+120x^4-15x(15-20x^2+8x^4)\tanh^{-1}(x)\right)}{225a^4(-1+x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(7/2),x]
[Out] (Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*ArcT
anh[x]))/(225*a^4*(-1 + x^2)^3)
```

Maple [A]
time = 1.80, size = 176, normalized size = 1.42

method	result
risch	$\frac{x(8x^4-20x^2+15)\ln(1+x)}{30a^3(x^2-1)^2\sqrt{-a(x^2-1)}} - \frac{120x^5\ln(1-x)+240x^4-300x^3\ln(1-x)-520x^2+225x\ln(1-x)+298}{450a^3(x^2-1)^2\sqrt{-a(x^2-1)}}$
default	$-\frac{(1+x)^2(-1+5\,\mathrm{arctanh}(x))\sqrt{-(x-1)(1+x)a}}{800(x-1)^3a^4} + \frac{5(1+x)(-1+3\,\mathrm{arctanh}(x))\sqrt{-(x-1)(1+x)a}}{288a^4(x-1)^2} - \frac{5(\mathrm{arct}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/800*(1+x)^2*(-1+5*arctanh(x))*(-(x-1)*(1+x)*a)^(1/2)/(x-1)^3/a^4+5/288*(
1+x)*(-1+3*arctanh(x))*(-(x-1)*(1+x)*a)^(1/2)/a^4/(x-1)^2-5/16*(arctanh(x)-
1)*(-(x-1)*(1+x)*a)^(1/2)/a^4/(x-1)-5/16*(1+arctanh(x))*(-(x-1)*(1+x)*a)^(1
/2)/(1+x)/a^4+5/288*(x-1)*(1+3*arctanh(x))*(-(x-1)*(1+x)*a)^(1/2)/(1+x)^2/a
^4-1/800*(x-1)^2*(1+5*arctanh(x))*(-(x-1)*(1+x)*a)^(1/2)/(1+x)^3/a^4
```

Maxima [A]
time = 0.26, size = 99, normalized size = 0.80

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{-ax^2+a}\,a^3} + \frac{4\,x}{(-ax^2+a)^{\frac{3}{2}}a^2} + \frac{3\,x}{(-ax^2+a)^{\frac{5}{2}}a}\right)\mathrm{artanh}\,(x) - \frac{8}{15\sqrt{-ax^2+a}\,a^3} - \frac{4}{45\,(-ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25\,(-ax^2+a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a*
x^2 + a)^(5/2)*a))*arctanh(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x^2
 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

Fricas [A]
time = 0.34, size = 82, normalized size = 0.66

$$\frac{\left(240 x^{4} - 520 x^{2} - 15 \left(8 x^{5} - 20 x^{3} + 15 x\right) \log\left(-\frac{x+1}{x-1}\right) + 298\right) \sqrt{-a x^{2} + a}}{450 \left(a^{4} x^{6} - 3 a^{4} x^{4} + 3 a^{4} x^{2} - a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log(-(x + 1)/(x - 1))
 + 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(x)}{\left(-a (x-1) (x+1)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(7/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(7/2), x)
```

Giac [A]
time = 0.45, size = 118, normalized size = 0.95

$$-\frac{\sqrt{-a x^{2} + a} \left(4 x^{2} \left(\frac{2 x^{2}}{a} - \frac{5}{a}\right) + \frac{15}{a}\right) x \log\left(-\frac{x+1}{x-1}\right)}{30 \left(a x^{2} - a\right)^{3}} - \frac{120 \left(a x^{2} - a\right)^{2} - 20 \left(a x^{2} - a\right) a + 9 a^{2}}{225 \left(a x^{2} - a\right)^{2} \sqrt{-a x^{2} + a}\, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a*x^2 + a)*(4*x^2*(2*x^2/a - 5/a) + 15/a)*x*log(-(x + 1)/(x - 1
))/(a*x^2 - a)^3 - 1/225*(120*(a*x^2 - a)^2 - 20*(a*x^2 - a)*a + 9*a^2)/((a
*x^2 - a)^2*sqrt(-a*x^2 + a)*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(x)}{{\left(a - a x^{2}\right)}^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x)/(a - a*x^2)^(7/2),x)
[Out] int(atanh(x)/(a - a*x^2)^(7/2), x)
```

3.522 $\int x^4\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=315

$$-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\tanh^{-1}(cx)}{5c^4}-\frac{2bex^3\tanh^{-1}(cx)}{15c^2}-\frac{2}{25}bex^5\tanh^{-1}(cx)+\frac{be\tanh\cdots}{5\cdots}$$

```
[Out] -2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c-2/25*a*e
*x^5-2/5*b*e*x*arctanh(c*x)/c^4-2/15*b*e*x^3*arctanh(c*x)/c^2-2/25*b*e*x^5*
arctanh(c*x)+1/5*b*e*arctanh(c*x)^2/c^5-1/20*(4*a+3*b)*e*ln(-c*x+1)/c^5+1/2
0*(4*a-3*b)*e*ln(c*x+1)/c^5-23/75*b*e*ln(-c^2*x^2+1)/c^5-1/20*b*e*ln(-c^2*x
^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(-c^2*x^2+1))/c^3+1/20*b*x^4*(d+e*ln(-c^2*x^2
+1))/c+1/5*x^5*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/10*b*ln(-c^2*x^2+1
)*(d+e*ln(-c^2*x^2+1))/c^5
```

Rubi [A]
time = 0.53, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6037, 272, 45, 6232, 6857, 1816, 647, 31, 6127, 6021, 266, 6095, 2525, 2437, 2338}

$$-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{2aex}{5c^4}-\frac{2aex^3}{15c^2}-\frac{2}{25}aex^5+\frac{be\tanh^{-1}(cx)^2}{5c^5}-\frac{2bex\tanh^{-1}(cx)}{5c^4}-\frac{77bex^2}{300c^3}+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}-\frac{2bex^3\tanh^{-1}(cx)}{15c^2}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}-\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}-\frac{23be\log\left(1-c^2x^2\right)}{75c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}-\frac{2}{25}bex^5\tanh^{-1}(cx)-\frac{9bex^4}{200c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcTanh[c*x])/(5*c^4) - (2*b*e*x^
3*ArcTanh[c*x])/(15*c^2) - (2*b*e*x^5*ArcTanh[c*x])/25 + (b*e*ArcTanh[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6232

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^4 \left(a+b\tanh^{-1}(cx)\right) \left(d+e\log\left(1-c^2x^2\right)\right) \, dx = \frac{bx^2(d+e\log(1-c^2x^2))}{10c^3} + \frac{bx^4(d+e\log(1-c^2x^2))}{20c} +$$

$$= \frac{bx^2(d+e\log(1-c^2x^2))}{10c^3} + \frac{bx^4(d+e\log(1-c^2x^2))}{20c} +$$

$$= \frac{bx^2(d+e\log(1-c^2x^2))}{10c^3} + \frac{bx^4(d+e\log(1-c^2x^2))}{20c} +$$

$$= \frac{bx^2(d+e\log(1-c^2x^2))}{10c^3} + \frac{bx^4(d+e\log(1-c^2x^2))}{20c} +$$

$$= -\frac{be\log^2(1-c^2x^2)}{20c^5} + \frac{bx^2(d+e\log(1-c^2x^2))}{10c^3} + \frac{bx^4(}{}$$

$$= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25}aex^5 - \frac{2}{25}bex^5$$

$$= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25}aex^5 - \frac{2bex^3 \text{ta}}{1}$$

$$= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25}aex^5 - \frac{2bex \tan}{5}$$

$$= -\frac{2aex}{5c^4} - \frac{19bex^2}{100c^3} - \frac{2aex^3}{15c^2} - \frac{9bex^4}{200c} - \frac{2}{25}aex^5 - \frac{2bex}{}$$

$$= -\frac{2aex}{5c^4} - \frac{77bex^2}{300c^3} - \frac{2aex^3}{15c^2} - \frac{9bex^4}{200c} - \frac{2}{25}aex^5 - \frac{2bex}{}$$

Mathematica [A]
time = 0.09, size = 236, normalized size = 0.75

$$\frac{-240acex + 2bc^2(30d-77e)x^2 - 80ac^3ex^3 + 3bc^4(10d-9e)x^4 + 24ac^5(5d-2e)x^5 - 8bcx(-15c^4dx^4 + 2e(15+5c^2x^2+3c^4x^4))\tanh^{-1}(cx) + 120be\tanh^{-1}(cx)^2 + 2(30bd-60ae-137be)\log(1-cx) + 2(30bd+60ae-137be)\log(1+cx) + 30c^2ex^2\left(4ac^3x^3 + b(2+c^2x^2) + 4bc^3x^3\tanh^{-1}(cx)\right)\log(1-c^2x^2) + 30be\log^2(1-c^2x^2)}{600c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
 - 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15 +
5*c^2*x^2 + 3*c^4*x^4))*ArcTanh[c*x] + 120*b*e*ArcTanh[c*x]^2 + 2*(30*b*d -
 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 + c*x
] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcTanh[c*x])
*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 17.53, size = 4757, normalized size = 15.10

method	result	size
risch	Expression too large to display	4341
default	Expression too large to display	4757

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)

[Out] -2/5*b*e*x*arctanh(c*x)/c^4-2/15*b*e*x^3*arctanh(c*x)/c^2-2/25*a*e*x^5-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c-2/25*b*e*x^5*arctanh(c*x)+1/10/c^5*b*e*(4*arctanh(c*x)*x^5*c^5+c^4*x^4+2*c^2*x^2+4*arctanh(c*x)-4*ln(1+(c*x+1)^2/(-c^2*x^2+1))-3)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+181/600*e/c^5*b+1/40*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^2*e-1/20*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x^4*e-1/10*I/c^3*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x^2*e-1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^4*e-1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^2*e+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2*e+1/5*x^5*a*d-1/10*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^5*e+1/10*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^4*e-1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*x^2*e-1/10*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(c*x)-3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-3/40*I/c^5*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-3/40*I/c^5*b*e*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-3/40*I/c^5*b*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)+3/20*I/c^5*b*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2*e+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^2*e+1/40*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^2*e+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*

```
x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^3+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)
^2/(-c^2*x^2+1))^2)^3+1/10*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^5*e+1/10*I*b*Pi*arctanh(c*x)*csgn(I*(c*x
+1)^2/(c^2*x^2-1))^3*x^5*e+1/10*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c
^2*x^2+1))^2)^3*x^5*e-1/10*I/c^5*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^3-1/10*I/c^5*b*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/10*I/c^5*b*Pi*
ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3+3/40*
I/c^5*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+
(c*x+1)^2/(-c^2*x^2+1))^2)^2-3/40*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*e-3/40*I/c^5*b*e*Pi*csgn(I/(1+(c*x+1)^
2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^
2)^2-3/20*I/c^5*b*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/
(c^2*x^2-1))^2+2/5/c^5*b*arctanh(c*x)*ln(2)*e-2/5/c^5*b*ln(2)*ln(1+(c*x+1)^
2/(-c^2*x^2+1))*e-1/10/c*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^4*e-1/5/c^3*b*ln(
1+(c*x+1)^2/(-c^2*x^2+1))*x^2*e+1/10/c*b*ln(2)*x^4*e+1/5/c^3*b*ln(2)*x^2*e+
2/5*b*ln(2)*arctanh(c*x)*x^5*e-2/5*b*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+
1))*x^5*e+1/20/c*b*x^4*d+1/5*b*arctanh(c*x)*x^5*d-46/75/c^5*b*arctanh(c*x)*
e+137/150/c^5*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))-1/5/c^5*b*ln(1+(c*x+1)^2/(-c
^2*x^2+1))*d+1/5/c^5*b*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))^2+1/5/c^5*b*arctanh(c
*x)*d+1/10/c^3*b*d*x^2-3/10/c^5*b*ln(2)*e+1/40*I/c*b*Pi*csgn(I/(1+(c*x+1)^2
/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2
)^2*x^4*e-1/5*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*
csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(
I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)-1/10*I
/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*
(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(c*x)+1/5*I/c^5*b*arctanh(c*x)*Pi*e*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/10*I/c^5*b
*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c
^2*x^2-1))+3/40*I/c^5*b*e*Pi*csgn(I*(c*x+1)^2/(...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.27, size = 320, normalized size = 1.02

$$\frac{1}{5}\,adx^5+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)b\,\mathrm{artanh}\left(cx\right)e+\frac{1}{20}\left(4\,x^5\,\mathrm{artanh}\left(cx\right)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bd+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)ae-\frac{\left(3\left(-10i\,\pi c^4+9\,c^4\right)x^4+2\left(-30i\,\pi c^2+77\,c^2\right)x^2+2\left(-30i\,\pi-15\,c^4x^4-30\,c^2x^2-60\,\log\left(cx-1\right)+137\right)\log\left(cx+1\right)+2\left(-30i\,\pi-15\,c^4x^4-30\,c^2x^2+137\right)\log\left(cx-1\right)\right)be}{600\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*arctanh(c*x)*
e + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1
)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
```

```
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/600*(3*
(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 + 2*(-30*I*pi -
15*c^4*x^4 - 30*c^2*x^2 - 60*log(c*x - 1) + 137)*log(c*x + 1) + 2*(-30*I*pi
 - 15*c^4*x^4 - 30*c^2*x^2 + 137)*log(c*x - 1))*b*e/c^5
```

Fricas [A]
time = 0.36, size = 390, normalized size = 1.24

$$\frac{120\,ac^5dx^5 + 30\,bc^4dx^4 + 60\,bc^2dx^2 + 30\,(b\cosh\left(1\right) + b\sinh\left(1\right))\log\left(-c^2x^2 + 1\right)^2 + 30\,(b\cosh\left(1\right) + b\sinh\left(1\right))\log\left(-\frac{cx+1}{cx-1}\right)^2 - (48\,ac^5x^5 + 27\,bc^4x^4 + 80\,ac^3x^3 + 154\,bc^2x^2 + 240\,acx)\cosh\left(1\right) + 2\,(30\,bd + (60\,ac^5x^5 + 15\,bc^4x^4 + 30\,bc^2x^2 - 137\,b)\cosh\left(1\right) + (60\,ac^5x^5 + 15\,bc^4x^4 + 30\,bc^2x^2 - 137\,b)\sinh\left(1\right))\log\left(-c^2x^2 + 1\right) + 4\,(15\,bc^5dx^5 - 2\,(3\,bc^5x^5 + 5\,bc^3x^3 + 15\,bcx - 15\,a)\cosh\left(1\right) + 15\,(bc^5x^5\cosh\left(1\right) + bc^5x^5\sinh\left(1\right))\log\left(-c^2x^2 + 1\right) - 2\,(3\,bc^5x^5 + 5\,bc^3x^3 + 15\,bcx - 15\,a)\sinh\left(1\right))\log\left(-\frac{cx+1}{cx-1}\right) - (48\,ac^5x^5 + 27\,bc^4x^4 + 80\,ac^3x^3 + 154\,bc^2x^2 + 240\,acx)\sinh\left(1\right)}{600\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/600*(120*a*c^5*d*x^5 + 30*b*c^4*d*x^4 + 60*b*c^2*d*x^2 + 30*(b*cosh(1) +
b*sinh(1))*log(-c^2*x^2 + 1)^2 + 30*(b*cosh(1) + b*sinh(1))*log(-(c*x + 1)/
(c*x - 1))^2 - (48*a*c^5*x^5 + 27*b*c^4*x^4 + 80*a*c^3*x^3 + 154*b*c^2*x^2
+ 240*a*c*x)*cosh(1) + 2*(30*b*d + (60*a*c^5*x^5 + 15*b*c^4*x^4 + 30*b*c^2*
x^2 - 137*b)*cosh(1) + (60*a*c^5*x^5 + 15*b*c^4*x^4 + 30*b*c^2*x^2 - 137*b)
*sinh(1))*log(-c^2*x^2 + 1) + 4*(15*b*c^5*d*x^5 - 2*(3*b*c^5*x^5 + 5*b*c^3*
x^3 + 15*b*c*x - 15*a)*cosh(1) + 15*(b*c^5*x^5*cosh(1) + b*c^5*x^5*sinh(1))
*log(-c^2*x^2 + 1) - 2*(3*b*c^5*x^5 + 5*b*c^3*x^3 + 15*b*c*x - 15*a)*sinh(1
))*log(-(c*x + 1)/(c*x - 1)) - (48*a*c^5*x^5 + 27*b*c^4*x^4 + 80*a*c^3*x^3
+ 154*b*c^2*x^2 + 240*a*c*x)*sinh(1))/c^5
```

Sympy [A]
time = 2.39, size = 338, normalized size = 1.07

$$\begin{cases} \frac{adx^5}{5} + \frac{aex^5\log\left(-c^2x^2+1\right)}{5} - \frac{2aex^5}{25} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae\operatorname{atanh}\left(cx\right)}{5c^5} + \frac{bdx^5\operatorname{atanh}\left(cx\right)}{5} + \frac{bex^5\log\left(-c^2x^2+1\right)\operatorname{atanh}\left(cx\right)}{5} - \frac{2bex^5\operatorname{atanh}\left(cx\right)}{25} + \frac{bdx^4}{20c} + \frac{bex^4\log\left(-c^2x^2+1\right)}{20c} - \frac{9bex^4}{200c} - \frac{2bex^3\operatorname{atanh}\left(cx\right)}{15c^2} + \frac{bdx^2}{10c^3} + \frac{bex^2\log\left(-c^2x^2+1\right)}{10c^3} - \frac{77bex^2}{300c^3} - \frac{2bex\operatorname{atanh}\left(cx\right)}{5c^4} + \frac{bd\log\left(-c^2x^2+1\right)}{10c^5} + \frac{be\log\left(-c^2x^2+1\right)^2}{20c^5} - \frac{137be\log\left(-c^2x^2+1\right)}{300c^5} + \frac{be\operatorname{atanh}^2\left(cx\right)}{5c^5} & \text{for}\: c \neq 0 \\ \frac{adx^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atanh(c*x)/(5*c**5) + b*d*x**
5*atanh(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*atanh(c*x)/5 - 2*b*e*x**5*ata
nh(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*b*e*
x**4/(200*c) - 2*b*e*x**3*atanh(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x
**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atanh(
c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x**2 + 1)
**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*atanh(c*x)**2/
(5*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

Giac [A]
time = 0.56, size = 313, normalized size = 0.99

$$-\frac{1}{10}\,bex^5\log\left(-cx+1\right)^2 + \frac{1}{25}\,(5\,ad - 2\,ae)x^5 + \frac{(10\,bd - 9\,be)x^4}{200\,c} - \frac{2\,aex^3}{15\,c^2} + \frac{1}{10}\left(bex^5 + \frac{be}{c^5}\right)\log\left(cx+1\right)^2 + \frac{1}{300}\left(6\,(5\,bd + 10\,ae - 2\,be)x^5 + \frac{15\,bex^4}{c} - \frac{20\,bex^3}{c^2} + \frac{30\,bex^2}{c^3} - \frac{60\,bex}{c^4}\right)\log\left(cx+1\right) - \frac{1}{300}\left(6\,(5\,bd - 10\,ae - 2\,be)x^5 - \frac{15\,bex^4}{c} - \frac{20\,bex^3}{c^2} - \frac{30\,bex^2}{c^3} - \frac{60\,bex}{c^4} - \frac{60\,be\log\left(cx-1\right)}{c^5}\right)\log\left(-cx+1\right) + \frac{(30\,bd - 77\,be)x^2}{300\,c^3} - \frac{2\,aex}{5\,c^4} - \frac{be\log\left(cx-1\right)^2}{10\,c^5} + \frac{(30\,bd + 60\,ae - 137\,be)\log\left(cx+1\right)}{300\,c^5} + \frac{(30\,bd - 60\,ae - 137\,be)\log\left(cx-1\right)}{300\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/10*b*e*x^5*log(-c*x + 1)^2 + 1/25*(5*a*d - 2*a*e)*x^5 + 1/200*(10*b*d -
9*b*e)*x^4/c - 2/15*a*e*x^3/c^2 + 1/10*(b*e*x^5 + b*e/c^5)*log(c*x + 1)^2 +
 1/300*(6*(5*b*d + 10*a*e - 2*b*e)*x^5 + 15*b*e*x^4/c - 20*b*e*x^3/c^2 + 30
*b*e*x^2/c^3 - 60*b*e*x/c^4)*log(c*x + 1) - 1/300*(6*(5*b*d - 10*a*e - 2*b*
e)*x^5 - 15*b*e*x^4/c - 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 - 60*b*e*x/c^4 - 60
*b*e*log(c*x - 1)/c^5)*log(-c*x + 1) + 1/300*(30*b*d - 77*b*e)*x^2/c^3 - 2/
5*a*e*x/c^4 - 1/10*b*e*log(c*x - 1)^2/c^5 + 1/300*(30*b*d + 60*a*e - 137*b*
e)*log(c*x + 1)/c^5 + 1/300*(30*b*d - 60*a*e - 137*b*e)*log(c*x - 1)/c^5
```

Mupad [B]
time = 5.59, size = 599, normalized size = 1.90

$$\frac{a\,d\,x^5}{5}-\frac{2\,a\,e\,x^5}{25}+\frac{b\,d\,x^5\,\ln\left(c\,x+1\right)}{10}-\frac{b\,d\,x^5\,\ln\left(1-c\,x\right)}{10}-\frac{b\,e\,x^5\,\ln\left(c\,x+1\right)}{25}+\frac{b\,e\,x^5\,\ln\left(1-c\,x\right)}{25}+\frac{b\,e\,{\ln\left(c\,x+1\right)}^2}{10\,c^5}+\frac{b\,e\,{\ln\left(1-c\,x\right)}^2}{10\,c^5}-\frac{2\,a\,e\,x}{5\,c^4}-\frac{2\,a\,e\,x^3}{15\,c^2}+\frac{b\,d\,x^4}{20\,c}+\frac{b\,d\,x^2}{10\,c^3}-\frac{9\,b\,e\,x^4}{200\,c}-\frac{77\,b\,e\,x^2}{300\,c^3}+\frac{a\,e\,x^5\,\ln\left(1-c^2\,x^2\right)}{5}-\frac{a\,e\,\ln\left(c\,x-1\right)}{5\,c^5}+\frac{a\,e\,\ln\left(c\,x+1\right)}{5\,c^5}+\frac{b\,d\,\ln\left(c\,x-1\right)}{10\,c^5}+\frac{b\,d\,\ln\left(c\,x+1\right)}{10\,c^5}-\frac{137\,b\,e\,\ln\left(c\,x-1\right)}{300\,c^5}-\frac{137\,b\,e\,\ln\left(c\,x+1\right)}{300\,c^5}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,x\,\ln\left(c\,x+1\right)}{5\,c^4}+\frac{b\,e\,x\,\ln\left(1-c\,x\right)}{5\,c^4}+\frac{b\,e\,x^4\,\ln\left(1-c^2\,x^2\right)}{20\,c}+\frac{b\,e\,x^2\,\ln\left(1-c^2\,x^2\right)}{10\,c^3}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{5\,c^4}\right)\,\ln\left(1-c^2\,x^2\right)}{10\,c^5}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{5\,c^4}\right)\,\ln\left(1-c^2\,x^2\right)}{10\,c^5}-\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)}{15\,c^2}+\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)}{15\,c^2}+\frac{b\,e\,x^5\,\ln\left(c\,x+1\right)\,\ln\left(1-c^2\,x^2\right)}{10}-\frac{b\,e\,x^5\,\ln\left(1-c\,x\right)\,\ln\left(1-c^2\,x^2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] (a*d*x^5)/5 - (2*a*e*x^5)/25 + (b*d*x^5*log(c*x + 1))/10 - (b*d*x^5*log(1 -
 c*x))/10 - (b*e*x^5*log(c*x + 1))/25 + (b*e*x^5*log(1 - c*x))/25 + (b*e*lo
g(c*x + 1)^2)/(10*c^5) + (b*e*log(1 - c*x)^2)/(10*c^5) - (2*a*e*x)/(5*c^4)
- (2*a*e*x^3)/(15*c^2) + (b*d*x^4)/(20*c) + (b*d*x^2)/(10*c^3) - (9*b*e*x^4
)/(200*c) - (77*b*e*x^2)/(300*c^3) + (a*e*x^5*log(1 - c^2*x^2))/5 - (a*e*lo
g(c*x - 1))/(5*c^5) + (a*e*log(c*x + 1))/(5*c^5) + (b*d*log(c*x - 1))/(10*c
^5) + (b*d*log(c*x + 1))/(10*c^5) - (137*b*e*log(c*x - 1))/(300*c^5) - (137
*b*e*log(c*x + 1))/(300*c^5) - (b*e*log(c*x + 1)*log(-(2*a*e - 2*a*c*e*x)/(
5*c^4)))/(10*c^5) - (b*e*log(c*x + 1)*log(-(2*a*e + 2*a*c*e*x)/(5*c^4)))/(1
0*c^5) - (b*e*log(1 - c*x)*log(-(2*a*e - 2*a*c*e*x)/(5*c^4)))/(10*c^5) - (b
*e*log(1 - c*x)*log(-(2*a*e + 2*a*c*e*x)/(5*c^4)))/(10*c^5) - (b*e*x*log(c*
x + 1))/(5*c^4) + (b*e*x*log(1 - c*x))/(5*c^4) + (b*e*x^4*log(1 - c^2*x^2))
/(20*c) + (b*e*x^2*log(1 - c^2*x^2))/(10*c^3) + (b*e*log(-(2*a*e - 2*a*c*e*
x)/(5*c^4))*log(1 - c^2*x^2))/(10*c^5) + (b*e*log(-(2*a*e + 2*a*c*e*x)/(5*c
^4))*log(1 - c^2*x^2))/(10*c^5) - (b*e*x^3*log(c*x + 1))/(15*c^2) + (b*e*x^
3*log(1 - c*x))/(15*c^2) + (b*e*x^5*log(c*x + 1)*log(1 - c^2*x^2))/10 - (b*
e*x^5*log(1 - c*x)*log(1 - c^2*x^2))/10
```

3.523 $\int x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=225

$$\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{b(2d-3e)\tanh^{-1}(cx)}{8c^4}+\frac{2be\tanh^{-1}(cx)}{3c^4}-\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{1}{8}$$

```
[Out] 1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3+1/24*b*(2*d-e)*x^3/c-1/18*b*e*x^3/c-1/8
*b*(2*d-3*e)*arctanh(c*x)/c^4+2/3*b*e*arctanh(c*x)/c^4-1/4*e*x^2*(a+b*arcta
nh(c*x))/c^2-1/8*e*x^4*(a+b*arctanh(c*x))+1/4*b*e*x*ln(-c^2*x^2+1)/c^3+1/12
*b*e*x^3*ln(-c^2*x^2+1)/c-1/4*e*(a+b*arctanh(c*x))*ln(-c^2*x^2+1)/c^4+1/4*x
^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))
```

Rubi [A]
time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2504, 2442, 45, 6230, 470, 327, 212, 2521, 2498, 2505, 308}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+b\tanh^{-1}(cx)\right)-\frac{b(2d-3e)\tanh^{-1}(cx)}{8c^4}+\frac{2be\tanh^{-1}(cx)}{3c^4}+\frac{bx(2d-3e)}{8c^3}-\frac{2bex}{3c^3}+\frac{bex^3\log\left(1-c^2x^2\right)}{12c}+\frac{bex\log\left(1-c^2x^2\right)}{4c^3}+\frac{bx^3(2d-e)}{24c}-\frac{bex^3}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*
x])/(3*c^4) - (e*x^2*(a + b*ArcTanh[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTanh[
c*x]))/8 + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcTanh[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcT
anh[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 6230

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= -\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tanh^{-1}(cx)\right)-\cdots\\
&= -\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tanh^{-1}(cx)\right)-\cdots\\
&= \frac{b(2d-e)x^3}{24c}-\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\tanh\cdots\right.\\
&= \frac{b(2d-3e)x}{8c^3}+\frac{b(2d-e)x^3}{24c}-\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\cdots\\
&= \frac{b(2d-3e)x}{8c^3}+\frac{b(2d-e)x^3}{24c}-\frac{b(2d-3e)\tanh^{-1}(cx)}{8c^4}-\cdots\\
&= \frac{b(2d-3e)x}{8c^3}-\frac{bex}{2c^3}+\frac{b(2d-e)x^3}{24c}-\frac{b(2d-3e)\tanh^{-1}(\cdots}{8c^4}\\
&= \frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{b(2d-3e\cdots}{\cdots}\\
&= \frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{b(2d-3e\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 192, normalized size = 0.85

$$\frac{6bc(6d-25e)x-36ac^2ex^2+2bc^3(6d-7e)x^3+18ac^4(2d-e)x^4-18bc^2x^2\left(-2c^2dx^2+e\left(2+c^2x^2\right)\right)\tanh^{-1}(cx)+3(6bd-12ae-25be)\log(1-cx)-3(6bd+12ae-25be)\log(1+cx)+12e\left(3ac^4x^4+bcx\left(3+c^2x^2\right)+3b\left(-1+c^4x^4\right)\tanh^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{144c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^4
*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcTanh[c*x]
 + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e)*L
og[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x^4)*
ArcTanh[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 16.66, size = 3739, normalized size = 16.62

method	result	size
default	Expression too large to display	3739
risch	Expression too large to display	8859

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*d*x/c^3+1/12*b*d*x^3/c-25/24*b*e*x/c^3-7/72*b*e*x^3/c+41/24*b*e*arcta
nh(c*x)/c^4+1/4*x^4*a*e*ln(-c^2*x^2+1)+1/4*x^4*a*d-1/8*x^4*a*e+41/36*e/c^4*
b+1/6/c^4*b*e*(3*arctanh(c*x)*x^3*c^3+3*arctanh(c*x)*x^2*c^2+c^2*x^2+3*arct
anh(c*x)*x*c+c*x+3*arctanh(c*x)+4)*(c*x-1)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1
/6*I/c^4*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/6*I/c^4*b*e*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/6*I/c^4*b*e*Pi*csgn(I*
(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3+1/8*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2
/(-c^2*x^2+1))^2)^3*x^4*e+1/24*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2
)^3*x^3*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x*e+1/8*I/c^3*b*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x*e+1/8*I/c^3
*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x*e-1/8*I/c^4*b*arctanh(c*x)*P
i*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/8*I/c^4*b*arctanh(c*x
)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/3*I/c^4*b*e*Pi*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/6*I/c^4*b*e*Pi*csg
n(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*
x^2+1))^2)^2-1/6*I/c^4*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2
*x^2+1)^(1/2))^2*Pi*e-1/6*I/c^4*b*e*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/3*I/c^4*b*P
i*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)
^2-1/6*I/c^4*b*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^
2/(-c^2*x^2+1))^2)+1/8*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*
x^4*e+1/8*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c
^2*x^2+1))^2)^3*x^4*e+1/24*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^3*e+1
/24*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x
```

```
^3*e-1/4*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2+1/8*I/c^4*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(c*x)-1/
8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))-1/8*I/c^4*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*e*arctanh(c*x)+1/
4*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c
*x+1)^2/(-c^2*x^2+1))^2)^2-1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)+1/6*I/c^4*b*e*Pi*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)+1/12*I/c*b*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^3*e-1/24*I/c*b*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c
^2*x^2+1))^2)^2*x^3*e+1/24*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c
*x+1)/(-c^2*x^2+1)^(1/2))^2*x^3*e+1/24*I/c*b*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x
^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^3*
e+1/4*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-
c^2*x^2+1)^(1/2))*x^4*e-1/8*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^4*e+1/8*I
*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))^2*x^4*e+1/8*I*b*Pi*arctanh(c*x)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^
2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^4*e-1/4*I
*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)
^2/(-c^2*x^2+1)))*x^4*e+1/8*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x
^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^4*e-1/12*I/c*b*Pi*csgn(I*(
1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x^3*e+1/2
4*I/c*b*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x
^2+1)))^2*x^3*e+1/4*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*x*e-1/8*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e+1/8*I/c^3*b*P
i*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x*e+1/
8*I/c^3*b*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e-1/4*I/c^3*b*Pi*csgn(I*(1+(c*x+1)^2/
(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x*e+1/8*I/c^3*b*Pi*cs
gn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x*e
+1/2*b*ln(2)*arctanh(c*x)*x^4*e-1/2*b*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2
+1))*x^4*e+1/6/c*b*ln(2)*x^3*e-1/6/c*b*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^3*e-1
/4/c^2*b*arctanh(c*x)*x^2*e+1/2/c^3*b*ln(2)*x*e-1/2/c^3*b*ln(1+(c*x+1)^2/(-
c^2*x^2+1))*x*e-1/2/c^4*b*arctanh(c*x)*ln(2)*e+1/2/c^4*b*arctanh(c*x)*e*ln(
1+(c*x+1)^2/(-c^2*x^2+1))-1/8*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2
-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(
c*x+1)^2/(-c^2*x^2+1))^2)*x^4*e-1/24*I/c*b*Pi*c...
```

Maxima [C] Result contains complex when optimal does not.

time = 0.27, size = 274, normalized size = 1.22

$$\frac{1}{4}\,adx^{4} + \frac{1}{8}\left(2\,x^{4}\log\left(-c^{2}x^{2}+1\right) - c^{2}\left(\frac{c^{2}x^{4}+2\,x^{2}}{c^{4}} + \frac{2\,\log\left(c^{2}x^{2}-1\right)}{c^{6}}\right)\right)b\,\mathrm{artanh}\left(cx\right)e + \frac{1}{24}\left(6\,x^{4}\,\mathrm{artanh}\left(cx\right) + c\left(\frac{2\left(c^{2}x^{3}+3\,x\right)}{c^{4}} - \frac{3\,\log\left(cx+1\right)}{c^{5}} + \frac{3\,\log\left(cx-1\right)}{c^{5}}\right)\right)bd + \frac{1}{8}\left(2\,x^{4}\log\left(-c^{2}x^{2}+1\right) - c^{2}\left(\frac{c^{2}x^{4}+2\,x^{2}}{c^{4}} + \frac{2\,\log\left(c^{2}x^{2}-1\right)}{c^{6}}\right)\right)ae - \frac{\left(2\left(-6i\pi c^{3}+7\,c^{3}\right)x^{3} + 6\left(-6i\pi c+25\,c\right)x + 3\left(6i\pi-4\,c^{3}x^{3}-12\,cx-25\right)\log\left(cx+1\right) + 3\left(-6i\pi-4\,c^{3}x^{3}-12\,cx+25\right)\log\left(cx-1\right)\right)be}{144\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2
*log(c^2*x^2 - 1)/c^6))*b*arctanh(c*x)*e + 1/24*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/8*(
2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c
^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x + 3*
(6*I*pi - 4*c^3*x^3 - 12*c*x - 25)*log(c*x + 1) + 3*(-6*I*pi - 4*c^3*x^3 -
12*c*x + 25)*log(c*x - 1))*b*e/c^4
```

Fricas [A]
time = 0.48, size = 306, normalized size = 1.36

$$\frac{36\,ac^{4}dx^{4} + 12\,bc^{3}dx^{3} + 36\,bcdx - 2\left(9\,ac^{4}x^{4} + 7\,bc^{3}x^{3} + 18\,ac^{2}x^{2} + 75\,bcx\right)\cosh\left(1\right) + 12\left(\left(3\,ac^{4}x^{4} + bc^{3}x^{3} + 3\,bcx - 3\,a\right)\cosh\left(1\right) + \left(3\,ac^{4}x^{4} + bc^{3}x^{3} + 3\,bcx - 3\,a\right)\sinh\left(1\right)\right)\log\left(-c^{2}x^{2}+1\right) + 3\left(6\,bc^{4}dx^{4} - 6\,bd - \left(3\,bc^{4}x^{4} + 6\,bc^{2}x^{2} - 25\,b\right)\cosh\left(1\right) + 6\left(\left(bc^{4}x^{4} - b\right)\cosh\left(1\right) + \left(bc^{4}x^{4} - b\right)\sinh\left(1\right)\right)\log\left(-c^{2}x^{2}+1\right) - \left(3\,bc^{4}x^{4} + 6\,bc^{2}x^{2} - 25\,b\right)\sinh\left(1\right)\right)\log\left(-\frac{cx+1}{cx-1}\right) - 2\left(9\,ac^{4}x^{4} + 7\,bc^{3}x^{3} + 18\,ac^{2}x^{2} + 75\,bcx\right)\sinh\left(1\right)}{144\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/144*(36*a*c^4*d*x^4 + 12*b*c^3*d*x^3 + 36*b*c*d*x - 2*(9*a*c^4*x^4 + 7*b*
c^3*x^3 + 18*a*c^2*x^2 + 75*b*c*x)*cosh(1) + 12*((3*a*c^4*x^4 + b*c^3*x^3 +
 3*b*c*x - 3*a)*cosh(1) + (3*a*c^4*x^4 + b*c^3*x^3 + 3*b*c*x - 3*a)*sinh(1)
)*log(-c^2*x^2 + 1) + 3*(6*b*c^4*d*x^4 - 6*b*d - (3*b*c^4*x^4 + 6*b*c^2*x^2
 - 25*b)*cosh(1) + 6*((b*c^4*x^4 - b)*cosh(1) + (b*c^4*x^4 - b)*sinh(1))*lo
g(-c^2*x^2 + 1) - (3*b*c^4*x^4 + 6*b*c^2*x^2 - 25*b)*sinh(1))*log(-(c*x + 1
)/(c*x - 1)) - 2*(9*a*c^4*x^4 + 7*b*c^3*x^3 + 18*a*c^2*x^2 + 75*b*c*x)*sinh
(1))/c^4
```

Sympy [A]
time = 1.48, size = 279, normalized size = 1.24

$$\begin{cases} \frac{adx^{4}}{4} + \frac{aex^{4}\log\left(-c^{2}x^{2}+1\right)}{4} - \frac{aex^{4}}{8} - \frac{aex^{2}}{4c^{2}} - \frac{ae\log\left(-c^{2}x^{2}+1\right)}{4c^{4}} + \frac{bdx^{4}\,\mathrm{atanh}\left(cx\right)}{4} + \frac{bex^{4}\log\left(-c^{2}x^{2}+1\right)\mathrm{atanh}\left(cx\right)}{4} - \frac{bex^{4}\,\mathrm{atanh}\left(cx\right)}{8} + \frac{bdx^{3}}{12c} + \frac{bex^{3}\log\left(-c^{2}x^{2}+1\right)}{12c} - \frac{7bex^{3}}{72c} - \frac{bex^{2}\,\mathrm{atanh}\left(cx\right)}{4c^{2}} + \frac{bdx}{4c^{3}} + \frac{bex\log\left(-c^{2}x^{2}+1\right)}{4c^{3}} - \frac{25bex}{24c^{3}} - \frac{bd\,\mathrm{atanh}\left(cx\right)}{4c^{4}} - \frac{be\log\left(-c^{2}x^{2}+1\right)\mathrm{atanh}\left(cx\right)}{4c^{4}} + \frac{25be\,\mathrm{atanh}\left(cx\right)}{24c^{4}} & \text{for}\: c \neq 0 \\ \frac{adx^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*x
**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atanh(c*x)/4 + b
*e*x**4*log(-c**2*x**2 + 1)*atanh(c*x)/4 - b*e*x**4*atanh(c*x)/8 + b*d*x**3
```

```
/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e*x**
2*atanh(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*c**3)
 - 25*b*e*x/(24*c**3) - b*d*atanh(c*x)/(4*c**4) - b*e*log(-c**2*x**2 + 1)*a
tanh(c*x)/(4*c**4) + 25*b*e*atanh(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, T
rue))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 1.70, size = 851, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - c*x)^2*((b*e)/(8*c^4) - (b*e*x^4)/8) - log(c*x + 1)^2*((b*e)/(8*c^4
) - (b*e*x^4)/8) + log(1 - c*x)*((x^4*(a*e - (b*d)/2 + (b*e)/4 + (b*e*(log(
c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/2))/4 - (x^2*((16*a*e - 8*b*d
+ 8*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/c - (16*a*e - 8*b
*d + 4*b*e + 8*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/c))/(3
2*c) + (b*e*x)/(4*c^3) + (b*e*x^3)/(12*c)) - x^2*((a*(e - 2*d + 2*e*(log(c*
x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(4*c^2) + (a*(d - e*(log(c*x +
1) + log(1 - c*x) - log(1 - c^2*x^2))))/(2*c^2)) - x*((b*(7*e - 6*d + 6*e*(
log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(24*c^3) + (3*b*e)/(4*c^3
)) - (a*x^4*(e - 2*d + 2*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))
))/8 - (log((x*(12*a*e - 6*b*d + 25*b*e + 6*b*e*(log(c*x + 1) + log(1 - c*x
) - log(1 - c^2*x^2))))/(24*c^2) - (25*b*e - 6*b*d + 6*b*e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2)))/(24*c^3) - (a*e*x)/(2*c^2))*(12*a*e - 6*b
*d + 25*b*e + 6*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(48*
c^4) - (log((x*(12*a*e + 6*b*d - 25*b*e - 6*b*e*(log(c*x + 1) + log(1 - c*x
) - log(1 - c^2*x^2))))/(24*c^2) - (25*b*e - 6*b*d + 6*b*e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2)))/(24*c^3) - (a*e*x)/(2*c^2))*(12*a*e + 6*b
*d - 25*b*e - 6*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(48*
c^4) + c*log(c*x + 1)*((x^4*(4*a*e + 2*b*d - b*e - 2*b*e*(log(c*x + 1) + lo
g(1 - c*x) - log(1 - c^2*x^2))))/(16*c) + (b*e*x)/(4*c^4) + (b*e*x^3)/(12*c
^2) - (b*e*x^2)/(8*c^3)) - (b*x^3*(7*e - 6*d + 6*e*(log(c*x + 1) + log(1 -
c*x) - log(1 - c^2*x^2))))/(72*c)
```

3.524 $\int x^2 \left(a+b \tanh^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx$

<u>**Optimal**</u>. Leaf size=247

$$-\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}(cx)+\frac{be\tanh^{-1}(cx)^2}{3c^3}-\frac{(2a+b)e\log(1-cx)}{6c^3}+\frac{(2a$$

```
[Out] -2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*b*e*x*arctanh(c*x)/c^2-2/9*b*
e*x^3*arctanh(c*x)+1/3*b*e*arctanh(c*x)^2/c^3-1/6*(2*a+b)*e*ln(-c*x+1)/c^3+
1/6*(2*a-b)*e*ln(c*x+1)/c^3-4/9*b*e*ln(-c^2*x^2+1)/c^3-1/12*b*e*ln(-c^2*x^2
+1)^2/c^3+1/6*b*x^2*(d+e*ln(-c^2*x^2+1))/c+1/3*x^3*(a+b*arctanh(c*x))*(d+e*
ln(-c^2*x^2+1))+1/6*b*ln(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/c^3
```

<u>**Rubi**</u> [A]

time = 0.43, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6037, 272, 45, 6232, 6857, 815, 647, 31, 6127, 6021, 266, 6095, 2525, 2437, 2338}

$$-\frac{e(2a+b)\log(1-cx)}{6c^3}+\frac{e(2a-b)\log(cx+1)}{6c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{2aex}{3c^2}-\frac{2}{9}aex^3+\frac{be\tanh^{-1}(cx)^2}{3c^3}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}-\frac{2bex\tanh^{-1}(cx)}{3c^2}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}-\frac{4be\log\left(1-c^2x^2\right)}{9c^3}-\frac{2}{9}bex^3\tanh^{-1}(cx)-\frac{5bex^2}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcTanh[
c*x])/(3*c^2) - (2*b*e*x^3*ArcTanh[c*x])/9 + (b*e*ArcTanh[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 45</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

<u>Rule 266</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6232

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+\right.$$

$$= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+\right.$$

$$= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+\right.$$

$$= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+\right.$$

$$= -\frac{be\log^2\left(1-c^2x^2\right)}{12c^3} + \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3(a$$

$$= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2}{9}bex^3\tanh^{-1}(cx) - \frac{be\log^2(}{12}$$

$$= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2bex\tanh^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3\tan$$

$$= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2bex\tanh^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3\tan$$

$$= -\frac{2aex}{3c^2} - \frac{5bex^2}{18c} - \frac{2}{9}aex^3 - \frac{2bex\tanh^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3\text{ta}$$

Mathematica [**A**]
time = 0.07, size = 183, normalized size = 0.74

$$\frac{-24acex+2bc^2(3d-5e)x^2+4ac^3(3d-2e)x^3+4bcx(3c^2dx^2-2e(3+c^2x^2))\tanh^{-1}(cx)+12be\tanh^{-1}(cx)^2+2(3bd-6ae-11be)\log(1-cx)+2(3bd+6ae-11be)\log(1+cx)+6c^2ex^2\left(b+2acx+2bcx\tanh^{-1}(cx)\right)\log\left(1-c^2x^2\right)+3be\log^2\left(1-c^2x^2\right)}{36c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x*
(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcTanh[c*x] + 12*b*e*ArcTanh[c*x]^2 + 2*
(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1 +
c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcTanh[c*x])*Log[1 - c^2*x^2] +
3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.62, size = 3994, normalized size = 16.17

method	result	size

default	Expression too large to display	3994
risch	Expression too large to display	4015

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b*e*x*arctanh(c*x)/c^2+1/6*b*d*x^2/c-2/9*a*e*x^3-2/3*a*e*x/c^2-5/18*b*
e*x^2/c-2/9*b*e*x^3*arctanh(c*x)+1/3*a*d*x^3+1/3*a*e*x^3*ln(-c^2*x^2+1)+5/1
8*e*b/c^3-1/12*I*b/c^3*e*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/6*I*b/c^3*Pi*e*csgn(I*
(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1/12*I*b
/c^3*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^
2+1))^2)+1/12*I*b/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2*e+1/12*I*b/c*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^2*e+1/12*I*b
/c*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^2*e+1/6*I*b/c^3*arctanh(c*x)
*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/6*I*b/c^3*arctanh(c*x)*Pi*e*csgn(I*
(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3+1/6*I*b/c^3*arctanh(c
*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/6*I*b/c^3*Pi*ln(1+(c*x+1)
^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/6*I*b*Pi*arctanh(c*x)*
csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^3*e+1/6*I*b*Pi*arctanh(c*x)*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^3*x^3*e+1/6*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^3*e-1/6*I*b/c^3*Pi*ln(1+(c*x+1)
^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^
2)^3-1/6*I*b/c^3*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(1+(c*x+1)^2/(-c^
2*x^2+1))^2)^3-1/6*I*b/c^3*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(
c*x+1)^2/(c^2*x^2-1))^2+1/12*I*b/c^3*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1/12*I*b/c^3*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*e-1/3*I*b
*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2
/(-c^2*x^2+1)))*x^3*e+1/6*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2
+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^3*e+1/6*I*b/c*Pi*csgn(I*(c*x
+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^2*e-1/12*I*b/c*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-
c^2*x^2+1))^2)^2*x^2*e+1/12*I*b/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(
c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2*e+1/12*I*b/c*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^2
*e-1/6*I*b/c*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/
(-c^2*x^2+1)))*x^2*e+1/12*I*b/c*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csg
n(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^2*e+1/3*I*b/c^3*arctanh(c*x)*Pi*e*csgn(
I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/6*I*b/c^3*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*Pi*e*arctanh(c*x)+1/6*I*b/c^3*arctanh(c*x)*Pi*e*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/6*I*b/c^3*csgn(I*(
```

```
c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1))^2)*Pi*e*arctanh(c*x)-1/3*I*b/c^3*arctanh(c*x)*Pi*e*csgn(I*(1+(c
*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/6*I*b/c^3*a
rctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(
-c^2*x^2+1))^2)-1/3*I*b/c^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi+1/6*I*b/c^3*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/6*I*b/c^3*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))
*Pi-1/6*I*b/c^3*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^
2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi+1/
3*I*b/c^3*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*
x^2+1)))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/6*I*b/c^3*csgn(I*(1+(c*x+1)^2/
(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*e*ln(1+(c*x+1)^2/(-c^
2*x^2+1))*Pi+1/12*I*b/c^3*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+
1))^2)+1/3*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*x^3*e-1/6*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^3*e+
1/6*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))^2*x^3*e+1/6*I*b*Pi*arctanh(c*x)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2
+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^3*e+
1/3*b/c*ln(2)*x^2*e-1/3*b/c*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^2*e+2/3*b/c^3*ar
ctanh(c*x)*ln(2)*e-2/3*b/c^3*ln(2)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e+2/3*b*ln(
2)*arctanh(c*x)*x^3*e-2/3*b*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x^3*e
+1/3*b/c^3*e*(2*arctanh(c*x)*x^3*c^3+c^2*x^2+2*arctanh(c*x)-2*ln(1+(c*x+1)^
2/(-c^2*x^2+1))-1)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/12*I*b/c^3*Pi*e*csgn(I*
(c*x+1)^2/(c^2*x^2-1))^3-1/12*I*b/c^3*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+
(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/12*I*b/c^3*e*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^
2+1))^2)^3-1/3*a*e/c^3*ln(c*x-1)+1/3*a*e/c^3*ln...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.28, size = 255, normalized size = 1.03

$$\frac{1}{3}adx^3 + \frac{1}{9}\left(3x^3\log\left(-c^2x^2+1\right) - c^2\left(\frac{2\left(c^2x^3+3x\right)}{c^4} - \frac{3\log\left(cx+1\right)}{c^5} + \frac{3\log\left(cx-1\right)}{c^5}\right)\right)b\,\mathrm{artanh}\left(cx\right)e + \frac{1}{6}\left(2x^3\,\mathrm{artanh}\left(cx\right) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bd + \frac{1}{9}\left(3x^3\log\left(-c^2x^2+1\right) - c^2\left(\frac{2\left(c^2x^3+3x\right)}{c^4} - \frac{3\log\left(cx+1\right)}{c^5} + \frac{3\log\left(cx-1\right)}{c^5}\right)\right)ae + \frac{\left(\left(3i\pi c^2 - 5c^2\right)x^2 + \left(3i\pi + 3c^2x^2 + 6\log\left(cx-1\right) - 11\right)\log\left(cx+1\right) + \left(3i\pi + 3c^2x^2 - 11\right)\log\left(cx-1\right)\right)be}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*arctanh(c*x)*e + 1/6*(2*x^3*arct
anh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2*x^
2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c
^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*log(c*x
 - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*e/c^3
```

Fricas [A]
time = 0.37, size = 306, normalized size = 1.24

$$\frac{12\,ac^3dx^3 + 6\,bc^2dx^2 + 3\,(b\cosh(1) + b\sinh(1))\log\left(-c^2x^2 + 1\right)^2 + 3\,(b\cosh(1) + b\sinh(1))\log\left(-\frac{cx+1}{cx-1}\right)^2 - 2\,(4\,ac^3x^3 + 5\,bc^2x^2 + 12\,acx)\cosh(1) + 2\,(3\,bd + (6\,ac^3x^3 + 3\,bc^2x^2 - 11\,b)\cosh(1) + (6\,ac^3x^3 + 3\,bc^2x^2 - 11\,b)\sinh(1))\log\left(-c^2x^2 + 1\right) + 2\,(3\,bc^3dx^3 - 2\,(bc^3x^3 + 3\,bcx - 3\,a)\cosh(1) + 3\,(bc^3x^3\cosh(1) + bc^3x^3\sinh(1))\log\left(-c^2x^2 + 1\right) - 2\,(bc^3x^3 + 3\,bcx - 3\,a)\sinh(1))\log\left(-\frac{cx+1}{cx-1}\right) - 2\,(4\,ac^3x^3 + 5\,bc^2x^2 + 12\,acx)\sinh(1)}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/36*(12*a*c^3*d*x^3 + 6*b*c^2*d*x^2 + 3*(b*cosh(1) + b*sinh(1))*log(-c^2*x
^2 + 1)^2 + 3*(b*cosh(1) + b*sinh(1))*log(-(c*x + 1)/(c*x - 1))^2 - 2*(4*a*
c^3*x^3 + 5*b*c^2*x^2 + 12*a*c*x)*cosh(1) + 2*(3*b*d + (6*a*c^3*x^3 + 3*b*c
^2*x^2 - 11*b)*cosh(1) + (6*a*c^3*x^3 + 3*b*c^2*x^2 - 11*b)*sinh(1))*log(-c
^2*x^2 + 1) + 2*(3*b*c^3*d*x^3 - 2*(b*c^3*x^3 + 3*b*c*x - 3*a)*cosh(1) + 3*
(b*c^3*x^3*cosh(1) + b*c^3*x^3*sinh(1))*log(-c^2*x^2 + 1) - 2*(b*c^3*x^3 +
3*b*c*x - 3*a)*sinh(1))*log(-(c*x + 1)/(c*x - 1)) - 2*(4*a*c^3*x^3 + 5*b*c^
2*x^2 + 12*a*c*x)*sinh(1))/c^3
```

Sympy [A]
time = 0.99, size = 258, normalized size = 1.04

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^3\log\left(-c^2x^2+1\right)}{3} - \frac{2aex^3}{9} - \frac{2aex}{3c^2} + \frac{2ae\,\operatorname{atanh}(cx)}{3c^3} + \frac{bdx^3\operatorname{atanh}(cx)}{3} + \frac{bex^3\log\left(-c^2x^2+1\right)\operatorname{atanh}(cx)}{3} - \frac{2bex^3\operatorname{atanh}(cx)}{9} + \frac{bdx^2}{6c} + \frac{bex^2\log\left(-c^2x^2+1\right)}{6c} - \frac{5bex^2}{18c} - \frac{2bex\,\operatorname{atanh}(cx)}{3c^2} + \frac{bd\log\left(-c^2x^2+1\right)}{6c^3} + \frac{be\log\left(-c^2x^2+1\right)^2}{12c^3} - \frac{11be\log\left(-c^2x^2+1\right)}{18c^3} + \frac{be\,\operatorname{atanh}^2(cx)}{3c^3} & \text{for}\ c \neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a
*e*x/(3*c**2) + 2*a*e*atanh(c*x)/(3*c**3) + b*d*x**3*atanh(c*x)/3 + b*e*x**
3*log(-c**2*x**2 + 1)*atanh(c*x)/3 - 2*b*e*x**3*atanh(c*x)/9 + b*d*x**2/(6*
c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*atanh
(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)
**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*atanh(c*x)**2/(3
*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

Giac [A]
time = 0.54, size = 244, normalized size = 0.99

$$-\frac{1}{6}\,bex^3\log\left(-cx+1\right)^2 + \frac{1}{9}\,(3\,ad - 2\,ae)x^3 + \frac{1}{6}\left(bex^3 + \frac{be}{c^3}\right)\log\left(cx+1\right)^2 + \frac{(3\,bd - 5\,be)x^2}{18\,c} + \frac{1}{18}\left((3\,bd + 6\,ae - 2\,be)x^3 + \frac{3\,bex^2}{c} - \frac{6\,bex}{c^2}\right)\log\left(cx+1\right) - \frac{1}{18}\left((3\,bd - 6\,ae - 2\,be)x^3 - \frac{3\,bex^2}{c} - \frac{6\,bex}{c^2} - \frac{6\,be\log\left(cx-1\right)}{c^3}\right)\log\left(-cx+1\right) - \frac{2\,aex}{3\,c^2} - \frac{be\log\left(cx-1\right)^2}{6\,c^3} + \frac{(3\,bd + 6\,ae - 11\,be)\log\left(cx+1\right)}{18\,c^3} + \frac{(3\,bd - 6\,ae - 11\,be)\log\left(cx-1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/6*b*e*x^3*log(-c*x + 1)^2 + 1/9*(3*a*d - 2*a*e)*x^3 + 1/6*(b*e*x^3 + b*e
/c^3)*log(c*x + 1)^2 + 1/18*(3*b*d - 5*b*e)*x^2/c + 1/18*((3*b*d + 6*a*e -
2*b*e)*x^3 + 3*b*e*x^2/c - 6*b*e*x/c^2)*log(c*x + 1) - 1/18*((3*b*d - 6*a*e
```

```
 - 2*b*e)*x^3 - 3*b*e*x^2/c - 6*b*e*x/c^2 - 6*b*e*log(c*x - 1)/c^3)*log(-c*
x + 1) - 2/3*a*e*x/c^2 - 1/6*b*e*log(c*x - 1)^2/c^3 + 1/18*(3*b*d + 6*a*e -
 11*b*e)*log(c*x + 1)/c^3 + 1/18*(3*b*d - 6*a*e - 11*b*e)*log(c*x - 1)/c^3
```

Mupad [B]
time = 2.81, size = 515, normalized size = 2.09

$$\frac{a\,d\,x^3}{3}-\frac{2\,a\,e\,x^3}{9}+\frac{b\,d\,x^3\,\ln\left(c\,x+1\right)}{6}-\frac{b\,d\,x^3\,\ln\left(1-c\,x\right)}{6}-\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)}{9}+\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)}{9}+\frac{b\,e\,{\ln\left(c\,x+1\right)}^2}{6\,c^3}+\frac{b\,e\,{\ln\left(1-c\,x\right)}^2}{6\,c^3}-\frac{2\,a\,e\,x}{3\,c^2}+\frac{b\,d\,x^2}{6\,c}-\frac{5\,b\,e\,x^2}{18\,c}+\frac{a\,e\,x^3\,\ln\left(1-c^2\,x^2\right)}{3}-\frac{a\,e\,\ln\left(c\,x-1\right)}{3\,c^3}+\frac{a\,e\,\ln\left(c\,x+1\right)}{3\,c^3}+\frac{b\,d\,\ln\left(c\,x-1\right)}{6\,c^3}+\frac{b\,d\,\ln\left(c\,x+1\right)}{6\,c^3}-\frac{11\,b\,e\,\ln\left(c\,x-1\right)}{18\,c^3}-\frac{11\,b\,e\,\ln\left(c\,x+1\right)}{18\,c^3}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,x\,\ln\left(c\,x+1\right)}{3\,c^2}+\frac{b\,e\,x\,\ln\left(1-c\,x\right)}{3\,c^2}+\frac{b\,e\,x^2\,\ln\left(1-c^2\,x^2\right)}{6\,c}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{3\,c^2}\right)\,\ln\left(1-c^2\,x^2\right)}{6\,c^3}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{3\,c^2}\right)\,\ln\left(1-c^2\,x^2\right)}{6\,c^3}+\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)\,\ln\left(1-c^2\,x^2\right)}{6}-\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)\,\ln\left(1-c^2\,x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] (a*d*x^3)/3 - (2*a*e*x^3)/9 + (b*d*x^3*log(c*x + 1))/6 - (b*d*x^3*log(1 - c
*x))/6 - (b*e*x^3*log(c*x + 1))/9 + (b*e*x^3*log(1 - c*x))/9 + (b*e*log(c*x
 + 1)^2)/(6*c^3) + (b*e*log(1 - c*x)^2)/(6*c^3) - (2*a*e*x)/(3*c^2) + (b*d*
x^2)/(6*c) - (5*b*e*x^2)/(18*c) + (a*e*x^3*log(1 - c^2*x^2))/3 - (a*e*log(c
*x - 1))/(3*c^3) + (a*e*log(c*x + 1))/(3*c^3) + (b*d*log(c*x - 1))/(6*c^3)
+ (b*d*log(c*x + 1))/(6*c^3) - (11*b*e*log(c*x - 1))/(18*c^3) - (11*b*e*log
(c*x + 1))/(18*c^3) - (b*e*log(c*x + 1)*log(-(2*a*e - 2*a*c*e*x)/(3*c^2)))/
(6*c^3) - (b*e*log(c*x + 1)*log(-(2*a*e + 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b
*e*log(1 - c*x)*log(-(2*a*e - 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b*e*log(1 - c
*x)*log(-(2*a*e + 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b*e*x*log(c*x + 1))/(3*c^
2) + (b*e*x*log(1 - c*x))/(3*c^2) + (b*e*x^2*log(1 - c^2*x^2))/(6*c) + (b*e
*log(-(2*a*e - 2*a*c*e*x)/(3*c^2))*log(1 - c^2*x^2))/(6*c^3) + (b*e*log(-(2
*a*e + 2*a*c*e*x)/(3*c^2))*log(1 - c^2*x^2))/(6*c^3) + (b*e*x^3*log(c*x + 1
)*log(1 - c^2*x^2))/6 - (b*e*x^3*log(1 - c*x)*log(1 - c^2*x^2))/6
```

3.525 $\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=140

$$\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be\tanh^{-1}(cx)}{c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+$$

```
[Out] 1/2*b*(d-e)*x/c-b*e*x/c-1/2*b*(d-e)*arctanh(c*x)/c^2+b*e*arctanh(c*x)/c^2+1
/2*d*x^2*(a+b*arctanh(c*x))-1/2*e*x^2*(a+b*arctanh(c*x))+1/2*b*e*x*ln(-c^2*
x^2+1)/c-1/2*e*(-c^2*x^2+1)*(a+b*arctanh(c*x))*ln(-c^2*x^2+1)/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2504, 2436, 2332, 6230, 327, 213, 2498, 212}

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{bex\log\left(1-c^2x^2\right)}{2c}+\frac{be\tanh^{-1}(cx)}{c^2}+\frac{bx(d-e)}{2c}-\frac{bex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*A
rcTanh[c*x])/c^2 + (d*x^2*(a + b*ArcTanh[c*x]))/2 - (e*x^2*(a + b*ArcTanh[c
*x]))/2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcTanh[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6230

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{e}{}$$
$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{e}{}$$
$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh\right.$$
$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{t}\right.$$
$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be\tanh^{-1}(cx}{c^2}$$

Mathematica [A]
time = 0.06, size = 129, normalized size = 0.92

$$\frac{2bc(d-3e)x+2ac^2(d-e)x^2+2bc^2(d-e)x^2\tanh^{-1}(cx)+(b(d-3e)-2ae)\log(1-cx)-(b(d-3e)+2ae)\log(1+cx)+2e\left(cx(b+acx)+b\left(-1+c^2x^2\right)\tanh^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcTanh[c*x]
 + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x]
+ 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcTanh[c*x])*Log[1 - c^2*x^2])/(
4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.23, size = 2951, normalized size = 21.08

method	result	size
default	Expression too large to display	2951
risch	Expression too large to display	8433

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*b*e*x/c+5/2*b*e*arctanh(c*x)/c^2+1/2*b*d*x/c-1/2*a*e*x^2+1/2*a*d*x^2-1
/2*b*arctanh(c*x)*x^2*e+1/2*a*e*x^2*ln(-c^2*x^2+1)+1/2*a*e/c^2+1/2*I*b*Pi*a
rctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/
2))*x^2*e-1/4*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^2*e+1/4*I*b*Pi*arctanh(
c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2
```

*e+1/4*I*b*Pi*arctanh(c*x)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x^2*e-1/2*I*b*Pi*arctanh(
c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1
)))*x^2*e+1/4*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn
(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*x^2*e+1/2*I*b/c*Pi*csgn(I*(c*x+1)^2/(c^2*x
^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x*e-1/4*I*b/c*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^
2*x*e+1/4*I*b/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))^2*x*e+1/4*I*b/c*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e-1/2*I*b/c*Pi*csgn(I*
(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x*e+1/4*
I*b/c*Pi*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2
+1)))^2*x*e-1/2*I*b/c^2*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/4*I*b/c^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(
c*x)-1/4*I*b/c^2*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csg
n(I*(c*x+1)^2/(c^2*x^2-1))-1/4*I*b/c^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*e*arctanh(
c*x)+1/2*I*b/c^2*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(
I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1/4*I*b/c^2*arctanh(c*x)*Pi*e*csgn(I*(1+(
c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)+1/4*I*b/c^2*
e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)+1/4*I*b/c*Pi*csgn(I
*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x*e-1/4*I*b/c^2*arctanh(c*x)*Pi*e*csgn(I*(
c*x+1)^2/(c^2*x^2-1))^3-1/4*I*b/c^2*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/4*I*b/c^2*arctanh(c*x)*Pi*e*csgn(
I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/2*I*b/c^2*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+1/4*I*b/c^2*e*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2
-1/4*I*b/c^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2
))^2*Pi*e-1/4*I*b/c^2*e*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2+1/2*I*b/c^2*Pi*e*csgn(I*(
1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2-1/4*I*b/c
^2*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+
1))^2)+1/4*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2*e+1/4*I*
b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2
)^3*x^2*e+1/4*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x^
2*e+1/4*I*b/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x*e+1/4*I*b/c*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3*x*e-1/4*I*b/c^2*Pi*e*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/4*I*b/c^2*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/4*I*b/c^2*e*Pi*csgn(I*(1+(c*x+1)^2/(-c^2
*x^2+1))^2)^3+b/c^2*e*(arctanh(c*x)*x*c+arctanh(c*x)+1)*(c*x-1)*ln((c*x+1)/
(-c^2*x^2+1)^(1/2))+b*ln(2)*arctanh(c*x)*x^2*e-b*arctanh(c*x)*ln(1+(c*x+1)^
2/(-c^2*x^2+1))*x^2*e+b/c*ln(2)*x*e-b/c*ln(1+(c*x+1)^2/(-c^2*x^2+1))*x*e-b/
c^2*arctanh(c*x)*ln(2)*e+b/c^2*arctanh(c*x)*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))-

```
1/2*a*e/c^2*ln(-c^2*x^2+1)+1/2*b*arctanh(c*x)*x^2*d-1/2*b/c^2*arctanh(c*x)*
d-b/c^2*ln(2)*e-1/2*b/c^2*d-1/4*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1))^2)*x^2*e-1/4*I*b/c*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)*x*e+1/4*I*b/c^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*Pi*e*arctanh(c*x)+3/2*e*b/c^2
```

Maxima [A]
time = 0.26, size = 174, normalized size = 1.24

$$\frac{1}{2}adx^2+\frac{1}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd-\frac{(c^2x^2-(c^2x^2-1)\log(-c^2x^2+1)-1)b\operatorname{artanh}(cx)e}{2c^2}-\frac{(c^2x^2-(c^2x^2-1)\log(-c^2x^2+1)-1)ae}{2c^2}-\frac{(3cx-(cx+1)\log(cx+1)-(cx-1)\log(-cx+1))be}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*b
*arctanh(c*x)*e/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*a
*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*b*e
/c^2
```

Fricas [A]
time = 0.37, size = 215, normalized size = 1.54

$$\frac{2ac^2dx^2+2bcdx-2(ac^2x^2+3bcx)\cosh(1)+2((ac^2x^2+bcx-a)\cosh(1)+(ac^2x^2+bcx-a)\sinh(1))\log(-c^2x^2+1)+(bc^2dx^2-bd-(bc^2x^2-3b)\cosh(1)+((bc^2x^2-b)\cosh(1)+(bc^2x^2-b)\sinh(1))\log(-c^2x^2+1)-(bc^2x^2-3b)\sinh(1))\log\left(-\frac{cx+1}{cx-1}\right)-2(ac^2x^2+3bcx)\sinh(1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*d*x^2 + 2*b*c*d*x - 2*(a*c^2*x^2 + 3*b*c*x)*cosh(1) + 2*((a*c^
2*x^2 + b*c*x - a)*cosh(1) + (a*c^2*x^2 + b*c*x - a)*sinh(1))*log(-c^2*x^2
+ 1) + (b*c^2*d*x^2 - b*d - (b*c^2*x^2 - 3*b)*cosh(1) + ((b*c^2*x^2 - b)*co
sh(1) + (b*c^2*x^2 - b)*sinh(1))*log(-c^2*x^2 + 1) - (b*c^2*x^2 - 3*b)*sinh
(1))*log(-(c*x + 1)/(c*x - 1)) - 2*(a*c^2*x^2 + 3*b*c*x)*sinh(1))/c^2
```

Sympy [A]
time = 1.58, size = 202, normalized size = 1.44

$$\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log(-c^2x^2+1)}{2}-\frac{aex^2}{2}-\frac{ae\log(-c^2x^2+1)}{2c^2}+\frac{bdx^2\operatorname{atanh}(cx)}{2}+\frac{bex^2\log(-c^2x^2+1)\operatorname{atanh}(cx)}{2}-\frac{bex^2\operatorname{atanh}(cx)}{2}+\frac{bdx}{2c}+\frac{bex\log(-c^2x^2+1)}{2c}-\frac{3bex}{2c}-\frac{bd\operatorname{atanh}(cx)}{2c^2}-\frac{be\log(-c^2x^2+1)\operatorname{atanh}(cx)}{2c^2}+\frac{3be\operatorname{atanh}(cx)}{2c^2} & \text{for } c\neq 0\\ \frac{adx^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*l
og(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atanh(c*x)/2 + b*e*x**2*log(-c**2*x*
*2 + 1)*atanh(c*x)/2 - b*e*x**2*atanh(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-c**
2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*atanh(c*x)/(2*c**2) - b*e*log(-c**2
*x**2 + 1)*atanh(c*x)/(2*c**2) + 3*b*e*atanh(c*x)/(2*c**2), Ne(c, 0)), (a*d
*x**2/2, True))
```

Giac [A]
time = 0.63, size = 209, normalized size = 1.49

$$-\frac{1}{4}\,bex^2\log\left(-cx+1\right)^2+\frac{1}{2}\,(ad-ae)x^2+\frac{1}{4}\left(bex^2-\frac{be}{c^2}\right)\log\left(cx+1\right)^2+\frac{1}{4}\left((bd+2\,ae-be)x^2+\frac{2\,bex}{c}\right)\log\left(cx+1\right)-\frac{be\log\left(cx-1\right)^2}{4\,c^2}-\frac{1}{4}\left((bd-2\,ae-be)x^2-\frac{2\,bex}{c}-\frac{2\,be\log\left(cx-1\right)}{c^2}\right)\log\left(-cx+1\right)+\frac{(bd-3\,be)x}{2\,c}-\frac{(bd+2\,ae-3\,be)\log\left(cx+1\right)}{4\,c^2}+\frac{(bd-2\,ae-3\,be)\log\left(cx-1\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/4*b*e*x^2*log(-c*x + 1)^2 + 1/2*(a*d - a*e)*x^2 + 1/4*(b*e*x^2 - b*e/c^2
)*log(c*x + 1)^2 + 1/4*((b*d + 2*a*e - b*e)*x^2 + 2*b*e*x/c)*log(c*x + 1) -
 1/4*b*e*log(c*x - 1)^2/c^2 - 1/4*((b*d - 2*a*e - b*e)*x^2 - 2*b*e*x/c - 2*
b*e*log(c*x - 1)/c^2)*log(-c*x + 1) + 1/2*(b*d - 3*b*e)*x/c - 1/4*(b*d + 2*
a*e - 3*b*e)*log(c*x + 1)/c^2 + 1/4*(b*d - 2*a*e - 3*b*e)*log(c*x - 1)/c^2
```

Mupad [B]
time = 1.42, size = 557, normalized size = 3.98

$$\ln\left(1-c\,x\right)^2\,\left(\frac{b\,e}{4\,c^2}-\frac{b\,e\,x^2}{4}\right)-\ln\left(c\,x+1\right)^2\,\left(\frac{b\,e}{4\,c^2}-\frac{b\,e\,x^2}{4}\right)+\ln\left(1-c\,x\right)\,\left(\frac{x^2\,\left(a\,e-\frac{b\,d}{2}+\frac{b\,e}{2}+\frac{b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)}{2}\right)}{2}+\frac{b\,e\,x}{2\,c}\right)+c\,\ln\left(c\,x+1\right)\,\left(\frac{x^2\,\left(2\,a\,e+b\,d-b\,e-b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{4\,c}+\frac{b\,e\,x}{2\,c^2}\right)-\frac{a\,x^2\,\left(e-d+e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{2}-\frac{\ln\left(\frac{x\,\left(2\,a\,e+b\,d-3\,b\,e-b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{2}-\frac{3\,b\,e-b\,d+b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)}{2\,c}-a\,e\,x\right)\,\left(2\,a\,e+b\,d-3\,b\,e-b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{4\,c^2}-\frac{\ln\left(\frac{x\,\left(2\,a\,e-b\,d+3\,b\,e+b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{2}-\frac{3\,b\,e-b\,d+b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)}{2\,c}-a\,e\,x\right)\,\left(2\,a\,e-b\,d+3\,b\,e+b\,e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{4\,c^2}-\frac{b\,x\,\left(3\,e-d+e\,\left(\ln\left(c\,x+1\right)+\ln\left(1-c\,x\right)-\ln\left(1-c^2\,x^2\right)\right)\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - c*x)^2*((b*e)/(4*c^2) - (b*e*x^2)/4) - log(c*x + 1)^2*((b*e)/(4*c^2
) - (b*e*x^2)/4) + log(1 - c*x)*((x^2*(a*e - (b*d)/2 + (b*e)/2 + (b*e*(log(
c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/2))/2 + (b*e*x)/(2*c)) + c*log
(c*x + 1)*((x^2*(2*a*e + b*d - b*e - b*e*(log(c*x + 1) + log(1 - c*x) - log
(1 - c^2*x^2))))/(4*c) + (b*e*x)/(2*c^2)) - (a*x^2*(e - d + e*(log(c*x + 1)
 + log(1 - c*x) - log(1 - c^2*x^2))))/2 - (log((x*(2*a*e + b*d - 3*b*e - b*
e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/2 - (3*b*e - b*d + b*e
*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/(2*c) - a*e*x)*(2*a*e +
b*d - 3*b*e - b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(4*c^2
) - (log((x*(2*a*e - b*d + 3*b*e + b*e*(log(c*x + 1) + log(1 - c*x) - log(1
 - c^2*x^2))))/2 - (3*b*e - b*d + b*e*(log(c*x + 1) + log(1 - c*x) - log(1
- c^2*x^2)))/(2*c) - a*e*x)*(2*a*e - b*d + 3*b*e + b*e*(log(c*x + 1) + log(
1 - c*x) - log(1 - c^2*x^2))))/(4*c^2) - (b*x*(3*e - d + e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2))))/(2*c)
```

3.526 $\int \left(a+b \tanh^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right) \, dx$

Optimal. Leaf size=104

$$-2aex-2bex\tanh^{-1}(cx)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c}+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)$$

```
[Out] -2*a*e*x-2*b*e*x*arctanh(c*x)+e*(a+b*arctanh(c*x))^2/b/c-b*e*ln(-c^2*x^2+1)
/c+x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/4*b*(d+e*ln(-c^2*x^2+1))^2/c
/e
```

Rubi [A]
time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6220, 2525, 2437, 2338, 6127, 6021, 266, 6095}

$$x\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}-2aex+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be\log\left(1-c^2x^2\right)}{c}-2bex\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcTanh[c*x] + (e*(a + b*ArcTanh[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6220

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[
2*e*g, Int[x^2*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tanh^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= x(a+b\tanh^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)-(bc)\int \frac{x(d\\
&= x(a+b\tanh^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)-\frac{1}{2}(bc)\text{Subst}\\
&= -2aex+\frac{e(a+b\tanh^{-1}(cx))^2}{bc}+x(a+b\tanh^{-1}(cx))\,(d\\
&= -2aex-2bex\tanh^{-1}(cx)+\frac{e(a+b\tanh^{-1}(cx))^2}{bc}+x(a\\
&= -2aex-2bex\tanh^{-1}(cx)+\frac{e(a+b\tanh^{-1}(cx))^2}{bc}-\frac{be\,l}{}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 144, normalized size = 1.38

$$adx-2aex+\frac{2ae\tanh^{-1}(cx)}{c}+bdx\tanh^{-1}(cx)-2bex\tanh^{-1}(cx)+\frac{be\tanh^{-1}(cx)^2}{c}+\frac{bd\log\left(1-c^2x^2\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c}+aex\log\left(1-c^2x^2\right)+bex\tanh^{-1}(cx)\log\left(1-c^2x^2\right)+\frac{be\log^2\left(1-c^2x^2\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*ArcTanh[c*x])/c + b*d*x*ArcTanh[c*x] - 2*b*e*x*Arc
Tanh[c*x] + (b*e*ArcTanh[c*x]^2)/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*Lo
g[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcTanh[c*x]*Log[1 - c^2
*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.36, size = 2529, normalized size = 24.32

method	result	size
default	Expression too large to display	2529
risch	Expression too large to display	3552

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*e*x+a*d*x-2*b*e*x*arctanh(c*x)+a*e*x*ln(-c^2*x^2+1)-a*e/c*ln(c*x-1)+a*
e/c*ln(c*x+1)+1/2*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x*e+1
/2*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+
1))^2)^3*x*e+1/2*I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3
*x*e+1/2*I*b/c*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/2*I*b/c*
```

arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^2
)^3+1/2*I*b/c*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-1/2*
I*b/c*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/2
*I*b/c*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c
*x+1)^2/(-c^2*x^2+1))^2)^3-1/2*I*b/c*Pi*ln(1+(c*x+1)^2/(-c^2*x^2+1))*e*csgn
(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)^3-2*b/c*ln(2)*ln(1+(c*x+1)^2/(-c^2*x^2+1))
*e+2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))*(arctanh(c*x)*x*c+arctanh(c*x)-ln(1+(c*
x+1)^2/(-c^2*x^2+1)))*b*e/c+2*b*ln(2)*arctanh(c*x)*x*e-2*b*arctanh(c*x)*ln(
1+(c*x+1)^2/(-c^2*x^2+1))*x*e+2*b/c*arctanh(c*x)*ln(2)*e+I*b/c*csgn(I*(1+(c
*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*e*ln(1+(c*x+1
)^2/(-c^2*x^2+1))*Pi+I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x*e-1/2*I*b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1+(c*x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh
(c*x)+1/2*I*b/c*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn
(I*(c*x+1)^2/(c^2*x^2-1))+1/2*I*b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*Pi*e*arctanh(c*x
)-I*b/c*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(1+(c*x
+1)^2/(-c^2*x^2+1))^2)^2+1/2*I*b/c*arctanh(c*x)*Pi*e*csgn(I*(1+(c*x+1)^2/(-
c^2*x^2+1)))^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)-I*b/c*csgn(I*(c*x+1)^2/
(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*e*ln(1+(c*x+1)^2/(-c^2*x^
2+1))*Pi+1/2*I*b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1))^
2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/2*I*
b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*e*ln
(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/2*I*b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1))^2)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*e*ln(1+(c*x+
1)^2/(-c^2*x^2+1))*Pi-1/2*I*b/c*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I
*(1+(c*x+1)^2/(-c^2*x^2+1)))^2*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi-1/2*I*b*Pi
*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
+(c*x+1)^2/(-c^2*x^2+1))^2)^2*x*e+1/2*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/
(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x*e+1/2*I*b*Pi*arctanh(c*
x)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*
x+1)^2/(-c^2*x^2+1))^2)^2*x*e-I*b*Pi*arctanh(c*x)*csgn(I*(1+(c*x+1)^2/(-c^2
*x^2+1))^2)^2*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))*x*e+1/2*I*b*Pi*arctanh(c*x
)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(1+(c*x+1)^2/(-c^2*x^2+1)))^2
*x*e+I*b/c*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x
+1)^2/(c^2*x^2-1))^2-1/2*I*b*Pi*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1
)^2/(-c^2*x^2+1))^2)*x*e-1/2*I*b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^
2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^2/(c
^2*x^2-1))*Pi*e*arctanh(c*x)+1/2*I*b/c*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1))^2)*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))^2)*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))*Pi+b*arctanh(c*x)*x*d+b/c*e*l
n(1+(c*x+1)^2/(-c^2*x^2+1))^2+b/c*arctanh(c*x)*d-2*b/c*arctanh(c*x)*e-b/c*l
n(1+(c*x+1)^2/(-c^2*x^2+1))*d+2*b/c*e*ln(1+(c*x+1)^2/(-c^2*x^2+1))

Maxima [C] Result contains complex when optimal does not.
time = 0.27, size = 181, normalized size = 1.74

$$-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)b\operatorname{artanh}(cx)e+adx-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)ae+\frac{(2cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))bd}{2c}+\frac{((i\pi+2\log(cx-1)-2)\log(cx+1)+(i\pi-2)\log(cx-1))be}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1)
)*b*arctanh(c*x)*e + a*d*x - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1
)/c^3) - x*log(-c^2*x^2 + 1))*a*e + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2)*lo
g(c*x - 1))*b*e/c
```

Fricas [A]
time = 0.44, size = 189, normalized size = 1.82

$$\frac{4acdx-8acx\cosh(1)-8acx\sinh(1)+(b\cosh(1)+b\sinh(1))\log\left(-c^2x^2+1\right)^2+(b\cosh(1)+b\sinh(1))\log\left(-\frac{cx+1}{cx-1}\right)^2+2\left(bd+2(acx-b)\cosh(1)+2(acx-b)\sinh(1)\right)\log\left(-c^2x^2+1\right)+2\left(bcdx-2(bcx-a)\cosh(1)+(bcx\cosh(1)+bcx\sinh(1))\log\left(-c^2x^2+1\right)-2(bcx-a)\sinh(1)\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*c*d*x - 8*a*c*x*cosh(1) - 8*a*c*x*sinh(1) + (b*cosh(1) + b*sinh(1)
)*log(-c^2*x^2 + 1)^2 + (b*cosh(1) + b*sinh(1))*log(-(c*x + 1)/(c*x - 1))^2
 + 2*(b*d + 2*(a*c*x - b)*cosh(1) + 2*(a*c*x - b)*sinh(1))*log(-c^2*x^2 + 1
) + 2*(b*c*d*x - 2*(b*c*x - a)*cosh(1) + (b*c*x*cosh(1) + b*c*x*sinh(1))*lo
g(-c^2*x^2 + 1) - 2*(b*c*x - a)*sinh(1))*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [A]
time = 0.67, size = 148, normalized size = 1.42

$$\begin{cases} adx+aex\log\left(-c^2x^2+1\right)-2aex+\frac{2ae\operatorname{atanh}(cx)}{c}+bdx\operatorname{atanh}(cx)+bex\log\left(-c^2x^2+1\right)\operatorname{atanh}(cx)-2bex\operatorname{atanh}(cx)+\frac{bd\log(-c^2x^2+1)}{2c}+\frac{be\log(-c^2x^2+1)^2}{4c}-\frac{be\log(-c^2x^2+1)}{c}+\frac{be\operatorname{atanh}^2(cx)}{c} & \text{for } c\neq 0 \\ adx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atanh(c*x)/c
 + b*d*x*atanh(c*x) + b*e*x*log(-c**2*x**2 + 1)*atanh(c*x) - 2*b*e*x*atanh(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*atanh(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

Giac [A]
time = 0.46, size = 165, normalized size = 1.59

$$-\frac{1}{2}bex\log\left(-cx+1\right)^2+\frac{1}{2}(bd+2ae-2be)x\log(cx+1)+\frac{1}{2}\left(bex+\frac{be}{c}\right)\log(cx+1)^2-\frac{be\log(cx-1)^2}{2c}+(ad-2ae)x-\frac{1}{2}\left((bd-2ae-2be)x-\frac{2be\log(cx-1)}{c}\right)\log(-cx+1)+\frac{(bd+2ae-2be)\log(cx+1)}{2c}+\frac{(bd-2ae-2be)\log(cx-1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/2*b*e*x*log(-c*x + 1)^2 + 1/2*(b*d + 2*a*e - 2*b*e)*x*log(c*x + 1) + 1/2
*(b*e*x + b*e/c)*log(c*x + 1)^2 - 1/2*b*e*log(c*x - 1)^2/c + (a*d - 2*a*e)*
x - 1/2*((b*d - 2*a*e - 2*b*e)*x - 2*b*e*log(c*x - 1)/c)*log(-c*x + 1) + 1/
2*(b*d + 2*a*e - 2*b*e)*log(c*x + 1)/c + 1/2*(b*d - 2*a*e - 2*b*e)*log(c*x
- 1)/c
```

Mupad [B]
time = 1.79, size = 385, normalized size = 3.70

$$a\,d\,x-2\,a\,e\,x+\frac{b\,e\,\ln\left(c\,x+1\right)^2}{2\,c}+\frac{b\,e\,\ln\left(1-c\,x\right)^2}{2\,c}+a\,e\,x\,\ln\left(1-c^2\,x^2\right)+\frac{b\,d\,x\,\ln\left(c\,x+1\right)}{2}-\frac{b\,d\,x\,\ln\left(1-c\,x\right)}{2}-b\,e\,x\,\ln\left(c\,x+1\right)+b\,e\,x\,\ln\left(1-c\,x\right)-\frac{a\,e\,\ln\left(c\,x-1\right)}{c}+\frac{a\,e\,\ln\left(c\,x+1\right)}{c}+\frac{b\,d\,\ln\left(c\,x-1\right)}{2\,c}+\frac{b\,d\,\ln\left(c\,x+1\right)}{2\,c}-\frac{b\,e\,\ln\left(c\,x-1\right)}{c}-\frac{b\,e\,\ln\left(c\,x+1\right)}{c}+\frac{b\,e\,x\,\ln\left(c\,x+1\right)\,\ln\left(1-c^2\,x^2\right)}{2}-\frac{b\,e\,x\,\ln\left(1-c\,x\right)\,\ln\left(1-c^2\,x^2\right)}{2}+\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\ln\left(-2\,a\,e-2\,a\,c\,e\,x\right)}{2\,c}+\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\ln\left(2\,a\,c\,e\,x-2\,a\,e\right)}{2\,c}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-2\,a\,e-2\,a\,c\,e\,x\right)}{2\,c}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(2\,a\,c\,e\,x-2\,a\,e\right)}{2\,c}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-2\,a\,e-2\,a\,c\,e\,x\right)}{2\,c}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(2\,a\,c\,e\,x-2\,a\,e\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] a*d*x - 2*a*e*x + (b*e*log(c*x + 1)^2)/(2*c) + (b*e*log(1 - c*x)^2)/(2*c) +
 a*e*x*log(1 - c^2*x^2) + (b*d*x*log(c*x + 1))/2 - (b*d*x*log(1 - c*x))/2 -
 b*e*x*log(c*x + 1) + b*e*x*log(1 - c*x) - (a*e*log(c*x - 1))/c + (a*e*log(
c*x + 1))/c + (b*d*log(c*x - 1))/(2*c) + (b*d*log(c*x + 1))/(2*c) - (b*e*lo
g(c*x - 1))/c - (b*e*log(c*x + 1))/c + (b*e*x*log(c*x + 1)*log(1 - c^2*x^2)
)/2 - (b*e*x*log(1 - c*x)*log(1 - c^2*x^2))/2 + (b*e*log(1 - c^2*x^2)*log(-
 2*a*e - 2*a*c*e*x))/(2*c) + (b*e*log(1 - c^2*x^2)*log(2*a*c*e*x - 2*a*e))/
(2*c) - (b*e*log(c*x + 1)*log(- 2*a*e - 2*a*c*e*x))/(2*c) - (b*e*log(c*x +
1)*log(2*a*c*e*x - 2*a*e))/(2*c) - (b*e*log(1 - c*x)*log(- 2*a*e - 2*a*c*e*
x))/(2*c) - (b*e*log(1 - c*x)*log(2*a*c*e*x - 2*a*e))/(2*c)
```

3.527 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

Optimal. Leaf size=216

$$ad\log(x)-\frac{1}{2}be\log(cx)\log^2(1-cx)+\frac{1}{2}be\log(-cx)\log^2(1+cx)-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}be\big(\log(1-cx)+\log$$

```
[Out] a*d*ln(x)-1/2*b*e*ln(c*x)*ln(-c*x+1)^2+1/2*b*e*ln(-c*x)*ln(c*x+1)^2-1/2*b*d
*polylog(2,-c*x)+1/2*b*e*(ln(-c*x+1)+ln(c*x+1)-ln(-c^2*x^2+1))*polylog(2,-c
*x)+1/2*b*d*polylog(2,c*x)-1/2*b*e*(ln(-c*x+1)+ln(c*x+1)-ln(-c^2*x^2+1))*po
lylog(2,c*x)-1/2*a*e*polylog(2,c^2*x^2)-b*e*ln(-c*x+1)*polylog(2,-c*x+1)+b*
e*ln(c*x+1)*polylog(2,c*x+1)+b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1)
```

Rubi [A]
time = 0.19, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6226, 6031, 6224, 2438, 6222, 2443, 2481, 2421, 6724}

$$-\frac{1}{2}ae\text{Li}_2(c^2x^2)+ad\log(x)+\frac{1}{2}be\text{Li}_2(-cx)\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)-\frac{1}{2}be\text{Li}_2(cx)\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+be\text{Li}_3(1-cx)-be\text{Li}_3(cx+1)-be\text{Li}_2(1-cx)\log(1-cx)+be\text{Li}_2(cx+1)\log(cx+1)-\frac{1}{2}be\log(cx)\log^2(1-cx)+\frac{1}{2}be\log(-cx)\log^2(cx+1)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] a*d*Log[x] - (b*e*Log[c*x]*Log[1 - c*x]^2)/2 + (b*e*Log[-(c*x)]*Log[1 + c*x
]^2)/2 - (b*d*PolyLog[2, -(c*x)])/2 + (b*e*(Log[1 - c*x] + Log[1 + c*x] - L
og[1 - c^2*x^2])*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2 - (b*e*(Lo
g[1 - c*x] + Log[1 + c*x] - Log[1 - c^2*x^2])*PolyLog[2, c*x])/2 - (a*e*Pol
yLog[2, c^2*x^2])/2 - b*e*Log[1 - c*x]*PolyLog[2, 1 - c*x] + b*e*Log[1 + c*
x]*PolyLog[2, 1 + c*x] + b*e*PolyLog[3, 1 - c*x] - b*e*PolyLog[3, 1 + c*x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6222

```
Int[(ArcTanh[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist
[Log[f + g*x^2] - Log[1 - c*x] - Log[1 + c*x], Int[ArcTanh[c*x]/x, x], x] +
 (-Dist[1/2, Int[Log[1 - c*x]^2/x, x], x] + Dist[1/2, Int[Log[1 + c*x]^2/x,
 x], x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

Rule 6224

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTanh[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[Log[f + g*x^2
]*(ArcTanh[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 6226

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcTanh[c*x])/x, x], x] + Dis
t[e, Int[Log[f + g*x^2]*((a + b*ArcTanh[c*x])/x), x], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(1\right.}{x}$$

$$= ad\log(x) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) + (ae)\int \frac{\log\left(1-\right.}{x}$$

$$= ad\log(x) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) - \frac{1}{2}ae\text{Li}_2\left(c^2x^2\right) -$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2($$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2($$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2($$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2($$

Mathematica [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 40.17, size = 1638, normalized size = 7.58

method	result	size
risch	Expression too large to display	1638

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

```
[Out] b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1)+1/2*b*e*ln(-c*x)*ln(c*x+1)^2+b*e
*ln(c*x+1)*polylog(2,c*x+1)-1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^2*csgn(I*(
c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^4*cs
gn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^
3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/2*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-
```

```
1))^3+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1)*(c*x+1))^3-1/2*I*Pi*ln(c*x)*ln(c*
x-1)*b*e*csgn(I*(c*x-1))^3+1/2*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1))^2
+1/2*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1)*(c*x+1))^2+1/4*Pi^2*ln(c*x)*
b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*Pi^2*ln
(c*x)*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/2*I*Pi*
ln(c*x)*a*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*dilog(c*x)*b
*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*dilog(c*x)*b*e*csgn(I
*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1))*cs
gn(I*(c*x-1)*(c*x+1))^2-I*Pi*ln(c*x)*ln(c*x-1)*b*e-1/4*I*Pi*dilog(c*x)*b*e*
csgn(I*(c*x-1)*(c*x+1))^3+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^3-(-
1/2*I*Pi*b*e*csgn(I*(c*x-1))^3+1/2*I*Pi*b*e*csgn(I*(c*x-1))^2+a*e-1/4*I*Pi*
b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/4*I*Pi*b*e*cs
gn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x+1))*csgn(I
*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^3-1/2*I*Pi*b*e*csg
n(I*(c*x-1)*(c*x+1))^2+1/2*d*b)*dilog(c*x+1)-1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c
*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b
*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e
*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*dilog(c*x)*b*e*csgn(I*(
c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/2*Pi^2*ln(c*x)*b*e*csgn(I
*(c*x-1))^2-1/2*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*ln(c*x)
*ln(c*x-1)*b*e*csgn(I*(c*x-1)*(c*x+1))^3+ln(c*x)*a*d-1/2*dilog(c*x)*b*d+dil
og(c*x)*a*e-3/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x-1)*(c*x+1))^
2-1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(c*x+1))^3+1/2*Pi^2
*ln(c*x)*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*ln(c*x)*l
n(c*x-1)*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/2*I*
Pi*dilog(c*x)*b*e*csgn(I*(c*x-1)*(c*x+1))^2+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*
x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x+1))*csgn(I
*(c*x-1)*(c*x+1))^2-1/2*I*Pi*ln(c*x)*b*d*csgn(I*(c*x-1))^3-1/2*I*Pi*dilog(c
*x)*b*e*csgn(I*(c*x-1))^3-I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*
Pi*ln(c*x)*b*d*csgn(I*(c*x-1))^2+1/2*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))^2+
1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^4*csgn(I*(c*x-1)*(c*x+1))^2+1/4*Pi^2*l
n(c*x)*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x-1)*(c*x+1))^3-1/2*I*Pi*ln(c*x)*b*d
-I*Pi*dilog(c*x)*b*e+1/2*Pi^2*ln(c*x)*b*e+ln(c*x)*ln(c*x-1)*a*e-1/2*ln(c*x)
*ln(c*x-1)*b*d-1/2*ln(c*x)*ln(c*x-1)^2*b*e-ln(c*x-1)*polylog(2,-c*x+1)*b*e+
I*Pi*ln(c*x)*a*e
```

Maxima [A]
time = 0.36, size = 156, normalized size = 0.72

$$-\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_{2}(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_{3}(-cx+1)\right)be+\frac{1}{2}\left(\log\left(cx+1\right)^{2}\log\left(-cx\right)+2\,\mathrm{Li}_{2}(cx+1)\log\left(cx+1\right)-2\,\mathrm{Li}_{3}(cx+1)\right)be+ad\log\left(x\right)-\frac{1}{2}\left(bd-2\,ae\right)\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_{2}(-cx+1)\right)+\frac{1}{2}\left(bd+2\,ae\right)\left(\log\left(cx+1\right)\log\left(-cx\right)+\mathrm{Li}_{2}(cx+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylo
g(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*dilog(c*x + 1)*log(
```

```
c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*(b*d - 2*a*e)*(log
(c*x)*log(-c*x + 1) + dilog(-c*x + 1)) + 1/2*(b*d + 2*a*e)*(log(c*x + 1)*lo
g(-c*x) + dilog(c*x + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas")
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx))\,(d + e\log(-c^2x^2 + 1))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{atanh}(c\,x))\,\left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x, x)
```

3.528 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{ce(a+b\tanh^{-1}(cx))^2}{b}-\frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x}+\frac{1}{2}bc(d+e\log(1-c^2x^2))\log\left(1-\frac{1}{1-\right.$$

```
[Out] -c*e*(a+b*arctanh(c*x))^2/b-(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x+1/2*b
*c*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(-c^2*x^
2+1))
```

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6228, 2525, 2458, 2379, 2438, 6095}

$$-\frac{(a+b\tanh^{-1}(cx))(e\log(1-c^2x^2)+d)}{x}-\frac{ce(a+b\tanh^{-1}(cx))^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{1-c^2x^2}\right)(e\log(1-c^2x^2)+d)-\frac{1}{2}bce\text{Li}_2\left(\frac{1}{1-c^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -((c*e*(a + b*ArcTanh[c*x])^2)/b) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^
2*x^2]))/x + (b*c*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/2 -
 (b*c*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/2
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6228

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{(a+b\tanh^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x} + (bc)\int \frac{d+\cdots}{x\cdots}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b} - \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(\cdots}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b} - \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(\cdots}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b} - \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(\cdots}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b} + bcd\log(x) - \frac{(a+b\tanh^{-1}(cx\cdots}{\cdots}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b} + bcd\log(x) - \frac{(a+b\tanh^{-1}(cx\cdots}{\cdots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(105) = 210.

time = 0.13, size = 332, normalized size = 3.16

$$-\frac{4ad+4bd\tanh^{-1}(cx)+8acex\tanh^{-1}(cx)+4bcex\tanh^{-1}(cx)^2-4bcdx\log(x)-bcex\log^2\left(-\frac{1}{c}+x\right)-bcex\log^2\left(\frac{1}{c}+x\right)-2bcex\log\left(\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1-cx)\right)+4bcex\log(x)\log(1-cx)-2bcex\log\left(-\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1+cx)\right)+4bcex\log(x)\log(1+cx)+4ae\log\left(1-c^2x^2\right)+2bcdx\log\left(1-c^2x^2\right)+4be\tanh^{-1}(cx)\log\left(1-c^2x^2\right)-4bcex\log(x)\log\left(1-c^2x^2\right)+2bcex\log\left(-\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)+2bcex\log\left(\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)+4bcex\text{PolyLog}(2,-cx)+4bcex\text{PolyLog}(2,cx)-2bcex\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)-2bcex\text{PolyLog}\left(2,\frac{1}{2}(1+cx)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -1/4*(4*a*d + 4*b*d*ArcTanh[c*x] + 8*a*c*e*x*ArcTanh[c*x] + 4*b*c*e*x*ArcTa
nh[c*x]^2 - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c^(
-1) + x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[x]*
Log[1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*Log[
x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2] + 4*b
*e*ArcTanh[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2] + 2*b*
c*e*x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*Log[1 -
 c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x] - 2*b*
c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2])/x
```

Maple [F]
time = 6.58, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln\left(-c^2x^2+1\right))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - (c^2*(log(c
*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e + 1/2*b*(log(-c*x +
1)^2/x - integrate(-((c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x^3
 - x^2), x))*e - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))\,(d + e \log(-c^2x^2 + 1))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))\,\left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)
```

3.529 $\int \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx$

Optimal. Leaf size=157

$$-ac^2e\log(x)+\frac{1}{2}(a+b)c^2e\log(1-cx)+\frac{1}{2}(a-b)c^2e\log(1+cx)-\frac{bc(d+e\log(1-c^2x^2))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\,(d+e$$

```
[Out] -a*c^2*e*ln(x)+1/2*(a+b)*c^2*e*ln(-c*x+1)+1/2*(a-b)*c^2*e*ln(c*x+1)-1/2*b*c
*(d+e*ln(-c^2*x^2+1))/x+1/2*b*c^2*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/2*(a+
b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2+1/2*b*c^2*e*polylog(2,-c*x)-1/2*b*
c^2*e*polylog(2,c*x)
```

Rubi [A]
time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6037, 331, 212, 6232, 815, 6031}

$$-\frac{(a+b\tanh^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{2x^2}+\frac{1}{2}c^2e(a+b)\log(1-cx)+\frac{1}{2}c^2e(a-b)\log(cx+1)-ac^2e\log(x)-\frac{bc(e\log(1-c^2x^2)+d)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\,(e\log(1-c^2x^2)+d)+\frac{1}{2}bc^2e\mathrm{Li}_2(-cx)-\frac{1}{2}bc^2e\mathrm{Li}_2(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] -(a*c^2*e*Log[x]) + ((a + b)*c^2*e*Log[1 - c*x])/2 + ((a - b)*c^2*e*Log[1 +
 c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d +
e*Log[1 - c^2*x^2]))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(2
*x^2) + (b*c^2*e*PolyLog[2, -(c*x)])/2 - (b*c^2*e*PolyLog[2, c*x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6232

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^3}\,dx = -\frac{bc(d+e\log(1-c^2x^2))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log(1\right.$$

$$= -\frac{bc(d+e\log(1-c^2x^2))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log(1\right.$$

$$= -\frac{bc(d+e\log(1-c^2x^2))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log(1\right.$$

$$= -ac^2e\log(x)+\frac{1}{2}(a+b)c^2e\log(1-cx)+\frac{1}{2}(a-b)c^2e\log$$

Mathematica [A]
time = 0.19, size = 152, normalized size = 0.97

$$\frac{1}{2}\left(-\frac{ad}{x^2}-2ac^2e\log(x)+(a+b)c^2e\log(1-cx)+(a-b)c^2e\log(1+cx)-\frac{bd(2\tanh^{-1}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))}{2x^2}-\frac{e(a+bcx+(b-bc^2x^2)\tanh^{-1}(cx))\log(1-c^2x^2)}{x^2}+bc^2e(\text{PolyLog}(2,-cx)-\text{PolyLog}(2,cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] (-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^2
*e*Log[1 + c*x] - (b*d*(2*ArcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Lo
```

```
g[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcTanh[c*x])*Log[1
 - c^2*x^2])/x^2 + b*c^2*e*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]))/2
```

Maple [F]
time = 17.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln\,(-c^2x^2+1))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d +
1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e + 1/4*b
*(log(-c*x + 1)^2/x^2 - 2*integrate(-((c*x - 1)*log(c*x + 1)^2 - c*x*log(-c
*x + 1))/(c*x^4 - x^3), x))*e - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(cx))\,(d+e\log\,(-c^2x^2+1))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^3,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)
```

3.530 $\int \frac{(a+b\tanh^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx$

Optimal. Leaf size=197

$$\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-bc^3e\log(x)+\frac{1}{3}bc^3e\log\left(1-c^2x^2\right)-\frac{bc(1-c^2x^2)\left(d+e\log\right.}{6x^2}$$

```
[Out] 2/3*c^2*e*(a+b*arctanh(c*x))/x-1/3*c^3*e*(a+b*arctanh(c*x))^2/b-b*c^3*e*ln(
x)+1/3*b*c^3*e*ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/x^2
-1/3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*ln(-c^2*x^2
+1))*ln(1-1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))
```

Rubi [A]
time = 0.29, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {6228, 2525, 2458, 2389, 2379, 2438, 2351, 31, 6129, 6037, 272, 36, 29, 6095}

$$-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-bc^3e\log(x)-\frac{bc(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{6x^2}+\frac{1}{6}bc^3\log\left(1-\frac{1}{1-c^2x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{1}{6}bc^3e\text{Li}_2\left(\frac{1}{1-c^2x^2}\right)+\frac{1}{3}bc^3e\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] (2*c^2*e*(a + b*ArcTanh[c*x]))/(3*x) - (c^3*e*(a + b*ArcTanh[c*x])^2)/(3*b)
 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*c*(1 - c^2*x^2)*(d +
e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/(3*x^3) + (b*c^3*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/
6 - (b*c^3*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6228

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{3}(bc)\int \frac{d+}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{6}(bc)\mathrm{Subst}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{(a+}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{(a+}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{bc(1}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 460 vs. 2(197) = 394.
time = 0.34, size = 460, normalized size = 2.34

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] ((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - 4*a*c^3*e*ArcTanh[c*x] - (2*b
*d*ArcTanh[c*x])/x^3 + (4*b*c^2*e*ArcTanh[c*x])/x - 2*b*c^3*e*ArcTanh[c*x]^
2 + 2*b*c^3*d*Log[x] - 2*b*c^3*e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 +
(b*c^3*e*Log[c^(-1) + x]^2)/2 + b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] -
2*b*c^3*e*Log[x]*Log[1 - c*x] + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] -
 2*b*c^3*e*Log[x]*Log[1 + c*x] - 4*b*c^3*e*Log[(c*x)/Sqrt[1 - c^2*x^2]] - b
*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2
])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcTanh[c*x]*Log[1 - c^2*x^2
])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1
 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[
2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2]
+ b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

Maple [F]
time = 16.76, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arctanh}(cx)) (d + e \ln(-c^2x^2 + 1))}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 + 1)
/x^3)*a*e + 1/6*b*(log(-c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x - 1)*log(
c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x^5 - x^4), x))*e - 1/3*a*d/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx)) (d + e \log(-c^2x^2 + 1))}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}(c\,x)\right)\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^4,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^4, x)
```

3.531 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

Optimal. Leaf size=244

$$\frac{ac^2e}{4x^2}+\frac{5bc^3e}{12x}-\frac{1}{4}bc^4e\tanh^{-1}(cx)+\frac{bc^2e\tanh^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}(3a+4b)c^4e\log(1-cx)+\frac{1}{12}(3a-4b)c^4e\log$$

```
[Out] 1/4*a*c^2*e/x^2+5/12*b*c^3*e/x-1/4*b*c^4*e*arctanh(c*x)+1/4*b*c^2*e*arctanh
(c*x)/x^2-1/2*a*c^4*e*ln(x)+1/12*(3*a+4*b)*c^4*e*ln(-c*x+1)+1/12*(3*a-4*b)*
c^4*e*ln(c*x+1)-1/12*b*c*(d+e*ln(-c^2*x^2+1))/x^3-1/4*b*c^3*(d+e*ln(-c^2*x^
2+1))/x+1/4*b*c^4*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/4*(a+b*arctanh(c*x))*
(d+e*ln(-c^2*x^2+1))/x^4+1/4*b*c^4*e*polylog(2,-c*x)-1/4*b*c^4*e*polylog(2,
c*x)
```

Rubi [A]
time = 0.18, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6037, 331, 212, 6232, 1816, 6191, 6031}

$$\frac{1}{12}c^4e(3a+4b)\log(1-cx)+\frac{1}{12}c^4e(3a-4b)\log(cx+1)-\frac{(a+b\tanh^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}-\frac{1}{2}ac^4e\log(x)+\frac{ac^2e}{4x^2}+\frac{1}{4}bc^4e\text{Li}_2(-cx)-\frac{1}{4}bc^4e\text{Li}_2(cx)-\frac{1}{4}bc^4e\tanh^{-1}(cx)+\frac{5bc^3e}{12x}-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{12x^3}+\frac{bc^2e\tanh^{-1}(cx)}{4x^2}+\frac{1}{4}bc^4\tanh^{-1}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc^3\left(e\log\left(1-c^2x^2\right)+d\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) - (b*c^4*e*ArcTanh[c*x])/4 + (b*c^2*
e*ArcTanh[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + ((3*a + 4*b)*c^4*e*Log[1 - c
*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 - (b*c*(d + e*Log[1 - c^2*x^2
]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/(4*x) + (b*c^4*ArcTanh[c*x]
*(d + e*Log[1 - c^2*x^2]))/4 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/(4*x^4) + (b*c^4*e*PolyLog[2, -(c*x)])/4 - (b*c^4*e*PolyLog[2, c*x])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6191

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rule 6232

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = -\frac{bc(d+e\log\left(1-c^2x^2\right))}{12x^3} - \frac{bc^3(d+e\log\left(1-c^2x^2\right))}{4x} + \frac{1}{4}b$$

$$= -\frac{bc(d+e\log\left(1-c^2x^2\right))}{12x^3} - \frac{bc^3(d+e\log\left(1-c^2x^2\right))}{4x} + \frac{1}{4}b$$

$$= -\frac{bc(d+e\log\left(1-c^2x^2\right))}{12x^3} - \frac{bc^3(d+e\log\left(1-c^2x^2\right))}{4x} + \frac{1}{4}b$$

$$= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3a+4b)c^4e\log(1-cx) -$$

$$= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} + \frac{bc^2e\tanh^{-1}(cx)}{4x^2} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3a+$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e\tanh^{-1}(cx)}{4x^2} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3a+$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} - \frac{1}{4}bc^4e\tanh^{-1}(cx) + \frac{bc^2e\tanh^{-1}(cx)}{4x^2} - \frac{1}{2}a$$

Mathematica [A]
time = 0.09, size = 299, normalized size = 1.23

$$-\frac{ad}{4x^4} + \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3ac^4e+4bc^4e)\log(1-cx) - \frac{1}{2}bc^4e\left(-\frac{\tanh^{-1}(cx)}{2c^2x^2} + \frac{1}{2}\left(-\frac{1}{cx} - \frac{1}{2}\log(1-cx) + \frac{1}{2}\log(1+cx)\right)\right) + bc^4d\left(-\frac{\tanh^{-1}(cx)}{4c^4x^4} + \frac{1}{4}\left(-\frac{1}{3c^3x^3} - \frac{1}{cx} - \frac{1}{2}\log(1-cx) + \frac{1}{2}\log(1+cx)\right)\right) + \frac{1}{12}(3ac^4e-4bc^4e)\log(1+cx) + \frac{e(-3a-bcx-3bc^3x^3-3b\tanh^{-1}(cx)+3bc^4x^4\tanh^{-1}(cx))\log\left(1-c^2x^2\right)}{12x^4} - \frac{1}{4}bc^4e(-\mathrm{PolyLog}(2,-cx)+\mathrm{PolyLog}(2,cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] -1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2 +
 ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcTanh[c*x]/(c
^2*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d*(-
1/4*ArcTanh[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]/2 +
 Log[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(-3*a
- b*c*x - 3*b*c^3*x^3 - 3*b*ArcTanh[c*x] + 3*b*c^4*x^4*ArcTanh[c*x])*Log[1
- c^2*x^2])/(12*x^4) - (b*c^4*e*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/4
```

Maple [F]
time = 66.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(cx\right)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxima
")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^
2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e + 1/8*b*(log(-c*x + 1)^2/x^4 - 4*integr
ate(-1/2*(2*(c*x - 1)*log(c*x + 1)^2 - c*x*log(-c*x + 1))/(c*x^6 - x^5), x)
)*e - 1/4*a*d/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx)) \left(d + e \log\left(-c^2x^2 + 1\right)\right)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))\,\left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^5,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^5, x)
```

3.532 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

Optimal. Leaf size=256

$$\frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+b\tanh^{-1}(cx)\right)^2}{5b}-\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log$$

```
[Out] 7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*arctanh(c*x))/x^3+2/5*c^4*e*(a+b*arctanh(c
*x))/x-1/5*c^5*e*(a+b*arctanh(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*ln(
-c^2*x^2+1)-1/20*b*c*(d+e*ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d+e*
ln(-c^2*x^2+1))/x^2-1/5*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5+1/10*b*
c^5*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1/(-c^
2*x^2+1))
```

Rubi [A]
time = 0.45, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6228, 2525, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 6129, 6037, 272, 36, 29, 6095}

$$-\frac{c^5e\left(a+b\tanh^{-1}(cx)\right)^2}{5b}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}-\frac{5}{6}bc^5e\log(x)+\frac{7bc^3e}{60x^2}-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{20x^4}+\frac{1}{10}bc^5\log\left(1-\frac{1}{1-c^2x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{1}{10}bc^5e\text{Li}_2\left(\frac{1}{1-c^2x^2}\right)+\frac{19}{60}bc^5e\log\left(1-c^2x^2\right)-\frac{bc^3\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcTanh[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTanh[c*x]))/(5*x) - (c^5*e*(a + b*ArcTanh[c*x])^2)/(5*b) - (5*b*c^
5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*c*(d + e*Log[1 - c^2*
x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(10*x^2) -
 ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^5) + (b*c^5*(d + e*Lo
g[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/10 - (b*c^5*e*PolyLog[2, (1 -
c^2*x^2)^(-1)])/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6228

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
```

```
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{5}(bc)\int \frac{d+}{x}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{10}(bc)\text{Subst}($$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log(1\right.}{5x^5}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{c^5e(a}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}-\frac{c^5e(a}{}$$

$$= \frac{bc^3e}{15x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x}$$

Mathematica [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

Maple [F]
time = 24.64, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(cx\right)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxima
")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arctanh(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) -
2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e + 1/10*b*(log(-c*
x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-c*
x + 1))/(c*x^7 - x^6), x))*e - 1/5*a*d/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))\,(d + e \log(-c^2x^2 + 1))}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^6,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^6, x)
```

3.533 $\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=512

$$\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh\right.$$

```
[Out] 1/2*b*(d-e)*x/c-b*e*x/c-1/2*b*(d-e)*arctanh(c*x)/c^2+1/2*d*x^2*(a+b*arctanh
(c*x))-1/2*e*x^2*(a+b*arctanh(c*x))-b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1)
)/c^2/g+1/2*b*e*x*ln(g*x^2+f)/c-1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(g*x^2+f)/
c^2/g+1/2*e*(g*x^2+f)*(a+b*arctanh(c*x))*ln(g*x^2+f)/g+1/2*b*e*(c^2*f+g)*ar
ctanh(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/c^
2/g+1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c
*(-f)^(1/2)+g^(1/2)))/c^2/g+1/2*b*e*(c^2*f+g)*polylog(2,1-2/(c*x+1))/c^2/g-
1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1
/2)-g^(1/2)))/c^2/g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+x*g^(1/2)
)/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/c^2/g+b*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/
2)/c/g^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {2504, 2436, 2332, 6230, 327, 213, 531, 2608, 2498, 211, 2520, 12, 6139, 6057, 2449, 2352, 2497}

$$\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e(f+gx^2)\log(f+gx^2)\left(a+b\tanh^{-1}(cx)\right)}{2g}-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be(c^2f+g)\,\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be(c^2f+g)\,\text{Li}_2\left(1-\frac{2c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\left(c\sqrt{-f}-\sqrt{g}\right)(cx+1)}\right)}{4c^2g}-\frac{be(c^2f+g)\,\text{Li}_2\left(1-\frac{2c\left(\sqrt{g}\,x+\sqrt{-f}\right)}{\left(\sqrt{-f}\,c+\sqrt{g}\right)(cx+1)}\right)}{4c^2g}-\frac{be(c^2f+g)\tanh^{-1}(cx)\log(f+gx^2)}{2c^2g}-\frac{be(c^2f+g)\log\left(\frac{2}{cx+1}\right)\tanh^{-1}(cx)}{c^2g}+\frac{be(c^2f+g)\tanh^{-1}(cx)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{2c^2g}+\frac{be(c^2f+g)\tanh^{-1}(cx)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{(cx+1)\left(c\sqrt{-f}+\sqrt{g}\right)}\right)}{2c^2g}+\frac{bx(d-e)}{2c}+\frac{bex\log(f+gx^2)}{2c}-\frac{bex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])
/(c*Sqrt[g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (d*x^2*(a + b*ArcTanh[c*x
]))/2 - (e*x^2*(a + b*ArcTanh[c*x]))/2 - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))]])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))]])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f + g)*ArcT
anh[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTanh[c*x])*Lo
g[f + g*x^2])/(2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)]])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))]])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))]])/(4*c^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
```

```
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6230

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e(\cdots}{\cdots}$$

$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e(\cdots}{\cdots}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\,\mathrm{ta}\cdots}{2c\cdots}$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\,\mathrm{ta}\cdots}{2c\cdots}$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\,\mathrm{ta}\cdots}{2c\cdots}$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\,\mathrm{ta}\cdots}{2c\cdots}$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\,\mathrm{ta}\cdots}{2c\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 4.46, size = 1376, normalized size = 2.69

Warning: Unable to verify antiderivative.

[In] Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]

[Out] (-4*b*c*e*g*x + 2*a*c^2*(d - e)*g*x^2 + 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + 2*a*c^2*e*f*Log[f + g*x^2] + 2*e*g*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcTanh[c*x])*Log[f + g*x^2] + b*d*(2*(c*g*x + c^2*f*ArcTanh[c*x]^2 + ArcTanh[c*x]*(-g + c^2*g*x^2 + 2*c^2*f*Log[1 + E^(-2*ArcTanh[c*x])]) - c^2*f*PolyLog[2, -E^(-2*ArcTanh[c*x])]) - c^2*f*(2*ArcTanh[c*x]*(-ArcTanh[c*x] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)]) + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g))] + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))])) + b*e*(-2*(c*g*x + c^2*f*ArcTanh[c*x]^2 + ArcTanh[c*x]*(-g + c^2*g*x^2 + 2*c^2*f*Log[1 + E^(-2*ArcTanh[c*x])]) - c^2*f*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + c^2*f*(2*ArcTanh[c*x]*(-ArcTanh[c*x] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] + Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)]) + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g))] + PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))])) + b*c^2*d*f*(2*ArcTanh[c*x]^2 - (4*I)*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 2*ArcTanh[c*x]*(ArcTanh[c*x] + 2*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*((-I)*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*(I*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*PolyLog[2, -E^(-2*ArcTanh[c*x])] - PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))]) + b*e*g*(2*ArcTanh[c*x]^2 - (4*I)*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 2*ArcTanh[c*x]*(ArcTanh[c*x] + 2*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*((-I)*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*(I*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]] + ArcTanh[c*x])*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*PolyLog[2, -E^(-2*ArcTanh[c*x])] - PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))]))/(4*c^2*g)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.21, size = 10161, normalized size = 19.85

method	result
risch	$-\frac{3bex}{2c}+\frac{bdx}{2c}-\frac{ae\,x^{2}}{2}+\frac{bd\ln(-cx+1)}{4c^{2}}-\frac{bd\ln(cx+1)}{4c^{2}}+\frac{ad\,x^{2}}{2}+\frac{be\ln(cx+1)}{4c^{2}}-\frac{eb\,\mathrm{dilog}\left(\frac{c\sqrt{-gf}-(-cx+1)g+g}{c\sqrt{-gf}+g}\right)f}{4g}+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/4*(2*c^2*g*integrate(x^3*log(c*x + 1)/(c^2*g*x^2 +
c^2*f), x) - 2*c^2*g*integrate(x^3*log(-c*x + 1)/(c^2*g*x^2 + c^2*f), x) -
2*c*g*(-I*f*(log(I*g*x/sqrt(f*g) + 1) - log(-I*g*x/sqrt(f*g) + 1))/(sqrt(f*
g)*c^2*g) - 2*x/(c^2*g)) - 2*g*integrate(x*log(c*x + 1)/(c^2*g*x^2 + c^2*f)
, x) + 2*g*integrate(x*log(-c*x + 1)/(c^2*g*x^2 + c^2*f), x) - (2*c*x + (c^
2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(-c*x + 1))*log(g*x^2 + f)/c^2)*
b*e - 1/2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arctanh(c*x) + a*d*x + (b*x*arctanh(c*x)*e + a*x*e)*log(g*x^
2 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{atanh}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int(x*(a + b*atanh(c*x))*(d + e*log(f + g*x^2)), x)
```

3.534 $\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=599

$$-2aex+\frac{2ae\sqrt{f}\,\mathrm{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-}\cdots}{\cdots}$$

[Out] -2*a*e*x-2*b*e*x*arctanh(c*x)-b*e*ln(-c^2*x^2+1)/c+x*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))+1/2*b*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))/c+1/2*b*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))/c+1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*ln(c*x+1)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*b*e*ln(c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*b*e*polylog(2,-(-c*x+1)*g^(1/2)/(c*(-f)^(1/2)-g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*b*e*polylog(2,-(c*x+1)*g^(1/2)/(c*(-f)^(1/2)-g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*polylog(2,(-c*x+1)*g^(1/2)/(c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*b*e*polylog(2,(c*x+1)*g^(1/2)/(c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/2)/g^(1/2)

Rubi [A]
time = 0.56, antiderivative size = 599, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6220, 2525, 2441, 2440, 2438, 6127, 6021, 266, 6121, 211, 6119, 2456}

$$\frac{2ae\sqrt{f}\,\mathrm{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-2aex+\frac{b\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}+\frac{be\,\mathrm{Li}_2\left(\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c}+\frac{be\sqrt{-f}\,\mathrm{Li}_2\left(-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\,\mathrm{Li}_2\left(\frac{\sqrt{g}(1-cx)}{\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\,\mathrm{Li}_2\left(-\frac{\sqrt{g}(cx+1)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\,\mathrm{Li}_2\left(\frac{\sqrt{g}(cx+1)}{\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\,\log(cx+1)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\,\log(cx+1)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}-2bex\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]

[Out] -2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - 2*b*e*x*ArcTanh[c*x] + (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*Log[1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) - (b*e*Log[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/(2*c) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog

```
[2, (Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*PolyLog
[2, (c^2*(f + g*x^2))/(c^2*f + g)])/(2*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6119

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6121

```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTanh[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6220

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[
2*e*g, Int[x^2*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int \frac{x(d+\cdots}{\cdots}$$

$$= x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\text{Subst}\Big(\cdots$$

$$= -2aex+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{b\log\cdots}{\cdots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+x(\cdots$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)-\frac{be\cdots}{\cdots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)-\frac{be\cdots}{\cdots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\cdots}{\cdots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\cdots}{\cdots}$$

$$= -2aex+\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.67, size = 1251, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + b*d
*x*ArcTanh[c*x] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e
*(x*ArcTanh[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] - (b*e*g*(((-Log[
-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2*x^2])*Log[f + g*x^2])/(2*g) +
(Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c
)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)])/(2*g)
+ (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)
] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (L
og[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)] +
 PolyLog[2, (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)])/(2*g) + (Lo
g[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)] + Pol
yLog[2, (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)])/(2*g)))/c - (b*e*(
4*c*x*ArcTanh[c*x] - 4*Log[1/Sqrt[1 - c^2*x^2]] + (Sqrt[c^2*f*g]*((-2*I)*Ar
cCos[(-(c^2*f) + g)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] + 4*ArcTan[S
qrt[c^2*f*g]/(c*g*x)]*ArcTanh[c*x] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] -
2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(g + I*Sqrt[c^2*f*g])*(1 + c*
x))/((c^2*f + g)*(c^2*f + I*c*Sqrt[c^2*f*g]*x))] - (ArcCos[(-(c^2*f) + g)/(
c^2*f + g)] + 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(I*g + Sqrt[c^2
*f*g])*(-1 + c*x))/((c^2*f + g)*((-I)*c^2*f + c*Sqrt[c^2*f*g]*x))] + (ArcCo
s[(-(c^2*f) + g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(
c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcTanh[c*x]*Sqrt[c^
2*f + g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + (ArcCos[(-(
c^2*f) + g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c*g*x
)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcTanh[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f +
 g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + I*(-PolyLog[2, (
(-(c^2*f) + g - (2*I)*Sqrt[c^2*f*g])*(I*c^2*f + c*Sqrt[c^2*f*g]*x))/((c^2*f
 + g)*((-I)*c^2*f + c*Sqrt[c^2*f*g]*x))] + PolyLog[2, ((-(c^2*f) + g + (2*I
)*Sqrt[c^2*f*g])*(I*c^2*f + c*Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c^2*f +
c*Sqrt[c^2*f*g]*x))])))/g))/(2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 23.53, size = 3574, normalized size = 5.97

method	result	size
risch	Expression too large to display	3574

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*e*x+a*d*x+1/2*b*d*ln(-c*x+1)/c+4*b*e/c-b*e/c*ln(c*x+1)+a*e*x*ln(g*x^2+
f)-d*b/c-1/4*I*e*b/c*Pi*ln(c*x+1)*csgn(I/c^2)*csgn(I*(c^2*f+((c*x+1)^2-2*c*
x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))-1/4*I*e*b*Pi*ln(c*x+1)*c
sgn(I/c^2)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)
^2-2*c*x-1)*g))*x+1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I/c^2)*csgn(I*(c^2*f+((-c*x+
```

1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))*x-1/4*I*e*b/c*
Pi*ln(-c*x+1)*csgn(I/c^2)*csgn(I*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))*csgn(I/c^2
*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))+1/2*d*x*b*ln(c*x+1)+1/2*d*b/c*ln(c*x+1)-1/
2*d*x*b*ln(-c*x+1)+1/2*e*b/c*ln(c^2*f-2*(c*x+1)*g+g*(c*x+1)^2+g)-1/2*e*b/c*
dilog((c*(-g*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))-1/2*e*b/c*dilog((c*(
-g*f)^(1/2)+(c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))-1/2*e*b/c*ln(c^2*f+((c*x+1)^2-
2*c*x-1)*g)-1/2*e*b*ln(c^2*f+((c*x+1)^2-2*c*x-1)*g)*x+1/2*e*b/c*ln(c^2*f-2*
(-c*x+1)*g+g*(-c*x+1)^2+g)-1/2*e*b/c*dilog((c*(-g*f)^(1/2)-(-c*x+1)*g+g)/(c
*(-g*f)^(1/2)+g))-1/2*e*b/c*dilog((c*(-g*f)^(1/2)+(-c*x+1)*g-g)/(c*(-g*f)^(
1/2)-g))-1/2*e*b/c*ln(c^2*f+((-c*x+1)^2+2*c*x-1)*g)+1/2*e*b*ln(c^2*f+((-c*x
+1)^2+2*c*x-1)*g)*x-e*b/c*ln(-c*x+1)-1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*
(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*
c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*ln(c*
x+1)*csgn(I*c^2)^3-1/4*I*e*b/c*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*
c*x-1)*g))^3-1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)
*g))^2*x+1/4*I*e*b*Pi*ln(c*x+1)*csgn(I*c^2)^3*x-1/4*I*e*b*Pi*csgn(I*(c^2*f+
((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*x-1/4*I
*e*b*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^3*x+1/4*I*e*b/c
*Pi*ln(-c*x+1)*csgn(I*c^2)^3-1/4*I*e*b/c*Pi*ln(-c*x+1)*csgn(I/c^2*(c^2*f+((
-c*x+1)^2+2*c*x-1)*g))^3-1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((-c*
x+1)^2+2*c*x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))
csgn(I/c^2(c^2*f+((-c*x+1)^2+2*c*x-1)*g))^2+1/4*I*e*b*Pi*ln(-c*x+1)*csgn(
I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))^3*x+1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I/c
^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))^2*x-1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I*c^2)^
3*x+1/4*I*e*b*Pi*csgn(I*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+(
(-c*x+1)^2+2*c*x-1)*g))^2*x-1/2*I*e*b/c*Pi*csgn(I*c)^2*csgn(I*c^2)+I*e*b/c*
Pi*csgn(I*c)*csgn(I*c^2)^2-1/2*e*b/(-g*f)^(1/2)*dilog((c*(-g*f)^(1/2)+(-c*x
+1)*g-g)/(c*(-g*f)^(1/2)-g))*f-e*b/(g*f)^(1/2)*arctan(1/2*(2*(-c*x+1)*g-2*g
)/c/(g*f)^(1/2))*f-1/2*e*b/c*ln(-c*x+1)*ln((c*(-g*f)^(1/2)-(-c*x+1)*g+g)/(c
*(-g*f)^(1/2)+g))-1/2*e*b/c*ln(-c*x+1)*ln((c*(-g*f)^(1/2)+(-c*x+1)*g-g)/(c*
(-g*f)^(1/2)-g))+1/2*e*b/c*ln(c^2*f+((-c*x+1)^2+2*c*x-1)*g)*ln(-c*x+1)-e*b/
c*ln(-c*x+1)*ln(c)+e*b*ln(-c*x+1)*ln(c)*x-1/2*e*b*ln(c^2*f+((-c*x+1)^2+2*c*
x-1)*g)*ln(-c*x+1)*x+2*e*a*f/(g*f)^(1/2)*arctan(x*g/(g*f)^(1/2))-e*b/(g*f)^
(1/2)*arctan(1/2*(2*(c*x+1)*g-2*g)/c/(g*f)^(1/2))*f-1/2*e*b/c*ln(c*x+1)*ln(
(c*(-g*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))-1/2*e*b/c*ln(c*x+1)*ln((c*
(-g*f)^(1/2)+(c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))+1/2*e*b/c*ln(c^2*f+((c*x+1)^2
-2*c*x-1)*g)*ln(c*x+1)-e*b/c*ln(c*x+1)*ln(c)+1/2*e*b/(-g*f)^(1/2)*dilog((c*
(-g*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))*f-1/2*e*b/(-g*f)^(1/2)*dilog(
(c*(-g*f)^(1/2)+(c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))*f+1/2*e*b*ln(c^2*f+((c*x+1
)^2-2*c*x-1)*g)*ln(c*x+1)*x-e*b*ln(c*x+1)*ln(c)*x+1/2*e*b/(-g*f)^(1/2)*dilo
g((c*(-g*f)^(1/2)-(-c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))*f+2*e*b/c*ln(c)-b*ln(c*
x+1)*x*e+b*ln(-c*x+1)*x*e-1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I*(c^2*f+((-c*x+1)^2
+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*l
n(c*x+1)*csgn(I*c)^2*csgn(I*c^2)*x-1/2*I*e*b*Pi*ln(c*x+1)*csgn(I*c)*csgn(I*
c^2)^2*x+1/4*I*e*b*Pi*ln(c*x+1)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(

```
I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I*(c
^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*x+1/
4*I*e*b*Pi*ln(c*x+1)*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^
2*x-1/2*I*e*b/c*Pi*ln(c*x+1)*csgn(I*c)*csgn(I*c^2)^2+1/4*I*e*b/c*Pi*ln(c*x+
1)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x
-1)*g))^2+1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*
csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))+1/4*I*e*b/c*Pi*ln(c*x+1)*csgn(I/c
^2)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*ln(c*x+1)*cs
gn(I*c)^2*csgn(I*c^2)+1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I*(c^2*f+((-c*x+1)^2+
2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))+1/4*I*e*b/c*Pi*ln(-
c*x+1)*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))^2+1/4*I*e*b/c
*Pi*ln(-c*x+1)*csgn(I*c)^2*csgn(I*c^2)-1/2*I*e*b/c*Pi*ln(-c*x+1)*csgn(I*c)*
csgn(I*c^2)^2+1/4*I*e*b/c*Pi*ln(-c*x+1)*csgn(I*(c^2*f+((-c*x+1)^2+2*c*x-1)*
g))*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))^2-1/4*I*e*b*Pi*csgn(I/c^2)*c
sgn(I*(c^2*f+((-c*x+1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2*c*x-1
)*g))*x-1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((-c*x+1)^2+2
*c*x-1)*g))^2*x-1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] a*d*x + (2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 +
f))*a*e + 1/2*b*(((c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*log(g*x
^2 + f)/c + integrate(-2*((c*g*x^2 + g*x)*log(c*x + 1) - (c*g*x^2 - g*x)*lo
g(-c*x + 1))/(c*g*x^2 + c*f), x))*e + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*b*d/c
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(g*x^2 + f),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{atanh}(c\,x))\,\left(d + e\,\ln\left(g\,x^2 + f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int((a + b*atanh(c*x))*(d + e*log(f + g*x^2)), x)
```

3.535 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal. Leaf size=93

$$ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}bd\text{PolyLog}(2,cx)+\frac{1}{2}ae\text{PolyLog}\Big(2,1+$$

[Out] b*e*CannotIntegrate(arctanh(c*x)*ln(g*x^2+f)/x,x)+a*d*ln(x)+1/2*a*e*ln(-g*x^2/f)*ln(g*x^2+f)-1/2*b*d*polylog(2,-c*x)+1/2*b*d*polylog(2,c*x)+1/2*a*e*polylog(2,1+g*x^2/f)

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]

[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 - (b*d*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])/2 + b*e*Defer[Int][(ArcTanh[c*x]*Log[f + g*x^2])/x, x]

Rubi steps

$$\begin{aligned}\int \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx &= d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+e\int \frac{(a+b\tanh^{-1}(cx))\log(f\\ &= ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+(ae)\int \frac{\log(f+}{x}\\ &= ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+\frac{1}{2}(ae)\text{Subst}\Big(\int\\ &= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\text{Li}_2(-cx\\ &= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\text{Li}_2(-cx\end{aligned}$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tanh^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Maple [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln{(g\,x^2+f)})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + integrate(1/2*b*(log(c*x + 1) - log(-c*x + 1))*e*log(g*x^2 + f
)/x + 1/2*b*d*(log(c*x + 1) - log(-c*x + 1))/x + a*e*log(g*x^2 + f)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(g*x^2 + f))
/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atanh}\,(cx))\,(d+e\log{(f+gx^2)})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(f + g*x**2))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))\,\left(d + e\,\ln\left(g\,x^2 + f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x,x)
[Out] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

3.536 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal. Leaf size=613

$$\frac{2ae\sqrt{g}\,\mathrm{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\,\log(1+cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\right.}{c\sqrt{-f}+\sqrt{}}\right.}{2\sqrt{-f}}$$

[Out] -(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+f))-1/2*b*c*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))-1/2*b*c*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))+1/2*b*c*e*polylog(2,1+g*x^2/f)-1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*ln(c*x+1)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)-1/2*b*e*ln(c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*ln(-c*x+1)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)-1/2*b*e*polylog(2,-(-c*x+1)*g^(1/2)/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)-1/2*b*e*polylog(2,-(c*x+1)*g^(1/2)/(c*(-f)^(1/2)-g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*polylog(2,(-c*x+1)*g^(1/2)/(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*b*e*polylog(2,(c*x+1)*g^(1/2)/(c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-f)^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)

Rubi [A]
time = 0.51, antiderivative size = 613, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6228, 2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438, 6121, 211, 6119, 2456}

$$\frac{2ae\sqrt{g}\,\mathrm{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{(a+b\tanh^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}-\frac{1}{2}bc\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bce\mathrm{Li}_2\left(\frac{c^2(gx^2+f)}{fc^2+g}\right)+\frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{1}{2}bce\mathrm{Li}_2\left(\frac{gx^2}{f}+1\right)-\frac{be\sqrt{g}\,\mathrm{Li}_2\left(-\frac{\sqrt{g}\,(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\,\mathrm{Li}_2\left(\frac{\sqrt{g}\,(1-cx)}{\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{be\sqrt{g}\,\mathrm{Li}_2\left(-\frac{\sqrt{g}\,(cx+1)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\,\mathrm{Li}_2\left(\frac{\sqrt{g}\,(cx+1)}{\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\,\log(cx+1)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{be\sqrt{g}\,\log(cx+1)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]

[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*Log[1 - c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*Log[1 - c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - ((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 + c*x

```
))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g
]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*c*e*PolyLog[2, (c^2
*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6119

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6121

```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTanh[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 6228

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + (bc)\int \frac{d+e}{x\,(}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\text{Subst}\Big($$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(\right.\right.}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(\right.\right.}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(\right.\right.}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-}\right.}{c\sqrt{-}}\right.}{2\sqrt{-f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-}\right.}{c\sqrt{-}}\right.}{2\sqrt{-f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\log(1-cx)\log\left(\frac{c\left(\sqrt{-}\right.}{c\sqrt{-}}\right.}{2\sqrt{-f}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.78, size = 1226, normalized size = 2.00

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] -((a*d)/x) - (b*d*ArcTanh[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2])
/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2]/
```

```
x) + (b*e*(-(((2*ArcTanh[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f +
 g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*Sqrt
[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (I*Sqr
t[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g]*x))/(c
*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log[1 - c^2*
x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x))/(I*c*Sqr
t[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt[f] + Sqrt[
g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g])] + PolyLog
[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyLog[2, (I*Sqrt
[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) + (c*g*((2*I)*ArcCos[(-(c^2*f) + g
)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - 4*ArcTan[(c*f)/(Sqrt[c^2*f*g
]*x)]*ArcTanh[c*x] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] + 2*ArcTan[(c*g*x)
/Sqrt[c^2*f*g]])*Log[((2*I)*c*f*(I*g + Sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f +
 g)*(c*f + I*Sqrt[c^2*f*g]*x))] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] - 2*A
rcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c*f*(g + I*Sqrt[c^2*f*g])*(1 + c*x))/(
(c^2*f + g)*(c*f + I*Sqrt[c^2*f*g]*x))] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g
)] + 2*(ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*L
og[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcTanh[c*x]*Sqrt[c^2*f + g]*Sqrt[c^2*f - g +
 (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)]
- 2*(ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[
(Sqrt[2]*E^ArcTanh[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f + g]*Sqrt[c^2*f - g + (c
^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + I*(PolyLog[2, ((-(c^2*f) + g - (2*I)*Sq
rt[c^2*f*g])*(I*c*f + Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c*f + Sqrt[c^2*f
*g]*x))] - PolyLog[2, ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(I*c*f + Sqrt[c
^2*f*g]*x))/((c^2*f + g)*((-I)*c*f + Sqrt[c^2*f*g]*x))])))/Sqrt[c^2*f*g]))/
2
```

Maple [**F**]
time = 1.89, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln{(g\,x^2+f)})}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d + (2*g*arctan
(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + 1/2*b*e*integrate((log(
c*x + 1) - log(-c*x + 1))*log(g*x^2 + f)/x^2, x) - a*d/x
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(g*x^2 + f))
/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x))\,\left(d + e\,\ln\left(g\,x^2 + f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

3.537 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal. Leaf size=470

$$\frac{bce\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be(c^2f+g)\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f}-\frac{be(c^2f+g)\tanh^{-1}(cx)\log\left(\frac{2c(}{(c\sqrt{}}\right.}{2f}$$

```
[Out] a*e*g*ln(x)/f+b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1))/f-1/2*a*e*g*ln(g*x^2
+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x+1/2*b*c^2*arctanh(c*x)*(d+e*ln(g*x^2+f))-
1/2*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2-1/2*b*e*(c^2*f+g)*arctanh(c*x)
*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/f-1/2*b*e*(c
^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^
(1/2)))/f-1/2*b*e*g*polylog(2,-c*x)/f+1/2*b*e*g*polylog(2,c*x)/f-1/2*b*e*(c
^2*f+g)*polylog(2,1-2/(c*x+1))/f+1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1
/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/f+1/4*b*e*(c^2*f+g)*polylog(
2,1-2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/f+b*c*e*arct
an(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {6037, 331, 212, 6232, 815, 649, 211, 266, 457, 78, 6857, 6031, 6139, 6057, 2449, 2352, 2497}

$$-\frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{2x^2}-\frac{aeg\log(f+gx^2)}{2f}+\frac{aeg\log(x)}{f}+\frac{bce\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{1}{2}bc^2\tanh^{-1}(cx)(d+e\log(f+gx^2))-\frac{be(c^2f+g)\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2f}+\frac{be(c^2f+g)\text{Li}_2\left(1-\frac{2c(\sqrt{-f}-\sqrt{g}\,x)}{(c\sqrt{-f}-\sqrt{g})(cx+1)}\right)}{4f}+\frac{be(c^2f+g)\text{Li}_2\left(1-\frac{2c(\sqrt{g}\,x+\sqrt{-f})}{(\sqrt{-f}\,c+\sqrt{g})(cx+1)}\right)}{4f}+\frac{be(c^2f+g)\log\left(\frac{2}{cx+1}\right)\tanh^{-1}(cx)}{f}-\frac{be(c^2f+g)\tanh^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}\,x)}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)}{2f}-\frac{be(c^2f+g)\tanh^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}\,x)}{(cx+1)(c\sqrt{-f}+\sqrt{g})}\right)}{2f}-\frac{bc(d+e\log(f+gx^2))}{2x}-\frac{begLi_2(-cx)}{2f}+\frac{begLi_2(cx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*x)])/f - (b*e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/
(2*f) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*
Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f) - (b*
c*(d + e*Log[f + g*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]
))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - (b*e*g*PolyL
og[2, -(c*x)])/(2*f) + (b*e*g*PolyLog[2, c*x])/(2*f) - (b*e*(c^2*f + g)*Pol
yLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqr
t[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(4*f) + (b*e*(c^2*
f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*
(1 + c*x))])/(4*f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6232

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = -\frac{bc(d+e\log\left(f+gx^2\right))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+\right.\right.$$

$$= -\frac{bc(d+e\log\left(f+gx^2\right))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+\right.\right.$$

$$= -\frac{bc(d+e\log\left(f+gx^2\right))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+\right.\right.$$

$$= \frac{aeg\log(x)}{f}-\frac{bc(d+e\log\left(f+gx^2\right))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d\right.$$

$$= \frac{aeg\log(x)}{f}-\frac{bc(d+e\log\left(f+gx^2\right))}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d\right.$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}-\frac{aeg\log\left(f+gx^2\right)}{2f}-$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be(c^2f+g)\tanh^{-1}(}{f}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be(c^2f+g)\tanh^{-1}(}{f}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be(c^2f+g)\tanh^{-1}(}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.98, size = 982, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -1/4*(2*a*d*f + 2*b*c*d*f*x - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x
)/Sqrt[f]] + 2*b*d*f*ArcTanh[c*x] - 2*b*c^2*d*f*x^2*ArcTanh[c*x] - 4*b*e*g*
```

```
x^2*ArcTanh[c*x]^2 - (4*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*
ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 4*b*e*g*x^2*ArcTanh[c*x]*Log[1 - E^(-2*
ArcTanh[c*x])] - 4*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])]
- (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*
ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*
ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[(c^2*(1 + E^
(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^
(2*ArcTanh[c*x])*(c^2*f + g))] + (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c
^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))
*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*x^
2*ArcTanh[c*x]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x
]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*e*g*x^2
*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f
]*Sqrt[g] - g)] + 2*b*e*g*x^2*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2
*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)] - 4*a*e*g*x^2*Log[x] + 2*a*e*f
*Log[f + g*x^2] + 2*b*c*e*f*x*Log[f + g*x^2] + 2*a*e*g*x^2*Log[f + g*x^2] +
 2*b*e*f*ArcTanh[c*x]*Log[f + g*x^2] - 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[f +
 g*x^2] + 2*b*c^2*e*f*x^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*b*e*g*x^2*Po
lyLog[2, E^(-2*ArcTanh[c*x])] + b*e*g*x^2*PolyLog[2, -((E^(2*ArcTanh[c*x])*
(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g))] + b*e*g*x^2*PolyLog[2, -(
(E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))] - b*c
^2*e*f*x^2*PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x
])*(c^2*f + g))] - b*c^2*e*f*x^2*PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*
g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))])/(f*x^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(408) = 816$.
time = 8.97, size = 961, normalized size = 2.04

method	result
risch	$\frac{aeg\ln(x)}{f} - \frac{aeg\ln(g\,x^2+f)}{2f} - \frac{bcd}{2x} - \frac{b\,c^2 d\ln(-cx+1)}{4} - \frac{da}{2x^2} - \frac{db\ln(cx+1)}{4x^2} + \frac{c^2\ln(cx+1)bd}{4} - \frac{c^2\ln(cx)db}{4} + \frac{\ln(-cx+1}{4x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*e*g*ln(x)/f-1/2*a*e*g*ln(g*x^2+f)/f-1/2*b*c*d/x-1/4*b*c^2*d*ln(-c*x+1)-1/
2*d*a/x^2-1/4/x^2*d*b*ln(c*x+1)+1/4*c^2*ln(c*x+1)*b*d-1/4*c^2*ln(c*x)*d*b+1
/4/x^2*ln(-c*x+1)*b*d-1/4*b*e*c^2*ln(c*x+1)*ln((c*(-g*f)^(1/2)+(c*x+1)*g-g)
/(c*(-g*f)^(1/2)-g))-1/4*g*b*e/f*dilog((c*(-g*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*
f)^(1/2)+g))-1/4*g*b*e/f*dilog((c*(-g*f)^(1/2)+(c*x+1)*g-g)/(c*(-g*f)^(1/2)
-g))-1/2*g*b*e*dilog(c*x+1)/f+1/4*b*e*c^2*ln(-c*x+1)*ln((c*(-g*f)^(1/2)-(-c
*x+1)*g+g)/(c*(-g*f)^(1/2)+g))+1/4*b*e*c^2*ln(-c*x+1)*ln((c*(-g*f)^(1/2)+(-
c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))+1/4*g*b*e/f*dilog((c*(-g*f)^(1/2)-(-c*x+1)*
```

```
g+g)/(c*(-g*f)^(1/2)+g))+1/4*g*b*e/f*dilog((c*(-g*f)^(1/2)+(-c*x+1)*g-g)/(c
*(-g*f)^(1/2)-g))+1/2*g*b*e*dilog(-c*x+1)/f-1/4*b*e*c^2*ln(c*x+1)*ln((c*(-g
*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))+1/4*d*b*c^2*ln(-c*x)-1/4*g*b*e/f
*ln(c*x+1)*ln((c*(-g*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))-1/4*g*b*e/f*
ln(c*x+1)*ln((c*(-g*f)^(1/2)+(c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))+g*e*b*c/(g*f)
^(1/2)*arctan(x*g/(g*f)^(1/2))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-g*f)^(1/2)-(-
c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-g*f)^(1/2)+(
-c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))+(-1/4*b*e/x^2*ln(c*x+1)-1/4*e*(b*x^2*ln(-c
*x+1)*c^2-b*c^2*ln(c*x+1)*x^2+2*b*c*x-b*ln(-c*x+1)+2*a)/x^2)*ln(g*x^2+f)+1/
4*b*e*c^2*dilog((c*(-g*f)^(1/2)-(-c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))+1/4*b*e*c
^2*dilog((c*(-g*f)^(1/2)+(-c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))-1/4*b*e*c^2*dilo
g((c*(-g*f)^(1/2)-(c*x+1)*g+g)/(c*(-g*f)^(1/2)+g))-1/4*b*e*c^2*dilog((c*(-g
*f)^(1/2)+(c*x+1)*g-g)/(c*(-g*f)^(1/2)-g))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d -
1/2*(g*(log(g*x^2 + f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e - 1/4*(2*c
^2*g*integrate(x^2*log(c*x + 1)/(g*x^3 + f*x), x) - 2*c^2*g*integrate(x^2*l
og(-c*x + 1)/(g*x^3 + f*x), x) + 2*I*c*g*(log(I*g*x/sqrt(f*g) + 1) - log(-I
*g*x/sqrt(f*g) + 1))/sqrt(f*g) - 2*g*integrate(log(c*x + 1)/(g*x^3 + f*x),
x) + 2*g*integrate(log(-c*x + 1)/(g*x^3 + f*x), x) + (2*c*x - (c^2*x^2 - 1)
*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x + 1))*log(g*x^2 + f)/x^2)*b*e - 1/2*
a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*arctanh(c*x)*e + a*e)*log(g*x^2 + f))
/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

```
[Out] int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

3.538 $\int \frac{\tanh^{-1}(cx)\left(a+b\tanh^{-1}(cx)\right)}{(1+cx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{a+b}{2c(1+cx)}+\frac{(a+b)\tanh^{-1}(cx)}{2c}-\frac{(a+b)\tanh^{-1}(cx)}{c(1+cx)}-\frac{b(1-cx)\tanh^{-1}(cx)^2}{2c(1+cx)}$$

```
[Out] 1/2*(-a-b)/c/(c*x+1)+1/2*(a+b)*arctanh(c*x)/c-(a+b)*arctanh(c*x)/c/(c*x+1)-
1/2*b*(-c*x+1)*arctanh(c*x)^2/c/(c*x+1)
```

Rubi [A]
time = 0.22, antiderivative size = 122, normalized size of antiderivative = 1.56, number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6063, 641, 46, 213, 6874, 6065, 6095}

$$-\frac{a}{2c(cx+1)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(cx+1)}-\frac{b}{2c(cx+1)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh^{-1}(cx)^2}{c(cx+1)}+\frac{b\tanh^{-1}(cx)}{2c}-\frac{b\tanh^{-1}(cx)}{c(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

```
[Out] -1/2*a/(c*(1 + c*x)) - b/(2*c*(1 + c*x)) + (a*ArcTanh[c*x])/(2*c) + (b*ArcT
anh[c*x])/(2*c) - (a*ArcTanh[c*x])/(c*(1 + c*x)) - (b*ArcTanh[c*x])/(c*(1 +
 c*x)) + (b*ArcTanh[c*x]^2)/(2*c) - (b*ArcTanh[c*x]^2)/(c*(1 + c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\tanh^{-1}(cx)\left(a+b\tanh^{-1}(cx)\right)}{(1+cx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)(a+b\tanh^{-1}(x))}{(1+x)^2}\,dx,x,cx\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a\tanh^{-1}(x)}{(1+x)^2}+\frac{b\tanh^{-1}(x)^2}{(1+x)^2}\right)\,dx,x,cx\right)}{c}$$

$$= \frac{a\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{(1+x)^2}\,dx,x,cx\right)}{c}+\frac{b\text{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{(1+x)^2}\,dx,x,cx\right)}{c}$$

$$= -\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}+\frac{a\text{Subst}\left(\int \frac{1}{(1+x)(1-x^2)}\,dx,x,cx\right)}{c}+$$

$$= -\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}+\frac{a\text{Subst}\left(\int \frac{1}{(1-x)(1+x)^2}\,dx,x,cx\right)}{c}+$$

$$= -\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}+$$

$$= -\frac{a}{2c(1+cx)}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh}{c(}$$

$$= -\frac{a}{2c(1+cx)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b\tan}{}$$

$$= -\frac{a}{2c(1+cx)}-\frac{b}{2c(1+cx)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh}{c(1+}$$

$$= -\frac{a}{2c(1+cx)}-\frac{b}{2c(1+cx)}+\frac{a\tanh^{-1}(cx)}{2c}+\frac{b\tanh^{-1}(cx)}{2c}-\frac{a\tanh}{c(1+}$$

Mathematica [**A**]
time = 0.04, size = 70, normalized size = 0.90

$$-\frac{4(a+b)\tanh^{-1}(cx)-2b(-1+cx)\tanh^{-1}(cx)^2+(a+b)(2+(1+cx)\log(1-cx)-(1+cx)\log(1+cx))}{4c(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

```
[Out] -1/4*(4*(a + b)*ArcTanh[c*x] - 2*b*(-1 + c*x)*ArcTanh[c*x]^2 + (a + b)*(2 +
 (1 + c*x)*Log[1 - c*x] - (1 + c*x)*Log[1 + c*x]))/(c*(1 + c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(76) = 152.
time = 1.60, size = 203, normalized size = 2.60

method	result
derivativedivides	$-\frac{a\,\mathrm{arctanh}(cx)}{cx+1}-\frac{a\ln(cx-1)}{4}-\frac{a}{2(cx+1)}+\frac{a\ln(cx+1)}{4}-\frac{b\,\mathrm{arctanh}(cx)^2}{cx+1}-\frac{b\,\mathrm{arctanh}(cx)\ln(cx-1)}{2}-\frac{b\,\mathrm{arctanh}(cx)}{cx+1}+\frac{b\,\mathrm{arctanh}(cx)\ln(cx}{2}$
default	$-\frac{a\,\mathrm{arctanh}(cx)}{cx+1}-\frac{a\ln(cx-1)}{4}-\frac{a}{2(cx+1)}+\frac{a\ln(cx+1)}{4}-\frac{b\,\mathrm{arctanh}(cx)^2}{cx+1}-\frac{b\,\mathrm{arctanh}(cx)\ln(cx-1)}{2}-\frac{b\,\mathrm{arctanh}(cx)}{cx+1}+\frac{b\,\mathrm{arctanh}(cx)\ln(cx}{2}$
risch	$\frac{b(cx-1)\ln(cx+1)^2}{8c(cx+1)}-\frac{(bcx\ln(-cx+1)-b\ln(-cx+1)+2a+2b)\ln(cx+1)}{4c(cx+1)}-\frac{-bx\ln(-cx+1)^2c+2\ln(cx-1)acx+2\ln(cx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a/(c*x+1)*arctanh(c*x)-1/4*a*ln(c*x-1)-1/2*a/(c*x+1)+1/4*a*ln(c*x+1)-
b/(c*x+1)*arctanh(c*x)^2-1/2*b*arctanh(c*x)*ln(c*x-1)-b/(c*x+1)*arctanh(c*x
)+1/2*b*arctanh(c*x)*ln(c*x+1)+1/4*b*ln(c*x-1)*ln(1/2*c*x+1/2)-1/8*b*ln(c*x
-1)^2-1/4*b*ln(c*x-1)-1/2*b/(c*x+1)+1/4*b*ln(c*x+1)-1/4*ln(-1/2*c*x+1/2)*ln
(1/2*c*x+1/2)*b+1/4*b*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/8*b*ln(c*x+1)^2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 226, normalized size = 2.90

$$-\frac{1}{8}\left(bc\left(\frac{2}{c^4x+c^3}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)+2a\left(\frac{2}{c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)+\frac{-2i\pi b+(i\pi b+(i\pi bc-bc)x+b)\log(cx+1)+(-i\pi b+(-i\pi bc+bc)x-b)\log(cx-1)+2b}{c^3x+c^2}\right)c-\frac{1}{4}\left(\left(c\left(\frac{2}{c^3x+c^2}-\frac{\log(cx+1)}{c^2}+\frac{\log(cx-1)}{c^2}\right)+\frac{4\,\mathrm{artanh}(cx)}{c^2x+c}\right)b+\frac{4a}{c^2x+c}\right)\mathrm{artanh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*c*(2/(c^4*x + c^3) - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) + 2*a*(2/
(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + (-2*I*pi*b + (I*pi*b
 + (I*pi*b*c - b*c)*x + b)*log(c*x + 1) + (-I*pi*b + (-I*pi*b*c + b*c)*x -
b)*log(c*x - 1) + 2*b)/(c^3*x + c^2))*c - 1/4*((c*(2/(c^3*x + c^2) - log(c*
x + 1)/c^2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*b + 4*a/(c^2*x
 + c))*arctanh(c*x)
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.95

$$\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)^2+2\left((a+b)cx-a-b\right)\log\left(-\frac{cx+1}{cx-1}\right)-4\,a-4\,b}{8\left(c^2x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1))^2 + 2*((a + b)*c*x - a - b)*log(
-(c*x + 1)/(c*x - 1)) - 4*a - 4*b)/(c^2*x + c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))\,\mathrm{atanh}\,(cx)}{(cx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c*x)*(a+b*atanh(c*x))/(c*x+1)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))*atanh(c*x)/(c*x + 1)**2, x)
```

Giac [A]
time = 0.41, size = 93, normalized size = 1.19

$$\frac{1}{8}\,c\left(\frac{(cx-1)b\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2} + \frac{2\,(cx-1)(a+b)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2} + \frac{2\,(cx-1)(a+b)}{(cx+1)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*c*((c*x - 1)*b*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2) + 2*(c*x - 1
)*(a + b)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2) + 2*(c*x - 1)*(a + b)/(
(c*x + 1)*c^2))
```

Mupad [B]
time = 1.15, size = 67, normalized size = 0.86

$$\frac{a\,\mathrm{atanh}(c\,x)+b\,\mathrm{atanh}(c\,x)+b\,{\mathrm{atanh}(c\,x)}^2}{2\,c}-\frac{a+b+2\,a\,\mathrm{atanh}(c\,x)+2\,b\,\mathrm{atanh}(c\,x)+2\,b\,{\mathrm{atanh}(c\,x)}^2}{2\,x\,c^2+2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atanh(c*x)*(a + b*atanh(c*x)))/(c*x + 1)^2,x)
```

```
[Out] (a*atanh(c*x) + b*atanh(c*x) + b*atanh(c*x)^2)/(2*c) - (a + b + 2*a*atanh(c
*x) + 2*b*atanh(c*x) + 2*b*atanh(c*x)^2)/(2*c + 2*c^2*x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```